Gold Rush Fever in Business Cycles

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First version: May 2006
This version: December 2006

Abstract

Gold rushes are periods of economic boom, generally associated with large increases in expenditures aimed at securing claims near new found veins of gold. An interesting aspect of gold rushes is that, from a social point of view, much of the increased activity is wasteful since it contributes simply to the expansion of the stock of money. In this paper, we explore whether business cycle fluctuations may sometimes be driven by a phenomenon akin to a gold rush. In particular, we present a model where the opening of new market opportunities causes an economic expansion by favoring competition for market share, which is essentially a dissolution of rents. We call such an episode a market rush. We construct a simple model of a market rush that can be embedded into an otherwise standard Dynamic General Equilibrium model, and show how market rushes can help explain important features of the data. We use a simulated-moment estimator to quantify the role of market rushes in fluctuations. We find that market rushes may account for over half the short run volatility in hours worked and a third of the short run volatility of output.

Key Words: Business Cycle, Investment, Imperfect Competition

JEL Class.: E32

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Introduction

Sutter's Mill near Coloma, California. January 24, 1848. James W. Marshall, a carpenter from New Jersey, found a gold nugget in a sawmill ditch. This was the starting point of one of the most famous Gold Rushes in history, the California Gold Rush of 1848-1858. More than 90,000 people made their way to California in the two years following Marshall's discovery, and more than 300,000 by 1854 —or one of about every 90 people then living in the United States. The population of San Francisco exploded from a mere 1,000 in 1848 to 20,000 full–time residents by 1850. More than a century later, the San Francisco 49ers NFL team is still named for the prospectors of the California Gold Rush. Another famous episode, which inspired Charlie Chaplin's movie "The Gold Rush" and Jack London's book The "Call of the Wild", is the Klondike Gold Rush of 1896–1904. Gold prospecting took place along the Klondike River near Dawson City in the Yukon Territory, Canada. An estimated 100,000 people participated in the gold rush and about 30,000 made it to Dawson City in 1898. By 1910, when the first census was taken, the population had declined to 9,000[1].

Gold rushes are periods of economic boom, generally associated with large increases in expenditures aimed at securing claims near new found veins of gold. An interesting aspect of many gold rushes is that, from a social point of view, much of the increased activity is wasteful since historically it mainly contributed to the expansion of the stock of money. In this paper, we explore whether business cycle fluctuations may sometimes be driven by a phenomenon akin to a gold rush. In particular, we present a dynamic general equilibrium model where the opening of new market opportunities causes an economic expansion by favoring competition for market share. We call such an episode a market rush. The market rush may mainly act to redistribute rents between firms with little external gain, in which case the net social value of such a market rush would be minor. The object of this paper is to present a simple model of a market rush that can be embedded into an otherwise standard Dynamic General Equilibrium model, to evaluate whether such a phenomenon is a significant contributor to business cycle fluctuations and to examine the social desirability of such fluctuations.

To capture the idea of a market rush, we present an expanding varieties model where agents compete to secure monopoly positions in new markets. However, in a first step and in contrast to standard growth models (see for example Romer [1987] and Romer [1990]) and to some business cycle models (see for example Devereux, Head and Lapham [1993]), we do not impose that an expansion in variety induces productivity gains, and we treat the growth in the potential set of varieties as technologically driven and exogenous. In this setting, when agents perceive an increase in the set of technologically feasible products, they invest to set up a prototype firm (or product)

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[1]See http://en.wikipedia.org/wiki/Gold_rushes for further facts and references.

with the hope of securing a monopoly position in the new market. It is therefore the perception of these new market opportunities that causes the market rush and the associated economic expansion. After the initial rush, there is a shake out period where one of the prototypes secures the dominant position in market. The long term effect of such a market rush depends on whether the expansion in variety has an external effect on productivity. In the case where it does not have an external effect, the induced cycle is socially wasteful as it only contributes to the redistribution of market rents. In contrast, when the expansion of variety does exert positive external effects, the induced cycle can have social value but will generally induce output fluctuations that are excessively large. A potential example of such a process is the dot.com frenzy of the late 90s, where large investments were made by firms trying to secure a position in the expanding internet market. At the end of this process, there was a large shake out as many firms went bankrupt and only a small percentage survived and obtained a substantial market position. The long run productivity gain associated with this process are still unclear.

This paper explores whether expectations about new market could be a significant contributor to macroeconomic fluctuations.[2] Since expectations of new markets are not directly observable, we need to set a demanding standard to evaluate such story. This is what we do. Not only do we provide a model that is capable of explaining important features of a Consumption–Output Vector Error Correcting Model (VECM) and their implication for hours, we also examine how well our model does when it is forced to compete with alternative explanations.

We begin the paper by presenting, in Section 1, a set of properties of the data that any good model of fluctuations should explain. Several of these features are well known and extensively discussed in Cochrane [1994]. As shown by Cochrane [1994], in a bivariate Output–Consumption VECM of the U.S. postwar economy, consumption is, at all horizons, almost solely associated with a permanent shock recovered using a long run restriction. In contrast, the associated temporary shock of the system is found to explain an important part of the short run volatility of output — i.e. the business cycle. We show that this temporary shock also explains much of the fluctuations in hours worked. We argue that these robust features of the data are quite challenging for business cycle models since even temporary shocks generally imply some reaction of consumption. Furthermore, the literature remains divided as to a structural interpretation for the temporary shock. As we think that a market rush is a potential candidate, we develop in Section 2 a simple model[3] which

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[2] This paper is related to several papers, both old and recent, which emphasize the role of expectations in affecting business cycles. The newest embodiment of the literature (Beaudry and Portier [2004a], Beaudry and Portier [2004b], Jaimovich and Rebelo [2006]and Christiano, Motto and Rostagno [2005a], Beaudry and Portier [2006]) emphasizes the role of expectations regarding future productivity growth in creating fluctuations. However, anecdotal evidence suggests that expectations about new market may be an important alternative.

[3]The type of models we present are ones where nominal rigidities play no role. Our interpretation of such models is that they can correspond to models with sticky prices in which monetary authorities follow rules that implement the flexible price outcomes.

can be solved analytically and whose properties can therefore be clearly stated. In this model, we show that current economic activity depends positively on the expectation of next period's activity and on the perceived opening of new markets. Hence, when agents believe that the economy is starting a prolonged period of market expansion, this induces an immediate increase in investment and an associated economic expansion. In contrast, when there are no new perceived market opportunities, the economy experiences a slump. Given the tractability of the model, we can solve it in the presence of a standard technology shock and our market expansion shock. The model puts sufficient structure on the data to isolate the market expansion shock: it suggests that the market expansion shock is the output innovation in a Consumption–Output VECM.

Since our simple analytical model performs well in explaining qualitative aspects of the data, in Section 3 we construct and estimate a more general and richer model to see whether it can explain quantitative features of the data. Since we allow for more than two shocks, the permanent and transitory components recovered from the data in Section 1 do not have an easy structural interpretation in this case, as they are combinations of fundamental shocks. We estimate the resulting (more complex) model using a simulated method of moments. Our findings from the larger model suggest that market expansion shocks are a non-negligible driving force underlying business cycle fluctuations. Section 4 examines the robustness of our results with respect to allowing for additional sources of fluctuations, such as investment specific technology shocks, temporary technology shocks and preference shocks (in an appendix, we also discuss the potential role of monetary shocks). Overall, we find that market rushes help explain a sizable fraction of hours and output fluctuations even when allowing for these alternative explanations. Section 4 offers concluding comments.

1   A Target Set of Observations

In this section, we present a set of observations which provides a rich though concise description of fluctuations in output, consumption and hours worked. Some of these observations are well known, and some are not. We believe that this set of observations captures many important features of fluctuations that any business cycle theory should aim to explain. We will use these observations to both motivate our interest in market rushes, and to test whether market rushes are a good candidate explanation of these features. In particular, we will emphasize both qualitative and quantitative properties of the data. After presenting the observations, we will begin by presenting a simple analytical model of market rushes that is capable of explaining the main qualitative features we highlight. It is in this sense that the empirical properties help motivate our interest in market rushes as a potentially relevant contributor to business cycle fluctuations. In the following sections, we explore the capacity of a model with market rushes to quantitatively explain the facts presented.

Our interest is in emphasizing important time series properties of output, consumption and hours worked. We could do this by directly examining the trivariate process. However, as is well known, such an approach can depend heavily on the treatment of hours worked as a stationary or non-stationary process. We therefore choose an approach that is robust to the treatment of hours worked. To this end, we begin by reviewing properties of the bi-variate process for consumption and output. More precisely, we study a Consumption–Output system with one cointegrating relation. The main properties of this system were originally discussed in Cochrane [1994]. As in Cochrane [1994], we use two schemes to orthogonalize the innovations of the process: a long run orthogonalization scheme *à la* Blanchard and Quah [1989], and a short run or impact scheme *à la* Sims [1980] . At this point, these two schemes should be viewed as devices for helping present properties of the data. There is no claim that these schemes identify structural shocks, and there are no claims that these data should be explained by a model which only has two shocks. One of the properties we will emphasize is that these two schemes deliver almost identical impulse response functions (IRFs). As we will show, this property can be traced back to a particular feature of the Wold representation for consumption and output, and we argue that this feature constitutes a qualitative property that business cycle models should try to replicate.

We begin by documenting the impulse responses associated with using a long run othogonalizing scheme *à la* Blanchard and Quah [1989], and show that almost all of the short run volatility of consumption is associated with the permanent shock, while this shock only accounts for half of the volatility of output. We then use a short run orthogonalizing scheme *à la* Sims [1980] to show that the resulting output innovation does not explain the long run properties of the two variables, nor the short run properties of consumption. We then formally test for the identity of the temporary shock recovered using the long run scheme and the output shock recovered using the short run scheme. This is done by showing that such an identity is in fact a zero restriction in the long run impact matrix of the Wold representation. We then turn our focus on relating these features with the behavior of hours and we show that the temporary/output innovation recovered from the consumption–output VECM explains most of the short run volatility of hours.

1.1   Long Run and Short Run Orthogonalization

We consider quarterly data for the US economy. The sample spans the period 1947Q1 to 2004Q4. Consumption, C, is defined as real personal consumption expenditures on nondurable goods and services and output, Y, is real gross domestic product. Both series are first deflated by the 15–64 U.S. population and expressed in logarithms.[4]

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[4]Consumption is defined as the sum of services and nondurable goods, while output is real gross domestic product. Each variable is expressed in per capita terms by dividing by the 15 to 64 population. The series are obtained from the following links. Real Personal Consumption Ex-

Standard Dickey–Fuller, likelihood ratio and cointegration tests indicate that C and Y are $I(1)$ processes and do cointegrate. We therefore model their joint behavior with Vector Error Correcting Model (VECM), where the cointegrating relation coefficients are [1;-1] (meaning that the consumption to output ratio is stationary). Likelihood ratio tests suggest that the VECM should include 3 lags. Omitting constants, the joint behavior of (C, Y) admits the following Wold representation

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = A(L) \begin{pmatrix} \mu_{1,t} \\ \mu_{2,t} \end{pmatrix},$$ (1)

where L is the lag operator, $A(L) = I + \sum_{i=1}^{\infty} A_i L^i$, and where the covariance matrix of μ is given by Ω. As the system possesses one common stochastic trend, $A(1)$ is not full rank. Given $A(1)$, it is possible to derive a representation of the data in terms of permanent and transitory components of the form

