Certifier Competition and Product Quality*

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Abstract

This paper models the effect of rating agency competition on the quality of rated securities. The quality of the securities is unknown. In each of two periods, an issuer exerts unobservable effort to improve the quality of a security and can hire rating agencies to rate it. The rating agencies observe noisy signals of quality and assign their ratings simultaneously and non-cooperatively. Each agency can be honest or strategic. An honest agency always rates according to its signal. A strategic agency can request a bribe to issue an undeserved rating. I compare equilibria across a regime of competition between two rating agencies and a monopolistic regime. In both regimes, all available agencies are hired in equilibrium, so under competition more ratings are observed. However, competing agencies do not fully internalize the return of a reputation for being honest. Whenever strategic agencies are not very concerned about their reputation, competition can induce more issuer effort than monopoly. Otherwise, a monopolistic agency induces more effort. Finally, the model is extended to compare the equilibria with agencies observing identical signals and agencies observing conditionally independent signals.

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1. Introduction

The recent financial crisis put credit rating agencies in the public spotlight.[1] Favorable and undeserved ratings were blamed for encouraging issuers of structured products to sell extremely low quality securities. Unlike issuers of other financial assets such as corporate bonds, issuers of structured products could design their securities to meet the requirements of the rating procedures.[2] A recent report by the U.K. Financial Services Authority highlights this issue:

> While a corporate bond issuer can make only limited adjustments to its balance sheet to improve its rating ... an originator of a structured credit product has an incentive and flexibility to design them in such a way as to obtain maximal ratings.[3]

In these markets, however, undeserved ratings not only left investors uninformed, but also encouraged issuers to originate and securitize high-risk loans.

The lack of competition in the credit rating market was blamed for the low quality of the rating process. Regulators in the United States and the European Union have argued that increased competition among rating agencies is desirable. A recent SEC report illustrates the position of the U.S. Congress:

> In enacting the Rating Agency Act, Congress found that "the 2 largest credit rating agencies [Moody's and S&P] serve the vast majority of the market, and additional competition is in the public interest."[4]

Will an increase in the number of available rating agencies lead to more informative ratings? Will informative ratings induce socially efficient choices of investment in product quality? To answer these questions, I construct a two-period certification model that captures the relationship between credit ratings and the quality of rated products under different structures of the credit rating market. In every period, an issuer can exert effort to increase the quality of her security.

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[1] Ashcraft *et al.* (2011) and Benmelech and Duglosz (2009b) provide detailed accounts of the role of credit rating agencies in the recent financial crisis.

[2] See the Coburn Levin Senate Report, part V, section B, and Benmelech and Duglosz (2009a) for a description of the rating process of a structured finance product.

[3] Fennell and Medvedev (2011).

[4] SEC (2012).

Quality can be any characteristic of the security that affects its value, such as expected return or riskiness. Neither the issuer's effort nor the security's quality can be observed by the buyers. However, the issuer can hire rating agencies to observe a signal correlated with the security's quality and then assign a rating. Rating agencies are long-lived agents. They can be strategic or committed to honesty. Strategic agencies face a classic trade-off: inflate their ratings and increase current revenues, or rate honestly and preserve a reputation for honesty. In this context, I compare equilibria across a regime of competition between two rating agencies and a monopolistic regime.

If two agencies are available in the market, I show that the issuer hires both agencies. This result matches the empirical evidence for structured finance ratings. Benmelech and Duglosz (2009b) show that over the period from 2004 to 2007 most structured finance tranches were receiving more than one rating. Rating agencies are not textbook competitors selling substitute goods. Rather, they are experts with competing opinions.

When reputation incentives are weak, that is, when rating agencies heavily discount future revenues, I show that competing agencies can provide more informative ratings than a monopolistic agency. As a result, in the competition regime the issuer has a stronger incentive to invest in the quality of her security. When reputation provides a strong discipline, a monopolistic agency induces more investment in quality.[5]

Even when reputation motives are weak and strategic agencies are likely to assign undeserved ratings, under the competition regime buyers have the opportunity to compare independent ratings. The presence of a low rating, for example, makes a high rating look suspicious. This opportunity to compare ratings ensures that buyers are better informed and has an indirect effect on the issuer's incentive to invest in quality. At the same time, a monopolistic agency has stronger incentives to maintain a reputation for honesty in order to induce the issuer to invest in quality. When buyers expect more effort from the issuer, they are willing to pay a higher price for a security with favorable ratings. As long as the rating fee is proportional to the

[5]Monopoly and competition might also differ in the total amount of information that all agencies obtain. I abstract from this issue and assume that in the two regimes the total amount of information observed by the rating agencies is constant.

expected quality of the security, a higher security price allows the agency to request a higher fee from the issuer. If multiple agencies rate the same security, each and every agency's reputation determines the issuer's decision of effort. Competing agencies do not fully internalize the effect of their reputation. Ultimately, this externality reduces the competitors' incentives to maintain a reputation for honesty.

My results imply that competition might increase the incentive to invest in the quality of a security in markets where rating agencies are only weakly disciplined by reputation motives. One such example are markets where a large number of new securities are issued in a short amount of time, as in the case of asset-backed securities in the years preceding the financial crisis. In these markets, many ratings are assigned before the investors acquire the information necessary to evaluate the ratings' quality. At the same time, competition among rating agencies is not desirable in every rating market. In markets where the volume of ratings is constant over time, competition might be detrimental to the quality of the ratings and the investments in securities' quality.

In an extension of my model, I characterize the equilibria in the case of competition between rating agencies observing identical signals of quality. When the signals are closely correlated with the quality of the security, identical signals result in stronger incentives to invest in quality than conditionally independent signals.

This paper presents the first model to simultaneously consider rating agencies' reputational incentives and issuers' investments in the quality of their securities. These issues have, so far, been considered separately. The certification models for products of endogenous quality developed by Albano and Lizzeri (2001) and Donaldson and Piacentino (2012) abstract from the reputational incentives of certifiers. Both papers conclude that a monopolistic regime in the market for certification induces inefficient amounts of investment in product quality. Under a monopolistic regime, firms under-invest in product quality, anticipating that any potential increase in revenues would be captured by the rating fees. A regime of competition would ensure lower certification fees and could mitigate this sort of hold-up problem. In my model, investment in a security's quality takes place only after the agencies set their fees. As a result, this inefficiency is not present.

A growing body of literature in quality certification considers competition among strategic certifiers. The focus is on certifiers concerned with their reputation for rating honestly. Unlike my model, this literature considers the certification of products of exogenous quality. In Bouvard and Levy (2012), firms can hire more than one rating agency to rate their security. The authors are concerned with the possibility of rating agencies developing different reputations among firms and buyers. They find, in line with my results, that reputation provides a weaker discipline for raters under competition than under monopoly. Strausz (2005), Lo (2010), and Camanho and Deb (2012) limit the firms to hiring, at most, one rating agency. All these models predict that monopoly always ensures more informative ratings than competition among certifiers.

In the model developed in this paper, reputational incentives are always desirable. A strategic agency that worries about its reputation will honestly report the signal privately observed. Reputational incentives are not always socially desirable. In Mariano (2011), rating agencies improve their reputation for expertise by disregarding private information and assigning ratings based on public information; competition among agencies exacerbates the inefficiency and results in informational losses.

In my model, the issuer does not gain from hiding unfavorable ratings as opposed to other models in which different assumptions give rise to rating shopping. Rating shopping refers to the issuers' strategy to cherry-pick the most favorable ratings. In Skreta and Veldkamp (2009) and Bolton et al. (2012), issuers hide unfavorable ratings to deceive naïve investors. In Sangiorgi et al. (2009), issuers publish only the most favorable ratings to attract investors who are legally required to invest only in high-rated securities. Rating shopping does not take place in my model because buyers are considered to be fully rational, and cannot be systematically deceived in equilibrium.

Finally, the informativeness of the signal observed in my model by the rating agencies is exogenous. The incentives for the raters to acquire information has been considered, among others, in Bizzotto (2012), Bouvard and Levy (2012), and Kashyap and Kovrijnykh (2013).

The theoretical literature on rating agencies is matched by a growing number of empirical studies.[6] These papers address questions including whether credit ratings influence the price of rated products, and the effect of new agencies on the quality of ratings by incumbents. Using a sample of residential mortgage-backed securities issued before the financial crisis, Ashcraft *et al.* (2012) show that credit ratings did influence prices. Becker and Milbourn (2011) measure the effect of the sequential entry of Fitch in separate corporate-ratings markets on the quality of the ratings from incumbent agencies. They show that when a new competitor enters the market, the informative content of the incumbents' ratings is reduced. They explain their finding by suggesting that reputational incentives decrease when competition increases. Xia (2012) considers the entry of Egan-Jones Rating Company, an investor-paid rating agency, in the market for corporate rating, and comes to the opposite conclusion. This study shows that the ratings of the incumbents became more strict and more responsive to information after the entry of a new competitor. Doherty *et al.* (2012) study the entry of S&P in the market for insurance ratings and focuses on the ratings of the new agency. They show that S&P set higher standards than incumbent agencies for securities that received the same rating.

The rest of the paper is structured as follows. Section 2 presents the monopoly case. The competition case is discussed in Section 3. Section 4 compares the different equilibria obtained in Section 2 and 3. Section 5 discusses an extension. Section 6 concludes. All proofs are contained in the appendix.

2. Monopoly

In this section, I characterize the equilibria in a market with a single rating agency. In the next sections, I will compare the equilibria in a competition regime to the monopoly benchmark.

[6] A comprehensive review of the theoretical literature on credit rating agencies is beyond the scope of this section; White (2010), Jeon ad Lovo (2013) and Dranove and Jin (2010) provide comprehensive reviews of the subject.

2.1 The Model

I characterize the perfect Bayesian equilibria of a two-period game. In every period, a new issuer has an indivisible unit of a security of quality $q_t \in \{B,G\}$ with $N \geq 2$ potential buyers. A monopolistic rating agency is the only agent active in both periods. The security is worthless to the issuer and the agency. To the buyers, the security is worth 1 if $q_t = G$ and 0 otherwise. The security's quality is endogenous and unobservable.

At the beginning of period $t \in \{1,2\}$, the rating agency announces its fee $\phi_t^M \geq 0$ to rate the security. The issuer decides whether to hire the rating agency. The agency is hired to assign a rating $r_t^M \in \{g,b\}$ identical to its signal of quality to be observed later on. Only after deciding whether to hire the agency does the issuer choose an effort $e_t \in [0,1]$. Let the issuer's effort cost be $c(e_t)\colon [0,1] \rightarrow \mathbb{R}_+$. Effort ensures $Pr\{q_t = G\} = e_t$ and satisfies $c \in C^1$, c', $c''' \geq 0$, and $c'' > 0$, for all $e_t \in [0,1]$.[7] Moreover, $c(0) = c'(0) = 0$ and $c'(1) > 1$. Once the security's quality is realized, the rating agency and the issuer observe at no cost a signal $s_t^M \in \{g,b\}$ correlated with q_t as follows:

$$Pr\{s_t^M = g\} = \begin{cases} 1 & \text{if } q_t = G, \\ \pi & \text{if } q_t = B, \end{cases}$$

for some $\pi \in (0,1)$. This order of actions is particularly suited to describe the rating process of structured finance products. These products are often modified (for example, by adding credit enhancements) after one or more agencies are hired to rate them. In general, the order describes any market in which the certifiers can commit to their certification fees before the sellers decide how much to invest in quality of their products.

The rating agency can be one of two types. With probability $\mu_1 \in (0,1)$, the rating agency is an *honest* type that always assigns a rating identical to the signal of quality. With probability $1-\mu_1$, the agency is *strategic*. Upon observing a b signal, a *strategic* agency can request a monetary bribe $\beta_t^M > 0$ to renege on the original contract. If the issuer pays the bribe, then $r_t^M = g$. Let $h_t^M \in [0,1]$ denote the probability that a *strategic* agency does not request a bribe upon observing a b signal. The initial reputation μ_1 satisfies the following condition.

[7] The unusual assumption $c''' \geq 0$ ensures the uniqueness of the issuer's optimal choice of effort.

Assumption 1. $\mu_1 > \frac{c''(0)-1+\pi}{c''(0)(1+\pi)/(1-\pi)-1}$.

In equilibrium, Assumption 1 ensures that in every period the issuer has an incentive to exert a strictly positive amount of effort. This assumption rules out equilibria in which the issuer exerts no effort.

Finally, the issuer can decide whether to publish the rating or conceal it.[8] The potential buyers only observe the rating, or the lack thereof, and the rating fee. They simultaneously bid for the security. Let $bid^i(r_t^M, \phi_t^M)$ denote buyer i_t's bid. The winning bid determines the security's price $p(r_t^M, \phi_t^M | \mu_t^M, h_t^{M*})$. At the end of every period, the quality of the security is observed by all the agents. Figure 1 summarizes the timeline.

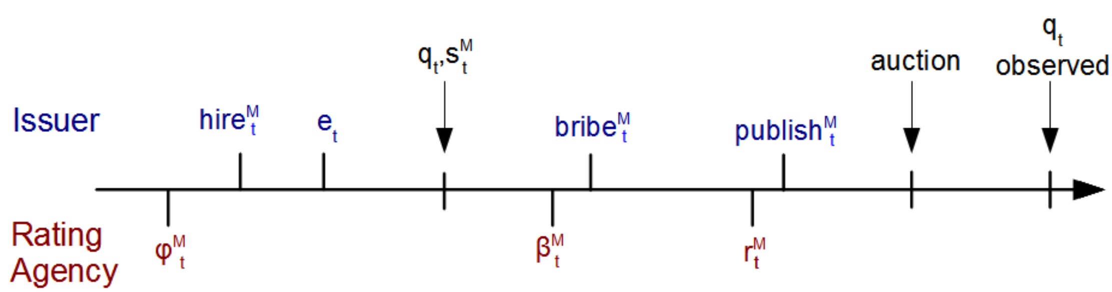

FIGURE 1. Timeline: Monopoly.

The issuer and buyers active in the second period observe the rating assigned as well as the quality of the security in the first period. $\mu_2(r_t^M, q_t)$ denotes their updated belief about the agency's type.

The equilibrium concept of Perfect Bayesian Equilibrium does not restrict out-of-the-equilibrium-path beliefs. Nevertheless, I impose a restriction on out-of-equilibrium beliefs to rule out equilibria in which buyers arbitrarily pay no attention to the rating.

Assumption 2. Upon observing an out-of-equilibrium rating fee ϕ_t^M, the issuer's and buyers' beliefs about the agency's type are identical to their prior beliefs. Moreover, upon observing an out-of-equilibrium rating, buyers hold beliefs consistent with the effort choice (e_t^*) and with the agency's rating strategy (h_t^{M*}).

[8]If a rating is not published, I use the notation $r_t^M = \emptyset$.

The payoff functions complete the model. The payoff of issuer I_t amounts to

$$U_t^I = p(r_t, \phi_t^M | \mu_t, h_t^{M*}) - I_t^\phi \phi_t^M - A_t^\beta I_t^\beta \beta_t^M - c(e_t),$$

where $I_t^\phi, I_t^\beta \in \{0,1\}$ denote, respectively, the issuer's decision to hire the agency and to pay a bribe, while $A_t^\beta \in \{0,1\}$ denotes the agency's decision to request a bribe. If buyer n purchases the security, the payoff amounts to

$$U_t^n = \mathbf{1}_{\{q_t = G\}} - p(r_t^M, \phi_t^M | \mu_t^M, h_t^{M*}).$$

If the buyer does not purchase the security, $U_t^n = 0$. Finally, the rating agency's payoff consists of a discounted sum of fees and bribes:

$$\begin{aligned} U^M &= u_1^M(\phi_1^M, \beta_1^M, A_1^\beta) + \delta u_2^M(\phi_2^M, \beta_2^M, A_2^\beta), \\ u_1^M(\phi_1^M, \beta_1^M, A_1^\beta) &= I_t^\phi \phi_t^M + A_t^\beta I_t^\beta \beta_t^M, \end{aligned}$$

for some $\delta > 0$. I allow the discount factor to be larger than 1 because the second period is a reduced form of all future periods in which the rating agency is active.

