Open Source without Free-Riding *

Philippe Aghion
Harvard University

Salvatore Modica
Università di Palermo

March, 2006

Abstract

There has been considerable investment in Open Source software projects, notably the Linux operating system, in the last years by competing firms who could have free-ridden on the others' efforts. We identify two characteristics —industry competitivity and input common usefuleness— which explain the phenomenon and point to open source production possibilities outside the software sector.

Keywords: R&D, Open Source, Free-Riding.
JEL classification: L1, O3.

1 Introduction

The cover story of *Business Week*, 31 January 2005 on the Linux computer operating system reports how "otherwise fierce competitors –think IBM and Hewlett-Packard– are demonstrating that they can benefit from embracing the open source philosophy of sharing work" (p.64). To economists it is a surprising story. Indeed the exciting thing about Open Source (OS) is that, as hackers like to put it, "Each contributes a brick and each gets back a complete house in return" (Ganesh Prasad 2001, [12]); problem is that the OS house is yours even if you do not spare your brick —only a little smaller. So why bother at all? But of course if no-one puts his brick there is no house to share. The purpose of this paper is to derive conditions under which this free-riding problem is overcome —to be precise: once the first brick is there—, for software as well as for other non-digital goods.

The idea we develop starts close to Linus Torvalds' explanation of the Open Source Software success: "Much software will be developed this way. It's especially good for infrastructure —stuff that affects everybody" (ib.). We build on this stuff-that-affects-everybody aspect, which is in principle not confined to software. From the story we abstract the fact that Linux is a widely shared input, whose improvements raise the productivity of other process-specific inputs with which it is combined and the profitability of all the firms involved. It may

*We wish to thank Jean Tirole...Financial support: Aghion ..., Modica MIUR

apply to any machine that many firms use in their production lines; or of things like brakes, used by producers of trucks, cars, motorcycles, trains, airplanes.

What we show is that if this productivity-enhancing effect is strong enough and the industry is sufficiently competitive, research on the common input under a General Public Licence (the licence that underlies the OS production mode) is not only non-zero, but more intense than under a proprietary monopolistic licence. This is after all what has been happening for the last few years in the Linux–versus–Windows confrontation.

An argument hinting at the plausibility of the result runs as follows: given non-rivalry of research output, both in the OS and monopolist production environment each one enjoys n times what he produces (where n are the firms contributing to OS or the monopolist's customers), the monopolist because he sells the same thing n times; therefore the latter's research investment must be of the same order of magnitude as the total investment in OS mode; on the other hand, in OS mode firms get research output at cost price, while with the monopolist around they pass a mark-up to him; this may make the difference in favor of OS.

More subtle is the fact that for this effect to emerge, competition among the firms involved must be strong enough. Stronger competition induces competitors to invest more in the common input because it raises the stake the innovator stands to win; and ex ante each one is more likely to be a winner if the probability that *someone* innovates increases; but this increase is exactly what higher common investment induces. Finally, competition leads to invest *more than* an outside monopolist because the stronger it is, the less competitors can afford to pay monopolistic rents, so the more inelastic their demand becomes, and the less the monopolist will produce.

Recent literature on Open Source focuses on rationalizing the programmers' contributions to OS Software. Notably Lerner-Tirole [10] indicate the signaling motive as a possible explanation, the signal being one of professional ability; this may be more appropriate to the early days of OS than to what constitutes today the main contributive source of OS software projects, namely funding by large corporations (see the quoted issue of *Business Week* or visit the chronologically first and last important OS web sites, `gnu.org` and `osdl.org`). [1] Closer to our point of view is the user-value explanation (where users may be firms). User value has certainly been decisive for the major open source projects (like Apache and Linux itself, see e.g. Lerner–Tirole [10]); and also outside the software industry the relevance of user-driven product development is widely recognized, see most notably von Hippel ([17]–[19]). The present paper points in effect to collective-user value.

Formally, we study two-stage models where firms pursue cost-reducing research in the first and engage in Hotelling price competition in the second stage. So the paper is linked to the literature on R&D spillovers, cartels and joint ventures, where the general structure of the models studied is the same as ours, with

[1] There are too many 'gates' to the open source community on the web to refer to a few for background information. A readily available note is Modica [11].

research conducted in the first stage and product competition in the second. In this line of research, the early results of Spence [14], Katz [8], d'Aspremont–Jacquemin [6], Kamien–Muller–Zang [7] (the last two interestingly compared by Amir [2]) and Suzumura [15] have been significantly extended by Amir et al. [3]. The essential difference in our setting lies in the structure of R&D expenses: in the cited literature a fraction of *all* of a firm's R&D results spill over to the others; our firms on the other hand may conduct *private* research on firm-specific inputs, whose results remain private, and at the same time, as an independent choice, they may contribute to research on shared inputs whose results are common property. This separation is what creates the obvious free-rider problem in the common-research dimension which we address in the paper. [2]

The next section contains our results: after introducing analysis in 2.1, we make a first point in 2.2 (Result 2, page 8) in a simple duopoly-versus-monopoly setting; the main result of the paper is presented in 2.3 (Result 4, page 14), and section 3 concludes.

