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The Compensation Principle and the National Income Test

Aldo Montesano

Abstract

According to a common opinion in economic literature, the National Income Test provides a necessary condition for potential Pareto dominance. This paper demonstrates that this statement is true in pure-exchange economies, but, in general, false in production economies.

KEYWORDS: compensation principle, national income test, potential welfare, potential pareto dominance

1 Introduction

Comparisons between allocations or situations (defining situations as economies with the same set of consumers) are useful in economics since we want to know (i) whether an allocation or situation is socially better than another, and (ii) how to test for it. Pareto dominance is a widely accepted tool for comparing allocations. However, this criterion induces an incomplete social preference system, since it is substantially based on unanimity. The need to compare alternatives when Pareto dominance does not apply gave rise to a vast literature. The Pareto dominance criterion does not apply if there are both gainers and losers in the comparison; the valuation of gains and losses can then be done in terms of utilities, assuming interpersonal utility comparisons are possible, or in terms of goods (or wealth), via potential compensations between individuals.

The (Kaldor-Hicks-Scitovsky-Samuelson-Chipman-Moore) *Compensation Principle* allows for comparisons between allocations without introducing any interpersonal utility comparisons. It is a generalization of the Pareto dominance criterion obtained by taking into account the possibility of Pareto dominance through reallocations. A standard tool for comparing two allocations in terms of potential Pareto dominance is the *National Income Test.* The link between potential Pareto dominance and National Income Test could provide an answer to whether – and to what extent – society appreciates an increase in national income.

An excellent historical presentation of this kind of analyses is provided by Chipman and Moore (1976) and Chipman (2007), who take under consideration the compensation principle, the National Income Test[1], and the cost-benefit analysis.

The National Income Test can be used both to check whether an allocation is Pareto optimal and to make comparisons between two allocations. Barone (1908), following some Pareto's hints, proposed the National Income Test to check Pareto optimality. He basically showed that if an allocation a is Pareto optimal and p is its shadow price vector, then the national income evaluated at prices p is

[1] The only addition to this presentation, with regard to the National Income Test, could be recalling the controversy between Wieser and Böhm-Bawerk. Wieser's thesis (1893, pp. 32-36) is that choice maximizes both utility and, at least for practical purposes, value (measured by summing prices times quantities of goods). Böhm-Bawerk (1921, vol. II, Exkurs VII, pp. 127-161) says that the true meaning of value is utility (so that there is only one maximization problem). The controversy between Wieser and Böhm-Bawerk is indicated, rather briefly, by Stigler (1950, p.316, footnote 51), who specifically mentions the so-called "paradox of value". This paradox emerges when we use marginal utilities as prices, so that marginal utility times quantity may decrease when quantity (and utility) increases, *i.e.* we can have $x' > x$, thus $u(x') > u(x)$, and $p'x' < px$, where $p = \mathrm{D}_x u(x)$ and $p' = \mathrm{D}_x u(x')$.

maximized by allocation a.[2] While this result is quite uncontroversial, many problems arise from the National Income Test when we compare allocations belonging to different situations.

Intuitively, when we compare consumption allocations $(x_i)_{i\in N}$ and $(x_i')_{i\in N}$, the National Income Test is based on the observation that if the value of aggregate consumption increases, (*i.e.* $\sum_{i\in N} px_i' > \sum_{i\in N} px_i$, using the competitive equilibrium prices that support allocation $(x_i)_{i\in N}$) then there exists a redistribution of consumption allocation $(x_i')_{i\in N}$ that increases the wealth of every consumer (*i.e.* there is an allocation $(\hat{x}_i')_{i\in N}$ with $\sum_{i\in N} \hat{x}_i' \le \sum_{i\in N} x_i'$ and $p\hat{x}_i' > px_i$ for all i) and thus makes all consumers better off. However, the increase in wealth is not a sufficient condition for generating an increase in utility. (In order to understand why it is insufficient, recall the Croesus paradox: imagine an economy with two goods, corn and gold, and consider an allocation with a sufficient amount of corn and a small quantity of gold, so that the price of gold is high in equilibrium; then introduce an alternative allocation with very small quantity of corn and large quantity of gold, so that wealth increases but welfare plummets.) Even if the National Income Test does not provide a sufficient condition, it provides a necessary condition. In other words, if $\sum_{i\in N} px_i' \le \sum_{i\in N} px_i$, then there exists no reallocation of $(x_i')_{i\in N}$ that Pareto dominates $(x_i)_{i\in N}$. In fact, every reallocation of $(x_i')_{i\in N}$ that increases the wealth of at least one consumer leaves at least one consumer with a smaller wealth (*i.e.* for every $(\hat{x}_i')_{i\in N}$ with $\sum_{i\in N} \hat{x}_i' \le \sum_{i\in N} x_i'$ and $p\hat{x}_i' > px_i$ for some i, there is at least another i with $p\hat{x}_i' < px_i$) and, because the indirect utility is assumed to be an increasing function of wealth, also with smaller