2.2 The Equilibria

I first consider the equilibrium of a single-period game, in which a *strategic* agency rates honestly with some exogenous probability $\overline{h} \in [0,1)$. Buyers form their beliefs by observing the rating fee and the rating, or the lack thereof. The price paid for the security depends on the buyers' beliefs about its quality as described in the next lemma.

Lemma 1. *The price of the security equals its expected value:*

$$\begin{aligned} p(g, \phi_t^M | \mu_t, \overline{h}) &= \frac{e_t^*(\mu_t, \overline{h})}{e_t^*(\mu_t, \overline{h}) + (1 - e_t^*(\mu_t, \overline{h})) Pr\{r_t^M = g, \phi_t^M | q_t = B, \mu_t, \overline{h}\}}, \\ p(b, \phi_t^M | \mu_t, \overline{h}) &= p(\emptyset, \phi_t^M | \mu_t, \overline{h}) = 0, \end{aligned} \tag{2.1}$$

where $e_t^*(\mu_t, \overline{h})$ denotes the equilibrium effort of the issuer.

An unfavorable rating and a lack of rating lead to the same price, so the issuer is indifferent about the publication of an unfavorable rating.[9]

The issuer decides how much effort to devote to a security's quality by weighing a deterministic cost and a stochastic benefit. If the agency is not hired, the issuer has no reason to exert costly effort. If instead the agency is hired, effort increases the probability of obtaining a favorable rating without incurring the cost of a bribe. The expected amount of the bribe determines the issuer's choice of effort. In equilibrium, if a *strategic* agency requests a bribe, it sets the highest bribe that the issuer is willing to pay. Therefore, by Lemma 1, the bribe amounts to

$$\beta_t^{M*}(\mu_t,\overline{h})=p(g|\mu_t,\overline{h}).$$

In equilibrium, the bribe is paid whenever it is requested. If the issuer pays to obtain a rating for the security, her payoff equals

$$U_t^I=(e_t+(1-e_t)\pi)p(g|\mu_t,\overline{h})-c(e_t)-\phi_t^M. \tag{2.2}$$

The issuer's choice of effort maximizes the utility defined in (2.2):

$$e_t^*(\mu_t,\overline{h})=\mathbf{e}((1-\pi)p(g|\mu_t,\overline{h})),\text{ where }\mathbf{e}(.):=c'^{-1}(.). \tag{2.3}$$

The issuer's effort and security price are mutually consistent. Assumption 1 implies that there is at most one unique positive level of effort, and a corresponding price, which satisfy the two equations.

Lemma 2. *A pair $e_t^*(\mu_t,\overline{h}),p(g|\mu_t,\overline{h})>0$ that satisfy (2.1) and (2.3) exists for any $\overline{h}\in[0,1]$ iff*

$$\mu_t(\phi_t)>\underline{\mu}^M:=1/(1-\pi)-1/c''(0). \tag{2.4}$$

These $e_t^*(\mu_t,\overline{h})$ and $p(g|\mu_t,\overline{h})$ are unique. $e_t^*(\mu_t,\overline{h})=p(g|\mu_t,\overline{h})=0$ satisfy (2.1), (2.3) for any $\mu_t(\phi_t)\in[0,1]$.

Lemma 2 states that the presence of a rating agency with a high reputation is necessary but not sufficient to ensure an issuer's effort. Nevertheless, the next assumption rules out equilibria in which the issuer and the buyers coordinate on $e_t^*(\mu_t,\overline{h})=p(g|\mu_t,\overline{h})=0$.

[9] As in equilibrium the security price does not depend on the rating fee, I drop ϕ_t from the argument of $p(r_t,\phi_t)$ in the rest of the section.

Assumption 3. Whenever the agency's reputation satisfies (2.4), the issuer and the buyers coordinate on $e_t^*(\mu_t,\overline{h})>0$ *and* $p(g|\mu_t,\overline{h})>0$.

A rating is valuable because it allows a profitable investment in quality. The monopolist agency, regardless of its type, extracts the entire surplus generated by investment in effort with its rating fee.[10]

Lemma 3. *In equilibrium, the rating agency, regardless of its type, requires the highest fee that the issuer is willing to pay:*

$$\phi_t^{M*}(\mu_t,\overline{h})=\Big(e_t^*(\mu_t,\overline{h})+(1-e_t^*(\mu_t,\overline{h}))\pi\Big)p(g|\mu_t,\overline{h})-c(e_t^*(\mu_t,\overline{h})).$$

Lemma 3 implies that the rating fee in equilibrium does not reveal any information about the type of the agency and concludes the description of the single-period equilibrium. Before considering the complete two-period model, I illustrate the effect of the expected honesty $\overline{h}$ on the investment in effort for a quadratic cost of effort.

Quadratic cost example. *Let* $c(e)=e^2$. *Figure 2 shows equilibrium price and effort for* $\overline{h}=0$ *and* $\overline{h}=1$.

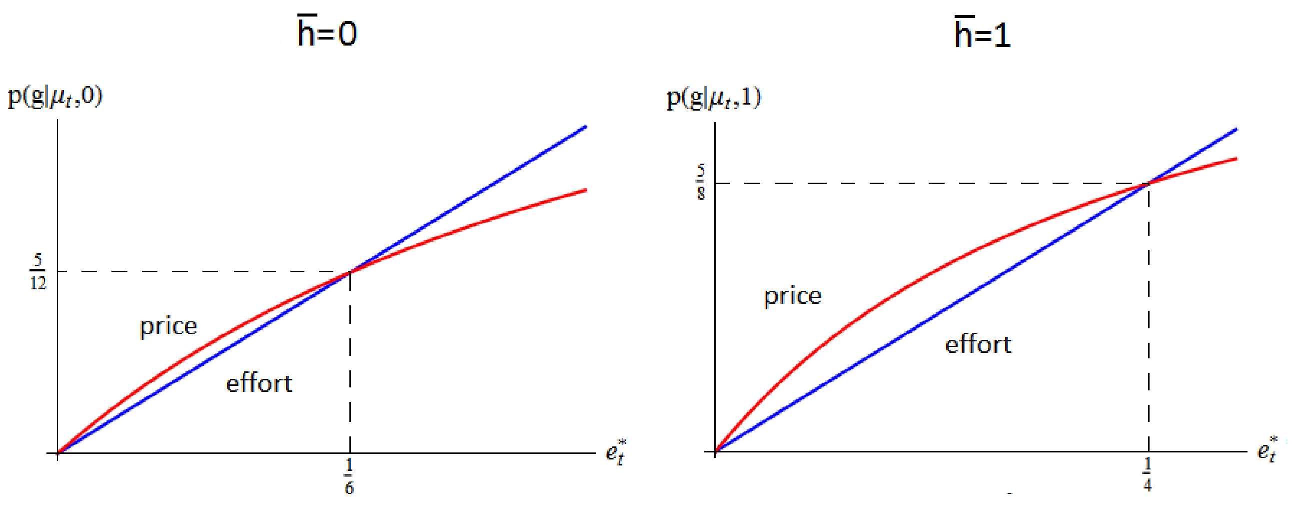

Figure 2. Effort choices and prices for $\pi=1/5$ and $\overline{\mu}=9/10$.

[10] I rule out equilibria in which the agency follows a weakly dominated strategy and requests a fee larger than what the issuer is willing to pay for a rating.

The intersections of the two curves correspond to the equilibria of the static game. For larger $\overline{h}$ the equilibrium with effort is characterized by a more effort and a higher price. The honesty of the agency determines the price that buyers are willing to pay for any expected level of effort. Indirectly, it also determines the equilibrium choice of effort.

I can now characterize the probability that a strategic agency will request a bribe. A *strategic* agency does not care about its reputation after the last period, and so requests a bribe whenever $s_2^M = b$. In the first period a *strategic* agency is faced with a trade-off: to collect a bribe or to assign an unfavorable rating in order to maintain a good reputation. In the last period the reputation determines the expected payoff for a strategic agency. This payoff is

$$u_2^{Ms}(\mu_2^M, h_2^{M*}) = \phi_2^*(\mu_2^M, h_2^{M*}) + Pr\{s_2 = b | e_2^*\} \beta_2^*(\mu_2^M, h_2^{M*}), \text{ where } h_2^{M*} = 0.$$

If the agency is hired in the first period, its reputation is updated to $\mu_2^M(r_1^M | h_1^{M*})$.[11] The updated reputation equals

$$\mu_2(r_1^M | h_1^{M*}) = \begin{cases} \frac{\mu_1}{\mu_1 + (1-\mu_1) h_1^{M*}} := \mu^b(h_1^{M*}) & \text{if } r_1^M \in \{b, \emptyset\}, \\ \frac{\mu_1 \pi}{\pi + (1-\pi)(1-\mu_1)(1-h_1^{M*})} := \mu^g(h_1^{M*}) & \text{if } r_1^M = g \text{ and } q_1 = B, \\ \mu_1 & \text{if } r_1^M = g \text{ and } q_1 = G. \end{cases} \tag{2.5}$$

Assumption 1 ensures that $\mu^g(0) > \underline{\mu}^M$. As $\mu^g(0)$ is the worst possible reputation, in every period, positive effort can be sustained in equilibrium. Proposition 4 characterizes the equilibria of the entire game.

Proposition 4. *In equilibrium, the rating agency is hired in every period. In the last period $h_2^{M*} = 0$, while in the initial period $h_1^{M*} = 0$ iff $\delta \leq \overline{\delta}^M$, where*

$$\overline{\delta}^M := \frac{\beta_1^M(\mu_1, 0)}{u_2^{Ms}(\mu^b(0), 0) - u_2^{Ms}(\mu^g(0), 0)}.$$

For $\delta > \overline{\delta}^M$, h_1^{M*} satisfies the implicit function

$$\delta u_2^{Ms}(\mu^b(h_1^{M*}), 0) = \beta_1^M(\mu_1, h_1^{M*}) + \delta u_2^M(\mu^g(h_1^{M*}), 0).$$

[11] If in equilibrium the agency is not hired in the first period, then $\mu_2^M(r_1^M | h_1^{M*}) = \mu_1$.

For a small discount factor ($\delta \leq \overline{\delta}^M$), reputation motives do not discipline a *strategic* rating agency. In contrast, for $\delta > \overline{\delta}^M$, a *strategic* rating agency rates honestly with a positive probability. The following corollary describes how the rating strategy depends on δ.

Corollary 5. *$\partial h_1^{M*}/\partial\delta > 0$ for $\delta > \overline{\delta}^M$ and $\lim_{\delta\to\infty} h_1^{M*} = 1$. Moreover $\partial\overline{\delta}^M/\partial\mu_1 > 0$.*

For larger values of δ, the *strategic* type mimics more closely the *honest* type. Corollary 5 also states that a *strategic* agency is more likely to request a bribe in the first period if the agency has a better reputation. I characterize the equilibrium strategies of the issuer and the buyers in case of quadratic cost function.

Quadratic cost example. *Figure 3 shows the probability that a strategic agency does not request a bribe upon observing $s_1^M = b$. As stated in Corollary 5, for larger values of δ, a strategic agency more closely mimics the ratings of the honest type. Figure 4 shows the equilibrium level of the issuer's effort as a function of the rating agency's discount factor. As h_1^{M*} increases, the corresponding level of effort increases.*

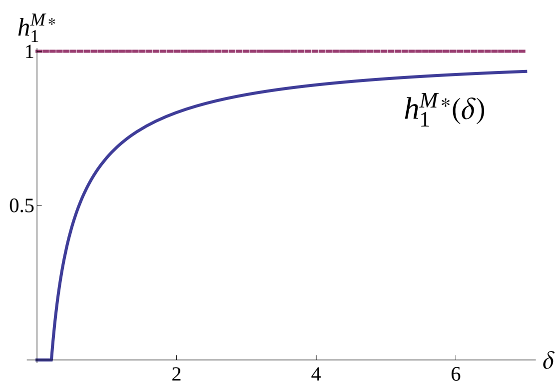

Figure 3. h_1^{M*} as a function of δ, for $\mu_1 = 9/10$ and $\pi = 1/5$.

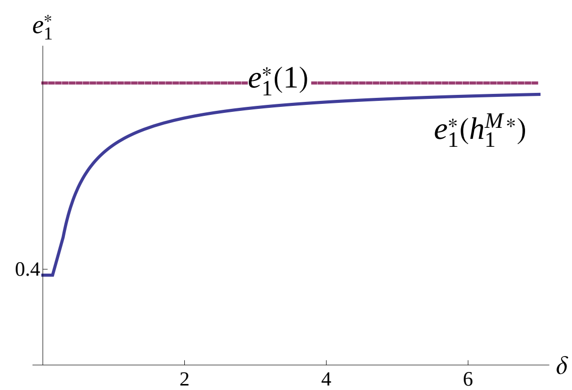

Figure 4. e_1^* as a function of δ, for $\mu_1 = 9/10$ and $\pi = 1/5$.

3. Competition

In this section, I present the equilibria in a market with two rating agencies that rate simultaneously and non-cooperatively.

3.1 The Model

In the competition regime, two rating agencies, denoted A^I and A^{II}, are present on the market. The types of the agencies are independent, and every agency is an *honest* type with probability μ_1. The two agencies act simultaneously and the sequence of actions is identical to the monopoly model. The issuer can hire one agency or both, and the agencies observe two conditionally independent signals, s_t^I and s_t^{II}, distributed as the monopoly one. Figure 5 describes the timeline. Assumption 1, 2, and 3 hold also under competition, and the payoffs are defined as in the monopoly case.

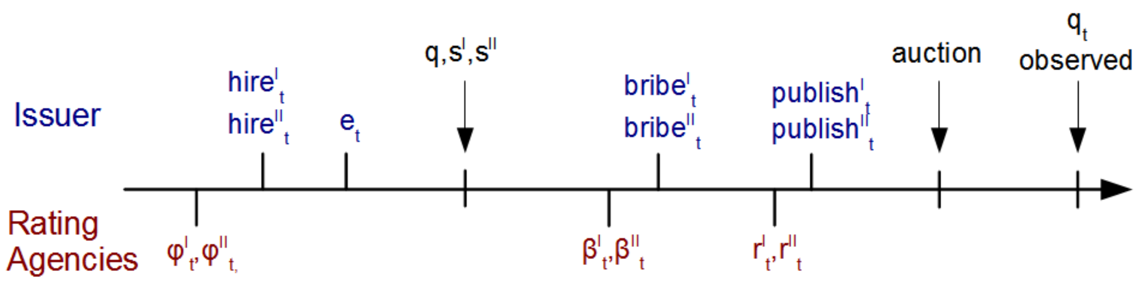

Figure 5. Timeline: Competition.

3.2 The Equilibria

I first consider a single-period model in which the *strategic* agencies rate honestly with probabilities $\overline{H}:=[\overline{h}^I,\overline{h}^{II}]\in[0,1)^2$. When two rating agencies are available, the issuer can hire one, two or none. The next lemma ensures that in equilibrium both rating agencies are hired.

