Memory and Screening

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Abstract

Past actions are often recorded and stored independently of the will of the actor. Credit registers, law enforcement agencies, Google and Facebook keep substantial amount of information about individuals. Typically, part of the information is erased from the public records with time, that is past behavior becomes forgotten. Shall good behavior be forgotten faster than bad behavior (as it is in many legal systems), or the other way around? We answer this question in a dynamic model with adverse selection. We show that in general it is optimal to keep records of bad behavior for a long time, while good behavior should be forgotten fast.

Introduction

Privacy concerns have been at the center of several parallel debates on electronic markets and the Internet. According to some, a privacy regulation that mandates the removal of data from the public domain is at unjustified and opens the door for "fraud[1]" (see also Posner, 1983, Nock, 1993). And indeed, in markets plagued by severe adverse selection (Akerlof 1970), more information available to participants tends to facilitate screening and increase efficient trade. On the other hand, privacy concerns are increasingly widespread among populations and governments, particularly after the revelations from Snowden's leakage, and are obviously one reason why the Antitrust debate has been so intensively focussing on Google in recent times, particularly in Europe. The spring 2014 ruling of the European Court of Justice supporting the right to "forgetfullness", i.e. the right to

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[1]From Posner's blog, 8th May 2005. http://www.becker-posner-blog.com/2005/05/index.html

obtain ones' records cancelled from the Web, at least under some conditions, hightened the discussion on how much "memory" should be allowed for, both in the private domains of electronic trading companies and in the public one of the Web. Regulation limiting data retention has been already in place for some time in many countries, but it is currently rather generic and definitley not based on efficiency principles rooted in any serious research.[2] It is also hard to apply because it does not explain how the limits to memory should be established for different types of information, commercial, individual, positive, negative, etc.

Our paper builds a dynamic model with adverse selection.to study this kind of questions. We ask what are the effects of forgetting information on past interactions on screening, equilibrium outcomes and welfare, differentiating between positive and negative information, a distinction that turns out to be critical. We show that in general it is optimal to keep records of bad behavior for a long time, while good behavior should be forgotten fast. We also study incentives of information providers and the possibe need for public intervention. The model and its results are general enough to apply to many different markets affected by adverse selection, but for concreteness we frame it as a credit market because in these markets the importance of adverse selection problems and the lack of consistent knowledge-based regulation on data retention are paricularly evident.

In most modern credit markets, credit registers collect and share data on borrowers' past behavior with other market participants, in an effort to reduce the well-known informational asymmetries that characterize lending relationships (Jappelli, Pagano 2002). These asymmetries take often the form of adverse selection. Reliable information on a borrower's past behavior serves as a screening device for lenders. A first dimention along which they vary is which type of information is collected. Some registers record only negative, or "black" information, that is, remarks on borrowers' past deficiencies (from credit remarks to defaults and bankruptcies). Others collect positive, or "white" information as well, which describes the borrower's overall position. Of the countries where a public credit register is operating, fifty-four percent of the registers share positive as well as negative information. As for private credit bureaus, the picture is reversed. About two thirds of these registers only share negative information about the borrowers. The data are typically erased from the records after a certain number of years, in accordance with privacy protection laws designed to enable people to make a fresh start. Indeed, from a legal point of view credit data fall under the umbrella of personal data and are thus

[2]For instance, consider the EU Directive on Data Protection (95/46/EC) which mandates that "[M]ember States shall provide that personal data must be [...] adequate, relevant and not excessive in relation to the purposes for which they are collected and/or further processed".

subject to the privacy protection provisions that regulate the extent to which personal information can be handled, shared and stored. In particular, an important by-product of privacy protection is the principle of data retention, which prescribes that personal data, as collected by any data user, should be retained for a limited period of time only, and by any means for no longer than what is necessary for the intended purpose.[3] This reguation, therefore, directly mandates the removal from the market of data that may play a key role in reducing the impact of information asymmetries in the market. The interesting thing for the purpose of this paper is that the implementation of these regulation combined with other public or private rules determined an incredible heterogeneity in the retention limits, i.e. in the memory of the information sharing systems. While up to 90 percent of the countries where a credit register is operating adopted such a privacy provision (Bottero and Spagnolo 2012), the period after which the informaton is erased ranges from three years (Germany, Italy, Swe- den) up to fourteen and fifteen years (Canada and Greece respectively).