[2] Formally, let $a = \left((x_i)_{i\in N}, (y_j)_{j\in M}\right)$ be a Pareto optimal allocation and (p, a) a competitive equilibrium, where N is the set of consumers, $(x_i)_{i\in N}$ is the consumption allocation, M is the set of producers, and $(y_j)_{j\in M}$ is the production allocation. Then, $p\sum_{i\in N} x_i = \max_{\sum_{i\in N} x_i' \in Y+\{\omega\}} p\sum_{i\in N} x_i'$, where $\sum_{i\in N} x_i$ is the aggregate consumption, Y is the aggregate production set (*i.e.* $Y = \sum_{j\in M} Y_j$ and $y_j, y_j' \in Y_j$), and ω is the resource vector. We can prove this proposition taking under consideration the profit maximization condition, *i.e.* $p\sum_{j\in M} y_j \ge p\sum_{j\in M} y_j'$ for every $\sum_{j\in M} y_j' \in Y$, and the feasibility condition, *i.e.* $\sum_{j\in M} y_j = \sum_{i\in N} x_i - \omega$, which imply $p\sum_{i\in N} x_i \ge p(\sum_{j\in M} y_j' + \omega)$ for every $(\sum_{j\in M} y_j' + \omega) \in Y + \{\omega\}$.

utility (*i.e.* if $p\hat{x}_i' < px_i$, then $u_i(\hat{x}_i') \le u_i^*(p, p\hat{x}_i') < u_i^*(p, px_i) = u_i(x_i)$ where $u_i^*(p, w_i) = \max_{px_i \le w_i} u_i(x_i)$ is the indirect utility function).

According to this reasoning, the National Income Test provides a necessary condition for the *Potential-Pareto-Dominance.*[3] However, this reasoning overlooks production as no reallocation of production is taken into account (in our reasoning we considered only changes of the consumption allocation $(x_i')_{i \in N}$, leaving production allocation unchanged). What happens when production reallocations are taken into account? This paper aims at demonstrating that, in this case, the National Income Test does not provide necessary condition for Potential-Pareto-Dominance. Consequently, for production economies the National Income Test provides neither sufficient nor necessary condition for Potential-Pareto-Dominance.

2 Definitions and notation

Consider an economy with *n* consumers, *m* producers and *k* goods:

$$\mathcal{E} = (X_i \subset \mathbb{R}^k \text{ and } u_i : X_i \to \mathbb{R}, i \in N; Y_j \subset \mathbb{R}^k, j \in M; \omega \in \mathbb{R}^k_+).$$

Consumers are indicated by $i \in N = \{1, ..., n\}$, producers by $j \in M = \{1, ..., m\}$, and goods by $h \in K = \{1, ..., k\}$. Consumers' preferences are represented by utility functions $u_i : X_i \to \mathbb{R}$ for all $i \in N$. An allocation $a = ((x_i)_{i \in N}, (y_j)_{j \in M})$ is attainable for the economy $\mathcal{E}$ if $x_i \in X_i$, where X_i is the consumption set of the *i*-th consumer, $y_j \in Y_j$, where Y_j is the production set of the *j*-th producer, and $\sum_{i \in N} x_i - \sum_{j \in M} y_j \le \omega$,[4] taking into account that the positive (negative) components of x_i and the negative (positive) components of y_j are inputs (outputs). The utilities generated by an allocation are represented by a vector $u(a) = (u_i(x_i))_{i \in N}$.

