Credit Frictions and Household Debt in the U.S. Business Cycle: A Bayesian Evaluation

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Abstract

The last two decades have been characterized by a significant increase in the amounts of collateralized household debt in the U.S. and the Euro Area, along with loosened financial constraints for households and businesses, originating from liberalized and deregulated financial markets. All of these facts have occurred during what has been labelled the "Great Moderation", a period of reduced macroeconomic volatility, the causes of which are the object of theoretical and applied research. This paper proposes the estimation of a DSGE model that answers the following questions: What is the role of credit frictions faced by the household sector in explaining business cycle dynamics? How relevant are those frictions in the transmission of exogenous shocks? How well does the model fit the data, when compared to a standard New Keynesian one? The model provides empirical evidence in favor of the existence of an original transmission mechanism for monetary policy, based on the existence of collateral constrained households. The role of monetary policy is amplified by the existence of nominal private debt, which generates some degree of non-neutrality in the model economy. Bayesian estimation of the structural parameters suggests the importance of such a new channel in terms of reducing the relative role traditionally assigned to nominal rigidities in the propagation of exogenous shocks.

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1 Introduction

The last two decades have been characterized by an increasing degree of financial innovation and market deregulation in the U.S.. The developments in the loan market heavily improved households' financing conditions. At the same time, the amount of collateralized household debt has significantly increased: as documented by Dynan, Elmendorf and Sichel (2006), the ratio of household debt to disposable personal income doubled during the period 1960-2004, and the private debt/GDP ratio has grown larger than one in 2005. Both facts are intrinsically related to the remarkable macroeconomic stabilization that has occurred in the U.S. in about the same period. The huge reduction in the standard deviation of GDP growth over the last 30 years is perhaps the most immediate source of such evidence. Reductions in loan-to-value ratios and in the collateralized value of loans turn out to be quantitatively relevant in explaining the reduced volatility of GDP growth, hours worked and household debt, as shown by Campbell and Hercowitz (2006), who make use of a calibrated general equilibrium model. The expansion of collateralized household borrowing and the increasing level of loans securitization have contributed to deepen the influence of credit market conditions on households' consumption and saving decisions, as well as on firms' production and investment choices. The latter has been extensively studied by macroeconomists, who have recognized the existence of *credit cycles* (see Kyiotaki and Moore (1997)), and, more precisely, of a *credit channel* of transmission of monetary policy shocks (see Bernanke and Gertler (1995), Bernanke, Gertler and Gilchrist (1996)). The emphasis is placed on the role of financially constrained *firms* in the transmission of monetary policy shocks to the real economy. Only recently, Iacoviello (2005) and Monacelli (2006) have investigated the role of borrowing constraints in monetary models of the business cycle, pointing to the existence of an additional transmission channel, based on the role of *households*.

This paper aims at quantifying the impact of credit market frictions on households' financing decisions as both an original driving force of the business cycle, and a potential transmission mechanism for monetary policy (alternative or complementary to the usual one). A large strand of the literature has recently focused on medium-to-large scale dynamic stochastic general equilibrium (DSGE) models, which have become the workhorses for policy analysis (see Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005) as two prominent examples). These models usually incorporate nominal rigidities, along with a large number of structural shocks, to match the empirically observed degree of money non-neutrality. No attempt has been made, so far, to incorporate financial frictions on the household side into an estimated general equilibrium model[1]. The contribution of this paper is twofold. First, on the theoretical side, collateralized household debt is introduced into a large-scale DSGE model, along with the existence of borrowing constraints. Second, the model is structurally estimated using Bayesian methods, in order to provide an empirical assessment of the role of financial frictions and nominal rigidities in the business cycle.

[1] Recently, Iacoviello and Neri (2007) have proposed an estimated two-sector model of the U.S. economy with collateral-constrained agents.

The starting point for the theoretical structure of this paper is, quite naturally, a standard New Keynesian model, incorporating some degree of nominal rigidities. Non-neutrality arises in such an environment, because of price and/or wage stickiness, associated to the presence of market power for final goods-producing firms. Nonetheless, a standard one-agent model is essentially unable to generate private debt as an equilibrium phenomenon. The crucial assumption of identical preferences across individuals, summarized by the existence of a *representative agent*, prevents any form of trade among agents, whose desired (and realized) consumption profiles all look alike. Therefore, this paper follows Campbell and Hercowitz (2006), Iacoviello (2005) and Monacelli (2006) and introduce a dual structure on the household side: agents belong to two different groups (labeled *patients* and *impatiens*) according to their intertemporal discount factor. Intuitively, this assumption captures the presence of relatively impatient agents in the economy, as opposed to standard consumption-smoothing individuals. This simple fact originates a shift of resources across consumers both intratemporally, and intertemporally: Household debt thus results as an equilibrium phenomenon. A second, crucial assumption is necessary, though, to ensure a positive amount of consumption to both classes of agents in equilibrium. Namely, agents cannot borrow without limits: a ceiling is put on the nominal quantity of resources that each (impatient) agent can borrow in every period, to avoid having the impatient consumer depleting all the available resources in the long run steady state[2].

Heterogeneity in the intertemporal discount factors and the existence of borrowing constraints are the two main assumptions that characterize the model, and represent two significant departures from the standard representative agent New Keynesian framework. Their introduction has important effects on the theoretical predictions of the model about business cycle fluctuations and the monetary transmission mechanism.

The model is estimated over a sample of more than forty year of U.S. quarterly data, using modern Bayesian techniques. The overall fit and forecasting performance of the model is compared to the one of a rather standard one-agent New Keynesian model without credit frictions and household borrowing. Interestingly, the estimated degree of price stickiness is lower than the one usually obtained in the literature, confirming the theoretical result that credit frictions on the household side can at least partially substitute for nominal rigidities in creating some original degree of non-neutrality. In order to account for the dramatically increased degree of deregulation and liberalization in the financial markets occurred after the early 1980's, sub-sample estimation is also performed. The existence of a structural break in the sample (particularly evident in the case of household debt) is accounted for by splitting the full sample into two sub-periods. The break point is set at 1982 Q4, when the Garn-St.Germain Act was passed by the U.S. federal government, allowing savings and loan associations to make commercial loans. That act strongly contributed to reduce equity requirements in the mortgage market. The time series behavior of total household debt clearly changed from 1983 Q1 onwards, thus suggesting the existence of two different sub-samples in the whole forty-year period considered. The model forecasts obtained in the first sub-sample estimation exercise

[2]See Iacoviello and Neri (2007), Monacelli (2006) and references therein for a thorough discussion.

are compared to the observed series, to gauge intuition on the scope of the financial deregulation process. Finally, posterior distributions of the structural parameters in the two sub-periods are compared.

The paper is organized as follows: Section 2 introduces the model; Section 3 analyzes the econometric exercise and provides comments on the main results, as well as on the general mechanics of the model. Section 4 concludes.

2 The model

The model economy is populated by two different types of agents, each characterized by an idiosyncratic discount factor, along the lines of Campbell and Hercowitz (2006). Agents are allowed to trade assets and goods among themselves, with the only limitation that the impatient agents face a collateral constraint whenever they borrow from the patient ones. Household debt thus results as an equilibrium phenomenon, originated by the intertemporal trade between the two groups of agents. Such a setup guarantees the existence of a unique steady state, with positive consumption for each kind of household (see Campbell and Hercowitz (2006), Monacelli (2006) and references therein for a thorough discussion). Households consume two goods: durables and nondurables. Durable goods serve two purposes: they can be either directly consumed or used as collateral when applying for a loan. Each good is produced in a different sector, so that a relative-price channel is introduced. The market structure of each sector is discussed in Section 2.2.

2.1 Households

2.1.1 The impatient agents

The representative *impatient* agent solves the following intertemporal maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \tag{1}$$

subject to the infinite sequence of budget constraints:

$$P_{c,t}\widehat{C}_t + P_{d,t}(\widehat{D}_t - (1-\delta)\widehat{D}_{t-1}) - \widehat{B}_t = -R_{t-1}\widehat{B}_{t-1} + W_t N_t + \widehat{T}_t \tag{2}$$

where $\widehat{B}_t$ is end-of-period t *nominal* private debt, issued by the impatient agent. The issuing of nominal debt is consistent with the empirical evidence for most countries. The budget constraint can be conveniently rewritten in *real* terms as follows:

$$\widehat{C}_t + q_t(\widehat{D}_t - (1-\delta)\widehat{D}_{t-1}) - \widehat{b}_t = -R_{t-1}\frac{\widehat{b}_{t-1}}{\pi_{c,t}} + \frac{W_t}{P_{c,t}}N_t + \frac{\widehat{T}_t}{P_{c,t}} \tag{3}$$

where $q_t \equiv \frac{P_{c,t}}{P_{d,t}}$ is the relative price of durables, $b_t \equiv \frac{\widehat{B}_t}{P_{c,t}}$ is *real* debt (in terms of durables), and $\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}$ is durable-goods inflation. The instantaneous utility function has the following form:

$$U(X_t, N_t) = \varepsilon_t^B \left(\log(X_t) - \frac{\upsilon \varepsilon_t^N}{1+\varphi} N_t^{1+\varphi} \right) \tag{4}$$

where ε_t^B is an exogenous iid shock to the discount factor, while ε_t^N is an exogenous shock to labor supply. Both shocks follow an AR(1) process:

$$\log \varepsilon_t^i = \rho_i \log \varepsilon_{t-1}^i + \eta_t^i \qquad i = B, N \tag{5}$$

The consumption aggregator has a constant elasticity of substitution (CES) specification:

$$X_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(\widehat{C}_t - \theta \widehat{C}_{t-1}^* \right)^{\frac{\eta-1}{\eta}} + \varepsilon_t^{D,b} \alpha \widehat{D}_t^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where ε_t^D is an exogenous preference shock attached to durable consumption, which follows an AR(1) process as in (5).[3] The term $\widehat{C}_{t-1}^*$ indicates last period *aggregate* consumption of nondurable goods, so that the representative impatient agent exhibits *external habit formation* in nondurable consumption. The existence of state-contingent securities guarantees that in equilibrium $\widehat{C}^* = \widehat{C}$. Finally, the law of accumulation for durable goods is:

$$\widehat{I}_t^D = \widehat{D}_t - (1-\delta)\widehat{D}_{t-1}$$

All the impatient agents are subject to a *collateral constraint*. In a preliminary, very simple form, the constraint can be specified as follows:

$$\widehat{B}_t \leq (1-\chi) P_{d,t} \widehat{D}_t \tag{7}$$

Following Campbell and Hercowitz (2006), Iacoviello (2005) and Monacelli (2006), we assume that the whole amount of debt is unsecured and has to be collateralized. The parameter $\chi \in [0,1]$ indicates the share of durable goods that cannot be used as a collateral: the term $(1-\chi)$ thus provides a proxy for the loan-to-value ratio. The presence of such a constraint is necessary to ensure the existence of a well-defined deterministic steady state. Without any ceiling to the available amount of borrowing, the impatient agent would in fact try to consume the whole amount of goods in the economy, and the patient agent would not be able to consume a positive amount of goods in equilibrium. Therefore, a limit must be imposed to private borrowing. Notice that private debt will not be nil in equilibrium, due to the accumulation of debt among different agents and across time. Thus, the presence of heterogeneous agents *per se* allows for the existence of private debt, but cannot guarantee that a unique, well-defined equilibrium is reached. The collateral constraint in real terms reads:

$$b_t \leq (1-\chi) q_t \widehat{D}_t \tag{8}$$

[3] Clearly, in the case of unitary elasticity of substitution ($\eta = 1$), the CES aggregator simplifies to a Cobb-Douglas function. The latter case is considered in the empirical analysis, withouh loss of generality.

It is immediate to show that the collateral constraints always binds in the deterministic steady state[4]. We will assume throughout that the collateral constraint is satisfied with equality in a sufficiently small neighborhood of the steady state too, so that the model can be solved by taking a log-linear approximation around the equilibrium[5].

