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The international diversification puzzle is not as bad as you think[1]

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ABSTRACT

In the data country portfolios are heavily biased toward domestic assets. Standard one-good international macro models predict that, due to the presence of non-diversifiable labor income risk, country portfolios should be heavily biased toward foreign assets; this discrepancy constitutes the international diversification puzzle (Baxter and Jermann, 1997). We show that a simple extension of one-good models help to reconcile theory and data. In particular we analytically solve for the equilibrium country portfolios in a two-country, two-goods model with non-diversifiable labor income and investment. In this set-up, consistently with the data, country portfolios contain a relatively small, but positive, share of foreign assets. The reason why international diversification is low is that terms of trade movements provide considerable insurance against country specific shocks and labor income risk (Cole and Obstfeld 1991, Acemoglu and Ventura, 2002, Pavlova and Rigobon, 2003). The reason why international diversification is positive is that foreign assets are crucial to share the financing of investment across countries. Finally in the model a country's portfolio share of foreign assets should depend on its trade/GDP ratio and on its capital income/GDP ratio. We show how this relation is qualitatively and quantitatively consistent with country portfolios in the cross section of OECD countries in the 1990s.

KEYWORDS: Home bias, international diversification

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1. Introduction

Although there has been rapid growth in international portfolio diversification in recent years, portfolios remain heavily biased towards domestic assets. For example, foreign assets accounted, on average, for only about 14% of the total value of the assets owned by U.S. residents during the 1990s. There is a large theoretical literature that explores whether observed low diversification should be interpreted as evidence of incomplete insurance against country-specific risk [references]. These papers share a common conclusion: relative to the prediction of frictionless models, too little diversification is observed in the data. In response, recent work on diversification has focused on introducing frictions that can rationalize observed portfolios. The set of candidate frictions is long and includes costs in goods trade, costs in asset trade, imperfect competition and price stickiness in product markets, and asymmetric information in financial markets [reference in each case].

In this paper, we take a different approach. We develop a frictionless model in which perfect risk sharing is in fact wholly consistent with relatively low levels of international diversification. We argue that previous theoretical benchmarks delivered the wrong answers because the models were too simple to capture the key diversification motives associated with country-specific business cycle risks. Our environment is the two-country extension of the stochastic growth model developed by Backus, Kehoe and Kydland (1992 and 1995, henceforth BKK), which is a workhorse model for quantitative international macroeconomics. While BKK allow for a complete set of Arrow securities to be traded between countries, we instead follow the tradition in the international diversification literature and assume that households only trade shares in domestic and foreign firms. BKK and others have shown that the international stochastic growth model is broadly consistent with a large set of international business cycle facts. We show that the same model rationalizes observed levels of international diversification. Since our model is frictionless, this finding casts doubt on the quantitative role of frictions in understanding observed portfolios.

One contribution of our paper is to show that given particular assumptions on preferences and technologies, equilibrium portfolio choices can be characterized analytically.[2] In this case, the equilibrium portfolio choice depends on only two parameters: (i) the relative preference in consumption for domestically-produced versus imported goods, and (ii) capital's share in production. When we calibrate these parameters to replicate appropriate features of the United States economy, our expression implies portfolios comprising around 80% domestic stocks and 20% foreign stocks. An additional interesting property of the model is that two stocks effectively complete markets, in the sense that consumption and leisure in both countries are identical state-by-state to the corresponding allocations for an economy in which a complete set of Arrow securities is traded internationally. We conclude that observed low levels of diversification should not be interpreted as inconsistent with the complete markets hypothesis.

We use the equilibrium portfolio expression to test our model by considering the extent to which differences in trade shares in a cross-section of countries predict differences in levels of diversification. We find that the theoretical relationship is both qualitatively and quantitatively consistent with the empirical pattern for relatively high-income economies. In particular, the ratio of trade to GDP is a powerful predictor of observed international diversification, while controlling for openness to trade, neither size nor GDP per capita significantly affect diversification.

To better understand the predictions of our model for portfolio choices we compare and contrast our economy to those considered by Lucas (1982), Baxter and Jermann (1997), and Cole and Obstfeld (1991). Lucas (1982) points out that in a symmetric one-good two-country model, perfect risk pooling involves agents of each country owning half the claims to the home endowment and half the claims to the foreign endowment. Baxter and Jermann (1997) extend Lucas' model in

[2] The assumptions required to derive an analytical expression for the portfolio choice are (i) preferences are separable between consumption and leisure and logarithmic in consumption, and (ii) all production technologies are Cobb-Douglas, which implies a unitary elsasticity of substitution between traded goods.

one direction by introducing production while retaining the single-good assumption. They show that if returns to capital and labor are highly correlated within a country, then agents can compensate for undiversifiable labor income risk by aggressively diversifying asset holdings. In their examples, fully diversified portfolios typically involve substantial short positions in domestic assets.

Cole and Obstfeld (1991) extend Lucas' analysis in a different direction. They retain the focus on an endowment economy, but assume that the two countries receive endowments of different goods that are imperfect substitutes. These goods are then traded, and agents consume bundles comprising both goods. They show that changes in relative endowments induce off-setting changes in the terms of trade. When preferences are log-separable between the two goods, the terms of trade responds one-for-one to changes in relative income, effectively delivering perfect risk-sharing. Thus, in sharp contrast to the results of Lucas or Baxter and Jermann, any level of diversification is consistent with complete risk-pooling, including portfolio autarky.

One important difference in our analysis relative to Baxter and Jermann (1997) is that we allow for imperfect substitutability between domestic and foreign-produced traded goods, following Cole and Obstfeld. Thus in our model, changes in international relative prices provide some insurance against country-specific shocks and, in the flavor of the Cole and Obstfeld indeterminacy result, the portfolio choice does not have to do all the heavy-lifting when it comes to delivering perfect risk-sharing. In contrast to Cole and Obstfeld, however, the presence of production and particularly investment in our model means that it is not the case that any portfolio automatically delivers perfect risk-sharing. The exact portfolio split determines who finances changes in investment, which induces a unique equilibrium portfolio choice characterized by a strong bias towards domestic assets. Home bias is optimal, since even though most of an extra unit of domestic investment comes directly out of the pocket of domestic shareholders, domestic investment also disproportionately raises demand for domestic goods, inducing a decline in terms of trade that equates appropriately measured consumption in the two countries.

In our analysis, we emphasize the role of terms of trade dynamics in portfolio choice considerations. However, it is well known that productivity shock driven business cycle models tend to deliver a high positive correlation between domestic output and the price of imports relative to exports, while the analogous correlation in the data is close to zero for many countries. We therefore extend the model to introduce a preference shocks as a second source of risk. We show that our low equilibrium diversification result survives in this extended model, while the presence of demand-side shocks brings the correlation between domestic output and the terms of trade in line with the data.

2. The Model

The modelling framework is the one developed by Backus, Kehoe and Kydland, 1995. There are two countries, each of which is populated by the same measure of identical, infinitely-lived households. Firms in each country use country-specific capital and labor to produce an intermediate good. The intermediate good produced in the domestic country is labeled a, while the good produced in the foreign country is labeled b. These are the only traded goods in the world economy. Intermediate-goods-producing firms are subject to country-specific productivity shocks. Within each country the intermediate goods a and b are combined to produce country-specific final consumption and investment goods. The final goods production technologies are asymmetric across countries, in that they are biased towards using a larger fraction of the locally-produced intermediate good. This bias allows the model to replicate empirical measures for the volume of trade relative to GDP.

We assume that the assets that are traded internationally are shares in the domestic and foreign representative intermediate-goods-producing firms. These firms make investment and employment decisions, and distribute any non-reinvested earnings to shareholders.

A. Preferences and technologies

In each period t the economy experiences one event $s_t \in S$. We denote by $s^t = (s_0, s_1, ..., s_t) \in S^t$ the history of events from date 0 to date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t)$. Let $s_r(s^t)$ denote the date r event in history s^t.

Period utility for a household in the domestic country after history s^t is given by[3]

$$(1) \quad U\left(c(s^t), n(s^t)\right) = \ln c(s^t) + v\left(1 - n(s^t)\right)$$

where $c(s^t)$ denotes consumption at date t given history s^t, and $n(s^t)$ denotes labor supply. Utility from leisure is given by the increasing and concave function $v(.)$, which satisfies $\lim_{n \to 1} v'(1-n) = \infty$. The assumption that utility is log-separable in consumption will play a role in deriving a closed-form expression for equilibrium portfolios in our baseline calibration. In contrast, the equilibrium portfolio in this case will not depend on the particular functional form for $v(.)$.