2 Models and Results

2.1 Outline

We study subgame perfect equilibria of two-stage games. Going backwards, in the second stage there is price competition among either two or n price-setting firms i in spatial models à la Hotelling: when there are two firms they are located at the extreme points of a unit-length segment; when there are n they are evenly located along a unit-length circle. They produce a good at unit cost c_i, which is sold to a unit mass of consumers who buy a unit each from the firm which they find more convenient on the basis of the firms' selling prices p_i and of their unit transportation cost $t > 0$. It is exactly as in Tirole [16] ch.7, except that we allow for $c_i \neq c_j$ (as in Aghion–Schankerman [1]).

In the first stage, on which present attention is focused, firms invest to pursue a cost-reducing innovation which would increase profits in stage two. The firms' production technology is not formally specified, but it is thought of as involving two types of inputs: firm-specific and shared ones. We then model research as consisting of two types of efforts, directed at the two types of inputs and resulting in two numerical indexes, x_i and z_i respectively, which may be interpreted as quality, or quantity of services; these influence the probability of cost reduction. Firm-specific x_i is always assumed to be produced at constant, unit marginal cost; the larger scale on which research on the common input takes place may determine on the other hand convex production costs $c(z_i)$.

[2] Cozzi [5] addresses the free-riding problem with non-rival goods such as research results in a growth model, but in a different setup: he has nonatomic firms, so the effect of individual firms' contributions is always null, and to escape free-riding a detection-punishment mechanism is in place; given detection, punishment takes the form of excluding the deviator from joint results. This is by definition impossible under GPL.

We are interested in the consequences of imposing a General Public Licence (GPL) on the common input. The GPL imposes first not to impose the restrictions of use, modification and distribution of the usual proprietary licenses; and second, cleverly, to release the modified good under GPL in turn. [3] The good released under a GPL is (and always be) 'Open Source'. In our setting, GPL makes the fruits of research on the common input a public good: non-rival, non-excludable. Individual efforts add up, and all enjoy the cumulative result. The problem with investing in the common input becomes that by not investing at all a firm can still appropriate the result of the others' efforts. The present paper addresses this free-riding problem.

The presumption underlying our results is that increased productivity of the shared GPL'd inputs, while not giving a relative advantage to any single firm (any one's effort increases the chances of competitors as well as its own), still determines some advantage to the family of firms as a whole, i.e. to the average firm. We typically find that if the latter effect is strong enough, then research effort is higher with Open Source than without. Result is not totally unsurprising because with relatively large production the free-riding problem should become severe.

We consider two non-open-source production systems: the first is a one-firm world, where the public good becomes perforce a private good; in the other the common input is provided by an external patent-protected monopolist, who sells it to the firms for profit and is the only one who can conduct research on its product. Comparison of the latter with Open Source is more like the Linux-versus-Window situation.

2.2 The Linear City: Duopoly Versus Monopoly

Here we make our point (Result 2) in the simplest possible setting, contrasting a duopoly against a monopoly. It is simple because in the latter case by construction there is only one possible institutional setting (since there is no input or knowledge to share).

Duopoly with GPL'd Common Input. In the linear city $[0, 1]$ there are two firms, firm 1 at 0 and firm 2 at 1. In the first stage players take into account the equilibrium continuation they anticipate for the second stage; so we begin with the latter.

Second Stage. We compute equilibrium profits when unit production costs are $C = (c_1, c_2)$. Prices, which are the firms' moves for this stage, are $p = (p_1, p_2)$. Recall from standard model that product differentiation is embedded in transportation cost t, so strength of competition is dually measured by $1/t$. We assume t being not so high as to leave some consumers unserved in equilibrium; then demands D_i at p are determined by the consumer who is indifferent between

[3] The GPL is at `http://www.gnu.org/licenses/gpl.html`. It is currently undergoing a revision process, details at `http://gplv3.fsf.org/`.

the two firms (cfr. Tirole [16] pp. 98, 279), yielding

$$D_i(p) = \frac{1}{2} - \frac{p_i - p_j}{2t} = \frac{p_j - p_i + t}{2t};$$

profits $\pi_i(p, C) = (p_i - c_i) \cdot D_i$ are then

$$\pi_i(p, C) = (p_i - c_i)\,\frac{p_j - p_i + t}{2t}\,. \tag{1}$$

In equilibrium it must be $\partial \pi_i / \partial p_i = 0$ for all i, that is $2p_i = p_j + c_i + t$ for $i \neq j = 1, 2$; so equilibrium p satisfies

$$\begin{aligned} 2p_1 - c_1 &= p_2 + t \\ 2p_2 - c_2 &= p_1 + t, \end{aligned}$$

whose solution is

$$p_i = \frac{2c_i + c_j}{3} + t, \quad i \neq j = 1, 2.$$

Since $p_i - c_i = t + (c_j - c_i)/3$ and $p_j - p_i = (c_j - c_i)/3$, from (1) equilibrium profits are

$$\begin{aligned} \pi_i(C) &= \left(\frac{c_j - c_i}{3} + t\right)\left(\frac{c_j - c_i}{6t} + \frac{1}{2}\right) \\ &= \frac{(c_j - c_i)^2}{18t} + \frac{c_j - c_i}{3} + \frac{t}{2}. \end{aligned} \tag{2}$$

First Stage. Here firms invest to increase the probability of an innovation which starting from a symmetric situation $c_1 = c_2 = c$ gives the innovator a cost advantage of δc, bringing down his cost from c to $(1 - \delta)c < c$. We rule out the possibility that both firms innovate.