The impatient agent thus maximizes (1) subject to (3) and (8) satisfied with equality. The corresponding set of first order conditions read:

$$q_t U_{c,t} = U_{d,t} + \beta(1-\delta)E_t\left\{U_{c,t+1}q_{t+1}\right\} + (1-\chi)\psi_t U_{c,t} q_t \tag{9}$$

$$\psi_t = 1 - \beta E_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\frac{R_t}{\pi_{c,t+1}}\right\} \tag{10}$$

Notice that ψ_t, the Lagrange multiplier attached to the collateral constraint, can be intuitively interpreted as the marginal value of borrowing. More precisely, any rise in ψ_t is equivalent to a tightening of the collateral constraint.

The set of optimality conditions is completed by the intratemporal trade-off relation between consumption and leisure. The form of such a condition depends on the labor market structure, and is the object of the next subsection. In brief, the presence of noncompetitive labor markets drives a wedge between the marginal rate of substitution between consumption and leisure and the real wage. The total supply of hours is thus inefficient and suboptimal, when compared to the one arising in a perfectly competitive labor market.

2.1.2 Labor Market Structure and Wage Setting

The labor force is made up of *impatient* agents only, as motivated in the previous subsection. The general setup of the labor market structure follows Erceg, Henderson and Levin (2000). There exists a continuum of impatient households (indexed with j) on the unit interval, each supplying a differentiated labor service. Each final-good producing firm uses all of the services in production. We assume the existence of a labor aggregator (union) that combines households' hours worked in the same proportion as the firms would choose. The labor market index N_t^i denotes the amount of labor input used by firm i:

$$N_t^i = \left(\int_0^1 \left(N_t^i(j)\right)^{\frac{1}{1+\lambda_W}} dj\right)^{1+\lambda_W}$$

where the term $\frac{1+\lambda_W}{\lambda_W}$ represents the elasticity of substitution across differentiated labor services. The labor aggregator minimizes the cost of producing a given amount of N_t, taking each household's wage $W_t(j)$ as given, and then sells units of N_t to the production sector at their unit cost W_t, which can be expressed as:

$$W_t \equiv \left(\int_0^1 \left(W_t(j)\right)^{-\frac{1}{\lambda_W}} dj\right)^{-\lambda_W}$$

[4] See Appendix.

[5] The size of the neighborhood directly influences the accuracy of the approximation and is related to the magnitude of the exogenous shocks considered.

Total demand for each household's labor service is given by:

$$N_t^i(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\lambda_W}{\lambda_W}} N_t^i$$

Each impatient household exerts its monopolistic power through the setting of a wage. However, wages do not adjust instantaneously to account for changed economic conditions. Each period, only a constant fraction of households receives a signal that allows for price changing. The probability that a specific household receives a signal in a given period t is equal to $(1-\xi_W)$. After receiving the signal, the household sets a new - optimal - nominal wage $\widetilde{W}_t$. For those households that cannot re-optimize, we assume a partial-indexation mechanism of the following type:

$$W_t^i = \left(\frac{P_{c,t-1}}{P_{c,t-2}}\right)^{\gamma_w} W_{t-1}^i$$

where $\gamma_w \in [0,1]$ denotes the degree of indexation to past nondurable inflation[6]. The two extreme cases of no indexation and full indexation correspond to $\gamma_w = 0$ and $\gamma_w = 1$, respectively. The optimality condition for the wage setters results in the following dynamic wage mark-up equation:

$$E_t\left\{\sum_{s=t}^{\infty} \beta^{s-t}\xi_w^{s-t} N_s(i)\left[-(1+\lambda_W)U_{N,s} + U_{C,s}\frac{\widetilde{W}_t}{P_{c,s}}\left(\frac{P_{c,s-1}}{P_{c,t-1}}\right)^{\gamma_w}\right]\right\} = 0 \tag{11}$$

The law of motion of the aggregate nominal wage thus reads:

$$W_t = \left((1-\xi_W)(\widetilde{W}_t)^{-\frac{1}{\lambda_{W,t}}} + \xi_W\left(\frac{P_{c,t-1}}{P_{c,t-2}}\right)^{-\frac{\gamma_w}{\lambda_{W,t}}}(W_{t-1})^{-\frac{1}{\lambda_{W,t}}}\right)^{-\lambda_{W,t}} \tag{12}$$

Log-linearizing equation (11) around the deterministic steady state gives the standard formula:

$$\begin{aligned}\widehat{w}_t = {} & \left(\frac{\beta}{1+\beta}\right)E_t\{\widehat{w}_{t+1}\} + \left(\frac{\beta}{1+\beta}\right)E_t\{\widehat{\pi}_{c,t+1}\} + \left(\frac{1}{1+\beta}\right)\widehat{w}_{t-1} \\ & - \left(\frac{1+\beta\gamma_w}{1+\beta}\right)\widehat{\pi}_{c,t} + \left(\frac{\gamma_w}{1+\beta}\right)\widehat{\pi}_{c,t-1} - \left(\frac{(1-\xi_W)(1-\beta\xi_W)}{(1+\beta)\,\xi_W\left(1+\frac{1+\lambda_W}{\lambda_W}\varphi\right)}\right)\widehat{\mu}_t^w\end{aligned} \tag{13}$$

where variables with a hat are expressed in log-deviations from their steady-state value. In particular, μ_t^w is the (variable) wage markup, which is defined as the wedge between

[6]Notice that in the impatient's budget constraint (3) the real wage is defined as the ratio between nominal wage (W_t) and nondurable price ($P_{c,t}$). According to this convention, the relevant price index for wage setting is $P_{c,t}$. Clearly, choosing a different definition for the real wage would not modify the results.

the real wage and the marginal rate of substitution between consumption and leisure[7]:

$$\mu_t^w = \frac{\left(-\frac{U_{n,t}}{U_{c,t}}\right)}{\frac{W_t}{P_{c,t}}}$$

2.1.3 The case for time-varying collateral constraints

The simple formulation for the collateral constraint provided by equation (8) can be easily extended to account for variations in equity requirements over time. The present subsection proposes an alternative formulation, which is meant to capture the evolution of U.S. financial market conditions over the last 30 years. Time series evidence confirms that, among other things, the average loan-to-value ratio has been moving around, reflecting a more general change in financial constraints faced by households and businesses. A natural way of capturing such evolution is suggested by the interpretation of the parameter χ. As already pointed out, χ indicates the share of durable goods that cannot be used as a collateral, so that $(1-\chi)$ approximately measures the loan-to-value ratio. In a dynamic setting, the loan-to-value ratio is better interpreted as a variable, which moves over time according to some exogenous process. The collateral constraint then modifies to:

$$b_t \leq \lambda_t q_t \widehat{D}_t \tag{14}$$

where λ_t denotes the loan-to-value ratio in period t, which evolves according to the following exogenous process:

$$\log \lambda_t = (1-\rho_\lambda)\log(1-\chi) + \rho_\lambda \log \lambda_{t-1} + \epsilon_t^\lambda$$

The term ϵ_t^λ is an iid shock and can be interpreted as a measure of exogenous changes in financial markets regulation. The impatient agent thus maximizes (1) subject to (3) and (14). Equation (9) is correspondingly modified as follows:

$$q_t U_{c,t} = U_{d,t} + \beta(1-\delta)E_t\left\{U_{c,t+1}q_{t+1}\right\} + \lambda_t \psi_t U_{c,t} q_t \tag{15}$$

The above formulation allows to analyze the reaction of the model economy to a generic shock hitting the financial sector. A loosening of financial constraints, or a financial-technology innovation are captured by positive shocks to ϵ_t^λ, which are transmitted to the real sector through the variable λ_t. Section ?? discusses the dynamic effects of such a shock in detail. Intuitively, whenever a positive shock ϵ_t^λ hits the economy, the loan-to-value ratio λ_t increases and the amount of borrowing available to the impatient agent (b_t) increases too, for a given level of collateral $\widehat{D}_t$.

[7]Notice that the marginal utility of consumption is the same across the impatient households, since we are assuming the existence of a complete set of state-contingent assets within this group. Moreover, the impatient agent's utility function does not distinguish between hours worked in the durable and in the nondurable sector. Therefore, with free mobility of labor the wage set by the impatient households will be unique. Differences in hours worked across sectors thus simply stem from the demand side.

2.1.4 The patient agents

The representative patient agent solves a standard intertemporal maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \gamma^t U(\widetilde{X}_t, \widetilde{N}_t)$$

subject to the infinite sequence of (real) budget constraints:

$$\widetilde{C}_t + q_t \widetilde{I}_t^d - \widetilde{b}_t + R_{t-1}\widetilde{b}_{t-1} - (Div_t + \widetilde{T}_t) = 0$$

One of the key assumptions of the model concerns the intertemporal discount factor. It is assumed that the patient agent attaches relatively more weight to the future than the impatient, or:

$$\gamma > \beta$$

Savers are assumed not to work at all, so that $\widetilde{N_t} = 0$ for all t. While this can be regarded as an extremely simplifying assumption, it should be observed that the model aims at capturing a specific feature of the U.S. economy, namely the existence of a large and positive amount of debt over time, held by a *small* part of the population, whose contribution to the labor force can be ignored at virtually no price[8].

The functional form of the utility function is thus slightly modified with respect to the impatient agent's one. In particular, we have:

$$U(\widetilde{X}_t, \widetilde{N_t}) = \varepsilon_t^S \log(\widetilde{X}_t)$$

where, again, ε_t^S is an exogenous shock to the discount factor driven by an exogenous AR(1) process:

$$\log \varepsilon_t^S = \rho_S \log \varepsilon_{t-1}^S + \eta_t^S \tag{16}$$

and, consistently with the previous specification $\widetilde{X}_t$ is the consumption aggregator:

$$\widetilde{X}_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(\widetilde{C}_t - \theta \widetilde{C}_{t-1}^* \right)^{\frac{\eta-1}{\eta}} + \varepsilon_t^{D,s} \alpha \widetilde{D}_t^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

To keep the specification as general as possible, the durable-specific preference shock is allowed to differ among the two groups of agents, exactly as the generic preference shock ε_t^j. Again, the existence of a full set of state-contingent securities ensures that in equilibrium $\widetilde{C}^* = \widetilde{C}$.

The first order conditions characterizing the patient agent's problem can be expressed as follows:

$$q_t = \frac{U_{\widetilde{d},t}}{U_{\widetilde{c},t}} + \gamma(1-\delta) E_t \left\{ \frac{U_{\widetilde{c},t+1}}{U_{\widetilde{c},t}} q_{t+1} \right\} \tag{17}$$

$$U_{\widetilde{c},t} = \gamma E_t \left\{ U_{\widetilde{c},t+1} R_t \frac{1}{\pi_{c,t+1}} \right\} \tag{18}$$

[8] See Campbell and Hercowitz (2006) for an analysis of asset ownership in the U.S. economy.

Equation (17) is a standard optimality condition for investment in a durable good: the purchase price of a durable good is equated to the immediate payoff of the purchase (the MRS between durable and nondurable consumption), plus the discounted expected resale value. Equation (18) is a standard Euler equation.

2.2 Firms

Production of durable and nondurable goods is modeled in the standard New Keynesian way. In each sector there exists of a perfectly competitive final-good firm which produces a single good out of a continuum of intermediate goods. The intermediate-good firms operate in a monopolistically competitive market, where each firm produces a single differentiated good and thus exerts some market power. Price stickiness is also introduced in a standard way, resorting to a Calvo scheme analogous to the one analyzed in the labor market case.