Lemma 6. *In equilibrium, the issuer hires both rating agencies. Accordingly, two favorable ratings are necessary to obtain a positive price:*

$$\begin{aligned} p(g,g,\Phi_t|M_t,\overline{H}) &= \frac{e_t^*(\Phi_t)}{e_t^*(\Phi_t)+(1-e_t^*(\Phi_t))Pr\{R_t=[g,g]|q_t=B,M_t,\overline{H}\}}, \\ p(R_t,\Phi_t|M_t,\overline{H}) &= 0 \textit{ if } r_t^i\in\{\emptyset,b\} \textit{ for some } i\in\{I,II\}. \end{aligned} \tag{3.1}$$

Multiple features of my model ensure that the issuer will not hire only a single agency.[12] First of all, an additional rating cannot decrease the price paid for the security because the issuer can hide any unfavorable ratings. Moreover, Assumptions 2 and 3 ensure that two favorable ratings are a stronger signal that $q_t{=}G$ than a single favorable rating. Finally, the rating agencies can observe the quality signal at no cost. This extreme assumption captures the low marginal cost of the rating process. Consider an hypothetical equilibrium in which the issuer is expected to exert effort and hire one agency. Agencies would compete to be hired and lower their fees. Fees would be low enough to make it convenient for the issuer to hire a second agency.

As the issuer is expected to hire both agencies, she is indifferent about publishing unfavorable ratings. If an unfavorable rating is published, buyers infer that $q_t{=}B$. If less than two ratings are published, the buyers correctly infer that the issuer is hiding one or more unfavorable ratings, and therefore $q_t{=}B$. This result contrasts with the models of rating shopping developed by Bolton *et al.* (2012) and Skreta and Veldkamp (2009)). In these models, hiding bad ratings is profitable because some buyers are naïve and do not suspect that only the best ratings are published. To my knowledge, Bouvard and Levy (2012) is the only other model in which a seller can hire more than one rating agency, and buyers are fully rational. In their model all the raters are hired if the cost for rating agencies to obtain a quality signal is small enough.[13]

A security price larger than zero requires two favorable ratings. Accordingly, the issuer bribes agency A^i only if the other agency observes a favorable signal ($s_t^{-i}{=}g$), or if the two agencies can be bribed at a total cost not exceeding the value of the favorable ratings ($\beta_t^I+\beta_t^{II}\leq p(g,g|M_t,\overline{H})$). As a result, a *strategic* agency faces a trade-off. A small bribe is paid even if the issuer needs to bribe the other agency at the same time, while a high bribe is paid only if the other agency observes a favorable signal. The next lemma characterizes the choice of bribes.

Lemma 7. *Either both agencies request "high" bribes $\beta_t^{I*}{=}\beta_t^{II*}{=}p(g,g|M_t,\overline{H})$ or they both request "low" bribes $\beta_t^{I*},\beta_t^{II*}{:}\beta_t^{I*}+\beta_t^{II*}{=}p(g,g|M_t,\overline{H})$.*

[12] As in monopoly, the security price does not depend on the rating fee, so I drop Φ_t from $p(R_t,\Phi_t)$.

[13] The other models of competition between rating agencies (Strausz (2005), Lo (2010), Camanho and Deb (2012), and Donaldson and Piacentino (2012)) exogenously limit the issuer to hiring only one agency.

In equilibrium, the rating agencies coordinate their bribes. A “low” bribe $\beta_t^i < p(g,g|M_t,\overline{H})$ maximizes A^i's expected revenue only if the other agency requests a low bribe $\beta_t^{-1} = p(g,g|M_t,\overline{H}) - \beta_t^i$. Moreover, low bribes are the best response only if every agency believes its competitor is strategic with a large enough probability. Rating agencies learn more about each others' types than buyers and issuer do. Let $\mu_t^{-i(i)}$ denote A^i's belief about the competitor's type, at the beginning of period t. The next lemma provides a necessary condition for low bribes to be selected in equilibrium.

Lemma 8. *Bribes $\beta_t^{I*}, \beta_t^{II*} : \beta_t^{I*} + \beta_t^{II*} = p(g,g|M_t,\overline{H})$ are selected in equilibrium only if*

$$\mu_t^{j(i)} \leq \frac{1-2\pi}{1-\pi}.$$

I proceed to characterize the single-period equilibrium in case the agencies request high bribes. By hiring the two agencies, the issuer ensures an expected payoff equal to

$$U^I = (e_t^* + (1-e_t^*)\pi^2) p(g,g|M_t,\overline{H}) - c(e_t^*) - \phi_t^I - \phi_t^{II}. \tag{3.2}$$

The choice of effort maximizes the expected payoff of the issuer:

$$e_t^*(\Phi_t) = \mathbf{e}((1-\pi^2) p(g,g|M_t,\overline{H})). \tag{3.3}$$

As stated in the next lemma, Assumption 1 ensures that an equilibrium with positive effort exists for any $\overline{H}$. As both ratings are necessary, I only consider equilibria in which the two rating agencies set identical rating fees.

Assumption 4. I consider only equilibria in which $\phi_t^I = \phi_t^{II}$ $\forall t$.

With their fees, the rating agencies can extract the entire surplus generated by the issuer's effort, as stated in the next lemma.

Lemma 9. *In the single-period game's equilibrium, the issuer exerts positive effort. The rating agencies require the highest fee that the issuer is willing to pay:*

$$\phi_t^I = \phi_t^{II} = [(e^* + (1-e^*)\pi^2) p(g,g|M_t,\overline{H}) - c(e^*)]/2. \tag{3.4}$$

In Appendix A, I characterize the equilibrium of the one-period game in which the rating agencies coordinate on “low” bribes. The next lemma states that the low-bribes equilibrium is characterized by less effort than the high-bribes one.

Lemma 10. *For any* $\overline{H}\in[0,1)^2$*, the equilibrium level of effort is lower if rating agencies coordinate on “low” bribes than in the case of high bribes.*

Low bribes induce lower effort for two reasons. First, the issuer can simultaneously bribe the two agencies only if the bribes are low. Therefore, when agencies request low bribes, a low-quality security has a larger probability of receiving two favorable ratings than in the case of high bribes. This results in a lower price for a security that receives two favorable ratings than in case of agencies requesting high bribes. A lower expected price, in turn, reduces the issuer’s incentive to invest in quality. Moreover, obtaining favorable ratings when the signals are unfavorable is cheaper under low bribes. By reducing the difference between the payoffs following favorable and unfavorable signals, low bribes reduce even more the incentive to invest in effort.

Quadratic cost example. *Figure 6 shows the equilibrium price following two favorable ratings and the issuer’s effort, when agencies coordinate on high and low bribes.*

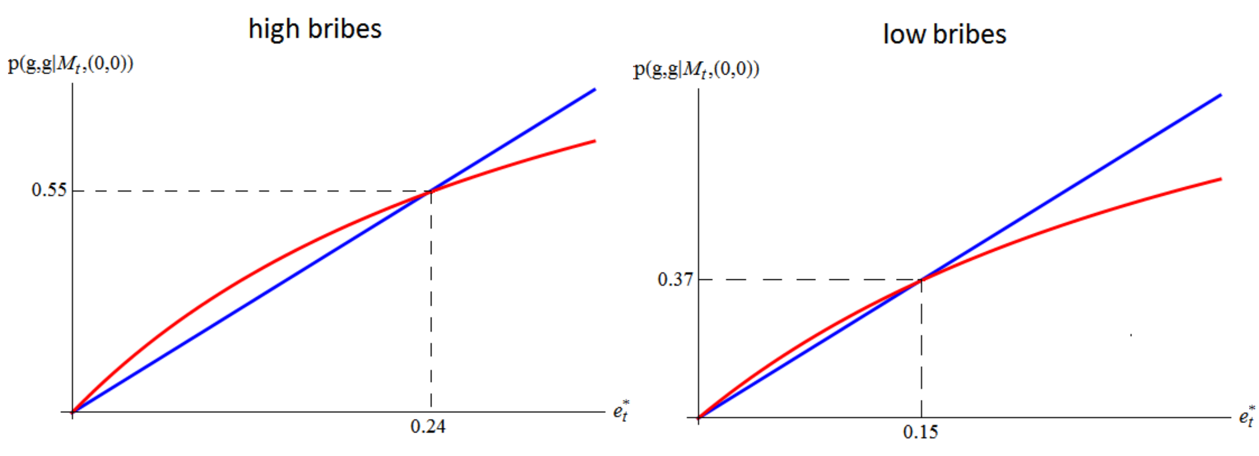

Figure 6. Equilibrium effort choice and market price for $\pi=1/8$ and $\mu_t^I=\mu_t^{II}=7/10$.

When the agencies are expected to request low bribes: (i) the buyers pay a lower price for any expected e^* *and (ii) the issuer exerts less effort for any expected price.*

Consider the equilibria of the two-period game. By Lemma 8, low bribes are mutually consistent only if the agencies' reputations satisfy $\mu_t^{-i(i)} \leq (1-2\pi)/(1-\pi)$. In general, low bribes ensure larger payoffs than high bribes if every agency believes that the competitor is *strategic* with a high probability. The next assumption allows me to focus on equilibria in which agencies coordinate on low bribes if their reputations for honesty are low.

Assumption 5. I only consider equilibria in which the rating agencies coordinate on low bribes in period t iff their reputations satisfy $\mu_t^{I(II)}$, $\mu_t^{II(I)} \leq \overline{\mu}$ for some $\overline{\mu} \in [0,(1-2\pi)/(1-\pi)]$.

In the rest of the section, I characterize the equilibrium in which the rating agencies request high bribes in both periods. Then, I show that this equilibrium ensures more effort in the first period than any equilibrium which involves low bribes. I start characterizing the equilibrium with high bribes from the rating decisions of a *strategic* agency. In the last period, a *strategic* agency requests a bribe whenever possible, while in the first period a *strategic* agency faces a trade-off between a better reputation and a bribe. The reputations are updated as to $mu_2^i(R_1,q_1)$.[14] The reputation in the second period satisfies

$$\mu_2^i(R_1,q_1) = \begin{cases} \frac{\mu_1}{\mu_1+(1-\mu_1)h_1^{i*}} := \mu^{i,b}(h_1^{i*}) & \text{if } r_1^i = b \text{ and } r_1^{-i} = g, \\ \frac{\mu_1(\pi+(1-\pi)(1-\mu_1)(1-h_1^{-i*}))}{\pi+(1-\pi)(1-\mu_1)\sum_{I,II}(1-h_1^{n*})} := \mu^{i,g}(H_1^*) & \text{if } r_1^i = r_1^{-i} = g \text{ and } q_1 = B, \\ \mu_1 & \text{otherwise.} \end{cases} \tag{3.5}$$

Because the two agencies rate simultaneously, a competitor's rating provides information about an agency's type. For example, the second-period agents interpret a rating $r_1^i = g$ for a security of quality $q_1 = B$ differently depending on the rating assigned by the other agency. As the issuer bribes an agency only if the other agency receives a signal g, a favorable rating is interpreted as an honest mistake if the other agency publishes an unfavorable rating $r_1^{-i} = b$. If instead the other agency also assigns a favorable rating, then the rating $r_1^i = g$ could be the result of a mistake or a bribe.

[14]Under competition, agencies learn more about each other than other agents do, but their strategies in the second period only depend on an agency's reputation among issuer and buyers.

Assumption 1 ensures that the agencies' reputations are sufficiently high to support effort in every period. Therefore, in the second period a *strategic*-agency's payoff amounts to

$$u_2^{is}(M_2) := \phi_2^i + (1-e_2^*)\pi(1-\pi)\beta_2^i = [(e_2^* + (1-e_2^*)\pi(2-\pi))p(g,g|M_2,0,0) - c(e_2^*)]/2.$$

The *strategic*-agency's payoff in the second period depends on the reputations of both agencies. A monopolistic agency that accepts a bribe in the first period only lowers its own expected revenues. In competition, a lower reputation for either agency lowers the revenues of both agencies. The next proposition characterizes the equilibrium in which the rating agencies coordinate on high bribes in both periods.

Proposition 11. *If in equilibrium the rating agencies coordinate on high bribes, every issuer hires both rating agencies. For every $i \in \{I,II\}$, the probability that a strategic agency rates honestly is $h_2^{i*} = 0$ and*

$$h_1^{i*} = \begin{cases} 0 & \text{if } \delta \leq \overline{\delta}^c, \\ h^c(\delta) & \text{otherwise,} \end{cases}$$

where $\overline{\delta}^c := \frac{p(g,g|M_1,0,0)}{u_2^{is}(\mu_2^{i,b}(0),\mu_1) - u_2^{is}(\mu^{i,g}(0,0),\mu^{i,g}(0,0))}$ and $h^c(\delta)$ is defined by the implicit function:

$$\delta = \frac{p(g,g|M_1,(h^c(\delta),h^c(\delta)))}{u_2^{is}(\mu^{i,b}(h^c(\delta)),\mu_1^{-i}) - u_2^{is}(\mu^{i,g}(h^c(\delta),h^c(\delta)),\mu^{-i,g}(h^c(\delta),h^c(\delta)))}. \tag{3.6}$$

In equilibrium the two *strategic*-type agencies follow the same rating strategy in both periods. Similarly to the monopoly case, for a low discount factor a *strategic* agency is not disciplined by the threat of losing its reputation, while for larger discount factors, the *strategic*-type mimics the *honest* type, as described in the next corollary.

Corollary 12. *$\partial h_1^{i*}/\partial\delta > 0$ iff $\delta \geq \overline{\delta}^c$, and $\lim_{\delta\to\infty} h_1^{i*} = 1$.*

The next lemma is composed of two parts. First, it states that in every equilibrium in which the rating agencies coordinate on low bribes in some period, they will coordinate on low bribes in the first period. Moreover, the *strategic* agencies coordinate on low bribes only if they strictly prefer to request a bribe. This is the case because an agency that is indifferent between a low bribe and an honest rating should strictly prefer to request a higher bribe. As a result, in any equilibrium that involves low bribes the issuer's effort in the first period is lower than in ther case of high bribes.

Lemma 13. *If in equilibrium the rating agencies coordinate on low bribes in the second period, they also coordinate on low bribes in the first period. Therefore, in any equilibrium in which agencies coordinate on low bribes,* $h_1^{I*}=h_1^{II*}=0$ *and the first-period effort* e_1^* *is weaker than in the equilibrium in which agencies request high bribes in every period.*

I proceed to compare equilibria in the regime of competition among rating agencies and in the monopoly regime.

4. Comparing equilibria

In this section, I compare the expected quality of the first-period security under monopoly and competition. The last-period security is not considered, as the last period amounts to a modeling device to account for reputational concerns of the rating agencies.

A different number of rating agencies can result in different amounts of information generated. My model has little to say about the process through which rating agencies collect their information. In fact, I even assume away any cost to obtain a signal correlated with quality. Along this line, I consider different market structures while holding constant the overall amount of information available to the rating agencies. I compare the effort choice of the issuer when a monopolistic agency observes two signals of quality and when each of two competing agencies observes a single signal.

The monopolistic regime and the regime of competition differ in the number of ratings assigned. Proposition 4 and Lemma 6 ensure that in both regime the issuer requests a rating from every available rating agency. The regimes also differ in the reputational concerns of the rating agencies. These two dimensions can be considered in turn. Lemma 14 focuses on the effect of increasing the number of agencies, while abstracting from their reputational concerns. This lemma compares equilibria under monopoly and under competition for an exogenously-given probability that a strategic agency requests a bribe.

Lemma 14. *Assume that every agency has a probability $\overline{\mu}$ to be an honest type, and each strategic agency rates honestly with probability $\overline{h}$. Two competing agencies, observing a single signal each and requesting high bribes, $\beta_t^I = \beta_t^{II} = p(g,g|.)$, induce more issuer's effort than a monopolistic agency observing two signals.*

Figure 7 gives an intuitive explanation. Under competition a strategic agency is less likely to be bribed. In fact, under competition the issuer agrees to pay a bribe to agency A^i only if agency A^{-i} observes a favorable signal, that is, $s_t^{-i} = g$. When both agencies observe an unfavorable signal, however, their ratings reveal the signals to the buyers, whether or not either agency requested a bribe. In contrast, a bribe is paid whenever it is requested under the monopoly regime.