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = \Gamma(L) \begin{pmatrix} \varepsilon_t^P \\ \varepsilon_t^T \end{pmatrix},$$ (2)

where the covariance matrix of $(\varepsilon^P, \varepsilon^T)$ is the identity matrix and $\Gamma(L) = \sum_{i=0}^{\infty} \Gamma_i L^i$. The Γ matrices solve

$$\begin{cases} \Gamma_0 \Gamma_0' & = \quad \Omega \\ \Gamma_i & = \quad A_i \Gamma_0 \quad \text{for} \quad i > 0 \end{cases}$$ (3)
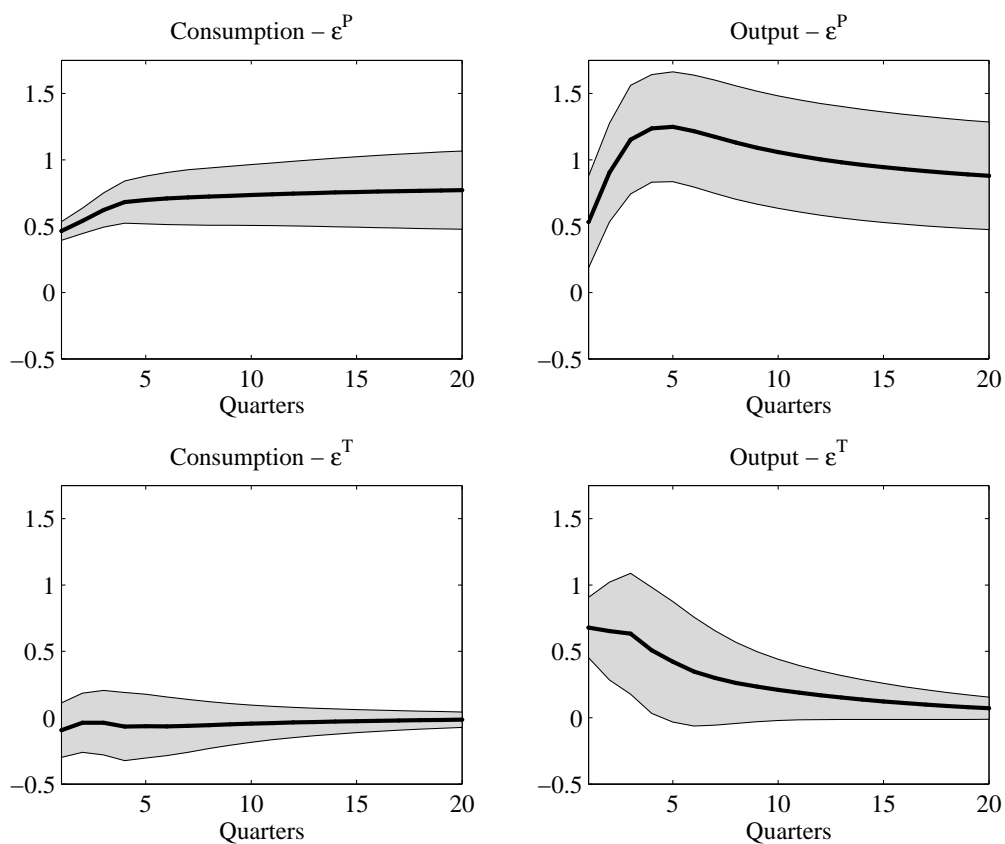
Note that once Γ_0 is known, all Γ_i are pinned down by the second set of relations. But, due to the symmetry of the covariance matrix Ω, the first part of the system only pin downs three parameters of Γ_0. One remains to be set. This is achieved by imposing an additional restriction. We impose that the $1, 2$ element of the long run matrix $\Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i$ equals zero, that is, we choose an orthogonalization where the disturbance ε^T has no long run impact on C and Y (the use of this type of orthogonalization was first proposed by Blanchard and Quah [1989]). Hence, ε^T is labeled as a temporary shock, while ε^P is a permanent one. Figure 1 graphs the impulse response functions of C and Y to both shocks as well as their associated 95% confidence bands, obtained by bootstrapping the VECM. Table 1 reports the corresponding variance decomposition of the process.

These results provide an interesting decomposition of macroeconomic fluctuations. The lower left panel of Figure 1 clearly shows that consumption responds very little to the transitory shock, which in turn accounts for less than 4% of consumption volatility at any horizon. Conversely consumption is very responsive to the permanent shock and most of the adjustment dynamics take place in less than one year. In other words, consumption is almost a pure random walk, that responds only to permanent shocks and has very little interesting dynamics. On the contrary, short run fluctuations

Figure 1: Responses of Output and Consumption to ε^P and ε^T

This figure shows the responses of consumption and output to temporary ε^T and permanent ε^P one percent shocks. These impulse response functions are computed from a VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VECM.

Table 1: The Contribution of the Shocks to the Volatility of Output and Consumption

Horizon	Output		Consumption	
	ε^T	ε^Y	ε^T	ε^Y
1	62%	80%	4%	0%
4	28 %	46 %	1%	1%
8	17 %	33%	1%	1 %
20	10 %	22 %	0%	2%
∞	0 %	4 %	0%	4%

This table shows the k-period ahead share of the forecast error variance of consumption and output that is attributable to the temporary shock ε^T in the long run orthogonalization and to the output innovation ε^Y in the short run one, for $k = 1,\ 4,\ 8,\ 20$ quarters and for $k \longrightarrow \infty$. Those shares are computed from a VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4.

in output are mainly associated with the temporary shocks, which explain more than 60% of output volatility on impact.

Now we consider an alternative orthogonalization that uses short run restrictions

$$\left(\begin{array}{c} \Delta C_t \\ \Delta Y_t \end{array} \right) = \widetilde{\Gamma}(L) \left(\begin{array}{c} \varepsilon_t^C \\ \varepsilon_t^Y \end{array} \right), \tag{4}$$

where $\widetilde{\Gamma}(L) = \sum_{i=0}^{\infty} \widetilde{\Gamma}_i L^i$ and the covariance matrix of $(\varepsilon^C, \varepsilon^Y)$ is the identity matrix. The $\widetilde{\Gamma}$ matrices are solution to a system of equations similar to (3). We however depart from (3) as we impose that the $1, 2$ element of $\widetilde{\Gamma}_0$ be equal to zero. Therefore, ε^Y can be called an output innovation, and by construction the contemporaneous response of C to ε^Y is zero.

Figure 2 graphs the impulse responses of C and Y associated with the second orthogonalization scheme. The associated variance decompositions are displayed in Table 1. The striking result from these estimations is that the consumption shock ε^C is almost identical to the permanent shock to consumption (ε^P in the long run orthogonalization scheme), so that the responses and variance decompositions are very similar to those obtained using the long run orthogonalization scheme. This observation is further confirmed by Figure 3, which plots ε^P against ε^C and ε^T against ε^Y. It is striking to observe that both shocks align along the 45° line, indicating that the consumption innovation is essentially identical to the permanent component.

We now want to link the behavior of hours worked to the above description of output and consumption. In particular, we want to ask how much of the variance of hours worked is associated with the temporary shock (or quasi–equivalently the output shock) versus the permanent shock recovered from the consumption–output VECM. It is of interest to evaluate the contribution of these two shocks to the volatility of hours since it allows us to see whether hours can best be described

Figure 2: Responses of Output and Consumption to ε^C and ε^Y
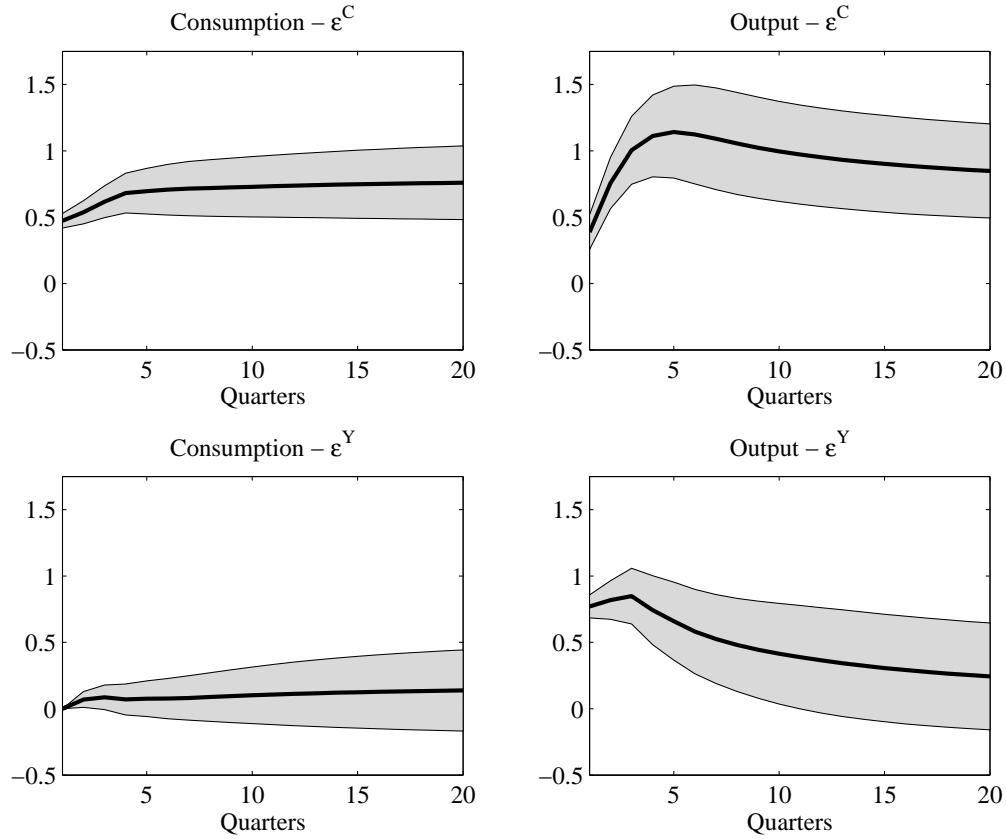
This figure shows the responses of consumption and output to consumption ε^C and output ε^Y one percent shocks obtained from a short run orthogonalization scheme. Those impulse response functions are computed from a VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VECM.

Figure 3: Plots of ε^C against ε^P and ε^Y against ε^T
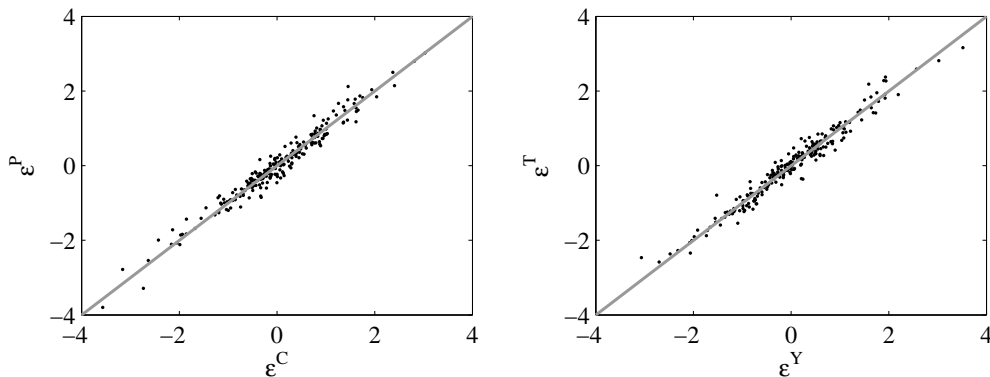
The left panel plots the estimated permanent innovation ε^P (from the long run orthogonalization scheme) against the consumption innovation ε^C (from the short run orthogonalization scheme). The right panel plots the estimated temporary innovation ε^T (from the long run orthogonalization scheme) against the output innovation ε^Y (from the short run orthogonalization scheme). In both panels, the straight line is the 45° line. These shocks are computed from a VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4.

as moving with the temporary component or the permanent component. To do so, we adopt the following approach. Once the innovations ε^P and ε^T are recovered from the bivariate C–Y VECM, we regress hours worked (in levels or differences) on current and lagged values of these two shocks plus a moving average error term denoted ε^H, which we call an hours specific shock.[5] An attractive feature of this approach is that it delivers results which are robust to the specification of hours worked (level or difference)[6]. More precisely, we run the regression

$$x_t = c + \sum_{k=0}^{K} \left(\alpha_k \varepsilon_{t-k}^P + \beta_k \varepsilon_{t-k}^T + \gamma_k \varepsilon_{t-k}^H \right), \tag{5}$$

where x_t denotes either the (log) hours per capita in levels or in differences. This model is estimated by maximum likelihood, choosing an arbitrarily large number of lags ($K = 40$). We then compute,

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[5]Such a two step strategy amounts to the estimation of the following restricted trivariate moving-average process:

$$\begin{pmatrix} C_t \\ Y_t \\ H_t \end{pmatrix} = \begin{pmatrix} A(L) & 0_{2,1} \\ B(L) & C(L) \end{pmatrix} \begin{pmatrix} \varepsilon_t^P \\ \varepsilon_t^T \\ \varepsilon_t^H \end{pmatrix},$$

where $A(L)$ is a 2×2 polynomial matrix, $0_{2,1}$ is a 2×1 vector of zeros, $B(L)$ is a 1×2 polynomial matrix and $C(L)$ is a polynomial in lag operator. $A(L)$, ε^P and ε^T are recovered from the first step bivariate VECM, while $B(L)$, $D(L)$ and ε^H are estimated using a truncated approximation of the third line of the above MA process (which is equation (5)).