2.2.1 Final-good producers

In each sector the individual, perfectly competitive final-good firm has the following production function:

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}^{\frac{1}{1+\lambda_{pj}}}(i)dj \right)^{1+\lambda_{pj}}$$

where $Y_{j,t}(j)$ denotes the the quantity of intermediate good of type j demanded by the final good producer in sector j $(i = C, D)$ at date t. The term $\frac{1+\lambda_{pj}}{\lambda_{pj}}$ denotes the elasticity of substitution between differentiated varieties, while λ_{pj} represents the price markup over marginal costs. The demand function for each intermediate good reads:

$$Y_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\frac{1+\lambda_{pj}}{\lambda_{pj}}} Y_{j,t}$$

where $P_{j,t}$ denotes the sectorial price index:

$$P_{j,t} = \left(\int_0^1 P_{j,t}^{-\frac{1}{\lambda_{pj}}}(i)di \right)^{-\lambda_{pj}} \tag{19}$$

2.2.2 Intermediate-good producers and price setting

Intermediate-good firms produce out of the following technology:

$$Y_{j,t}(i) = \varepsilon_t^j \omega N_{jt}(i)$$

where ε_t^a is a productivity shock following an AR(1) process:

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$$

with η_t^a IID-Normal. The term ω denotes the fraction of impatient agents in the economy. Solving the firms' static profit-maximization problem yields the following definition of real marginal costs:

$$mc_t^j = \frac{W_t^j}{\varepsilon_t^j}$$

Labor and capital are assumed to be fully mobile across sectors, so that the nominal wage and the rental rate of capital are unique. Firms change their prices à la Calvo, i.e. after receiving a random price-change signal, exactly as in the case of wage-setting operated by the impatient agents. The probability that a given firm receives the signal in each period is constant and equal to $(1-\xi_{pj})$. In analogy with the wage setting scheme, partial indexation to past inflation is assumed for those firms that do not receive the signal. The law of motion of the price index in each sector follows from definition (19):

$$P_{j,t} = \left((1-\xi_{pj}) \left(\widetilde{P}_{j,t} \right)^{-\frac{1}{\lambda_{pj}}} + \xi_{pj} \left(\frac{P_{j,t-1}}{P_{j,t-2}} \right)^{-\frac{\gamma_{pj}}{\lambda_{pj}}} (P_{j,t-1})^{-\frac{1}{\lambda_{pj}}} \right)^{-\lambda_{pj}}$$

where $\widetilde{P}_{j,t}$ denotes the newly-optimized price. Solving the profit-maximization problem and log-linearizing yields the following log-linear New Keynesian Phillips Curve(s):

$$\begin{aligned} \widehat{\pi}_{j,t} = {} & \left(\frac{\gamma_{pj}}{1+\gamma_{pj}} \right) \widehat{\pi}_{j,t-1} + \left(\frac{\gamma}{1+\gamma\gamma_{pj}} \right) E_t \left\{ \widehat{\pi}_{j,t+1} \right\} + \\ & + \left(\frac{(1-\xi_{pj})(1-\gamma\xi_{pj})}{(1+\gamma)\xi_{pj}} \right) \widehat{mc}_{jt} \end{aligned} \tag{20}$$

where mc_{jt} is the *real* marginal cost for firm j in period t.

2.3 Monetary policy

The monetary authority sets the short-term nominal interest rate R_t according to the following Taylor-type rule:

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}} \right)^{\rho_r} (\pi_{c,t-1})^{\phi_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\phi_{\Delta y}} e^{\xi_t^R} \tag{21}$$

where R is the steady state level of the gross nominal interest rate[9] and ξ_t^R denotes the monetary policy shock, which is assumed to be iid normally distributed. Noticeably, the monetary authority can specify rule (21) by targeting aggregate inflation π_t, or sectorial inflation $\pi_{j,t}$, with $j = c, d$. In particular, it is possible to recover the following relations between π_t and $\pi_{j.t}$:

$$\pi_t = \pi_{j,t} \frac{g_{j,t}}{g_{j,t-1}}$$

[9] The value of R is immediately computed by evaluating the Euler equation (18) in a zero-inflation steady state:

$$R = \frac{1}{\gamma}$$

See Appendix for details.

where

$$g_{c,t} \equiv \frac{P_t}{P_{c,t}} = \left[(1-\alpha) + \alpha q_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

and

$$g_{d,t} \equiv \frac{P_t}{P_{d,t}} = \left[\alpha + (1-\alpha)\, q_t^{-(1-\eta)}\right]^{\frac{1}{1-\eta}}$$

2.4 Market clearing

The goods market clearing conditions in the two sectors read:

$$Y_{c,t} = \omega \widehat{C}_t + (1-\omega)\, \widetilde{C}_t + (1-\omega)\,(I_{c,t} + I_{d,t}) \tag{22}$$

and

$$Y_{d,t} = \omega \widehat{I}_{d,t} + (1-\omega) \widetilde{I}_{d,t} \tag{23}$$

where clearly:

$$Y_{j,t} \equiv \int_0^1 Y_{j,t}(i)di = \omega \varepsilon_t^j \int_0^1 N_{j,t}(i)di = \omega N_{j,t}$$

The equilibrium condition for the bonds market requires that all the debt issued by the impatient agents is held by the patient ones:

$$\omega \widehat{B}_t + (1-\omega)\, \widetilde{B}_t = 0$$

Finally, the labor market clearing condition reads:

$$N_{c,t} + N_{d,t} = N_t$$

3 The estimation exercise

The overall structure of the artificial economy is enriched with a number of exogenous structural shocks, according to the recent literature on Bayesian estimation of DSGE models[10]. The application of standard log-linearization solution methods permits to solve the model and cast it in state-space form; it is then immediate to compute the likelihood function using the Kalman filter. After specifying independent prior distributions for the structural parameters, the application of Markov Chain Monte Carlo (MCMC) methods delivers estimates of the posterior distributions.

The model is estimated on quarterly U.S. data: The set of observables includes non-durable consumption, residential investment, household debt, nominal interest rate, consumer price inflation and real output[11]. The full sample goes from 1965 Q1 through 2006 Q4.

[10] See An and Schorfheide (2007) for an excellent review.

[11] See Appendix for a detailed description of the dataset.

The purpose of the estimation exercise is to provide a quantitative answer to the main questions that motivate this paper. First, the model is estimated on the whole sample, to obtain posterior distributions for the structural parameters, and an estimate of the marginal likelihood. Such information characterizes the empirical fit of the proposed model, which is compared to an alternative specification. As it is clear from the previous sections, the existence of two types of agents and the presence of collateralized household debt makes the model virtually impossible to nest on a more general one. Nonetheless, a one-agent, two-sector model with durable and nondurable consumption (but without household debt and credit frictions) seems to be a reasonable benchmark in the evaluation of the relative fit and forecasting performance of the proposed model. A comparison is given in Section 3.2.

A second question concerns the relative importance of credit frictions in explaining business cycle fluctuations. More precisely, the model emphasizes the role of exogenous shocks to the financial architecture of the economy in shaping cyclical fluctuations. In this vein, shocks to the loan-to-value ratio should mimic exogenous changes in financial legislation, or deregulation processes in the credit market, thus accounting for the huge reduction in equity requirements that characterized the U.S. over the last 25 years. In terms of timing, Campbell and Hercowitz (2006) identify the Monetary Control Act of 1980 and the Garn-St.Germain Act of 1982 as two crucial events that somehow initiated a new era in the U.S. equity requirement legislation. The Garn-St.Germain Act, by allowing savings and loan associations to make commercial loans, strongly contributed to reduce equity requirements in the mortgage market. Although other events occurred in the mortgage markets in about the same period that dramatically accelerated the development of a secondary market (see Gerardi, Rosen and Willen (2007) for a discussion), a look at the time series behavior of household debt suggests that some form of structural break is very likely to have occurred after 1982 Q4 (the quarter of the Garn-St.Germain Act's passage).

[insert Figure 1 here]

To account for such a break, the model is estimated on two sub-samples. The first sub-period goes from 1965 Q1 through 1982 Q4, while the second one goes from 1983 Q1 through the end of the sample. Posterior estimates of the structural parameters are compared to check for possible differences or breaks. Moreover, a forecasting exercise is performed using estimates over the first sub-sample, in order to track the theoretical behavior that the model would have attributed to the observable variables over the next sub-period, in absence of relevant exogenous changes. In particular, the contribution of financial shocks in explaining business cycle fluctuations can be analyzed.

3.1 Full-sample Estimation: Calibration and Priors

Some of the structural parameters have to be calibrated and excluded from the estimation set. In particular, the agents' intertemporal discount factors are chosen as follows: the patient agent's impatience rate γ is calibrated in such a way to obtain a steady-state value of the net nominal interest rate equal to 1% on a quarterly basis. The impatient agent's rate, β, is instead fixed at 0.96: this calibration is in line with the literature on heterogeneous agents models (see Krusell and Smith (1998), Campbell and Hercowitz (2006) and Iacoviello and Neri (2007))[12]. The elasticity of substitution between durable and nondurable goods is set to one, thus implying the simplifying case of a Cobb-Douglas function. The relative share of durable goods in the aggregator, α, is set to 0.4. Such a value is picked in order to obtain an equilibrium ratio between residential investment and output equal to 0.4, as empirically measured in the sample. The long-run average for the loan-to-value ratio is fixed at 0.75, which fairly replicates the available data for cars and housing purchases. The parameter ϕ in the durable-goods adjustment cost function is chosen to approximately replicate the response of durables to a monetary shock. The depreciation rate δ is parametrized to an annual value of 10%, following Campbell and Hercowitz (2006) and Monacelli (2006). Finally, the elasticities of substitution among differentiated goods and labor types are calibrated to yield a steady-state markup of, respectively, 16% in the nondurable sector, 10% in the durable sector, and 5% in the labor market.

The specification of independent priors is summarized in columns 3, 4 and 5 of Table 1. Priors are quite loose and not much informative in general. A Beta distribution is assumed for those parameters that can only assume values in the unit interval. In particular, the mean of the habit persistence parameter, θ, is set to 0.65, consistently with existing estimates (see Christiano, Eichenbaum and Evans (2005)). The inverse elasticity of substitution is assumed to follow a Gamma distribution, with mean 2 and standard deviation 0.75. The choice of prior distributions for the Calvo parameters in the two sectors reflects the observed data: prices in the nondurable sector are in fact assumed to be quite sticky ($\xi_{p,c}$ has mean 0.75), whereas they are likely to be much more flexible in the durable sector ($\xi_{p,d}$ has mean 0.05). This is consistent with using data on residential fixed investment as the empirical counterpart of durable investment in the model. Correspondingly, price indexation has a higher prior mean in the nondurable than in the durable sector. The wage stickiness parameter is assumed to follow a Beta distribution with mean 0.75 and standard deviation 0.05. About monetary policy, the parameters describing the Taylor rule are centered around standard values, with the Taylor principle always being satisfied by construction (ϕ_π is Normally distributed with mean 1.5 and standard deviation 0.1). Finally, noninformative priors are used for the standard deviations of the six structural shocks, which are assumed to follow a Uniform distribution over the interval [0, 0.5]. The persistence parameters are instead concentrated in the tail of a Beta distribution with mean 0.75 and standard deviation 0.05.

[12] Notice that β cannot be determined by using steady state ratios, nor does it influence the interest rate. Thus, some degrees of freedom are left in its choice

3.2 Full-sample Estimation: Posteriors and Business Cycle Indicators

Columns 6, 7 and 8 in Table 1 report respectively the mean, the 10th and the 90th percentiles of the posterior distributions.

The estimated degree of habit persistence is close to 0.5, while the inverse labor supply elasticity is close to 3. Looking at the parameters defining nominal rigidities, the mean of the estimated degree of price stickiness in the nondurable sector is 0.67. Such a value is lower than the one estimated, for instance, by Smets and Wouters (2003) in a prototypical New Keynesian model with nominal rigidities but without credit frictions ($\xi_{p,c} = 0.908$). Whether such a value represents a good description of the real sector is unclear from available studies based on disaggregated price data. Bils and Klenow (2004) estimate that the median time between two price changes is 4.3 months, but the number varies dramatically across sectors, ranging from less than one month (in the case of gasoline) to more than 6 years. Moreover, the inclusion of sales in the measurement turns out to be crucial. Recently in fact, Nakamura and Steinsson (2006) have argued that the median duration ranges between 8 and 11 months if sales and price changes due to product substitution are excluded from the sample. In terms of model parameters, the two studies imply a value of $\xi_{p,c}$ respectively equal to 0.3 and 0.68. Thus, the estimated posterior mean of the price stickiness parameter is closer to the results in Nakamura and Steinsson (2006) than to those in Bils and Klenow (2004). Price indexation is slightly higher than existing estimates ($\gamma_{p,c} = 0.6569$). In the case of durable goods, the mean of the posterior distribution is indeed very low ($\xi_{p,d} = 0.0199$), reflecting a very high frequency of adjustments. The estimated price indexation parameter is also quite low ($\gamma_{p,d} = 0.0228$). The wage stickiness indicator, ξ_w, has a mean of 0.94, which is quite higher than the estimates reported in Smets and Wouters (2003), but very similar to those reported by Iacoviello and Neri (2007), who estimate a two-sector model with credit frictions. The median of the wage indexation parameter is 0.0456.