Let x_i and z_i denote research output respectively for firm-specific input and GPL'd common input. We assume that the probability that i innovates is influenced by individual x_i and cumulative $Z \equiv z_i + z_j$ (public good aspect); and that the latter also increases the probability that someone innovates (cake-increasing effect). More precisely, we take the probability q_i that i innovates as given by

$$q_i \;=\; (1 - \eta)\frac{x_i}{X} \;+\; \eta \frac{x_i}{X} f(Z) \tag{3}$$

where $X = x_1 + x_2$, f is concave, increasing from $f(0) \in [0, 1]$ to 1 as Z goes from zero to infinity; $\eta \in [0, 1]$ is the parameter reflecting the influence of the productivity of the common input on innovation probability $q_1 + q_2 = 1 - \eta + \eta f(Z)$, because

$$\frac{\partial}{\partial \eta} \frac{\partial (q_1 + q_2)}{\partial Z} = f'(Z) > 0\,.$$

Note that as mentioned before an increase in z_i also raises q_j. [4] [5]

Moves for firm i in this stage are pairs $s_i = (x_i, z_i)$; the profile will be denoted by $s = (x, z)$, where $x = (x_1, x_2)$, $z = (z_1, z_2)$. Of course the probability q_i above is a $q_i(s)$. Consider firm i: with probability q_i it will be the innovator and will have a cost advantage of δc; with probability q_j the innovator is j and i will bear a cost disadvantage of the same amount; with probability $1 - q_1 - q_2$ no-one will innovate and both will have cost c. By perfectness firms choose moves taking second stage profits as given by (2), which in the three events above become respectively (w, l, d for win, lose and draw)

$$\pi^w \equiv \frac{(\delta c)^2}{18t} + \frac{\delta c}{3} + \frac{t}{2}, \quad \pi^l \equiv \frac{(\delta c)^2}{18t} - \frac{\delta c}{3} + \frac{t}{2}, \quad \pi^d \equiv \frac{t}{2}.$$

We shall always assume unitary research costs for the private input. In the present (sub)section we assume the same also for the shared input, for simplicity. Then firm i's payoff u_i in the first stage is given by

$$\begin{aligned} u_i(s) &= q_i \pi^w + q_j \pi^l + (1 - q_i - q_j)\pi^d - x_i - z_i \\ &= \frac{t}{2} + (q_i + q_j)\frac{(\delta c)^2}{18t} + (q_i - q_j)\frac{\delta c}{3} - x_i - z_i \\ &= \frac{t}{2} + \frac{\delta c}{3}\big(1 - \eta + \eta f(Z)\big)\Big[\frac{\delta c}{6t} + \frac{x_i - x_j}{X}\Big] - x_i - z_i. \end{aligned} \tag{4}$$

In this expression it is clear how competition 'raises stakes' as we said in introduction (from the t in denominator), and how it moreover enhances importance of Z:

$$\frac{\partial}{\partial(1/t)}\frac{\partial u_i}{\partial Z} = \eta f'(Z)\frac{(\delta c)^2}{18} > 0\,,$$

in fact clearly $\partial u_i / \partial Z$ goes to infinity with $1/t$.

Partial derivatives of u_i with respect to x_i and z_i are

$$\begin{aligned} \frac{\partial u_i}{\partial x_i} &= \frac{\delta c}{3}\big(1 - \eta + \eta f(Z)\big)\frac{2x_j}{X^2} - 1 \\ \frac{\partial u_i}{\partial z_i} &= \frac{\delta c}{3}\,\eta\, f'(Z)\,\big[\frac{\delta c}{6t} + \frac{x_i - x_j}{X}\big] - 1; \end{aligned}$$

[4] The idea of a probability of winning based on the fraction of invested resources has its roots in the literature on rent–seeking games, see Baye–Hoppe [4]. We add η and $f(Z)$.

[5] (Model Structure) In the models we present innovation lowers cost, and investment in research raises the probability of innovation (equations (3) and (13)). A plausible alternative to this reduced form would be research directly influencing costs. On the other hand, with price competition on the second stage profits depend on cost differences, and these are unaffected by lowering each term as a result of common research. So equilibrium would have null investment in common input. This is not due to the form of competition in the second stage: if one models the latter as a Cournot game, profits depend negatively on own cost and positively on competitors' costs; investing in common input would then be dominated by investment in private input because the latter lowers own cost without lowering others' costs; and again equilibrium would have zero common-input investment. Therefore these alternative models run against the empirical evidence showing positive expenditure on research on GLP'd inputs (such as Linux, see `www.osdl.org`).

equating these to zero and summing over i we arrive at equilibrium relations: $X = (\delta c/3)\big[1-\eta+\eta f(Z)\big]$ from the first set; and from the partials with respect to z_i one obtains

$$\frac{(\delta c)^2}{18}\,\frac{1}{t}\,\eta \;=\; \frac{1}{f'(Z)}\,. \tag{5}$$

Since the right member increases with Z (by concavity of f), this relation gives some comparative statics in the total research effort Z devoted to the open source common input which seem worth reporting (inspect $\partial u_i/\partial z_i$ above for the first two statements):

Result 1 (OS in Linear city)**.** *Assuming finite $f'(0)$ and fixing t, for η small enough $Z = 0$ (and X is proportional to δc). Fixing η, for t small enough (strong competition) $Z > 0$. When $Z > 0$ it increases with the common benefit index η, with strength of competition $1/t$, and with cost reduction δ.*

The only non-obvious result here is the one concerning competition, and the intuition for this was anticipated in the introduction: competition raises the stake for the winner. At this point we see clearly how it works: it is the $q_1 + q_2$ term in equation (4).

