Competition amongst Contests *

Ghazala Azmat †
and
Marc Möller ‡

Abstract

This article analyses the allocation of prizes in contests. While existing models consider a single contest with an exogenously given set of players, in our model several contests compete for participants. As a consequence, prizes not only induce incentive effects but also participation effects. We show that contests that aim to maximize players' aggregate effort will award their entire prize budget to the winner. In contrast, multiple prizes will be awarded in contests that aim to maximize participation and the share of the prize budget awarded to the winner increases in the contests' randomness. We also provide empirical evidence for this relationship using data from professional road running. In addition, we show that prize structures might be used to screen between players of differing ability.

JEL *classification*: D44, J31, D82.

Keywords: Contests, allocation of prizes, participation, incentives, screening

*We are especially grateful to Michele Piccione and Alan Manning for their guidance and support. We also thank Jean–Pierre Benoit, Michael Bognanno, Jordi Blanes i Vidal, Pablo Casas–Arce, Vicente Cuñat, Maria–Ángeles de Frutos, Christopher Harris, Nagore Iriberri, Belen Jerez, Michael Peters and participants at various seminars and conferences for valuable discussions and suggestions.

†Department of Economics and Business, University Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. Email: ghazala.azmat@upf.edu, Tel.: +34-93542-1757, Fax: +34-93542-1766.

‡Department of Economics, University Carlos III Madrid, Calle Madrid 126, 28903 Getafe Madrid, Spain. Email: mmoller@eco.uc3m.es, Tel.: +34-91624-5744, Fax: +34-91624-9329.

1 Introduction

The world is full of contests. In investment banks, financial analysts compete for promotions; in architectural competitions, architects contend for design contracts; and in sports contests, athletes compete for prize–money. How many hierarchical levels should an investment bank implement? How should an architectural competition be set up? And how should a sports contest distribute its prize–budget across ranks?

A recent literature has provided some answers to these type of questions by considering the implications of the design of a contest on the players' incentives to exert effort. A common assumption in this literature is that the set of players is given exogeneously. However, as potential participants often have to choose between several contests, the set of players itself might be influenced by the contest's design. For example, an investment banker may decide to work for bank A rather than bank B because it offers a steeper hierarchical structure and, in turn, a fast–tracking career. Due to differences in contest rules, an architect may be reluctant to devote his time and effort to the design proposal for the World Trade Center Site Memorial and instead participate in the design competitions for the London 2012 Olympic Park. Similarly, a marathon runner may enter the New York Marathon instead of the Chicago Marathon because it awards a larger fraction of its prize money to suboptimal performances. In this paper we study optimal contest design when contest designers have to provide contestants with both, incentives to exert effort *and* incentives to participate.

Referring to the recent contests for European 3G telecom licenses, Paul Klemperer (2002) notes that "a key determinant of success of the European telecom auctions was how well their designs attracted entry [...]". From the viewpoint of a contest designer, attracting entry is important. Some contests benefit directly from the participation of certain key–players or aim to maximize the total number of contestants. For example, architectural competitions greatly benefit from the mere presence of prominent architects and big–city marathons boost their media interest by securing the participation of elite runners.[1] Other contests aim to maximize participants' aggregate effort and

[1]In 2006 organizers of three major marathons went head to head in their bid for the world record holder, Paula Radcliffe, and the U.S. record holder, Deena Castor (see "Marathons: Top Races are Vying for the Elite Runners", *International Herald Tribune*, June 12, 2006).

will benefit from entry indirectly through its positive effect on aggregate effort. In this paper we study optimal contest design from both of these perspectives.

We consider a complete information model in which two contests compete for the participation of a given set of $N \geq 3$ risk–neutral players with linear costs of effort. Players make their contest choice simultaneously and these choices depend of the contests' allocation of prizes. Once the set of participants has been determined in each contest, players exert effort in order to win a prize. The contests' outcome depends on both, players' efforts and the impact of exogenous factors (i.e., the level of randomness). This level of randomness may vary substantially across different types of contest. For example, while randomness plays a small role in a chess competition, it is the main determinant in a poker tournament. Similarly the outcome of a labour market tournament might be more random in industries characterized by high volatility, e.g. the finanical market.

We begin our analysis by focusing on participation. We show that when organizers aim to maximize participation, in equilibrium, contests will award multiple identical prizes. We distinguish between three different types of contests: lotteries, where the contests' outcome is completely random; all pay auctions, where the contests' outcome depends only on effort; and imperfectly discriminating contests that depend on both, randomness and effort. Our results are very general, i.e. they hold for an arbitrary number of players, any number of prizes and easily generalize to models with more than two contests. Overall, we find that the equilibrium number of prizes is decreasing in the contests' randomness. This implies that contests in which the impact of exogeneous factors is more important will tend to offer a higher share of their prize budget to the winner.

Using data from professional road running, we provide empirical evidence for this relationship. We argue that as the race distance increases, the impact of exogeneous factors on the race outcome becomes more important.[2] We find that as the race distance increases, there is a monotonic increase in the ratio between first and second prize. For example, as the race moves from 5km to 42km, the ratio between the first

[2]While in a 5km race the prediction of the winner based on past performance turns out to be correct in 43% of the cases, this number reduces to 20% for a marathon. For details see Section 6.

and the second prize increases by 4 percentage points. We find qualitatively similar results when using alternative measures of competitiveness and after controlling for various important factors.

When turning our attention to the maximization of players' aggregate effort, we find that awarding multiple prize rather than a single first prize has two effects. It *directly* decreases effort for a given set of participants (incentive effect) and it *indirectly* increases effort through its positive effect on participation (participation effect). Under the assumptions of our model we show that the incentive effect outweighs the participation effect. Hence when contest designers aim to maximize players' aggregate effort, in equilibrium contests will award their entire prize budget to the winner.

Our theory can provide an explanation for the significant differences in prize structures observed in reality. Contests that aim to maximize aggregate effort, as it is the case in science and engineering competitions, are likely to implement the winner–takes–all principle.[3] In contrast, when participation itself is important, for example in sports contests, multiple prizes will be awarded.

This paper also shows that prize structures might be used to screen players of differing ability. When players are heterogeneous a contest might want to select the most able players. We show that high ability players are more likely to enter contests with steep prize structures than low ability players. This insight is especially important in labour market settings where firms aim to attract the most productive workers. It shows that firms with steep hierarchies can be expected to have a higher quality workforce.

This paper is the first to model how contests compete for participants. The existing literature on contest design has focused on single contests with an exogenously given set of participants. In their seminal paper, Moldovanu and Sela (2001) show that the optimal allocation of prizes depends critically on the shape of players' cost of effort functions. Multiple prizes become optimal when the costs of effort are sufficiently convex. Multiple prizes have also been justified and derived from players' risk aversion (Krishna

[3]The NASA 2007 Astronaut Glove Competition awards a single first prize of $250000 to the designer of the best performing glove. Similarly, the DARPA (Defense Advanced Research Projects Agency) 2005 Grand Challenge awarded $2 million to the fastest driverless car on a 132–mile desert course.

and Morgan (1998)) and players' heterogeneity (Szymanski and Valletti (2005)) but under the restrictive assumption that the number of players is small ($N \leq 4$). Other papers provide arguments for the use of a single (Clark and Riis (1998b), Glazer and Hassin (1988)) or large (Rosen (2001)) first prize or few prizes (Barut and Kovenock (1998)). In addition, issues considered by this literature include simultaneous versus sequential designs (Clark and Riis (1998a)), the splitting of a contest into sub–contests (Moldovanu and Sela (2007)) and optimal seeding in elimination tournaments (Groh et al. (2008)).

Although some papers endogenize the set of participants, they maintain the focus on a single contest. Taylor (1995) and Fullerton and McAfee (1999), for example, study how the set of participants, and hence the expected winning performance, in a research tournament varies with its entry fee.

Competition for participants has attracted some attention in the literature on auction and mechanism design. McAfee (1993), Peters and Severinov (1997) and Burguet and Sakovics (1999) for example, consider models in which auctions compete for bidders. However, while in our model contests compete via their prize allocation, in these papers, prizes are fixed and auctions compete by using their reservation price. More related, Moldovanu et al. (2008) consider quantity competition between two auction sites. Although their model is different in its setup it shares a common feature with ours. In the same way in which in our model contests increase participation by awarding multiple prizes (at the cost of undermining incentives), in their model auctions increase the number of bidders by raising their supply (at the cost of lowering prices).

Finally, the literature on labor tournaments is also relevant here. Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Mookherjee (1984) have shown that the introduction of some form of contest among workers could provide optimal incentives to exert effort inside a firm. While Green and Stokey (1983) and Mookherjee (1984) take the set of workers as exogenously given, Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) assume a competitive labor market in which each firm hires a fixed number of workers. While in these papers each worker faces a fixed number of opponents, our results are driven by the fact that a player's set of opponents itself depends on the contest design.

The paper is organized as follows. In Section 2 we describe the theoretical model. Section 3 considers the case where contest designers aim to maximize participation while Section 4 contains our results about aggregate effort. In Section 5 we consider the possibility of screening. Section 6 tests the predictions of Section 3 using data on professional road running. Section 7 concludes. Some proofs and all empirical tables are contained in the Appendix.

2 The model

We consider two contests, $i \in \{1, 2\}$, and $N \geq 3$ players. Apart from possible differences in the allocation of prizes, contests are homogeneous. We assume that contests face the same budget V. Contests must choose how to distribute their budget across ranks. In particular, contest i chooses a *prize structure*, i.e. a vector of non–negative real numbers $v_i = (v_i^1, v_i^2, \ldots, v_i^N)$ such that v_i^m is (weakly) decreasing in m and $\sum_{m=1}^N v_i^m = 1$. The m'th prize awarded by contest i has the value $v_i^m V$. Note that in order to focus on the participation effects implied by a contest's prize structure we rule out the possibility that contests pay participants for attendance. Our results remain valid when we allow for attendance pay (see discussion at the end of Section 3). It will become clear that the contests' competition in prize structures resembles price competition a la Bertrand. As a consequence our results generalize to an arbitrary number of contests.