RELATED LITERATURE

Privacy issues have already been the subject of econonomic analysis from several point There are instances in which privacy regulations can alleviate market failures that arise even when assuming rational agents (Acquisti, 2010; Acquisti, Varian, 2005; Calzolari, Pavan, 2006; Hermalin, Katz, 2006; Hirshleifer, 1971; Taylor, 2004a, 2004b). .... The effects of data retention provisions on credit market outcomes were also studied by Elul and Gottardi (2011) and Bottero and Spagnolo (2013), but in very different environments where moral hazard played a major role...

1 Environment

The economy is populated by numerous competitive lenders and a unit mass of borrowers $i \in [0, 1]$. All agents are risk neutral. The time is continuous $t \in [0, \infty)$ and the discounting factor is δ. Lenders have enough resources to finance all potential borrowers at any period t. Each period t a borrower i may face an investment opportunity which requires investment 1 in the beginning of the period and delivers R at the end of the period if the borrower works

[3] Data retention was first mentioned by the Council of Europe in 1973, where it was declared that "[T]hose responsible of these [personal data] files [...] should refrain from storing information which is not necessary for the given purposes". This principle was later on incorporated into the 1995 EU Directive (95/46/EC) on Data Protection, which states that "[M]ember States shall provide that personal data must be [...] adequate, relevant and not excessive in relation to the purposes for which they are collected and/or further processed".

and it delivers 0 if he shirks. The arrival of investment opportunities to each borrower is described by a Poisson process with arrival intensity λ. When a borrower faces an investment opportunity he seeks a loan on the market. Each borrower can either be of good type (G) with probability μ, or of bad type (B) with complementary probability. The good borrower always repays the loan while the bad borrower defaults.[4] Borrowers have no wealth to begin with and have to borrow from numerous competitive lenders. Whenever, borrowers successfully exercise the investment opportunity they consume all the proceeds, so that they start with no resources in the next period.

2 Economy with negative records

We assume that the only source of information about past behavior of borrowers is the credit bureau (credit register). We assume that each time a borrower faces an investment opportunity he is matched with random lender. We consider different policies of the credit bureau. To begin with we assume that the credit bureau only records past defaults by borrowers, we refer to this case as *negative records*. The credit bureau deletes each negative record for a borrower after T periods. At any $t \geq T$ for any borrower $i \in [0,1]$ his credit history h_i^t specifies if he defaulted D at each of the past periods between t and $t-T$ or not N. Essentially h_i^t is a function defined on interval $[t-T, t]$ and taking values D or N.

Lenders observe credit histories h_i^t for all borrowers $i \in [0,1]$ at any t and form beliefs about the borrowers type, which is given by the posterior probability $\mu(h_i^t) \in [0,1]$ that the borrower is of good type. If borrower i faces an investment opportunity at t and asks for a loan the competitive lenders are ready to provide the loan if they expect to recover their investment, that is if $\mu(h_i^t)R \geq 1$. In this case they charge the rate of interest $R_i^t = 1/\mu(h_i^t)$.

2.1 Equilibrium

We look for a stationary equilibrium, where each good borrower always repays a loan and each bad borrower always defaults. The lender use Bayes' rule and credit histories of borrowers to update their beliefs about borrowers' types.

First, note that a good borrower never defaults on a loan, therefore if borrower i has defaulted at least once in the last T periods the lenders believe his is of bad type: $\mu(h_i^t) = 0$.

[4]In an extension we consider a continuum of borrower's types that differ by the size of private benefit they obtain when they default instead of repaying the loan. Naturally the good borrower has low private benefit and bad borrower has high private benefit.