Households supply labor to domestically located perfectly-competitive intermediate-goods-producing firms ($i-$firms). $I-$firms in the domestic country produce good a, while those in the foreign country produce good b. These firms hold the capital in the economy and operate a Cobb-Douglas production technology:

$$(2) \quad F\left(z(s^t), k(s^{t-1}), n(s^t)\right) = e^{z(s^t)} k(s^{t-1})^{\theta} n(s^t)^{1-\theta},$$

where $z(s^t)$ is an exogenous productivity shock. The vector of shocks $\widehat{z}(s^t) = [z(s^t), z^*(s^t)]$ evolves stochastically. For now, the only assumption we make about this process is that it is symmetric. In the baseline version of the model, productivity shocks are the only source of uncertainty in the

[3]The equations describing the foreign country are largely identical to those for the domestic country. We use star superscripts to denote foreign variables.

model. Thus $s_t(s^t) = \widehat{z}_t(s^t)$ and the probabilities $\pi(s^t)$ are those implied by the stochastic process for productivity.

Each period, households receive dividends from their stock holdings in the domestic and foreign $i-$firms, and buy and sell shares to adjust their portfolios. After completing asset trade, households sell their holdings of intermediate goods to domestically located final-goods-producing firms ($f-$firms). The $f-$firms are perfectly competitive and produce final goods using intermediate goods a and b as inputs to a Cobb-Douglas technology:

$$
\begin{aligned}
G\left(a(s^t), b(s^t)\right) &= a(s^t)^{\omega} b(s^t)^{(1-\omega)} \\
G^*\left(a^*(s^t), b^*(s^t)\right) &= a^*(s^t)^{(1-\omega)} b^*(s^t)^{\omega}
\end{aligned} \tag{3}
$$

where $\omega > 0.5$ determines the size of the local input bias in the composition of domestically produced final goods.

Note that the Cobb-Douglas assumption implies a unitary elasticity of substitution between between domestically-produced goods and imports. The Cobb-Douglas assumption, in conjunction with the assumption that utility is logarithmic in consumption, will allow us to derive a closed-form expression for the equilibrium portfolio. Note, however, that a unitary elasticity is within the range of existing estimates: BKK (1995) set this elasticity to 1.5 in the benchmark calibration, while Heathcote and Perri (2002) estimate the elasticity to be 0.9. In a sensitivity analysis we will explore numerically the implications of deviating from the logarithmic utility, unitary elasticity baseline.

We now define two relative prices that will be useful in the subsequent analysis. Let $t(s^t)$ denote the terms of trade, defined as the price of good b relative to good a. Because the law of one price applies to traded, goods, this relative price is the same in both countries:

$$
t(s^t) = \frac{q_b(s^t)}{q_a(s^t)} = \frac{q_b^*(s^t)}{q_a^*(s^t)} \tag{4}
$$

Let $e(s^t)$ denote the real exchange rate, defined as the price of foreign relative to domestic consumption. Because the law of one price applies to intermediate goods, $e(s^t)$ is equal to the foreign price of good a (or good b) relative to foreign consumption divided by the domestic price of the same good relative to domestic consumption:

$$(5) \quad e(s^t) = \frac{q_a(s^t)}{q_a^*(s^t)} = \frac{q_b(s^t)}{q_b^*(s^t)}$$

B. Households' problem

The budget constraint for the domestic household is given by

$$(6) \qquad c(s^t) + P(s^t)\left(\lambda_H(s^t) - \lambda_H(s^{t-1})\right) + e(s^t)P^*(s^t)\left(\lambda_F(s^t) - \lambda_F(s^{t-1})\right)$$
$$= \quad q_a(s^t)w(s^t)n(s^t) + \lambda_H(s^{t-1})d(s^t) + \lambda_F(s^{t-1})e(s^t)d^*(s^t) \qquad \forall t \geq 0, s^t$$

Here $P(s^t)$ is the price at s^t of (ex dividend) shares in the domestic firm in units of domestic consumption, $P^*(s^t)$ is the price of shares in the foreign firm in units of foreign consumption, $\lambda_H(s^t)$ ($\lambda_H^*(s^t)$) denotes the fraction of the domestic firm purchased by the domestic (foreign) agent, $\lambda_F(s^t)$ ($\lambda_F^*(s^t)$) denotes the fraction of the foreign firm bought by the domestic (foreign) agent, $d(s^t)$ and $d^*(s^t)$ denote domestic and foreign dividend payments per share, and $w(s^t)$ denotes the domestic wage in units of the domestically-produced intermediate good. The budget constraint for the foreign household is

$$(7) \qquad c^*(s^t) + P^*(s^t)\left(\lambda_F^*(s^t) - \lambda_F^*(s^{t-1})\right) + (1/e(s^t))P(s^t)\left(\lambda_H^*(s^t) - \lambda_H^*(s^{t-1})\right)$$
$$= \quad q_b^*(s^t)w^*(s^t)n^*(s^t) + \lambda_F^*(s^{t-1})d^*(s^t) + \lambda_H^*(s^{t-1})(1/e(s^t))d(s^t) \qquad \forall t \geq 0, s^t$$

We assume that at the start of period 0, the domestic (foreign) household owns the entire domestic (foreign) firm: thus $\lambda_H(s^{-1}) = 1$, $\lambda_F(s^{-1}) = 0$, $\lambda_F^*(s^{-1}) = 1$ and $\lambda_H^*(s^{-1}) = 0$.

At date 0, domestic households choose $\lambda_H(s^t)$, $\lambda_F(s^t)$, $c(s^t) \geq 0$ and $n(s^t) \in [0,1]$ for all s^t and for all $t \geq 0$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t U\left(c(s^t), 1 - n(s^t)\right) \tag{8}$$

subject to 6.

The domestic households' first-order condition for domestic and foreign stock purchases are, respectively,

$$\begin{aligned} U_c(s^t)P(s^t) &= \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[d(s^t, s_{t+1}) + P(s^t, s_{t+1})\right] \\ U_c(s^t)e(s^t)P^*(s^t) &= \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) e(s^t, s_{t+1}) \left[d^*(s^t, s_{t+1}) + P^*(s^t, s_{t+1})\right] \end{aligned} \tag{9}$$

where (s^t, s_{t+1}) denotes the $t+1$ length history s^t followed by s_{t+1}

The domestic household's first-order condition for hours is

$$\begin{aligned} U_c(s^t) q_a(s^t) w(s^t) + U_n(s^t) &\geq 0 \\ &= \text{if } n(s^t) > 0 \end{aligned} \tag{10}$$

Analogously, the foreign households' first-order condition for domestic and foreign stock purchases and hours are, respectively,

$$\begin{aligned} U_c^*(s^t) \frac{P(s^t)}{e(s^t)} &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{d\left(s^t, s_{t+1}\right) + P(s^t, s_{t+1})}{e\left(s^t, s_{t+1}\right)}\right] \\ U_c^*(s^t) P^*(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[d^*(s^t, s_{t+1}) + P^*(s^t, s_{t+1})\right] \end{aligned} \tag{11}$$

and

$$(12)\quad U_c^*(s^t)q_b^*(s^t)w^*(s^t)+U_n^*(s^t) \geq 0$$

$$= \text{ if } n^*(s^t)>0.$$

C. Intermediate firms' problem

The domestic $i-$firm's maximization problem is given by

$$\max_{\{k(s^t),n(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t} Q(s^t)d(s^t)$$

subject to $k(s^t)$, $n(s^t)\geq 0$, taking as given $k(s^{-1})$, where $Q(s^t)$ is the price the firm uses to value dividends at s^t relative to consumption at date 0, and dividends (in units of the final good) are given by

$$(13)\quad d(s^t)=q_a(s^t)\left[F\left(z(s^t),k(s^{t-1}),n(s^t)\right)-w(s^t)n(s^t)\right]-\left[k(s^t)-(1-\delta)k(s^{t-1})\right].$$

In this expression δ is the depreciation rate for capital. Analogously, foreign firms use prices $Q^*(s^t)$ to price dividends in state s^t, where foreign dividends are given by

$$(14)\quad d^*(s^t)=q_b^*(s^t)\left[F\left(z^*(s^t),k^*(s^{t-1}),n^*(s^t)\right)-w^*(s^t)n^*(s^t)\right]-\left[k^*(s^t)-(1-\delta)k^*(s^{t-1})\right].$$

The domestic and foreign $i-$firms' first order conditions for $n(s^t)$ and $n^*(s^t)$ are

$$(15)\quad w(s^t)=\frac{(1-\theta)F\left(z(s^t),k(s^{t-1}),n(s^t)\right)}{n(s^t)}$$

$$(16)\quad w^*(s^t)=\frac{(1-\theta)F\left(z^*(s^t),k^*(s^{t-1}),n^*(s^t)\right)}{n^*(s^t)}.$$

The corresponding first order conditions for $k(s^t)$ and $k^*(s^t)$ are

$$(17) \quad Q(s^t) = \sum_{s_{t+1} \in S} Q(s^t, s_{t+1}) \left[q_a(s^t, s_{t+1}) \frac{\theta F\left(z(s^t, s_{t+1}), k(s^t), n(s^t, s_{t+1})\right)}{k(s^t)} + (1-\delta) \right]$$

$$(18) \quad Q^*(s^t) = \sum_{s_{t+1} \in S} Q^*(s^t, s_{t+1}) \left[q_b^*(s^t, s_{t+1}) \frac{\theta F\left(z^*(s^t, s_{t+1}), k^*(s^t), n^*(s^t, s_{t+1})\right)}{k^*(s^t)} + (1-\delta) \right]$$

