Marco Lippi: AGGREGATION, FUNDAMENTALNESS AND DYNAMIC FACTOR MODELS.

Plan of the talk.

1. Some issues concerning structural VAR models. More precisely, concerning the interpretation of the structural shocks or of the impulse-response functions.

2. Serious problems arise with aggregation of heterogeneous agents.

3. Serious problems arise with fundamentalness.

4. There is some overlapping between aggregation problems and fundamentalness.

5. Dynamic factor models as a way out of fundamentalness problems.

6. The talk will try to give the technical flavour of the problems.

7. The talk is a promotion of the paper Forni, Giannone, Lippi and Reichlin (2005) Opening the Black Box: Structural Factor Models with large cross-sections, but I will insist on very elementary examples.

Fundamentalness. Example.

(See on fundamantalness

Hansen, L.P and T.J. Sargent (1991) Two problems in interpreting vector autoregressions. In *Rational Expectations Econometrics*, L.P. Hansen and T.J. Sargent, eds. Boulder: Westview, pp.77-119,

Lippi, M. and L. Reichlin (1993) The dynamic effects of aggregate demand and supply disturbances: Comment. *American Economic Review* **83**, pp.644-52,

Lippi, M. and L. Reichlin (1994) VAR analysis, non fundamental representation, Blaschke matrices. *Journal of Econometrics* **63**, pp.307-25.)

Suppose that w_t is the wage rate, the same for all firms and workers. Suppose that at the beginning of each quarter a committee gathers, negotiates and decides the increase u_t. However, there is a permanent agreement to smooth increases, so that the increase u_t will take place in two quarters, $a_0 u_t$ this quarter, $a_1 u_t$ the next quarter, with $a_0 + a_1 = 1$. As a consequence

$$\Delta w_t = a_0 u_t + a_1 u_{t-1}.$$

Moreover, for the sake of simplicity, assume that u_t is a white noise.

Now,

(i) The econometrician does not observe u_t, nor has information about the coefficients a_0 and a_1.

(ii) He/she only observes w_t and therefore Δw_t.

The questions are:

(I) Can we recover u_t ?

(II) Can we recover a_0 and a_1 ?

The answer is negative, even though, in this simple case, we are able to strongly restrict the range of possible solutions.

Solution of the problem:

(1) We can consistently estimate the autocovariance function of Δw_t, and

$$\begin{aligned}\gamma_0 =& a_0^2\sigma_u^2 + a_1^2\sigma_u^2 \\ \gamma_1 =& a_0 a_1 \sigma_u^2\end{aligned}$$

(2) Normalize temporarily by setting $a_0 = 1$ and get

$$\frac{\gamma_0}{\gamma_1} = \frac{1 + a_1^2}{a_1},$$

that is

$$a_1^2 - \Gamma a_1 + 1 = 0,$$

where $\Gamma = \gamma_0/\gamma_1$.

(3) This equation has two solutions, α and $1/\alpha$, with $|\alpha| > 1$.

(4) In conclusion,

$$\Delta w_t = (1 - \alpha L)U_t = (1 - \alpha^{-1}L)U_t^*.$$

Comments:

(1) We normalize by setting $u_t = a^{-1}U_t$, with $a + \alpha a = 1$ (the same for the starred quantities).

(2) We cannot identify which representation (shock) is the structural one, unless more information is available. In our example, the knowledge that, say, $a_0 > a_1$ would be sufficient to choose α instead of α^{-1}. So, typically, agents may know more than the econometrician.