This also implies that before the default record is deleted this borrower will never be able to borrow. This observation considerably simplifies our analysis, because only the borrower who have no default records in last T periods will be able to borrow. For brevity we say that these borrowers have N record, while borrowers who have defaulted $\tau \in [0, T]$ periods ago have $D(\tau)$ record.

Proposition 1. *A stationary equilibrium with lending exists if and only if*

$$R(N) = 1 + \frac{1-\mu}{\mu(1+\lambda T)} \leq R. \tag{1}$$

The equilibrium is unique.

Proof. Suppose a stationary equilibrium exists. Denote by $\mu(N)$ the mass of good borrowers with N record in a stationary equilibrium. Because good borrowers never default all good borrowers have N record and $\mu(N) = \mu$. Denote by $\eta(N)$ the mass of bad borrowers with an N record and by $\eta(D(\tau))$ the mass of bad borrowers with $D(\tau)$ record in a stationary equilibrium. In a stationary equilibrium, each instance mass $\eta(D(T))$ of bad borrowers has the default record deleted from the credit bureau and enters the mass of bad borrowers with N records. At the same time each instance fraction λ of bad borrowers with N record faces an investment opportunity, borrows, defaults and obtains $D(0)$ records. In a stationary equilibrium the mass $\eta(N)$ is constant, that is $\dot{\eta}(N) = \eta(D(T)) - \lambda\eta(N) = 0$. Since all default records are kept for T periods, in a stationary equilibrium $\eta(D(\tau)) = \eta(D(T))$ for any $\tau \in [0, T]$ and we obtain $\int_0^T \eta(D(\tau))d\tau = T\lambda\eta(N)$. Since the total mass of bad borrowers is $1-\mu$ we get $\eta(N)+T\lambda\eta(N) = 1-\mu$ and rearranging we get $\eta(N) = \frac{1-\mu}{1+\lambda T}$. The lenders will only lend a borrower with an N record if the probability that the borrower repays is high enough $\frac{\mu}{\mu+\eta(N)}R \geq 1$, which is equivalent to (1). Thus (1) is necessary and sufficient for an equilibrium with lending to exist. The equilibrium is also unique, because borrowers with $D(\tau)$ records can't borrow and the interest rate for borrowers with N record is uniquely defined $R(N) = 1 + \frac{1-\mu}{\mu(1+\lambda T)}$. QED.

The result is very intuitive, it requires longer memory (higher T) to make sure the adverse selection among the borrowers with N record is not severe enough to brake down the market for loans. The longer the default records are stored, the longer a bad borrower is excluded from the market for loans and the easier it is for good borrowers to borrow. Indeed, the interest rate $R(N)$ decreases with T.

3 Economy with positive records

Now we assume that the credit bureau only records past repayments by borrowers but not defaults, we refer to this case as *positive records.* The credit bureau deletes each positive record for a borrower after T periods. At any $t \geq T$ for any borrower $i \in [0,1]$ his credit history h_i^t specifies if he repaid a loan S at each of the past periods between t and $t-T$ or not N. Essentially h_i^t is a function defined on interval $[t-T, t]$ and taking values S or N.

3.1 Equilibrium

As before we look for a stationary equilibrium where lenders use Bayes' rule and credit histories of borrowers to update their beliefs about borrowers' types.

First, note that a good borrower always repays a loan, therefore if borrower i has repaid at least once in the last T periods the lenders believe his is of good type: $\mu(h_i^t) = 1$. This also implies that before the last successful repayment record is deleted this borrower will be believed to be of good type. This observation simplifies our analysis, because the latest successful repayment record is a sufficient statistic for the lender's beliefs about borrower's type, indeed if the most recent successful repayment record by a borrower is $\tau < T$ periods old it does not matter for lenders if he has other successful repayment records that are between τ and T periods old: in any case he is believed to be of good type. Thus, without loss of generality we only keep track of the most recent successful repayment records: the credit bureau's record of a borrower with the most recent successful repayment $\tau \leq T$ periods is denoted by $S(\tau)$, $\tau \in [0, T]$. A borrower who has no successful repayment record in the last T periods has record N.