The One-Firm City With a single firm it does not matter whether the unit-length city is linear or circular; we continue with the unit interval, monopolist located in the middle (the location she would choose).

Start as usual from the price-setting (second) stage. Denoting by x for a moment the position on the segment, consumer at x with reservation utility $R > c$ is willing to buy if $p+t\left|x-\frac{1}{2}\right| \le R$; so demand at p is $2(R-p)/t$, half on each side; but this as long as $2(R-p)/t \le 1$, for otherwise demand is 1. That is, monopolist's demand $D(p)$ is given by

$$D(p) = \min\big\{2\frac{R-p}{t}, 1\big\}.$$

Monopolist's problem is to maximize profits, $\max_p (p-c)D(p)$. If the p which solves the unrestricted problem

$$\max_p 2(p-c)\,\frac{R-p}{t}$$

is such that $D(p) \le 1$, then that p is the solution to the original problem; otherwise optimal price is the lowest such that $D(p) \le 1$. The unrestricted optimum price is easily seen to be $p = (R+c)/2$, so $D(p) \le 1$ reads

$$\frac{R-c}{t} \le 1\,. \tag{6}$$

If this is satisfied —competition $1/t$ weak enough— then $(R+c)/2$ is the optimal price, and resulting profits are $\pi(c) = 2(p-c)(R-p)/t = (R-c)^2/2t$; if on the other hand equation (6) fails, then monopolist will set the p at which $D(p) = 1$, i.e. $2(R-p)/t = 1$, or $p = R-t/2$, with profits $\pi(c) = R-c-t/2$. Recapitulating, second stage equilibrium profits for the monopolist are

$$\pi(c) = \begin{cases} \frac{(R-c)^2}{2t} & \textit{if } \frac{R-c}{t} \leq 1 \\ R - c - \frac{t}{2} & \textit{otherwise}\,. \end{cases} \tag{7}$$

We turn to investment (first) stage. Innovation probability (3) is independent of x_i now, for $x_i/X = 1$ for any x_i; omitting useless subscripts, we then have

$$q(z) = 1 - \eta + \eta f(z)\,.$$

The monopolist has no reason to set $x > 0$, and his problem becomes

$$\max_z \; (1 - q(z))\,\pi(c) \;+\; q(z)\,\pi((1-\delta)c) - z.^6 \tag{8}$$

Now notice that if (6) fails for c it fails for any $c' < c$; we shall neglect the case where it holds for c but fails for $(1-\delta)c$; we thus consider the two typical cases, (i) weak competition with (6) valid (for c and $(1-\delta)c$), and (ii) strong competition with (6) failing.

After plugging first and second line of (7) into (8) in turn, one finds the following optimality conditions:

$$\begin{cases} \dfrac{\delta c\,(2(R-c) + \delta c)}{2}\dfrac{\eta}{t} = \dfrac{1}{f'(z)} & \textit{if } \dfrac{R-c}{t} \leq 1 \\ \delta c\,\eta \;=\; \dfrac{1}{f'(z)} & \textit{otherwise}\,. \end{cases} \tag{9}$$

Comparison of this with (5) yields the linear city comparison we were after:

Result 2 (Linear city, OS versus Monopoly)**.** *If competition (as measured by $1/t$) is strong enough, then research on open source common input Z is larger than the z provided by a monopolist. For weak enough competition the opposite occurs.*

In this case what happens is that for small enough t the monopolist's marginal gain from z is independent of t, while the innovating competitor gains more the smaller is t.

2.3 The Circular City

Here we present the main result of the paper (Result 4), which concerns comparison of two alternative institutional settings: one with n competing firms with GPL'd common input, the other with the same firms but where research on common input is provided by an external patented monopolist.

[6]Notation is slightly imprecise: here c and $(1-\delta)c$ are specific cost values, in (7) c denotes a variable —apologies.

Circular City with GPL'd Common Input. The n firms are evenly located around the circle, and in second-stage price competition a fraction q_l will have low cost, the rest will have high cost. The relative proportions will be endogenously determined by research investments in the first-stage of the game.