The state-contingent consumption prices $Q(s^t)$ and $Q^*(s^t)$ obviously play an important role in $i-$firms' state-contingent decisions regarding how to divide earnings between investment and dividend payments. We assume that these firms use a weighted sum of the discount factors of the representative domestic and foreign households to price the marginal cost of foregoing current dividends in favor of extra investmest. Thus

$$\begin{array}{lrcl} (19) & Q(s^t) & = & \upsilon(s^t) \dfrac{\pi(s^t)\beta^t U_c(s^t)}{U_c(s^0)} + (1-\upsilon(s^t)) \dfrac{\pi(s^t)\beta^t \left(U_c^*(s^t)/e(s^t)\right)}{\left(U_c^*(s^0)/e(s^0)\right)} \\ (20) & Q^*(s^t) & = & \upsilon^*(s^t) \dfrac{\pi(s^t)\beta^t U_c^*(s^t)}{U_c^*(s^0)} + (1-\upsilon^*(s^t)) \dfrac{\pi(s^t)\beta^t e(s^t) U_c(s^t)}{e(s^0) U_c(s^0)} \end{array}$$

where, for example, $\upsilon(s^t)$ is the relative weight placed on the preferences of domestic shareholders by domestic firms.[4]

D. Final goods firms' problem

The $f-$firm's static maximization problem in the domestic country after history s^t is given by

$$\max_{a(s^t), b(s^t)} \left\{ G(a(s^t), b(s^t)) - q_a(s^t) a(s^t) - q_b(s^t) b(s^t) \right\}$$

[4] Note that each agent takes $Q(s^t)$ as given, understanding that their individual atomistic portfolio choices will not affect aggregate investment decisions.

subject to $a(s^t)$, $b(s^t) \geq 0$.

The first order conditions for domestic and foreign $f-$firms may be written as

$$(21) \qquad \begin{array}{ll} q_a(s^t) = \frac{\omega G(a(s^t),b(s^t))}{a(s^t)} & q_b(s^t) = \frac{(1-\omega)G(a(s^t),b(s^t))}{b(s^t)} \\ q_b^*(s^t) = \frac{\omega G^*\left(a^*(s^t),b^*(s^t)\right)}{b^*(s^t)} & q_a^*(s^t) = \frac{(1-\omega)G^*\left(a^*(s^t),b^*(s^t)\right)}{a^*(s^t)} \end{array}$$

E. Definition of equilibrium

An equilibrium is a set of quantities $c(s^t)$, $c^*(s^t)$, $k(s^t)$, $k^*(s^t)$, $n(s^t)$, $n^*(s^t)$, $a(s^t)$, $a^*(s^t)$, $b(s^t)$, $b^*(s^t)$, $\lambda_H(s^t)$, $\lambda_H^*(s^t)$, $\lambda_F(s^t)$, $\lambda_F^*(s^t)$, prices $P(s^t)$, $P^*(s^t)$, $r(s^t)$, $r^*(s^t)$, $w(s^t)$, $w^*(s^t)$, $Q(s^t)$, $Q^*(s^t)$, $q_a(s^t)$, $q_a^*(s^t)$, $q_b(s^t)$, $q_b^*(s^t)$, productivity shocks $z(s^t)$, $z^*(s^t)$ and probabilities $\pi(s^t)$ for all s^t and for all $t \geq 0$ which satisfy the following conditions:

1. The first order conditions for intermediate-goods purchases by $f-$firms (equation 21)
2. The first-order conditions for labor demand by $i-$firms (equations 15 and 16)
3. The first-order conditions for labor supply by households (equations 10 and 12)
4. The first-order conditions for capital accumulation for $i-$firms (equations 17 and 18),
5. The market clearing conditions for intermediate goods a and b :

$$(22) \qquad \begin{aligned} a(s^t) + a^*(s^t) &= F\left(z(s^t), k(s^{t-1}), n(s^t)\right) \\ b(s^t) + b^*(s^t) &= F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right). \end{aligned}$$

6. The market clearing conditions for final goods:

$$(23) \qquad \begin{aligned} c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) &= G\left(a(s^t), b(s^t)\right) \\ c^*(s^t) + k^*(s^t) - (1-\delta)k^*(s^{t-1}) &= G^*\left(a^*(s^t), b^*(s^t)\right). \end{aligned}$$

7. The market clearing condition for stocks:

$$\text{(24)} \quad \lambda_H(s^t) + \lambda_H^*(s^t) = 1 \qquad \lambda_F(s^t) + \lambda_F^*(s^t) = 1.$$

8. The households' budget constraints (equations 6 and 7)
9. Households' first-order conditions for stock purchases (equations 9 and 11).
10. The probabilities $\pi(s^t)$ are consistent with the stochastics process for $(z(s^t), z^*(s^t))$

3. Equilibrium portfolios

PROPOSITION 1: Suppose that at time zero, productivity is equal to its unconditional mean value in both countries $(z(s^0) = z^*(s^0) = 1)$ and that initial capital is equalized across countries, $k(s^{-1}) = k^*(s^{-1}) > 0$. Then there is an equilibrium in this economy with the property that portfolios in both countries exhibit a constant level of diversification given by

$$\begin{aligned} \lambda_F(s^t) &= \lambda_H^*(s^t) = 1 - \lambda_H(s^t) = 1 - \lambda_F^*(s^t) \\ &= 1 - \lambda = \frac{1-\omega}{1+\theta-2\omega\theta} \qquad \forall t, s^t \end{aligned} \tag{25}$$

PROOF: See the appendix

CORROLLARY: In this equilibrium stock prices are given by

$$\text{(26)} \quad P(s^t) = k(s^t), \ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.$$

We prove this result by showing that these portfolios decentralize the solution to an equal-weighted planner's problem in the same environment. In particular, we consider the problem of a planner who seeks to maximize the equally-weighted expected utilities of the domestic and foreign agents, subject only to resource constraints of the form 22 and 23. We then describe a set of candidate prices such that if the conditions that define a solution to the planner's problem are satisfied, then the conditions that define a competitive equilibrium in the stock trade economy are also satisfied when portfolios are given by the expression in equation 25.[5]

PROOF: FOR APPENDIX

Let $G(s^t)$ and $F(s^t)$ be compact notations for $G(a(s^t), b(s^t))$ and $F\left(z(s^t), k(s^{t-1}), n(s^t)\right)$.

The equations that characterize a solution to the planner's problem are:

1. First order conditions for hours:

$$
\begin{aligned}
U_c(s^t)\frac{\omega G(s^t)}{a(s^t)}\frac{(1-\theta)F(s^t)}{n(s^t)} + U_n(s^t) &\geq 0 \\
U_c^*(s^t)\frac{\omega G^*(s^t)}{b^*(s^t)}\frac{(1-\theta)F^*(s^t)}{n^*(s^t)} + U_n^*(s^t) &\geq 0
\end{aligned}
$$

2. First order conditions for allocating intermediate goods across countries:

$$
\begin{aligned}
U_c(s^t)\omega G(s^t)/a(s^t) &= U_c^*(s^t)(1-\omega)G^*(s^t)/a^*(s^t) \\
U_c(s^t)(1-\omega)G(s^t)/b(s^t) &= U_c^*(s^t)\omega G^*(s^t)/b^*(s^t)
\end{aligned}
$$

3. First order conditions for investment:

$$
\begin{aligned}
\widetilde{Q}(s^t) &= \sum_{s_{t+1}\in S}\widetilde{Q}(s^t, s_{t+1})\left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})}\frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta)\right] \\
\widetilde{Q}^*(s^t) &= \sum_{s_{t+1}\in S}\widetilde{Q}^*(s^t, s_{t+1})\left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})}\frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta)\right]
\end{aligned}
$$

[5] We have not shown the reverse, namely that any competitive equilibrium in the stock trade economy is efficient.

where

$$\widetilde{Q}(s^t) = \frac{1}{2}\pi(s^t)\beta^t U_c(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t)\frac{(1-\omega)}{\omega}\frac{G^*(s^t)}{G(s^t)}\frac{a(s^t)}{a^*(s^t)}$$

$$\widetilde{Q}^*(s^t) = \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c(s^t)\frac{\omega}{(1-\omega)}\frac{G(s^t)}{G^*(s^t)}\frac{a^*(s^t)}{a(s^t)}$$

4. Resource constraints of the form 22 and 23.

We now show that given the candidate portfolio rules in equation 25 we can take allocations that satisfy the set of equations defining the solution to the planner's problem, and construct a set of prices such that given these prices, if an allocation satisfies the set of equations defining the planner's problem, then it also satisfies the set of equations defining the equilibrium in the stock trade economy, i.e. we can decentralize the complete markets allocations with only trade in two stocks.