Proposition 2. *A stationary equilibrium with lending exists if and only if*

$$R(N) = 1 + \frac{1-\mu}{\mu} e^{\lambda T} \leq R. \tag{2}$$

The equilibrium is unique.

Proof. Denote by $\mu(N)$ and $\eta(N)$ the masses of good and bad borrowers with N record correspondingly in a stationary equilibrium. Because bad borrowers never succeed they never get a positive record and all have N record, so that $\eta(N) = 1-\mu$. Denote by $\mu(S(\tau))$ the mass of good borrowers with $S(\tau)$ record in a stationary equilibrium. In a stationary equilibrium, at each instance mass $\mu(s(T))$ of good borrowers has their records deleted from the credit bureau and enters the mass of good borrowers with N records. At the same time at each instance fraction λ of good borrowers with N record faces an investment opportunity, borrows, repays and obtains $S(0)$ records. In a stationary equilibrium the mass $\mu(N)$ is constant, that is $\dot{\mu}(N) = \mu(S(T)) - \lambda\mu(N) = 0$. Consider good borrowers with $S(0)$ record, each instance mass $\lambda\mu$ of good borrowers gets loans, repays them and gets $S(0)$ record. Hence, in a stationary equilibrium we have a condition $\dot{S}(0) = \lambda\mu - S(0)$. Consider good borrowers with $S(\tau)$ record $\tau \in (0, T)$ each instance Δt fraction $\lambda\Delta t$ of them gets $S(0)$ record, while the rest $\mu(S(\tau)) - \lambda\Delta t\mu(S(\tau))$ gets $S(\tau+\Delta t)$ record, that is $\mu(S(\tau+\Delta t)) = \mu(S(\tau)) - \lambda\Delta t\mu(S(\tau))$. Hence we get $\dot{\mu}(S(\tau)) = -\lambda\mu(S(\tau))$ and $\mu(S(\tau)) = \mu(S(0))e^{-\lambda\tau}$, $\tau \in [0, T]$. This allows to express $\mu(N) = \mu e^{-\lambda T}$. A stationary equilibrium is unique, it exists if and only if the interest rate for borrowers with N record is not too high: $R(N) = 1 + \frac{\eta(N)}{\mu(N)} = 1 + \frac{1-\mu}{\mu}e^{\lambda T} \leq R$. QED.

Note that the effect of memory T is the opposite of the one that we had in case of negative records. Indeed, longer memory (higher T) to makes the adverse selection among the borrowers with N record worse and may led to a brake down in the market for loans. The idea is the following, the longer positive records are stored, the easier it is for a good borrower to maintain positive reputation. In other words the longer a good borrower can stay out of the pool of borrowers with N records. Since defaults are not recorded all bad borrowers are in the pool of borrowers with N record. Thus the higher the T the worse is the pool and the harder it is for borrowers with N record to borrow: the interest rate $R(N)$ goes up. If T is high enough the market for loans breaks down and a stationary equilibrium does not exits. Note, however, that a non-stationary equilibrium may exist in this case. In an extension we study this possibility. All in all this section shows that improvements in record keeping maybe harmful for certain markets. This may actually imply that informational technologies that facilitate storage and sharing of information may not necessarily be beneficial for certain markets.

4 Economy with positive and negative records

In the section we generalize the results obtained before and consider an economy in which successful repayments are recorded for T periods and defaults are recorded for T' periods. We immediately obtain:

Proposition 3. *A stationary equilibrium with lending exists if and only if*

$$R(N) = 1 + \frac{1-\mu}{\mu}\frac{e^{\lambda T}}{1+\lambda T'} \leq R. \tag{3}$$

The equilibrium is unique.