In models with a single contest it has been shown that it may be optimal to award second prizes when players are heterogeneous (Szymanski and Valletti (2005)), risk averse (Krishna and Morgan (1998)), or have convex costs of effort (Moldovanu and Sela (2001)). In order to identify competition for participants as the reason for the emergence of multiple prizes, we instead assume that players are identical, risk neutral, and have linear costs of effort.[4]

Each player can participate in, at most, one of the two contests because of time or other resource constraints. In each contest, participants exert effort in order to win a prize. A player who enters contest i, exerts effort $e_n \geq 0$, and wins the m'th prize,

[4]In Section 5 we allow players to differ in their marginal cost of effort.

receives the payoff $U_n^i = v_i^m V - Ce_n$.

The parameter $C > 0$ denotes the players' constant marginal cost of effort. Assuming that players have a zero outside option we can normalize, without a loss of generality, by setting $V = C = 1$.

The timing is as follows. First, contests simultaneously choose their prize structures. We denote the subgame, which starts after contests have announced the prize structures v_1 and v_2, as the (v_1, v_2) entry game. Second, players simultaneously decide which contest to enter.[5] Third, players simultaneously choose their effort levels.

In general a contest's outcome might depend on players' efforts and on exogenous/random factors. We will use the parameter r to measure the relative importance of these two factors. For $r = \infty$ the contests' outcome is determined entirely by players' efforts. In this case, the player with the highest effort wins the first prize, the player with the second highest effort wins the second prize, and so on. For $r = 0$ the contests' outcome is completely random. Here, every player is equally likely to win any of the prizes irrespective of his effort choice. In order to determine the contests' outcome in the intermediate case, $0 < r < \infty$, where both, players efforts and exogenous factors play a role, we employ Tullock's (1980) widely used contest success function (see Skaderpas (1996) for axiomatization and Nti (1997) for properties). In particular, letting $\mathcal{N}_i$ denote contest i's set of participants and N_i its cardinality, prizes in contest i are distributed as follows. The probability that player $n \in \mathcal{N}_i$ wins the first prize v_i^1 is given by

$$p_n^1 = \frac{e_n^r}{\sum_{k \in \mathcal{N}_i} e_k^r}. \tag{1}$$

Conditional on player m winning the first prize, player n wins the second prize v_i^2 with probability

$$p_{n|m}^2 = \frac{e_n^r}{\sum_{k \in \mathcal{N}_i - \{m\}} e_k^r}. \tag{2}$$

[5]While our results remain unchanged when contests are allowed to choose their prize structure sequentially, the assumption that entry takes place simultaneously is important as it rules out coordination. Note however that when entry is sequential contests have an incentive to conceal the entry of earlier players from later players. Hence our results remain valid under sequential entry as long as players cannot communicate with each other.

Hence the (unconditional) probability that player n wins the second prize is given by

$$p_n^2 = \sum_{m \in \mathcal{N}_i - \{n\}} p_m^1 p_{n|m}^2. \tag{3}$$

Note that for $0 < r < \infty$ each player wins the contest with positive probability and this probability is increasing in his own effort and decreasing in the efforts of his rivals. Also note that the importance of the level of randomness in determining the contests' outcome is decreasing in r.

As participation is assumed to be costless, players prefer to participate in *some* contest rather than to not participate at all. Player n will therefore enter contest 1 with probability $q_n(v_1, v_2) \in [0, 1]$ and contest 2 with probability $1 - q_n(v_1, v_2)$. As players are identical we restrict our attention to the symmetric equilibria of the entry game, where $q_n(v_1, v_2) = q^*(v_1, v_2)$ for all players.

While players always choose contests and effort in order to maximize their expected payoff, with respect to the contest organizers we will distinguish between two objectives. In Section 3 we consider the case where organizers aim to maximize expected participation, while in Section 4 we turn out attention to the maximization of expected aggregate effort.

3 Participation

Contests need to attract participants, without participants there is no contest. Participation increases (aggregate) incentives and often raises contests' revenues directly. For example, the design proposal of a famous architect, once realized, will attract tourists to the building/city that staged the architectural contest. Similarly sports contests will yield higher media revenues if they are able to secure the participation of star athletes.

From a more theoretical perspective, the incentive effects of a prize structure for a given set of players have been well understood. However, the participation effects of a prize structure when the set of players is endogeneous have not been considered so far. In this section we therefore concentrate on participation by assuming that contest organizers set prize structures in order to maximize the expected number of participants. In particular, contest 1 chooses v_1 to maximize $Nq^*(v_1, v_2)$, while contest 2 chooses v_2 to maximize $N(1 - q^*(v_1, v_2))$.

The probability $q^*(v_1, v_2)$ with which players enter contest 1 in equilibrium will be derived as follows. We first consider the effort choice for all players $n \in \mathcal{N}_i$ participating in contest i given the prize structure v_i. This allows us to determine a player's expected payoff in contest i conditional on contest i having N_i participants, $E[U_n^i|N_i]$. Next, assuming that all players enter contest 1 with the same probability q, we can then obtain a player's expected payoffs from entering contest 1 or contest 2, respectively:

$$E[U_n^1] = \sum_{m=1}^{N} \tbinom{N-1}{m-1} q^{m-1}(1-q)^{N-m} E[U_n^1|m] \tag{4}$$

$$E[U_n^2] = \sum_{m=1}^{N} \tbinom{N-1}{m-1} (1-q)^{m-1} q^{N-m} E[U_n^2|m]. \tag{5}$$

With a few exceptions $q^*(v_1, v_2)$ will be the unique solution of the equation

$$\Delta(q) \equiv E[U_n^1] - E[U_n^2] = 0. \tag{6}$$

The rest of this section derives the equilibrium prize structure for different contest forms. We begin by looking at the two extreme cases where the contests' outcome are completely random or determined entirely by players' efforts. We end the section by allowing for both effort and randomness to affect the contests' outcome. The main insight of this section is that a decrease in the contests' randomness leads to an increase in the number of prizes awarded in equilibrium and to a decrease in the share awarded to the winner.

3.1 Lotteries: $r = 0$

We start by considering the extreme case, $r = 0$, where the contests' outcome is completely random. Given a prize structure v_i let $\bar{v}_i(m) = \frac{1}{m}\sum_{m'=1}^{m} v_i^{m'}$ denote the average of the m highest prizes. In contest i each player $n \in \mathcal{N}_i$ is equally likely to win any of the prizes $v_i^1, \ldots, v_i^{N_i}$, irrespective of the players' efforts while the prizes $v_i^{N_i+1}, \ldots, v_i^N$ will remain unawarded. Hence in equilibrium all players will exert zero effort and player n's expected payoff in contest i conditional on contest i having N_i participants is

$$E[U_n^i|N_i] = \bar{v}_i(N_i) \tag{7}$$

for all $n \in \mathcal{N}_i$. Our first result characterizes the players' equilibrium contest choice for $r = 0$.

Lemma 1 *Consider the case $r = 0$. If $v_i \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ for some $i \in \{1, 2\}$ then the (v_1, v_2) entry game has a unique symmetric equilibrium $q^*(v_1, v_2)$. The expected number of participants in contest i is strictly larger than in contest j if and only if $P^0(v_i) > P^0(v_j)$ where $P^0(v) \equiv \sum_{m=1}^{N} \binom{N-1}{m-1} \bar{v}(m)$.*

Proof: Note that $\Delta(\frac{1}{2}) = \frac{1}{2^{N-1}}(P^0(v_1) - P^0(v_2))$. If $v_1 \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}) \neq v_2$ then Δ is strictly decreasing in q with $\Delta(0) = v_1^1 - \frac{1}{N} > 0$ and $\Delta(1) = \frac{1}{N} - v_2^1 < 0$. Hence $\Delta(q^*) = 0$ defines a unique symmetric equilibrium $q^* \in (0, 1)$. Moreover, $q^* > (<) \frac{1}{2}$ if and only if $\Delta(\frac{1}{2}) > (<) 0$. If $v_1 = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}) \neq v_2$ then $\Delta(\frac{1}{2}) < 0$ and $q^* = 0$ while for $v_1 \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}) = v_2$ it holds that $\Delta(\frac{1}{2}) > 0$ and $q^* = 1$. ■

To understand the intuition for this result consider the following example. Suppose that contest 1 has a more competitive prize structure than contest 2, in the sense that it concentrates a larger fraction of its prize budget towards low ranks, i.e. $\bar{v}_1(m) \geq \bar{v}_2(m)$ for all $m \in \{1, 2 \ldots, N\}$ with strict inequality for some m. Hence $P^0(v_1) > P^0(v_2)$ and Lemma 1 implies that the more competitive contest expects a higher number of participants. Players prefer the more competitive prize structure as it maximizes the prize money that will be shared (equally) when the number of participants turns out to be small. Note that this result depends crucially on the fact that players are assumed to be risk neutral. The immediate implication of Lemma 1 is that contests have an incentive to choose a more competitive prize structure than their rival. Hence we get the following result:

Proposition 1 *Consider the case $r = 0$. In equilibrium both contests award their entire prize budget to the winner, i.e. $v_1 = v_2 = (1, 0, \ldots, 0)$.*

Proof: The prize structure v^* that maximizes $P^0(v)$ is unique and $v^* = (1, 0, \ldots, 0)$. Hence Lemma 1 implies that in equilibrium $v_1 = v_2 = v^*$ and both contests expect the same number of participants, i.e. $q^*(v_1, v_2) = \frac{1}{2}$. ■

The idea is that, in the same way as firms undercut each others prices in order to increase the demand for their goods, contests try to take participants from each other by choosing competitive prize structures. The resulting Bertrand style competition leads to "winner–takes–all" contests. Note that the existence of *several* contests is crucial for this result. In a model with a single contest, players would be completely indifferent with respect to different prize structures and expect a payoff of $\frac{1}{N}$.