Then, the following sets can be introduced:
the set of the possible allocations of economy $\mathcal{E}$:

[3] For instance, Ruiz-Castillo (1987), summarizing Chipman's and Moore's results, says, p. 35: "It is true that under very general conditions, an increase in national income according to the Laspeyres criterion is a necessary condition for an improvement in potential welfare in the second situation". Also Keenan and Snow, 1999, p. 218: "*X* Kaldor superior to $\overline{x}$ requires that the old cost of *X*, $p(\overline{x})X$, exceeds the old cost of the old aggregate, $p(\overline{x})\sum_h \overline{x}^h$".

[4] The following notation is adopted: $a \le b$ means that $b - a$ is a nonnegative vector; this vector is semipositive if $a < b$. Analogously for $a \ge b$ and $a > b$.

$$A=\left\{a: x_i \in X_i \text{ for } i \in N,\ y_j \in Y_j \text{ for } j \in M, \text{ and } \sum\nolimits_{i\in N} x_i - \sum\nolimits_{j\in M} y_j \le \omega\right\};$$

the utility possibility set:

$$U=\left\{u \in \mathbb{R}^n : u = u(a); a \in A\right\};$$

the utility possibility frontier:

$$U_s=\left\{u \in U : u' > u \Rightarrow u' \notin U\right\};$$

the set of Pareto-optimal (maximal utilities) allocations:

$$P=\left\{a \in A : u(a) \in U_s\right\}.$$

The compensation principle introduces a comparison between different allocations and/or economies with the same consumers. These different economies represent different situations for the given set of consumers. Thus, an alternative situation is taken under consideration, which refers, in general, to a different set of producers $M' = \left\{1',...,m'\right\}$, consumption and production sets, and also to a different set of goods. However, we can define the set of goods K as the set of all goods, that includes the goods of both economies. Of course, if a good does not exist in $\mathcal{E}$ or in $\mathcal{E}'$, then its quantity is zero for all allocations in $\mathcal{E}$ or in $\mathcal{E}'$. We also assume that the utility functions are defined on the union of the consumption sets, *i.e.* $u_i : X_i \cup X_i' \to \mathbb{R}$ for every $i \in N$.

Then, let us to introduce the alternative situation as the economy

$$\mathcal{E}' = (X_i' \subset \mathbb{R}^k \text{ and } u_i : X_i' \to \mathbb{R}, i \in N; Y_{j'}' \subset \mathbb{R}^k, j' \in M'; \omega' \in \mathbb{R}^k_+),$$

and the set of the possible allocation of $\mathcal{E}'$

$$A'=\left\{a' : x_i' \in X_i' \text{ for } i \in N,\ y_{j'}' \in Y_{j'}' \subset \mathbb{R}^k \text{ for } j' \in M', \text{ and } \sum\nolimits_{i\in N} x_i' - \sum\nolimits_{j'\in M'} y_{j'}' \le \omega'\right\}.$$

The compensation principle was introduced by Kaldor (1939), Hicks (1940 and 1941) and Scitovsky (1941) by comparing two allocations. It is indicated as the *KHSC* compensation criterion.

Kaldor Direct Potential-Pareto-Preference: let $a \in A$ and $a' \in A'$, then $a' \succ_D a$ if there is an $\hat{a}' \in A'$, with $\sum\nolimits_{i\in N} (\hat{x}_i' - x_i') - \sum\nolimits_{j'\in M'} (\hat{y}_{j'}' - y_{j'}') \le 0$, such that $u(\hat{a}') > u(a)$, *i.e.* if there is an allocation $\hat{a}'$ in A' that does not require larger resources than a' and generates higher utilities than a.

Hicks Inverse Potential-Pareto-Preference: let $a \in A$ and $a' \in A'$, then $a' \succ_I a$ if there is no $\hat{a} \in A$ such that $u(\hat{a}) > u(a')$, *i.e.* $a' \succ_I a$ if $a \not\succ_D a'$.

Scitovsky Double Criterion of Potential-Pareto-Preference: let $a \in A$ and $a' \in A'$, then $a' \succ_{DI} a$ if both $a' \succ_D a$ and $a' \succ_I a$.

A more compelling definition of Potential-Pareto-Preference was introduced by Samuelson (1950) and analyzed by Chipman and Moore (1971 and 1976): this will be indicated as the *SCM* compensation criterion. It states that a situation $\mathcal{E}'$

exhibits higher potential welfare than situation $\mathcal{E}$ if any attainable allocation of $\mathcal{E}$ is Pareto dominated by some attainable allocation of $\mathcal{E}'$. (Sometimes this criterion is qualified as "strong" and Kaldor's as "weak").