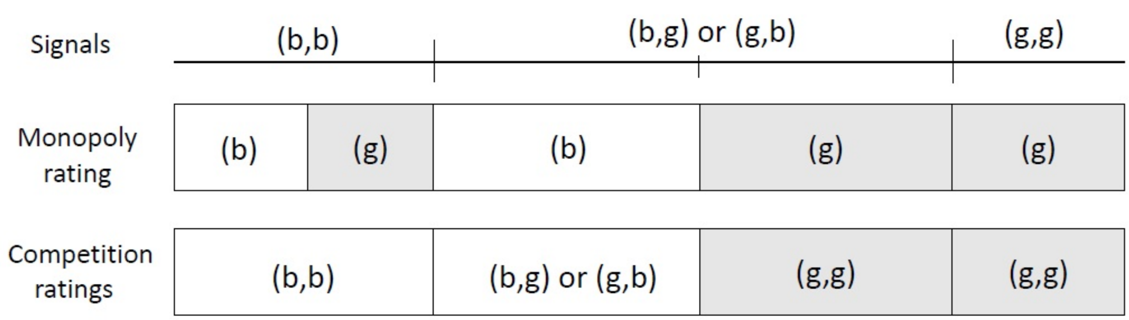

FIGURE 7. Ratings for exogenous $\overline{\mu}$ and $\overline{h}$.

The next proposition compares monopoly and competition assuming that Assumption 1 holds for $\pi = \widetilde{\pi}^2$.

Proposition 15. *Let competing agencies coordinate on high bribes. If $\pi > 1/3$, there is a unique discount factor δ^* at which a monopolistic agency and two competing agencies both ensure the same effort in the first period. The monopolistic agency ensures a higher effort iff $\delta > \delta^*$.*

Proposition 15 is the main result of the paper. It states that monopoly can induce more effort than competition with high bribes if the reputational incentives are strong. It also gives a sufficient condition to ensure that, for a large enough discount factor, the monopolist is indeed more informative than the two competitors. Intuitively, if the signal is often wrong (large π) the reputation update of the monopolist is larger than the reputation update of the competitors. At the same time, the monopolist's reputational motives are stronger because the payoff of the monopolist is more dependent on its reputation than the payoffs of competitors.

Quadratic cost example. *Figure 8 describes the equilibrium choice of effort for a quadratic cost function.*

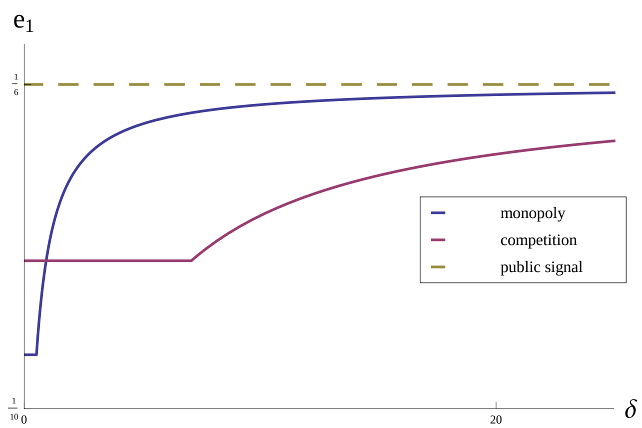

Figure 8. Effort in period 1 for μ_1=15/16 and π=1/2.

Effort choice under monopoly is lower for low values of δ. In particular if $\delta < \min\left\{\overline{\delta}^M, \overline{\delta}^C\right\}$, in both regimes a strategic agency requests a bribe whenever possible. In this case, Lemma 14 ensures that the buyers are more informed and the issuer has more incentive to invest in effort under rating agency competition. As the issuer's choice of effort is a continuous function of the agencies' discount factor, competition induces more effort than monopoly for any δ below a threshold.

5. Extension: Identical Signals

Will an issuer exert more effort when competing agencies observe identical signals of quality or when they observe conditionally independent signals of quality? Should policy makers incentivize standardization in the methods used to evaluate financial products? On the one hand, the presence of a competitor observing the same signal of quality can deter an agency from assigning undeserved ratings. On the other hand, agencies following different procedures might have stronger reputational incentives. In this section, I consider competition among rating agencies which observe the same signals of quality. I proceed then to compare equilibria in a regime of competition where agencies observe identical signals, and where agencies observe conditionally independent signals.

Consider the single-period game with exogenous rating strategies $\overline{H}$ and quality signals $s_t^I = s_t^{II} := s_t$. Lemma 6 does not depend on the correlation between the signals observed by the rating agencies. As a result, two ratings are necessary to ensure a price larger than zero even if the rating agencies observe the same signal.

A security may receive two favorable ratings if each of the agencies observes a favorable signal. Two favorable ratings can also be the result of two bribes. When two agencies observe the same signal, the issuer either bribes both agencies, or neither of them. The issuer will pay the bribes only if $\beta_t^I + \beta_t^{II} \leq p(g,g|M_t,\overline{H})$. I only consider symmetric equilibria, in which agencies request bribes $\beta_t^I = \beta_t^{II} = p(g,g|M_t,\overline{H})/2$. The next lemma provides a sufficient condition to ensure that there is an equilibrium with positive effort.

Lemma 16. *A pair $p(g,g|M_t,\overline{H})>0$ and $e^*>0$ that satisfies (2.1) and*

$$e_t^*(\Phi_t) = \mathbf{e}((1-\pi)p(g,g|M_t,\overline{H})) \tag{5.1}$$

exists for any $\overline{H}$ iff

$$(1-\mu_t^I)(1-\mu_t^{II}) < 1/c''(0) - \pi/(1-\pi). \tag{5.2}$$

This pair is unique. $e_t^* = p(g,g|M_t,\overline{H}) = 0$ satisfy (2.1) and (5.1) for any M_t.

Lemma 17 compares the equilibria of a single period game under competition with different signal structures. I hold constant the overall amount of information received by the rating agencies in the two regimes. This amounts to assuming that under both regimes two signals of quality are generated.

Lemma 17. *In the equilibrium of the single period game, the issuer exerts more effort when agencies receive identical signals than when agencies receive conditionally independent signals if*

$$\frac{\sum_i (1-\mu_t^i)(1-\overline{h}^i)}{\prod_i (1-\mu_t^i)(1-\overline{h}^i)} \geq \frac{1+\pi}{\pi}.$$

When agencies observe conditionally independent signals, they either coordinate on high or low bribes, as described in Section 4. If agencies coordinate on low bribes, the issuer has less incentive to exert effort than in the regime of identical signals of quality. Consider the equilibria in which the agencies coordinate on high bribes. Figure 9 compares the ratings in the two informational regimes. When the two signals are different, buyers are more likely to observe an unfavorable rating if each agency observes both signals. In contrast, if the two signals are identical and unfavorable, agencies that observe independent signals are more reliable: as the agencies coordinate on high bribes, they cannot be bribed at the same time. As a result, in case of two unfavorable signals, the agencies can be bribed only in the regime of identical signals.

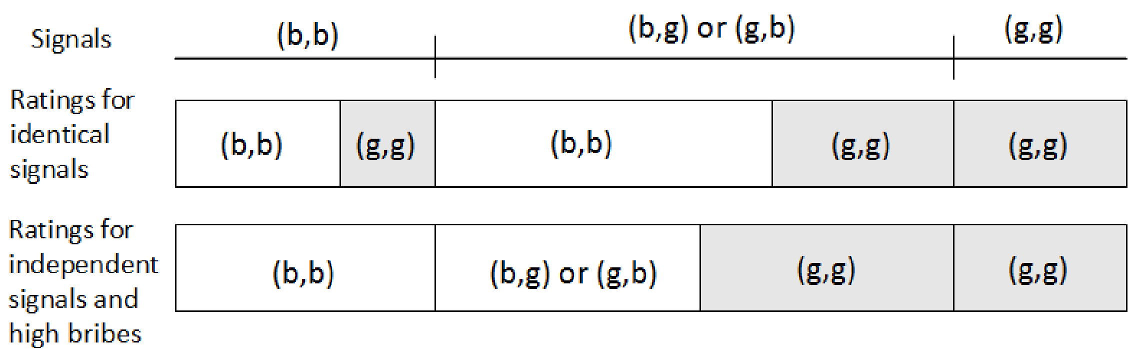

FIGURE 9. Ratings for exogenous $\overline{H}$ under different informational regimes.

Consider the two-period game. In the last period, the reputation of the rating agencies among the other agents is updated to $\mu_2^i(R_1,q_1)$.[15] In this case, the second period reputation satisfies

[15]Agencies can learn about each others' type more than the other agents do, but in equilibrium the belief about the type of the competing agency will not be determinant for the choice of action of each agency.

$$\mu_2^i(R_1,q_1)=\begin{cases}\frac{\mu_1}{Pr\left\{R_1=(b,b)|H_1^*,s_1=b\right\}}:=\mu^{i,b}(H_1^*) & \text{for } r_1^I=r_1^{II}=b,\\ \frac{\mu_1\pi}{Pr\left\{R_1=(g,g)|H_1^*,q_1=B\right\}}:=\mu^{i,g}(H_1^*) & \text{for } r_1^I=r_1^{II}=g \text{ and } q_1=B,\\ \mu_1 & \text{otherwise.}\end{cases} \tag{5.3}$$

The payoff of a strategic agency in the second period depends on the reputation of both agencies. The agency's reputation among the issuer and the buyers determines, respectively, the choice of effort and the willingness to pay for the security. The belief about the type of the competitor determines the expected probability of receiving a bribe. The continuation payoff equals

$$u_2^i(M_2,\mu_2^{-i(i)}):=\phi_2^i+(1-e_2)(1-\pi)(1-\mu_2^{-i(i)})\beta_2^i/2.$$

The next proposition characterizes the equilibrium of the game.

Proposition 18. *In equilibrium, for every $i\in\{I,II\}$,*

$$h_1^i=\begin{cases}0 & \text{if } \delta\le\overline{\delta}^s,\\ h_1^s(\delta) & \text{otherwise.}\end{cases}$$

$\overline{\delta}^s$ *and $h_1^s(\delta)$ are defined, respectively, by:*

$$\overline{\delta}^s=\frac{p(g,g|M_1,0,0)}{u_2^i(\mu_2^{i,b}(0),\mu_2^{i,b}(0),0)-u_2^i(\mu^{i,g}(0,0),\mu^{j,g}(0,0),0)},$$
$$\delta=\frac{p(g,g|M_1,h_1^s(\delta),h_1^{c_s}(\delta))}{u_2^i(\mu^{i,b}(h_1^s(\delta)),\mu_2^{i,b}(h_1^s(\delta)),0)-u_2^i(\mu^{i,g}(h_1^s(\delta),h_1^{c_s}(\delta)),0)}.$$

As in the regime of conditionally independent signals, there is a threshold discount factor $\overline{\delta}^s$. If and only if $\delta<\overline{\delta}^s$, a strategic agency strictly prefers to obtain a bribe in the first period. Note that $\overline{\delta}^s$ and $h_1^s(\delta)$ are defined by the indifference condition of a strategic rating agency. The next corollary is the equivalent of Corollary 12.

Corollary 19. *If $\delta>\overline{\delta}^{c_s}$, then $\partial h_1^{i*}/\partial\delta>0$ and $\lim_{\delta\to\infty}h_1^{i*}=1$, $\forall i$.*

Proposition 20 compares the equilibrium effort under monopoly and under competition. I consider only the case of issuers endowed with a quadratic cost function.

Proposition 20. *Let $c(e_t){=}e_t^2$. If every competing rating agency observes both signals, there is a unique $\delta^{**}{<}\overline{\delta}^{c_s}$ for which a monopolistic agency and competing agencies induce the same effort choice in the first period. The issuer exerts more effort in the monopoly regime iff $\delta{>}\delta^{**}$.*

The competition regime ensures more effort than monopoly for low discount factors. Proposition 20 ensures that this is the case regardless of the signal structure. Figure 10 shows a numerical example.

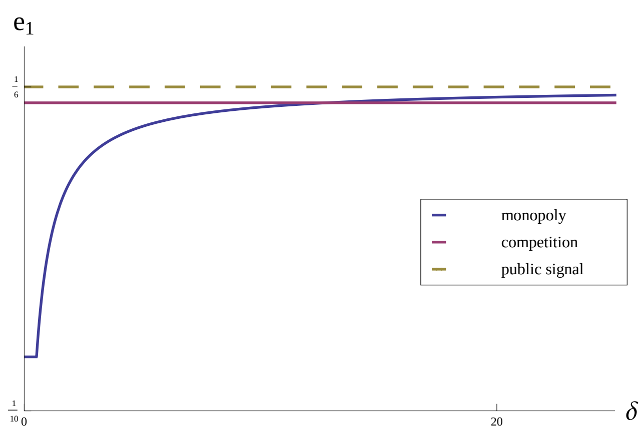

FIGURE 10. First-period effort under monopoly and competition (identical signals).

The next lemma compares the threshold discount factors obtained by comparing effort under competition for the two signal structures and effort under monopoly.

Lemma 21. *Let $c(e_t){=}e_t^2$. Then $\delta^{**}{>}\delta^{*}$ iff $\mu_1{>}\frac{1-2\pi}{1+\pi}$.*

Identical-signal competition ensures more effort than monopoly for a larger set of δ than independent-signal competition for large values of μ_1 and π. Identical signals ensure that each agency observes more information. If the agencies are likely to be honest, identical signals ensure more informative ratings than independent signals.

6. Conclusion

My paper represents the first attempt to simultaneously consider rating agencies' reputational incentives and issuers' investments in the quality of their securities. I show that competition among agencies can ensure more investment in a security's quality than monopoly whenever the rating agencies have weak reputational incentives. Reputational incentives can be weak for many reasons. For example, while rating new or complex securities, rating agencies are likely to make many honest mistakes. As a result, inflated ratings will pass unnoticed and will not hurt the agencies' reputations for rating honestly. If the number of new securities to be rated is very high, the reputational incentives can be weak because many ratings are issued before the returns of the securities are observed. The markets for asset-backed securities in the years preceding the financial crisis are an example of market in which a large volume of new and complex securities were rated in a short amount of time, and rating agencies were likely to lack reputational concerns. Therefore, my model supports the regulators' claims that increasing rating competition is in the public interest in these markets.

Other certification models with endogenous product quality (Albano and Lizzeri (2001) and Donaldson and Piacentino (2012)) conclude that competition always ensures more investment in security quality than monopoly does. These models differ from mine primarily because their certifiers do not have reputational incentives. Models that do consider reputational incentives, such as Strautz (2005), Lo (2010) and Camanho and Deb (2012), conclude that a monopolistic agency always ensures more information for investors than competing agencies. Unlike my model, these papers consider ratings for products of exogenous quality and limit the issuer to hiring at most a single rating agency.

My analysis could be extended to consider competition between more than two rating agencies. In my setting, the issuer hires all the available rating agencies, regardless of their number. The analysis, however, becomes significantly more complicated when more than two rating agencies are available. This is because strategic agencies can coordinate their bribes in many different ways, and the number of possible equilibria increases quite rapidly.

In my model, the structure of the securities market is exogenously determined. After agencies announce their rating fees, a single issuer decides on her effort. It would be interesting, however, to explicitly model the issuer's decision to enter the securities market, and therefore consider the effect of the rating fees on the structure of the securities market. In fact, the rating agencies could potentially demand low fees in order to encourage multiple issuers to enter the market, or high fees that result in a monopoly in the securities market. Ultimately, this extension could provide conditions under which external certifiers should be expected to induce a socially desirable market structure for rated products.