Proof. From the proofs of first two propositions we know that $\mu(N) = \mu e^{-\lambda T}$ and $\eta(N) = \frac{1-\mu}{1+\lambda T}$. Hence borrowers with N record can borrow at interest rate $R(N) = 1 + \frac{1-\mu}{\mu}\frac{e^{\lambda T}}{1+\lambda T'}$ if and only if $R(N) \leq R$. QED.

Essentially, long memory for defaults makes it easier for borrowers with no records

5 Variable size of investment

Thus far we have assumed that each borrower needs one unit of resources to implement the project. Here we generalize the analysis and assume that each borrower can invest $I \in [0, \infty)$ in the project, that will deliver return $R(I) = \frac{1}{1-\gamma} I^{1-\gamma}$, in case of success and zero in case of failure. We assume $\gamma \in (0, 1)$. We study the stationary equilibrium. t is condition (3) holds.

First, it is clear that a borrower who is known to be of bad type can't borrow. Consider a good borrower who is known to be of good type he faces an interest rate of 1. He maximizes his expected payoff $R(I) - I$ and he borrows $I_1 = 1$ such that $R'(I_1) = 1$. If a good borrower is believed to be of good type only with probability μ he can borrow at interest rate $1/\mu$. In this case he optimally borrows $I_2 = \mu^{1/\gamma}$ such that $R'(I_2) = 1/\mu$. It is also easy to see that a bad borrower who is believed to be of good type with probability μ prefers to "pool" with the good borrower and also borrows I_2. After these calculations we can easily characterize the credit market and welfare. We compute the welfare in a stationary equilibrium as the sum of instantaneous payoffs of all agents in the economy:

$$W = \int_{i \in G} [R(I(i)) - I(i)] di - \int_{i \in B} I(i) di, \tag{4}$$

where with a slight abuse of notation we denote by G the set of good borrowers and by B the set of bad borrowers.

We consider the general case of a credit bureau that keeps positive records for T periods and negative records for T' periods. We assume that T and T' satisfy (3) so that the stationary equilibrium exits and is unique. The borrowers with an N record face an interest rate $R(N) = 1 + \frac{1-\mu}{\mu} \frac{e^{\lambda T}}{1+\lambda T'}$ and borrow $I(N) = R(N)^{\frac{-1}{\gamma}}$. Mass $\mu(N) = \mu e^{-\lambda T}$ of these borrowers is good and repays the loans and mass $\eta(N) = \frac{1-\mu}{1+\lambda T'}$ defaults. Borrowers with positive records can borrow at an interest rate of 1, each of them borrows $I = 1$ if faces an investment opportunity. In a stationary equilibrium there is mass $\mu(1 - e^{-\lambda T})$ of these borrowers. Clearly borrowers with negative records can't borrow.

Theorem 1. *The per period welfare in a stationary equilibrium with lending is*

$$W = \frac{\lambda \mu \gamma}{1-\gamma} \left[1 - e^{-\lambda T} + e^{-\lambda T} \left(1 + \frac{1-\mu}{\mu} \frac{e^{\lambda T}}{1+\lambda T'} \right)^{\frac{\gamma-1}{\gamma}} \right]. \tag{5}$$

The welfare increases with the length of negative records T' and decreases with the length of positive records T.

Proof. Each of the borrowers faces an investment opportunity with probability λ at any moment of time $t \in [0, \infty)$. Hence, per period welfare of all borrowers can be expressed as $W = \lambda\mu(1 - e^{-\lambda T})\frac{\gamma}{1-\gamma} + \lambda\mu(N)R(I) - \lambda(\mu(N) + \eta(N))I$. Note that in equilibrium I is chosen optimally by a good borrower with an N record and we can rewrite $\mu(N)R(I) - (\mu(N) + \eta(N))I = \mu(N)\frac{\gamma}{1-\gamma}R(N)^{\frac{\gamma-1}{\gamma}}$. Finally, substituting for $R(N)$ in the expression for welfare we get (5).