Samuelson and Chipman-Moore Potential-Pareto-Preference: $\mathcal{E}' \succ_{SCM} \mathcal{E}$ if for every a of $\mathcal{E}$ there is an a' of $\mathcal{E}'$ such that $u(a') > u(a)$. This relationship is equivalent to $U_s \subset (U' - \mathbb{R}^n_+) \setminus U_s'$, *i.e.* the utility possibility frontier of $\mathcal{E}$ is inferior to the utility possibility frontier of $\mathcal{E}'$.

3 The National Income Test for Pure-Exchange Economies

The current approach to checking potential Pareto dominance is known as the "National Income Test" (*e.g.* Varian, 1992, pp. 407-410). Its main assumption requires allocation a to be a competitive equilibrium allocation of a "nice" economy without externalities[5].

The following proposition summarizes the National Income Test for a pure-exchange economy $\mathcal{E} = (X_i \subset \mathbb{R}^k \text{ and } u_i : X_i \to \mathbb{R},\ i \in N,\ \omega \in \mathbb{R}^k_+)$.

Proposition 1. If (p, a), where $a = (x_i)_{i \in N}$, is a competitive equilibrium of a "nice" pure-exchange economy without externalities and $a' = (x_i')_{i \in N}$, then the National Income Test provides a necessary condition for the direct potential preference, *i.e.* $a' \succ_D a$ only if $\sum_{i \in N} px_i' > \sum_{i \in N} px_i$. Moreover, if $a' = (x_i')_{i \in N}$ is a Pareto-optimal allocation and p' is its shadow price vector, then the National Income Test provides a sufficient condition for the inverse potential preference, *i.e.* $a' \succ_I a$ if $\sum_{i \in N} p'x_i \le \sum_{i \in N} p'x_i'$.[6]

Proof: If $a' \succ_D a$, then there is an allocation $\hat{a}' = (\hat{x}_i')_{i \in N} \in A'$, with $\sum_{i=1}^{n} (\hat{x}_i' - x_i') \le 0$, such that $\hat{x}_i' \succsim_i x_i$ for all i and $\hat{x}_i' \succ_i x_i$ for some i. Thus, $p\hat{x}_i' \ge px_i$ for all i, with $p\hat{x}_i' > px_i$ for some i, so that $\sum_{i \in N} px_i' \ge \sum_{i \in N} p\hat{x}_i' > \sum_{i \in N} px_i$. Correlatively, if (p', a') is a competitive equilibrium, since $a \succ_D a'$ only if $\sum_{i \in N} p'x_i > \sum_{i \in N} p'x_i'$, then $a \not\succ_D a'$, *i.e.* $a' \succ_I a$, if $\sum_{i \in N} p'x_i \le \sum_{i \in N} p'x_i'$. □

[5] An economy is "nice" if analytical problems are not involved. For instance, a pure-exchange economy without externalities $\mathcal{E} = (X_i \subset \mathbb{R}^k \text{ and } u_i(x_i),\ i \in N;\ \omega \in \mathbb{R}^k_+)$ is "nice" if free disposal is assumed, $X_i = \mathbb{R}^k_+$, and the functions $u_i : X_i \to \mathbb{R}$ are continuous, monotonically increasing and quasi-concave for all $i \in N$.

[6] The last implication was introduced by Hicks (1940, p. 111) and demonstrated by Samuelson (1950, p. 7-8) using the Edgeworth-Pareto box diagram.

4 The National Income Test for Production Economies

Proposition 1 states that the National Income Test provides a necessary condition for the direct potential preference in pure-exchange economies. When production economies are examined (and the production sets as well as the amount of resources differ in the two economies), then the National Income Test does not generally hold, *i.e.* it does not even provide necessary condition for the direct potential preference, as shown by the following examples 1 and 2.