[6] It is well known (see for instance the discussions in Gali [1999], Gali and Rabanal [2004], Chari, Kehoe and McGrattan [2004], Christiano, Eichenbaum and Vigfusson [2004]) that specification choice (levels versus first differences) matters a lot for VARs with hours worked. Results show that our procedure is robust to this specification choice.

for each horizon k the share of the overall volatility of hours worked accounted for by ε^P, ε^T and by the hours specific shock ε^H. Results are reported in Table 2. The numbers reported in the table

Table 2: Variance Decomposition of Hours Worked Levels

Horizon	Level Specification			Difference Specification		
	ε^P	ε^T	ε^H	ε^P	ε^T	ε^H
1	19 %	75 %	6 %	21 %	74 %	5 %
4	37 %	56 %	7 %	46 %	52 %	2 %
8	61 %	32 %	7 %	66 %	32 %	2 %
20	60 %	21 %	19 %	69 %	28 %	3 %
40	54 %	20 %	26 %	57 %	38 %	5 %

This table shows the k-period ahead share of the forecast error variance of hours worked to the temporary ε^T, the permanent ε^P and the hours specific shock ε^H. Those shares are computed using a two-step procedure. First ε^T and ε^P are derived from the estimation of a VECM (C,Y) with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4. Then hours worked (in levels or difference depending on the specification) are projected on current and past values of those innovations plus a moving average term in ε^H.

clearly indicate that hours worked are primarily explained by the transitory component at business cycle frequencies.

To summarize, there are four properties of the data that we want to highlight: *(i)* the permanent shock (ε^P) recovered using a long run restriction in a consumption–output VECM is essentially the same shock as that corresponding to a consumption shock (ε^C) recovered using an impact restriction, *(ii)* the response of consumption to a temporary shock is extremely close to zero at all horizons, and there are almost no dynamics in the response of consumption to a permanent shock, as it jumps almost instantaneously to its long run level, *(iii)* the temporary shock (or the output shock in the short run orthogonalization) is responsible for a significant share of output volatility at business cycle frequencies and *(iv)* hours are largely explained by the transitory shock at business cycle frequencies. These facts emphasize that a substantial fraction of the business cycle action seems to be related to changes in investment and hours worked, without any short or long run implications for consumption. We have investigated the robustness of these findings both against changes in the specification of the VECM — by estimating rather than imposing the cointegration relation, adding additional lags or estimating the VECM in levels — and against the data used to estimate the VECM — we considered total consumption rather than the consumption of nondurables and services, output as measured by consumption plus investment only — and in all these cases we found no major changes in patterns.[7] Since we have emphasized the quasi

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[7]All these results are reported in the technical appendix to this paper, available from

equivalence between the shocks recovered using a long run restriction, and shocks recovered using an impact restrictions, in the next subsection we provide a formal test for the equality between ε^Y and ε^T.

1.2 Testing the Equality of ε^Y and ε^T

This section proposes a test for the equality between ε^Y and ε^T (or equivalently between ε^C and ε^P as the shocks are pairwise orthogonal). We show that, for the VECM under consideration, this equality corresponds to a particular zero in the long run impact matrix of the Wold decomposition. Consider the Wold representation (1), and consider the following representation of the process:

$$\left(\begin{array}{c} \Delta C_t \\ \Delta Y_t \end{array} \right) = B(L) \left(\begin{array}{c} \nu_{1,t} \\ \nu_{2,t} \end{array} \right), \tag{6}$$

where $B(L) = \sum_{i=0}^{\infty} B_i L^i$ and the variance covariance matrix of (ν_1, ν_2) is the identity matrix. We want here to perform an overidentified orthogonalization and impose at the same time that *(i)* the shock ν_2 has no impact effect on C and *(ii)* no long run effect on C and Y. More precisely, we look for a matrix S such that $\mu = S\nu$ and $B_i = A_i S$. Imposing a zero impact effect of ν_2 on C implies

$$s_{12} = 0. \tag{7}$$

The matrix giving the long run effect of ν on both variables is given by $\widehat{B} = \widehat{A}S$, where $\widehat{A} = \sum_{i=0}^{\infty} A_i$. Imposing the long run restriction $\widehat{b}_{12} = 0$ implies

$$\widehat{a}_{11}s_{12} + \widehat{a}_{12}s_{22} = 0. \tag{8}$$

When the two series cointegrate, the matrix $\widehat{A}$ rewrites

$$\widehat{A} = \left(\begin{array}{cc} \widehat{a}_{11} & k\widehat{a}_{11} \\ \widehat{a}_{21} & k\widehat{a}_{21} \end{array} \right),$$

where k is a real number.

When $\widehat{a}_{12} \neq 0$ — which occurs when both k and $\widehat{a}_{11}$ are non zero — equations (7) and (8) imply that the second column of S is composed of zeros, meaning that S is not a full rank matrix. In other words, the two restrictions cannot hold at the same time. On the contrary, if $\widehat{a}_{12} = 0$ — when either k or $\widehat{a}_{11}$ are zero — the long run and short run constraints are simultaneously satisfied. This suggests that a convenient way of testing whether both the short and long run constraints are satisfied is to test for the nullity of a particular coefficient of the long run matrix $\widehat{A}$ of the Wold representation of the process, $\widehat{a}_{12}$. The following proposition states this result in a more general case where the two series need not cointegrate.

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PROPOSITION 1 *Consider a bivariate process whose Wold decomposition is given by*

$$\left(\begin{array}{c} \Delta X_t^1 \\ \Delta X_t^2 \end{array} \right) = A(L) \left(\begin{array}{c} \mu_{1,t} \\ \mu_{2,t} \end{array} \right),$$

with $A(L) = \sum_{i=0}^{\infty} A_i L^i$, with $A_0 = I$, where the covariance matrix of μ is given by Ω, and where the matrix of long run effect $\widehat{A} = A(1) = \sum_{i=0}^{\infty} A_i$ is non singular. Consider a structural representation

$$\left(\begin{array}{c} \Delta X_t^1 \\ \Delta X_t^2 \end{array} \right) = B(L) \left(\begin{array}{c} \nu_{1,t} \\ \nu_{2,t} \end{array} \right),$$

where $B(L) = \sum_{i=0}^{\infty} B_i L^i$, the covariance matrix of ν is the identity matrix and $\mu = S\nu$. Then, the two following statements are equivalent

(a) *If the second structural shock ν_2 has no short run impact on X_1, i.e. $s_{12} = 0$, then it has no long run impact on X_1, i.e. $\widehat{b}_{12} = 0$, and conversely.*

(b) *The $(1,2)$ element of the long run effect matrix of the Wold decomposition is zero, i.e. $\widehat{a}_{12} = 0$*

Proof : We first prove that *(a)* implies *(b)*. Assume that

$$S = \left(\begin{array}{cc} s_{11} & 0 \\ s_{21} & s_{22} \end{array} \right) \quad \text{and} \quad \widehat{B} = \left(\begin{array}{cc} \widehat{b}_{11} & 0 \\ \widehat{b}_{21} & \widehat{b}_{22} \end{array} \right).$$

Inverting S, we obtain

$$S^{-1} = \left(\begin{array}{cc} 1/s_{11} & 0 \\ -s_{21}/(s_{11}s_{22}) & 1/s_{22} \end{array} \right).$$

The long run effect matrix of the Wold decomposition is $\widehat{A} = \widehat{B} \times S^{-1}$ and one can easily check that $\widehat{a}_{12} = 0$.

We then prove that *(b)* implies *(a)*. We have the relation $\widehat{A}S = \widehat{B}$. We assume that $\widehat{a}_{12} = 0$. Then, $\widehat{b}_{12} = \widehat{a}_{11}s_{12}$. As $\widehat{A}$ is assumed to be nonsingular, $\widehat{a}_{11} \neq 0$, so that $\widehat{b}_{12} = 0 \Longleftrightarrow s_{12} = 0$. *Q.E.D.*

To formally test whether $\varepsilon^P = \varepsilon^C$, we test for the nullity of $\widehat{a}_{12}$. The confidence intervals are obtained from 1000 bootstraps of the long run matrix. The coefficient $\widehat{a}_{12} = 0$ takes an average value of 0.2024 with a 95% confidence interval $[-0.2, 0.8]$. At a 5% significance level, one cannot reject the hypothesis that the consumption shock is identical to the permanent shock.

2   An Analytical Model of Market Rushes

In this section, we present a simple analytical model of market rushes. The main element is that, in each period agents receive information about potential new varieties of goods which could become

profitable to produce. In response to these expectations of profits, agents invest in putting on the market a prototype of the new good. Since many agents may invest in such startups, they engage in a winner takes all competition for securing the market of a newly created variety. The winning firm then becomes a monopolist on the market. This position may then be lost randomly at an exogenous rate. Expansion in variety may or may not have a long run impact on productivity, so that the market rush is not forced to satisfy the gold rush analogy. We first present the model, then characterize its solution and discuss the equilibrium allocation properties. We contrast the equilibrium allocations with those obtained from a social planner problem. Finally, we show that a particular version of the model replicates the four properties of the data that we previously highlighted.

2.1 Model

Firms : There exists a raw final good, denoted Q_t, produced by a representative firm using labor h_t and a set of intermediate goods X_{jt} with mass N_t according to a constant returns to scale technology represented by the production function

$$Q_t = (\Theta_t h_t)^\alpha N_t^\xi \left(\int_0^{N_t} X_t(j)^\chi \mathrm{d}j \right)^{\frac{1-\alpha}{\chi}}, \tag{9}$$

where Θ_t is an index of disembodied exogenous technological progress and $\alpha \in (0,1)$. $\chi \leqslant 1$ determines the elasticity of substitution between intermediate goods and ξ is a parameter that determines the long run effect of variety expansion. Since this final good will also serve to produce intermediate goods, we will refer to Q_t as the gross amount of final good. Also note that the raw final good will serve as the numéraire. The representative firm is price taker on the markets.

Each existing intermediate good is produced by a monopolist. Just like in many expanding variety models, the production of one unit of intermediate good requires one unit of the raw final good as input. Since the final good serves as a numéraire, this leads to a situation where the price of each intermediate good is given by $P_t(j) = \frac{1}{\chi}$. Therefore, the quantity of intermediate good j, $X_t(j)$, produced in equilibrium is given by

$$X_t(j) = (\chi(1-\alpha))^{\frac{1}{\alpha}} \Theta_t N_t^{\frac{\xi-1+(1-\alpha)/\chi}{\alpha}} h_t. \tag{10}$$

and the profits, $\Pi_t(j)$, generated by intermediate firm j are given by

$$\Pi_t(j) = \pi_0 \Theta_t N_t^{\frac{\xi-1+(1-\alpha)/\chi}{\alpha}} h_t, \tag{11}$$

where $\pi_0 = \left(\frac{1-\chi}{\chi} \right) (\chi(1-\alpha))^{\frac{1}{\alpha}}$. Equalization of the real wage with marginal product of labor implies

$$W_t = A\Theta_t N_t^{\frac{\xi+(1-\alpha)(1-\chi)/\chi}{\alpha}}, \tag{12}$$

14

where $A = \alpha(\chi(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}$.

Value added, Y_t, is then given by the quantity of raw final good, Q_t, net of that quantity used to produce the intermediate goods, $X_t(j)$. Once we substitute out for $X_t(j)$, and take away the amount of Q_t used in the production of $X_t(t)$, we obtain

$$
\begin{aligned}
Y_t &= Q_t - \int_0^{N_t} P_t(j)X_t(j)\mathrm{d}j \\
&= A\Theta_t N_t^{\frac{\xi+(1-\alpha)(1-\chi)/\chi}{\alpha}} h_t.
\end{aligned}
\tag{13}
$$

Note that when $\xi = -(1-\alpha)(1-\chi)/\chi$, an expansion in variety exerts no effect on labor productivity. In this case, value–added reduces to

$$
Y_t = A\Theta_t h_t
\tag{14}
$$

The net amount of raw final good can serve for consumption, C_t, and startup expenditures, S_t, purposes.

$$
Y_t = C_t + S_t.
\tag{15}
$$

Variety Dynamics : In each period, there is an exogenous probability η_t that a potential new variety appears in the economy. In such a case, any entrepreneur who desires to produce this potential new variety has to pay a fixed cost of one unit of the final good to setup the startup. In order to obtain a tractable solution, we consider allocations for which it is always optimal to exploit the whole range of intermediate goods. In practice, we assume that there is no difference between the potential number of varieties and the actual one in equilibrium. We later check that the model parameters are such that full adoption is indeed optimal. Let S_t denote total expenditures on setup costs. A time $t+1$ the startup will become a functioning new firm with a product monopoly with an endogenous probability ρ_t. Likewise, an existing monopoly becomes obsolete at an exogenous probability μ. Therefore, the dynamics for the number of products is given by

$$
N_{t+1} = (1 - \mu + \eta_t)N_t.
\tag{16}
$$

In the above, μN_t represents the existing products that are destroyed, while there will be $\eta_t N_t$ openings which can be filled by startups. η_t follows a random process, with unconditional mean μ. Note that η is a news shock, since it brings information about future profitable varieties but does not immediately affect the production function.