Second Stage. At this stage q_l is given, and i's competitors $i-1$ and $i+1$ look the same to her. Denoting by $\mathbb{E}p_{-i}$ their common expected price, firm i's demand is derived as in Tirole [16] ch.7:

$$D_i(p_i, \mathbb{E}p_{-i}) = \frac{\mathbb{E}p_{-i} - p_i + t/n}{t}\ ; \tag{10}$$

so firm i, with production cost c_i, solves

$$\max_{p_i}\ (p_i - c_i)\, \frac{\mathbb{E}p_{-i} - p_i + t/n}{t}\ ;$$

equating derivative to zero one obtains $p_i = \big(\mathbb{E}p_{-i} + c_i + t/n\big)/2$. We consider price-symmetric equilibrium, where $\mathbb{E}p_{-i} = \mathbb{E}p_i \equiv \mathbb{E}p$; since $\mathbb{E}c_i = \mathbb{E}c$, by taking expectations in the p_i equation one gets $\mathbb{E}p = \mathbb{E}c + t/n$; and substituting this for $\mathbb{E}p_{-i}$ gives

$$p_i = \frac{\mathbb{E}c + c_i}{2} + \frac{t}{n}\,. \tag{11}$$

In our context $c_i \in \{c_l, c_h\}$ for all i (they will be $c_h = c,\ c_l = (1-\delta)c$), with relative proportions $q = (q_l, q_h),\ q_h = 1 - q_l$. Letting $\Delta c = c_h - c_l$, we now compute (second-stage) equilibrium profits, which will depend on the cost vector C and on q. Since $(c_h + \mathbb{E}c)/2 = c_h - q_l\Delta c/2$ and $(c_l + \mathbb{E}c)/2 = c_l + q_h\Delta c/2$, from (11) one has, assuming symmetry within groups,

$$p_l = c_l + \frac{t}{n} + q_h\frac{\Delta c}{2}\,, \quad p_h = c_h + \frac{t}{n} - q_l\frac{\Delta c}{2}\,;$$

so $p_h - p_l = \Delta c/2$; on the other hand $p_h - \mathbb{E}p = q_l(p_h - p_l)$ and $\mathbb{E}p - p_l = q_h(p_h - p_l)$; therefore from (10) equilibrium demands are

$$D_l = \frac{1}{n} + \frac{q_h\Delta c}{2t}\,, \quad D_h = \frac{1}{n} - \frac{q_l\Delta c}{2t}\,;$$

hence from $\pi_i \equiv (p_i - c_i)D_i$, equilibrium profits are given by

$$\pi_l(\Delta c, q) = t\left(\frac{1}{n} + q_h\frac{\Delta c}{2t}\right)^2, \quad \pi_h(\Delta c, q) = t\left(\frac{1}{n} - q_l\frac{\Delta c}{2t}\right)^2 \tag{12}$$

where we have made explicit that they depend on C only through Δc.

First Stage. As before, moves in this stage are pairs (x_i, z_i), and innovation reduces production cost from c to $(1-\delta)c$ —so in our current notation $\Delta c = \delta c$. Firm i invests to influence the probability of being low-cost in stage two.

We assume that assignment of positions occur after stage 1, so that at that stage firm i does not know her neighbors' type (high or low cost). Hence she views stage 2 as a price-competition game as the one we have described above, with payoffs given by equation (12) (with $\Delta c = \delta c$).

The probability q_i that firm i will be low-cost depends on her research investments. As before we denote sums by capitals: $X = \sum_i x_i$, $Z = \sum_i z_i$; and for q_i we take the following specification:

$$q_i = n\,\frac{x_i}{X}\left(1 - \eta + \eta f(Z)\right). \tag{13}$$

This is analogous to (3) of page 5: x_i is divided by the average X/n instead of X because here q_i denotes the probability that i is *among* the innovators, not *the* innovator. Again the relevant variables are individual investment on firm-specific input x_i and cumulative investment in shared open-source input Z; and again q_i depends on the profile of moves, which will be denoted by $s = (x, z)$, $x = (x_i), z = (z_i)$. From now on we shall assume $f(0) < 1/2$.

Given q_i, $i = 1, \ldots, n$, the fraction of low cost firms will be their average:

$$q_l = q_l(Z) = n^{-1} \sum\nolimits_i q_i = 1 - \eta + \eta\, f(Z)\,,$$

and $q_h = 1 - q_l$; expected number of low-cost firms will be $\sum_i q_i = nq_l$. Note that $q_i > q_l$ iff $x_i/X > 1/n$.

Here q_l is the analogous of innovation probability in the linear city, indeed it has the same expression; and the same is the role of η: $\frac{\partial}{\partial\eta}\frac{\partial q_l}{\partial Z} > 0$.

In principle it may be $q_i \notin [0, 1]$; we are thinking of firm i as small enough, so that x_i cannot be too far from X/n in the relevant range, and the problem does not bite. Assuming now z_i-production costs $c(z_i)$ convex with $c'(0) = 0$, first-stage payoffs are then

$$u_i(s) = q_i(s)\pi_l + (1 - q_i(s))\pi_h - x_i - c(z_i)\,, \tag{14}$$

where q_i and second-stage profits are given by equations (13) and (12). After a little algebra, letting $\alpha = \delta c/2t$ and $\psi_i = nx_i/X$ one obtains

$$u_i(s) = t\left[\frac{1}{n^2} + \frac{2\alpha}{n} q_l(\psi_i - 1) + \alpha^2\big(\psi_i q_l - q_l^2(2\psi_i - 1)\big)\right] - x_i - z_i\,.$$

Here again competition has the same roles as in the linear city case: the raising stakes effect is clear by inspection, and it will be $\frac{\partial}{\partial(1/t)}\frac{\partial u_i}{\partial Z} > 0$ in symmetric equilibrium (because $q_l < 1/2$ in equilibrium).