Appendix

Appendix A. Low-Bribes Equilibrium

I first consider the single-period game for a given $\overline{H}$. I focus on equilibria in which the rating agencies request identical bribes $\beta_t^I, \beta_t^{II} : \beta_t^I + \beta_t^{II} = p(g,g|M_t,\overline{H})$. The price of the security is defined by Lemma 6. If the issuer hires the two agencies, she ensures an expected payoff equal to

$$U^I = (e_t^* + (1-e_t^*)(\pi^2 + (1/2)\pi(1-\pi)\Sigma_i(1-\mu_t^i)(1-\overline{h}^i)))p(g,g|M_t,\overline{H}) - c(e_t^*) - \phi_t^I - \phi_t^{II}. \quad (6.1)$$

The optimal effort choice maximizes the issuer's expected payoff

$$e_t^*(\Phi_t) = \mathbf{e}((1-\pi)(1+\pi-(\pi/2)\Sigma_i(1-\mu_t^i)(1-\overline{h}^i))p(g,g|M_t,\overline{H})). \quad (6.2)$$

Assumption 1 ensures that an equilibrium with positive effort exists for any $\overline{H} \in [0,1)^2$. In equilibrium, the rating agencies extract all the surplus generated by the issuer's effort, as described in the next lemma.

Lemma 22. *An equilibrium with positive effort exists for any $\overline{H} \in [0,1)^2$. In equilibrium, the rating agencies require the highest fee that the issuer is willing to pay*

$$\phi_t^I = \phi_t^{II} = (1/2)(e^* + (1-e^*)(\pi^2 + (\pi(1-\pi)/2)\Sigma_i(1-\mu_t^i)(1-\overline{h}^i)))p(g,g|M_t,\overline{H}) - c(e^*)).$$

Appendix B. Proofs

Proof of Lemma 1. Assumptions 2 and 3 imply that all buyers hold identical beliefs on and out-of the equilibrium path. Therefore, in any PBE:

$$|i_t : bid_t^i = Pr\{q_t = G|r_t,\mu_t,\overline{h},\phi_t^M\}| \geq 2 \quad \forall r_t,\mu_t,\overline{h},\phi_t^M, \text{ and}$$
$$bid_t^i \leq Pr\{q_t = G|r_t,\mu_t,\overline{h},\phi_t^M\} \quad \forall i,r_t,\mu_t,\overline{h},\phi_t^M.$$

By construction, $Pr\{q_t = B|r_t^M = b\} = 1$ therefore $p(b,\phi_t|\mu_t,\overline{h}) = 0$, $\forall \phi_t,\mu_t$. $p(g,\phi_t|\overline{\mu},\overline{h})$ is defined by Bayes Rule, and as $p(g,\phi_t|\mu_t,\overline{h}) \geq p(b,\phi_t|\mu_t,\overline{h})$ the rate is hidden only if $r_t = b$ or if $r_t = g$ and $p(g,\phi_t|\mu_t,\overline{h}) = 0$. Moreover, if the agency is not hired, then $e_t^* = 0$; therefore $p(\emptyset,\phi_t|\mu_t,\overline{h}) = 0 \ \forall \phi_t,\mu_t$ □

Proof of Lemma 2. (2.1), (2.3) hold at the same time iff $p(g|\mu_t,\overline{h}) = f(\mathbf{e}((1-\pi)p(g|\mu_t,\overline{h})))$, where $f(x) := x/(x+(1-x)a)$, and $a := \pi + (1-\pi)(1-\mu_t)(1-\overline{h})$. As $a \in (0,1)$ and $\mathbf{e}((1-\pi)p(g|\mu_t,\overline{h})) \in [0,1)$, then $f''(x) < 0$ (i). Moreover, $c''' \geq 0$ and $c'' > 0$ imply

$$\partial^2 \mathbf{e}((1-\pi)p)/\partial p^2 = -c'''(\mathbf{e}((1-\pi)p))(1-\pi)^2/(c''(\mathbf{e}((1-\pi)p)))^3 \leq 0 \quad (6.3)$$

(i) and (6.3) imply

$$\partial f(\mathbf{e}((1-\pi)p(g|.)))/\partial p(g|.)>0>\partial^2 f(\mathbf{e}((1-\pi)p(g|.)))/\partial^2 p(g|.) \tag{6.4}$$

(6.4) implies $|\{x{:}x=f(\mathbf{e}((1-\pi)x))\}|\leq 2$. $0=f(\mathbf{e}((1-\pi)0))$ implies that $p(g|\mu_t,\overline{h})=e^*=0$ satisfy (2.1) and (2.3) and that there is at most a unique pair $p(g|\mu_t,\overline{h})>0$ and $e^*>0$ that satisfies (2.1) and (2.3). As $f(\mathbf{e}((1-\pi)1))\leq 1$, then a necessary and sufficient condition for the existence of a unique $x\in(0,1)$ s.t. $x=f(\mathbf{e}((1-\pi)x))$ is

$$\partial f(\mathbf{e}((1-\pi)x))/\partial x|_{x=0}>1 \tag{6.5}$$

If (6.5) does not hold, then $x>f(\mathbf{e}((1-\pi)x))\forall x\in(0,1)$. (6.5) holds for all $\overline{h}\in[0,1]$ iff

$$\mu_t>1/(1-\pi)-1/c''(0) \qquad \square$$

Let

$$\overline{\phi}_t:=(e_t^*+(1-e_t^*)\pi)p(g|\mu_t,\overline{h})-c(e_t^*).$$

I rule out pathological equilibria in which some type of rating agency requests $\phi_t^*>\overline{\phi}_t$. In any equilibrium in which $\mu(\phi_t)=\mu_t$ $\forall\mu_t$, both types of agency set $\phi_t^*=\overline{\phi}_t$. If instead in equilibrium $\mu(\phi_t')\neq\mu_t$ for some ϕ_t', by Assumption 2, ϕ_t' must be on the equilibrium path for some type of agency. But this implies that there is at least another ϕ_t'' on the equilibrium path s.t. $\mu(\phi_t')\neq\mu(\phi_t'')$. WLG let $\mu(\phi_t')>\mu(\phi_t'')$ (i). If the honest type strictly prefers ϕ_t'' over ϕ_t', then $\mu(\phi_t')=0$, which contradicts (i). If the honest type is indifferent or strictly prefers ϕ_t' over ϕ_t'' then the strategic type strictly prefers ϕ_t' over ϕ_t'' because $\beta(\mu(\phi_t'),\overline{h})>\beta(\mu(\phi_t''),\overline{h})$, which implies that either $\mu(\phi_t'')=1$ (which contradicts (i)) or ϕ_t'' is out of the equilibrium path, which is also a contradiction

Proof of Proposition 4. First of all $h_2^*=0$. In the last period a *strategic* agency has no incentive to maintain a reputation for honesty. To characterize the equilibrium, I need to pin down $h_1^*$and to show that it is optimal for the rating agency - regardless of its type - to set a fee in each period s.t. the issuer is willing to hire the agency. For the moment, I assume that the latter condition is satisfied and I pin down h_1^{M*}. For $s_1=b$, an honest rate ensures a continuation payoff $\delta u_2^{Ms}(\mu^b(h_1^*))$. If instead a bribe is paid, the continuation payoff is $p(g|\mu_1,h_1^*)+\delta u_2^{Ms}(\mu^g(h_1^*))$.
Step 1.$p(g|\mu_t,h_t^*)$ satisfies the implicit function

$$F(p(g|.),\mu_t,h_t^*):=p(g|.)-f(\mathbf{e}((1-\pi)p(g|.)))=0$$

where $f(.)$ is defined in the Proof of 2. As shown in Lemma 2, $F_p(p^*(g|\mu_t,h_t^*),\mu_t,h_t^*)|_{p=}>0 \quad \forall\mu_t,h_t^*$. Moreover, the Implicit Function Theorem ensures that

$$\partial p(g|\mu_t,h_t^*)/\partial\mu_t=-F_\mu(p(g|.),\mu_t)/F_p(p(g|.),\mu_t)>0.$$

Step 2. Note that

$$\partial(p(g|\mu_1,h_1^*)/\partial h_1^*>0 \quad (i), \quad \partial\mu^g(h_1^*)/\partial h_1^*>0 \quad (ii)$$

$$\partial u_2^s(\mu_2)/\partial\mu_2=(\partial p(g|\mu_2,0)/\partial\mu_2)(1-(1-\pi)p(g|\mu_2,0)\partial\mathbf{e}((1-\pi)x)/\partial x|_{x=p(g|\mu_2,0)})>0 \quad (iii)$$

(iii) holds as $\frac{\partial p(g|\mu_2,0)}{\partial\mu_2}>0$ by Step 1 and

$$p(g|.)\partial\mathbf{e}((1-\pi)x)/\partial x|_{x=p(g|.)}<\mathbf{e}((1-\pi)p(g|.)) \quad (iv)$$

$\mathbf{e}((1-\pi)p(g|.))<1$. (iv) in turn, holds as $\partial^2\mathbf{e}((1-\pi)x)/\partial x^2<0$ and $\mathbf{e}(0)=0$. (i), (ii), and (iii) imply

$$\partial(p(g|\mu_1,h_1^*)+\delta u_2^M(\mu^g(h_1^*))/\partial h_1^*>0.$$

Step 3. $\partial u_2^s(\mu^b(h_1^*)))/\partial h_1^*<0$: as $\partial\mu^b(h_1^*)/\partial h_1^*<0$ and $\partial u_2^M(\mu_2)/\partial\mu_2>0$ by in step 2.
Step 4. From steps 2 and 3, either:

$$p(g|\mu_1,h_1)+\delta u_2^M(\mu^b(h_1))>\delta u_2^M(\mu^b(h_1)),\ \forall h_1\in[0,1)$$

in which case the agency strictly prefers to request a bribe and $h_1^{M*}=0$, or there is a unique $h^{M*}\in[0,1]$ s.t.

$$p(g|\mu_1,h^{M*})=\delta(u_2^s(\mu^b(h^{M*}))-u_2^s(\mu^b(h^{M*})))$$

in which case $h_1^M=h^{M*}$. Let $\overline{\delta}^M$ be defined by

$$p(g|\mu_1,0)+\overline{\delta}^M u_2^M(\mu^g(0))=\overline{\delta}^M u_2^M(\mu^b(0)).$$

As $u_2^M(\mu^g(0))<u_2^M(\mu^b(0))$, for $\delta<\overline{\delta}^M$ $h_1^M=0$, while for $\delta\geq\overline{\delta}^M$ $h_1^M=h^{M*}$ holds.
Step 5. All is left to show is that is it optimal to set a fee s.t. the issuer is willing to hire the agency. In the last period, Lemma 3 ensures that the agency - regardless of its type - will set a fee equal to the highest willingness to pay of the issuer. In the first period, assume that each type sets a fee s.t. it is not optimal to hire an agency; then the payoff of a honest type equals $u_2^C(\mu_1)=(e^*+(1-e^*)\pi)p(g|\mu_1,0)-c(e^*)$. By deviating to $\overline{\phi}_1:=(e_t^*+(1-e_t^*)\pi)p(g|\mu_t,h_1^*)-c(e_t^*)$, the strategic type has a profitable deviation. In fact $\overline{\phi}_1>0$ and $E(u_2^C(\mu_1)|)>\mu_1$(i). (i) is the case because
a) $$E(\mu_2)=e^*\mu_1+(1-e^*)\pi\mu^b(h^{M*})+(1-e^*)(1-\pi)\mu^g(h^{M*})>$$
$$>e^*\mu_1+(1-e^*)(\pi+(1-\pi)(1-\mu_1^b)(1-h_1^*))\mu(h^{M*})+((1-e^*)(1-\pi)(1-\mu_1(1-h_1^{M*}))\mu_1^g(h^{M*})>\mu_1.$$
b) Define

$$\overline{u}_2^C(\mu_2):=(e_2^*(\mu_1)+(1-e_2^*(\mu_1))\pi)p(g|\mu_1,0)-c(e_2^*(\mu_1)),$$

then $\partial^2\overline{u}_2^C(\mu_2)/\partial\mu_2>0$. But $u_2^C(\mu_2)\geq\overline{u}_2^C(\mu_2)$ and $\partial(u_2^C(\mu_2)-\overline{u}_2^C(\mu_2))/\partial\mu_2<0$ iff $\mu_2<\mu_1$, therefore $\partial^2u_2^C(\mu_2)/\partial\mu_2^2|_{\mu_2=\mu_1}>0$.
As in the first period fees s.t the issuer is not willing to hire the agency can be ruled out, Lemma 3 ensures that the agency - regardless of its type - will set a fee equal to the highest willingness to pay of the issuer □

Proof of Lemma 5. Let $F(h_1^*,\delta):=\delta-p(g|\mu_1,h_1^*)/(u_2^{Ms}(\mu^b(h_1^*))-u_2^{Ms}(\mu^g(h_1^*)))$. By the Implicit Fun. Theorem:
$\partial h_1^*/\partial\delta=-\frac{\partial F(h_1^*,\delta)/\partial\delta}{\partial F(h_1^*,\delta)/\partial h_1^*}$ and $\partial F(h_1^*,\delta)/\partial\delta>0$ while
$\partial F(h_1^*,\delta)/\partial h_1^*=-\frac{(dp(g|\mu_1,h_1^*)/dh_1^*)(u_2^{Ms}(\mu^b(h_1^*))-u_2^{Ms}(\mu^g(h_1^*)))-p(g|\mu_1,h_1^*)((du_2^{Ms}(\mu^b(h_1^*))/d\mu^b(h_1^*))(d\mu^b(h_1^*)/dh_1^*)+..}{(u_2^{Ms}(\mu^b(h_1^*))-u_2^{Ms}(\mu^g(h_1^*)))^2}$
$\frac{-(du_2(\mu^g(h_1^*))/d\mu^g(h_1^*))(d\mu^g(h_1^*)/dh_1^*))}{.}<0$
The last inequality holds as: $\partial p(g|\mu_1,h_1^*)/\partial h>0$, $du_2^{Ms}/d\mu_2>0$, and $\partial\mu^g(h_1^*)/\partial h_1^*>0>\partial\mu^b(h_1^*)/\partial h_1^*$.
To show $\lim_{\delta\to\infty}h_1^*$ it is enough to show that $u_2^{Ms}(\mu^b(h_1^*))-u_2^{Ms}(\mu^g(h_1^*))=0$ iff $h_1^*=1$. The *if* part holds as $\mu^b(1)=\mu^g(1)$ the *only if* part holds as: $h_1^*<1\to\mu^b(h_1^*)>\mu^g(h_1^*)$ and as shown above $\partial u_2^{Ms}/\partial\mu_2>0$. Finally, $\partial\overline{\delta}^M/d\mu_1>0$ as $\partial p(g|\mu_1,0)/\partial\mu_1>0$ and $\partial(u_2^{Ms}(\mu^b(0))-u_2^{Ms}(\mu^g(0)))/\partial\mu_1=\partial(1-u_2^{Ms}(\mu^g(0)))/\partial\mu_1<0$ □