The S_t startups of period t compete to secure the $\eta_t N_t$ new monopoly positions. We assume that in equilibrium $S_t > \eta_t N_t$, which can later be verified as being satisfied. The $\eta_t N_t$ successful startups are uniformly drawn from among the S_t existing ones. Therefore, the probability that a startup at time t will become a functioning firm at $t+1$ is given by $\rho_t = \frac{\eta_t N_t}{S_t}$.

Households : The preferences of the representative household are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\log(C_{t+\tau}) + \psi(\overline{h} - h_{t+\tau}) \right), \tag{17}$$

where $0 < \beta < 1$ is a constant discount factor, C_t denotes consumption in period t and h_t is the quantity of labor the household supplies. The household chooses how much to consume, supply labor, hold equity ($\mathcal{E}_t$) in existing firms, and invest in startups (S_t) by maximizing (17) subject to the following sequence of budget constraints

$$C_t + P_t^{\mathcal{E}} \mathcal{E}_t + S_t = W_t h_t + \mathcal{E}_t \Pi_t + (1 - \mu) P_t^{\mathcal{E}} \mathcal{E}_{t-1} + \rho_{t-1} P_t^{\mathcal{E}} S_{t-1}, \tag{18}$$

where $P_t^{\mathcal{E}}$ is the beginning of period price of equity, prior to dividend payments and W_t is the wage rate. Dividends per equity share are assumed to be equal to period–profits Π_t.

The first order conditions imply:

$$\psi C_t = W_t \tag{19}$$

$$\frac{1}{C_t}(P_t^{\mathcal{E}} - \Pi_t) = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} (1 - \mu) P_{t+1}^{\mathcal{E}} \right] \tag{20}$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \rho_t P_{t+1}^{\mathcal{E}} \right]. \tag{21}$$

2.2 Equilibrium Allocations

The three last first order conditions can be combined to give:

$$\frac{1}{\rho_t C_t} = \beta \mathbb{E}_t \left[\frac{\Pi_{t+1}}{C_{t+1}} \right] + \beta \mathbb{E}_t \left[\frac{(1 - \mu)}{\rho_{t+1} C_{t+1}} \right], \tag{22}$$

which can be simply restated as

$$1 = \beta \rho_t \mathbb{E}_t \sum_{\tau=0}^{\infty} \left[\beta^\tau \frac{C_t}{C_{t+\tau+1}} (1 - \mu)^\tau \Pi_{t+\tau+1} \right].$$

This condition can be interpreted as a free entry condition, whereby the setup cost of a startup (one unit of the final good) is equal to the expected discounted sum of future profits. Using the labor demand condition (12) and the profit equation (11), the free entry condition (22) rewrites as

$$\left(\frac{S_t}{C_t} \right) = \beta \frac{\psi \pi_0}{A} \left(\frac{\eta_t}{1 - \mu + \eta_t} \right) \mathbb{E}_t h_{t+1} + \beta \left(\frac{1 - \mu}{1 - \mu + \eta_t} \right) \mathbb{E}_t \left[\left(\frac{\eta_t}{\eta_{t+1}} \right) \left(\frac{S_{t+1}}{C_{t+1}} \right) \right]. \tag{23}$$

Using the labor demand condition (12) and the resource constraint (15), we get

$$\left(\frac{S_t}{C_t} \right) = \psi h_t - 1. \tag{24}$$

The free entry condition can therefore we written as:

$$(h_t - \psi^{-1}) = \beta \delta_t \frac{\pi_0}{A} \mathbb{E}_t h_{t+1} + \beta \delta_t \mathbb{E}_t \left[\left(\frac{1}{\delta_{t+1}} - 1 \right) (h_{t+1} - \psi^{-1}) \right], \tag{25}$$

where $\delta_t = \eta_t/(1 - \mu + \eta_t)$ is an increasing function of the fraction of newly opened markets η_t.

Equation (25) is a key equation of the model. It shows that current employment h_t depends on h_{t+1}, δ_t and δ_{t+1}, and therefore indirectly depends on all the future expected δs. As δ_t brings news about the future, employment is purely forward looking. The reason why future employment favors current employment can be easily given an economic intuition: higher future employment reflects higher expected profits, which therefore stimulates new entries today. Note that the model possesses a lot of neutrality, as the determination of employment does not depend on either current or future changes in disembodied technological change Θ_t.

By repeated substitution, the above equation can be written as a function of current and future values of δ only. Given the nonlinearity of equation (25),[8] it is useful to compute a log–linear approximation around the deterministic steady–state value of employment h. The latter is given by:[9]

$$h = \frac{\psi^{-1}(1 - \beta(1 - \mu))}{(1 - \beta\mu\frac{\pi_0}{A} - \beta(1 - \mu))},$$

and the log–linear approximation takes the form

$$\widehat{h}_t = \gamma \mathbb{E}_t \widehat{h}_{t+1} + \left(\frac{h - \psi^{-1}}{h} \right) \mathbb{E}_t \left[\widehat{\delta}_t - \beta \widehat{\delta}_{t+1} \right]$$

where $\widehat{h}_t$ now represents relative deviations from the steady state and $\gamma \equiv \beta\mu(\pi_0/A) + \beta(1 - \mu)$ with $\gamma \in (0, 1)$. Solving forward, this can be written as

$$\widehat{h}_t = \left(\frac{h - \psi^{-1}}{h} \right) \left(\widehat{\delta}_t - \mu\beta \left(\frac{A - \pi_0}{A} \right) \mathbb{E}_t \left[\sum_{i=0}^{\infty} \gamma^i \widehat{\delta}_{t+1+i} \right] \right). \tag{26}$$

Note that, as $\gamma \in (0, 1)$, the model possesses a unique determinate equilibrium path.

Once the equilibrium path of h is computed, output is directly obtained from equation (13). Finally, combining labor demand (12) and labor supply (19), we obtain an expression for aggregate consumption:

$$C_t = \frac{A}{\psi} \Theta_t N_t^{\frac{\xi + (1-\alpha)(1-\chi)/\chi}{\alpha}}. \tag{27}$$

Equation (26) reveals that a positive $\widehat{\delta}_t$, – *i.e.* an acceleration of variety expansion, causes an instantaneous increase in hours worked, output and investment in startups S. This boom arises as the result of the prospects of profits derived from securing those new monopoly positions. This

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[8]We show in the technical appendix to this paper that it is possible to obtain an exact analytical solution to the model in the case of i.i.d. shocks.

[9]Note that we used the fact that $\mathbb{E}_t(\eta_t) = \mu$, which implies that $\delta = \mu$ in steady state.

occurs irrespective of any current change in the technology or in the number of varieties. Such an expansion is therefore akin to a "demand driven" or "investment driven" boom. In this analytical model, consumption does not increase following a variety increase since total wealth is not increased.

2.3 Comparison to the social optimum

Optimality properties of those allocations are worth discussing, and it is useful to compute the socially optimal allocations as a benchmark. The social planner problem is given by [10]

$$
\text{Max} \quad \mathbb{E}_t \quad \sum_{i=0}^{\infty} \left[\log C_{t+i} + \psi(\overline{h} - h_{t+i}) \right]
$$

$$
\text{s.t.} \quad C_t \leq \widehat{A} \Theta_t N_t^{\frac{\xi + (1-\alpha)(1/\chi - 1)}{\alpha}} h_t - \eta_t N_t,
$$

(28)

with $\widehat{A} = \alpha(1-\alpha)^{\frac{\alpha}{(1-\alpha)}}$ and where we have already solved for the optimal use in intermediate goods. The first order condition of the social planner program is given by

$$
\frac{\widehat{A} \Theta_t N_t^{\frac{\xi + (1-\alpha)(1/\chi - 1)}{\alpha}}}{\widehat{A} \Theta_t N_t^{\frac{\xi + (1-\alpha)(1/\chi - 1)}{\alpha}} h_t - \eta_t N_t} = \psi.
$$

(29)

There are many sources of inefficiency in the decentralized allocations. One obvious source is the presence of imperfect competition: *ceteris paribus*, the social planner will produce more of each intermediate good. Another one is the congestion effect associated with investment in startups, because only a fraction ρ_t of startups are successful. The social planner internalizes this congestion effect, and does not duplicate the fixed cost of startups, as the number of startups created is equal to the number of available slots.[11] Because of these imperfections, the decentralized allocation differs from the optimal allocation along a balanced growth path.

The difference between the market and the socially optimal allocations that we want to highlight regards the response to expected future market shocks. It is remarkable that the socially optimal allocation decision for employment (29) is static, and only depends on η_t (positively). This stands in sharp contrast with the market outcome, as summarized by equation (25), in which all future values of η appear. To understand this difference, let us consider an increase in period t in the expected level of η_{t+1}. We assume that full adoption is always optimal in both the decentralized and the socially optimal allocations. In the decentralized economy, larger η_{t+1} means more startup investment in $t+1$ and more firms in $t+2$. Those firms will affect other firms profits in period

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[10]We assume again here that parameters are such that it is always socially optimal to invest in a new variety. One necessary condition for full adoption to be socially optimal is that the long run effect of variety expansion is positive, – *i.e* $\xi > -(1-\alpha)(1-\chi)/\chi$

[11]Note that we assume here that parameter values are such that it is always optimal to adopt all the new varieties. Another potential source of sub–optimality would be an over or under adoption of new goods by the market. As shown in Benassy [1998] in a somewhat different setup with endogenous growth, the parameter ξ is then crucial in determining whether the decentralized allocations show too much or too little of new goods adoption.

$t + 2$ and onwards. Therefore, a period t startup will face more competitors in $t + 2$, which reduces its current value, and therefore decreases startup investment and output[12]. Such an expectation is not relevant for the social planner, which does not respond to news about future values of η. Therefore, in that simple analytical model, part of economic fluctuations are driven by investors (rational) forecast about future profitability that are inefficient from a social point of view.[13]

2.4   A Gold Rush Configuration

We now make a set of specific assumptions on some parameters of the model. The objective is to obtain a Wold representation of that simple analytical model that provides a structural interpretation to the shocks we recovered in Section 1. We first assume that disembodied technical change, Θ_t, follows (in log) a random walk without drift: $\log \Theta_t = \log \Theta_{t-1} + \varepsilon_t^\Theta$. Second, we assume that variety expansion exerts no effect on productivity in the long run. This is achieved by setting that $\xi = -(1 - \alpha)(1 - \chi)/\chi$. The aggregate production function is then given by (14). Finally, we assume that variety expansion shocks η_t follow an AR(1) process of the form

$$\log(\eta_t) = \rho \log(\eta_{t-1}) + (1 - \rho) \log(\mu) + \varepsilon_t^N,$$

where ε_t^N are *i.i.d.* with mean 0. In this case, the solution for hours worked is given by

$$\widehat{h}_t = \omega \widehat{\eta}_t \quad \text{with } \omega \equiv \frac{h - \psi^{-1}}{h} \frac{(1 - \mu)(1 - \beta\rho)}{1 - \gamma\rho}.$$

Under those assumptions, the logs of consumption and output are given by:

$$\log(Y_t) \;\; = \;\; k_y + \log(\Theta_t) + \log(h_t) \tag{30}$$

$$\log(C_t) \;\; = \;\; k_c + \log(\Theta_t), \tag{31}$$

where k_c and k_y are constants. Using equation (25) to replace h_t with its solution, it is straightforward to write the $MA(\infty)$ representation of the system. For instance, ignoring constant terms, we obtain

$$\begin{pmatrix} \Delta \log(C_t) \\ \Delta \log(Y_t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & \omega \frac{1-L}{1-\rho L} \end{pmatrix} \begin{pmatrix} \varepsilon_t^\Theta \\ \varepsilon_t^N \end{pmatrix} = C(L) \begin{pmatrix} \varepsilon_t^\Theta \\ \varepsilon_t^N \end{pmatrix}.$$

This particular version of the model shares a lot of dynamic properties with the data. First of all the system clearly shows that consumption and output do cointegrate (C(1) is not full rank) with cointegrating vector [1;-1]. Second, it shows that consumption is actually a random walk, that is only affected —in the short run as well as in the long run— by technology shocks, ε^Θ. Output is also

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[12]This is due to the typical "business stealing" effect found in the endogenous growth literature, for example in Aghion and Howitt [1992], and originally discussed in Spence [1976a] and Spence [1976b].