(3) The representation $\Delta w_t = (1 - \alpha L)U_t$ is called fundamental, and so is the shock U_t. The shock U_t^* is not fundamental. Because $1 - \alpha L$ has an inverse in L, that is

$$(1 - \alpha L)^{-1} = 1 + \alpha L + \alpha^2 L^2 + \cdots,$$

then U_t belongs to the space spanned by

$$\Delta w_t, \quad \Delta w_{t-1}, \quad \Delta w_{t-2}, \quad \ldots$$

whereas, denoting by F the forward operator, $F = L^{-1}$,

$$\frac{1}{1 - \alpha^{-1}L} = \frac{\alpha}{\alpha - L} = \frac{-\alpha F}{1 - \alpha F} = -\alpha F - \alpha^2 F^2 - \alpha^3 F^3 + \cdots,$$

so that U_t^* lies in the space spanned by

$$\Delta w_{t+1}, \quad \Delta w_{t+2}, \quad \Delta w_{t+3}, \quad \ldots$$

(4) You may feel that what I am saying contradicts common practice with ARIMA models. But you should remember that ARIMA models have been introduced with the task of forecasting stochastic processes. Now, as long as forecasting is concerned, more precisely, as long as forecasting w_t based on past values of w_t, we **must** choose the fundamental representation. However, my point in this talk and in related work of mine is that

forecasting the process w_t

finding the structural representation for w_t

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are not the same thing, contrary to what usually empirical macroeconomists seem to believe.

What I mean here is that usually the fundamentalness problem is solved by macroeconomists by choosing the fundamental representatation, **as though** their problem was forecasting, whereas they are looking for structural representations and shocks. This is what we do when we estimate and identify structural VAR models.

Firstly we estimate

$$A(L)\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} u_t \\ v_t \end{pmatrix}, \quad A(L) = I - A_1 L - \cdots - A_p L^p,$$

whete A_j is a 2×2 matrix. **Secondly**, we invert

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = B(L)\begin{pmatrix} u_t \\ v_t \end{pmatrix},$$

with $B(L) = A(L)^{-1}$. **Thirdly**, we transform the shocks $(u_t\ v_t)'$ by an invertible matrix C,

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \left[B(L)C^{-1}\right]\left[C\begin{pmatrix} u_t \\ v_t \end{pmatrix}\right] = D(L)\begin{pmatrix} a_t \\ b_t \end{pmatrix},$$

in such a way that, for example,

(A) a_t and b_t are unit variance and orthogonal (this is a very common condition)

(B) $D_{21}(0) = 0$, that is, the shock a_t has no contemporaneous impact on y_t.

It is well known that conditions (A) and (B) are sufficient to identify the matrix D.

However, what we usually do not say is that we start with a very strong assumption, that is

The structural shocks a_t and b_t are fundamental.

For,

$$\begin{aligned}\begin{pmatrix} a_t \\ b_t \end{pmatrix} &= C \begin{pmatrix} u_t \\ v_t \end{pmatrix} \\ &= C \left[\begin{pmatrix} x_t \\ y_t \end{pmatrix} - A_1 \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} - \cdots - A_p \begin{pmatrix} x_{t-p} \\ y_{t-p} \end{pmatrix} \right],\end{aligned}$$

so that a_t and b_t belong to the space spanned by the **past** of the observable variables x_t and y_t. But, as we have seen with our elementary example, fundamentalness is not necessarily a consequence of our knowledge about the variables x_t and y_t. In particular, there may be shocks that agents see and consider in their maximization schemes, but that an econometrician only observing present and past of the variables x_t and y_t cannot recover.

Here insert literature.

Going back to our estimation-identification procedure, the steps should be

(I) Again start with

$$A(L)\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} u_t \\ v_t \end{pmatrix}, \quad A(L) = I - A_1 L - \cdots - A_p L^p$$

and

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = B(L)\begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

(II) Then orthonormalize the residuals in any way:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = B(L)\begin{pmatrix} u_t \\ v_t \end{pmatrix} = \left[B(L)S^{-1}\right]\left[S\begin{pmatrix} u_t \\ v_t \end{pmatrix}\right] = E(L)\begin{pmatrix} c_t \\ d_t \end{pmatrix}.$$

(III) Now consider the matrices in L, $C(L)$, such that

$$C(L)C'(F) = I. \qquad (*)$$

Then insert:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = [B(L)C(L)]\left[C'(F)\begin{pmatrix} u_t \\ v_t \end{pmatrix}\right] = D(L)\begin{pmatrix} a_t \\ b_t \end{pmatrix},$$

and impose that $D_{21}(0) = 0$.