The estimated monetary policy rule parameters are $\phi_\pi = 1.68$, $\phi_y = 1.10$ and $\rho_{\xi_r} = 0.34$.

Estimated technology shocks are more persistent and volatile in the nondurable than in the durable goods sector ($\rho_{zc} = 0.9551$, $\rho_{zd} = 0.6564$, $\sigma_{zc} = 0.0137$, $\sigma_{zd} = 0.0068$). The estimated standard deviation of financial markets shocks - captured by shocks to the loan-to-value ratio - is almost a half as large as the one of a monetary shock, while the estimated persistence parameter is much higher ($\rho_{ltv} = 0.8224$). The estimates thus suggest the overall significant role played by regulatory interventions, liberalizations and financial markets innovations in the full 40-year sample. The relative importance of such factors in the two subsamples is analyzed in detail in the next subsection.

[insert Table 1 here]

Tables 2 to 4 summarize the main business cycle properties of the estimated model and compare them to those of the actual data. In terms of second moments, the model captures almost exactly the behavior of household debt over the sample: the observed standard deviation lies inside the 90% confidence interval, as reported in the third row of Table 2. The same holds true for the short-term nominal interest rate. The estimated standard deviations of nondurable consumption, durable investment and real output are higher than the observed ones, although the order of magnitude is reasonably comparable. The last column of Table 2 reports the estimated standard deviations obtained using a one-agent tow-sector New Keynesian model with the same type of nominal rigidities, but without any sort of credit frictions. Clearly, household debt is not part of such a model. Comparing the median estimates shows that the enriched model does not perform worse than a standard one in terms of capturing cyclical volatilities, with the clear advantage of accounting for household debt. Table 3 reports first order autocorrelations. Again, the model captures fairly well the overall dynamics, although for some variables the actual autocorrelations lie outside the estimated error bands. Table 4 reports variance decomposition obtained using the median of the estimated posterior distributions. Most of the dynamics of the real variables are explained by technology shocks in the nondurable goods sector, while supply shocks in the durable sector only matter in the explanation of residential fixed investment volatility, as simple economic intuition would suggest. Noticeably, the estimated importance of supply shocks is very likely to capture the combined effect of pure productivity shocks and cost-push shocks originating, for instance, from variations in the elasticity of substitution among goods[13]. Interestingly, preference shocks hitting the impatient agent are in general much more important than shocks to the patient agent. Shocks to the loan-to-value ratio only explain a very low portion of the variance for most of the variables, with two significant exceptions: durable investment and household debt. This result provides empirical evidence to emphasize the role played by the impatient agent's consumption-financing decisions in the model. Preference shocks modify the impatient agent's demand for nondurable and durable consumption, which in turn is affected by financial shocks, that loosen the tightness of equity requirements, and monetary shocks, that alter the cost of borrowing. The data thus support of the existence of an original transmission channel based on the presence of impatient, collateral-constrained agents.

3.3 Full-sample Estimation: Impulse Response Analysis

Figures 2a to 2d plot the impulse responses of the main variables to a positive technology shock in the nondurable sector. Both nondurable consumption and durable investment increase on impact, as does household debt. Intuitively, an increase in productivity raises labor demand in the nondurable sector, at least partially[14]. The impatient agent (who represents the only worker in the economy) will then increase labour supply to obtain a

[13]The analysis of cost-push shocks is easily implemented by making the elasticity of substitution among goods vary over time. The set of observables should be correspondingly augmented - or measurement errors introduced - to overcome the well-knwon problem of stochastic singularity.

[14]In the limit-case of completely fixed prices, firms in the nondurable sector would be forced to keep production fixed, for no change in aggregate demand could occur. Labor demand would not vary in that case.

higher wage income and finance nondurable and durable consumption.

[insert figures 2 here]

The effects of an increase in productivity in the durable sector are shown in Figures 3a to 3d. Clearly, durable consumption is increased by both supply and demand effects in this case, whereas household debt falls on impact, due to the reduced need for collateral, ceteris paribus. In both cases, the behavior of the impatient agent violates the standard consumption-smoothing assumption. As this type of agent prefers current to future consumption, any increase in her total disposable income today will imply higher current, as opposed to future consumption (of both goods, which substitute according to the elasticity parameter η). The impatient agent can finance a surge in consumption in two ways. The first one is a standard neoclassical increase in labor supply: since only the impatient agent works, an increase in productivity under flexible prices induces the firms to demand more labour, and the consumer to work more. The second, nonstandard channel stems from the demand side: consumption can be increased via the trading of private debt, which is unsecured and must be collateralized. As already noticed, durable goods play a dual role in the economy, since they can be used as collaterals, but they also enter the individual utility function. Whenever the impatient agent is driven to demand more consumption today, she tries to increase her present disposable income by issuing more debt. This is only possible if new durable goods are accumulated as collaterals.

[insert figures 3 here]

Figures 4a to 4d show the effects of a positive shock to the loan-to-value ratio. Such shocks loosen equity requirements and therefore act in favour of the impatient agent's consumption decision. More precisely, an exogenous increase in the loan-to-value ratio works exactly as a positive income shock for the impatient agent. The presence of collateral constraints links durable and nondurable consumption decisions to the possibility of borrowing, and endogenously determines the optimal amount of debt. More precisely, the *cost* of borrowing is related to the short-term interest rate[15]: any decrease in R_t amounts to a decrease in the cost of servicing existing debt, and is therefore equivalent to a positive income shock (just as the technology shock analyzed above). In exactly the same way, a monetary tightening that reduces R_t implies a reduction in borrowing and a lower consumption for the impatient agents[16]. Figures 5a to 5d plot the impulse responses to a monetary tightening.

[insert figures 4 here]

[insert figures 5 here]

[15] The model considers a simplifed framework in which there is no term structure of interest rates, so that the cost of borrowing is represented by the short-term nominal interest rate. Such a simplification could be easily overcome by introducing an additional equation that links the long-term interest rate on loans (the mortgage rate) to the policy rate.

[16] Interestingly, such a channel builds on the *nominal* nature of debt: were debt indexed to inflation, such a non-neutrality effect would disappear.

3.4 Sub-sample Estimation

Prior distributions for the first sub-sample (1965 Q1: 1982 Q4) are specified exactly as in the full-sample estimation exercise. The results obtained using the Metropolis-Hastings algorithm are presented in columns 5, 6 and 7 of Table 6. The estimated means do not differ much from the full-sample estimates, but for the parameters that define the exogenous driving process of the loan-to-value ratio. The estimated persistence parameter is higher in the first sub-sample (0.81917 as opposed to 0.7246) and the standard deviation is also larger (0.006 versus 0.004). These preliminary results seem to support the intuition that changes in the loan-to-value ratio were relatively longer-lasting and larger in the first part of the sample, probably reflecting a quite low frequency of changes in equity requirements.

Posterior estimates on the first sub-sample are used to define prior distributions and initial values for the structural parameters over the second sub-period. Intuitively, this corresponds to the Bayesian prior updating that a rational agent would have operated at the beginning of 1983, using all the available information up to that time. Formally, it can be interpreted as using the first sub-sample as a *training sample* in the specification of prior distributions. The means and the 90% confidence intervals of the posterior distributions are reported in the last three columns of Table 6. The amplitude of technology shocks in both sectors is significantly reduced in the second sub-period, confirming the presence of a more stable macroeconomic environment, as discussed by the large existing literature on the so-called "Great Moderation". As it is well-known, the conduct of monetary policy by the Federal Reserve Bank also experienced a change since the early 80's. In particular, the estimated monetary policy parameters show some signs of a change in the behavior of the Fed, as testified by the increase in ϕ_π, which captures the response of the short-term nominal interest rate to a change in the inflation rate. The posterior mean raises from 1.64 to 1.74. Changes in macroeconomic and financial conditions are also testified by significant changes in the estimated standard deviation and persistence of the loan-to-value ratio shock. In particular, σ_{ltv} decreases significantly from 0.006 to 0.003, while the autocorrelation parameter ρ_{ltv} increases from 0.81 to 0.83. Hence, exogenous shocks hitting the financial architecture of the economy turn out to be of lower magnitude and higher persistence in the second sub-period with respect to the first one. The results thus support the idea that loosening financial constraints helped reducing overall macroeconomic volatility, as proposed by Campbell and Hercowitz (2006) and Dynan, Elmendorf and Sichel (2006) among others. Interestingly, posterior estimates suggest that the role of individual preference shocks did not change significantly across the two periods. Both σ_b and σ_l are in fact quite stable, as well as ρ_b and ρ_g. Such conclusions are supported by the analysis of business cycle properties across samples. Table 7 reports the estimated standard deviation of the observable variables over the two sub-samples. Again, the model clearly tracks the significant reduction in macroeconomic volatility that characterizes the "Great Moderation" era, approximated here by the post-1982 period. Table 8 compares variance decompositions across the two periods. Interestingly, the portion of variance explained by preference shocks increases after 1982, the more so for shocks that hit the impatient agent. However, both monetary and financial shocks become less important in the same period. Summarizing, loosened financial constraints seem to be crucial in the explanation of macroeconomic stabilization after the financial market reforms of the early 80s. Preference shocks to the collateral-constrained also played a role. Although these shocks are exogenous and not motivated in the model, it is conceivable that more favorable conditions in the financial markets helped modifying consumer preferences.

Finally, a forecasting exercise is performed, as a by-product of sub-sample estimation. Figure 6 plots the forecasts obtained using the posterior median of the structural parameters in the first sub-sample, along with error bands. The forecast reflects the theoretical predictions of the model under the hypothesis of an unchanged economic environment across samples. In particular, no changes in the financial regulatory system are taken into account. The comparison of forecasts with observed series thus captures two main features. On the one hand, the model can forecast quite precisely the average behavior of future variables. This provides evidence in favour of the theoretical content of the proposed model. On the other hand, the distance between forecasts and actual data follows from the existence of some additional sources of variation in the second sub-period. As already observed, sub-sample estimates point towards financial shocks as a potential source of variation.

[insert Figure 6 here]

3.5 Fit of the model

TBA

see Figures 7 in the Appendix.

4 Conclusions

The last two decades have been characterized by a significant increase in the amounts of collateralized household debt in the U.S., along with loosened financial constraints for households and businesses, originating from liberalized and deregulated financial markets. All of these facts have occurred during what has been labelled the "Great Moderation", a period of reduced macroeconomic volatility, the causes of which are the object of theoretical and applied research.

This paper builds and estimates a DSGE model that makes sense of several features. First, the role of credit frictions faced by households is analyzed in detail. The presence of collateral constraints faced by the relatively more impatient agents introduces an original transmission mechanism that amplifies exogenous shocks. In particular, the existence of *nominal* private debt originates a source of non-neutrality that in principle does not require nominal rigidities of any sort to operate. Both monetary and financial shocks are transmitted to the economy through modifications in equity requirements and borrowing costs. Estimates indicate that shocks to the financial sector are an important source of business cycle dynamics. The role of traditional nominal rigidities such as price and wage stickiness is considerably de-emphasized when credit frictions are taken into account. Sub-sample estimates support the role attributed to financial deregulation and liberalization in contributing to the overall reduction in macroeconomic volatility observed during the "Great moderation" era. The contribution of this paper is twofold. On the theoretical side, it introduces the presence of time-varying collateral constraints, accounting for the observed variability of equity requirements over both the short and the long run. On the empirical side, it enriches existing estimated DSGE models (see Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005) and Iacoviello and Neri (2007)) with new empirical evidence on the relative role of nominal and real rigidities.

Future extensions require that the analysis of term structure considerations be added to the picture. Monetary policy affects the cost of borrowing through the control of the inflation rate (which in turn determines the *real* value of outstanding household debt), and through variations in the short-term nominal interest rate. The cost of borrowing is typically pinned down by long-term rates such as the mortgage rate, which usually augment expected short-term rates by a term premium. The enriched model could potentially account for variations in risk premia and the possibility of default.