One example where extremely competitive prize structures are observed in reality are investment banks' promotional contests. A recent review of financial packages at Wall Street approximated that the average entry level annual payments to an analysts was $150,000 while a top Managing Director could receive as much as $8 million (including bonuses), making the "top prize" more than 53 times the "lowest prize".[6] It is often argued that a financial analyst's career path in an investment bank's promotional contest is marked by uncertainty. For example, Nassim Taleb (2005), a legend amongst option traders, states that "[...] one can make money in the financial market totally out of randomness." So far the literature on contest design has explained the steep hierarchies found in the financial industry by referring to the firms' attempt to provide employees with optimal incentives to exert effort. In the light of Proposition 1 they can be seen as a consequence of investment banks' competition in the labour market. Those firms that offer the steepest hierarchies and the fast track careers attract the most graduates from top ranked MBA programs.

3.2 All Pay Auctions: $r = \infty$

We now move on to consider the case $r = \infty$ where the contests' outcome is determined entirely by the players' efforts. An important example for this type of contest is an all–pay auction. In an all–pay auction all bidders pay their bids and then the goods are allocated according to the ranking of bids. In the literature on contest design, all–pay

[6] "How much am I worth? M&A banker, leading investment bank, Wall Street", *Institutional Investor Magazine*, 2006.

auctions have been frequently used as a modelling device (see for example Moldovanu and Sela (2001 and 2006)).

Barut and Kovenock (1998) have characterized the equilibria of an all–pay auction with identical risk–neutral players and several not necessarily identical prizes. Their results apply here. If contest i has a single participant, i.e. if $N_i = 1$, then he will exert zero effort and win the first prize v_i^1 with certainty. For $N_i = 2$ the equilibrium depends on v_i. If $v_i^1 = v_i^2$ then both players will exert zero effort and obtain the payoff v_i^2. Otherwise there is a unique equilibrium in which both players choose their efforts randomly from the interval $[0, v_i^1 - v_i^2]$ and obtain the expected payoff $E[U_n^i|N_i] = v_i^2$. More generally Barut and Kovenock (1998) show that in a contest with N_i participants each player $n \in \mathcal{N}_i$ obtains the expected payoff

$$E[U_n^i|N_i] = v_i^{N_i}. \tag{8}$$

Our next lemma characterizes the players' equilibrium contest choice for $r = \infty$.

Lemma 2 *Consider the case $r = \infty$. If $v_i \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ for some $i \in \{1, 2\}$ then the (v_1, v_2) entry game has a unique symmetric equilibrium $q^*(v_1, v_2) \in (0, 1)$. The expected number of participants in contest i is strictly larger than in contest j if and only if $P^\infty(v_i) > P^\infty(v_j)$ where $P^\infty(v) \equiv \sum_{m=1}^{N} \binom{N-1}{m-1} v^m$.*

Proof: Note that $\Delta(\frac{1}{2}) = \frac{1}{2^{N-1}}(P^\infty(v_1) - P^\infty(v_2))$. If $v_i \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ for some $i \in \{1, 2\}$ then Δ is strictly decreasing in q with $\Delta(0) = v_1^1 - v_2^N > 0$ and $\Delta(1) = v_1^N - v_2^1 < 0$. Hence $\Delta(q^*) = 0$ defines a unique symmetric equilibrium $q^* \in (0, 1)$. Moreover, $q^* > (<) \frac{1}{2}$ if and only if $\Delta(\frac{1}{2}) > (<) 0$. ■

Note that when contest 1 chooses to award its entire prize budget to the winner, it holds that $P^\infty(v_1) - P^\infty(v_2) < 0$ for all $v_2 \neq (1, 0, \ldots, 0)$. Hence Lemma 2 implies that for $r = \infty$ a winner–takes–all contest attracts less participation than any other contest.

The important question that arises is why, contrary to the case where $r = 0$, do players prefer contests that award several prizes when $r = \infty$? Note that for $r = \infty$, competition amongst players is very strong. In order to win a contest a player has to exert more effort than his rivals and all possible gains in prize money are spent in the form of effort costs. Hence players prefer contests that mitigate this competition by awarding a positive fraction of their prize budget to higher ranks. If players prefer several prizes to a single first prize then we must ask, what is the prize structure that is most attractive to participants? Our next result characterizes the equilibrium prize structure for $r = \infty$.

Proposition 2 *Consider the case $r = \infty$. Let $N^* = \frac{N+1}{2}$ if N is odd and $N^* = \frac{N+2}{2}$ if N is even. In equilibrium both contests will award N^* identical prizes i.e. $v_1^m = v_2^m = \frac{1}{N^*}$ for $m = 1, \ldots, N^*$ and $v_1^m = v_2^m = 0$ for $m = N^* + 1, \ldots, N$.*

Proof: Lemma 2 implies that in equilibrium both contests will choose the prize structure v^* that maximizes $P^\infty(v)$. Note that for N odd the binomial coefficient $\binom{N-1}{m-1}$ increases in m for all $m < N^*$, is maximized at $m = N^*$, and decreases for all $m > N^*$. Hence v^* is the prize structure that maximizes v^{N^*} and is as specified in Proposition 2. The argument for N even is similar. ■

To understand the intuition for this result consider for example the case where $N = 7$. As a contest can always immitate the prize structure of the other contest, in equilibrium both contests have to expect an equal number of participants. Hence in equilibrium players enter both contests with equal probability $q^* = \frac{1}{2}$. The likelihood that a player who enters contest i finds himself in a contest with m participants expecting the payoff v_i^m is given by $\frac{1}{2^6}\binom{6}{m-1}$ and is maximized for $m = 4$. Hence players prefer the prize structure which maximizes v_i^4 and in equilibrium both contests will choose $v_i = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, 0)$.

Proposition 2 shows that competition for participants is able to explain the use of multiple, identical prizes. One example where participation is of utmost importance are TV shows that try to attract telephone participation by its audience. These shows frequently promise a certain number of identical prizes to those participants who manage to call first.

3.3 Imperfectly Discriminating Contests: $r \in (0, \infty)$

We now show that the insights we obtained above extend to contests where outcomes depend on both, players' efforts and exogenous/random factors. In particular we will consider the case $r \in (0, \infty)$ and show that in equilibrium contests will award multiple prizes if and only if r is sufficiently large. To see this suppose that contests award first and second prizes only, i.e. $v_i = (v_i^1, 1 - v_i^1, 0, \ldots, 0)$. If contest i has a single participant, i.e. if $N_i = 1$ then he will exert zero effort and win the first prize v_i^1 with certainty, i.e. $E[U_n^i|N_i] = v_i^1$. For $N_i \geq 2$ each player $n \in \mathcal{N}_i$ who participates in contest i chooses effort e_n in order to solve

$$\max_{e_n \geq 0} \left[p_n^1(e_n, e_{-n}) v_i^1 + p_n^2(e_n, e_{-n})(1 - v_i^1) - e_n \right]. \tag{9}$$

A symmetric pure startegy equilibrium can be derived by calculating the first order condition and substituting $e_n = e^*$ for all $n \in \mathcal{N}_i$. We find that

$$e^* = \frac{r}{N_i} \left(\frac{N_i - 1}{N_i} - \frac{1 - v_i^1}{N_i - 1} \right) \tag{10}$$

and in equilibrium each player $n \in \mathcal{N}_i$ expects the payoff

$$E[U_n^i|N_i] = \frac{1}{N_i} \left(1 - r \left(\frac{N_i - 1}{N_i} - \frac{1 - v_i^1}{N_i - 1} \right) \right). \tag{11}$$

Note that this equilibrium is unique and it exists if $r \leq \frac{N_i}{N_i - 1}$.[7] Our next result provides a necessary and sufficient condition under which the contest with the steeper prize structure expects the higher number of participants.

Lemma 3 *Suppose that* $0 < r \leq \frac{N}{N-1}$ *and* $v_i = (v_i^1, 1 - v_i^1, 0, \ldots, 0)$ *with* $v_1^1 > v_2^1$. *In the unique symmetric equilibrium of the* (v_1, v_2) *entry game the expected number of participants in contest* 1 *is strictly smaller (larger) than in contest* 2 *if and only if* $r > (<)\ \bar{r}$ *where*

$$\bar{r} \equiv \left(\sum_{m=1}^{N-1} \frac{\binom{N-1}{m}}{m(m+1)} \right)^{-1} \in (0, 1). \tag{12}$$

[7] It is well known from the literature on rent seeking with a single prize ($v_i^1 = 1$) that for $r > 1$ a symmetric pure strategy equilibrium might fail to exist. See Perez–Castrillo and Verdier (1992) for details.

Proof: $\Delta(q)$ is strictly decreasing in q with $\Delta(0) = v_1^1 - E[U_n^2|N] > 0$ and $\Delta(1) = E[U_n^1|N] - v_2^1 < 0$. Hence $\Delta(q^*) = 0$ defines a unique symmetric equilibrium. Note that $\Delta(\frac{1}{2}) = 0$ if and only if $r = \bar{r}$. Also note that

$$\frac{\partial \Delta}{\partial r}|_{q=\frac{1}{2}} = \frac{1}{2^{N-1}} \sum_{m=1}^{N-1} \binom{N-1}{m} \frac{v_2^1 - v_1^1}{m(m+1)} < 0. \tag{13}$$

Hence $q^* < (>) \frac{1}{2}$ if and only if $r > (<) \bar{r}$. ■

Lemma 3 applies to the important cases where players' returns to effort are decreasing ($r < 1$) or constant ($r = 1$). Whether, in equilibrium, the more competitive contest 1 is more attractive to participants than the less competitive contest 2, depends on the parameter r. When the contests' outcome is sufficiently random ($r < \bar{r}$), contest 1 expects higher participation than contest 2. On the other hand, when the role played by players' efforts in the determination of the contests' outcome is sufficiently important ($r > \bar{r}$) then contest 2 attracts more participants.