Proof of Lemma 6. In equilibrium, suppose that following fees Φ_t the issuer is expected to hire only one agency and $e^*(\Phi_t)>0$. Then by Assumption 3 $e^*(\Phi_t)>0$ and $p(g,\Phi_t|.)>0$ must be mutually consistent as in monopoly. Assumption 3 requires that also $p(g,g,\Phi_t|.)$ must also be mutually consistent with the effort choice. Suppose $p(g,g,\Phi_t|.)\leq p(g,\Phi_t|)$: then the effort choice is identical whether the agency hires one or two agencies. But for identical effort, it must be the case that $p(g,g,\Phi_t|.)>p(g,\Phi_t|)$ as $\mu_t^I,\mu_t^{II}>0$. Therefore, it must be the case that $p(g,g,\Phi_t|.)>p(g,\Phi_t|)$. Suppose there is an equilibrium in which the issuer hires only one agency. Then either the fees ϕ_t^I and ϕ_t^{II} are such that the issuer is indifferent to hire either of the agencies and $\phi_t^I>0$ and $\phi_t^{II}>0$, in which case every agency has an incentive to charge an ϵ- smaller fee, or $\phi_t^I\geq 0$ and $\phi_t^{II}\geq 0$ with at least one weak inequality holding as an equality; then any agency charging $\phi_t^i=0$ could deviate to a strictly positive, small enough fee and ensure a larger profit. If in equilibrium the issuer does not hire the agencies, he exerts no effort, $p(r_t|\overline{\mu},\overline{h})=0\forall r_t$ and the Lemma holds. If in equilibrium the issuer hires the two agencies, if $p(R_t|M_t,H_t)>0$ for $r_t^i=\emptyset$ for some $i\in\{I,II\}$, then the issuer hides an unfavorable rating, making $p(R_t|M_t,H_t)>0$ not consistent with the equilibrium strategies. $p(R_t|M_t,H_t)=0$ if $r_t^i=b$ for some $i\in\{I,II\}$ by construction. $p(g,g|M_t,H_t^e)$ is defined by Bayes Rule □

Proof of Lemma 7. If $\beta_t^{-i*}\geq p(g,g,\Phi|.)$, then β_t^i is paid only if $\beta_t^i\leq p(g,g,\Phi|)$ and $s^{-i}=g$. So $\beta_t^i=p(g,g,\Phi|.)$ maximizes A^i's expected payoff. If instead $\beta_t^{-i}<p(g,g,\Phi|.)$, the payoff of A^i is maximized by:

$$\beta_t^i=\begin{cases} p(g,g,\Phi|.)-\beta_t^{-i} & \text{if } (\pi+(1-\pi)(1-\mu_t^{-i})(1-\overline{h}^{-i}))(p(g,g,\Phi|.)-\beta_t^{-i})\geq \pi p(g,g,\Phi|.),\\ p(g,g,\Phi|.) & \text{otherwise.}\end{cases}$$

So, in equilibrium either $\beta_t^{I*}=\beta_t^{II*}=p(g,g,\Phi|.)$ or $\beta_t^i+\beta_t^j=p(g,g,\Phi|.)$ □

Proof of Lemma 8. (3.1) and (3.3) hold at the same time iff $p(g,g|M_t,\overline{H})=f^H(\mathbf{e}((1-\pi^2)p(g,g|M_t,\overline{H})))$, where $f^H(x):=x/(x+(1-x)a^H)$, and $a^H:=\pi^2+\sum_{I,II}(1-\pi)\pi(1-\mu_t^i)(1-\overline{h}^i)$. Applying the same steps followed in Lemma 2, the necessary and sufficient condition for the existence of an $x\in(0,1){:}x=f^H(\mathbf{e}((1-\pi^2)x))$ is: $\partial f^H(\mathbf{e}((1-\pi^2)x))/\partial x|_{x=0}>1$ (i). (i) holds for every $\overline{H}\in[0,1]^2$ iff

$$\mathbf{e}'(0)(1-\pi^2)/(\pi^2+\pi(1-\pi)(2-(\mu_t^I+\mu_t^{II})))>1$$

which is equivalent to $\leftrightarrow\mu_t^I+\mu_t^{II}>(2-\pi)/(1-\pi)-(1+\pi)/\pi c''(0)$ □

Proof of Lemma 9. Let $\overline{\phi}_t:=(e_t^*+(1-e_t^*)\pi)p(g,g|M_t,\overline{H})-c(e_t^*)$. I rule out pathological equilibria in which some type of rating agency requests $\phi_t^*>\overline{\phi}_t$. In any equilibrium in which $\mu(\phi_t)=\mu_t\forall\mu_t$ Assumption 5 ensures that both types of agency set $\phi_t^*=\overline{\phi}_t/2$. If instead in equilibrium $\mu^i(\phi_t')\neq\mu_t^i$ for some ϕ_t', by Assumption 2 ϕ_t' must be on the equilibrium path for some type of agency A^i. But this implies that there exists at least another ϕ_t'' on the equilibrium path s.t. $\mu^i(\phi_t')\neq\mu^i(\phi_t'')$. WLG let $\mu^I(\phi_t')>\mu^I(\phi_t'')$ (i).

If the honest type strictly prefers ϕ_t'' over ϕ_t', then $\mu^i(\phi_t')$=0, which contradicts (i). If the honest type is indifferent or strictly prefers ϕ_t' over ϕ_t'', then the strategic type strictly prefers ϕ_t' over ϕ_t'' because $\beta(M(\phi_t'),\overline{H})>\beta(M(\phi_t''),\overline{H})$ which implies that either $\mu^i(\phi_t'')=1$ (contradicting (i)) or ϕ_t'' is out of the equilibrium path which is also a contradiction

□

Proof of Lemma 10. Using the definitions in the proofs of Lemma 8 and Lemma 24

$$a^H<a^L\rightarrow f^H(\mathbf{e}((1-\pi^2)p(g,g|M_t,\overline{H})))>f^L(\mathbf{e}((1-\pi^2)p(g,g|M_t,\overline{H}))) \qquad \forall M\in(0,1)^2,\overline{H}\in[0,1)^2 \qquad (i)$$

Moreover:

$$f^L(\mathbf{e}((1-\pi^2)p(g,g|M_t,\overline{H})))>f^L(\mathbf{e}((1-\pi)(1+\pi-(\pi/2)\Sigma_{I,II}(1-\mu_t^i)(1-\overline{h}^i)p(g,g|M_t,\overline{H})))\forall M\in(0,1)^2,\overline{H}\in[0,1)^2 \quad (II)$$

Therefore, if $p^H>0$ and $p^L>0$ and $p^H=f^H(\mathbf{e}((1-\pi^2)p^H))$ while

$$p^L=f^L(\mathbf{e}((1-\pi)(1+\pi-(\pi/2)\Sigma_{I,II}(1-\mu_t^i)(1-\overline{h}^i)p^L)),$$

then $p^H>p^L$. Therefore, the equilibrium price following two favorable ratings is larger if the agencies coordinate on high bribes. Also, the equilibrium effort is larger if agencies coordinate on high bribes, as

$$\mathbf{e}((1-\pi^2)p)>\mathbf{e}((1-\pi)(1+\pi-(\pi/2)\Sigma_{I,II}(1-\mu_t^i)(1-\overline{h}^i)p) \qquad \forall p$$

□

Proof of Proposition 11. First of all $h_2^{I*}=h_2^{II*}=0$. In the last period a *strategic* agency has no incentive to maintain a reputation for honesty. To characterize the equilibrium, I need to pin down h_1^{I*},h_1^{II*} and to show that it is optimal for the rating agency - regardless of its type - to set a fee in each period s.t. the issuer is willing to hire the agency. For the moment, I assume that the latter condition is satisfied and I pin down h_1^{i*}. In equilibrium, β_1^i is paid only if $s_1^{-i}=g$, in which case a *strategic* agency faces a trade-off between demanding a bribe and obtaining $p(g,g|M_1,H_1^*)+\delta u_2^{is}(\mu^{i,g}(H_1^*),\mu^{j,g}(H_1^*))$ and rating honestly to obtain $\delta u_2^{is}(\mu^{i,b}(h_1^{i*}),\mu_1)$.

Step 1. $p(g,g|M_t,H_t^*)\in C^1$ is increasing in μ_t^i, and h_t^{i*} $\forall i$ (analogous to Proof of Prop. 4).

Step 2. $\partial(p(g,g|M_1,H_1^*)+\delta u_2^{is}(\mu^{i,g}(H_1^*),\mu^{-i,g}(H_1^*)))/\partial h_1^i>0$. This is the case as $\partial\mu^{i,g}(H_1^*)/\partial h_1^i>0$, and $\partial u_2^i(M_2,0,0)/\partial\mu_2^i=(\partial p(g,g|M_2,0,0)/\partial\mu_2^i)\partial((1/2)((e_2+(1-e_2)\pi(2-\pi))p(g,g|.)-c(e_2)))/\partial p(g,g|.)=$
$=(1/2)(\partial p(g,g|M_2,0,0)/\partial\mu_2^i)\Big((e_2+(1-e_2)\pi(2-\pi)-\partial\mathbf{e}((1-\pi^2)x)/\partial x|_{x=p(g,g|M_2,0,0)}p(g,g|.))\Big)>0.$

The inequality holds as $\partial p(g,g|M_2,0,0)/\partial\mu_2^i>0$ (step 1), and by the Proof of Prop. 4 for $x=p(g,g|M_2,0,0)\}$:

$$\partial(\mathbf{e}(1-\pi^2)x)/\partial x<\mathbf{e}((1-\pi^2)x).$$

Step 3. $\partial u_2^i(\mu_2^{i,b}(h_1^{i*}),\mu_1)/\partial h_1^i+\partial u_2^i(\mu_2^{i,b}(h_1^{i*}),\mu_1)/\partial h_1^{-i}=\partial u_2^i(\mu_2^{i,b}(h_1^{i*}),\mu_1)/\partial h_1^i<0.$

This is the case as $\partial\mu^{i,b}(h_1^{i*})/\partial h_1^*<0$.

Step 4. From steps 2 and 3, in the first period the two agencies will choose the same h_1^*. Suppose not and $h_1^{I*}>h_1^{II*}$ (i), then $h_1^{I*}>0$ implies

$$p(g,g|M_1,H_1^*)+\delta u_2^{Is}(\mu^{I,g}(H_1^*),\mu^{II,g}(H_1^*))=\delta u_2^I(\mu_2^{I,b}(h_1^{I*}))$$

but then (i) implies: $u_2^I(\mu_2^{I,b}(h_1^{I*}))<u_2^{II}(\mu_2^{II,b}(h_1^{II*}))$ and therefore

$$\delta u_2^{II}(\mu_2^{II,b}(h_1^{II*}))>p(g,g|M_1,H_1^*)+\delta u_2^{Is}(\mu^{I,g}(H_1^*),\mu^{II,g}(H_1^*))=$$
$$=p(g,g|M_1,H_1^*)+\delta u_2^{IIs}(\mu^{I,g}(H_1^*),\mu^{II,g}(H_1^*))$$

Therefore, either

$$p(g,g|M_1,H_1)+\delta u_2^{is}(\mu^{i,g}(H_1),\mu^{-i,g}(H_1))>\delta u_2^i(\mu_2^{i,b}(h_1^i))\ \forall h_1^i\in[0,1),$$

in which case each agency strictly prefers to request a bribe and $h_1^{I*}=h_1^{II*}=0$, or there is a unique value $h_1(\delta)$ s.t.

$$p(g,g|M_1,H_1)+\delta u_2^{is}(\mu^{i,g}(H_1),\mu^{-i,g}(H_1))=\delta u_2^i(\mu_2^{i,b}(h_1^i)),$$

for $h_1^I=h_1^{II}=h^c(\delta)$. Let $\overline{\delta}^c$ be defined by

$$p(g,g|M_1,[0,0])+\overline{\delta}^c u_2^{is}(\mu^{i,g}(0,0),\mu^{-i,g}(0,0))=\overline{\delta}^c u_2^i(\mu_2^{i,b}(0)).$$

For $\delta<\overline{\delta}^c$ $h_1^i=0$, while for $\delta\geq\overline{\delta}^c$ $h_1^{I*}=h_t^{II*}=h^c(\delta)$.

Step 5. All is left to show is that is it optimal to set a fee s.t. the issuer is willing to hire the agency. In the last period, Lemma 9 ensures that the agency - regardless of its type - will set a fee equal to the highest willingness to pay of the issuer. I rule out equilibria in which both agencies set a rating fee higher than the fee defined in Lemma 9. In the first period, assume that each type of agency A^i sets a fee s.t. it is not optimal to hire an agency: then the payoff of an *honest* agency equals $u_2^C(\mu_1)=(e^*+(1-e^*)\pi)p(g,g|M_1,[0,0])-c(e^*)$. By deviating to $\overline{\phi}_1:=(1/2)((e_t^*+(1-e_t^*)\pi)p(g,g|M_1,H_1^*)-c(e_t^*))$ the strategic type has a profitable deviation (the proof is identical to the proof of Proposition 4, step 5) □

Proof of Lemma 12. Let

$$F(h^c(\delta),\delta):=\delta-p(g,g|M_1,h^c(\delta),h^c(\delta))/(u_2^{is}(\mu^{i,b}(h^c(\delta)),\mu_1)-u_2^{is}(\mu^{i,g}(h^c(\delta),h^c(\delta)),\mu^{-i,g}(h^c(\delta),h^c(\delta)))),$$

(3.6) implies that $F(h_1^c(\delta),\delta)=0$. By the Implicit Function Theorem,

$$\partial h^c(\delta)/\partial\delta=-\frac{\partial F(h^c(\delta),\delta)/\partial\delta}{\partial F(h^c(\delta),\delta)/\partial h^c(\delta)},$$

where $\partial F(h^c(\delta),\delta)/\partial\delta=1>0$, and

$$\partial F(h^c(\delta),\delta)/\partial h^c(\delta)=-\frac{\partial p(g,g|.)/\partial h^c(\delta)}{u_2^i(\mu^{i,b},\mu_1)-u_2^i(\mu^{i,g},\mu^{j,g})}+$$
$$-\frac{p(g,g|.)((\partial u_2^i(\mu^{i,b},\mu_1)/\partial\mu^{i,b})(\partial p(g,g|.)/\partial h^c)-(\partial u_2^i(M^{g,g})/\partial\mu^{i,g})(\partial\mu^{i,g}/\partial h^c(\delta))-)\partial u_2^i(M^{g,g})/\partial\mu^{-i,g})\partial\mu^{-i,g}/\partial h^c)}{(u_2^i(\mu^{i,b},\mu_1)-u_2^i(M^{g,g}))^2}<0$$

The second inequality holds as: $\partial p(g,g|.)/\partial h^c>0$ and $\partial u_2^{is}/\partial\mu_2^i>0$, $\partial u_2^{is}/\partial\mu_2^{-i}>0$ and $\partial\mu^{i,g}/\partial h^{i*}>0>\partial\mu^{i,b}/\partial h^{i*}$. Therefore $\partial h^c(\delta)/\partial\delta>0$. To show $\lim_{\delta\to\infty}h^c(\delta)=1$ it is enough to show that $u_2^i(\mu^{i,b},\mu_1)-u_2^i(\mu^{i,g},\mu^{-i,g})=0$ iff $h_1^{i*}=h_1^{-i*}=1$. The *if* part holds as $\mu^{i,b}=\mu^{i,g}=\mu^{-i,g}=\mu_1$ for $h_1^{i*}=h_1^{-i*}=1$. The *only if* part holds as: $h^c(\delta)<1\rightarrow\mu^{i,b}(h^c(\delta))>\mu^{i,g}(h^c(\delta),h^c(\delta))$ and $\mu^{i,g}(h^c(\delta),h^c(\delta))<\mu_1$.
Moreover $\partial p(g,g|.)/\partial\mu^i,\partial p(g,g|.)/\partial\mu^{-i}>0$ and as shown above $\partial u_2^i/\partial p(g,g|.)>0$ □

Proof of Lemma 13. For the first part of Lemma 13, assume that rating agencies select $\beta_1^I=\beta_1^{II}=p(g,g|M_1,H_1)$. This implies that $\mu_1>\overline{\mu}$. But if $q_1=G$ then $\mu_2^I=\mu_2^{II}=\mu_1$, while if $q_1=B$ for $R_1=(b,b)$

$\mu_2^{i(-i)}=\mu_1$ so $\mu_2^{i(-i)}>\mu_1\forall 1$, for $r_1^i=g$, $r_1^j=b$ $r^{j(i)}=\mu_1/(\mu_1+(1-\mu_1)h_t^{i*})>\mu_1$, while for $R_1=(g,g)$ $\mu_2^{i(-i)}=\mu_1$ or $\mu_2^{-i(i)}=\mu_1$. Therefore $\max\{\mu_2^{I(II)},\mu_2^{II(I)}\}\geq\mu_1$, therefore by Assumption 6 if rating agencies select $\beta_1^I=\beta_1^{II}=p(g,g|M_1,H_1)$, then $\beta_2^{I*}=\beta_2^{II*}=p(g,g|M_2,H_2)$. For the second part of the theorem in equilibrium, rating agencies choose bribes $\beta_t^I,\beta_t^{II}:\beta_t^I+\beta_t^{II}=p(g,g|M_t,H_t)$ only if they strictly prefer to receive a bribe, that is only if $h_t^{I*}=h_t^{II*}=0$. This is the case because an agency A^i that is indifferent between honest rating and the bribe $\beta_t^i=p(g,g|M_t,H_t)/2$ will strictly prefer to request bribe $\beta^i=p(g,g|M_t,H_t)$.