In a production economy, reallocation concerns not only consumption but also production allocation, so the aggregate consumption is not bounded by the total amount of resources as in a pure-exchange economy. (*I.e.* the resource constraint for a reallocation does not require $\sum_{i=1}^{n}(\hat{x}_i' - x_i') \le 0$ in a production economy, but $\sum_{i\in N}(\hat{x}_i' - x_i') - \sum_{j'\in M'}(\hat{y}_{j'}' - y_{j'}') \le 0$). Consequently, the fact that the inequality $\sum_{i\in N} px_i' \le \sum_{i\in N} px_i$ holds does not imply that such an inequality holds for all reallocations of a'. There may exist a reallocation $\hat{a}'$ such that $\sum_{i\in N} p\hat{x}_i' > \sum_{i\in N} px_i$, and we cannot exclude that it Pareto dominates a. Thus, even if the national income decreases (*i.e.* $\sum_{i\in N} px_i' < \sum_{i\in N} px_i$), a reallocation of a' that Pareto dominates a may exist. Therefore, the National Income Test (*i.e.* the condition $\sum_{i\in N} px_i' > \sum_{i\in N} px_i$) is not a necessary condition for the direct potential preference (*i.e.* for the existence of $\hat{a}' = ((\hat{x}_i')_{i\in N}, (\hat{y}_{j'}')_{j'\in M'}) \in A'$, with $\sum_{i\in N}(\hat{x}_i' - x_i') - \sum_{j'\in M'}(\hat{y}_{j'}' - y_{j'}') \le 0$, such that $\sum_{i\in N} p\hat{x}_i' > \sum_{i\in N} px_i$), since the inequality $\sum_{i\in N} px_i' \ge \sum_{i\in N} p\hat{x}_i'$, which holds for a pure-exchange economy, does not generally hold for a production economy.[7]

Example 1: $\mathcal{E}$ is a pure-exchange economy with two consumers and two goods; utility functions $u_1 = x_{11}x_{12}$ and $u_2 = x_{22} + \sqrt{x_{22}^{\,2} + 40x_{21}}$, where x_{ih} is the quantity of good *h* consumed by individual *i*; and resources $\omega = (2,\ 20.1)$. Then

[7] Analogously, with regard to the compensating variation, which is the basis of cost-benefit analysis, it is normally said that $a' \succ_D a$ only if the compensating variation is positive (*e.g.* Ruiz-Castillo, 1987, p. 41), *i.e.* only if $\sum_{i\in N}(p'x_i' - \min_{u_i(\hat{x}_i)\ge u_i(x_i)} p'\hat{x}_i) > 0$. This condition holds for pure-exchange economies. It does not generally hold for production economies, since the condition $\sum_{i\in N}(\hat{x}_i' - x_i') - \sum_{j'\in M'}(\hat{y}_{j'}' - y_{j'}') \le 0$ does not imply $\sum_{i\in N} p'(\hat{x}_i' - x_i') \le 0$. However, it holds in case of technology with constant returns to scale, *i.e.* if $\sum_{j'\in M'} p'(\hat{y}_{j'}' - y_{j'}') = 0$.

$\mathcal{E} = ((X_i = \mathbb{R}^2_+)_{i=1,2},\ u_1 = x_{11}x_{12}, u_2 = x_{22} + \sqrt{x_{22}^2 + 40x_{21}}; \omega = (2, 20.1))$.

Allocation $a = (x_1, x_2)$ with $x_1 = (2,\ 18)$ and $x_2 = (0,\ 2.1)$ is Pareto-optimal and is competitive (*e.g.* with respect to the endowments $\omega_i = x_i$ for $i = 1,2$), with prices $p = (9,\ 1)$. Thus, consumers obtain utilities $u(a) = (36,\ 4.2)$, and the "National Income" is $p(x_1 + x_2) = 38.1$. The utility possibility set is

$U = \{u \in \mathbb{R}^2_+ : 40u_1 + 20.1u_2^2 \le 1608 \text{ for } u_2 \in [0, 1.9002];\ 0.5u_2^2 - 20.1u_2 + \sqrt{80u_1u_2} \le 40 \text{ for } u_2 \in [1.9002, 2.0997] \text{ and } u_2 \in [38.1003, 42.1002];\ \text{and } u_1 + u_2 \le 40.2 \text{ for } u_2 \in [2.0997, 38.1003]\}$.

Let us introduce an alternative economy $\mathcal{E}'$ that differs from $\mathcal{E}$ in terms of production set, $Y = \{y \in \mathbb{R}^2 : y_1 + y_2 \le 0, y_1 \le 0\}$, and the amount of resources:

$\mathcal{E}' = ((X_i = \mathbb{R}^2_+)_{i=1,2},\ u_1 = x_{11}x_{12}, u_2 = x_{22} + \sqrt{x_{22}^2 + 40x_{21}}; y \in \mathbb{R}^2, y_1 + y_2 \le 0, y_1 \le 0; \omega' = (21,1))$.