Setting partial derivatives of u_i with respect to z_i equal to zero gives, recalling that $\partial q_l/\partial z_i = \eta f'(Z)$,

$$\frac{2c'(z_i)}{\delta c \eta f'} = \frac{2}{n}(\psi_i - 1) + \alpha\big(\psi_i - 2(2\psi_i - 1)q_l\big)\,;$$

and since $\sum_i \psi_i = n$, by summing over i one obtains (after re-substituting for α and ψ_i and rearranging) the equilibrium condition

$$\frac{\eta f'}{t}\,\frac{(\delta c)^2}{4}\big[2\eta(1 - f) - 1\big] = \frac{\sum_i c'(z_i)}{n}\,.$$

Since FOC is really $\partial u_i/\partial z_i \leq 0$, we see that if η is small enough $2\eta(1-f)-1 < 0$, whence optimal $z_i = 0$ for all i and therefore $Z = 0$. On the other hand, concentrating on symmetric equilibrium in the x_i 's, the assumption $f(0) < 1/2$ ensures that for η not too small $\partial u_i/\partial z_i > 0$ at $Z = 0$, so FOC holds with equality and the above equilibrium condition may be written as

$$\frac{(\delta c)^2}{4}\,\frac{\eta}{t} = \frac{n^{-1}\sum_i c'(z_i)}{f'(Z)\,(2\eta(1-f(Z))-1)}\,. \tag{15}$$

Notice why $f(0) < 1/2$ is needed: if not, even for $\eta = 1$ it would be $Z = 0$. For comparative statics we note that in x_i–symmetric equilibrium the numerator on the right is $c'(n^{-1}Z)$, so again the right hand side of this expression is increasing in Z; this gives the following (easily checked by inspection):

Result 3. *Assume $f(0) < 1/2$ and $c(\cdot)$ convex with $c'(0) = 0$, and consider x_i–symmetric equilibria. Then for η less than some η_0 one has $Z = 0$; otherwise, for competition $1/t$ strong enough it is positive and increasing in n, η, $1/t$, and δ.*

The last part of comparative statics is as in Proposition 1, while the first part is somewhat weaker: for η small enough $Z = 0$ regardless of t, because of the extra term $2\eta(1-f(Z))-1$. The reason is that here a firm cannot 'win': it can only be in a set of winners; and by increasing z_i firm i raises the probability of being there, but at the same time makes this set larger. On the other hand notice the positive dependence of Z upon the size of the market, an effect which could not emerge with the number of firms fixed at two.

Circular City with Outside Monopolist. We now study the $n+1$–player model where besides the n firms around the circle there is a monopolist, player 0, who sells the services denoted by z throughout the paper to the n competitors. Non-rivalry is a physical characteristic of research output, but non-excludability is not —it may often be eliminated by law. Thinking of software for example, Windows and Linux are both operating systems, both non-rival in nature, but one is made excludable by a proprietary licence, the other is a public good by GPL. We now model the proprietary system, then compare it with the GPL model of last paragraph.

For the n competitors $i = 1, \ldots, n$, stage two (price competition) is still as in the previous case, with profits depending on Δc and q given by (12). But there are two changes in the first stage. The first is in $q_i(s)$, the probability with which i will be low-cost in stage two (cfr. (13)): it is now no longer cumulative Z which enters the formula, but just the amount z_i which firm i acquires from the monopolist; in other words we now have

$$q_i = n\,\frac{x_i}{X}\left(1-\eta+\eta f(z_i)\right), \tag{16}$$

and as a consequence, $q_l = n^{-1}\sum_i q_i$ is now given by

$$q_l = \sum_i \frac{x_i}{X}\left(1-\eta+\eta f(z_i)\right). \tag{17}$$

We will actually end up considering the $\eta = 1$ to simplify arguments, because we already know that for small η open source does not work well anyway; but for the moment we keep it explicit to facilitate comparison with previous expressions.

The second difference is that firm i no longer produces z_i, but buys it at the price p the monopolist sets. Thus firm i's first-stage payoff is (cfr. (14) for comparison)

$$u_i(s) = q_i(s)\pi_l + (1 - q_i)\pi_h - x_i - pz_i\,, \tag{18}$$

where now q_i is to be read from equation (16) above. Using notation ψ_i and α from page 10, and also letting $\mu_i = 1 - \eta + \eta f(z_i)$ (so that $q_i = \psi_i\,\mu_i$), substitution from (12) now leads to the following expression for $u_i(s)$:

$$u_i(s) = t\left[\frac{1}{n^2} + \frac{2\alpha}{n}\big(\psi_i\mu_i - q_l\big) + \alpha^2\big(\psi_i\mu_i(1 - 2q_l) + q_l^2\big)\right] - x_i - pz_i\,.$$

The FOC with respect to z_i gives, fixing the other firms' investments z_{-i}, price p as function of z_i, whose inverse is firm i's demand of z_i at price p. We then set $\partial u_i/\partial z_i = 0$; since $\mu'(z_i) = \eta f'(z_i)$ and $\partial q_l/\partial z_i = n^{-1}\psi_i\eta f'(z_i)$, after substituting for $\alpha = \delta c/2t$ this yields

$$p = \psi_i\eta f'(z_i)\left[\frac{(\delta c)^2}{4t}\Big(1 - 2q_l(z_i, z_{-i})\,\frac{n-1}{n} - 2\frac{\psi_i\mu_i(z_i)}{n}\Big) + \delta c\,\frac{n-1}{n^2}\right]. \tag{19}$$