Because $\gamma \in (0, 1)$ welfare increases with T'. It remains to show that welfare falls with T. In order to show this denote $z = e^{\lambda T}$, $b = \frac{1-\mu}{\mu}\frac{1}{1+\lambda T'}$, and compute

$$\frac{\partial W}{\partial z} = \frac{-\lambda\mu\gamma}{(1-\gamma)z^2(1+bz)^{1/\gamma}}\left[(1+bz)^{1/\gamma} - 1 - bz/\gamma\right]. \tag{6}$$

Note that $y = bz > 0$ and expression $g(y) = (1+y)^{1/\gamma} - 1 - y/\gamma > 0$ for any $y > 0$. Indeed $g(y) = 0$ and $g'(y) = \frac{1}{\gamma}(1+y)^{\frac{1-\gamma}{\gamma}} - \frac{1}{\gamma} > 0$ for $y \in (0, \infty)$. It follows that $\frac{\partial W}{\partial z} < 0$ and, therefore, welfare decreases with T. QED.

It is intuitive that welfare increases with the length of negative memory: the longer past defaults are recorded the longer bad borrowers stay out of the lending market and prevents them from inefficiently financing their projects. This in turn lowers the interest rate paid by borrowers with no records. Lower interest rate allows good borrowers to borrow more and leads to higher returns.

The fact that keeping records of successful past repayments harms welfare is less obvious. If positive records are kept for time interval T fraction $1 - e^{-\lambda T}$ of good borrowers in a stationary equilibrium has at least one positive repayment recorded in the last T periods. These borrowers enjoy very good reputation and can borrow cheaply, hence in case the face an investment opportunity they borrow a lot and implement large projects. However, there is a negative effect of T. Since eventually, a fraction $e^{-\lambda T}$ of good borrowers eds up not having any positive records and is pooled with bad borrowers who have no records. The larger the T the worse is the pool of borrowers with no records and the higher is the interest rate charged to these borrowers $R(N)$. Facing high interest rate, good borrowers with no records cut back on their investment and implement smaller projects when face an investment opportunity. If T increases the pool deteriorates and the interest rate goes up, on top of that the mass of good borrowers that have no records also decreases, so the productive investment by the borrowers with no records drops substantially. It turns out that the negative effect of an increase in T is stronger that the positive effect, so that the overall welfare decreases when T goes up.

All in all, the theorem suggests that the welfare is the highest when past defaults are recorded for a long time while successful repayments are not recorded at all.

6 Private credit bureau

The normative analysis of credit bureau's policies conducted in previous sections shows what consequences these policies have for credit market and welfare. In reality there are different kinds credit bureaus: public institutions like central banks and private credit bureaus that can potentially be regulated. In this section we analyze the incentives of profit maximizing credit bureau and discuss possibilities for regulation.

The credit bureaus can be compensated either by lenders (so caller "user-pays" business model) or by borrowers (so-called "seller-pays" model). We first consider the "user-pays" model where a credit bureau charges lenders a fixed fee f whenever they want to retrieve the credit record of a potential borrower.

6.1 Monopolistic credit bureau

Suppose there is only one credit bureau that collects and sells information about the borrowers. The credit bureau decides on the fee f and on the record keeping policy: that is it chooses f, T and T' in order mo maximize the per period profit taking into account this choice affects the equilibrium on the lending market. For simplicity we assume that at any $t \in [0, \infty)$ all borrowers that face an investment opportunity seek a loan. Consider a borrower who applied for a loan. In principle, the lender is ready to lend only if the borrower has a reasonable credit record. For simplicity, we assume that the fee f is paid by the lender at the moment when the loan is granted. Since lenders are competitive they will have to recoup the fee f with an appropriate increase in the interest rates to make sure they make zero profits since. Since for a loan of size I the lender has to effectively provide $I + f$ the interest rate charged by competitive lenders can be computed as $R(p) = \frac{I+f}{I}\frac{1}{p}$ where p is the probability that the loan is repaid. For simplicity we call p the risk category of a borrower.