To understand the intuition for this result note that players prefer steep prize structures when they happen to meet few opponents whereas they prefer flat prize structures when they happen to meet many opponents. A steeper prize structure raises the pie to be shared when the number of opponents turns out to be low but leads to stronger competition and hence lower payoffs when the number of opponents turns out to be high. As r increases, the contests' outcome becomes more sensitive to changes in players' efforts thereby increasing competition. Hence when r is sufficiently high, players prefer flat prize structures that will mitigate competition.

As the threshold $\bar{r}$ is independent of the prize structures v_1 and v_2, Lemma 3 has the following implication for the contests' equilibrium prize structures:

Proposition 3 *Suppose that contests cannot award more than two prizes. If $r \in (0, \bar{r})$ then in equilibrium both contests will award a single first prize, i.e. $v_1^* = v_2^* = (1, 0, \ldots, 0)$. If $r \in (\bar{r}, \frac{N}{N-1})$ then in equilibrium each contest will award two identical prizes, i.e. $v_1^* = v_2^* = (\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0)$.*

Proof: See Appendix 1. ■

Proposition 3 shows that the equilibrium prize structure depends on the contests' discriminatory power r. Contests in which the impact of exogenous factors is sufficiently important in determining the contest's outcome ($r < \bar{r}$), tend to award a single first prize while contests whose outcome is determined to a large extent by players' efforts ($r > \bar{r}$) will award several identical prizes. In the proof of Proposition 3 contained in Appendix 1 we show that this result remains valid when contests are allowed to award more than two prizes. For example when $N \geq 4$ and contests are allowed to award three prizes the equilibrium prize structure is $(1, 0, \ldots, 0)$ if $r < \bar{r}$, $(\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0)$ if $\bar{r} < r < \bar{\bar{r}}$, and $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots, 0)$ if $r > \bar{\bar{r}}$ where

$$\bar{\bar{r}} \equiv \frac{N-1}{2} \left(\sum_{m=2}^{N-1} \frac{\binom{N-1}{m}}{(m-1)(m+1)} - \frac{N-1}{4} \right)^{-1} > \bar{r}. \tag{14}$$

Note that the results of this section remain valid when contests are allowed to pay players for their attendance. To see this suppose that in an initial stage contests can approach individual players and offer attendance pay which players can either accept or reject. After this initial stage the timing is as specified in Section 2. In the subgame that starts after each contest has signed up $N^s \leq \frac{N}{2}$ players for a total attendance payment of $A \leq V$ competition in prize structures will take place as described in Propositions 1–3 if we substitute N by $N - 2N^s$ and the contests' prize budget is reduced to $V - A$.

In this section we have shown that when contests aim to maximize participation, competition in prize structures, due to its Bertrand style nature, leads to extreme outcomes. Contests either implement the winner–takes–all principle or award multiple identical prizes. In reality, contests are likely to care about factors other than participation, for example, they may be concerned about players' aggregate effort. However, the main mechanism identified in this section will still be present, such that contests in which exogenous/random factors play a larger role will tend to implement steeper prize structures. In Section 6 we provide empirical evidence for this relationship using data from professional road running.

4 Aggregate effort

Most of the literature on contest design aims to determine the prize structure that maximizes players' aggregate effort. While the existing literature assumes that the set of participants is exogeneously given, our analysis so far indicates that the allocation of prizes will not only influence the players' incentives to exert effort but also their incentives to participate. In Section 3 we have shown that second and higher order prizes, although harmful for incentives to exert effort, might increase a contest's participation. As for a given prize structure, aggregate effort is increasing in the number of participants second prizes might become optimal once participation is endogenous.

To show this more clearly, consider again the case where $0 < r \leq \frac{N}{N-1}$ and suppose that contest i, offering the prize structure $v_i = (v_i^1, 1-v_i^1, 0, \ldots, 0)$, has attracted $N_i \geq 2$ participants. From (10) it follows that in equilibrium, aggregate effort in contest i, Σe^i, is given by

$$\Sigma e^i = N_i e^* = r\left(\frac{N_i - 1}{N_i} - \frac{1 - v_i^1}{N_i - 1}\right). \tag{15}$$

Note that Σe^i increases in v_i^1. That is, for a given number of participants aggregate effort increases in the fraction of prize budget awarded to the winner. Ex post, once entry has taken place, aggregate effort would therefore be maximized by awarding a single first prize. This is the standard result of the literature and is not surprising here as players are identical, risk neutral and have linear costs of effort.

Also note however, that Σe^i increases in N_i. That is, aggregate effort increases in the number of participants. If players enter contest 1 with probability $q \in [0, 1]$ then expected aggregate efforts in contest 1 and contest 2 are given by

$$E[\Sigma e^1] = r\sum_{m=2}^{N} q^m (1-q)^{N-m} \tbinom{N}{m} \left(\frac{m-1}{m} - \frac{1 - v_1^1}{m-1}\right) \tag{16}$$

$$E[\Sigma e^2] = r\sum_{m=2}^{N} (1-q)^m q^{N-m} \tbinom{N}{m} \left(\frac{m-1}{m} - \frac{1 - v_2^1}{m-1}\right). \tag{17}$$

Our analysis in Section 3 has shown that in equilibrium q will depend on the contests' prize structures. Second prizes therefore have a direct and an indirect effect on aggregate effort. On the one hand, they decrease aggregate effort directly through their detrimental effect on incentives to exert effort for a given set of participants. On the other hand, they influence expected participation and thereby affect aggregate effort indirectly. For $r < \bar{r}$ the overall effect is immediate. In this case Lemma 3 has shown that second prizes decrease participation thereby leading to an overall reduction of expected aggregate effort. However, for $r > \bar{r}$ participation is increased by the use of second prizes and the overall effect might be an increase in expected aggregate effort. Our next result shows that this is not the case:

Proposition 4 *Suppose that* $0 < r \leq \frac{N}{N-1}$ *and* $v_i = (v_i^1, 1 - v_i^1, 0, \ldots, 0)$ *with* $v_1^1 > v_2^1$. *In the unique symmetric equilibrium of the* (v_1, v_2) *entry game expected aggregate effort is strictly higher in contest* 1, *i.e.* $E[\Sigma e^1] > E[\Sigma e^2]$.

Proof: See Appendix 1. ■

Proposition 4 shows that second prizes cannot be used to increase expected aggregate effort. The negative incentive effect of second prizes outweighs the possibly positive participation effect and the overall effect is a reduction in expected aggregate effort. Proposition 4 is important as it provides justification for the literature's focus on an exogeneously given set of participants. It shows that when players are homogeneous, risk neutral, and have linear costs of effort, winner–takes–all contests maximize aggregate incentives even when participation is endogenous.

5 Screening

The screening of workers of differing abilities has been an important theme in the literature on explicit incentive contracts. For example, Lazear (1986) has shown that firms might choose fixed salaries and piece rates in order to screen between workers of low and high productivity. However, the possibility of screening through compensation schemes that are based on relative performance has so far been ignored by this literature. Moreover, due to its focus on an exogeneously given set of participants, screening has not been considered in the literature on contest design. Hence whether contest organizers might employ steep prize structures in order to attract the most able participants is still an open question.

In this section we show that contests might indeed use their prize structure in order to screen between players of high and low ability. To keep the analysis simple we consider the case where $N = 3$ and $r = \infty$ but we expect our results to hold more generally.

As before we assume that players are identical ex ante. However, in an initial stage nature determines whether a player has high or low ability. Both types are equally likely and abilities are distributed independently across players. A low ability player's marginal cost of effort is $C_L = 1$ while for a high ability player it is $C_H = c \in (0, 1)$. A player's ability is private information but players learn their rivals' abilities after they have entered a contest but before they exert effort.

To see that prize structures might be used to screen players suppose that contest 1 offers a single first prize, i.e. $v_i = (1, 0, 0)$ while contest 2 offers two identical prizes, i.e. $v_2 = (\frac{1}{2}, \frac{1}{2}, 0)$. Consider contest 1 and suppose that N_1 players have entered. Baye, Kovenock, and de Vries (1996) have characterized the equilibria of a single prize all–pay auction allowing for asymmetries amongst players. Their results apply here. If contest 1 has a single participant, $N_1 = 1$ then he exerts zero effort and wins the first prize with certainty so that $E[U_H^1|N_1] = E[U_L^1|N_1] = 1$. For $N_1 = 2$ there is a unique equilibrium which depends on the players' abilities. When players have identical abilities, both players randomize uniformly over $[0, \frac{1}{C}]$ and $E[U_H^1|N_1] = E[U_L^1|N_1] = 0$. When players differ in abilities then $E[U_H^1|N_1] = 1 - c$ and $E[U_L^1|N_1] = 0$. Finally for $N_1 = 3$ we have $E[U_H^1|N_1] = 1 - c$ and $E[U_L^1|N_1] = 0$ if one player has high ability and two players have low ability. Otherwise $E[U_H^1|N_1] = E[U_L^1|N_1] = 0$.