We check here that the above FOC is sufficient for a maximum of u_i; one has in fact

$$\frac{1}{\psi_i}\,\frac{\partial^2 u_i}{\partial z_i^2} \;=\; \mu_i''\,[\cdot] - \mu_i'\,\frac{(\delta c)^2}{2nt}\Big((n-1)q_l' + \psi_i\mu_i'\Big)\,, \tag{20}$$

where the bracketed expression is the one of (19), positive whenever (19) holds; our assertion then follows by recalling that μ_i' and q_l' are positive while μ_i'' is negative. Next, let $\tilde{\zeta}_i(p; z_{-i})$ be the inverse of the function defined in (19), and the profile $(\zeta_i)(p)$ be a fixed point of the map $(\tilde{\zeta}_i)(p)$ (which exists because the z_i's can be uniformly bounded above in searching for an equilibrium).

We now turn to the monopolist. Given the non-rivalry of his product, whatever he produces for one firm can be re-used for all n. Since in principle the various firms may demand different amounts of the monopolist's service, he will have to produce the highest required; but with that he is able to serve the whole market. In other words, he produces $\max_i z_i$ and sells $\sum_i z_i$. We continue to assume convex z-production costs $c(z)$ here. Thus the monopolist's problem is the following:

$$\max_p\; p\sum\nolimits_i \zeta_i(p) \;-\; c(\max_i \zeta_i(p))\,.$$

We shall consider the symmetric equilibrium, with $x_i = n^{-1}X$ and $z_i = z$ for all i. Having checked that first order conditions are sufficient, see the discussion around equation (20), existence of this equilibrium follows from existence of a solution to equation (22) below, which is easily established using the regular curvature of the functions involved. In such an equilibrium the monopolist produces z and sells nz, so that his problem becomes

$$\max_z \; nz\, p(z) - c(z)\,, \text{ with FOC } \; p + zp' \leq n^{-1}\, c'(z) \tag{21}$$

where p is given by equation (19). At this point we take $\eta = 1$. Then $\mu_i = f(z) = q_l$; also by symmetry $\psi_i = 1$ all i, so that p and p' read, letting $A = (\delta c)^2/4t$,

$$p(z) = \; Af'(z)\left[1 - 2f(z) + \tfrac{4t}{\delta c}\tfrac{n-1}{n^2}\right]\,,$$
$$p'(z) = \; A\left[f''(z)\big(1 - 2f(z) + \tfrac{4t}{\delta c}\tfrac{n-1}{n^2}\big) \; - 2f'(z)^2\right]\,.$$

We now take t small (strong competition). For such t's the above expressions are approximately as follows:

$$p(z) \simeq A\, f'(z)\big(1 - 2f(z)\big), \quad p'(z) \simeq A\left[f''(z)\big(1 - 2f(z)\big) - 2f'(z)^2\right]\,.$$

Given $f(0) < 1/2$, which we maintain from previous paragraph (if it fails monopolist produces zero), $p(0) > 0$, whence from (21) the monopolist will produce positive z and meet FOC with equality. Using the above approximations the FOC reads $A(1-2f)[f' + z\big(f'' - 2(1-2f)^{-1}f'^2\big)] = n^{-1}c'$, that is

$$A\left[1 + z\frac{f''(z) - 2(1-2f)^{-1}f'^2}{f'(z)}\right] \;=\; \frac{n^{-1}c'(z)}{f'(z)\big(1 - 2f(z)\big)}\,. \tag{22}$$

Since $f'' < 0$ the left member is smaller than A. Since we are assuming $c(\cdot)$ convex the right member is increasing in z; therefore (for small t) equilibrium z^{em} with external monopolist is smaller than the z^o determined by

$$A = \; \frac{n^{-1}c'(z^o)}{f'(z^o)\big(1 - 2f(z^o)\big)} \equiv \mathcal{M}(z^o)\,.$$

We now go back to the circular city with open source common input. The situation was that each firm would enjoy cumulative contribution Z by producing, in symmetric equilibrium, $n^{-1}Z$. In the linear cost version we had in the previous section the equilibrium quantity was determined by relation (15). Notice that with $\eta = 1$ the left member is just A, and denominator in right member $f'(1-2f)$, like what we have here; numerator was 1, the constant marginal cost of z. With $c(z_i)$ replacing z_i in payoff (14), in symmetric equilibrium the 1 in that numerator gets replaced by $c'(n^{-1}Z)$; that is, equilibrium Z in GPL economy is defined by

$$A = \; \frac{c'(n^{-1}Z)}{f'(Z)\big(1 - 2f(Z)\big)} \equiv \mathcal{C}(Z)\,.$$

How does Z compare with z^o? We assume $c(\cdot)$ convex with (weakly) convex marginal cost (e.g. if c is a power function $c(z) = z^\alpha$, we are saying $\alpha \geq 2$); then (elementary calculus) $n^{-1}c'(z) \geq c'(n^{-1}z)$. Thus we have $A = \mathcal{C}(Z) \leq \mathcal{M}(Z)$; and since $\mathcal{M}(z^o) = A$ and $\mathcal{M}$ is increasing, $z^o \leq Z$; since we already know that monopoly equilibrium z^{em} is less than z^o, conclusion is:

Result 4 (Linux-versus-Windows)**.** *Take $\eta = 1$ in the expressions* (13) *and* (16) *for q_i, and assume $f(\cdot)$ concave with $f(0) < 1/2$ and $c(\cdot)$ increasing convex along with c'. Then in symmetric equilibrium the OS economy produces more research on the common input than that provided by a patent-protected monopolist if competition strong enough.*

There are two effects at play here that counteract the monopolist's advantage that non-rivalry gives him. One is that marginal costs are increasing: the monopolist has to produce the whole market quantity, while each competitor goes away with an n-th of it. The other is the rent-paying effect mentioned in the introduction: the stronger the competition the less affordable is paying monopolistic rents. The end result is that when these two effect cumulate, Open Source works better than the proprietary system.

3 Conclusions

We are discussing R&D on shared inputs where sequential innovation matters. What we have shown is that, given a primary invention, the amount of subsequent research is higher under GPL than under a proprietary licence held by a monopolist if the productivity increase due to improvements in the common input is substantial and the industry is sufficiently competitive. Releasing an invention under a GPL may be the inventor's choice, as it has been for Linux (Saint-Paul [13] has a result about the possibility of voluntary information sharing in a related context), but typically it is not. On the other hand imposing GPL by law on any set of goods obviously de-incentivates innovative activity; thus there is a patent policy trade-off between fostering initial inventions or subsequent product development. Having identified the contexts in which a GPL would lead to better sequential improvements than a proprietary licence, a policy route which seems worth exploring (for such products) is the one hinted at by Kremer [9], which consists of granting patents to inventors, and then proceeding with patent buyouts by the government when this is judged beneficial (Kremer dose not mention GPL, but the obvious step after a buyout would be to release the patent's content under a GPL).

On the empirical side, our model has testable implications which we propose to investigate at a later stage. The typical regression one would want to set up has expenditure on open source projects as dependent variable; this should be extracted from the public firms' balance sheets. As explanatory variables we have highlighted the extent to which the OS'd input enters the group of firms' technological processes and the strength of competition in the industry; and our finding that the two effects are complementary could be tested in the usual way. Of course one should control for other variables which we have not mentioned, possibly cost differentials between the OS'd input and the proprietary alternative. Such variables may play a role, but as this paper has pointed out, without the shared-input and competitivity conditions that we have highlighted the free riding problem would prevent any cost advantage from being exploited and frustrate any other mutually beneficial opportunity.

References

[1] Aghion, Philippe and Mark Schankerman (2004): On the Welfare Effects and Political Economy of Competition-enhancing Policies, *The Economic Journal* **114**: 800-824

[2] Amir, Rabah (2000): Modelling Imperfectly Appropriable R&D via Spillovers, *International Journal of Industrial Organization* **18**: 1013-1032

[3] Amir, Rabah, Igor Evstigneev and John Wooders (2003): Noncooperative versus Cooperative R&D with Endogenous Spillover Rates, *Games and Economic Behavior* **42**: 183–207

[4] Baye, Michael R. and Heidrun C. Hoppe (2003): The strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games, *Games and Economic Behavior* **44**:217–226

[5] Cozzi, G. (1999): R&D Cooperation and Growth, *Journal of Economic Theory* **86**: 17–49

[6] d'Aspremont, Claude and Jacquemin, A. (1988): Cooperative and Noncooperative R&D in Duopoly with Spillovers, *American Economic Review* **78**: 1133–1137

[7] Kamien, Morton, Eitan Muller and Israel Zang (1992): Research Joint Ventures and R&D Cartels, *American Economic Review* **82**: 1293–1306

[8] Katz, Michael (1986): An Analysis of Cooperative Research and Development, *RAND Journal of Economics* **17**: 527–543

[9] Kremer, M. (1998): Patent Buyouts: a Mechanism for Encouraging Innovation, *Quarterly Journal of Economics* **93**: 1137–1167

[10] Lerner, Josh and Jean Tirole (2002): Some Simple Economics of Open Source, *Journal of Industrial Economics* **50**: 197–234

[11] Modica, Salvatore (2006): Notes on the Economic History of Linux, mimeo, at textttwww.unipa.it/modica

[12] Prasad, Ganesh: Open Source-onomics: Examining some pseudo-economic arguments about Open Source `http://www.freeos.com/articles/4087`

[13] Saint-Paul, G. (2003): Cumulative Innovation and Information Sharing in Business Networks, IDEI Working Paper.

[14] Spence, Michael (1984): Cost Reduction, Competition and Industry Performance, *Econometrica* **52**: 101–112

[15] Suzumura, K. (1992): Cooperative and Noncooperative R&D Spillovers in Oligopoly, *American Economic Review* **82**:1307–1320

[16] Tirole, Jean (1988): *Theory of Industrial Organization*, MIT Press

[17] von Hippel, Eric (1986): Lead Users: a Source of Novel Product Concepts, *Management Science* **32**: 791–805

[18] von Hippel, Eric (1998): Economics of Product Development by Users: Impact of 'Sticky' Local Information, *Management Science* **44**: 629–644

[19] von Hippel, Eric (2002): Horizontal Innovation Networks —by and for Users, *MIT Sloan School of Management* WP No. 4366–02