Consider a good borrower who is in risk category p. He faces an interest rate $R(p)$ which he takes as given. He maximizes his expected payoff and borrows $I(p) = R(p)^{\frac{-1}{\gamma}}$. Therefore the equilibrium interest rate for risk category p is given by

$$R(p)^{\frac{-1}{\gamma}}(R(p)p - 1) = f. \tag{7}$$

Simplifying assumption. We assume that the fee paid to the credit register is proportional to the amount borrowed, that is the total fee $f = I(p)\phi$. In this case we get $R(p) = \frac{1+\phi}{p}$. In a stationary equilibrium mass $\mu(1-e^{-\lambda T})$ of good borrowers has $p = 1$ and borrows $I(1) = (1+\phi)^{\frac{-1}{\gamma}}$. Mass $\mu e^{-\lambda T} + \frac{1-\mu}{1+\lambda T'}$ has $p_N = \frac{\mu(1+\lambda T')}{\mu(1+\lambda T')+(1-\mu)e^{\lambda T}}$ and borrows $I(p_N) = \left(\frac{1+\phi}{p_N}\right)^{\frac{-1}{\gamma}}$. The credit bureau receives the following revenue per period:

$$\pi = \lambda\phi(1+\phi)^{\frac{-1}{\gamma}}\mu\left[1-e^{-\lambda T}+e^{-\lambda T}\left(1+\frac{1-\mu}{\mu}\frac{e^{\lambda T}}{1+\lambda T'}\right)^{\frac{\gamma-1}{\gamma}}\right]. \tag{8}$$

Note that the credit bureau's profit is affected by T and T' in a similar way as the the social welfare. This implies that monopolistic credit bureau will choose socially optimal T and T'. The credit bureau will also set a positive fee $\phi^* = \frac{\gamma}{1-\gamma}$, which is not socially optimal. The optimal fee depends on the elasticity of investment with respect to the interest rate $\epsilon_R^I = \frac{-1}{\gamma}$, higher elasticity (smaller γ) leads to lower fees (smaller ϕ^*).

7 General model

The effects outlined above are general and can be found in other markets. In this section we describe the general environment. Each moment of time $t \in [0,\infty)$ there is a unit mass of sellers $i \in [0,1]$ each offering a product for sale at price P_i. The product can be of high or low quality $\theta \in \{\theta^H, \theta^L\}$. Fraction μ of sellers offers a product of high quality θ^H and fraction $1-\mu$ offers a low quality product $\theta^L < \theta^H$. The quality of the product the seller $i \in [0,1]$ is selling θ_i is the seller's private information. High quality product costs $c^H < \theta^H$ to the seller, while low quality product costs $c^L < c^H$. For brevity we assume $\theta^L = c^L$. Each period t a unit mass of new buyers is on the market, each buyer wants to buy one unit of product. A buyer derives utility θ from buying a product of quality θ. Before the purchase the buyers do not know the quality of the product. After they purchased the product they know it's quality.

Informational intermediary

After a buyer has purchased the product and learned it's quality he can leave his feedback about it to the information intermediary: the feedback can be positive S or negative D with no loss of generality. The information intermediary records buyers' feedback about each of the sellers $i \in [0,1]$ and makes these records available to the new buyers.[5] We

[5]Many online platforms like eBay, Amazon, Airbnb, Tripadvisor and the like allow users to leave feedback about the products and services they purchase. More generally, the internet allows buyers to share their opinions about the products they purchased in forums, blogs or specialized websites. All this information is potentially available to anyone searching the web, in this case it is natural to consider the search engine as an informational intermediary that retrieves certain information from the internet on

assume that the intermediary can't keep the records forever and erases them after a finite period of time: he erases positive records after time interval T and negative records after time interval T'.