Now consider contest 2 and suppose that N_2 players have entered. Clark and Riis (1998) have characterized the unique equilibrium of an all–pay auction with multiple identical prizes allowing for asymmetries amongst players.[8] If $N_2 < 3$ effort will be zero and each player will win a prize with certainty so that $E[U_H^2|N_2] = E[U_L^2|N_2] = \frac{1}{2}$. For $N_2 = 3$ the equilibrium depends on players' abilities. If all players have the same ability then $E[U_H^2|N_2] = E[U_L^2|N_2] = 0$. Otherwise $E[U_H^2|N_2] = \frac{1-c}{2}$ and $E[U_L^2|N_2] = 0$.

We are now prepared to derive the equilibrium in the (v_1, v_2) entry game. A player's

[8] A complete characterization of equilibrium in an all–pay auction with multiple non–idential prizes and asymmetric players is still to be found. For a first step in this direction see Cohen and Sela (2008).

strategy consists of the probabilities, $q_H \in [0,1]$ and $q_L \in [0,1]$, of entering contest 1 conditional on having high or low ability. As players are identical ex ante, we restrict our attention to *symmetric* Bayesian Nash equilibria i.e. we assume that q_H and q_L are the same for all players. A low ability player's expected payoff from entering contest 1 is then given by

$$U_L^1 = 1 \cdot (1-q)^2. \tag{18}$$

where $q \equiv \frac{q_H+q_L}{2}$ denotes a player's ex ante probability of entering contest 1. Choosing contest 2 instead he expects

$$U_L^2 = \frac{1}{2} \cdot [q^2 + (1-q_L)q + (1-q_H)q] = \frac{1}{2}q(2-q). \tag{19}$$

Note that a low ability player's choice between contest 1 and contest 2 only depends on the expected *total* number of rivals determined by q. As low ability players expect positive payoffs only when the number of players in a contest fails to exceed the number of prizes, their preferences are independent of the distribution of abilities given by q_H and q_L. A high ability player's expected utility from entering contest 1 is given by

$$U_H^1 \quad = \quad (1-q)^2 + (1-c)[q_L(1-q) + \frac{1}{4}q_L^2]. \tag{20}$$

Choosing contest 2 instead he expects

$$U_H^2 = \frac{1}{2}[q^2 + (1-q_L)q + (1-q_H)q] + \frac{1}{2}(1-c)[\frac{1}{4}(1-q_L)^2 + \frac{1}{2}(1-q_H)(1-q_L)]. \tag{21}$$

Our next result shows that in equilibrium the one prize contest is more attractive to high ability players than the two prize contest.

Proposition 5 *Suppose that* $N = 3$*,* $r = \infty$*,* $v_1 = (1,0,0)$ *and* $v_2 = (\frac{1}{2}, \frac{1}{2}, 0)$*. In the unique symmetric equilibrium of the* (v_1, v_2) *entry game high and low ability players enter contest* 1 *with probability* $q_H^* = \frac{1}{3}(5-\sqrt{10}) \approx 0.61$ *and* $q_L^* = \frac{1}{3}(1-2\sqrt{3}+\sqrt{10}) \approx 0.23$ *respectively.*

Proof: See Appendix 1. ■

Proposition 5 suggests that contests might use their prize structure to screen players. Players sort (partly) according to their abilities. High ability players are more likely to enter contests with steep prize structures than low ability players. As a consequence contests which aim to attract the most able players will tend to implement the winner–takes–all princple. This result is particularly relevant in a labour market setting. It implies that firms with steep hierarchies will attract the most productive workers.

6 Empirical Framework

The theory outlined in the previous sections predicts a positive relationship between the impact of exogenous factors on a contest's final outcome (i.e. the level of randomness) and a contest's competitiveness (i.e. the share of prize budget awarded to the winner). A similar relationship has been shown to exist in the labor tournament models of Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). However, while our theory is based on the contests' competition for participants, these models focus on the maximization of players' aggregate effort. Given that firms need to attract workers *and* provide incentives to exert effort, both aspects can be expected to be present in labor tournament data. Indeed, using firm level data, Eriksson (1999) finds that the dispersion of pay between job levels is greater in firms which operate in noisy environments. In order to abstract from the competing aspects of these models, we use sports data, i.e. professional road running, where the provision of incentives to exert effort is less of an issue then it is in firm level data. Given their dependence on media interest and sponsor support, sports contests typically strive to attract the most famous athletes. Sports data therefore provides the perfect framework to test our theory. Running data works particularly well as running contests are organised at a disaggregate or "firm" level instead of being governed by a federation as it is for example the case for tennis and golf.

Sports contests tend to be invariably rank ordered and the measurement of individual performance is generally straight forward, making sports data increasingly fashionable to test contest theory. Nevertheless, the empirical literature on contest design is scarce and the few papers that do exist test, whether prize levels and prize differentials have incentive effects. For example, Ehrenberg and Bognanno (1990a, b) use individual player and aggregate event data from US and European Professional Golf Associations to test whether prizes affect players' performance. For a recent review of the literature that uses sports data to test contest theory, see Frick (2003).

There are two papers that share our focus on professional road running. Both papers seek to test the hypothesis that prize structures affect finishing times. Maloney and McCormick (2000) use 115 foot races with different distances in the US and find that the average prize and prize spread have negative effects on the finishing times. Lynch and Zax (2000) use 135 races and also find that finishing times are faster in races offering higher prize money. However, Lynch and Zax conclude that the effect is not due to the provision of stronger incentives but rather a result of sorting of runners according to abilities. Once fixed effects are controlled for, the incentive effect disappears. This finding supports our projection that in professional road running, the provision of incentives to exert effort is less important than the attraction of the most able runners.

In order to provide empirical evidence for the positive relationship between a contest's level of randomness and its competitiveness, we have collected a dataset containing 368 road running contests. Road running contests differ in their race course but are (almost) identical with regard to their organisational set–up. We use the distance of a race as a measure of race randomness and argue that longer races are more likely to be affected by exogenous factors (and so have a higher level of randomness) than shorter races.[9]

6.1 Race distance as a measure of randomness

There are three strands of support for the assertion that longer races have a higher level of randomness. Firstly, the longer the race, the stronger the influence of external factors (e.g. weather conditions, race course profile, nutrition) on the runners' performance. This was evident during the 2004 Olympic Games in Athens. In the women's marathon the highly acclaimed world recorder holder, Paula Radcliffe, was predicted to win.

[9]To combat the concern that very short races may be quite random as they are often decided by millisecond differences, we restrict our analysis to distances of 5km or more.

However, after a consistent lead, at the 23rd mile mark, Paula stopped and sat crying on the side path suffering the symptoms of heat exhaustion.

Secondly, the longer the race, the less accurate is the estimate of a runner's ability based on past performance as longer races are run less frequently. For example, although it is possible to run a 5km race each week, elite runners typically restrict themselves to two marathons per year (see Noakes (1985)).

Finally, there exists statistical evidence showing that longer races have a higher level of randomness than shorter races. This evidence has been kindly provided to us by Ken Young, a statistician at the "Association of Road Racing Statisticians" (www.arrs.net). Using a data set containing more than 500,000 performances, Ken Young predicts the outcome of several hundred road running contests of varying distances between 1999 and 2003. As an example, Table 1 in the Appendix reports his results for the Men's races in 1999.[10]

Two distinct methods were used to predict the winner of a given race. A regression based handicapping (HA) evaluation attempts to predict each runner's finishing time based on past performance. The predicted time was assumed to be normally distributed for each runner and the numerical integration yielded the probability that each runner would win the race. The second method was a Point Level (PL) evaluation based on a rating system similar to the Elo system in chess or the ATP ranking in tennis, in which runners take points from runners they beat and lose points to runners they are beaten by.

Averaging over 274 Men's races with distances between 5km and 42km, the PL prediction of the winner was correct in 43% of the "Short" races (distance $\leq$ 10km), 41% of the "Medium" races (10km $<$ distance $<$ 42km) and 20% of the Long races (distance $\geq$ 42km). For the HA prediction the numbers are 45%, 46%, and 21% respectively. Hence, while Short and Medium distance races are similar in terms of randomness, Long distance races appear to be much more random.

[10]The complete set of results is available on http://www.econ.upf.edu/azmat/.

6.2 Data Description

The empirical investigation is done using data on professional road running from the Road Race Management Directory (2004). This Directory provides a detailed account of the prize structures, summaries, invitation guidelines, and contacts for almost 500 races. It is an important source of information for elite runners planning their race season. With the exception of a few, most of the races took place in the United States. The event listings are arranged in chronological order beginning in April 2004 and extend through to April 30th 2005. In our analysis we only include races that have at least $600 in prize money and a race distance greater or equal to 5km, leaving us with 368 races. The Directory provides us with information on the event name, event date, city, state and previous year's number of participants. The prize money information includes the total amount of prize money as well as the prize money breakdown. We focus on the Men's races by including only the Men's prize money distributions.

The Directory contains further information that may influence runners' race selection. In particular, it includes data on whether a race was a championship, took place on a cross country or mountain course, and the race's winning performance in the previous year. In order to make finishing times in races over different distances comparable with each other, we use the Riegel formula (see Riegel (1981)) to calculate 10km equivalent finishing times.[11]

Finally, given that the weather conditions play a role in the outcome of an outdoor race, we collect information on the weather using an internet site called Weatherbase (www.weatherbase.com). We can get information on the average temperature and average rainfall in the month that the race takes place.[12] Table 2 presents the summary statistics for three race distance categories: "Short" (distance $\leq$ 10km); "Medium" (10km $<$ distance $<$ 42km) and "Long" (distance $\geq$ 42km). In general, races tend to be clustered, the most frequent being 5km, 10km, 16km, 21km and 42km. Most runners specialize and run either Short or Long distance races, while Medium distance

[11]This formula predicts an athlete's finishing time t in a race of distance d on the basis of his finishing time T in a race of distance D as $t = T(\frac{d}{D})^{1.06}$. It is used by the IAAF to construct scoring tables of equivalent athletic performances.