Let intermediate goods prices be given by equations 21. Then condition (1) for the stock trade economy is satisfied. Let wages be given by equations 15 and 16. Then condition (2) for the stock trade economy is satisfied. Substituting these prices into condition (1) from the planner's problem gives condition (3) for the stock trade economy. Let the real exchange rate by given by equation 5. Then combining conditions (2) and (3) from the planner's problem gives condition (4) for the stock trade economy. Condition (4) from the planner's problem translates directly into conditions (5) and (6) for the stock trade economy. Condition (7) - stock market clearing - follows immediately from the symmetry of the candidate stock purchase rules.

Condition (8) is that households' budget constraints are satisfied. Given constant portfolios, the domestic household's budget constraint simplifies to

$$c(s^t) = q_a(s^t)w(s^t)n(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t)$$

Substituting in the candidate function for $w(s^t)$, the resource constraint for intermediate goods, and the definitions for dividends (and suppressing the state-contingent notation) gives

$$c = q_a(1-\theta)(a+a^*) + \lambda\left(q_a\theta(a+a^*) - x\right) + (1-\lambda)e\left(q_b^*\theta(b+b^*) - x^*\right)$$

Using the candidate expression for the real exchange rate gives

$$c = (1-\theta+\lambda\theta)\left(q_a a + e q_a^* a^*\right) - \lambda x + (1-\lambda)\theta\left(q_b b + e q_b^* b^*\right) - (1-\lambda)e x^*$$

Now using the candidate expressions for intermediate goods prices and collecting terms gives

$$c = \left[\omega + (1-\lambda)(\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(\theta - 2\omega\theta\right)\right]G^* - \lambda x - (1-\lambda)e x^*$$

Using the resource constraint for final goods firms gives

$$\begin{aligned} G &= \left[\omega + (1-\lambda)(\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(\theta - 2\omega\theta\right)\right]G^* \\ &\quad +(1-\lambda)(G-c) - (1-\lambda)e(G^* - c^*) \end{aligned}$$

Given the candidate expression for the real exchange rate, and exploiting the assumption that utility is logarithmic in consumption, condition (2) for the planners problem implies

$$c = ec^*.$$

Thus the budget constraint can be rewritten as

$$G = \left[\omega + (1-\lambda)(1+\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(1+\theta - 2\omega\theta\right)\right]G^*$$

Finally substituting in the candidate portfolio expressions confirms that the domestic consumer's budget constraint is satisfied. The foreign consumer's budget constraint is satisfied by Walras' Law.

Condition (9) is household's inter-temporal first order conditions for stock purchases. Substituting condition (2) from the planner's problem into condition (3), the planner's first order conditions for investment may be rewritten as

$$
\begin{aligned}
U_c(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta) \right] \\
U_c^*(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta) \right]
\end{aligned}
$$

Multiplying both sides of the first (second) of these two equations by $k(s^t)$ $(k^*(s^t))$ gives

$$
\begin{aligned}
U_c(s^t)k(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \theta F(s^t, s_{t+1}) + (1-\delta)k(s^t) \right] \\
U_c^*(s^t)k^*(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \theta F^*(s^t, s_{t+1}) + (1-\delta)k^*(s^t) \right]
\end{aligned}
$$

Let stock prices by given by

$$(27) \quad P(s^t) = k(s^t), \; P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.$$

Substituting these candidate prices for stocks, the prices for intermediate goods, the wage, and the expressions for dividends into the planner's first order conditions for investment gives the domestic household's first order condition for domestic stock purchases, and the foreign household's first order condition for foreign stock purchases. The remaining two first-order conditions for stock purchases follow immediately by substituting condition (2) from the planner's problem into these two conditions.

4. Intuition for the result

What explains the finding that two stocks are sufficient to effectively complete markets in this economy, and how should we understand the particular expression for the portfolios that deliver perfect risk sharing in equation 25? We now build intuition for these results from two different perspectives. First, we take a macroeconomic general equilibrium perspective, and combine a set of equilibrium conditions that link differences between domestic and foreign aggregate demand and aggregate supply in this economy. These equations shed light on how changes in relative prices coupled with modest levels of international portfolio diversification allow agents to achieve perfect risk-sharing. We then take a more micro agent-based perspective, and explore how, from a price-taking individual's point of view, returns to labor and to domestic and foreign stocks covary in such a way that agents prefer to bias portfolios towards domestic assets.

A. Macroeconomic Intuition

We now develop two key equations that are helpful for understanding the macroeconomics of portfolio choice.

The first equation relates differences in the relative value of absorption across countries to differences in the relative value of output. This equation is independent of preferences and the asset market structure, and follows solely from our assumption that the final-goods firms production technology is Cobb-Douglas, or equivalently, that the elasticity of substitution between the two traded goods is one.

From equations 22, 5 and 21, domestic GDP (in units of the final good) is given by

$$\begin{aligned} y(s^t) &= q_a(s^t)\left(a(s^t) + a^*(s^t)\right) = q_a(s^t)a(s^t) + e(s^t)q_a^*(s^t)a^*(s^t) \\ &= \omega G(s^t) + e(s^t)(1-\omega)G^*(s^t) \end{aligned}$$

Similarly, foreign GDP is given by

$$y^*(s^t) = \frac{1}{e(s^t)}(1-\omega)G(s^t) + \omega G^*(s^t)$$

Let $\Delta y(s^t)$ denote the difference between the value of domestic and foreign GDP in units of the domestic final good. Combining the two expressions above, $\Delta y(s^t)$ is linearly related to the difference between domestic and foreign absorption:

$$\begin{aligned} (28) \quad \Delta y(s^t) &= (2\omega - 1)\left(G(s^t) - e(s^t)G^*(s^t)\right) \\ &= (2\omega - 1)\left(\Delta c(s^t) + \Delta x(s^t)\right) \end{aligned}$$

The fact that countries devote a constant fraction of total final demand to each of the two intermediate goods means that changes in final demand (driven, for example, by changes in relative investment) affect the relative value of intermediate output. When the technologies for producing domestic and foreign final goods are the same ($\omega = 0.5$), changes to relative foreign versus domestic demand for consumption or investment do not impact the relative value of the outputs of goods a and b. When final goods are produced only with good a ($\omega = 1$), an increase in domestic demand translates into an equal-sized increase in the relative price of good a (assuming no supply response). For intermediate values for ω, the stronger the preference for home-produced goods, the larger the impact on the relative value of domestic output from an increase in relative domestic demand. Note that there would be no analogue to equation 28 in a model economy in which both countries produced the same good.

The second key equation uses budget constraints to express the difference between foreign and domestic consumption as a function of the difference in investment and the difference in GDP. Assuming constant portfolios, where λ denotes the fraction of the domestic (foreign) firm owned by domestic (foreign) households, domestic consumption is given by

$$\begin{aligned} c(s^t) &= q_a(s^t)w(s^t)n(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t) \\ &= (1-\theta)y(s^t) + \lambda\left(\theta y(s^t) - x(s^t)\right) + (1-\lambda)e(s^t)\left(\theta y^*(s^t) - x^*(s^t)\right) \qquad (29) \end{aligned}$$

where the second line follows from the assumption that the intermediate-goods firms production is Cobb-Douglas in capital and labor, and the definitions for dividends. Given a similar expression for foreign consumption, the difference between the value of consumption across countries is given by

$$(30) \quad \Delta c(s^t) = (1-2(1-\lambda)\theta)\Delta y(s^t) + (1-2\lambda)\Delta x(s^t)$$

Note that in the case of complete home bias ($\lambda = 1$), the relative value of consumption across countries would simply be the difference between relative output and relative investment. For $\lambda < 1$ financial flows mean that some fraction of changes in relative output and investment are financed by foreigners. Note that equation 30 would still hold true in a one good model.

The hallmark of complete risk-sharing given that the utility function is log-separable in consumption is

$$c(s^t) = e(s^t)c^*(s^t) \text{ or } \Delta c(s^t) = 0 \ \forall s^t.$$

Equation 30 indicates that if $\Delta y(s^t)$ and $\Delta x(s^t)$ follow independent stochastic processes, then no constant value for λ will deliver $\Delta c(s^t) = 0$, the perfect risk-sharing condition. However, recall that given the production technology in this economy, equation 28 consitutes an additional linear equilibrium relationship between $\Delta y(s^t)$, $\Delta(c^t)$ and $\Delta x(s^t)$. In particular, we can substitute equation 28 into equation 30 to express the difference in consumption as a function solely of the difference in investment:[6]

$$\Delta c(s^t) = (1 - 2(1-\lambda)\theta)\,(2\omega - 1)(\Delta c(s^t) + \Delta c(s^t)) + (1 - 2\lambda)\Delta x(s^t)$$

which implies that

$$(31) \quad \mu\Delta c(s^t) = \underbrace{(1-2\lambda)}_{direct\ foreign\ financing}\Delta x(s^t) \; + \; \underbrace{(2\omega - 1)\,(1 - 2(1-\lambda)\theta)}_{indirect\ foreign\ financing}\Delta x(s^t)$$

where μ is a constant.