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Appendix

Tables

Table 1. Prior and Posterior Distributions

		PRIOR			POSTERIOR		
Parameter	Description	Distr.	Mean	Std.Dev.	Mean	10%	90%
θ	cons. habit	Beta	0.65	0.1	0.5662	0.5391	0.5915
φ	inv. el.labor supply	Gamma	2	0.75	2.8902	2.7226	3.0225
$\xi_{p,c}$	Calvo prices (nond.)	Beta	0.75	0.05	0.668	0.6107	0.7024
$\xi_{p,d}$	Calvo prices (dur.)	Beta	0.05	0.04	0.0199	0.0002	0.0326
ξ_w	Calvo wages	Beta	0.75	0.05	0.9483	0.9371	0.9611
$\gamma_{p,c}$	price index. (nond.)	Beta	0.75	0.05	0.6569	0.5687	0.7387
$\gamma_{p,d}$	price index. (dur.)	Beta	0.75	0.05	0.0228	0.0003	0.0461
γ_w	wage indexation	Beta	0.75	0.15	0.1370	0.0456	0.2265
ϕ_π	Taylor rule	Normal	1.5	0.1	1.6830	1.5762	1.7679
$\phi_{\Delta y}$	Taylor rule	Normal	0.5	0.2	1.1074	0.9246	1.2821
ρ_{ξ_r}	Taylor rule	U[0,1]	0.5	0.2887	0.4458	0.3421	0.5324
ρ_{zc}	Tech. shock (nond.)	Beta	0.75	0.05	0.9551	0.9482	0.9611
ρ_{zd}	Tech. shock (dur.)	Beta	0.75	0.05	0.6564	0.5950	0.7067
ρ_b	Pref. shock (imp.)	Beta	0.75	0.05	0.9586	0.9554	0.9611
ρ_g	Pref. shock (pat.)	Beta	0.75	0.05	0.9147	0.8889	0.9402
ρ_{ltv}	Ltv-shock (nond.)	Beta	0.75	0.05	0.8224	0.7746	0.8825
σ_{zc}	Tech. shock (nond.)	U[0,0.5]	0.25	0.1443	0.0137	0.0110	0.0155
σ_{zd}	Tech. shock (dur.)	U[0,0.5]	0.25	0.1443	0.0068	0.0060	0.0076
σ_{ξ_r}	Mon. pol. shock	U[0,0.5]	0.25	0.1443	0.0086	0.0068	0.0101
σ_{ltv}	Ltv-shock	U[0,0.5]	0.25	0.1443	0.0043	0.0038	0.0049
σ_b	Pref. shock (imp.)	U[0,0.5]	0.25	0.1443	0.0729	0.0619	0.0836
σ_g	Pref. shock (pat.)	U[0,0.5]	0.25	0.1443	0.0414	0.0353	0.0465

Table 2. Business Cycle Properties: Standard deviation

	Data	Two-agent model			One-agent NK model		
		Median	10%	90%	Median	10%	90%
C	0.0387	0.0712	0.0670	0.0815	0.0498	0.0431	0.0555
I	0.1527	0.7564	0.6844	0.8248	0.6167	0.5445	0.6917
$\widehat{B}$	0.0976	0.0901	0.0843	0.0998	-	-	-
R	0.0064	0.0075	0.0067	0.0084	0.0081	0.0071	0.0092
π	0.0051	0.0078	0.0073	0.0086	0.0080	0.0070	0.0088
Y	0.0322	0.0781	0.0716	0.0912	0.0693	0.0572	0.0766

Table 3. Business Cycle Properties

First Order Autocorrelation

	Data	Model		
		10%	50%	90%
C	0.9832	0.98809	0.9891	0.99055
I	0.9511	0.94283	0.94854	0.95369
$\widehat{B}$	0.9849	0.93909	0.95155	0.95679
R	0.9416	0.91437	0.92472	0.93281
π	0.8124	0.83057	0.85388	0.88085
Y	0.9655	0.98606	0.98746	0.98878

Table 4. Variance Decomposition

	ε_g	ε_b	ε_{zc}	ε_{zd}	η_r	η_{ltv}
C	0.0653	0.1928	0.5146	0.0172	0.1908	0.0013
I	0.2172	0.1717	0.2516	0.1203	0.2264	0.0130
$\widehat{B}$	0.0599	0.1878	0.3969	0.0790	0.1869	0.0776
R	0.0742	0.2041	0.0912	0.1012	0.5139	0.0083
π	0.0215	0.1436	0.3417	0.0813	0.3967	0.0008
Y	0.0061	0.1293	0.5563	0.0098	0.2844	0.0014

Table 5. Priors and Posteriors: Sub-sample Estimation

	Full Sample			(1965 Q1: 1982 Q4)			(1983 Q1: 2006 Q4)		
Parameter	Mean	10%	90%	Mean	10%	90%	Mean	10%	90%
θ	0.5662	0.5391	0.5915	0.6334	0.6085	0.6558	0.6370	0.6241	0.6521
φ	2.8902	2.7226	3.0225	2.9957	2.9207	3.0706	3.0247	2.9928	3.0869
$\xi_{p,c}$	0.668	0.6107	0.7024	0.6298	0.5674	0.6923	0.5953	0.5063	0.6603
$\xi_{p,d}$	0.0199	0.0002	0.0326	0.0304	0.0002	0.0668	0.0509	0.0004	0.0866
ξ_w	0.9483	0.9371	0.9611	0.9322	0.9089	0.9610	0.9569	0.9525	0.9611
$\gamma_{p,c}$	0.6569	0.5687	0.7387	0.7193	0.6321	0.8036	0.6863	0.6238	0.7921
$\gamma_{p,d}$	0.0228	0.0003	0.0461	0.0509	0.0004	0.1192	0.0498	0.0056	0.0769
γ_w	0.1370	0.0456	0.2265	0.2899	0.1044	0.4528	0.1366	0.0348	0.1721
ϕ_π	1.6830	1.5762	1.7679	1.6418	1.5165	1.7721	1.7489	1.6598	1.8692
$\phi_{\Delta y}$	1.1074	0.9246	1.2821	1.0174	0.8074	1.2226	1.0313	0.7742	1.1602
ρ_{ξ_r}	0.4458	0.3421	0.5324	0.4054	0.2145	0.5823	0.3676	0.2894	0.4759
ρ_{zc}	0.9551	0.9482	0.9611	0.7907	0.7129	0.8674	0.8353	0.8306	0.8787
ρ_{zd}	0.6564	0.5950	0.7067	0.6217	0.5456	0.6864	0.7277	0.7047	0.7591
ρ_b	0.9586	0.9554	0.9611	0.8689	0.8243	0.9188	0.8706	0.8507	0.9106
ρ_g	0.9147	0.8889	0.9402	0.8360	0.7814	0.8848	0.8665	0.8621	0.8904
ρ_{ltv}	0.8224	0.7746	0.8825	0.8192	0.7372	0.9042	0.8324	0.8239	0.8310
σ_{zc}	0.0137	0.0110	0.0155	0.0163	0.0114	0.0219	0.0096	0.0064	0.0124
σ_{zd}	0.0068	0.0060	0.0076	0.0077	0.0059	0.0092	0.0060	0.0054	0.0067
σ_{ξ_r}	0.0086	0.0068	0.0101	0.0086	0.0065	0.0108	0.0065	0.0047	0.0079
σ_{ltv}	0.0043	0.0038	0.0049	0.0060	0.0045	0.0074	0.0033	0.0027	0.0040
σ_b	0.0729	0.0619	0.0836	0.0541	0.0513	0.0828	0.0524	0.0433	0.0579
σ_g	0.0414	0.0353	0.0465	0.0669	0.0362	0.0721	0.0466	0.0387	0.0568

Table 6. Sub-sample Estimation: Standard Deviation

	(1965 Q1: 1982 Q4)				(1983 Q1: 2006 Q4)			
	Data	Median	10%	90%	Data	Median	10%	90%
C	0.0204	0.0454	0.0378	0.0558	0.0175	0.0359	0.0331	0.0381
I	0.1791	0.9720	0.8177	1.1094	0.0925	0.7705	0.7297	0.8214
$\widehat{B}$	0.0392	0.0799	0.0709	0.0924	0.0479	0.0609	0.0559	0.0660
R	0.0053	0.0087	0.0068	0.0099	0.0050	0.0047	0.0044	0.0052
π	0.0045	0.0091	0.0070	0.0102	0.0023	0.0050	0.0046	0.0053
Y	0.0239	0.0511	0.0393	0.0637	0.0189	0.0357	0.0308	0.0390

Table 7. Sub-sample Estimation: Variance Decomposition												
	(1965 Q1: 1982 Q4)						(1983 Q1: 2006 Q4)					
	ε_g	ε_b	ε_{zc}	ε_{zd}	η_r	η_{ltv}	ε_g	ε_b	ε_{zc}	ε_{zd}	η_r	η_{ltv}
C	0.225	0.158	0.259	0.037	0.288	0.005	0.291	0.203	0.205	0.054	0.245	0.002
I	0.245	0.184	0.254	0.108	0.160	0.014	0.293	0.289	0.197	0.090	0.128	0.007
$\widehat{B}$	0.069	0.251	0.137	0.116	0.197	0.189	0.110	0.365	0.121	0.146	0.159	0.092
R	0.088	0.078	0.135	0.116	0.549	0.015	0.179	0.110	0.151	0.142	0.401	0.012
π	0.021	0.041	0.401	0.087	0.427	0.002	0.037	0.400	0.515	0.131	0.277	0.001
Y	0.013	0.012	0.491	0.017	0.460	0.004	0.027	0.004	0.476	0.022	0.465	0.003

Figures

Figure 1. Real per capita Household Debt (1965 Q1: 2006 Q4).

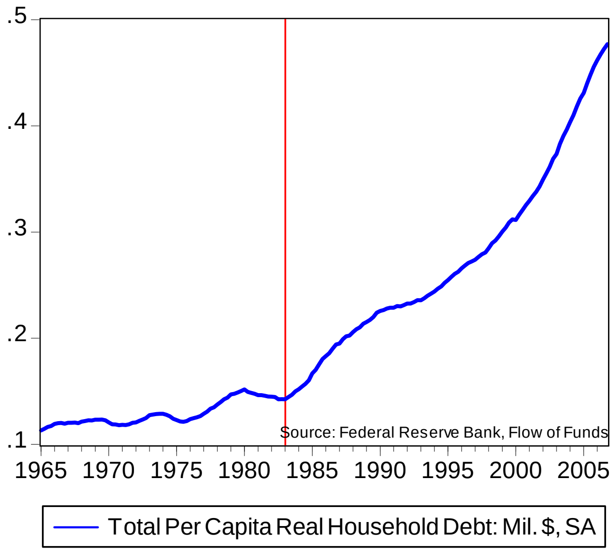


Figure 2. Impulse Responses to a positive technology shock in the nondurable sector[17]

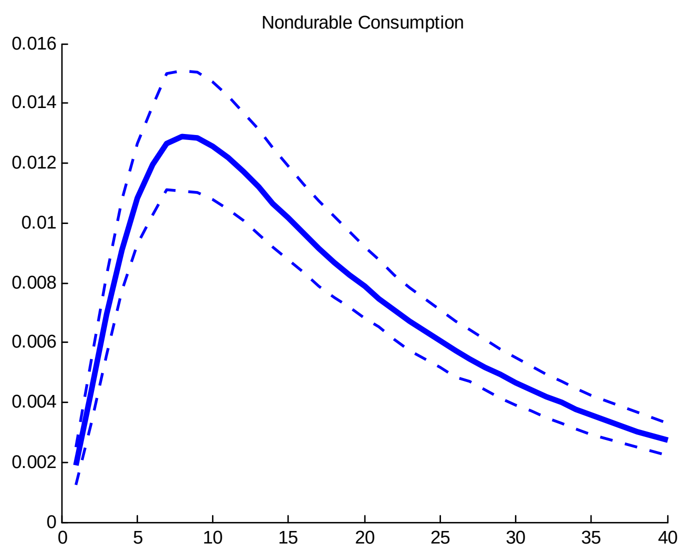


Figure 2a: IRF of Nondurable Consumption to a positive technology shock in the nondurable sector

[17] Solid lines plot the repsonses computed using the median of the posterior distribution of the structural parameters. Dashed lines plot the corresponding 10th and 90th percentiles.