Note that after the purchase the buyer has no incentive to leave a feedback. Yet, in reality many buyers do leave feedback. We abstract from buyer's motivation to leave feedback and simply assume that a buyer after purchasing a high quality product leaves positive feedback S with probability $\lambda^S \in (0,1)$, with complementary probability he leaves no feedback. Analogously, after purchasing a low quality product the buyer leaves negative feedback D with probability $\lambda^D \in (0,1)$ and no feedback with complementary probability. Since each instance $t \in [0,\infty)$ each seller sells only one product he gets at most one feedback per period. For each seller $i \in [0,1]$ at each instance t the intermediary provides seller's history h_i^t which contains positive feedback if any in the last T periods and negative feedback in the last T' periods. Each new buyer who arrives to the market at t for each seller i observes the seller's history h_i^t and updates the belief about the quality of the seller's product. The belief is characterized by probability $\mu(h_i^t)$ that the seller's product is of high quality. We assume that sellers are competitive, that is all sellers with identical history recorded h^t compete and set the same price.

As in the case of the credit bureau example high quality seller never gets a negative feedback and a low quality seller never gets a positive feedback, therefore it is sufficient to consider three classes of sellers: those with no record, with a positive record and a negative record. We refer to sellers that have no positive and no negative records in their history as sellers with an N record. A seller who has received a positive feedback $\tau \leq T$ periods ago is believed to be a high quality seller and we label such a seller with $S(\tau)$ record. Analogously, a seller who has received a negative feedback $\tau \leq T'$ periods ago is believed to be a low quality seller and we label such a seller seller with $D(\tau)$ record.

Competition between sellers with an $S(\tau)$ record drives the price to marginal cost of a high quality seller $P(S(\tau)) = c^H$ which is smaller than the buyer's willingness to pay θ^H. Analogously, competition between sellers with an $D(\tau)$ record drives the price to marginal cost of a low quality seller $P(D(\tau)) = c^L$ which is equal to the buyer's willingness to pay θ^L. Competition between sellers with an N record is less straightforward. Two cases are possible, either high quality sellers with an N record participate in trade or not. The first case is trivial: if high quality sellers with an N record do not participate in trade then in a stationary equilibrium only low quality sellers are trading.[6] We consider the case when

users' request.

[6]Such a stationary equilibrium always exists. Indeed, if buyers believe that only low quality sellers are present on the market they are ready to pay at most $\theta_L = c^L$ for a product. Given that the cost of high

high quality sellers with an N record participate in trade. In this case all sellers with an N record set the same price equal to marginal cost of high quality seller $P(N) = c^H$. The buyers are willing to buy from sellers with an N record at a price $P(N)$ only if this price is below their willingness to pay $\mu(N)\theta^H + (1-\mu(N))\theta^L$. The following proposition characterizes the stationary equilibrium of the game.

Proposition 4. *A stationary equilibrium with high quality sellers participating in trade exists if and only if*

$$P(N) = \frac{\mu(1+\lambda^D T')\theta^H + (1-\mu)e^{\lambda^S T}\theta^L}{\mu(1+\lambda^D T') + (1-\mu)e^{\lambda^S T}} \geq c^H. \tag{9}$$

The stationary equilibrium with high quality sellers participating in trade is unique.

The proof of the proposition is analogous to the proof of Proposition 3 and is omitted here. Intuitively, high quality sellers participate in trade only if they can charge a price high enough to compensate their costs. High quality sellers never get a negative feedback, hence in the worst case they have no feedback, that is they have an N record, and they can set price $P(N)$. If this price does not cover their costs c^H good quality sellers leave the market.

Note that in a model with imperfect competition high quality sellers with positive feedback $S(\tau)$, $\tau < T$ could charge price $P(S) = \theta^H > c^H$ and earn positive profits. Therefore, potentially a high quality seller could bear short term losses if he expects to make profits in the future. We leave this case for future research and concentrate on competitive case here.

The equilibrium is more likely to exist if bad feedback is recorded for a long time (high T') and if good feedback is erased fast (low T).

We expect the results about welfare and profit maximizing intermediary to be similar to those obtained for the credit market.

quality product is $c^H > c^L$ the sellers of high quality products will not participate in trade.

8 Appendix