□

Proof of Lemma 14. To prove the lemma I first prove an intermediate result:

Result: For any $\widetilde{\pi}\in[0,1]$, two signals with $\pi=\widetilde{\pi}$ have the same informative content of a single signal with $\pi=\widetilde{\pi}^2$.

Proof:Let s^1,s^2 be conditionally independent signals with $\pi=\widetilde{\pi}$ while s^3 is a signal with $\pi=\widetilde{\pi}^2$ for some $\widetilde{\pi}\in[0,1]$. Then $Pr\{(s^1,s^2)=(g,g)|q=G\}=Pr\{s^3=g|q=G\}=1$, and $Pr\{(s^1,s^2)=(g,g)|q=B\}=Pr\{s^3=g|q=B\}=\widetilde{\pi}^2$. So receiving signals $(s^1,s^2)=(g,g)$ has the same informative content as signal $s^3=g$. Moreover signals $(s^1,s^2)\neq(g,g)$ have the same informative content of signal $s^3=b$ as $Pr\{(s^1,s^2)\neq(g,g)|q=G\}=Pr\{s^3=b|q=G\}=0$ In monopoly $e_t=e^*((1-\pi^2)p(g))$ and

$$p(g)=e/e+(1-e)Pr\{r=g|q=B,\mu_t,\overline{h}\}.$$

In competition, if rating agencies coordinate on $\beta_t^i=\beta_t^j$ then $e_t=e^*((1-\pi^2)p(g))$ and

$$p(g,g)=e/e+(1-e)Pr\{R=(g,g)|q=B,M_t,\overline{H}\}.$$

The lemma holds as $Pr\{(r^I,r^{II})=(g,g)|q=B,(\overline{\mu},\overline{\mu}),(\overline{h},\overline{h})\}<Pr\{r^M=g|q=B,\overline{\mu},\overline{h}\}$ $\forall\overline{\mu},\overline{h}$ □

Proof of Proposition 15.

Step 1. $e_1^M=e_1^C$ iff $h_1^M=f(\pi,h_1^C)$, where $f(\pi,h_1^C):=\frac{1-\pi}{1+\pi}+\frac{2\pi}{1+\pi}h_1^C$.

Step 2. $\mu^g(f(\pi,h_1^C))<\mu^{g,g}(h_1^C,h_1^C)$ $\forall\pi,h_1^C$ as:

$\mu^g(f(\pi,h_1^C))=\frac{\mu_1\pi^2}{y^M(\mu_t,h_t)}=(\mu_1\pi^2/y^C(M_t,H_t))<(\mu_1\pi(\pi+(1-\pi)(1-\mu_1)(1-h_1^{Ci}))/y^C(\mu_t^I,\mu_t^{II},h_t^I,h_t^{II}))=\mu^{g,g}(h_1^C,h_1^C)$.

Where $y^M(\mu_t,h_t):=\pi^2+(1-\pi^2)(1-\mu_t)(1-h_t)$ and

$y^C(\mu_t^I,\mu_t^{II}h_t^I,h_t^{II}):=\pi^2+\pi(1-\pi)\Sigma_{I,II}(1-\mu_t^i)(1-h_t^i)$.

Step 3. If $\pi>1/3$ then $\mu^b(f(\pi,h_1^C))>(\mu^{b,g}(h_1^C,h_1^C)+\mu_1)/2$ for $h_1^C\in(\max(0,(2\mu_1\pi-(1-\pi))/(2\mu_1\pi-2\pi)),1)$.

Step 4: $\pi>\frac{1}{3}$ is sufficient to ensure $\partial u_2^M(p(g|\mu_2))/\partial\mu_2>\partial u_2^C(p(g,g|\mu_2))/\partial\mu_2|_{p(g|\mu_2)=p(g,g|M_2)}$ $\forall p(g|\mu_2)$

$\partial u_2^M(p(g|\mu_2))/\partial\mu_2=(\partial u_2^M(p(g|.))/\partial p(g|.))(\partial p(g|)/\partial y_2^M)(\partial y_2^M/\partial\mu_2^M)=$

$=-(1-(1-\pi^2)p(g|.)(\partial e_2/\partial b))(\partial p(g|.)/\partial y_2^M)(1-\pi^2)$.

$\partial u_2^{C,I}(p(g,g|M_2^C))/\partial M_2^C=(\partial u_2^{C,I}(p(g,g|M_2^C))/\partial\mu_2^I)+(\partial u_2^{C,I}(p(g,g|M_2^C))/\partial\mu_2^{II})=$

$=(\partial u_2^{C,I}(p(g,g|M_2^C))/\partial p(g,g|.))(\partial p(g,g|.)/\partial y_2^C)((\partial y_2^C/\partial\mu_2^I)+(\partial y_2^C/\partial\mu_2^I))=$

$=-(\frac{1}{2}(e_2+(1-e_2)(2\pi-\pi^2))-\pi(1-\pi)p(g,g|.)(\partial e_2/\partial p(g,g|.)))(\partial p(g,g|.)/\partial y_2^C)(2\pi(1-\pi))$

Note that $-(\partial p(g|.)/\partial y_2^M)=-(\partial p(g,g|.)/\partial y_2^C)>0$. So:

$\partial u_2^M(p(g|\mu_2))/\partial\mu_2>\partial u_2^C(p(g,g|\mu_2^C))/\partial\mu_2^C\leftrightarrow(1-(1-\pi^2)b(\partial e_2/\partial p))(1+\pi)>(\frac{1}{2}(e_2+(1-e_2)\pi(2-\pi))-\pi(1-\pi)b(\partial e_2/\partial p))2\pi$

A sufficient condition to ensure that the last inequality holds is:
$(1-(1-\pi^2)e_2)(1+\pi)>((1/2)(e_2+(1-e_2)\pi(2-\pi))-\pi(1-\pi)e_2)2\pi$. This condition holds $\forall e_2\in[0,1]$ if $\pi>1/3$.
Step 5. $u_2^M(\mu^b(f(\pi,h_1^C)))-u_2^M(\mu^g(f(\pi,h_1^C)))=\int_{\mu^g(f(\pi,h_1^C))}^{\mu^b(f(\pi,h_1^C))}(\partial u_2^M/\partial p(g|\mu_2^M))(\partial p(g|\mu_2^M)/\partial y_2^M)(\partial y_2^M/\partial\mu_2)d\mu_2>$
$>\int_{\mu^g(f(\pi,h_1^C))}^{\mu^b(f(\pi,h_1^C))}(\partial u_2^C/\partial p(g,g|\mu_2^C))(\partial p(g,g|\mu_2^C)/\partial y_2^C)(\partial y_2^C/\partial\mu_2)d\mu_2$.
By step 4, the inequality holds for $\pi>1/3$. By steps 2 and 3, the following inequality holds for $h_1^C\in(\max\{0,(1-\pi-2\mu_1\pi)/2\pi(1-\mu)\},1)$.

$$\int_{\mu^g(f(\pi,h_1^C))}^{\mu^b(f(\pi,h_1^C))}(\partial u_2^C/\partial p(g,g|\mu_2^C))(\partial p(g,g|\mu_2^C)/\partial y_2^C)(\partial y_2^C/\partial\mu_2)d\mu_2>$$

$$\int_{\mu^{g,g}(h_1^C,h_1^C)}^{(\mu^{b,g}(h_1^C,h_1^C)+\mu_1)/2}(\partial u_2^C/\partial p(g,g|\mu_2^C))(\partial p(g,g|\mu_2^C)/\partial y_2^C)(\partial y_2^C/\partial\mu_2)d\mu_2$$

$$=u_2^C(\mu^{b,g}(0),\mu_1)-u_2^C(\mu^{g,g}(0),\mu^{g,g}(0)).$$

.
Step 6: By Corollary (12) for δ large enough $h_1^C\in(\max\{0,(1-\pi-2\mu_1\pi)/2\pi(1-\mu_1)\},1)$.
By step 5, $h_1^C\in(\max\{0,(1-\pi-2\mu_1\pi)/2\pi(1-\mu)\},1)$ and $\pi>\frac{1}{3}$ ensure $h_1^M>f(\pi,h_1^C)$ and therefore $e_1^M>e_1^C$
As $h_1^C\in[0,1]$ is increasing in δ and every $h_1^C\in[0,1]$ is chosen in equilibrium for some δ, and for $\delta<\min\{\overline{\delta}^M,\overline{\delta}^C\}e_1^M<e_1^C$.
By Corollary (12) and Corollary (5) e_1^M and e_1^C are continuous functions of δ. Therefore there is a unique δ^* s.t. $e_1^M=e_1^C$ and $e_1^M>e_1^C$ iff $\delta>\delta^*$ $\square$

Proof of Lemma 16. (2.1) and (5.1) hold at the same time iff
$p(g,g|M_t,\overline{H})=f^L(\mathbf{e}((1-\pi)p(g,g|M_t,\overline{H})))$, where $f^L(x):=x/(x+(1-x)a^S)$, and
$a^S:=\pi+(1-\pi)\Pi_{I,II}(1-\mu_t^i)(1-\overline{h}^i)$.
Applying the same steps followed in Lemma 2, the necessary and sufficient condition for the existence of an $x\in(0,1):x=f^L(\mathbf{e}((1-\pi)x))$ is:
$\partial f^L(\mathbf{e}((1-\pi)x))/\partial x|_{x=0}>1$ (i). (i) holds for every $\overline{H}\in[0,1]^2$ iff $(1-\mu_t^I)(1-\mu_t^{II})<1/c''(0)-\pi/(1-\pi)$ $\square$

Proof of Lemma 17.
Whether agencies observe independent signals (and demand high bribes) or correlated signals, the issues sets $e^*=\mathbf{e}((1-\pi^2)p(g,g|M_t,\overline{H}))$. Therefore, effort is larger with identical signals iff:
$Pr\{R_t=[g,g]|M_t,\overline{H},e^*,indep\}>Pr\{R_t=[g,g]|M_t,\overline{H},e^*,ident\}\leftrightarrow$
$\leftrightarrow\pi^2+\pi(1-\pi)\Sigma_i(1-\mu_t^i)(1-\overline{h}^i)>\pi^2+(1-\pi^2)\Pi_i(1-\mu_t^i)(1-\overline{h}^i)\leftrightarrow$
$\Sigma_i(1-\mu_t^i)(1-\overline{h}^i)/\Pi_i(1-\mu_t^i)(1-\overline{h}^i)>(1+\pi)/\pi$.
For $\mu_t^I=\mu_t^{II}:=\mu_t$ and $\overline{h}^I=\overline{h}^{II}:=\overline{h}$ the condition reduces to: $(1-\mu_t)(1-\overline{h})<2\pi/(1+\pi)$ $\square$

Proof of Proposition 18. The only equilibrium strategies left to define are h_1^I,h_1^{II}. I focus on equilibria in which $h_1^I{=}h_1^{II}$ (I will refer to them as h^{c_s}). I show that if $\delta\leq\overline{\delta}^{c_s}$, for a $\overline{\delta}^{c_s}$ defined below, $h^{c_s}{=}0$. If instead $\delta{>}\overline{\delta}^{c_s}$, there is a unique h^{c_s} such that for $h_t^{i*}{=}h_t^{j*}{=}h^{c_s}$ a *strategic* agency is indifferent to request a bribe or not. In equilibrium β_t^i is paid only if A^j is also requesting a bribe, in which case a *strategic* agency faces a tradeoff between demanding a bribe and obtaining a continuation payoff equal to $p(g,g|.){+}\delta u_2^i(\mu^{i,g},\mu^{j,g})$ and rating honestly and getting $\delta u_2^i(\mu^{i,b},\mu_1^{i,b})$.
Step 1. $p(g,g|M_t,H_t^e){\in}C^1$ is increasing in μ_t^i, μ_t^j, h_t^{ie} and h_t^{je} (analogous to proof of Prop. 4)
Step 2.$\partial(p(g,g|.){+}\delta u_2^i(\mu^{i,g},\mu^{j,g}))/\partial h_1^i{+}\partial(p(g,g|.){+}\delta u_2^i(\mu^{i,g},\mu^{j,g}))/\partial h_1^j|_{h_1^I=h_1^{II}=h^{c_s}}{>}0$:
I prove this inequality in two intermediate steps.
(5.3) ensures$(\partial\mu^{i,g}/\partial h_1^j){>}0$ $\forall i,j{\in}\{I,II\}$, and
$\partial u_2^i(M_2,0,0)/\partial\mu_2^i{=}\ (\partial p(g,g|M_2,0,0)/\partial\mu_2^i)\partial(1/2)((e_2{+}(1{-}e_2)(\pi{+}(1{-}\pi)(1{-}\mu^{j(i)})))p(g,g|.){-}c(e_2))/\partial p(g.g|.){=}$
$(\partial p(g,g|M_2,0,0)/\partial\mu_2^i)(e_2/2{+}((1{-}e_2)/2)(\pi{+}(1{-}\pi)(1{-}\mu^{j(i)})){+}$
${-}\partial e^*((1{-}\pi^2)x)/\partial x|_{x=p(g,g|M_2,0,0)}(1{-}\mu^{j(i)})(1{-}\pi)p(g,g|.)){>}0$
The inequality holds as $\partial p(g,g|M_2,0,0)/\partial\mu_2^i{>}0$, and $\partial e^*((1{-}\pi^2)x)/\partial x|_{x=p(g,g|M_2,0,0)}p(g,g|){<}e_2$
(see Proof of Prop. 4).
Finally, $\partial p(g,g|M_1,h^c,h^c)/\partial h_1^i{>}0$ $\forall i{\in}\{I,II\}$.
Step 3. $\partial u_2^i(\mu_2^{i,b},\mu_1^{i,b})/\partial h_1^i{+}\partial u_2^i(\mu_2^{i,b},\mu_1^{i,b})/\partial h_1^j{<}0$: the inequality holds as $\partial\mu^{i,b}/\partial h_1^i{<}0{=}\partial u_2^i(\mu_2^{i,b},\mu_1)/\partial h_1^j$. The rest of the proof is identical to step 2).
Step 4. Analogous to Step 4 in Proof of Prop. 4 □

Proof of Corollary 19. The Proof follows the same steps of the Proof of Corollary 12 □