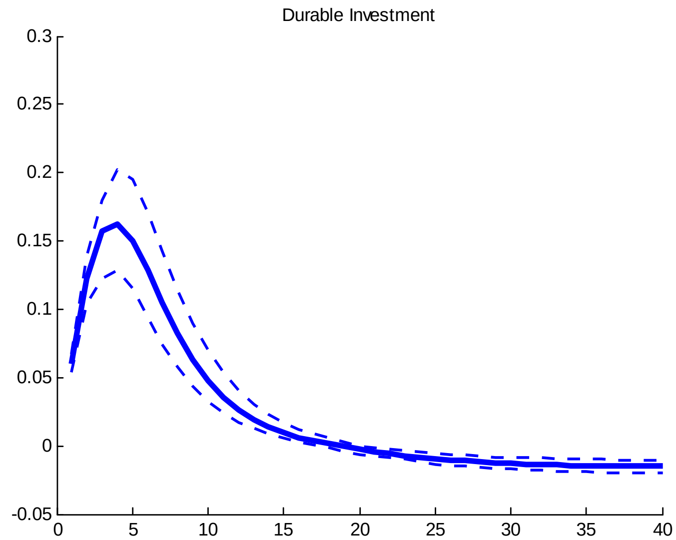


Figure 2b: IRF of Durable Investment to a positive technology shock in the nondurable sector

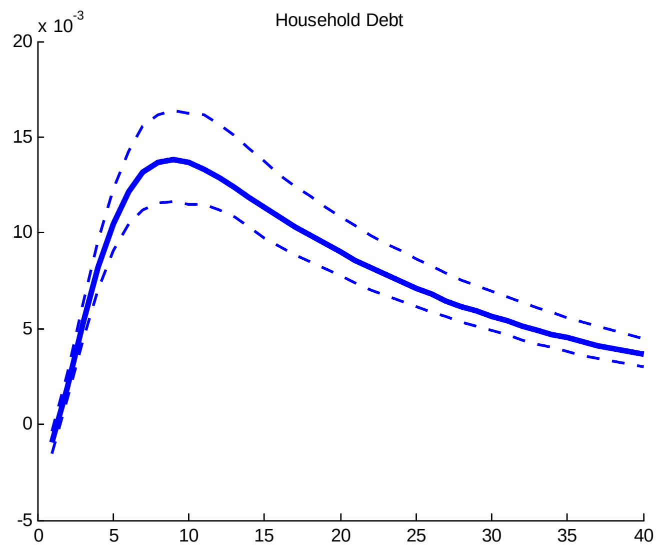


Figure 2c: IRF of Household Debt to a positive technology shock in the nondurable sector

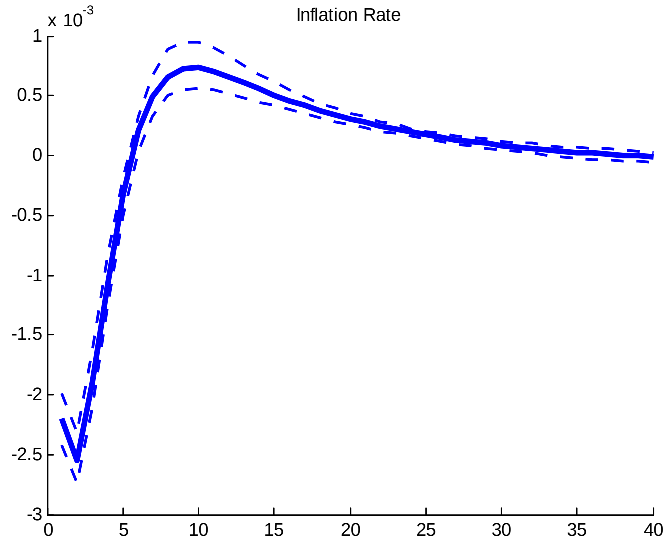


Figure 2d: IRF of Inflation Rate to a positive technology shock in the nondurable sector

Figure 3. Impulse Responses to a positive technology shock in the durable sector

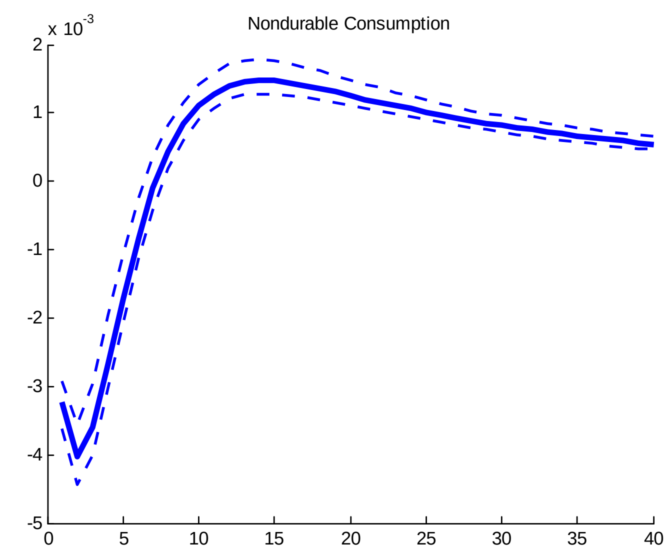


Figure 3a: IRF of Nondurable Consumption to a positive technology shock in the durable sector

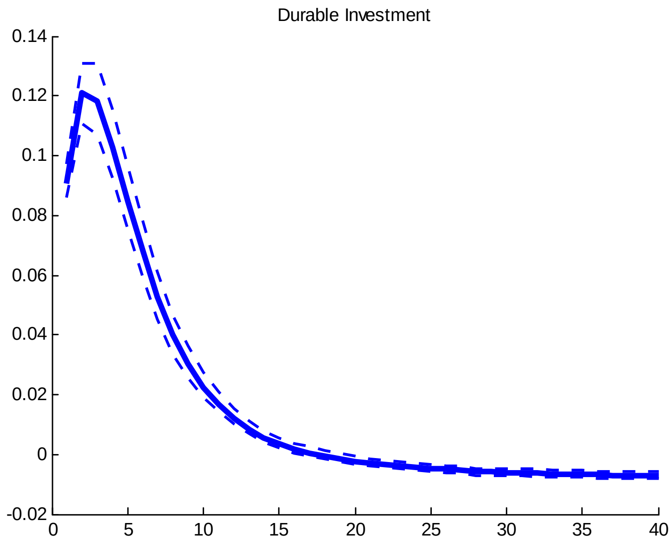


Figure 3b: IRF of Durable Investment to a positive technology shock in the durable sector

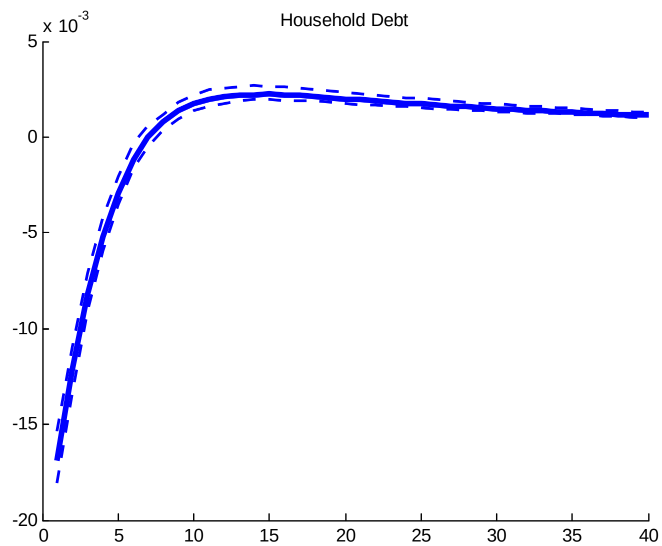


Figure 3c: IRF of Household Debt to a positive technology shock in the durable sector

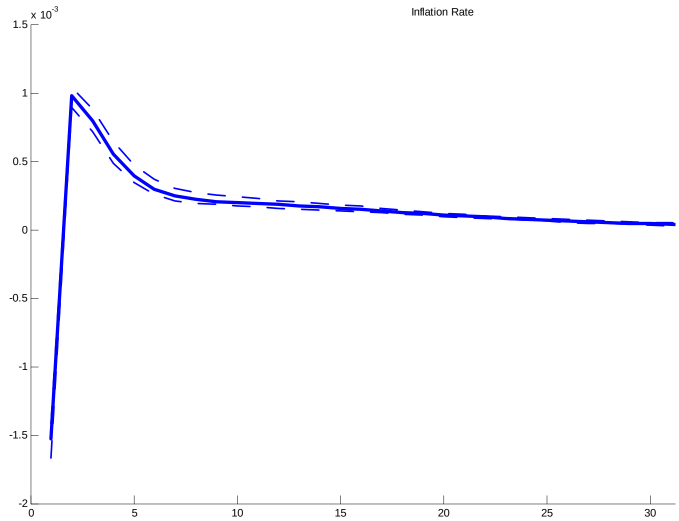


Figure 3d: IRF of Inflation Rate to a positive technology shock in the durable sector

Figure 4. Impulse Responses to a loan-to-value ratio shock

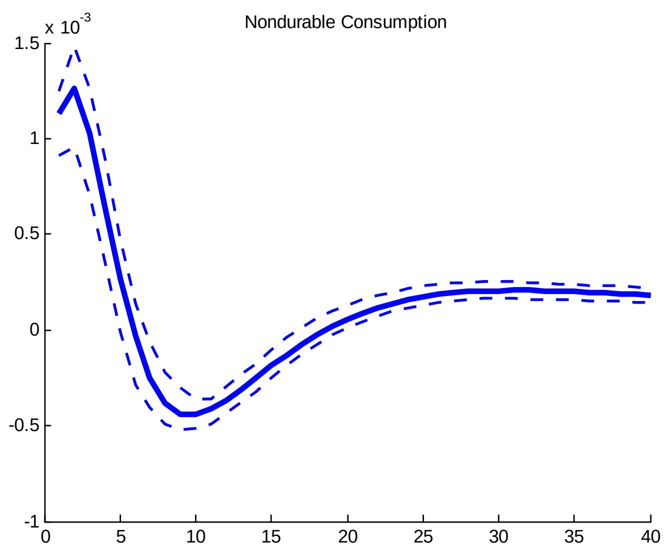


Figure 4a: IRF of Nondurable Consumption to a positive shock to the loan-to-value ratio

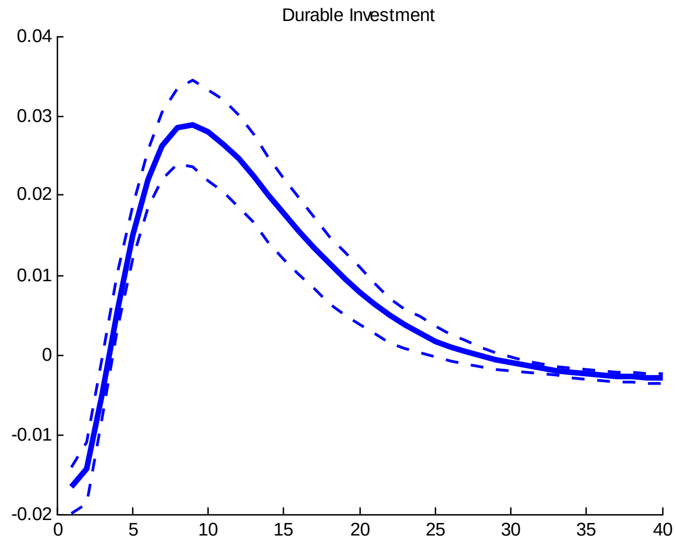


Figure 4b: IRF of Durable Investment to a positive shock to the loan-to-value ratio