Comments:

(a) Matrices fulfilling $(*)$ generalize orthogonal matrices. They are called Blaschke matrices.

(b) The vector

$$\begin{pmatrix} a_t \\ b_t \end{pmatrix} = C'(F) \begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

is a white noise. For

$$E \begin{pmatrix} a_t & b_t \end{pmatrix} \begin{pmatrix} a_t \\ b_t \end{pmatrix} = E \begin{pmatrix} u_t & v_t \end{pmatrix} C(L)C'(F) \begin{pmatrix} u_t \\ v_t \end{pmatrix} = I$$

(c) The condition that $D_{21}(0) = 0$ is no longer sufficient to identify the structural shocks. For, consider a very simple class of Blaschke matrices

$$C(L) = K \begin{pmatrix} \dfrac{\alpha - L}{1 - \alpha L} & 0 \\ 0 & 1 \end{pmatrix},$$

depending on two parameters, the angle of K and α. Obviously, the condition $D_{21}(0) = 0$ is not sufficient to identify both of them.

Alternatively, observing the vector $(\, x_t \quad y_t \,)'$ is equivalent to observing the matrix autocovariance function

$$\Gamma_k = E\,(\, x_t \quad y_t \,) \begin{pmatrix} x_{t-k} \\ y_{t-k} \end{pmatrix},$$

and therefore the spectral density

$$f(\theta) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \Gamma_k e^{-ik\theta}.$$

It is well known that if

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = A(L) \begin{pmatrix} g_t \\ h_t \end{pmatrix},$$

wehere $\begin{pmatrix} g_t \\ h_t \end{pmatrix}$ is an ortonormal white noise, is a representation of $(\, x_t \quad y_t \,)$, then

$$f(\theta) = \frac{1}{2\pi} A(e^{-i\theta}) A'(e^{i\theta}).$$

Now, if $C(L)$ is such that $C(L)C'(F) = I$, that is $C(e^{-i\theta})C'(e^{i\theta})$ I, then

$$
\begin{aligned}
\begin{pmatrix} x_t \\ y_t \end{pmatrix} &= A(L) \begin{pmatrix} g_t \\ h_t \end{pmatrix} \\
&= [A(L)C(L)] \left[C'(F) \begin{pmatrix} g_t \\ h_t \end{pmatrix} \right] = A^*(L) \begin{pmatrix} g_t^* \\ h_t^* \end{pmatrix}
\end{aligned}
$$

is another representation.

In conclusion, if possible non-fundamentalness of the structural shocks is taken into consideration, the situation is one of dramatic **underidentification.**

A solution to this problem requires that further conditions are introduced, like the shape of the impulse-response functions, positiveness (the sign, in general) of impulse-response at given lags, etc.

But this is not the approach I want to consider here.

The interpretation of VAR models should also take into account serious aggregation problems. See Forni, M. and M. Lippi (1997). *Aggregation and the microeconomic foundations of dynamic macroeconomics.* Oxford: Clarendon press.

Suppose that this is the micromodel:

$$\begin{pmatrix} \Delta y_t^j \\ x_t^j \end{pmatrix} = \begin{pmatrix} b_{11}^j(L) & (1-L)b_{12}^j(L) \\ b_{21}^j(L) & b_{22}^j(L) \end{pmatrix} \begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

dove

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = \begin{pmatrix} u_1 & u_2 & \cdots & u_h & v_1 & v_2 & \cdots & v_k \end{pmatrix}'$$