[12]We also collected data on average wind speed. However, the data was incomplete. Our results remain the same with and without conditioning on the average wind speed.

races are run by both types.

From the summary statistics in Table 2 we see that there are some obvious differences between the three distance categories. In particular, the mean total prize money (in US$) increases as the distance increases ($2,990, $5,664 and $23,207, respectively).[13] The average number of participants also increases with distance (3,359, 5,268 and 5,324, respectively). It is important to note that although the "size" of these contests increases with distance, typically the number of elite runners is similar.[14] In addition, we do not worry about congestion affects in the populated races because elite runners will run separately (and typically before) the non-elite runners.

There is consistency in the weather variables when we look across the race types. In addition, there is a similar probability that the race has a championship status and the average Riegel measure of performance is almost identical. This is reassuring as it implies that the "quality" of runners is independent of the race distance.

6.3 Analysis

To obtain estimates for the differences in prize structure, we estimate the following compensation equations using 368 men's races:

$$Y_i = \alpha + \beta D_i + \varepsilon_i. \tag{22}$$

Y_i represents the competitiveness of the prize structure and D_i denotes the distance (and acts as our measure of randomness) for race i. We use various measures of competitiveness Y: (1) a concentration index (C. I.), similar to the Herfindahl-Hirschman index, calculated from the top three prizes, i.e. $Y = \frac{(1st)^2+(2nd)^2+(3rd)^2}{(1st+2nd+3rd)^2}$, (2) the ratio between first and second prize, (3) the ratio between first and third prize and (4) the

[13] We use the sum of the top 10 Men's prizes as the "total prize money". This variable is more important for the race choice of elite Men's runners than the race's total prize budget as prize money that is to be distributed to Women's or Age–group runners is not accessible to them. For comparison of prize money across countries, we convert all prizes into US dollars using monthly historical exchange rates for 2004–2005 (www.gocurrency.com).

[14] Our participation data contains elite and non–elite runners. Unfortunately the number of elite runners was unavailable. Our participation variable therefore only gives a rough measure for the popularity of the event amongst elite runners.

ratio between first prize and total prize money. We expect these measures to increase with the race distance.

For the distance D we use a continuous measure, i.e. a km by km increase in distance, as well as a comparison between Short, Medium and Long distance races. As mentioned earlier, races tend to be clustered and so it is more informative to look at how the prize structure changes when we compare each group. In doing so, we can estimate the percentage point change in competitiveness when going from Short to Medium or Long races.

We report the results for all four measures of competitiveness, using the two different distance measures in Table 3. Overall, the results support the hypothesis that as the distance increases, the prize structure becomes steeper. In particular, using our concentration index we observe that as the distance of a race increases by 1km, there is a 0.1% increase in competitiveness. This implies, for example, that the prize structure of a marathon is almost 4% more concentrated towards the first prize than the prize structure of a 5km race. Similarly, we find that when the race changes from being Short to Long, there is a 3.2% increase in the concentration index. The coefficient of moving from Short to Medium is positive but insignificant. This is reassuring, as with Ken Young's analysis these races had a similar degree of randomness.

When we look at the other measures of competitiveness, we observe very similar patterns. In particular, we find that as the distance increases, the gap between the first prize and the second or third prize widens. When the distance increases by 1km, there is a 0.1% rise in the ratio between the first and the second or third prize. When we look across different race types, we see that the ratio between the first and second prize increases by 3.0%, while the ratio between the first and the third prize increases by 2.5% when moving from Short to Long. The proportion of total prize money that goes to the winner also increases with the distance but results are not significant.

Next, we extend the analysis of looking at the simple correlation to account for various factors that may affect runners' race selection and hence the prize structure. In particular, we may be concerned that the popularity of a certain race in the world of running may be important. For example, if the race is a championship race or if it offers a fast race course (where records can be established) then the race may be attractive for elite runners, irrespective of its prize structure. In addition, weather conditions may play a role. We control for these factors by estimating the following equation:

$$Y_i = \alpha + \beta D_i + \delta X_i + \varepsilon_i. \tag{23}$$

X includes average temperature, average rainfall, an indicator identifying whether the race was a championship, the number of race participants, total prize money, the 10km Riegel equivalent of the previous edition's winning time and an indicator for whether the race is a cross country or a mountain race. It is reassuring to see that the results remain very similar to the results without controls. In fact, as we can see in Table 4, the coefficients for all of the prize structure measures and both measures of distance are almost identical with and without controls.

When we look at the effect that the control variables have on competitiveness, it is only the average rainfall in the month of the race that has a consistently significant negative effect on the spread of prizes. However, neither the significance nor the size of the coefficients have been affected by including controls.

7 Conclusion

The optimal allocation of prizes has been a dominant theme of the recent literature on contest design. Existing models have determined the prize structure that maximizes aggregate effort for an exogeneously given set of participants. In this paper we have allowed the set of participants itself to depend on the contest's design. In most real world examples, several contests compete for a common set of potential participants. As a consequence prizes not only affect players' incentives to exert effort but also their incentives to participate.

While the existing literature has struggled to explain the wide spread occurrence of multiple prizes in our model multiple prizes arise naturally from the contests' need to attract participants. We therefore consider our theory as complementary to the one that focuses exclusively on the provision of incentives.

Appendix 1: Proofs

Proof of Proposition 3

Proposition 3 follows directly from Lemma 3 and the fact that $\bar{r}$ is independent of the prize structures v_1 and v_2. Here we show that the main insight of Proposition 3 remains valid when contests are allowed to award more than two prizes. In particular, we consider the case where contests can distribute their prize budget between three prizes. For a higher number of prizes the proof is similar although more tedious.

Suppose that contest i has chosen the prize structure $v_i = (v_i^1, v_i^2, v_i^3, 0, \ldots, 0)$ and $N_i \geq 3$ players participate. Conditional on player l winning the first prize and player m winning the second prize, player $n \in \mathcal{N}_i$ wins the third prize v_i^3 with probability

$$p^3_{n|lm} = \frac{e_n^r}{\sum_{k \in \mathcal{N}_i - \{l,m\}} e_k^r}. \tag{24}$$

Hence the (unconditional) probability that player n wins the third prize is given by

$$p_n^3 = \sum_{l,m \in \mathcal{N}_i - \{n\}, l \neq m} p_l^1 p^2_{m|l} p^3_{n|lm}. \tag{25}$$

where p_l^1 and $p^2_{m|l}$ are as defined in (1) and (2) respectively. Each player $n \in \mathcal{N}_i$ chooses effort e_n in order to solve

$$\max_{e_n \geq 0} \left[p_n^1(e_n, e_{-n}) v_i^1 + p_n^2(e_n, e_{-n}) v_i^2 + p_n^3(e_n, e_{-n}) v_i^3 - e_n \right]. \tag{26}$$

A symmetric pure startegy equilibrium can be derived by calculating the first order condition and substituting $e_n = e^*$ for all $n \in \mathcal{N}_i$. We find that

$$e^* = \frac{r}{N_i} \left[\frac{N_i - 1}{N_i} v_i^1 + \left(\frac{N_i - 1}{N_i} - \frac{1}{N_i - 1} \right) v_i^2 + \left(\frac{N_i - 3}{N_i - 2} - \frac{1}{N_i - 1} - \frac{1}{N_i} \right) v_i^3 \right] \tag{27}$$

and in equilibrium each player $n \in \mathcal{N}_i$ expects the payoff

$$E[U_n^i | N_i] = \frac{1}{N_i} - e^*. \tag{28}$$

Note that this equilibrium is unique and it exists if $r \leq \frac{N_i}{N_i - 1}$. From our earlier analysis we have $E[U_n^i|N_i] = v_i^1$ for $N_i = 1$ and $E[U_n^i|N_i] = (\frac{1}{2} - \frac{r}{4})v_i^1 + (\frac{1}{2} + \frac{r}{4})v_i^2$ for $N_i = 2$. In equilibrium contest 1 expects a strictly higher (lower) number of participants than contest 2 if and only if $\Delta(\frac{1}{2}) > (<)\ 0$ where $\Delta(q)$ is as defined in (6). Hence in equilibrium contests will choose v^1, v^2 and v^3 to maximize $P^r(v^1, v^2, v^3) = \alpha(r)v^1 + \beta(r)v^2 + \gamma(r)v^3$ where

$$\alpha(r) = 1 + (N-1)(\frac{1}{2} - \frac{r}{4}) - r\sum_{m=2}^{N-1} \binom{N-1}{m} \frac{m}{(m+1)^2} \quad (29)$$

$$\beta(r) = (N-1)(\frac{1}{2} + \frac{r}{4}) - r\sum_{m=2}^{N-1} \binom{N-1}{m} (\frac{m}{(m+1)^2} - \frac{1}{m(m+1)}) \quad (30)$$

$$\gamma(r) = -r\sum_{m=2}^{N-1} \binom{N-1}{m} \frac{1}{m+1} (\frac{m-2}{m-1} - \frac{1}{m} - \frac{1}{m+1}) \quad (31)$$

Note that $\beta(r) > \alpha(r)$ if and only if $r > \bar{r}$. Moreover $\gamma(r) > \beta(r)$ if and only if $N \geq 4$ and $r > \bar{\bar{r}}$ where $\bar{\bar{r}}$ is as defined in (14). As P^r is linear in its arguments this implies that in equilibrium contests will choose $v_i^1 = 1$ if $0 < r < \bar{r}$. The equilibrium will be $v_i^1 = v_i^2 = \frac{1}{2}$ if $N = 3$ and $r > \bar{r}$ or if $N \geq 4$ and $\bar{r} < r < \bar{\bar{r}}$. Finally contests will choose $v_i^1 = v_i^2 = v_i^3 = \frac{1}{3}$ if $N \geq 4$ and $r > \bar{\bar{r}}$. ■