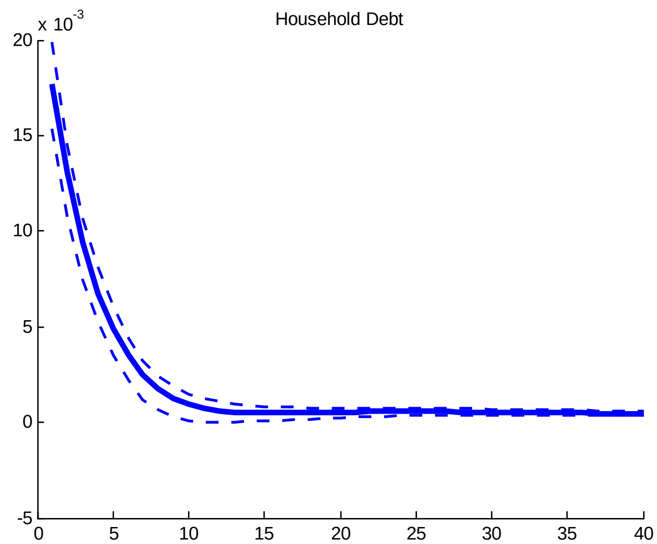


Figure 4c: IRF of Household Debt to a positive shock to the loan-to-value ratio

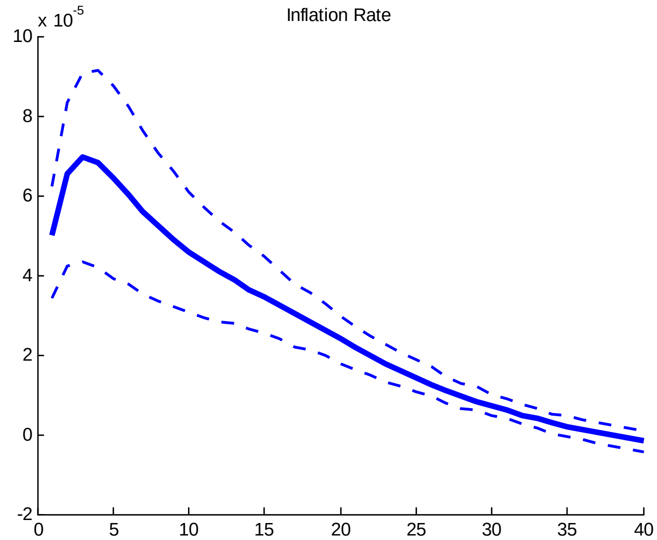


Figure 4d: IRF of Inflation Rate to a positive shock to the loan-to-value ratio

Figure 5. Impulse Response Functions to a positive monetary policy shock (monetary tightening)

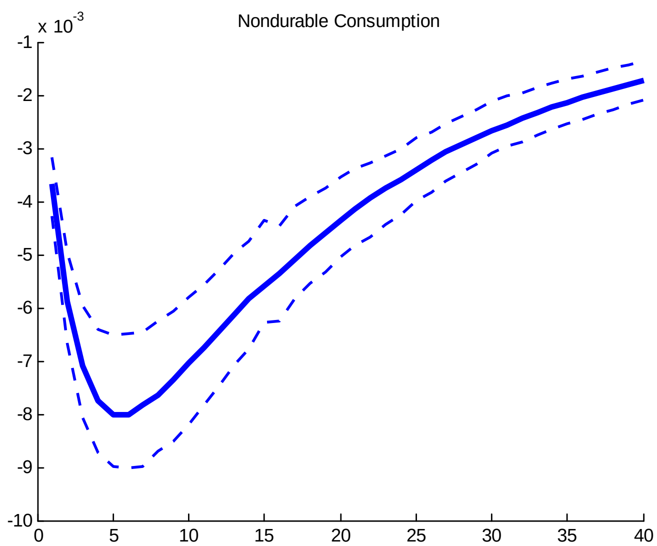


Figure 5a: IRF of Nondurable Consumption to a positive monetary policy shock

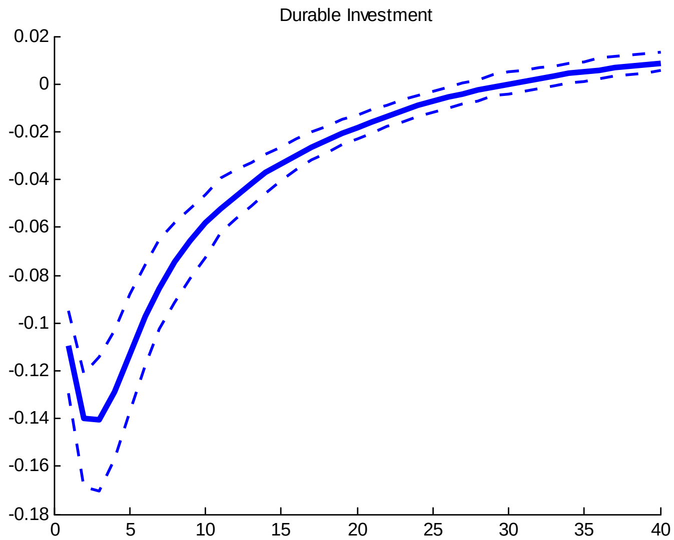


Figure 5b: IRF of Durable Investment to a positive monetary policy shock

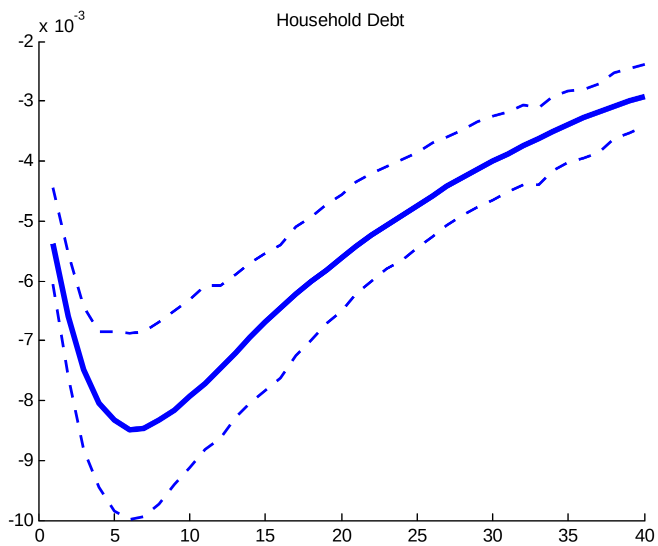


Figure 5c: IRF of Household Debt to a positive monetary policy shock

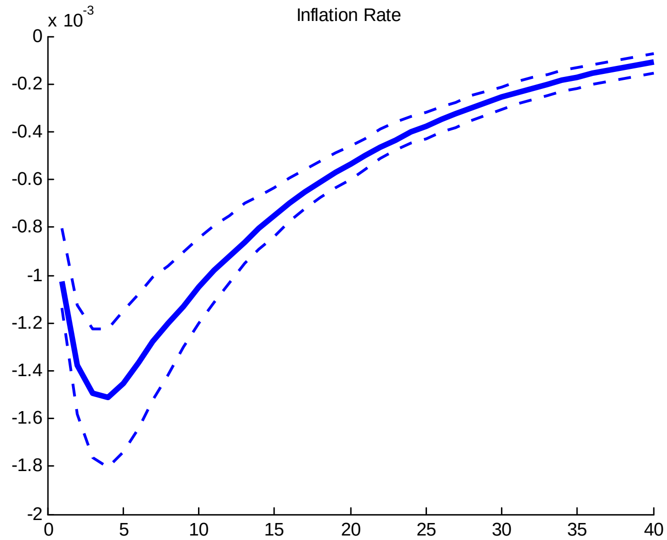
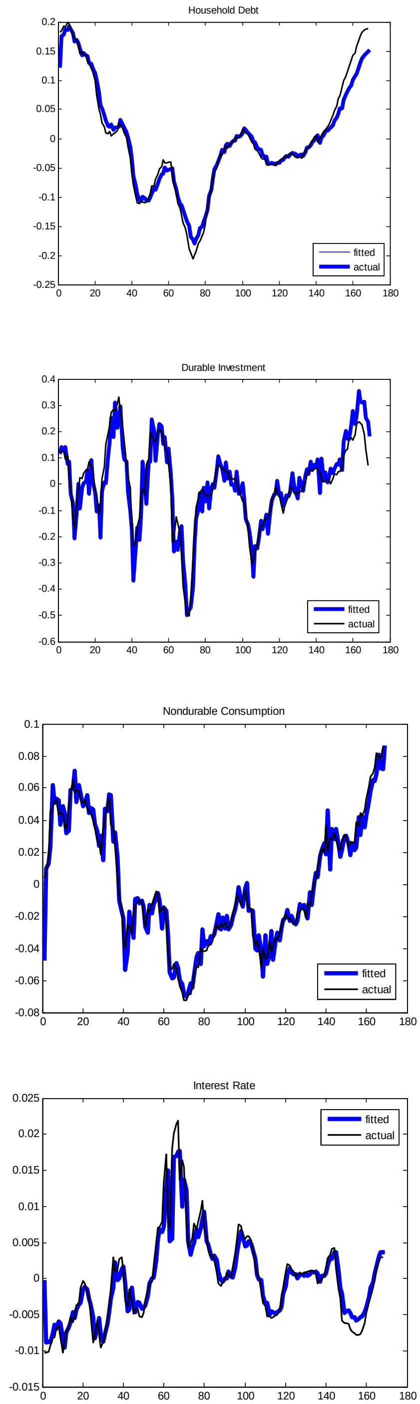


Figure 5d: IRF of Inflation Rate to a positive monetary policy shock

Figure 6. Fit of the model (full sample)



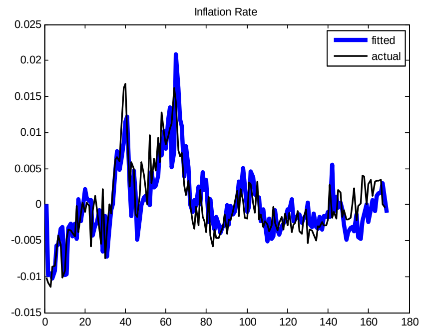
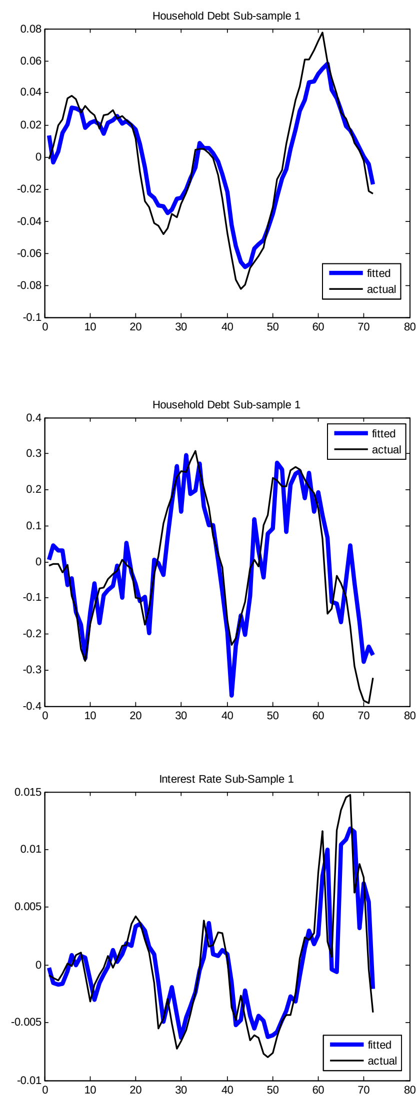


Figure 7. Fit of the model (sub-sample 1)