[13]The very result that it is socially optimal not to respond to such news is of course not general, and depends on the utility and production function specification. The general result is not that it is socially optimal not to respond to news about η, but that the decentralized allocations are inefficient in responding to news shocks.

affected in the short run by the temporary shock, ε^N. Hence, computing, sequentially, our short run and long run orthogonalization with this model would imply $\varepsilon^P = \varepsilon^C = \varepsilon^\Theta$ and $\varepsilon^T = \varepsilon^Y = \varepsilon^N$. Finally, it is the temporary shock ε^T (which is indeed ε^N) that explains all the variance in hours worked at any horizon. Such a model therefore allows for a structural interpretation of the results we obtained in Section 1. Permanent shocks to C and Y are indeed technology shocks. Consumption does not respond to variety expansion shocks, which however account for a lot of output fluctuations and all the fluctuations in hours worked. Variety expansion shocks create market rushes that are indeed gold rushes, generating inefficient business cycles as the social planner would choose not to respond to them. In effect, these shocks only trigger rent seeking activities, as startups are means of appropriating a part of the economy pure profits.

Although simple, this model illustrates how the market mechanism we have put forward has the potential to account for what we observe in the data. In the next section, we consider an extended version of the model in which we introduce capital accumulation, and real frictions. We use the estimated responses from the long run orthogonalization scheme to estimate the size of the technological and variety expansion shocks. Once those parameters are estimated, we are able to assess the ability of the model to account quantitatively for the facts we documented in Section 1, and therefore decompose economic fluctuations in a meaningful way, using our model as a measurement tool.

3   Quantitative Assessment

In this section, we first present the extended model before describing the calibration and estimation procedure. Then we comment on the estimated parameters and derive some implications of the estimated model. Finally, we discuss the quantitative success of alternative explanations of the facts.

3.1   Model, Calibration and Estimation Procedure

Our emphasis in this work is on the existence of a new type of shock, namely a market rush. In order to gauge the quantitative importance of this shock in the business cycle, we enrich the propagation mechanisms of our baseline model, and estimate it with U.S. data.

The Extended Model: We extend the model by including capital accumulation, two types of intermediate goods and habit persistence in consumption. The final good is now produced with

capital, K_t, labor, h_t and two types of intermediate goods $X_t(i)$ and $Z_t(j)$, according to

$$Q_t = K_t^{1-\alpha_x-\alpha_z-\alpha_h}(\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left(\int_0^{N_{x,t}} X_t(i)^\chi \mathrm{d}i \right)^{\frac{\alpha_x}{\chi}} N_{z,t}^{\widetilde{\xi}} \left(\int_0^{N_{z,t}} Z_t(j)^\chi \mathrm{d}j \right)^{\frac{\alpha_z}{\chi}}, \qquad (32)$$

with $\alpha_x, \alpha_z, \alpha_h \in (0,1)$, $\alpha_x + \alpha_z + \alpha_h < 1$ and $\chi \geqslant 1$. We impose that $\overline{\xi} = -\alpha_x(1-\chi)/\chi$ so that variety expansion in intermediate goods X has no long–run impact, and that $\widetilde{\xi} = (\chi(1-\alpha_x)-\alpha_z)/\chi$, so that the equilibrium aggregate value added production function is linear in the number of intermediate goods of type Z ($N_{z,t}$). Θ_t denotes Harrod neutral technical progress, the log of which is assumed to follow a random walk with drift $\gamma \geqslant 1$.

As in the analytical model, the numbers of available varieties evolve exogenously according to

$$\begin{aligned} N_{x,t+1} &= (1 - \mu + \eta_t^x)N_{x,t} \\ N_{z,t+1} &= (1 - \mu + \eta_t^z)N_{z,t}. \end{aligned}$$

In equilibrium, full adoption will always be optimal. We assume that the stochastic processes for productivity and the market shocks are given by

$$\begin{aligned} \log(\eta_t^x) &= \rho_x \log(\eta_{t-1}^x) + (1 - \rho_x) \log(\overline{\eta}^x) + \varepsilon_t^x \\ \log(\eta_t^z) &= \rho_z \log(\eta_{t-1}^z) + (1 - \rho_z) \log(\overline{\eta}^z) + \varepsilon_t^z \\ \log(\Theta_t) &= \log(\gamma) + \log(\Theta_{t-1}) + \varepsilon_t^\Theta. \end{aligned}$$

We assume that ε^x, ε^z and ε^Θ are Gaussian white noises with zero means and respective variances σ_x^2, σ_z^2 and σ_Θ^2.

Capital accumulation is governed by the law of motion

$$K_{t+1} = (1 - \delta) K_t + \left[1 - \mathcal{S} \left(\frac{I_t}{I_{t-1}} \right) \right] I_t,$$

where $\delta \in (0,1)$ is the constant depreciation rate. The function $\mathcal{S}(\cdot)$ accounts for the presence of adjustments costs in capital accumulation. We assume that $\mathcal{S}(\cdot)$ satisfies $\mathcal{S}(\gamma) = \mathcal{S}'(\gamma) = 0$ and $\varphi = \mathcal{S}''(\gamma)\gamma^2 > 0$. It follows that the steady state of the model does not depend on the parameter φ while its dynamic properties do. Notice that following Christiano, Eichenbaum and Evans [2005b], Christiano and Fisher [2003] and Eichenbaum and Fisher [2005], we adopt the dynamic investment adjustment cost specification. In this environment, it is the growth rate of investment which is penalized when varied in the neighborhood of its steady state value. In contrast, the standard specification penalizes the investment–to–capital ratio. The dynamic specification for adjustment costs is a significant source of internal propagation mechanisms as it generates a hump–shaped response of investment to various shocks.

Finally, we introduce habit persistence in consumption, and the intratemporal utility function is given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left[\log(C_{t+\tau} - bC_{t+\tau-1}) + \psi(\overline{h} - h_{t+\tau}) \right].$$

Note that introducing adjustment costs to investment and habit persistence, while not affecting the main qualitative properties of the model we have presented in Section 2, will improve the ability of the model to capture the shape of the impulse response function. The model then solves as in the preceding section, except that no analytical solution can be found.

Calibration: Our quantitative strategy is to calibrate those parameters for which we have estimates or that we can obtain by matching balanced growth path ratios with observed averages. The time period is a quarter. The discount factor is set such that the household discounts the future at a 3% annual rate. We assume constant markups of 20%, so that $\chi = 0.833$. The depreciation rate is equal to 2.5% per quarter, as is common the literature. We assume that the two sets of intermediate goods differ only with regards to the long run impact of a variety expansion, and therefore assume $\alpha_x = \alpha_z$.[14] The parameters α_h and α_x are set such that the model generates a labor share and a share of intermediate goods in value added of, respectively, 60% (Cooley and Prescott [1995]) and 50% (Jorgenson, Gollop and Fraumeni [1987]). μ is set such that the model generates a consumption share of 70%. The calibrated parameters are summarized in Table 3.

Table 3: Calibrated Parameters

Preferences		
Discount factor	β	0.9926
Technology		
Elasticity of output to intermediate goods	$\alpha_x + \alpha_z$	0.3529
Elasticity of output to hours worked	α_h	0.4235
Depreciation rate	δ	0.0250
Elasticity of substitution bw intermediates	χ	0.8333
Rate of technology growth	γ	1.0060
Monopoly death rate	μ	0.0086

Estimation Procedure: We estimate the following seven parameters: the standard deviation of the technological shock innovation σ_Θ, the persistence parameters ρ_x and ρ_z, and the standard deviations σ_x and σ_z of the two market shocks, the habit persistence parameter b and the adjustment cost parameter φ. These parameters are chosen in order to match the output impulse responses of the long run VECM that we have presented in section 1, and that are displayed in Figure 1. As the

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[14]This assumption amounts to the choice of the relative variance σ_x^2/σ_z^2 and can be made without loss of generality.

long run orthogonalization scheme cannot recover the model structural shocks (three shocks in the model and only two innovations in the VECM), we cannot directly match the model's theoretical responses with the empirical responses to ε^P and ε^T. Therefore, we follow a simulated method of moments approach. Let $\Psi = (\sigma_\Theta, \rho_x, \sigma_x, \rho_z, \sigma_z, b, \varphi)$ be the parameters to be estimated, and let M be the column vector of estimated moments to match. We denote by $\mathcal{M}(\Psi)$ a column vector of the same moments obtained from simulating the model with parameters Ψ. The set of estimated parameters $\widehat{\Psi}$ is then chosen so as to minimize the distance $\mathcal{D}$

$$\mathcal{D} = (\mathcal{M}(\Psi) - M)' W (\mathcal{M}(\Psi) - M),$$

where W is a weighting matrix that is given by the inverse of the covariance matrix of the estimators of M. The simulated moments $\mathcal{M}(\Psi)$ are obtained by simulating the model over 20 times 232 periods,[15] which is the length of our data sample.

The last issue concerns the choice of moments to match. We aim at matching the impulse responses of output obtained from model generated data to both the permanent and the transitory responses presented in Section 1 based on real data. We use the first twenty quarters of the impulse responses in these exercises. We leave as tests of the model its capacity to reproduce the responses of output to the short run orthonalization scheme and consumption in both scheme. Using the long run scheme, the response of output to a permanent shock displays a hump,[16] and for that reason, we have supplemented the model with habit persistence and adjustment costs in investment. There are therefore forty moments to match. We have two ways of testing our model. The first is by making use of the over–identifying restrictions associated with the estimation procedure (seven parameters for forty moments), by means of a J–test, following Hansen [1982]. The second one is to check whether or not the estimated model possesses the properties of the data that we have highlighted in Section 1, namely the identity between the short and long run orthogonalization schemes and the importance of the temporary shock in explaining the short run patterns in output and hours worked, but not of consumption. To do so, we will compute two distance statistics, $\mathcal{D}(C)$ and $\mathcal{D}(C, Y)$. $\mathcal{D}(C)$ is the a measure of the distance between the first twenty coefficients of consumption IRF as estimated in the data and as estimated from the simulated data. The statistic is computed as:

$$\mathcal{D}(C) = (\mathcal{M}(C, \widehat{\Psi}) - M(C))' W_c (\mathcal{M}(C, \widehat{\Psi}) - M(C)),$$

where $M(C)$ is a vector collecting the impulse responses of consumption to both the permanent and the transitory shocks obtained from the VECM, while $\mathcal{M}(C, \widehat{\Psi})$ collects the same impulse responses

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[15] Michaelides and Ng [1997] have shown that efficiency gains are negligible for a number of simulations larger than 10.

[16] Cogley and Nason [1995] have also proposed the estimation of a (C, Y) VECM, and show that the response of output to the temporary shock is hump–shaped. We find in this study that it is the response to the permanent shock that is hump–shaped. This difference comes from the sample period and the choice of the output variable, that is Net Domestic Product in Cooley and Nason and Gross Domestic Product(GDP) in our work. We prefer the use of GDP as it is the most commonly used measure of output in the literature.

obtained by simulating the theoretical model for the estimated values of the parameters. W_c is the inverse of the covariance matrix of these moments obtained from the VECM. Similarly, $\mathcal{D}(C,Y)$ represents the distance between the model and the data for both consumption and output IRFs.

3.2 Estimation Results

Table (4) reports the estimated values of the seven parameters of interest, together with the values of the J–statistics for over–identification. First of all note that the model is not rejected by the data as the associated J–state is low. A key result is that σ_z is not significantly different from zero. Market shocks therefore do not affect productivity in the long run, which makes market rushes inefficient in the model.

Table 4: Estimated Parameters

Persistence of the X market shocks	ρ_x	0.9166
		(0.0336)
Standard dev. of X market shocks	σ_x	0.2865
		(0.0317)
Persistence of the Z market shocks	ρ_x	0.9164
		(0.6459)
Standard dev. of Z market shocks	σ_z	0.0245
		(0.1534)
Standard dev. of the technology shocks	σ_Θ	0.0131
		(0.0015)
Habit persistence parameter	b	0.5900
		(0.1208)
Adjustment cost parameter	φ	0.4376
		(0.3267)
J–Stat		17.40
		[0.99]
$\mathcal{D}(C)$		42.51
		[0.12]
$\mathcal{D}(C,Y)$		92.78
		[0.06]

This table first presents the estimated parameters, as obtained from a Simulated Method od Moments estimation of the model. Standard errors are in parenthesis. The last three lines display J–statistics and Distance statistics. These statistics are distributed chi-square, and the p–value for testing their nullity is given in brackets.