Note, immediately, that if $\Delta x(s^t) = 0\ \forall s^t$, then $\Delta c(s^t) = 0$ irrespective of the choice for λ. In this case, movements in the terms of trade provide automatic and perfect insurance against fluctuations in the relative quantities of intermediate goods supplied, as in Cole and Obstfeld (1991). In an environment with fluctuating investment, there is a unique value for λ such that the right hand side of equation 31 is always equal to zero. In particular, simple algebra confirms that this value is defined in equation 25.

We now use equation 31 to understand the effect of an investment shock $\Delta x(s^t)$ on relative consumption, $\Delta c(s^t)$. Absent any diversification, an increase in $x(s^t)$ would reduce $c(s^t)$ proportionately. For $\lambda < 1$ some of the cost of additional investment is paid for by foreign shareholders directly (the first term on the right hand side) or indirectly through changes in relative prices (the second term). The direct foreign financing effect depends on the difference between the fraction of domestic stock held by foreigners relative to domestic agents $((1-\lambda) - \lambda)$. The indirect effect works as follows: an increase in relative domestic investment increases the relative value of domestic output in proportion to the factor $(2\omega - 1)$ (see eq. 28). This captures the fact that an increase in

[6] Alternatively, one could substitute out investment to derive an equation linking $\Delta y(s^t)$ to $\Delta c(s^t)$.

relative demand for domestic final goods has a positive effect on the terms of trade for the domestic economy. The fraction of this additional output that accrues as income to domestic shareholders is given by the term $(1-2(1-\lambda)\theta)$, which in turn amounts to labor's share of income $(1-\theta)$ plus the difference between domestic and foreign shareholder's claims to domestic capital income $(\lambda\theta - (1-\lambda)\theta)$. The equilibrium value for λ is the one for which the direct effect and the indirect effects exactly offset, so that changes in relative investment have no effect on relative consumption.

Why is diversification low?

If the lion's share of income goes to labor ($\theta < 0.5$) and, and if preferences are biased towards domestically-produced goods ($\omega > 0.5$), then the indirect effect of an increase in relative domestic investment on relative consumption is positive (*i.e.* the second term in equation 31 is positive). It is positive because the change in the terms of trade triggered by an increase in domestic demand favors domestic agents. This implies that domestic residents can afford to finance the bulk of an increase domestic investment while still equalizing consumption across countries.

Diversification and the trade share

A larger trade share implies a smaller value for ω. Equation 25 indicates that diversification is decreasing in ω, and thus increasing in the trade share. The intuition is as follows. As $\omega \to 0.5$, equation 28 implies a weaker terms of trade response to changes in relative final demand. Thus for any given diversification level, the positive indirect financing effect of demand changes that works through relative price changes is going to be smaller. Thus to achieve consumption equalization, the negative direct effect (given by $1-2\lambda$ in equation 31) must also be smaller, which implies that diversification $(1-\lambda)$ has to be closer to 0.5. Conversely, with stronger home bias in preferences (a small trade share), increases in relative domestic investment induce, via large changes in the terms of trade, large increases in the value of income in the country doing the investment. Thus little external investment financing (via diversification) is required.

Diversification and labor's share

A larger labor share strengthens the indirect impact of changes in investment on consumption, so the direct effect also has to be larger in absolute value. This means that diversification is going to be lower. In other words, with a larger labor share, the change in relative output induced by a change in the terms of trade has a larger impact on relative income (since a smaller part of income goes to capital and thus potentially to foreigners). Hence less external financing (the direct effect of diversification) is needed to achieve consumption equalization.

B. Microeconomic intuition

The key to understanding optimal portfolio choice from the perspective of an individual agent is to understand how the returns to domestic and foreign stocks covary with non-diversifiable labor income. If returns to domestic stocks co-vary negatively with labor earnings, then domestic stocks will offer a good hedge against labor income risk, and agents will prefer a portfolio biased towards domestic firms. In Section XX we described an equilibrium in which perfect risk sharing is achieved, and in which home bias is in fact observed. This suggests that domestic stock returns do in fact co-vary negatively with labor income. At first sight, this might seem a rather puzzling result, given that the production technology is Cobb-Douglas, suggesting a constant division of output between factors. We now explain how two key features of the BKK environment, durable capital and relative price dynamics, interact to give rise to this negative covariance.

First, recall that perfect risk sharing means equalizing the value of consumption across countries, state by state: $c(s^t) = e(s^t)c^*(s^t)$.

The difference between the value of domestic and foreign earnings (in units of the domestic final good) is

$$q_a(s^t)w(s^t)n(s^t) - e(s^t)q_b^*(s^t)w^*(s^t)n^*(s^t) = q_a(s^t)(1-\theta)\left(F(s^t) - t(s^t)F^*(s^t)\right)$$

Thus the relative value of domestic earnings rises in response to an increase in $z(s^t)$ relative to $z^*(s^t)$ if and only if the increase in the relative production of good a relative to good b exceeds the increase in the terms of trade (ie the price of good b relative to good a). In our economy this condition is satisfied: thus a positive domestic productivity shock is good news for domestic workers.

Now to rationalize the finding that agents prefer to bias their portfolios towards domestic stocks we need to show that in response to a positive domestic productivity shock, the return to domestic stocks declines relative to the return to foreign stocks, and thus that domestic stocks offer a good hedge against non-diversifiable labor income risk.

The return on domestic stocks between $t-1$ and t is

$$r(s^t) = \frac{d(s^t) + P(s^t)}{P(s^{t-1})}$$

The return on foreign stocks (in units of the domestic final good) is

$$r^*(s^t) = \frac{e(s^t)}{e(s^{t-1})}\frac{d^*(s^t) + P^*(s^t)}{P^*(s^{t-1})}$$

Using the expressions for equilibrium stock prices - $P(s^t) = k(s^t)$ and $P^*(s^t) = k^*(s^t)$ - along with the definitions for dividends, these returns can alternatively be expressed as

$$\begin{aligned} r(s^t) &= \frac{\theta q_a(s^t)F(s^t)}{k(s^{t-1})} + 1 - \delta \\ r^*(s^t) &= \frac{e(s^t)}{e(s^{t-1})}\left(\frac{\theta q_b^*(s^t)F^*(s^t)}{k^*(s^{t-1})} + 1 - \delta\right) \end{aligned}$$

The difference between the aggregate returns to domestic versus foreign stocks is

$$\begin{aligned} & r(s^t)P(s^{t-1}) - r^*(s^t)e(s^{t-1})P^*(s^{t-1}) \\ = \quad & \theta q_a(s^t)\left(F(s^t) - t(s^t)F^*(s^t)\right) + (1-\delta)\left(k(s^{t-1}) - e(s^t)k^*(s^{t-1})\right) \end{aligned}$$

The first term in this expression captures the change in relative income from capital, and it has exactly the same flavor as the change in relative earnings: through this term, a positive domestic productivity shock will increase the relative return on domestic stocks as long as the terms of trade does not respond too strongly. However, there is also a second term in the expression for relative returns. This captures the fact that part of the return to buying a stock is the change in its price.[7] A positive domestic productivity shock drives up the real exchange rate $e(s^t)$ and thus drives down the relative value of undepreciated domestic capital (recall that since final consumption and investment are perfectly substitutable in production, the relative price of capital - and of stocks - is equal to the relative price of consumption, *i.e.* the real exchange rate). Whether relative returns to domestic stocks rise or fall in response to a positive productivity shock depends on whether the first or second term dominates. In the model of this paper, the second term dominates, meaning that investors in domestic stocks lose more from seeing domestic capital devalued than they gain from higher rental rates.

Impulse responses

To further our understanding of how perfect risk sharing is achieved with time-invariant and home-biased portfolios, it is helpful to pick some parameter values, and examine the response of macro variables to a productivity shock in this economy. Figure XX plots impulse responses to a

[7] One might expect a third term in the expression for relative returns: additional domestic investment reduces current dividend payments. However, this extra investment translates into an equal-sized increase in the current stock price, which is why $k(s^t)$ and $k^*(s^t)$ do not appear in the expressions for returns.

persistent (but mean reverting) postive productivity shock in the domestic country. In this example, productivity follows independent AR(1) processes in each country, and the autoregressive parameter (at a quarterly frequency) is 0.95.