The allocation $a' = (x_1', x_2', y')$, with $x_1' = (1,\ 1)$, $x_2' = (0,\ 20)$, and $y' = (-20,\ 20)$, is Pareto-optimal and competitive (*e.g.* with respect to the endowments $\omega_1' = (1,\ 1)$ and $\omega_2' = (20,\ 0)$), with prices $p' = (1,\ 1)$. Consumers obtain utilities $u(a') = (1,\ 40)$. The utility possibility set is

$U' = \{u \in \mathbb{R}^2_+ : u_2^2 + 80\sqrt{u_1} \le 880 \text{ for } u_2 \in [0, 20] \text{ and } u_2 + 4\sqrt{u_1} \le 44 \text{ for } u_2 \in [20, 44]\}$.

We find that $a' \succ_D a$, because in the economy $\mathcal{E}'$ there exists an allocation $\hat{a}' = (\hat{x}_1', \hat{x}_2', \hat{y}')$, with $\hat{x}_1' = (9,\ 9)$, $\hat{x}_2' = (4,\ 0)$, $\hat{y}' = (-8,\ 8)$, which Pareto-dominates allocation a since $u(\hat{a}') = (81,\ 4\sqrt{10}) > (36,\ 4.2) = u(a)$. Moreover, it is also $a' \succ_I a$ since there is no allocation $\hat{a}$ in the economy $\mathcal{E}$ such that $u(\hat{a}) > u(a')$. Consequently a' is potentially-Pareto-preferred to allocation a according to the Scitovsky double criterion. Nevertheless, the National Income test gives the wrong indication, since we find that

$$p(x_1' + x_2') = 30 < 38.1 = p(x_1 + x_2)$$

and

$$p'(x_1' + x_2') = 22 < 22.1 = p'(x_1 + x_2) .$$

According to the necessary condition of the National Income Test, $a' \succ_D a$ would imply $p(x_1' + x_2') > p(x_1 + x_2)$. On the contrary, we have in this example both $a' \succ_D a$ and $p(x_1' + x_2') < p(x_1 + x_2)$. Notice also that, according to the National Income Test as reported by Ruiz-Castillo, 1987, p.39, the relations $p'(x_1' + x_2') < p'(x_1 + x_2)$ and $p(x_1' + x_2') < p(x_1 + x_2)$ recommend to select allocation $a=(x_1,x_2)$ versus allocation $a'=(x_1',x_2')$. This recommendation occurs to be wrong since a' is potentially Pareto preferred to a.

Figure 1 shows the shape of utility possibility sets U and U'.