Proof of Proposition 4

Define $\delta(q) \equiv E[\Sigma e^1] - E[\Sigma e^2]$. δ is strictly increasing in q with $\delta(0) = r(\frac{1-v_2^1}{N-1} - \frac{N-1}{N}) < 0$ and $\delta(\frac{1}{2}) = \frac{r}{2^N}\sum_{m=2}^{N}\binom{N}{m}\frac{v_1^1 - v_2^1}{m-1} > 0$. Hence there exists a unique $q^e < \frac{1}{2}$ such that $\delta(q^e) = 0$. For $r \leq \bar{r}$ Lemma 3 has shown that $q^*(v_1, v_2) \geq \frac{1}{2}$ which implies that $\delta(q^*(v_1, v_2)) > 0$. Hence suppose that $r > \bar{r}$ which implies that $q^*(v_1, v_2) < \frac{1}{2}$. Suppose that all players enter contest 1 with probability q^e so that expected aggregate effort is the same in each contest. Some algebra shows that $\Delta(q^e) > 0$ which implies that $q^e < q^*(v_1, v_2)$ and hence $\delta(q^*(v_1, v_2)) > 0$. ■

Proof of Proposition 5

Step 1: $U_L^1 - U_L^2$ is strictly decreasing in q with $U_L^1 - U_L^2 = 0 \Leftrightarrow q = 1 - \frac{1}{\sqrt{3}}$. Suppose that $q_L > 2 - \frac{2}{\sqrt{3}}$. Then $q > 1 - \frac{1}{\sqrt{3}}$ so that low abilities strictly prefer contest 2. Hence in any equilibrium it has to hold that $q_L^* \leq 2 - \frac{2}{\sqrt{3}}$.

Step 2: $U_H^1 - U_H^2$ strictly decreases in q_L and q_H. For $q_L = 0$ we have $U_H^1 - U_H^2 = 0 \Leftrightarrow q_H = \bar{q}_H \equiv \frac{1}{3}(5 + c - \sqrt{10 + c(1+c)})$. In any equilibrium it thus has to hold that $q_H \leq \bar{q}_H$.

$\bar{q}_H$ is strictly increasing in $c \in (0,1)$ with $\lim_{c\to 1} \bar{q}_H = 2 - \frac{2}{\sqrt{3}}$. Hence in any equilibrium $q_H^* < 2 - \frac{2}{\sqrt{3}}$.

Step 3: Suppose that there exists an equilibrium in which $q_L = 0$. From Step 2 it follows that $q = \frac{q_H}{2} < 1 - \frac{1}{\sqrt{3}}$. Hence Step 1 implies that $U_L^1 - U_L^2 > 0$, a contradiction. Hence in equilibrium it has to hold that $q_L^* > 0$.

Step 4: Step 1 and Step 3 together imply that in any equilibrium low ability players mix with $q_L = 2 - \frac{2}{\sqrt{3}} - q_H$. For $q_H = 0$ and $q_L = 2 - \frac{2}{\sqrt{3}}$ one finds $U_H^1 - U_H^2 = \frac{5}{8}(1-c) > 0$. Hence in any equilibrium it holds that $q_H^* > 0$.

Steps 1-4 imply that in equilibrium low and high ability players have to be indifferent between entering contest 1 and entering contest 2. Hence the equilibrium (q_L^*, q_H^*) solves the system of equations $U_H^1 = U_H^2$ and $U_L^1 = U_L^2$. The solution is unique and $(q_H^*, q_L^*) = (\frac{1}{3}(5 - \sqrt{10}), \frac{1}{3}(1 - 2\sqrt{3} + \sqrt{10}))$. ■

Appendix 2: Empirical Tables

Table 1: Ken Young's prediction of Men's winner (1999)

Date	Race Name	Distance (km)	HA Prob	HA WP	PL CI	PL WP
3/5/1999	IAAF World Indoor Champs (JPN)	3.0	80	1	796	1
3/5/1999	NCAA Indoor Champs (IN/USA)	5.0	70	2	398	1
3/6/1999	Gate River Run (FL/USA)	15.0	78	1	434	4
3/6/1999	NCAA Indoor Champs (IN/USA)	3.0	78	1	458	4
3/14/1999	Los Angeles (CA/USA)	42.2	54	2	650	4
3/27/1999	Azalea Trail (AL/USA)	10.0	97	1	432	1
4/11/1999	Cherry Blossom (DC/USA)	16.1	43	3	677	4
4/17/1999	Stramilano (ITA)	21.1	80	1	864	1
4/18/1999	Rotterdam (HOL)	42.2	71	2	709	1
4/19/1999	Boston (MA/USA)	42.2	37	2	727	4
4/25/1999	Sallie Mae (DC/USA)	10.0	66	2	728	1
5/2/1999	Pittsburgh (PA/USA)	42.2	47	1	355	4
5/16/1999	Volvo Midland Run (NJ/USA)	16.1	59	4	376	6
5/16/1999	Bay to Breakers (CA/USA)	12.0	50	3	676	1
5/31/1999	Bolder Boulder (CO/USA)	10.0	25	9	673	13
6/2/1999	NCAA Champs (ID/USA)	10.0	35	5	379	5
6/4/1999	NCAA Champs (ID/USA)	5.0	78	1	456	1
6/12/1999	Stockholm (SWE)	42.2	47	2	433	1
6/19/1999	Grandma's (MN/USA)	42.2	40	2	392	2
6/27/1999	Fairfield (CT/USA)	21.1	60	6	572	4
7/4/1999	Peachtree (GA/USA)	10.0	68	1	831	2
7/4/1999	Golden Gala (ITA) 5000m	5.0	52	1	1003	1
7/17/1999	Crazy 8's (TN/USA)	8.0	60	5	714	2
7/25/1999	Wharf to Wharf (CA/USA)	9.7	75	1	673	3
7/31/1999	Quad-Cities Bix (IA/USA)	11.3	88	1	767	1
8/15/1999	Falmouth (MA/USA)	11.3	72	2	845	2
8/21/1999	Parkersburg (WV/USA)	21.1	57	1	338	1
8/24/1999	IAAF World Champs (ESP)	10.0	74	2	960	1
8/28/1999	IAAF World Champs (ESP)	5.0	68	1	992	2
8/28/1999	IAAF World Champs (ESP)	42.2	7	5	699	18
9/3/1999	Ivo Van Damme (BEL)	10.0	42	13	843	12
9/26/1999	Berlin (GER)	42.2	57	1	586	1
10/24/1999	Chicago (IL/USA)	42.2	66	1	752	1
11/7/1999	New York City (NY/USA)	42.2	57	1	702	9
12/5/1999	California International (CA/USA)	42.2	12	8	378	11

Data kindly provided by Ken Young, Association of Road Racing Statisticians. For the handicapping (HA) evaluation, "HA Prob" denotes the probability with which the predicted winner was expected to win and "HA WP" reports the placing he actually obtained. Using a Point Level (PL) system the average rating for the five highest ranked runners in the race was compared to the average rating for the ten highest ranked runners in the world at the time of the race in order to construct the competition index (CI). The higher the index the better the quality of the field. The column "PL WP" reports the actual placing obtained by the highest ranked runner.

Table 2: Descriptive statistics

	Short (Distance $\leq$ 10km)				
Variable	Observations	Mean	Std. Dev.	Min	Max
Rain (cm)	175	7.29	3.19	0	17
Temperature (^{o}C)	175	21.44	6.65	3	33
Championship	175	0.07	0.26	0	1
Total (US $)	175	2,989.68	6,389.31	125	60,000
Size	175	3,358.98	7,324.30	18	55,000
Riegel 2003 (Sec)	175	1,821.77	96.91	1,647	2,276
Trail	175	0.01	0.11	0	1
	Medium (10km $<$ Distance $<$ 42km)				
Variable	Observations	Mean	Std. Dev.	Min	Max
Rain (cm)	97	7.06	2.92	0	16
Temperature (^{o}C)	97	20.27	6.00	0	32
Championship	97	0.12	0.33	0	1
Total (US $)	97	5,664.21	10,011.01	175	70,000
Size	97	5,267.56	10,671.53	100	80,000
Riegel 2003 (Sec)	97	1,849.14	203.44	1,653	3,056
Trail	97	0.05	0.22	0	1
	Long (Distance $\geq$ 42km)				
Variable	Observations	Mean	Std. Dev.	Min	Max
Rain (cm)	96	6.49	3.25	0	16
Temperature (^{o}C)	95	18.44	5.13	9	33
Championship	96	0.11	0.32	0	1
Total (US $)	96	23,206.75	46,878.09	225	270,000
Size	96	6,277.22	8,707.80	100	46,000
Riegel 2003 (Sec)	96	1,883.97	180.92	1,629	2,628
Trail	96	0.03	0.17	0	1

Notes: Means and standard deviations for each race distance category, “Short”, “Medium” and “Long”, respectively. “Championship” refers to whether or not the race held a championship title. “Total” is the total amount of the prize budget (all values are expressed in real US dollars evaluated at monthly historical exchange rate for 2004-2005). “Size” refers to the number of contestants in the race. “Riegel 2003” calculates the 10km equivalent race finishing times. “Trail” refers to whether the race took place on a cross country or mountain course.