The impulse responses of the VECM estimated on the artificial data generated from the model are presented on Figure 4, together with the ones estimated with the data. The confidence bands are

the one computed from the data. Note that the IRFs of the model lie within the confidence bands, which confirms that the model does a good job not only on impact and in the long run (as shown by the low level of the J–stat), but also for much of the dynamics. This is confirmed by the $\mathcal{D}(C)$ statistic. This statistic is distributed according to a chi-square with 33 degrees of freedom under the null hypothesis of equality between the IRFs obtained from the VECM on both historical and simulated data for consumption. The test statistic value is 42.51 with an associated p–value of 12%. The implications of the model for consumption dynamics are not rejected by the data. Similarly, we perform the same test for the joint behavior of consumption and output. Again, the model is not strongly at odds with the data ($\mathcal{D}(C, Y)$=92.78 with p–value 6%).

The model already displays two of the three properties of the data that we put forward previously: (i) there is virtually no dynamics in the response of consumption to the permanent shock, as it affects permanently and almost instantaneously the level of consumption and (ii) the temporary shock is responsible for a significant share of output volatility at business cycle frequencies.

It is now of interest to test whether the model also possesses the first property: (i) the permanent shock to consumption ε^P is approximately identical to the ε^C shock recovered from a consumption–output VECM. We therefore perform our test for the equality between ε^Y and ε^T in the data generated by the model. We then generate 1000 replications of the model simulations. In 87% of the cases we have the property that the $(1, 2)$ element of the long run effect matrix of the Wold decomposition of a (C, Y) VECM is not significantly different from zero. Figure 5 reports the estimated impulse response functions as obtained from the VECM in the data and using the simulated data of the model assuming a short–run orthogonalization scheme. Again, the fit is extremely good. In Figure 6 we report the IRFs of output and consumption as obtained from the short–run and long–run orthogonalization scheme of the VECM estimated with simulated data. The figure clearly shows that the profiles of the IRFs are very similar. The model is therefore able to reproduce those three salient features of the data that we have put forward in section 1.

Table 5 reports the variance decomposition of hours worked both in the data and in the model, as computed from the same regression as equation (5). As can be seen from the table, hours volatility is primarily accounted for by the transitory shock over the short run horizon. In fact the model predicts that 65% of the overall volatility of hours can be accounted for by transitory shocks at the one period horizon, to be compared to the 75% found in the data. Again the model seems to perform remarkably well along this dimension.

3.3   Business Cycle Accounting

Once estimated, the model can be used to evaluate the importance of the market rush phenomena in the U.S. business cycle. In effect, the model allows for a meaningful (structural) variance

Figure 4: Impulse Response Functions: Data versus Model (Long Run Orthogonalization Scheme)
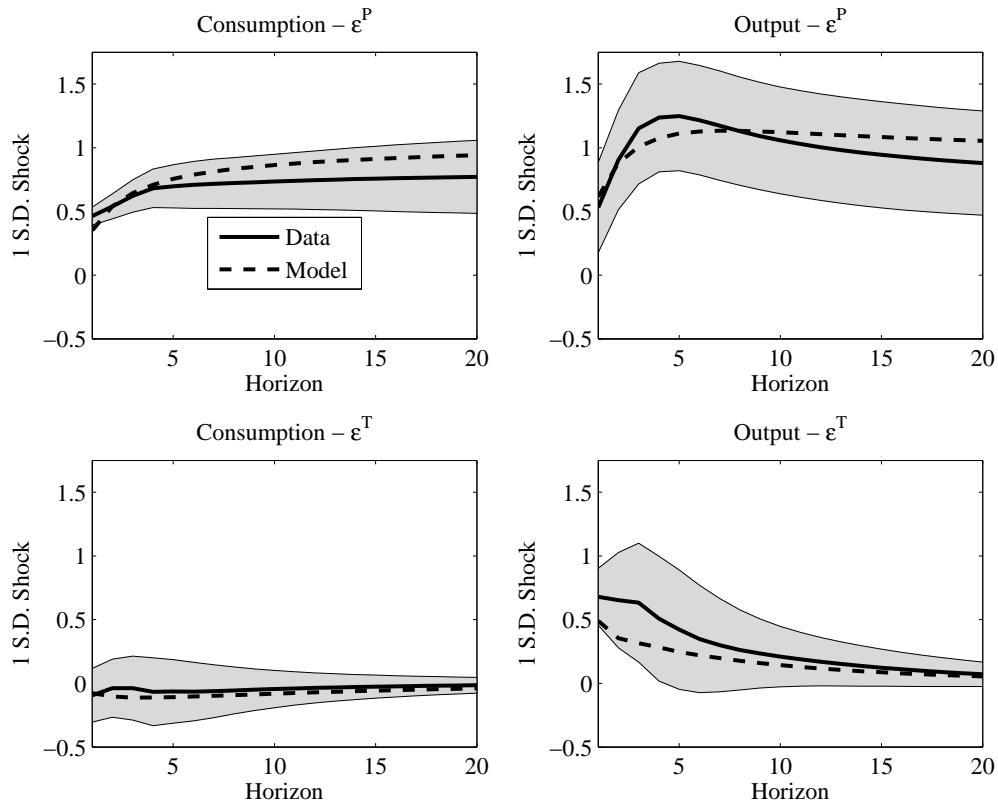
This figure compares the responses of consumption and output to permanent and transitory shocks (long run orthogonalization scheme), as estimated from the data (continuous line) and from model simulated data (dashed line). More precisely, the dashed line is the average over 1000 replications of the model simulation, VECM estimation and orthogonalization. The shaded area represents the 95% confidence intervals obtained from 1000 bootstraps of the VECM estimated with actual data.

Figure 5: Impulse Response Functions: Data versus Model (Short Run Orthogonalization Scheme)
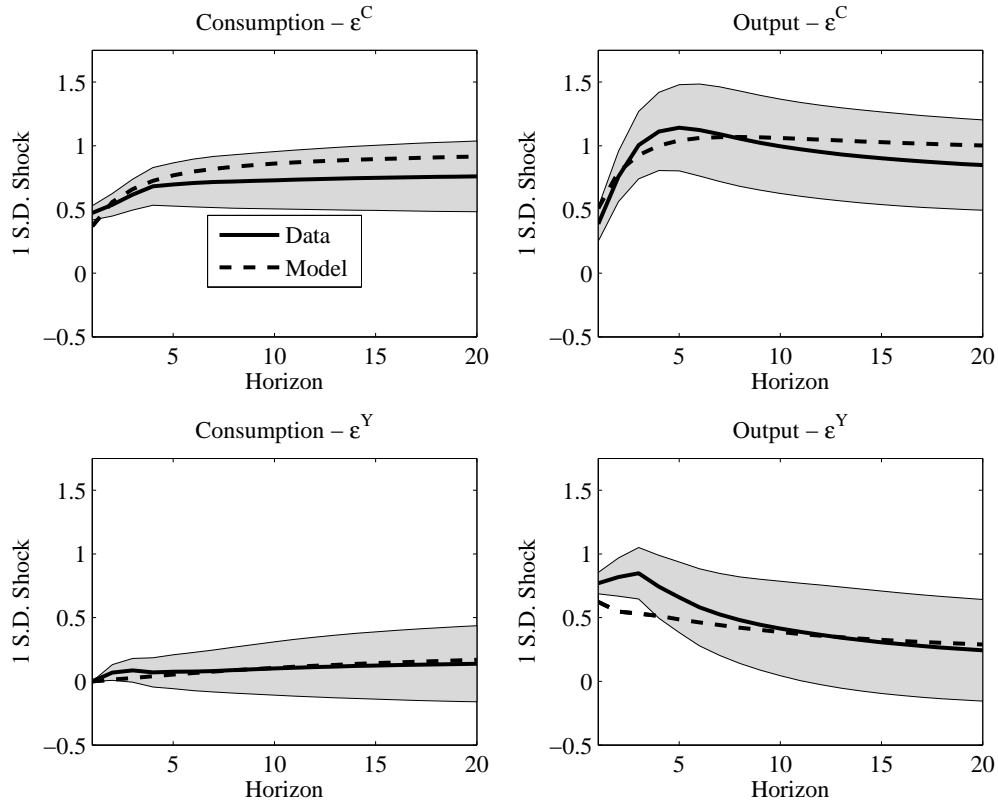
This figure compares the responses of consumption and output to consumption and output shocks (short run orthogonalization scheme), as estimated from the data (continuous line) and from model simulated data (dashed line). More precisely, the dashed line is the average over 1000 replications of the model simulation, VECM estimation and orthogonalization. The shaded area represents the 95% confidence intervals obtained from 1000 bootstraps of the VECM estimated with actual data.

Figure 6: Theoretical Impulse Response Functions (Long Run versus Short Run)
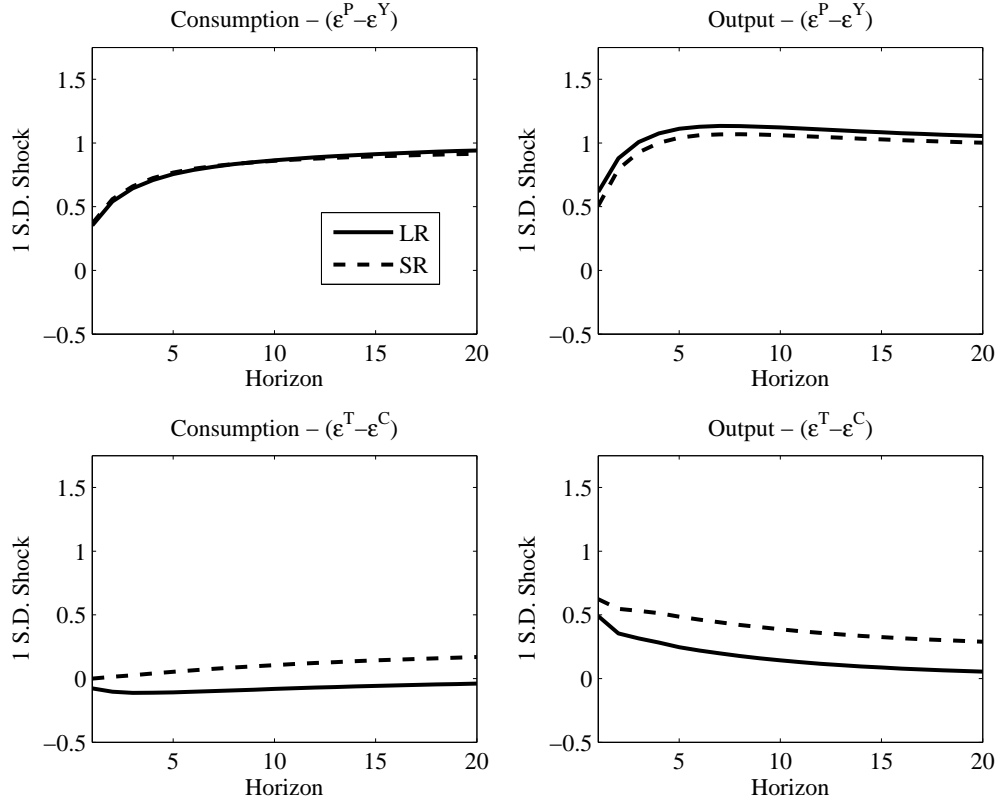
This figure compares the responses of consumption and output to a permanent (continuous line, labeled LR for Long Run) or consumption shock (dashed line, labeled SR for Short Run) and to a transitory (continuous line) or output shock (dashed line), as estimated from model simulated data. More precisely, each line is the average over 1000 replications of the model simulation, VECM estimation and short or long run orthogonalization.

Table 5: Variance Decomposition of Hours Worked

	Data			Model		
Horizon	ε^{P}	ε^{T}	ε^{H}	ε^{P}	ε^{T}	ε^{H}
1	19 %	75 %	6 %	35 %	65 %	0%
4	37 %	56 %	7 %	19 %	81 %	0%
8	61 %	32 %	7 %	24 %	76 %	0 %
20	60 %	21 %	19 %	28 %	72 %	0 %
40	54 %	20 %	26 %	28 %	72 %	0 %

This table shows the k-period ahead share of the forecast error variance of hours worked to the temporary ε^T, the permanent ε^P and the hours specific shock ε^H, as estimated from the data and from the simulated data. Those shares are computed using a two-step procedure. First ε^T and ε^P are derived from the estimation of a VECM (C, Y) with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4 for the actual data and 232 periods for simulated data. Then hours worked (in levels or difference) depending on the specification) are projected on current and past values of those innovations plus a moving average term in ε^H. In the case of the model, those numbers are averages over the 20 replications used during the estimation process.

decomposition of fluctuations as reported in Table 6.