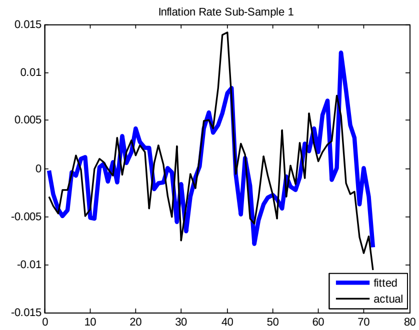
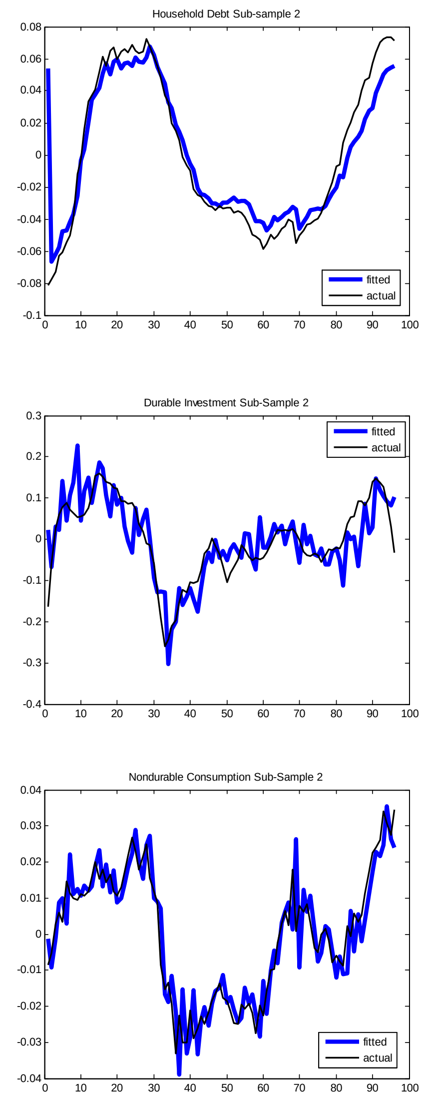
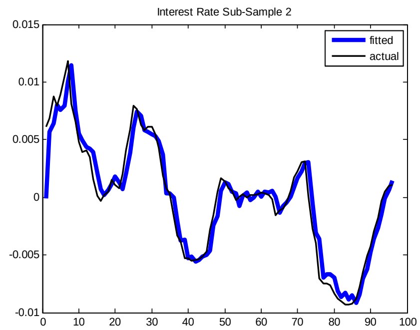
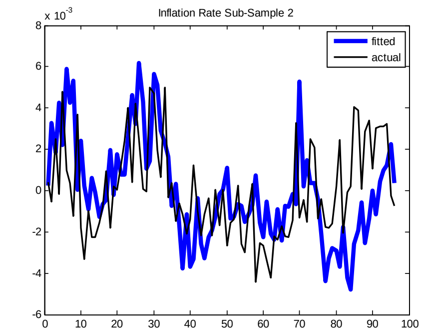


Figure 8. Fit of the model (sub-sample 2)




Interest Rate Sub-Sample 2
fitted
actual
0.015
0.01
0.005
0
-0.005
-0.01
0
10
20
30
40
50
60
70
80
90
100


x 10^-3
Inflation Rate Sub-Sample 2
fitted
actual
8
6
4
2
0
-2
-4
-6
0
10
20
30
40
50
60
70
80
90
100

Data

The dataset includes quarterly data on: nondurable consumption, residential fixed investment, total household debt, short-term nominal interest rate, consumer price inflation, GDP. The sample is 1965 Q1: 2006 Q4. A detailed description of the original data, their source and the transformation applied follows.

- Nondurable consumption: Real Personal Consumption Expenditure: Nondurable Goods (Billions of Chained 2000 Dollars); Source: Bureau of Economic Analysis.
- Residential Fixed Investment: Real Private Residential Fixed Investment; Source: Bureau of Economic Analysis.
- Total Household Debt: Total Outstanding Household Debt-Domestic Nonfinancial Sector. Source: Federal Reserve Bank, Flow of Funds.
- Short-term nominal interest rate: 3-month Treasury bill secondary market rate. Source: Federal Reserve Bank, Board of Governors.
- Consumer Price Inflation: Quarter-on-quarter log-difference, Gross Domestic Product, Implicit Price Deflator. Source: Bureau of Economic Analysis.
- Gross Domestic Product: Real Gross Domestic Product (Billions of Chained 2000 Dollars). Source: Bureau of Economic Analysis.

All series are seasonally adjusted. Nondurable consumption, residential fixed investment, household debt and GDP are expressed in per capita terms by dividing with the population over 16 (Civilian Noninstitutional Population, Source: Bureau of Labor Statistics). The nominal interest rate and the inflation rate are expressed on a quarterly basis, consistently with their definition in the model. The data are expressed in log.

Detrending. The model is a purely business cycle one, and therefore does not display any trend. Once the model is log-linearized around the deterministic steady-state, all variables can be treated as deviations around the mean (the steady state). Therefore, to make the data comparable with the model-generated series, a detrending procedure must be chosen. Following Smets and Wouters (2003), all variables are linearly detrended, while inflation and the nominal interest rate are detrended by the same linear trend in inflation.

The Deterministic Steady State

In this section we derive the steady-state version of the model equations. First, it is immediate to show that the collateral constraint always binds in equilibrium. In fact, by evaluating the Euler equation (18) in steady state, one obtains:

$$1 = \gamma R$$

or

$$R = \frac{1}{\gamma}$$

then, evaluating equation (10) in steady state gives:

$$\begin{aligned} \psi &= 1 - \beta R \\ &= 1 - \frac{\beta}{\gamma} > 0 \end{aligned}$$

where the last inequality follows from the crucial assumption about the two intertemporal discount factors:

$$\beta < \gamma$$

Therefore, the Lagrange multiplier ψ attached to the collateral constraint is strictly positive in steady state, which implies that the constraint holds with equality. Clearly, the result holds true in a sufficiently small neighborhood of the deterministic steady state; this allows to treat the collateral constraint as binding when solving the model up to a log-linear approximation.

Next, we turn to the computation of durable and nondurable consumption. We calibrate the parameter υ in the utility function in such a way to obtain a total amount of hours worked equal to 0.3 in equilibrium[18]. It is immediate to notice that under price and wage (perfect) flexibility, the two blocks of equations for price and wage setting modify substantially. First, when $\xi_W = 0$ and $\eta_t^W = 0$, all agents are allowed to change their wage every period. Therefore, the wage setting condition boils down to the usual, competitive equivalence between the real wage and the marginal rate of substitution between consumption and leisure. However, the presence of a wage markup drives a wedge between the two terms. Thus, the optimality condition for the impatient agent is replaced by:

$$-\frac{U_N}{U_C} = w = \frac{1}{1 + \lambda_{pc}}$$

where $w \equiv \frac{W}{P_c}$ is the real wage in terms of nondurable consumption, and

$$-\frac{U_N}{U_C} = wq = \frac{q}{1 + \lambda_{pd}}$$

Therefore, the relative price q is pinned down by the following equation:

$$q = \frac{1 + \lambda_{pd}}{1 + \lambda_{pc}}$$

Next, we turn to the computation of C and D. Evaluating (9) in steady state and using (4) and (6) gives:

$$\widehat{C}/\widehat{D} = \{q\left[1 - \beta\left(1 - \delta\right) - \left(1 - \chi\right)\psi\right]\}^{\eta}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1-\theta}\right) \tag{24}$$

[18] We are adopting the usual normalization that the total endowment of hours equals one.

By evaluating the collateral constraint - holding with equality - in steady state we obtain:

$$\frac{b}{\widehat{D}} = (1-\chi)\, q \tag{25}$$

Finally, the level of $\widehat{D}$ is pinned down using the impatient agent's budget constraint (3) together with (24) and (25):

$$\widehat{D} = \frac{wN}{\left(\widehat{C}/\widehat{D}\right) q + \delta - (1-R)(1-\chi)}$$

Then, clearly:

$$\widehat{C} = \left(\frac{\widehat{C}}{\widehat{D}}\right) \widehat{D}$$

Next we focus on the patient agent. Evaluating the market clearing conditions (22) and (23) in steady state and using the production functions gives:

$$Y_d = \omega\delta\widehat{D} + (1-\omega)\,\delta\widetilde{D} = \omega N_d \tag{26}$$

and

$$Y_c = \omega\widehat{C} + (1-\omega)\,\widetilde{C} = \omega N_c \tag{27}$$

where, by construction:

$$N_c + N_d = N = 0.3$$

Using (26):

$$\omega N_d = \omega\delta\widehat{D} + (1-\omega)\,\delta\widetilde{D}$$

Rearranging gives:

$$\widetilde{D} = \frac{\omega\left(N - \delta\widehat{D}\right)}{(1-\omega)\,\delta} \tag{28}$$

Analogously, using (27) yields:

$$\widetilde{D} = \frac{\omega\left(N - N_d - \widehat{C}\right)}{(1-\omega)\left(\widetilde{C}/\widetilde{D}\right)} \tag{29}$$

where the value of $\widetilde{C}/\widetilde{D}$ is obtained by using the patient agent's Euler equation (17):

$$\widetilde{C}/\widetilde{D} = [1-\gamma(1-\delta)]^{\eta}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1-\theta}\right) \tag{30}$$

Then, equating (28) and (29) and solving for N_d gives:

$$N_d = \frac{\delta\left(N - \widehat{C} + \widehat{D}\left(\widetilde{C}/\widetilde{D}\right)\right)}{\widetilde{C}/\widetilde{D} + \delta} \tag{31}$$

and,clearly:

$$N_c = N - N_d$$

Then, using either (28) or (29) one obtains $\widetilde{D}$. Finally, the level of $\widetilde{C}$ is immediately obtained by multiplying expression (30) by $\widetilde{D}$. Finally, the level of output in each sector can be easily obtained using (26) and (27).

4.1 Assessing Convergence in the RWMH algorithm

The model is solved up to a log-linear approximation around the deterministic steady state. Once the solution is obtained, the model can be cast in state-space form, and the likelihood function can be computed using the Kalman filter. More precisely, the posterior distributions can be computed once independent prior distributions are specified for each one of the structural parameters to be estimated.

Markov Chain Monte Carlo (MCMC) methods are used to simulate draws from an unknown target distribution, through the generation of a Markov chain, the stationary density of which is assumed to coincide with the target density. A natural question concerns the evaluation of convergence, and the definition of some convergence diagnostics. Following Robert and Casella (1998), one can distinguish between: (i) convergence of the MC to its stationary distribution (which implies exploring the correct distribution of interest and the whole space), (ii) convergence of empirical averages to the appropriate expected values (i.e. the posterior population moments) and (iii) convergence to *iid* sampling. This subsection briefly describes two approaches to the problem of evaluating convergence[19].

Geweke (1992) suggests an empirical evaluation method based on the following intuition. Consider a vector of parameters θ, and a function of interest $g(\theta)$. We are interested in estimating $g(\theta)$ based on the sample draws. For a sufficiently large number of draws, the estimate of $g(\theta)$ based on, say, the first half of the draws, should coincide with the estimate based on the last half. A difference in the two estimates indicates that (i) either too few draws have been taken, or that (ii) the effect of the initial - arbitrary - draw θ^0 is contaminating quite a large part of the draws. Therefore, the total number of draws, S, is divided into a given number of subsets. More precisely, after discarding a fraction S_0 of the initial draws as burn-in replications, the remaining S_1 are divided into, say three subsets: S_A, S_B, S_C. Then, the middle set of replications, S_B, is dropped out, in order to make it more likely for S_A and S_C to be independent of one another. Finally, denoting $\widehat{g_{S_A}}$ and $\widehat{g_{S_C}}$ the estimates of $E\left[g(\theta)|y\right]$ using S_A and S_C respectively, it is possible to construct the numerical standard errors of the two estimates as $\frac{\widehat{\sigma}_A}{\sqrt{S_A}}$ and $\frac{\widehat{\sigma}_C}{\sqrt{S_C}}$. Then a central limit theorem can be invoked to establish that

$$CD \to N(0,1)$$

where

$$CD = \frac{\widehat{g_{S_A}} - \widehat{g_{S_C}}}{\frac{\widehat{\sigma}_A}{\sqrt{S_A}} + \frac{\widehat{\sigma}_C}{\sqrt{S_C}}}.$$

[19] See Koop (2003) and Brooks and Gelman (1998).

The method suggested by Brooks and Gelman (1998) is a generalization of the original method of Gelman and Rubin (1992). The method assumes that m parallel chains have been simulated, each starting at a different point, with overdispersion of the starting points over the target distribution. Convergence is assessed by comparing *between* and *within* variances.