In the period of the shock, the relative returns to domestic labor increase, and the gap between relative earnings persists through time. The differential can persist because labor is immobile internationally. In the period of the shock, returns to domestic stocks exceed returns to foreign stocks, reflecting a decline in the relative value of domestic capital. After the first period, however, the relative returns to domestic and foreign stocks are equalized. The intuition for this result follows from the fact that new capital is produced using freely traded intermediate inputs, and efficiency dictates allocating these inputs so as to equate expected returns.[8]

Because agents do not adjust their portfolios in response to the shock, the decline in the relative value of domestic stocks on impact means that relative financial wealth for home-biased domestic agents declines relative to the wealth of foreigners. This means that in the periods immediately following the shock, even though returns are equalized, the total asset income accruing to foreign agents is larger, because they are holding more financial wealth in total. This additional asset income for foreigners exactly offsets their lower labor income, and the relative value of consumption

[8] More formally, the first order conditions for investment on the part of firms are

$$\begin{aligned} \frac{1}{c(s^t)} &= \beta E_{s^t}\left[\frac{1}{c(s^t)}\left(q_a(s^t,s_{t+1})\frac{\theta F(s^t,s_{t+1})}{k(s^t)}+1-\delta\right)\right] \\ \frac{1}{c^*(s^t)} &= E_{s^t}\left[\frac{1}{c^*(s^t,s_{t+1})}\left(q_b^*(s^t,s_{t+1})\frac{\theta F^*(s^t,s_{t+1})}{k^*(s^t)}+1-\delta\right)\right] \end{aligned}$$

Since risk sharing is complete at the candidate equilibrium, the real-exchange-rate-adjusted marginal utility of consumption is equalized across agents. Thus the first order condition for the foreign firm can be written

$$\frac{1}{c(s^t)} = E_{s^t}\left[\frac{1}{c(s^t,s_{t+1})}\frac{e(s^t,s_{t+1})}{e(s^t)}\left(q_b^*(s^t,s_{t+1})\frac{\theta F^*(s^t,s_{t+1})}{k^*(s^t)}+1-\delta\right)\right]$$

Comparing these two first order conditions,

$$E_{s^t}\left[\frac{r(s^t,s_{t+1})}{c(s^t,s_{t+1})}\right] = E_{s^t}\left[\frac{r^*(s^t,s_{t+1})}{c(s^t,s_{t+1})}\right]$$

and thus, to a first order approximation, expected returns are equalized across countries.

is equalized.

Over time several things happen. The relative value of domestic investment declines, and eventually goes negative. At the point where this happens, aggregate domestic demand falls below foreign demand, and the relative value of domestic output - and domestic labor earnings - goes negative, as implied by equation 28. Prior to this point, domestic agents are saving their excess labor earnings, as evidenced by the fact that their relative wealth is rising over time, notwithstanding that returns are equalized. Note that domestic agents can save without actively buying more stocks: rather they simply tolerate a period of relatively low dividend payments as domestic firms invest, and the value of domestic stocks increases. Once relative labor earnings turn negative, domestic agents start dis-saving, and net exports turn negative as domestic agents run down their relative wealth to maintain equal consumption.

To summarize, from the point of view of an individual worker / investor, optimal portfolio choice can be interpreted in the usual way as depending on the covariances between non-diversifiable labor income and the returns on domestic and foreign stocks. The key feature of this environment, however, is that these covariances are endogenous and depend critically on the dynamics of investment and relative prices. An important message from the preceeding analysis is that the model makes clear predictions about the signs of these covariances, and, perhaps surprisingly, returns to domestic labor and capital tend to co-move negatively even though the model is frictionless and the only shocks are Hicks-neutral innovations to TFP.

5. Sensitivity Analysis

The equilibrium portfolios described in Proposition XX are insensitive to many details of the model, including the process for productivity shocks, but they do hinge on two key assumptions: first, that the elasticity of substitution between traded intermediate goods is unity - so that the G functions are Cobb-Douglas - and second, that utility is logarithmic in consumption. We now consider the effects of alternative values for these parameters. To do so, we need to fully calibrate the model, solve the model numerically, and compute average values for diversification in simulations.

MORE DETAILS NEEDED

A. Elasticity of substitution

A unitary elasticity of substitution is towards the low end of estimates in the existing literature. Figure XX shows how the average level of diversification changes as this parameter, denoted σ, is varied from x to y. Increasing the elasticity in the neighbourhood of $\sigma = 1$ increases home bias. We conclude that for elasticities commonly used in the business cycle literature, theory predicts strong home bias. The intuition for why increasing substitutability strengthens home bias is as follows. First, the larger is σ, the less relative prices change in response to shocks. This means that following a positive shock, the increase in the relative value of domestic labor earnings becomes larger and, at the same time, the increase in relative foreign stock returns becomes smaller. Thus agents must overweight domestic stocks to an even greater extent in order to hedge against changes in relative productivity.

There is an asymtote in average equilibrium diversification around $\sigma = 3$, and for larger values of σ portfolios exhibit extremely high diversification. The reason is that for high enough elasticities, relative domestic stock returns and domestic labor income covary positively and so foreign stocks offer the best hedge against labor income risk. This change in the sign of covariance is easiest to understand in the extreme case of an infinite elasticity, *i.e.* a one good model. In this case, neither the terms of trade nor the real exchange rate will move in response to a shock and there will be an unambiguous increase in relative returns to domestic stocks in response to an increase in domestic productivity (see equation XX).

B. Risk aversion

Increasing the coefficient of relative risk aversion above one also tends to strengthen home bias (see Figure XX). Chaging the risk aversion coefficient does not impact either of the two equilibrium relationships (equations 28 and 30) developed in Section XX. It does, however, change the pattern of co-movement between domestic and foreign consumption consistent with perfect risk-sharing. In particular, higher risk aversion corresponds to a lower inter-temporal elasticity of substitution for consumption. Because desired consumption is less sensitive to changes in relative prices, in choosing portfolios agents want to ensure that their total income does not decline too much in periods when domestic productivity falls. This pushes agents further towards domestic stocks, whose relative return rises in periods when domestic productivity and wages decline.

6. Relation to Previous Literature

Lucas (1982), Cantor and Mark (1988) and Baxter and Jermann (1997) consider one-good models. In our set-up this implies that the real exchange rate is always equal to one, and that equation 28 does not apply. Lucas shows that in an endowment economy with common preferences across countries, perfect risk pooling is achieved when agents hold 50 percent of both domestic and foreign shares in each period, where shares are claims to future endowment streams. Since equation 28 does not apply in a one good model, the portfolio choice rule **??** does not apply to Lucas' economy. Nonetheless it is interesting to note that substituting $\omega = 0.5$ into equation **??** we reproduce Lucas' 50-50 portfolio split result.

Cantor and Mark extend Lucas' analysis to a simple environment with production. However, they make several assumptions that ensure that their economy inherits the properties of Lucas'. In particular, (i) domestic and foreign agents have the same log separable preferences over consumption and leisure, (ii) productivity shocks are assumed to be iid through time, (iii) firms must purchase capital and rent labor one period before production takes place, and (iv) there is 100% depreciation.

When their two economies are the same size, assumptions (ii) and (iii) ensure that in an efficient allocation capital and labor are always equalized across countries. Thus to deliver perfect risk-sharing, the optimal portfolio choice simply has to ensure an equal division of next period output, which is ensured with Lucas' 50-50 portfolio split.

Baxter and Jermann (1997) consider a one-good model with more conventional timing according to which labor and capital rental rates are both stochastic and both driven by total-factor-productivity shocks to a Cobb-Douglas production technology. They argue that since returns to capital and labor are highly correlated, agents can effectively diversify country-specific labor income risk (that cannot be directly diversified by simply working abroad) by aggressively diversifying claims to capital. The logic for their argument becomes transparent by considering 30, which we reproduce here.

$$\Delta c = (1 - 2(1-\lambda)\theta)\, \Delta GDP + (1-2\lambda)\Delta x \tag{32}$$

In the Baxter and Jermann one-good case, the real exchange rate is one, so Δc is simply equal $c - c^*$. Baxter and Jermann assume that capital stocks are exogenous. One way to translate this into our model would be to assume firms in both countries target a constant capital stock, in which case $\Delta x = 0$. In this case, to deliver perfect risk-sharing ($\Delta c = 0$) we only need to pick a value for λ such that the coefficient on ΔGDP is zero. The implied value for λ is

$$1 - \lambda = \frac{1}{2\theta}$$

which is exactly the expression described by equation (2) in Baxter and Jermann.

If capital's share θ is set to a third, the value for λ that delivers equal consumption in the two countries is -0.5. Thus, as Baxter and Jermann point out, a diversified portfolio involves a negative position in domestic assets. Note that equation 32 suggests that there will always exist a portfolio that delivers perfect risk sharing as long as Δx is strictly proportional to ΔGDP. Thus, as an alternative to assuming $\Delta x = 0$, we could assume, for example, that firms invest a fixed fraction of output, so that $x(s^t) = \kappa GDP(x^t)$. In this case, in a one-good world, $\Delta x = \kappa \Delta GDP$. Now consumption equalization requires that

$$(33)\quad \Delta c = \left[(1 - 2(1-\lambda)\theta) + (1-2\lambda)\kappa\right] \Delta GDP = 0$$

which implies

$$\lambda = \frac{2\theta - 1}{2(\theta - \kappa)}.$$

As an example, if the investment rate κ is equal to 0.2 and capital's share is 1/3, the value for λ that delivers consumption equalization is -1.25, implying an even larger short position in domestic assets than the one predicted by Baxter and Jermann. The intuition is simply that now asset income is a less effective hedge, since following an increase in foreign output, foreign investment rises, reducing income from foreign dividends.