As expected from the estimation results, the market shock that exerts a permanent effect on output does not contribute to the dynamics of the model. Indeed, the market shock that has no impact on productivity, ε^x, accounts for more than one third of output volatility[17] and about 85% of hours worked on impact. On the contrary, and as expected, consumption is almost solely explained by the permanent technology shock.

Figures 7, 8 and 9 show the theoretical responses of the main variables of the model to the structural shocks. Note that responses are qualitatively different from one shock to another, which allows for a proper identification of the contribution of each of them. The responses to the permanent shock (Figure 7) display comovements of investment, output and consumption, as well as a negative initial response of hours, explained by the role of habit persistence in consumption. The response to the unproductive market shock (Figure 8) displays a boom in output, hours worked and investment in the short–run, while consumption hardly responds. It is clear from this figure that such a shock is likely to contribute to the identified transitory shock of the (C, Y) VECM. The market shock drives the discounted sum of expected future profits upwards, making it worthwhile to invest in startups. This creates a boom in total investment, and also leads firms to raise their demand for

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[17]It is worth noting that the response of output to the transitory shock hinges on the response of all components of investment except residential investment (see the technical appendix). We take this observation as an additional fact in favor of our story in which entrepreneurs investment play a major role.

Table 6: Contribution of Shocks to the Business Cycle in the Estimated Model

Horizon	Output			Consumption			Hours		
	ε^Θ	ε^x	ε^z	ε^Θ	ε^x	ε^z	ε^Θ	ε^x	ε^z
1	64 %	36 %	0 %	94 %	6 %	0 %	15 %	85 %	0%
4	86 %	14 %	0 %	95 %	5 %	0 %	19 %	81 %	0%
8	92 %	8 %	0 %	96 %	4 %	0 %	32 %	68 %	0%
20	96 %	3 %	1 %	98 %	1 %	1 %	40 %	59 %	1%
∞	96 %	0 %	4 %	96 %	0 %	4 %	41 %	57 %	2%

This table reports the forecast error variance decomposition of consumption, output and hours worked when the estimated model is used as the forecasting model.

Figure 7: Model Response to a Permanent Technology Shock ε^Θ
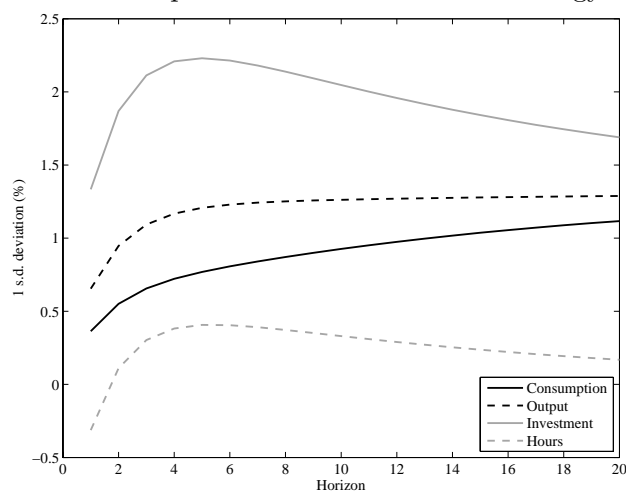
This figure displays the responses of consumption, total investment, hours worked and output to a technology innovation of one standard-deviation, as computed from the estimated model.

labor. Therefore output increases — creating an expansion without any changes in productivity. Furthermore, as this shock essentially creates a competition for rents, it is unproductive and consumption almost does not respond. The response to the productive market shock (Figure 9) is quite different, which again allows for a proper identification. First the shock exerts a permanent effect on consumption, investment and output. Second, output does not move much in the short run, which makes this shock unlikely to contribute to the temporary shock of the VECM.

Figure 8: Model Response to a Non-Productive Market Shock ε^x
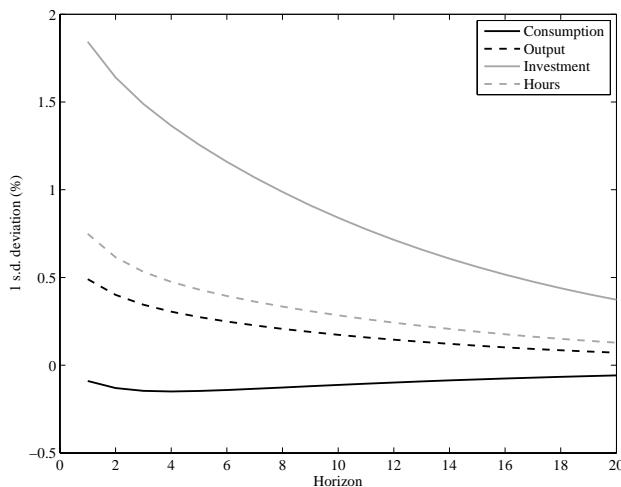
This figure displays the response of consumption, total investment, hours worked and output to a non-productive market shock of one standard-deviation, as computed from the estimated model.

Table 7 displays some statistics of the Hodrick-Prescot (HP) filtered simulated series. As can be seen from the table, the model performs well in matching the ranking of volatilities and the comovements of the US business cycle, although investment and hours appear to be not volatile enough.

3.4 Alternative Models

In this section we explore the robustness of the result of the previous section which indicated that non–productive market shocks (market rushes) may be an significant source of macroeconomic fluctuations. Our approach is to add alternative shocks to our model, one at a time, re–estimate it and then see whether the resulting structural variance decompositions substantially change our estimates of the relative contribution of market rushes for output, consumption and hours fluctuations. We consider four types of shocks.[18] First we investigate the implications of adding an

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[18]Obviously we do not claim to have exhausted the list of possible explanations, but want to point out what we think are the four more widespread explanations in the profession, as we can identify them from introspection and

Figure 9: Model Response to a Productive Market Shock ε^z

This figure displays the response of consumption, total investment, hours worked and output to a productive varities innovation of one standard-deviation, as computed from the estimated model.
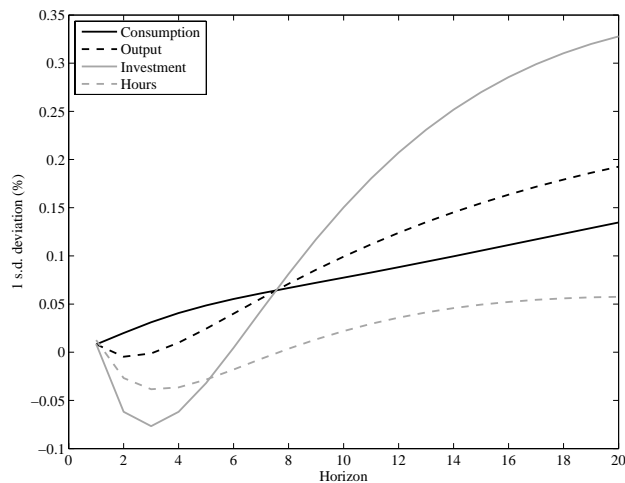
Table 7: Some Moments of the Simulated Data (HP–filtered)

	Data			Model		
	σ_x	$\rho(\cdot, y)$	$\rho(\cdot, h)$	σ_x	$\rho(\cdot, y)$	$\rho(\cdot, h)$
y	1.69	–	–	1.44	–	–
c	0.78	0.78	–	0.78	0.79	–
I Total	6.81	0.86	–	3.56	0.95	–
h	1.90	0.88	–	1.12	0.66	–
y/h	0.87	0.50	0.09	1.09	0.64	-0.14

This table reports moments calculed both model generated data and actual data. Series are taken in logs and detrended applying the HP–filter with smoothing parameter $\lambda = 1600$.

investment specific shock in our model. We examine this case in considerable detail since it has received substantial attention in the literature and we recognize that the omission of investment specific shocks could create a substantial bias in favor of finding that market rushes are important even if they are not. The two other real shocks we consider are a temporary total factor productivity shock and a preference shock. A fourth possibility involves examining the role of a monetary shock in a New–Keynesian type of model. As this fourth possibility requires writing a substantially different model, we leave the exposition of the model and results to the technical appendix. As can be seen in the appendix, we do not find that allowing a monetary shock to compete with our market rush shock changes our findings regarding the relevance of market rushes for fluctuations.

One simplification we make in analyzing these cases is that we remove the Z shock (the market shock with productivity effects), and thereby focus on models with three shocks: a permanent TFP shock, a non–productive market shock and a third shock which can be either an investment specific shock, a temporary TFP shock or a preference shock. The omission of the Z shock appears reasonable since we saw in the last section that it is not playing any substantial role in fluctuations. The production function therefore reduces to[19]

$$Q_t = K_t^{1-\alpha_x-\alpha_h}(\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left(\int_0^{N_{x,t}} X_t(i)^\chi \mathrm{d}i \right)^{\frac{\alpha_x}{\chi}}.$$

3.4.1   Investment Specific Shock

The first shock we consider is an investment specific shock *à la* Greenwood, Hercowitz and Krussell [2000] or Fisher [2002]. We modify the resource constraint, which is now given by

$$Y_t = C_t + S_t + e^{-\zeta_t}I_t,$$

where ζ is the investment specific shock. In his VAR study, Fisher [2002] argues that investment specific shocks ought to exert a permanent effect and should be modeled as random walk. We will consider both the case where the investment specific shock is modeled as a random walk and the case where it is modeled as a stationary autoregressive process.

We first estimate a model with a permanent investment specific shock, our market shock and a permanent TFP shock. As a first pass, the model is estimated by matching the response of output to both the permanent and the transitory component, as was done in the previous section. This experiment is labeled PIS–1 in Table 8 which reports the estimation results and in Table 9 which reports the associated variance decomposition. In this version the model fits the data very

comments during seminars.

[19]An implication of this simplification is that the elasticity of output with respect to the intermediate good, α_x, is now greater as it is still calibrated to match on the share of intermediate goods in output. This implies a mechanical reduction in the volatility of the market shock η_t^x (see footnote 14).

well. The model does relatively well in accounting for the joint dynamics of consumption and output (as indicated by the value of the $\mathcal{D}(C, Y)$). What is interesting is that the volatility of the permanent investment specific shock is estimated to be very close to zero. Its contribution to output, consumption and hours worked volatility is essentially zero. In contrast, our market shock remains a very important component in explaining hours and output.

We replicate the estimation of this model by now matching both output and consumption responses, instead of only matching the output response. This experiment is labeled PIS–2 in the tables. The model is not rejected by the data (p–value=0.86), and the volatility of the investment specific shock is now close to that usually obtained by fitting the relative price of investment. Nevertheless, the variance decomposition reported in Table 9 shows that the investment specific shock is not an important source of business cycle fluctuations, while the market shock remains important. These results suggest that our market shock is not simply picking up some component of output and consumption fluctuations that is induced by permanent investment specific shocks.

These pessimistic results regarding the contribution of the investment specific shock to fluctuations may come from its specification as a random walk. We therefore also consider a stationary representation of the investment specific shock. The TIS–1 experiment then corresponds to a situation where the model parameters are estimated so as to match the impulse responses of output. One difficulty with this model is that it estimates a volatility for the innovation to the investment specific shock that is about three times as high as in the data. The interesting result for our conjecture, as shown in Table 9, is that the investment specific shock does not undermine the contribution of the market shock to accounting for the business cycle. The market shock still accounts for 42% of output volatility in the short–run while the investment specific shock only accounts for less than 5% of the output volatility at the same horizon. Furthermore, the same invariance result obtains for hours worked and for consumption.