Table 3: Prize structure without controls

PANEL A				
	Measures of Competitiveness			
Dependent Variable	C. I.	1:2	1:3	1:Total
Distance (km)	0.0008	0.0008	0.0007	0.0006
	[0.0003]**	[0.0002]**	[0.0002]**	[0.0004]
Constant	0.394	0.6241	0.7321	0.4511
	[0.0075]**	[0.0058]**	[0.0063]**	[0.0102]**
Observations	368	368	368	368

PANEL B				
	Measures of Competitiveness			
Dependent Variable	C. I.	1:2	1:3	1:Total
Medium	0.0119	0.0115	0.0112	0.0162
	[0.0113]	[0.0086]	[0.0094]	[0.0153]
Long	0.0316	0.0299	0.025	0.0245
	[0.0113]**	[0.0087]**	[0.0094]**	[0.0153]
Constant	0.3989	0.6292	0.7358	0.4531
	[0.0067]**	[0.0052]**	[0.0056]**	[0.0091]**
Observations	368	368	368	368

Notes: Standard errors are in parentheses. (*) and (**) represent significance at the 95 and 99 percent level. Omitted group in Panel B is Short Distance.

Table 4: Prize structure with controls

PANEL A

Dep. Var.	C. I.	C. I.	1:2	1:2	1:3	1:3	1:Total	1:Total
	Measures of Competitiveness							
Dist.	0.0008	0.0007	0.0008	0.0008	0.0007	0.0006	0.0006	0.0004
	[0.0003]**	[0.0003]*	[0.0002]**	[0.0003]**	[0.0002]**	[0.0003]*	[0.0004]	[0.0004]
Rain		-0.0059		-0.0037		-0.0037		-0.008
		[0.0015]**		[0.0011]**		[0.0012]**		[0.0019]**
Temp.		0.0011		0.0007		0.0005		0.0005
		[0.0007]		[0.0006]		[0.0006]		[0.0010]
Champ.		0.0006		-0.0034		-0.0039		-0.0206
		[0.0156]		[0.0121]		[0.0131]		[0.0203]
Prize		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000]†
Size		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000]
Riegel		0.0001		0		0.0001		0.0002
		[0.0000]		[0.0000]		[0.0000]†		[0.0001]**
Trail		-0.0148		-0.0067		-0.0398		-0.0991
		[0.0351]		[0.0272]		[0.0294]		[0.0456]*
Const.	0.394	0.302	0.6241	0.6191	0.7321	0.6413	0.4511	0.0984
	[0.0075]**	[0.0726]**	[0.0058]**	[0.0562]**	[0.0063]**	[0.0609]**	[0.0102]**	[0.0943]
Obs.	368	368	368	368	368	368	368	368

PANEL B

Dep. Var.	C. I.	C. I.	1:2	1:2	1:3	1:3	1:Total	1:Total
	Measures of Competitiveness							
Medium	0.0119	0.011	0.0115	0.0109	0.0112	0.0112	0.0162	0.0163
	[0.0113]	[0.0112]	[0.0086]	[0.0087]	[0.0094]	[0.0094]	[0.0153]	[0.0145]
Long	0.0316	0.0291	0.0299	0.0289	0.025	0.0233	0.0245	0.0183
	[0.0113]**	[0.0122]*	[0.0087]**	[0.0094]**	[0.0094]**	[0.0102]*	[0.0153]	[0.0158]
Rain		-0.006		-0.0038		-0.0038		-0.0081
		[0.0015]**		[0.0011]**		[0.0012]**		[0.0019]**
Temp.		0.0011		0.0007		0.0005		0.0006
		[0.0007]		[0.0006]		[0.0006]		[0.0010]
Champ.		0		-0.0037		-0.0043		-0.0215
		[0.0156]		[0.0121]		[0.0131]		[0.0203]
Prize		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000]†
Size		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000]
Riegel		0.0001		0		0.0001		0.0002
		[0.0000]†		[0.0000]		[0.0000]*		[0.0000]**
Trail		-0.0163		-0.0092		-0.0424		-0.1021
		[0.0351]		[0.0273]		[0.0295]		[0.0456]*
Const.	0.3989	0.3021	0.6292	0.6168	0.7358	0.6389	0.4531	0.096
	[0.0067]**	[0.0726]**	[0.0052]**	[0.0564]**	[0.0056]**	[0.0610]**	[0.0091]**	[0.0944]
Obs.	368	368	368	368	368	368	368	368

Notes: Standard errors are in parentheses. (†), (*) and (**) represent significance at the 90, 95 and 99 percent level, respectively. For description of dependent variables see Table 2.

References

[1] Barut, Y., Kovenock, D. (1998) The Symmetric Multiple Prize All–Pay Auction with Complete Information. *European Journal of Political Economy* **14**(4), 627-644.

[2] Baye, M., R., Kovenock, D., de Vries, C., G. (1996) The All–Pay Auction with Complete Information. *Economic Theory* **8**, 291-305.

[3] Burguet, R., Sákovics, J. (1999) Imperfect Competition in Auction Designs. *International Economic Review* **40**(1), 231-247.

[4] Clark, D., J., Riis, C. (1998a) Competition over more than one Prize. *American Economic Review* **88**, 276-289.

[5] Clark, D., J., Riis, C. (1998b) Influence and the Discretionary Allocation of Several Prizes. *European Journal of Political Economy* **14**, 605-625.

[6] Cohen, C., Sela, A. (2008) Allocation of Prizes in Asymmetric All–Pay Auctions. *European Journal of Political Economy*, forthcoming.

[7] Ehrenberg, R., G, Bognanno, M., L. (1990a) Do Tounaments have Incentive Effects? *Journal of Political Economy* **98** (6), 1307-1324.

[8] Ehrenberg, R., G, Bognanno, M., L. (1990b) The Incentive Effects of Tournaments Revisted: Evidence from the European PGA Tour. *Industrial and Labor Relations Review* **43** (6), 74-88.

[9] Eriksson, T. (1999) Executive Compensation and Tournament Theory: Empirical Tests on Danish Data. *Journal of labor Economics* **17** (2), 262-280.

[10] Frick, B. (2003) Contest Theory and Sport? *Oxford Review of Economic Policy* **19** (4), 512-529.

[11] Fullerton, R., L., McAfee, R., P. (1999) Auctioning Entry into Tournaments. *Journal of Political Economy* **107**(3), 573-605.

[12] Glazer, A., Hassin, R. (1988) Optimal Contests. *Economic Inquiry* **26**, 133-143.

[13] Green, J., Stokey, N. (1983) A Comparison of Tournaments and Contracts. *Journal of Political Economy* **91**, 349-364.

[14] Groh, C., Moldovanu, B., Sela, A., Sunde, U. (2008) Optimal Seedings in Elimination Tournaments. *Economic Theory*, forthcoming.

[15] Klemperer, P. (2002) How (not) to run auctions: The European 3G telecom auctions. *European Economic Review* **46**, 829-845.

[16] Krishna, V., Morgan, J. (1998) The Winner–Take–All Principle in Small Tournaments. *Advances in Applied Microeconomics* **7**, 61-74.

[17] Lazear, E., P. (1986) Salaries and Piece Rates. *Journal of Business* **59**(3), 405-431.

[18] Lazear, E., P., Rosen, S. (1981) Rank–Order Tournaments as Optimal Labor Contracts. *Journal of Political Economy* **89**(5), 841-864.

[19] Lynch, J., Zax, J., S. (2000) The Rewards to Running: Prize Structure and Performance in Professional Road Racing. *Journal of Sports Economics* **1**(4), 323-340.

[20] McAfee, R., P. (1993) Mechanism Design by Competing Sellers. *Econometrica* **61**, 1281-1312.

[21] Maloney, M., T., McCormick, R., E. (2000) The Response of Workers to Wages in Tornaments: Evidence from Foot Races. *Journal of Sports Economics* **1**(2), 99-123.

[22] Moldovanu, B., Sela, A. (2001) The Optimal Allocation of Prizes in Contests. *American Economic Review* **91**(3), 542-558.

[23] Moldovanu, B., Sela, A. (2006) Contest Architecture. *Journal of Economic Theory* **126**(1), 70-97.

[24] Moldovanu, B., Sela, A., Shi, X. (2007) Contests for Status. *Journal of Political Economy* **115**, 338-363.

[25] Moldovanu, B., Sela, A., Shi, X. (2008) Competing Auctions with Endogenous Quantities. *Journal of Economic Theory*, forthcoming.

[26] Mookherjee, D. (1984) Optimal Incentive Schemes with Many Agents. *Review of Economic Studies* **51**, 433-446.

[27] Nalebuff, B., J., Stiglitz, J., E. (1983) Prizes and Incentives: Towards a General Theory of Compensation and Competition. *Bell Journal of Economics* **14**(1), 21-43.

[28] Noakes, T., D. (1985) *Lore of Running*, Oxford: Oxford University Press.

[29] Nti, K., O. (1997) Comparative Statics of Contests and Rent–Seeking Games. *International Economic Review* **38**, 43-59.

[30] Perez–Castrillo, D., Verdier, T. (1992) A General Analysis of Rent–Seeking Games. *Public Choice* **73**, 335-350.

[31] Peters, M., Severinov, S. (1997) Competition among Sellers Who Offer Auctions Instead of Prices. *Journal of Economic Theory* **75**, 114-179.

[32] Rosen, S. (2001) Prizes and Incentives in Elimination Tournaments. *American Economic Review* **76**(4), 701-715.

[33] Skaderpas, S. (1996) Contest Success Functions. *Economic Theory* **7**, 283-290.

[34] Szymanski, S., Valletti, T. (2005) Incentive Effects of Second Prizes. *European Journal of Political Economy* **21**, 467-481.

[35] Taleb, N., N. (2005) *Fooled by Randomness: The Hidden Role of Chance in Life and in the Markets*, New York: Random House.

[36] Taylor, C., R. (1995) Digging for Goldon Carrots: An Analysis of Research Tournaments. *American Economic Review* **85**(4), 872-890.

[37] Tullock, G. (1980) Efficient Rent Seeking. in: J. Buchanan et al. (Eds), *Towards a Theory of the Rent–Seeking Society*, Texas A&M University Press, College Station.