Proof of Proposition 20. Assume that there is a δ^{**} such that for that value of the discount factor, monopoly and competition ensure that same effort in the first period.
Step 1. Same effort requires: $e_1^M{=}e_1^C{\leftrightarrow}p(g|\mu_1,h_1^{M*}){=}p(g,g|\mu_1,\mu_1,h_1^{c*},h_1^{c*}){\leftrightarrow}h_1^{M*}{=}f^S(\mu_1,h_1^{c*}){:=}1{-}(1{-}\mu_1)(1{-}h_1^{c*})^2$.
Step 2. Same effort implies same reputation updates, that is: $\mu^g(f^S(\mu_1,h_1^{c*})){=}\mu^{i,g}(h_1^{c*},h_1^{c*})$
and $\mu^b(f^S(\mu_1,h_1^C)){=}\mu^{i,b}(h_1^C,h_1^C)$.
This is the case as:
$\mu^g(f^S(\mu_1,h_1^{c*})){=}\mu_1\pi^2/(\pi^2{+}(1{-}\pi^2)(1{-}\mu_1)(1{-}h_1^{M*})){=}\mu_1\pi^2/(\pi^2{+}(1{-}\pi^2)(1{-}\mu_1)^2(1{-}h_1^{c*})^2){=}\mu^{i,g}(h_1^{c*},h_1^{c*})$.
$\mu^b(f^S(\mu_1,h_1^{c*})){=}\mu_1/(1{-}(1{-}\mu_1)(1{-}h_1^M)){=}\mu_1/(1{-}(1{-}\mu_1)^2(1{-}h_1^{c*})^2){=}\mu^{i,b}(h_1^{c*},h_1^{c*})$.
Step 3.
$Pr\{r_1{=}g|q_1{=}B,h_t{=}0\}{:=}y^M(\mu_t,0){=}\pi^2{+}(1{-}\pi^2)(1{-}\mu_t){>}\pi^2{+}(1{-}\pi^2)(1{-}\mu_t)^2{=}$
${=}Pr\{R_1{=}(g,g)|q_1{=}B,H_t{=}(0,0)\}{:=}y^M(\mu_t,0)$, $\forall\mu_t{<}1$.
And $\frac{\partial y^M(\mu_2,0)/\partial\mu_2}{\partial y^{Cs}(\mu_2,0)/\partial\mu_2}{=}\frac{1}{2(1-\mu_2)}{>}1{\leftrightarrow}\mu_2{>}1/2$. By Assumption 1, $\mu_2{>}\underline{\mu}^M{:=}1/c''(0){=}1/2$.
Step 4. By Steps 2 and 3, $h_1^{M*}{=}f^S(\mu_1,h_1^{c*}){\rightarrow}y^M(\mu^b,0){-}y^M(\mu^g,0){>}y^C(\mu^{i,b},0){-}y^C(\mu^{i,g},0)$ and
$y^M(\mu^b,0){>}y^{Cs}(\mu^{i,b},\mu^{i,b},0,0)$, and $y^M(\mu^g,0){>}y^{Cs}(\mu^{i,g},\mu^{i,g},0,0)$.

Step 5. Let $F(y,p):=p-\frac{(1-\pi^2)p/2}{(1-\pi^2)p/2+(1-(1-\pi^2)p/2)y}$, then by the Implicit Function Theorem $p=p(y)$ and $p'(y)=\frac{1-2/(1-\pi^2)}{(1-y_t)^2}<0$ and
$p''(y_t)=\frac{1-2/(1-\pi^2)}{(1-y_t)^3}<0$.
By step 4, for identical effort in the first period,
$p(g|\mu^b,0)-p(g|\mu^g,0)>p(g,g|\mu^{i,b},\mu^{i,b},0,0)-p(g,g|\mu^{i,g},\mu^{i,g},0,0)$.
Step 6. For $\mu^{i(j)}=0$: $u_2^C(p_2)=(p_2-c(e_2))/2=u_2^M(p_2)/2$.
As utility $u_2^C(p_2)=u_2^M(p_2)/2$ and $d^2u_2^M(b_2)/d^2b_2=-(1-\pi^2)/2<0$, by step 5:
$u_2^i(\mu^{i,g}(h_1^{Cs},h_1^{Cs}),\mu^{i,g}(h_1^{Cs},h_1^{Cs}),0)-u_2^i(\mu^{i,b}(h_1^{Cs},h_1^{Cs}),\mu_1^j,\mu^{b(j)}(h_1(\delta))<$
$<\frac{1}{2}(u_2^i(\mu^b(h_1^*(\delta)))-u_2^i(\mu^g(h_1^*(\delta))))$.
As for identical effort in the first period $\beta_1^C=\beta_1^M/2$, then if the monopolist is indifferent to offer a bribe, the competitors strictly prefer to offer a bribe.
Step 7. This in turn implies that, for any $\delta>\overline{\delta}^c$, monopoly induces more effort than competition. This is the case because if competitors are indifferent to lying, or to being honest for h^{c*}, then the monopolist must choose in equilibrium an $h_1^{M*}>f^S(\mu_1,h_1^{c*})$. The continuity of $h_1^{M*}(\delta)$ ensures that there is a unique δ^{**} s.t. $e_1^M=e_1^C$. Moreover, $e_1^M>e_1^C\leftrightarrow\delta>\delta^{**}$ □

Proof of Lemma 21. First, I show $\delta^*<\overline{\delta}^c$. At first, I assume there is a δ^* for which $e_1^M=e_1^C$, and I show that for $\delta=\delta^*$ if in equilibrium a (strategic) monopolist rating agency is indifferent to request a bribe then a (strategic) competing rating agency which strictly prefers to request a bribe. Therefore, δ^* is unique and the monopoly ensures a higher level of effort in the first period iff $\delta>\delta^*$. I use this observation and the continuity of h_1^{M*} w.r.t. δ to show that δ^* exists and satisfies $\delta^*<\overline{\delta}^C$.
Step 1. $e_1^M=e_1^C\leftrightarrow h_1^M=f(h_1^C):=(1-\pi+2\pi h_1^C)/(1+\pi)$.
$e_t^M=e_t^C\leftrightarrow$ $p(g|\mu_t,h_t^{M*})=p(g,g|M_t,H_t^*)$.
Step 2. condition 1 implies
$h_1^M=f(h_1^C)\to y^M(\mu^g(h_1^M),0)-y^M(\mu^b(h_1^M),0)>y^C(\mu^{g,g}(h_1^C,h_1^C),0)-y^C((\mu^{g,b}(h_1^C,h_1^C)+\mu_1)/2,0)$.
Where $y^M(\mu_t,h_t):=\pi^2+(1-\pi^2)(1-\mu_t)(1-h_t)$ and
$y^C(\mu_t^I,\mu_t^{II}h_t^I,h_t^{II}):=\pi^2+\pi(1-\pi)\Sigma_{I,II}(1-\mu_t^i)(1-h_t^i)$.
This is the case as
$h_1^M=f(h_1^C)\to\mu^b-\mu^g>(\mu^{g,b}+\mu_1)/2-\mu^{g,g}$ for any $\mu_1\geq\max\left\{0,g(h_1^C,\pi)\right\}$ (condition 2).
Where $g(h_1^C,\pi):=(-2+4h_1^C+3\pi-4h_1^C\pi-\pi^2+2h_1^C\pi^2)/(-4+4h_1^C+4\pi-4h_1^C\pi-2\pi^2+2h_1^C\pi^2)$.
Note that $\partial g(h_1^C,\pi)/\partial\pi<0$, $\partial g(h_1^C,\pi)/\partial h_1^C<0$ and $g(h_1^C,\pi)<1/2$, $\forall h_1^C,\pi$.
Assumption 2 ensures that condition 2 holds.

$$\mu^b(h_1^M)-\mu^g(h_1^M)>(\mu^{g,b}(h_1^C,h_1^C)+\mu_1)/2-\mu^{g,g}(h_1^C,h_1^C)\to$$
$$(1-\pi^2)(\mu^b(h_1^M)-\mu^g(h_1^M))>2\pi(1-\pi)((\mu^{g,b}(h_1^C,h_1^C)+\mu_1)/2-\mu^{g,g}(h_1^C,h_1^C))\to$$
$$y^M(\mu^g(h_1^M),0)-y^M(\mu^b(h_1^M),0)>y^C(\mu^{g,g}(h_1^C,h_1^C),0)-y^C((\mu^{g,b}(h_1^C,h_1^C)+\mu_1)/2,0).$$

Step 3. $h_1^M=f(h_1^C)\to y^M(\mu^g(h_1^M),0)>y^C(\mu^{g,g}(h_1^C,h_1^C),0)$.
3.a $y^M(\mu_t,0)>y^C(\mu_t,\mu_t,0,0)$ $\forall\mu_t<1$.

3.b $\mu^g(f(h_1^C)){=}\mu_1\pi^2/y^M(\mu_1,f(h_1^C)){=}\mu_1\pi^2/y^C(\mu_1,h_1^C){<}(\mu_1\pi(\pi{+}(1{-}\pi)(1{-}\mu_1)(1{-}h_1^C)))/y^C(\mu,h){=}\mu^{g,g}(h_1^C,h_1^C)$.
Step 4. $p(g|\mu_t,h_t){=}k(y^M(\mu_t,h_t))$ and $p(g,g|M_t,H_t){=}k(y^C(\mu_t,\mu_t,h_t^I,h_t^{II}))$ where $k(x){:=}(1{-}(2/(1{-}\pi^2))x)/(1{-}x)$.
So the price is is decreasing and concave in y_t.
Therefore, by Steps 2 and 3: $p(g|\mu^b(h_1^M),0){-}p(g|\mu^g(h_1^M)){>}p(g,g|\mu^{b,g}(h_1^C,h_1^C)){-}p(g,g|\mu^{g,g}(h_1^C,h_1^C))$.
Step 5. The utility in the second period can be expressed as a function of the price in case of favorable ratings

$$u_2^M(p){:=}\mathbf{e}((1{-}\pi^2)p){+}(1{-}\mathbf{e}((1{-}\pi^2)p)\pi^2)p{-}c(p){+}(1{-}\mathbf{e}((1{-}\pi^2)p)(1{-}\pi^2)p{=}p{-}c(\mathbf{e}((1{-}\pi^2)p)),$$
$$u_2^C(p){:=}(1/2)(\mathbf{e}((1{-}\pi^2)p){+}(1{-}\mathbf{e}((1{-}\pi^2)p)\pi^2))p{-}c(\mathbf{e}((1{-}\pi^2)p)){+}(1{-}\mathbf{e}((1{-}\pi^2)p)\pi(1{-}\pi)p,$$

where $\mathbf{e}((1{-}\pi^2)p){=}(1{-}\pi^2)p/2$. Then

$$\partial u_2^M(p(g|.))/\partial p(g|.){>}\partial u_2^C(p(g,g|.))/\partial p(g,g|.){>}0$$

$\partial u_2^M(p)/\partial p{=}1{-}(1{-}\pi^2)^2p/2{>}0$ and $d^2u_2^M(p)/dp^2{=}{-}(1{-}\pi^2)^2/2{<}0$.
$\partial u_2^C(p)/\partial p{=}(1{-}\pi^2)/2((1{-}\pi^2)/2{-}\pi(1{-}\pi))p{+}(1{-}(1{-}\pi^2)p/2)\pi(1{-}\pi){>}0$. So $\partial u_2^M(p)/\partial p{>}\partial u_2^C(p)/\partial p$ $\forall p$
Step 6. By steps 4 and 5 $e_1^M{=}e_1^C$ implies

$$p(g|\mu^b(h_1^M),0){=}p(g,g|\mu^{b,g}(h_1^C,h_1^C))$$

and if $p(g|\mu_1,h_1^M,0){\geq}\delta(u_2^M(\mu^b(h_1^M)){-}u_2^M(\mu^g(h_1^M)))$ then

$$p(g,g|\mu^{b,g}(h_1^C,h_1^C)){>}\delta(u_2^C(\mu^{g,b}){-}u_2^C(\mu^{g,g})).$$

This implies that for $\delta{\geq}\overline{\delta}^C$ $h_1^{M*}{>}f(h_1^{C*})$. As e^M is a continuous function of δ and $e^M{<}e^C$ for $\delta{=}0$, then it must be the case that δ^* exists and satisfies $\delta^*{<}\overline{\delta}^C$
As $\delta^*{<}\overline{\delta}^C$ and $\delta^{**}{<}\overline{\delta}^C\delta^{**}{>}\delta^*$ iff $f^S(0){>}f(0)$ ($f^S(0)$ is defined in the Proof of Proposition 20) which is equivalent to $\mu_1{>}(1{-}2\pi)/(1{+}\pi)$ □

Proof of Lemma 22. (3.1) and (6.2) hold at the same time iff

$$p(g,g|M_t,\overline{H}){=}f^L(\mathbf{e}((1{-}\pi)(1{+}\pi{-}(\pi/2)\Sigma_{I,II}(1{-}\mu_t^i)(1{-}\overline{h}^i)p(g,g|M_t,\overline{H}))),$$

where $f^L(x){:=}x/(x{+}(1{-}x)a^L)$, and $a^L{:=}\pi^2{+}\Sigma_{I,II}(1{-}\pi)\pi(1{-}\mu_t^i)(1{-}\overline{h}^i){+}(1{-}\pi)^2\Pi_{I,II}(1{-}\mu_t^i)(1{-}\overline{h}^i)$.
Applying the same steps followed in Lemma 2, the necessary and sufficient condition for the existence of an $x{\in}(0{,}1){:}x{=}f^L(\mathbf{e}((1{-}\pi)(1{+}\pi{-}(\pi/2)\Sigma_{I,II}(1{-}\mu_t^i)(1{-}\overline{h}^i)x))$ is:
$\partial f^L(\mathbf{e}((1{-}\pi)(1{+}\pi{-}\frac{\pi}{2}\sum_{I,II}(1{-}\mu_t^i)(1{-}\overline{h}^i)x)))/\partial x|_{x=0}{>}1$ (i). (i) holds for every $\overline{H}{\in}[0{,}1]^2$ iff

$$\mu_t^i{+}\mu_t^j{-}\mu_t^i\mu_t^j(1{-}\pi)c''(0)/(c''(0){+}\pi){>}c''(0)/(c''(0){+}\pi)(1{-}\pi){-}(1{-}\pi)/(c''(0){+}\pi)$$ □

Let

$$\overline{\phi}_t{:=}(e_t^*{+}(1{-}e_t^*)(\pi{+}\pi(1{-}\pi)\Sigma_{I,II}(1{-}\mu_t^i)(1{-}\overline{h}^i)/2)p(g,g|M_t,\overline{H}){-}c(e_t^*).$$

I rule out pathological equilibria in which some type of rating agency requests $\phi_t^*{>}\overline{\phi}_t$. In any equilibrium in which $\mu(\phi_t){=}\mu_t\forall\mu_t$ Assumption 5 ensures that both types of agency set $\phi_t^*{=}\overline{\phi}_t/2$. If instead in equilibrium $\mu^i(\phi_t')\neq\mu_t^i$ for some ϕ_t', by Assumption 2 ϕ_t' must be on the equilibrium path for some type of agency A^i. But this implies that there exists at least another ϕ_t'' on the equilibrium path s.t. $\mu^i(\phi_t')\neq\mu^i(\phi_t'')$. W.l.g. let $\mu^I(\phi_t')>\mu^I(\phi_t'')$ (i). If the honest type strictly prefers ϕ_t'' over ϕ_t', then $\mu^i(\phi_t')=0$ which contradicts (i). If the honest type is indifferent or strictly prefers ϕ_t' over ϕ_t'', then the strategic type strictly prefers ϕ_t' over ϕ_t'' because $\beta(M(\phi_t'),\overline{H})>\beta(M(\phi_t''),\overline{H})$, which implies that either $\mu^i(\phi_t'')=1$ which contradicts (i) or ϕ_t'' is out of the equilibrium path, which is also a contradiction

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