Now aggregate

$$\begin{pmatrix} \Delta y_t \\ x_t^p \end{pmatrix} = \begin{pmatrix} B_{11}^p(L) & (1-L)B_{12}^p(L) \\ B_{21}^p(L) & B_{22}^p(L) \end{pmatrix} \begin{pmatrix} u_t \\ v_t \end{pmatrix},$$

where the index p means that $a_{hk}^p(L)$ depends on all the individual b_{hk}^j:

$$B_{hk}^p(L) = \sum_j b_{hk}^j(L).$$

Now consider the Wold representation

$$\begin{pmatrix} \Delta y_t^p \\ x_t^p \end{pmatrix} = \begin{pmatrix} C_{11}^p(L) & C_{12}^p(L) \\ C_{21}^p(L) & C_{22}^p(L) \end{pmatrix} \begin{pmatrix} E_t \\ F_t \end{pmatrix},$$

where now the shock vector is two-dimensional, and chose the identification rule

$$\begin{pmatrix} \Delta y_t^p \\ x_t^p \end{pmatrix} = \begin{pmatrix} A_{11}^p(L) & (1-L)A_{12}^p(L) \\ A_{21}^p(L) & A_{22}^p(L) \end{pmatrix} \begin{pmatrix} U_t \\ V_t \end{pmatrix}$$

The issue is under what conditions u_t depends only on the u_{jt} e V_t only on v_{jt}.

The answer is disappointing. In order to have "consistent aggregation", it is necessary and sufficient that there exists a 2×2 matrix $d(L)$, **fundamental**, and such that

$$d(L) \begin{pmatrix} B_{11}^p(L) & B_{12}^p(L) \\ B_{21}^p(L) & B_{22}^p(L) \end{pmatrix} \qquad \textbf{is diagonal}.$$

This implies that

$$\frac{a_{11,1}^p(L)}{a_{21,1}^p(L)} = \frac{a_{11,s}^p(L)}{a_{21,s}^p(L)} \qquad \frac{a_{12,1}^p(L)}{a_{22,1}^p(L)} = \frac{a_{12,s}^p(L)}{a_{22,s}^p(L)}$$

for all s.

Dynamic factor models.

See the references in the paper "Opening Black Box".

We have a set of macroeconomic variables, driven by a small number q of common shocks plus idiosyncratic shocks. Let $q = 2$ for simplicity:

$$x_{it} = \chi_{it} + \xi_{it} = a_{i1}(L)u_{1t} + a_{i2}(L)u_{2t} + \xi_{it},$$

where u_t is an orthonormal white noise, ξ_{it} is orthogonal to u_t and to ξ_{jt} for $j \neq i$ at any lead and lag.

Suppose that the variables of interest are the first two, and that we accept an interpretation of the idiosyncratic variables as measurement errors, so that we are only interested in

$$\chi_{1t} = a_{11}(L)u_{1t} + a_{12}(L)u_{2t}$$

$$\chi_{2t} = a_{11}(L)u_{1t} + a_{12}(L)u_{2t}$$

I claim that if we are able to estimate the spectral density of the vector

$$\chi_t = \begin{pmatrix} \chi_{1t} & \chi_{2t} & \cdots & \chi_{nt} \end{pmatrix}$$

where $n > 2$, then we may solve the fundamentalness problem.

On the other hand, estimating the spectral density of the common-component is precisely what dynamic factor models do. I will only give an idea of the result. Reduce further q to 1, suppose that we know the spectral density of χ_t and that the latter is consistent with

$$x_{it} = a_i(1 - \alpha_i L)u_t.$$

This means that

$$\begin{aligned} f_{jk}(\theta) &= \frac{1}{2\pi}(g_{jk,1}e^{-i\theta} + g_{jk,0} + g_{jk,-1}e^{i\theta}) \\ &= \frac{1}{2\pi}a_j a_k(1 - \alpha_j e^{-i\theta})(1 - \alpha_k e^{i\theta}) \end{aligned}$$

Now, if you look at the first variable, the spectral density $f_{11}(\theta)$ is consistent with α_1 but also with α_1^{-1}. The same for the second. But the cross-spectrum $f_{12}(\theta)$ is decisive: assuming for simplicity that $\alpha_1 \neq \alpha_2$, there is only one choice of the α which is consistent.