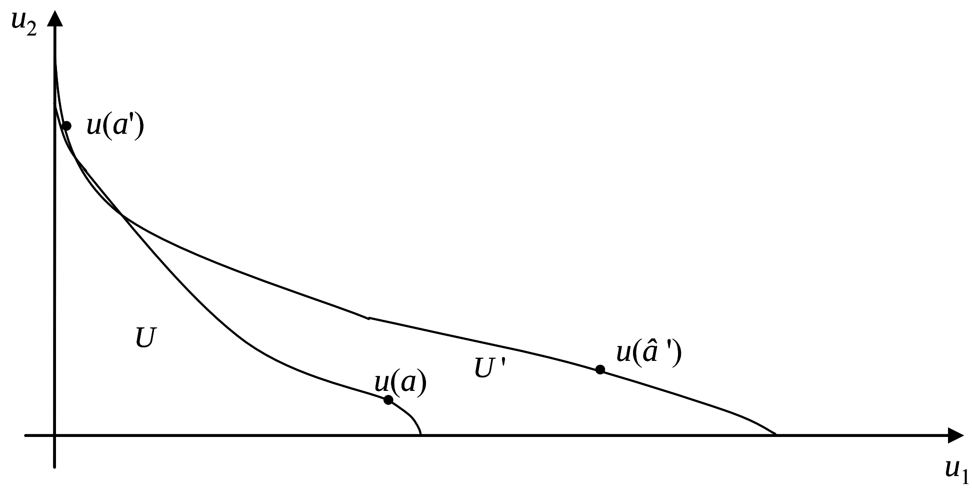

Figure 1

Figure 1 also shows that economy $\mathcal{E}'$ does not exhibit higher potential welfare than economy $\mathcal{E}$ according to the Samuelson and Chipman-Moore criterion. However, Example 2 shows that the National Test necessary condition (*i.e.* $a'\succ_D a$ only if $\sum_{i\in N} px_i' > \sum_{i\in N} px_i$) is invalid even in the case of Samuelson and Chipman-Moore potential-Pareto-preference (*i.e.* we can find that $a'\succ_D a$ and $\sum_{i\in N} px_i' < \sum_{i\in N} px_i$ with $\mathcal{E}'\succ_{SCM}\mathcal{E}$, where $a=((x_i)_{i\in N},(y_j)_{j\in M})$, $a'=((x_i')_{i\in N},(y_{j'}')_{j'\in M'})$, (p,a) is a competitive equilibrium of $\mathcal{E}$, and a' is an allocation of $\mathcal{E}'$).

Example 2. The only difference that Example 2 introduces with respect to Example 1 is that there is $\omega=(2,\ 19.9)$ in place of $(2,\ 20.1)$, so that

$$\mathcal{E}=((X_i=\mathbb{R}^2_+)_{i=1,2},\ u_1=x_{11}x_{12}, u_2=x_{22}+\sqrt{x_{22}^{\ 2}+40x_{21}};\omega=(2,19.9))$$

and

$U=\{u\in\mathbb{R}^2_+: 40u_1+19.9u_2^{\ 2}\le 1592$ for $u_2\in[0,1.9177]; 0.5u_2^{\ 2}-19.9u_2+\sqrt{80u_1u_2}\le$ 40 for $u_2\in[1.9177,2.1233]$ and $u_2\in[37.6767,41.7177]$; and $u_1+u_2\le 39.8$ for $u_2\in[2.1233,37.6767]\}$.

Allocation $a=(x_1,x_2)$ with $x_1=(2,\ 18)$ and $x_2=(0,\ 1.9)$ is Pareto-optimal and competitive (*e.g.* with respect to the endowments $\omega_i=x_i$ for $i=1,2$), with prices $p = (9,\ 1)$. Thus, consumers obtain utilities $u(a) = (36,\ 3.8)$ and the "National Income" is $p(x_1 + x_2) = 37.9$. Considering the economy $\mathcal{E}'$ as in Example 1, we find that $\mathcal{E}' \succ_{SCM} \mathcal{E}$, as represented in Figure 2. However, with respect to the allocation a' (introduced in Example 1) we find that

$$p(x_1' + x_2') = 30 < 37.9 = p(x_1 + x_2),$$

which contradicts the supposed necessity of the National Income Test.

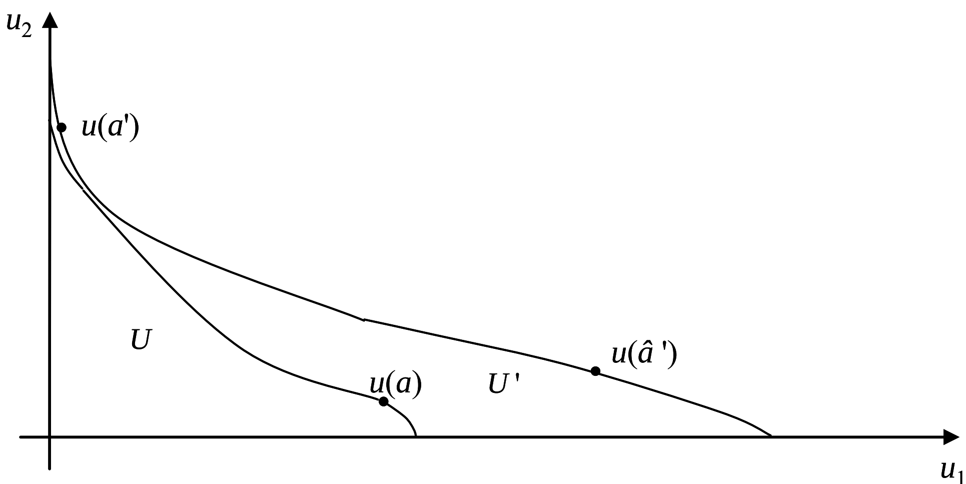

Figure 2

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