Our model enriches the Baxter and Jermann analysis along two dimensions. First, we explicitly endogenize investment. In both examples discussed above, the investment rules are arbitrary, whereas we assume firms make investment and dividend decisions with the interests of their shareholders at heart. Second, we assume that the two countries produce different traded goods that are imperfectly-substitutable when it comes to producing the final consumption-investment good.

The model that is closest to ours is Cole and Obstfeld (1991). They consider a two-country endowment economy, and a version with production in which the two goods may be consumed or used as capital inputs to produce in the next period. Like Cantor and Mark (1988) they assume 100 percent capital depreciation. They show that with when domestic and foreign agents share the same log-separable preferences for consuming the two goods, and (in the production version of the model) when production technologies are Cobb-Douglas in the quantities of the two goods allocated for investment, then a regime of portfolio autarky (100 percent home bias or $\lambda = 1$) delivers the same allocations as a world with a complete set of internationally-traded assets.

It is straightforward to revisit the logic for their results in the context of our model. In particularly, considering an endowment economy effectively sets $\Delta x = 0$, in which case equations 28 and 30 become two independent equations in two unknowns, Δc and ΔGDP. The only possible solution is $\Delta c = \Delta GDP = 0$. Thus for any choice for λ, including the portfolio autarky value $\lambda = 1$ emphasized by Cole and Obstfeld, perfect risk-pooling is achieved. The reason is simply that differences in relative quantities of output are automatically offset one-for-one by differences in the real exchange rate, so $GDP = rxGDP^*$. In the production version of the model, Cole and Obstfeld's assumptions of log separable preferences and full depreciation imply that consumption, investment and dividends are all fixed fractions of output, so that $\Delta x = \kappa \Delta GDP$ and, once again, equations 28 and 30 are two independent equations in two unknowns, Δc and ΔGDP. Thus total dividend income in any given period is again independent of the initial portfolio split. In this sense changes in the real exchange rate provide automatic insurance against country-specific income changes.

In contrast to the Cole and Obstfeld result, only one portfolio delivers perfect risk-pooling in our economy. Furthermore, portfolio autarky is only efficient in the case when there is complete specialization in tastes, so that $\omega = 1$. The reason for these differences relative to their results is that with partial depreciation, investment is no longer a fixed fraction of output, and changing the initial portfolio therefore changes the properties of the stream of asset income. However, efficiency can still be achieved for $\omega < 1$ provided the initial portfolio contains an appropriately-weighted mix of both domestic and foreign stock.

Nonetheless, despite these differences, the logic for our results is broadly the same as that in Cole and Obstfeld. In our model, changes in the terms of trade provide automatic insurance against shocks to the relative supply or relative demand for the two traded goods. Were it not for the risk of changes to relative investment, this insurance would automatically deliver perfect risk-pooling, irrespective of portfolio choices. However, in a world with partial depreciation and persistent productivity shocks, efficient investment will not be either constant or a constant fraction of output; rather, as in a standard growth model, positive persistent productivity shocks will be associated with a surge in investment. Thus one way to think about the role of portfolio diversification is to ensure that the cost of funding changes in investment is efficiently split between domestic and foreign residents. As we discussed in the previous section, relatively little diversification is required to achieve this, since an increase in domestic investment demand raises the relative price of domestically produced goods, and thus raises domestic income. Hence it is optimal for domestic residents to finance (by holding most of domestic equity) most of the extra domestic investment.

7. Preference Shocks

In the model we have discussed so far the key reason why perfect risk sharing is obtained with a fairly low level of diversification is that relative price movements provide insurance against country specific productivity shocks. In a model with only productivity shocks this insurance mechanism result in strongly procyclical terms of trade (whenevr a country has a negative productivity shock the price of its goos raise, couning terms of trade to fall). In table 1 we report trems of trade cyclical property for the g7 countries: the table show that terms of trade are, on average, procyclical but as strongly as the model with only productivity shocks predicts. version of the model considered perfect risk sharing is obtained and this in turn implies that the correlation between the real exchange rate e and relative consumption $\frac{c}{c^*}$is exactly equal to 1, regardless of the process for productivity shocks. Also the insurance mechanism from the terms of trade implies a correlation between the terms of trade (the price of good b relative to good a) and relative output $\frac{y}{y^*}$ very close to 1 for plausible estimates of the process for productivity. In most developed countries both these correlations are close to 0 or slightly negative (see Backus Smith, 1993). In this section we argue that counterfactual high unconditional correlations are not necessary elements for obtaining a low equilibrium diversification. To do so we solve for equilibrium diversification in the a version of the model with taste shocks like in Stockman and Tesar (1995). We show that with taste shocks the equilibrium diversification is still low but the correlation between relative output and terns of trade (and the correlation between relative consumption and real exchange rate) are also low, in line with the data. To understand why the correlation is low consider the effect of a positive taste shock, say, in country 1. In response to the shock consumer in country 1 will want to consume more; but since productivity in country 1 is unchanged the price of good a will tend to raise inducing more labor input in country 1. In equilibrium thus relative output will raise but terms of trade will fall inducing a negative correlation between the two. This negative correlation tends to offset the positive correlation arising form productivity shocks and the conditional correlation will be close to 0. Note also that the price movements also provide insurance agsinst the taste shocks a s well as the price of the good produced by the country with the taste shock raises, and that explain why international diversification in the presence of taste shocks remains low. The essential element for our result is a negative correlation between relative output and terms of trade, *conditional* on a productivity shock.

To be completed

8. Explaining the cross section of country portfolios

In the theoretical section we solve for the equilibrium level of international diversification in a world with two equal-sized countries. The equilibrium portfolio share of foreign stocks, $\phi = (1 - \lambda)$, is given by

$\phi = \frac{1-\omega}{1+\theta-2\omega\theta}$

where $1 - \omega = \eta$ is the share of imports/exports in GDP. This suggests that in the data we should observe a relation between import shares and shares of foreign assets in country portfolios. In particular, the relation we should observe is

$$\phi = \frac{\eta}{1+\theta-2(1-\eta)\theta} = \frac{\eta}{1-\theta+2\eta\theta}$$

Taking the reciprocal of this relation we obtain

$$\frac{1}{\phi} = 2\theta + (1-\theta)\frac{1}{\eta} \tag{34}$$

which implies that there should be a linear relationship between the reciprocal of the share of foreign assets in country portfolios and the reciprocal of the trade share in GDP. It is easy to explore whether a similar relationship holds in the data.

A. Data issues

We need country-level data on trade shares and on international diversification positions. Trade shares are obtained from the World Bank World Development Indicators: in particular η_i, the trade share for country i, is measured as

$$\eta_i = average\left(\frac{\text{Imports}_i}{GDP_i}, \frac{\text{Exports}_i}{GDP_i}\right)$$

For foreign asset shares we need a measure of total foreign assets divided by a measure of the total value of domestically-owned assets. There are three possible sources: IMF international asset position data, the Kraay, Loayza, Serven, and Ventura (2000) dataset, and the Lane and Milesi-Ferretti (2001) dataset. Coverage of these datasets is similar and consists of 70-80 countries. All these datasets distinguish between bond and stock positions. The Kraay and al. dataset is the only one that also contains capital stock data (constructed by cumulating investment). All these datasets have a time series dimension.

In our first exercise we use the Kraay and al. (2000) dataset and measure the ϕ_i, the international diversification position for country i as

$$\phi_i = average\left(\frac{FA_i}{k + FA_i - FL_i}, \frac{FL_i}{k + FA_i - FL_i}\right)$$

where FA_i is the value of foreign assets (including foreign direct investment, portfolio investment, loans and other investments) owned by domestic residents, FL_i is the value of all the claims that foreigners have on country residents, and k_i is the value of the capital stock in country i. We then compute averages of the constructed ϕ_i and η_i for the years 1990-1997 so as to obtain a cross-section of 53 countries[9] that we use to estimate equation (34).

B. Results

Results are reported in tables 1 and 2 below and figures 1 and 2. Equation (34) is estimated using OLS and median (robust) regression. We perform separate estimations for the entire cross section of countries and for a subgroup of rich countries. Controls for size and for GDP per capita are also used. Estimated coefficients from the data are then compared to the coefficients implied by the model.

[9] We eliminate all countries for which we have fewer than 5 observations.

In the whole cross section of countries, trade shares are a minor factor in explaining the cross section of country portfolios, while GDP per capita and country size play a bigger role. In particular, rich countries are more diversified while large countries are less diversified. Thus, for poor countries the model significantly overpredicts observed diversification. We suspect that this lack of diversification reflects some features of poorer countries that are not captured in our model, such as the presence of capital controls and under-developed financial markets.

By contrast, among the group of relatively high income economies for whom our model should be most appropriate, we find that trade share is a key factor in explaining the cross section of country portfolios. For these economies the relation between trade share and import share in the data is remarkably close to the one predicted by the model (see figure 2), and neither size nor GDP per capita seem to significantly affect diversification.