One may however be worried that the investment specific shock may not be well identified by this procedure since it considers only output responses at the estimation stage. We therefore add the impulse responses of consumption to our list of moments to match so as to add information to the system. This experiment is labeled TIS–2 in the tables. This experiment still gives some strong support to the model (p–value=0.87). Now the estimated process of the investment specific shock is very much in line with what would result from an estimation from the relative price of investment series. The key results in terms of variance decomposition are left unaffected. The market shock still accounts for 40% of output volatility in the short–run while the investment specific shock only accounts for less than 3%. Likewise hours worked are mainly explained by the market shock (94% on impact), while consumption is only explained by the technological shock. In other words, the investment specific shock appears to explain little of economic fluctuations when one allows for our market shock, while the market shock continues to explain a substantial fraction of fluctuations

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when the investment specific shock is included. [20]

Table 8: Estimation Results

	PIS–1	PIS–2	TIS–1	TIS–2	T.T.	T.P.
b	0.6108	0.3125	0.6457	0.3062	0.3420	0.3877
	(0.1229)	(0.1921)	(0.1180)	(0.2184)	(0.1869)	(0.1472)
φ	0.4195	0.2534	0.6099	0.2775	0.3125	0.3699
	(0.3227)	(0.3201)	(0.6675)	(0.4235)	(0.2645)	(0.3228)
σ_Θ	0.0131	0.0088	0.0126	0.0089	0.0062	0.0075
	(0.0017)	(0.1592)	(0.0017)	(0.0016)	(0.0044)	(0.0037)
ρ_x	0.9117	0.8919	0.9143	0.8967	0.9195	0.9075
	(0.0323)	(0.0395)	(0.0374)	(0.0420)	(0.0234)	(0.0259)
σ_x	0.1575	0.1859	0.1594	0.1775	0.1768	0.1825
	(0.0217)	(0.0349)	(0.0197)	(0.0266)	(0.0278)	(0.0297)
ρ_T	$-$	$-$	0.5328	0.8478	0.9143	0.8799
			(0.2742)	(0.4974)	(0.1148)	(0.1959)
σ_T	0.0003	0.0038	0.0118	0.0032	0.0046	0.0068
	(0.0243)	(0.0082)	(0.0137)	(0.0048)	(0.0021)	(0.0030)
J–stat	17.31	60.96	14.89	59.48	54.65	50.56
	[0.99]	[0.86]	[1.00])	[0.87]	[0.95]	[0.98]
$\mathcal{D}(C,Y)$	99.42		92.34			
	[0.03]		[0.06])			

Note: ρ_T and σ_T denote respectively the persistence parameter and the volatility of the third shock. Standard errors are in parenthesis, p–value in brackets.

We further pursue our experiments with investment specific shocks by examining a model in which the volatility of the market shock is constrained to be null and where only a permanent investment specific shock is introduced to complement the permanent technology shock. The model is again estimated by the simulated minimum distance estimation. In order to maximize the chances of this alternative model to match the facts, the estimation aims at matching the impulses response functions of both consumption and output over the first twenty quarters. We find that this model is unable to fit the data very well (J–stat=271.4, p–value=0.00). This may actually come as no surprise as the wealth effect of a permanent investment specific shock generates a strong reaction of consumption in the short run.[21] As a alternative, we also evaluate a model is which there is no

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[20]At first pass, these observations may appear at odds with the results in Fisher [2002] which suggest, using an identified VAR approach, that investment specific shocks explain a substantial fraction of fluctuations. One way to reconcile these two sets of observations is to recognize that the investment specific shocks identified in Fisher [2002] may be capturing part of the effects we associate with market rushes. In particular, if changes in the quality of equipment proceed and signal the opening of new markets, then the shocks identified in Fisher [2002] could be generating macroeconomic fluctuations mainly through their effect on market rushes as opposed to working through standard capital accumulation incentives.

[21]In the technical appendix to this paper, we present an analytic model with investment specific shocks and show that it cannot reproduce the four facts we have highlighted in the first section, regardless of the assumption that one

market shock and in which the investment specific shock is assumed to be stationary. The estimated the model is again not well supported by the data (J–stat=99.3, p–value=0.03). More worrisome is that the investment specific shock is estimated to be about seven times more volatile than its empirical counterpart.[22] As an final strategy, we impose the process for this shock as estimated from the series of the relative price of investment. When we do so, the model obviously does not fit the data very well, but the match is not overwhelmingly rejected by the data either (J–stat=106.1, p–value=0.02). When evaluating the role of investment specific shocks in the resulting estimated model, we find that they now can account for a substantial fraction of fluctuations in the very short run. Hence, we conclude this subsection by recognizing that a temporary investment shock could be responsible for a substantial fraction of output fluctuations, but that such an explanation does not appear robust to the inclusion of our market shock.

3.4.2 Transitory Technological Shock

We now consider a version of the model in which we allow for a temporary shock to total factor productivity in addition to a permanent TFP shock and our market shock. Technology is now given by

$$Q_t = e^{\zeta_t} K_t^{1-\alpha_x-\alpha_h} (\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left(\int_0^{N_{x,t}} X_t(i)^{\chi} \mathrm{d}i \right)^{\frac{\alpha_x}{\chi}},$$

where ζ is a stationary AR(1) autoregressive process. The parameters are estimated so as to match the impulse responses of consumption and output. The experiment is labelled T.T. in the tables. As can be seen from Table 9, the introduction of this shock does not undermine the contribution of the market shock to output volatility in the business cycle. The market shock still account for about 38% of output volatility in the short–run and more than 95% of that of hours worked. In fact, the transitory technology shock acts as a substitute for the permanent technology shock in explaining fluctuations. This can be seen from the variance decomposition of consumption that clearly shows that consumption is mainly accounted for by technology shocks — about 93% in the short–run — and that the split between the two shocks is about half–half. This can be explained from the estimated AR(1) process of the technology shock that clearly shows that the persistence of this shock is high ($\rho_T = 0.91$). Hence, it exerts a strong wealth effect on the consumption decision and it is not surprising that this shock competes with the permanent TFP one.

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makes about the stationarity or not of the two technological disturbances. This impossibility result in an admittedly specific model translates into rejection in the richer model we are studying here.

[22]When an AR(1) is fitted on the HP–filtered relative price of investment — the conventional measure of the investment specific shock — we obtain a persistence parameter of 0.8444 and a volatility of the innovation process of 0.0042. This series is the "total investment" deflator used in Fisher [2002]. We thank Jonas Fisher for kindly providing us with this series.

Table 9: Variance Decomposition

Horizon	Output			Consumption			Hours		
	ε^Θ	ν^x	ζ	ε^Θ	ν^x	ζ	ε^Θ	ν^x	ζ
PIS–1: ζ=Permanent Investment Specific Shock									
1	64 %	36 %	0 %	95 %	5 %	0 %	15 %	85 %	0 %
4	87 %	13 %	0 %	96 %	4 %	0 %	20 %	80 %	0 %
8	93 %	7 %	0 %	97 %	3 %	0 %	34 %	66 %	0 %
20	97 %	3 %	0 %	99 %	1 %	0 %	42 %	58 %	0 %
∞	100 %	0 %	0 %	100 %	0 %	0 %	44 %	56 %	0 %
PIS–2: ζ=Permanent Investment Specific Shock									
1	55 %	45 %	0 %	84 %	16 %	0 %	0 %	99 %	1 %
4	76 %	22 %	2 %	87 %	13 %	0 %	12 %	82 %	6 %
8	84 %	13 %	3 %	90 %	10 %	0 %	19 %	72 %	9 %
20	90 %	6 %	4 %	95 %	4 %	1 %	24 %	64 %	11 %
∞	96 %	0 %	4 %	96 %	0 %	4 %	26 %	63 %	11 %
TIS–1: ζ=Temporary Investment Specific Shock									
1	53 %	42 %	5 %	93 %	6 %	1 %	19 %	73 %	8 %
4	76 %	15 %	9 %	95 %	5 %	0 %	12 %	60 %	28 %
8	87 %	9 %	4 %	96 %	4 %	0 %	23 %	57 %	20 %
20	95 %	4 %	1 %	98 %	2 %	0 %	32 %	52 %	16 %
∞	100 %	0 %	0 %	100 %	0 %	0 %	34 %	50 %	16 %
TIS–2: ζ=Temporary Investment Specific Shock									
1	56 %	42 %	2 %	84 %	15 %	1 %	0 %	94 %	6 %
4	75 %	20 %	5 %	87 %	12 %	1 %	11 %	73 %	16 %
8	83 %	13 %	4 %	91 %	8 %	1 %	19 %	67 %	14 %
20	92 %	6 %	2 %	96 %	3 %	1 %	25 %	63 %	12 %
∞	100 %	0 %	0 %	100 %	0 %	0 %	26 %	62 %	12 %
T.T.: ζ=Temporary Technology Shock									
1	21 %	38 %	41 %	44 %	17 %	39 %	0 %	98 %	2 %
4	29 %	20 %	50 %	51 %	17 %	32 %	4 %	72 %	23 %
8	37 %	15 %	48 %	57 %	13 %	30 %	7 %	66 %	27 %
20	54 %	10 %	36 %	69 %	6 %	25 %	9 %	68 %	23 %
∞	99 %	0 %	0 %	100 %	0 %	0 %	10 %	66 %	24 %
T.P.: ζ=Temporary Preference Shock									
1	27 %	39 %	34 %	55 %	15 %	30 %	1 %	53 %	46 %
4	40 %	20 %	40 %	64 %	13 %	23 %	3 %	34 %	63 %
8	50 %	14 %	36 %	71 %	10 %	19 %	5 %	31 %	64 %
20	70 %	8 %	22 %	82 %	5 %	13 %	7 %	33 %	60 %
∞	100 %	0 %	0 %	100 %	0 %	0 %	8 %	33 %	59 %

3.4.3 Transitory Preference Shock

We now introduce a transitory preference shock that shifts the labor supply. Preferences are now given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left[\log(C_{t+\tau} - bC_{t+\tau-1}) + \psi e^{\zeta_{t+\tau}} (\overline{h} - h_{t+\tau}) \right],$$

where ζ is the temporary preference shock, that is assumed to follow a AR(1) process. The model is estimated so as to match the impulse responses of consumption and output. Again we find that the market shock accounts for about 40% of output volatility in the short–run and does not contribute much to the volatility of consumption (less than 15%). As far as output and consumption are concerned, the introduction of the preference shock mainly undermines the role of the technology shock. For instance, out of the 61% of output volatility not explained by the market shock about 35% is accounted for by the preference shock. The only dimension along which the preference shock competes with the market shock is in the determination of hours worked. About half of hours worked volatility can be accounted for by the preference shock. We however do not view it as seriously calling into question the potential role of the market shock in the business cycle, as the preference shock mainly reduces the explanatory power of the permanent shock rather than undermining the role of the market shock.

3.5 Discussion

One important question relates to the interpretation of "a new market" and the associated empirical observations with regards to its cyclical properties.[23] Our metaphor of new markets describes all new ways of introducing new products given existing technology or using new technologies, although our estimations seem to favor the former interpretation rather than the later, as we do not estimate any significant long run effect of new goods creation. Broadly speaking, a new market ranges from producing a newly invented product (say cellular phones) to producing old goods with newly developed uses (fiber–optic cable networks once the use of the internet has exploded) or new ways of designing old products (say producing shirts of a fashionable new color). Given this broad interpretation, it is difficult to obtain a comprehensive measure of our new market margin. In a very narrow sense, one could associate new markets with new firms, and therefore look at Net Business Formation. Net Business Formation is without ambiguity procyclical in the U.S., which is also one of our model predictions if we literally associate N with the number of firms. The problem is that the evidence suggests that smaller firms typically make up the majority of entrants and exits, which is insufficient to account for a large share of hours worked and output variance at short horizons. A less restrictive interpretation is to look at variations in the number of establishments and franchises

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[23]We have here benefitted from comments and discussion with Nir Jaimovich.

as an additional channel affecting the number of "operating units". The Business Employment Dynamics database documents job gains and job losses at the establishments level at the quarterly frequency for the period between the third quarter of 1992 and the second quarter of 2005. Using these observations, Jaimovich [2004] finds that more than 20% of the cyclical fluctuations in job creation is accounted for by opening establishments, which is already a sizable number. Another dimension which could be associated to the new market margin is variation in the number of franchises. As Lafontaine and Blair [2005] show, numerous firms in a variety of industries have adopted franchising as a method of operation. Sales of goods and services through the franchising format amounted to more than 13% of real Gross Domestic Product in the 1980s and 34% of retail sales in 1986. Jaimovich [2004] documents that the variations in the number of franchises are procyclical at the business cycle frequency, which is again in line with the predictions of our model. We take this empirical evidence, together with anecdotal evidence and the evidence obtained by estimating our model, as supporting the idea that agents expectations about the possibility of new markets is likely an important driving force of the business cycle.

4   Conclusion

This paper explores whether business cycles fluctuations may sometimes be driven by expectations of new market openings. We first reviewed a set of observations showing that the business cycle is, to a large extent, associated with a non-permanent effect on output and hours and that does not move consumption at all. We then propose a structural interpretation to that shock, in a model where the opening of new market opportunities causes an economic expansion by favoring competition for market share. We call such an episode a market rush. We study a simple analytical model that clearly displays the important qualitative features of the data we have previously highlighted. Then, a quantitative model built along the same lines and estimated by a simulated method of moments is exposed. The model suggests that market rush phenomenon is a significant contributor to business cycle fluctuations and this is robust to the inclusion of many alternative shocks discussed in the literature.

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