9. Conclusion

TO BE COMPLETED

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Table 1. All countries. Independent Variable is $1/(1-\lambda)_i$

	Lin. Reg.		Rob. Reg.		Model $\theta = 0.36$
$\frac{1}{(1-\omega)_i}$	**1.04**	0.18	**0.91**	**0.58**	0.64
	(0.30)	(0.28)	(0.21)	(0.22)	
Constant	**3.21**	-8.95	**2.21**	5.67	0.72
	(1.44)	(9.09)	(1.04)	(7.01)	
Log Ypc		**-2.36**		**-2.16**	
		(0.59)		(0.46)	
Log Pop		**2.22**		**1.08**	
		(0.44)		(0.44)	
Obs	53	53	53	53	
Adj. R^2	0.19	0.56	0.10	0.31	

Note: The numbers in parentheses are standard errors. The coefficients in bold are significant at at least the 5% level.

Table 2. Rich Countries. Independent Variable is $1/(1-\lambda)_i$

	Lin. Reg.		Rob. Reg.		Model $\theta = 0.36$
$\frac{1}{(1-\omega)_i}$	**0.91**	**1.10**	**1.08**	**1.15**	0.64
	(0.14)	(0.20)	(0.21)	(0.23)	
Constant	0.64	13.1	0.06	26.2	0.72
	(0.62)	(20.1)	(0.92)	(31.1)	
Log Ypc		-0.56		-2.40	
		(2.02)		(3.16)	
Log Pop		-0.45		-0.14	
		(0.36)		(0.52)	
Obs	20	20	20	20	
R^2	0.70	0.70	0.31	0.45	

Note: The numbers in parentheses are standard errors. The coefficients in bold are significant at at least the 5% level.

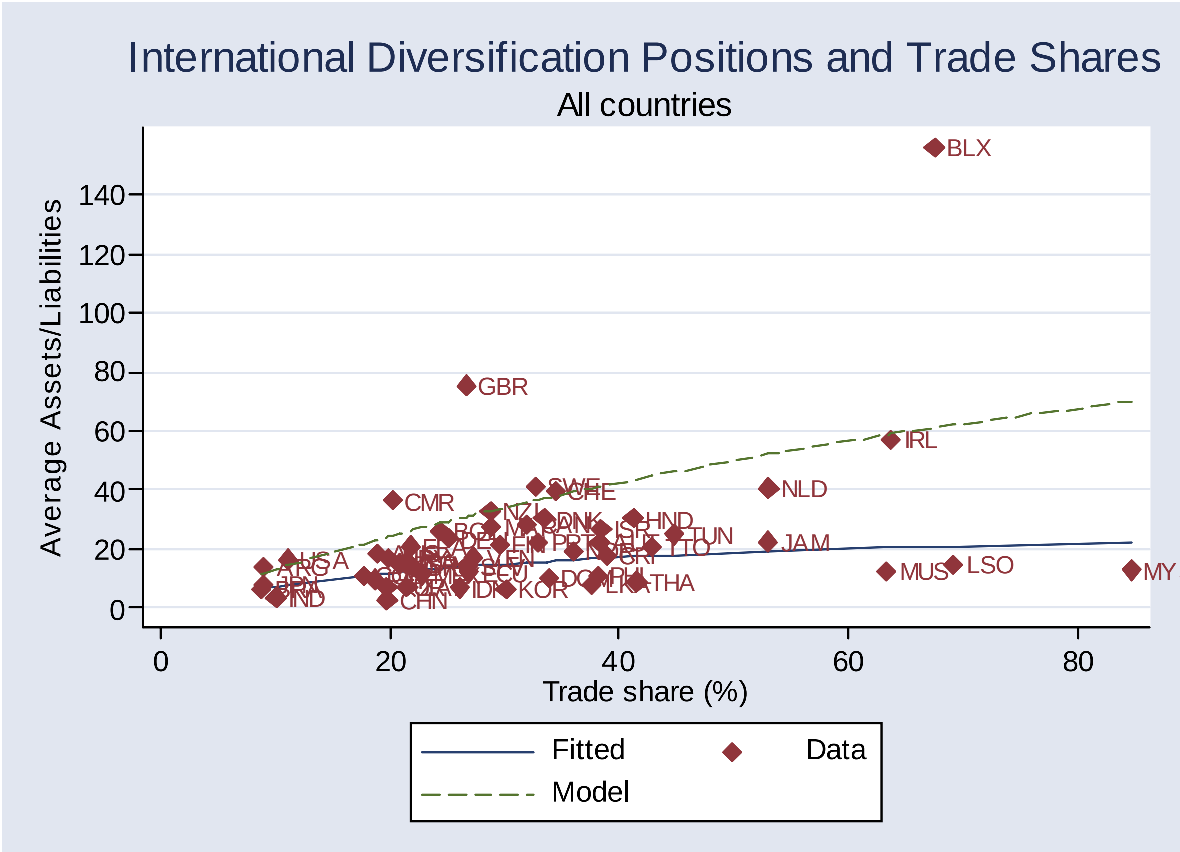
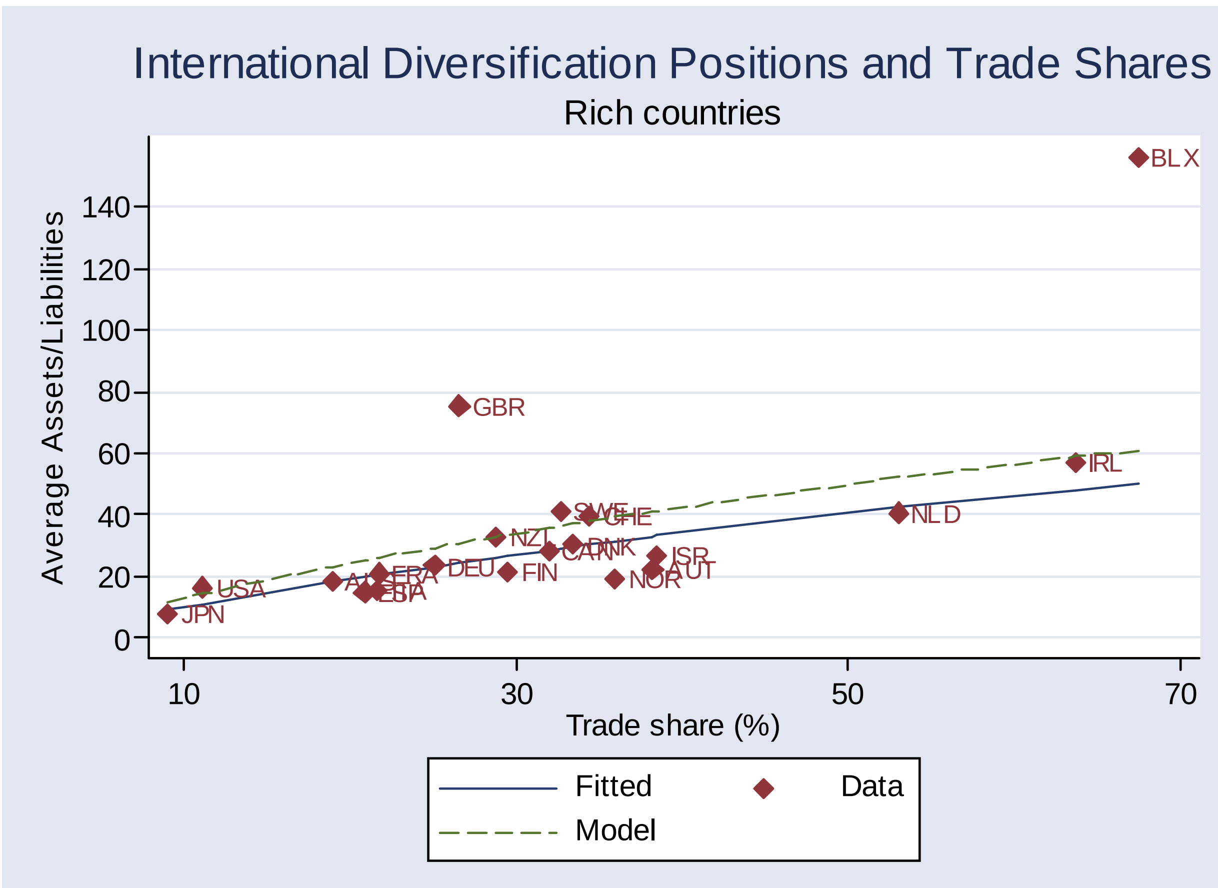

International Diversification Positions and Trade Shares
All countries
Average Assets/Liabilities
Trade share (%)
0
20
40
60
80
100
120
140
BLX
GBR
IRL
NLD
CMR
SWE
CHE
HND
TUN
JAM
TTO
MUS
LSO
MYS
USA
ARG
IND
CHN
KOR
THA
Fitted
Data
Model

Figure 1:


International Diversification Positions and Trade Shares
Rich countries
Average Assets/Liabilities
Trade share (%)
0
20
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10
30
50
70
BLX
GBR
IRL
NLD
SWE
CHE
NZL
DNK
CAN
ISR
AUT
FIN
NOR
DEU
FRA
USA
JPN
Fitted
Data
Model

Figure 2: