Which Structural Parameters Are "Structural"? Identifying the Sources of Instabilities in Economic Models

Atsushi Inoue and Barbara Rossi

University of British Columbia and Duke University

November 26, 2007

Abstract

The objective of this paper is to identify which parameters of a model are stable over time. Existing works only test whether a *given* subset of parameters is stable, but cannot be used to find *which* subset of parameters is stable. We propose a new procedure that is informative on the nature of instabilities affecting macroeconomic models, and sheds light on the economic interpretation and causes of such instabilities. Furthermore, our procedure provides clear guidelines on which parts of the model are reliable for policy analysis and which are possibly misspecified. Our empirical findings suggest that instabilities during the Great Moderation were mainly concentrated in Euler and IS equations as well as the monetary policy reaction function. Such results offer important insights to guide the future theoretical development of macroeconomic models.

PRELIMINARY AND INCOMPLETE

Keywords: Structural Change, Model Evaluation, Information Criteria, Great Moderation.

Acknowledgments: We are grateful to seminar participants at the Empirical Macro Study Group at Duke University, Princeton University, the Federal Reserve Banks fo Phildelphia, St. Louis Fed, and San Francisco, the University of California, Irvine, the University of British Columbia, and the University of Washington.

J.E.L. Codes: C22, C52, C53

1 Introduction

The objective of this paper is to identify which parameters of a model are stable over time. This is an important question especially in the context of structural models, as one of their main advantages is to offer a framework to qualitatively evaluate the effects of alternative economic policies without being subject to the Lucas' critique. However, such experiments make sense only if the parameters of the model are constant over time: parameter instabilities are a signal of possible model misspecification, which, for example, remains a concern for researchers estimating Dynamic Stochastic General Equilibrium (DSGE) models (see Del Negro, Schorfheide, Smets and Wouters, 2004). If neglected, such instabilities render the structural model untrustworthy for evaluating the consequences of alternative policies. But, among the components of the rich structure of a model, which are the stable and which are the unstable components?[1] In other words: "Which parameters are stable?". Our paper provides an answer to this important question. Such answer provides important information for both empirical and theoretical researchers regarding which parts of the model rely on stable parameters and which parts don't. The former are exactly the features of the model that policy-makers can rely upon when doing policy evaluation, and the latter are those that could possibly be mis-specified and therefore require further theoretical modeling efforts.

One of the contributions of this paper is to consider both a vector autoregression (VAR) and a representative New Keynesian model that has the basic features of the models now becoming popular in central banks and academia, and identify their set of stable parameters during the Great Moderation. Our empirical results strikingly show that parameters in the Euler and IS equations as well as those in the monetary policy reaction function are not stable. There is less evidence of instability in the Phillips curve, although this result turns out to depend on the model used for the analysis. While the instability in the parameters in the monetary policy reaction function is a well-known fact, the finding that instabilities also affect the parameters in the Euler and IS equations is new. These results provides important and useful guidelines for future research on modeling structural macroeconomic models. Our results also contribute to the debate on the Great Moderation by showing that, in a standard structural VAR, instabilities are concentrated both in the transmission mechanism (in particular, the coefficients in the monetary policy reaction function), as well as the variance of the shock to output growth. Therefore, overall, our empirical findings

[1] This question was raised by both Sims (2001) and Stock (2001) in their comment to the Cogley and Sargent (2001) paper, and eventually addressed by Cogley and Sargent (2005).

suggest that not only the variance of the unpredictable shocks has decreased during the Great Moderation (as in Stock and Watson, 2002), but that the transmission mechanism has changed as well (similarly to Boivin and Giannoni, 2006). Our findings reconcile results obtained in previous empirical work by using descriptive counterfactual VAR techniques, while, at the same time, being informative on which specific parameters significantly changed.

Since there are no appropriate econometric techniques to address the issue considered in this paper, another substantial contribution of this paper is to propose a new methodology for identifying the subset of parameters of a model that are stable among the set of the model's parameters. Our methodology can be applied to both reduced form and structural models. It has the advantage of identifying which "blocks" of the model contain parameters that are "time invariant", and which are not. It therefore provides directions as to which parts of the model should be modified in order to build a structural model whose parameter are time-invariant. Two such techniques are discussed. A first technique, which we refer to as "ESS" procedure ("Estimate of the Set of Stable parameters"), estimates the set of stable parameters by taking into account estimation uncertainty; the second, which we call the "ICS" procedure ("Information Criterion for the set of Stable parameters"), provides a set of parameters that contains the stable ones with probability one. The advantage of the ESS procedure relative to the ICS procedure is that it provides researchers with an estimate of the set of stable parameters that equals the true set of stable parameters with a pre-specified probability level.[2] We recognize, though, that in some situation this may be undesirable, and we offer the ICS procedure, which identifies the set of stable parameters with probability approaching one asymptotically. By construction, the ICS procedure is less powerful than the ESS procedure.

Many researchers have now realized the importance of identifying which parameters are time-varying among a set of parameters. In fact, there is considerable interest in estimating structural macroeconomic models with time-varying parameters (see, among others, Owyang and Ramey (2004), Cogley and Sargent (2005), Primiceri (2005), Boivin and Giannoni (2006), Li (2006), Fernandez-Villaverde and Rubio-Ramirez (2007), and Justiniano and Primiceri (2007)); in testing for structural breaks in macroeconomic data (see among others Gurkaynak et al. (2005), Fernald (2007) and Ireland (2001)); and in interpreting the causes

[2] Note that, with some probability (that is controlled by the size of the test) it is possible that a parameter is stable and it is not included in the set of stable parameters. There is nothing worrisome in this, as it is exactly what would happen in a standard hypothesis testing procedure, where with some pre-specified probability the null is rejected even if true.

of the time variation in macroeconomic aggregates (for example, the Great Moderation phenomenon) by relating it to parameter changes in the structural model (Stock and Watson (2002, 2003), and Cogley and Sargent (2001, 2005)). In particular, Cogley and Sbordone (2005) investigate the stability of the estimated parameters of a Phillips curve relationship in the face of changes elsewhere in the economy, and Ireland (2001) attempts to identify which parameters have been subject to breaks by applying standard structural break tests to each of the parameters separately. However, when used repeatedly to test structural change in more than one subset of parameters, such tests lead to size distortions in the overall procedure. Fernandez-Villaverde and Rubio-Ramirez (2007) also question whether the parameters of a representative New Keynesian model are invariant to shifts in monetary policy by allowing some of the parameters to shift. As pointed out by Cogley (2007), however, their results are difficult to interpret because they evaluate which parameters are unstable by looking at subsets of such parameters changing "one-at-a-time".[3] Similarly, in the effort of shedding light on the causes of the Great Moderation, Stock and Watson (2002) allow either the parameters in the monetary policy reaction function and/or the variance of the shocks to change over time, while keeping constant the slope of the Phillips curve and the slope of the IS equation. But what if the latter had changed as well? The advantage of our procedure relative to this literature is to provide a tool that can be used by researchers in exactly those situations and does not rely on a "one-at-a-time", ad-hoc approach.[4] From a technical point of view, the method proposed in this paper is related to recent contributions in the econometric literature on structural break tests. Andrews (1993), Andrews and Ploberger (1994) and Nyblom (1989) propose tests for structural breaks in the parameters, but their tests are for a null hypothesis on a specific subset of the parameters. Our paper instead allows the researcher to search for the subset of parameters that do not have structural breaks. Our procedure is also more distantly related to the literature on sequential model selection and

[3] Fernandez-Villaverde and Rubio-Ramirez (2007) allowed only a subset of the parameters at a time to be time-varying. Cogley's (2007) criticism is that it is difficult to interpret their results, as we don't know which of the parameters really changed and therefore the maintained assumptions in the "one-at-a-time" approach might make inference invalid. However, if one tried to allow all the parameters to be time-varying and repeatedly use structural break tests to identify which parameters are time-varying, one would incur into size distortions, as we discuss in this paper. Estrella and Fuhrer (1999) also attempt to interpret results of structural breaks in a joint system of New Keynesian equations versus results of structural breaks in only the monetary policy block by using the "one-at-a-time" approach criticized by Cogley (2007).

[4] A similar question to the one addressed in this paper is considered by Del Negro and Schorfheide (2007). They quantify the degree of misspecification of macroeconomic models due to time-varying parameters, and provide a diagnostic tool that allows researchers to parameterize the discrepancies between theory and data.

hypotheses testing, in particular the works by Hansen et al. (2005) and Pantula (1989).

The paper is organized as follows. The next section provides an overview of the new techniques that we propose, and Sections 3 presents a small Monte Carlo Analysis. Sections 4 and 5 present the main empirical results of the paper, and Section 6 concludes.

2 The econometric procedures

This section presents our econometric procedure to estimate the set of stable parameters. We will show that our testing procedure controls size, is consistent and provides an estimate of the set of stable parameters that contains the true set with a pre-specified probability. The Appendix compares our procedure to a naïve procedure based on discarding parameters when their individual tests for parameter stability reject the null, and shows that such naïve procedure leads to size distortions.

2.1 Notation and definitions

Consider a general parametric model with parameters $\theta_t = (\beta_t, \delta) \in \Theta \subseteq \mathbb{R}^{p+q}$ for $t = 1, 2, ...,$ where $\beta_t \in B \subseteq \mathbb{R}^p$ and $\delta \in D \subseteq \mathbb{R}^q$. Let the parameters β_t be time-varying, and the parameters δ be stable. To formalize the problem, let $s \in \{0,1\}^{p+q}$ denote a parameter selection vector and $\theta(s)$ denotes a subset of θ selected by the selection vector s, where s_i denotes the i-th element of such vector.[5] We also let s^* denote the population selection vector that selects only the constant parameters: $\theta(s^*) = \delta$. Note that it is possible that s^* is the vector of ones, in which case all parameters belong to the stable confidence set, or s^* is the vector of zeros, in which case none of the parameters belongs to the stable confidence set. The problem considered in this paper is that it is not known which parameters are time-varying and which are stable. In other words, s^* is unknown. We will propose a sequential procedure that uses sample information to estimate s^* by an estimator $\widehat{s} \in \{0,1\}^{p+q}$. With our procedure, the estimator $\widehat{s}$ will be equal to s^* with a pre-specified probability level.

Let the observed sample be $W = \{W_t : 1 \leq t \leq T\}$ and $\mathcal{T}_T(W; s)$ be a size-α, consistent test statistic for testing the null hypothesis that the parameters $\theta(s)$ are constant over time: $H_0(s) : \theta_t(s) = \theta(s)$ versus the alternative that the parameters $\theta(s)$ are time-varying.[6]

[5]For example, when $p + q = 3$, $s_0 = (0, 1, 0)$ indicates that the second element of θ is not time-varying and the first and third elements are time-varying in population.

[6]For example, in the case of a one-time structural break at an unknown fraction of the sample size $[\pi T]$: $H_A(s) : \theta_t(s) = \theta_1(s) \cdot 1(t \leq \pi T) + \theta_2(s) \cdot 1(t > \pi T)$, where $\pi \in \Pi \subset (0, 1)$.

For notational simplicity, we will omit the dependence of the test statistic on the observed sample, and use $\mathcal{T}_T(s)$. Also, let e_i be the $(p+q)\times 1$ vector whose i-th element is one and the other elements are zero, $\mathbf{1}_{(p+q)\times 1}$ be the $(p+q)\times 1$ vector of ones, $\mathbf{0}_{(p+q)\times 1}$ be the $(p+q)\times 1$ vector of zeros, and I_n denote the $(n\times n)$ identity matrix. Then, $\mathcal{T}_T(e_i)$ will denote the individual stability test on the i-th parameter $\theta(e_i)$, and $\mathcal{T}_T\left(\mathbf{1}_{(p+q)\times 1}\right)$ will denote the joint test for testing the null hypothesis that the all the parameters in θ are constant. Finally, let $k_\alpha(s)$ denote the size-α critical value of $\mathcal{T}_T(s)$, and $pv(s)$ denotes its p-value. For example, in the aforementioned case of a one-time structural break at an unknown time, when using Andrews' (1993) QLR test statistic, $k_\alpha(s)$ is the critical value of such test at level α for testing a number of restrictions equal to the number of nonzero elements in s.

In what follows, we will propose two procedures: the first is to construct an estimate of the set of stable parameters that equals the true set of stable parameters with a pre-specified probability level (which we call the "ESS procedure"), and the second is a method to estimate the set of stable parameters with probability approaching one asymptotically (which we call the "ICS procedure").

2.2 Estimating the set of stable parameters: the ESS procedure

We propose the following recursive procedure: first, test the joint null hypothesis that all parameters are stable. If the test does not reject, then all the parameters belong to the set of stable parameters. If it does, calculate the p-values of the individual test statistics for testing whether each of the parameters are stable.[7] Start by eliminating from the set of stable parameters the parameter with the lowest p-value, then test whether the remaining parameters are jointly stable.[8] If they are, then the set of stable parameters includes such parameters; otherwise, eliminate the parameter with the second lowest p-value from the set, and continue this procedure until the joint test on the remaining parameters does not reject stability: this will identify the set of constant parameters. We formalize the ESS ("Estimate of the Set of Stable parameters") procedure in the following Algorithm:

[7] It is important that such individual tests do not rely on the maintained assumption that the other parameters be constant over time. Therefore, the individual tests should be implementing by allowing all the other parameters to have a structural break.

[8] Alternative procedures could involve calculating the F-test for every subset of parameters and use that to choose which parameters to eliminate in the sequential procedure. However, this procedure is more computationally burdensome, especially when applied to the estimation of DSGE models, so we will not consider it here.

Algorithm 1 (The CIS procedure) *Step 0. Initially, let* $s_0 = \mathbf{1}_{(p+q)\times 1}$. *Test* $H_0^{(0)}(s_0)$ *against* $H_A^{(0)}(s_0)$ *at significance level* α *by using the test* $\mathcal{T}_T(s_0)$. *If the test does not reject, let* $\widehat{s}_{ESS} = s_0$. *If the test rejects, calculate the vector of test statistics* $\mathcal{T}_T(e_i)$ *for* $i = 1, ..., p+q$, *and order them such that to their p-values are increasing:* $pv(e_1) \leq pv(e_2) \leq ... \leq pv(e_{p+q})$. *Without loss of generality, let* e_1 *identify the parameter with the smallest p-value.*[9] *Continue to step 1.*

Step 1. Without loss of generality, let $s_1 = [0, \mathbf{1}_{1\times(p+q-1)}]'$. *Test* $H_0^{(1)}(s_1)$ *against* $H_A^{(1)}(s_1)$ *at significance level* α *by using* $\mathcal{T}_T(s_1)$. *If the test does not reject, let* $\widehat{s}_{ESS} = s_1$. *If the test rejects, let* e_2 *identify the parameter with the second smallest p-value, and continue to step 2.*

...

Step j. Without loss of generality,, let $s_j = [\mathbf{0}_{1\times j}, \mathbf{1}_{1\times(p+q-j)}]'$. *Test* $H_0^{(j)}(s_j)$ *against* $H_A^{(j)}(s_j)$ *at significance level* α *by using* $\mathcal{T}_T(s_j)$. *If the test does not reject, let* $\widehat{s}_{ESS} = s_j$. *If the test rejects, let* e_j *identify the parameter with the j-th smallest p-value, and continue to step (j+1).*

...

Step (p+q-1). Without loss of generality, let $s_{p+q-1} = [\mathbf{0}_{1\times(p+q-1)}, 1]'$. *Test* $H_0^{(p+q-1)}(s_{p+q-1})$ *against* $H_A^{(p+q-1)}(s_{p+q-1})$ *at significance level* α *by using* $\mathcal{T}_T(s_{p+q-1})$. *If the test does not reject, let* $\widehat{s}_{ESS} = s_{p+q-1}$. *If the test rejects, let* $\widehat{s}_{ESS} = \mathbf{0}_{(p+q)\times 1}$.

Appendix A shows that the algorithm provides an estimate of the set of stable parameters that equals the true set with probability $(1-\alpha)$. In words, $\widehat{s}_{ESS}$ defined in Algorithm 1 equals the true set of stable parameters with probability $(1-\alpha)$. Importantly, note that there are size distortions in existing tests for structural breaks when used repeatedly to test structural change in more than one subsets of parameters, in the sense that such tests would find a structural break eventually in one of the parameters with probability approaching one. Proposition (4) in Appendix A formally proves that this is the case.

[9] Actually, one does not even need to compute p-values: one may simply pick the estimated test statistic which is the largest. Because the degrees of freedom for testing each parameter individually are the same, and therefore the critical values are, the largest test statistic has the smallest p-value.

2.3 Consistent methods for estimating the set of stable parameters: the ICS procedure

We also consider a procedure that estimates the set of stable parameters with probability approaching one asymptotically (rather than with $(1-\alpha)$ probability level). The procedure is based on a simplified information criterion. While information criteria do not suffer from asymptotic size distortions and can be used to consistently estimate the set of stable parameters, they would be computationally demanding. For example, when there are $(p+q)$ structural parameters, the standard information criterion procedure requires that the model be estimated $2^{(p+q)}$ times. Instead we propose a practical procedure to estimate the set of stable parameters consistently. The idea is to replace the critical values in the ESS procedure by diverging ones. By doing so, the criterion will be more conservative but it will estimate the set of stable parameters with probability approaching one asymptotically.

Let $|s|$ denote the number of parameters selected by the selection vector s: $|s| = \Sigma_{i=1}^{p+q} s_i$. Let ν_T denote a sequence such that $\nu_T \to \infty$ as $T \to \infty$ and $\nu_T = o(T)$: this will be our penalty function. Common choices are: BIC-type penalty (for which $\nu_T = \ln T$) and Hannan-Quinn-type penalty (for which $\nu_T = \varsigma \ln \ln T$ for $\varsigma > 2$).[10] We formalize the ICS ("Information Criteria for the set of Stable parameters") procedure in the following Algorithm:

Algorithm 2 (The ICS procedure) *Step 0. Initially, let* $s_0 = \mathbf{1}_{(p+q)\times 1}$*. Test* $H_0^{(0)}(s_0)$ *against* $H_A^{(0)}(s_0)$ *by using the test* $\mathcal{T}_T(s_0)$ *with critical value* $|s_0|\nu_T$*. If the test does not reject, let* $\widehat{s}_{ICS} = s_0$*. If the test rejects, calculate the vector of test statistics* $\mathcal{T}_T(e_i;\alpha)$ *for* $i = 1, ..., p+q$*, and order them such that their p-values are increasing. Without loss of generality, let* e_1 *identify the parameter with the smallest p-value. Continue to step 1.*
Step 1. Let $s_1 = [0, \mathbf{1}_{1\times(p+q-1)}]'$*. Test* $H_0^{(1)}(s_1)$ *against* $H_A^{(1)}(s_1)$ *by using* $\mathcal{T}_T(s_1)$ *with critical value* $|s_1|\nu_T$*. If the test does not reject, let* $\widehat{s}_{ICS} = s_1$*. If the test rejects, let* e_2 *identify the parameter with the smallest p-value among the parameters associated with* s_1 *and continue to step 2.*
...
Step j. Let $s_j = [\mathbf{0}_{1\times j}, \mathbf{1}_{1\times(p+q-j)}]'$*. Test* $H_0^{(j)}(s_j)$ *against* $H_A^{(j)}(s_j)$ *by using* $\mathcal{T}_T(s_j)$ *with critical value* $|s_j|\nu_T$*. If the test does not reject, let* $\widehat{s}_{ICS} = s_j$*. If the test rejects, let* e_j *identify the parameter with the smallest p-value among the parameters associated with* s_j

[10] Note that the AIC-type penalty ($\nu_T = 2$) would result in an inconsistent selection criterion and is ruled out by our assumptions on ν_T.

and continue to step (j+1).

...

Step (p+q-1). Let $s_{p+q-1} = [\mathbf{0}_{1\times(p+q-1)}, 1]'$. *Test* $H_0^{(p+q-1)}(s_{p+q-1})$ *against* $H_A^{(p+q-1)}(s_{p+q-1})$ *by using* $\mathcal{T}_T(s_{p+q-1})$ *with critical value* ν_T. *If the test does not reject, let* $\widehat{s}_{ICS} = s_{p+q-1}$. *If the test rejects, let* $\widehat{s}_{ICS} = \mathbf{0}_{(p+q)\times 1}$.

In words, $\widehat{s}_{ICS}$ identified by Algorithm (2) is the greatest set of parameters for which the test does not reject the null hypothesis of parameter stability. See Appendix A for a proof of the consistency of the ICS procedure.

3 A small Monte Carlo analysis

To investigate the finite-sample properties of the proposed methods, we conduct a small Monte Carlo analysis. We consider three data-generating processes (DGP). In the first DGP, there is no unstable parameter.

$$Y_t = \Phi_t Y_{t-1} + u_t, \tag{1}$$

where $\Phi = 0.9I_3$, $u_t \overset{iid}{\sim} N(\mathbf{0}_{3\times1}, \Omega)$,

$$\Omega = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix}. \tag{2}$$

In the second DGP, there is a one-time change in some elements in the coefficient matrix:

$$Y_t = \begin{cases} \Phi_1 Y_{t-1} + u_t & t \leq [0.5T] \\ \Phi_2 Y_{t-1} + u_t & t > [0.5T], \end{cases} \tag{3}$$

where $\Phi_1 = 0.9I_3$, $\Phi_2 = 0.4I_3$, $u_t \overset{iid}{\sim} N(\mathbf{0}_{3\times1}, \Omega)$.

In the third DGP, there is a one-time change in some elements in the covariance matrix:

$$Y_t = \begin{cases} \Phi Y_{t-1} + u_{1t} & t \leq [0.5T] \\ \Phi Y_{t-1} + u_{2t} & t > [0.5T], \end{cases} \tag{4}$$

where $\Phi = 0.9I_3$, $u_{1t} \overset{iid}{\sim} N(0_{3\times1}, \Omega_1)$, $u_{2t} \overset{iid}{\sim} N(\mathbf{0}_{3\times1}, \Omega_2)$,

$$\Omega_1 = \begin{bmatrix} 1.8 & 0.5 & 0 \\ 0.5 & 1.8 & 0.5 \\ 0 & 0.5 & 1.8 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 0.8 & 0.5 & 0 \\ 0.5 & 0.8 & 0.5 \\ 0 & 0.5 & 0.8 \end{bmatrix}.$$

We estimate confidence sets of stable parameters using four procedures: (i) the naïve procedure; (ii) the ESS procedure; and (iii)–(v) the ICS procedures with the SIC, Hannan-Quinn and AIC penalty terms, respectively. In the naïve procedure, a parameter is included in the confidence set if its individual Wald test for structural break (estimated by allowing all the other coefficients to have a break) fails to reject the null that the parameter is stable. As shown in Appendix A, Proposition 3, this procedure will lead to size distortions. The ICS procedures are based on the SIC penalty term, $|s|ln(T)$, the HQ-type penalty term, $2|s|ln(ln(T))$ and the AIC penalty term, $2|s|$ where $|s|$ is the number of stable parameters and T is the sample size. These procedures are all based on Wald statistics that test parameters from the first sub-sample ($t = 1, 2, ..., [0.5T]$) are the same as those from the second sub-sample ($t = [0.5T] + 1, ..., T$). The break fraction is assumed to be known to make the experiment computationally less demanding.

Table 1 reports the actual coverage probabilities of each procedure. The nominal level is 0.95. The coverage probabilities of the naïve procedures are far from their nominal level, which confirms Proposition 3 in Appendix A. For relatively small sample sizes, the actual coverage probabilities of the ESS procedure tend to be slightly smaller than the nominal one, but improve substantially as the sample size grows. The reason why the actual coverage probability of our ESS procedure can be smaller than its nominal level in finite samples is because even though the Wald test is consistent, its power can be less than one in small samples. Proposition 5 in Appendix A shows that the ESS procedure is consistent when the SIC and HQ penalty terms are used while it is not consistent when the AIC penalty term is used. For moderate sample sizes, the HQ-based ICS procedure tends to work well. The last three columns confirm this. Although the ICS procedure with the AIC penalty term worked better in this experiment, it should be noted that the AIC-based ICS procedure is not consistent and the coverage probability is data-dependent.

Overall the results are consistent with our asymptotic theory. Even though the actual coverage probability is not exactly the same as the nominal one, our procedures tend to perform better than the naïve procedure.

INSERT TABLE 1

4 Time variation in a VAR with structural breaks in the parameters

We consider a VAR with structural breaks in both coefficients and volatilities. Models of this type have been estimated by Primiceri (2005), Boivin and Giannoni (2006), and Cogley and Sargent (2005), among others, to investigate the sources of the Great Moderation. We use our procedure to investigate which parameters are time varying – the conditional mean parameters or the volatilities – and which equations of the model contain most of the instability. Thus, our procedure sheds light on whether the time series have responded with time-invariant impulse responses to possibly time-varying shocks or whether the impulse responses have themselves changed over time. First, we consider a reduced form VAR with GDP growth, inflation, and the nominal interest rate. Then, we estimate the subset of time-varying reduced-form VAR parameters, and report their estimates. Finally, we consider a structural VAR in the spirit of Stock and Watson (2002) and Boivin and Giannoni (2006), where the shocks are identified according to a Cholesky decomposition, and discuss the subset of time-varying structural parameters.

We consider the following reduced-form VAR, where y_t is per-capita GDP growth in real terms, π_t is the GDP price deflator, and r_t is the three month U.S. Treasury bill interest rate:[11]

$$\begin{aligned}\begin{pmatrix} r_t \\ y_t \\ \pi_t \end{pmatrix} &= \underbrace{\begin{pmatrix} k_{11} \\ k_{22} \\ k_{33} \end{pmatrix}}_{K} + \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A} \begin{pmatrix} r_{t-1} \\ y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} u_{r,t} \\ u_{y,t} \\ u_{\pi,t} \end{pmatrix}}_{u_t} \qquad (5) \\ \text{where } V(u_t) &= \Omega = \begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}.\end{aligned}$$

The lag length is chosen according to the BIC (implemented with a maximum lag length of four lags) and is equal to one. We let both sets of parameters, that is those in the conditional mean (K, A) and those in the covariance matrix (Ω), to possibly change over time according to a one-time structural break at an unknown date. We use Andrews' (1993) QLR test for

[11] For comparability, the time series of the variables are the same as in the previous section. That is, they are calculated in deviations from their steady state levels.

structural breaks in our procedure.[12] The break is estimated to happen in 1985:2 in the reduced form VAR, and we use such a date in our analysis.

Table 2 shows the results. Panel A in Table 2 shows that Andrews's (1993) test strongly rejects the joint hypothesis of stability in all the parameters. Interestingly, and similarly to Cogley and Sargent (2001, 2005), the same test applied to the parameters in the conditional mean ($k's$ and $\alpha's$) does not reject the null hypothesis of structural stability at the 10% significance level, but a test on the variance parameters ($\omega \in \Omega$) does reject. However, we cannot really rely on such tests, as they repeatedly test hypotheses without taking into account the recursive nature of the procedure. Furthermore, such tests do not identify which parameters are time-varying. We therefore apply our procedure described in Algorithm 1. We find that the biggest evidence of parameter instability comes from the variance of all three reduced form shocks. In fact, Panel A in Table 2 shows that the only parameters that do not belong to the set of stable parameters are $\omega_{11}, \omega_{22}, \omega_{33}$. All other parameters in the conditional mean of the VAR ($k's$ and $a's$) appear to be stable, as well as the contemporaneous relationships between the endogenous variables ($\omega_{12}, \omega_{13}, \omega_{23}$).

INSERT TABLE 2 HERE

However, the covariances of the reduced form shocks do not separately identify the transmission mechanism from the variance of the structural shocks. To overcome this problem, we consider a structural VAR where we identify the structural shocks according to a recursive VAR identification used, among others, by Stock and Watson (2002), Primiceri (2005) and Boivin and Giannoni (2006). The Cholesky decomposition follows the order inflation, output and the interest rate:

$$\underbrace{\begin{pmatrix} u_{r,t} \\ u_{y,t} \\ u_{\pi,t} \end{pmatrix}}_{u_t} = \underbrace{\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 0 & \sigma_{22} & \sigma_{23} \\ 0 & 0 & \sigma_{33} \end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} \eta_{r,t} \\ \eta_{y,t} \\ \eta_{\pi,t} \end{pmatrix}}_{\eta_t} \tag{6}$$

where $\eta_t \sim iid(0, I)$ are the structural shocks. Again, we allow the parameters to have a break at an unknown date. The interpretation of (6) is as follows: the first equation

[12] Although single and once-and-for-all shifts in parameters are an important modeling device, they are only one way to model time-variation. However, tests of these type have power against a variety of types of structural breaks, and therefore the analysis in this section is somehow robust to alternative models for the time variation in the parameters.

represents the monetary policy rule, the second equation is the IS equation and the third is a Phillips curve.

Panel B in Table 2 shows the results, and Table 3 reports the structural VAR parameter estimates obtained over the full sample as well as in the two sub-samples. Again, by construction, a joint test on all parameters rejects the null of parameter stability, and simple joint tests on all $\sigma's$ rejects the null of stability. However, the structural VAR analysis uncovers the very interesting result that the evidence of time variation is concentrated both in the transmission mechanism and in the impulse: the set of stable parameters does *not* contain $\{\sigma_{11,}\sigma_{12},\sigma_{13}\}$, the coefficients in the monetary policy reaction function, as well as σ_{22}, the variance of the shock to GDP. This evidence suggests that changes in monetary policy are clearly linked to the Great Moderation, as well as an exogenous reduction in the variance of the IS curve. The relationship that appears to be the most stable over time is that of inflation (i.e. the Phillips curve): in fact, this is the only equation for which all the parameters, $\{k_{33},a_{31,}a_{32},a_{33},\sigma_{33}\}$, are in the set of stable parameters.

Further descriptive evidence in favor of our claims comes from the analysis of the impulse response functions before and after the structural break, reported in Figure 1. Our analysis uncovers that, while the pattern of the responses and their shapes is broadly consistent with economic theories and previous works, such responses did change at the time of the Great Moderation. For example, the size of the response of output to inflation changed substantially on impact, and the response of the interest rate to the inflation shock too (it was much more persistent and volatile before1985 than after). Also, the response of inflation to the output shock changes sign. Regarding the effects of a monetary policy shock onto macroeconomic variables, the response of inflation became less persistent and the response of the interest rate became more persistent after 1985; in addition, unanticipated increases in interest rates lead to deeper recessions pre-1985 than afterwards. These pictures, which are broadly qualitatively consistent with those in Boivin and Giannoni (2006),[13] lend additional support to the thesis that the transmission mechanism (and not only the variances of the shocks) changed during the Great Moderation. In fact, had the variances of the shocks changed without any simultaneous change in the transmission mechanism, the shape of the responses would have remained the same. The figures suggest that this is not the case.

INSERT TABLES 3 AND 4 AND FIGURE 1 HERE

[13] Their sample and choice of variables is slightly different from ours, which explains the slight differences in the empirical results, but overall the shape of the impulse responses are similar to theirs.

5 Time-variation in a representative New Keynesian model

While the structural VAR analysis in the previous section identifies structural shocks of interest with minimal identifying assumptions, it might be possible that more precise results could be obtained by imposing additional structure on the problem. While, on the one hand, imposing wrong restrictions would lead to a misspecified model, on the other hand considering a more detailed model might shed additional light on the problem and would allow us to compare our results with those in the literature. We therefore also consider a structural model with the basic features of many recent representative New Keynesian models.

The model is developed in Ireland (2007), and includes a generalized Taylor rule for monetary policy that allows the Central Bank's inflation target to adjust in response to other shocks hitting the economy. This feature is particularly appealing for our purposes, as it provides an additional way of allowing for time-variation in monetary policy. In fact, our objective is to estimate the set of stable parameters, which may include not only monetary policy parameters but also preference and technology parameters: we would like our conclusions to be robust to possible misspecification of the monetary policy rule, including possible time variation in the long-run inflation target of the Central Bank. The model also allows for a variety of features that have been found to be important to match theoretical models with the empirical data, namely habit formation, forward-looking price setting, and adjustment costs. We will refer to this (general) model as the "endogenous inflation target" model. As a special case, the model includes the "exogenous inflation target" model considered by Clarida, Gali and Gertler (2000), among others, where the Central Bank's inflation target is constant over time. The log-linearized model is directly from Ireland (2007) and it is included in Appendix A for reference. The data are quarterly time series of per-capita GDP growth in real terms, the first difference of the GDP price deflator, and the three month U.S. Treasury bill interest rate minus inflation from 1959:1 through 2004:2.

Our analysis focuses on the situation in which there is a single, unanticipated, and once-and-for-all shift in some of the parameters of the structural model at an unknown time, and in which there is an immediate convergence to a rational-expectations equilibrium after the regime change. If the time of the change were known and if we had a strong suspicion about which parameters could possibly have been affected by the change, we could re-estimate the model in the two sub-samples and test whether there was a break. However, while we have a variety of potential candidates (including monetary policy, preference, and Phillips curve parameters), we don't know exactly which parameters could have been affected by the break. We therefore use our procedure. As a representative test for structural break, we use Andrews' (1993) QLR test.[14] For computational simplicity, in Algorithm 1 we fixed the break date to be the estimated one, and applied a standard Chow test for structural break; however, since we evaluate the Chow test statistic at the estimated break date, the p-values are calculated using Andrews' (1993) critical values. We estimate all the parameters in Ireland's (2007) model including the slope of the Phillips curve, except for the discount factor, which is calibrated to standard values.[15] Unreported results show that a joint test on all the parameters strongly rejects the null hypothesis of parameter stability. The estimated time of the break (given by the date associated with the highest value of the QLR test statistic) is 1980:4.

Table 4 shows the empirical results. It reports p-values of t-tests for structural breaks on individual parameters and p-values of our recursive procedure, Algorithm 1. In order to give an economic interpretation of the sources of time variation, we divide the parameters in three groups: (i) those influencing the Euler equation ($\gamma, \rho_a, \sigma_z, \sigma_a$); (ii) those influencing the Phillips curve either in the standard Phillips curve relationship (α) or measuring the persistence and standard deviation of the cost-push shock (ρ_e, σ_e); and (iii) those influencing monetary policy (either the usual output gap and inflation aversion parameters (ρ_{gy}, ρ_π) or the long-term inflation target parameters ($\sigma_\pi, \delta_z, \delta_e$), or the serial correlation and standard deviation of the transitory monetary policy shock, ρ_v, σ_v).

[14] Note that the LR-like statistics in Andrews (1993), eq. (4.5), simply becomes the likelihood ratio test calculated as the difference between the constrained model (that is, the loglikelihood estimated over the full sample) and the unconstrained one (that is, the weighted average of the loglikelihood estimated separately over all possible two sub-samples, weighted according to the percentage of observations in each of the two sub-samples). Andrews' (1993) is very computationally convenient in this case because once the break date is estimated, we only need to estimate the DSGE model twice (before and after the break) and not take averages over all possible unknown break dates, as we would do for the Andrews and Ploberger's (1994) Exp-W and Ave-W tests.

[15] Although the discount factor β is calibrated and constant, note however that we are not imposing a constant discount factor because the discount factor itself is multiplied by a preference shock, modeled as an AR(1). We let both the serial correlation and the variance of the preference shock to possibly change following a structural break, and we estimate them from the data. Therefore, the discount factor is allowed to be time-varying.

Ireland (2007) also calibrates ψ; later in this section we also consider the case in which ψ is calibrated in order to compare our results to his.

INSERT TABLE 4

The empirical results are striking. According to our Algorithm 1, for the endogenous inflation target model (shown in panel A), the set of stable parameters is $(\gamma, \sigma_\pi, \sigma_e)$. Strong evidence of time variation comes from the parameters in the monetary policy reaction function of the Central Bank. The table shows that such instabilities affect both the parameters in the standard monetary policy reaction function (ρ_π, ρ_{gy}) and the parameters governing the long-term inflation target of the Central Bank (δ_z, δ_e), but does not affect the identified monetary policy shock itself (as σ_π belongs to the set of stable parameters). That is, once the behavior of the Central Bank is modeled by including a long-run time-varying inflation target, there is no evidence that the monetary policy shock has itself changed over time. In fact, when we impose an exogenous long-run inflation target (see panel B), σ_π does not belong to the set of stable parameters anymore. However, the way in which the Central Bank has responded to supply shocks over the last few decades is time-varying. The other parameters that are constant are γ and σ_e, respectively the parameters governing the habit formation and the variance of the cost-push shock. The most remarkable result is that time variation afflicts not only the parameters in the monetary policy reaction function, but also most of the "structural" parameters in the Euler and IS equations. There is a sense in which, therefore, such parameters are not "structural".

For completeness, and in order to compare our results to the full-sample analysis in Ireland (2007), we also consider the case in which ψ is calibrated. Interestingly, Table 4 (panel B) shows that when we calibrate ψ, the weakest evidence of time variation is in the parameter of the Phillips curve, α, which belongs to the set of stable parameters. The latter result reinforces that in Cogley and Sbordone (2005), who also claim that the estimated parameters of the Phillips curve are stable in the face of changes elsewhere in the economy. However, our results are more general than theirs, in the sense that: (i) we allowed all the parameters to be possibly time-varying and chose the set of stable parameters according to statistical criteria; (ii) we do not have to make a maintained assumption as to the nature of the time variation (our tests are admissible for one-time structural changes, but have power against other types of time-variation); (iii) we do not make any maintained assumption as to the VAR underlying the data. In addition, Figure 2 plots impulse response functions estimated by allowing the set of unstable parameters to have a structural break. We focus on the endogenous inflation target case. Panel A shows the impulse responses before the break and Panel B shows the impulse responses after the break. The magnitude of the impulse responses of each of the variables to the preference shock change considerably before and after the break, as well as the responses to the inflation target shock. Interestingly, in the case of the responses to the cost-push shock, not only the magnitude of the impulse responses change, but also their shape. This is additional empirical evidence suggesting that the transmission mechanism in the U.S. economy has changed starting the early Eighties, and that this has likely played a role in the Great Moderation.

INSERT FIGURE 2

To investigate which parameters are most responsible for the Great Moderation, we will consider changes in the standard deviation of five structural shocks in Ireland's (2007) model: the inflation target shock, the technology shock, the preference shock, the cost-push shock and the transitory monetary shock. Following Stock and Watson (2002), we impose a break in 1984:Q1. We consider the endogenous inflation target model where all parameters (including ψ) are free to vary, as well as the case in which ψ is calibrated. Table 5 reports structural parameter estimates and standard deviations in both sub-samples; Panel A reports results for the case in which ψ is estimated, and panel B reports results for the case in which ψ is calibrated. Based on these estimates we obtain the standard error of the structural shocks in Ireland's (2007) model. The last column of Table 6 shows the ratio of the standard deviation in the second sample period over the one in the first time period. Again, Panel A reports results for the case in which ψ is estimated, and panel B reports results for the case in which ψ is calibrated. The inflation target shock has experienced the largest reduction, and the reduction is larger than any of the reductions in the standard deviations in Stock and Watson's (2002) Table 8. However, when we calculate the relative contribution of each shock to the total reduction in the variance of GDP by using the DSGE model, we find that the technology shock seems to be explaining most of such decrease, although the transitory monetary policy played a role as well. Note however that the shock to the long run inflation target seems to have contributed to an increase in the volatility of the GDP growth.

INSERT TABLES 5 AND 6

Finally, to assess the robustness of the results in this section, we estimate a VAR using the same data as in Ireland (2007).[16] Table 7 shows the results of our test and Table 8 shows parameter estimates. First we consider the reduced form VAR, eq. (5). Andrews's (1993) test rejects the joint stability of all the parameters; the same test does not reject the

[16] That is, we consider the real interest rate, the rate of growth of inflation, and the output growth.

stability of the parameters in the conditional mean ($k's$ and $\alpha's$), whereas it does reject the stability of the variance parameters ($\omega \in \Omega$). Therefore, as in the previous section and as pointed out in Cogley and Sargent (2001, 2005), instabilities in the reduced-form VAR seem to be concentrated in the variance parameters. Our recursive procedure, again, shows that the biggest evidence of parameter instability comes from the variance of the three shocks. When we instead consider the structural VAR (6), our analysis uncovers, as in the previous section, the very interesting result that the variances of the structural shocks are constant (i.e. all σ_{ii} are constant, $i = 1, 2, 3$) and the instability is concentrated in the transmission mechanism, and more specifically in the monetary policy reaction function and IS equations, whereas the equation for inflation (the Phillips curve) is, again, stable over time.

INSERT TABLES 7 AND 8

6 Conclusions

This paper investigates which of the "structural" parameters of a representative DSGE and VAR models are stable over time for the U.S. postwar economy. We do so by developing new econometric tools that allow researchers to identify the set of stable parameters of a model. Empirically, our conclusions are that instabilities are mostly a concern for the monetary policy reaction function, the IS and the Euler equations. For the VAR, we also find empirical evidence in favor of stability in the Phillips curve. When interpreting these findings to shed light on the Great Moderation, our structural VAR analysis leads us to conclude that the Great Moderation was a combination of changes in the monetary policy reaction function and IS curve, as well as a break in the variance of the exogenous shocks to GDP growth. Results based on the DSGE model similarly find that time variation occurred simultaneously in such equations as well as in the variances of selected structural shocks.

Overall, our paper identified the instabilities that characterize estimated macroeconomic models and shed light on the economic interpretation and cause of such instabilities. It provides clear guidelines on which parts of the model are reliable for policy analysis and which are possibly mis-specified. Such results offer important insights to guide the future theoretical development of macroeconomic models.

References

[1] Andrews, D.W.K. (1993), “Tests for Parameter Instability and Structural Change with Unknown Change Point”, *Econometrica* 61, 821–856.

[2] Andrews, D.W.K., and W. Ploberger (1994), “Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative”, *Econometrica* 62(6), 1383-1414.

[3] Boivin, J., and M. Giannoni (2006), “Has Monetary Policy Become More Effective”, *The Review of Economics and Statistics* 88(3), 445-462.

[4] Clarida, R., J. Gali, and M. Gertler (2000), “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory”, *The Quarterly Journal of Economics* 115(1), 147-180.

[5] Cogley, T. and T. Sargent (2001), “Evolving Post-World War II U.S. Inflation Dynamics”, in: B. Bernanke and K. Rogoff (eds.), *NBER Macroeconomics Annual.*

[6] Cogley, T., and T.J. Sargent (2005), “Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.”, *Review of Economic Dynamics* 8(2), 262-302.

[7] Cogley, T. (2007), “Comment on ‘How Structural Are Structural Parameters?’ by Jesus Fernadez-Villaverde and Juan Rubio-Ramirez”, in: D. Acemoglu, K. Rogoff and M. Woodford (eds), *NBER Macroeconomics Annual.*

[8] Cogley, T., and A. Sbordone (2005), “A Search for a Structural Phillips Curve”, *Federal Reserve Bank of New York Staff Reports* No. 203.

[9] Del Negro, M., and F. Schorfheide (2007), “Monetary Policy Analysis with Potentially Misspecified Models”, *NBER Working Paper* 13099.

[10] Del Negro, M., F. Schorfheide, F. Smets and R. Wouters (2004), “On the Fit and Forecasting Performance of New Keynesian Models”, *mimeo.*

[11] Elliott, G., and U. Muller (2006), “Efficient Tests for General Persistent Time Variation in Regression Coefficients”, *Review of Economic Studies* 73, 907-940.

[12] Estrella, A., and J.C. Fuhrer (1999), “Are “Deep” Parameters Stable? The Lucas Critique as an Empirical Hypothesis, *Federal Reserve of Boston Working Paper 99-4.*

[13] Fernald, J. (2007), "Trend Breaks, Long-Run Restrictions, and Contractionary Technology Improvements", *Journal of Monetary Economics*, forthcoming.

[14] Fernandez-Villaverde, J. and J. Rubio-Ramirez (2007), "How Structural Are Structural Parameters?", in: D. Acemoglu, K. Rogoff and M. Woodford (eds.), *NBER Macroeconomics Annual*, MIT Press.

[15] Gurkaynak, R.S., B. Sack, and E. Swanson (2005), "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models", *American Economic Review* 95, 425-436.

[16] Hansen, P.R., A. Lunde, and J.M. Nason (2005), "Model Confidence Sets for Forecasting Models", *mimeo.*

[17] Hayashi, F. (2000), *Econometrics*, Princeton University Press.

[18] Ireland, P.N. (2001), "Sticky-price Models of the Business Cycle: Specification and Stability", *Journal of Monetary Economics* 47, 3-18.

[19] Ireland, P.N. (2007), "Changes in the Federal Reserve's Inflation Target: Causes and Consequences", *mimeo.*

[20] Justiniano, A., and G. Primiceri (2007), "The Time Varying Volatility of Macroeconomic Fluctuations", *mimeo.*

[21] Li, H. (2006), "Estimation and Testing of Euler Equation Models with Time-varying Reduced-form Coefficients", *Journal of Econometrics*, forthcoming.

[22] Nyblom, J. (1989), "Testing for the Constancy of Parameters Over Time", Journal of the American Statistical Association 84(405), 223-230.

[23] Owyang, M.T., and G. Ramey (2004), "Regime Switching and Monetary Policy Measurement", *Journal of Monetary Economics* 51, 1577-97.

[24] Pantula, S.G. (1989), "Testing for Unit Roots in Time Series Data", *Econometric Theory* (5), 256-271.

[25] Primiceri, G. (2005), "Time Varying Structural Vector Autoregressions and Monetary Policy", *The Review of Economic Studies* 72(3), 821-852.

[26] Sims, C. (2001), "Comment to: Evolving Post-World War II U.S. Inflation Dynamics", in: B. Bernanke and K. Rogoff (eds.), *NBER Macroeconomics Annual*.

[27] Stock, J.H. (2001), "Comment to: Evolving Post-World War II U.S. Inflation Dynamics", in: B. Bernanke and K. Rogoff (eds.), *NBER Macroeconomics Annual*.

[28] Stock, J. H. and M. W. Watson (2002), "Has the Business Cycle Changed and Why?," in: M. Gertler and K. Rogoff (eds.), *NBER Macroeconomics Annual.*

[29] Stock, J. H. and M. W. Watson (2003), "Has the Business Cycle Changed? Evidence and Explanations", *FRB Kansas City Symposium*, Jackson Hole.

7 Appendix A - Propositions and proofs

7.1 The ESS procedure

For any two vectors a and b, let $\max(a,b)$ denote the vector whose i-th element is the maximum of $a(i)$ and $b(i)$, where $a(i)$ denotes the i-th element of a. We make the following assumption:

Assumption 1. For all $s^* \in \{0,1\}^{p+q}$ *such that* $s^* \neq 0_{1\times(p+q)}$, $\mathcal{T}_T(s) \underset{d}{\Rightarrow} D(s)$ *if* $s = s^*$ *and* $\mathcal{T}_T(s) \underset{p}{\rightarrow} \infty$ *if* $|\max(s,s^*)| > |s^*|$, *where* $|s|$ *denotes the number of components in* s *that are different from zero.*

Remarks. Assumption 1 requires that $\mathcal{T}_T(s)$ has a well-defined asymptotic distribution under the null hypothesis, and diverges to positive infinity when testing a subset of parameters which includes at least one unstable parameter (under the alternative hypothesis of parameter instability). This assumption is satisfied by most tests for structural breaks, for example Andrews' (1993) QLR test, Andrews and Ploberger's (1994) Exp-W and Mean-W tests, and Nyblom's (1989) test.[17]

The following Proposition shows that, by selecting the parameters associated with $\widehat{s}_{ESS}$ identified in Algorithm (1) one obtains a confidence set of the stable parameters that has coverage $(1-\alpha)$.

Proposition 3 *Let Assumption 1 hold, and let* $\widehat{s}$ *be estimated as described by Algorithm (1). Then:*

$$\lim_{T\rightarrow\infty} \Pr\{\hat{s}_{ESS} = s^*\} \; = \; 1-\alpha \tag{7}$$

for any $s^* \in \{0,1\}^{p+q}$ *such that* $s^* \neq 0_{1\times(p+q)}$,

$$\lim_{T\rightarrow\infty} \Pr\{\hat{s}_{ESS} = s^*\} \; = \; 1 \tag{8}$$

for $s^* = 0_{1\times(p+q)}$, *and*

$$\lim_{T\rightarrow\infty} \Pr\{\hat{s}_{ESS} \neq s^* \text{ and } \hat{s}_{ESS} \geq s^*\} = 0. \tag{9}$$

for any $s^* \in \{0,1\}^{p+q}$.

[17] A test for structural break that does not satisfy our assumption is Elliott and Muller's (2005) qLL test.

Proof of Proposition (3). Let $k_\alpha(s)$ denote the critical value of $\mathcal{T}_T(s;\alpha)$ of the null distribution $D(s)$ at the level of significance, α. Recall that $p = p + q - |s^*|$ is the number of unstable parameters. First, suppose that $p = 0$. Then $\underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s_0\right) < k_\alpha\left(s_0\right)\right) = \underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s^*\right) < k_\alpha\left(s^*\right)\right) = 1-\alpha$, thus proving (7) under $H_0^{(0)}\left(s_0\right)$. When $p = 0$, (8) does not apply and (9) trivially holds. Next, suppose that $p > 0$ and $q > 0$. Note that $|\max(s_j, s^*)| > |s^*| = q$ for any s_j and $j = 0, 1, 2..., p-1$. By the consistency of $\mathcal{T}_T(s_j)$ for s_j such that $|\max(s_j, s^*)| > |s^*|$, the null hypotheses in steps $0, 1, 2, ..., p-1$ are all rejected and each of the p unstable parameters is selected in these p steps with probability approaching one. Therefore the null model in step p, s_p, converges in probability to s^* and (9) holds. Because $\underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s_p\right) < k_\alpha\left(s_p\right)\right) = \underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s^*\right) < k_\alpha\left(s^*\right)\right) = 1-\alpha$, (7) holds. When $p > 0$ and $q > 0$, (9) does not apply. Lastly, suppose that $q = 0$. Then the null hypotheses in steps $0, 1, 2, ..., p$ are all rejected and each of the p unstable parameters is selected in these steps with probability approaching one. Therefore $\hat{s}_{ESS}$ converges in probability to s^*, and (8) and (9) hold. ■

Proposition 4 *Let the naïve testing procedure be as follows:* $\widetilde{s} = s$*, where the i-th component of* s*,* $\widetilde{s}\left(i\right)$*, is such that:*

$$\widetilde{s}\left(i\right) = \begin{cases} 1 \text{ if } & \mathcal{T}_T\left(e_i\right) < k_\alpha\left(e_i\right) \\ 0 & \text{otherwise} \end{cases}$$

Then $\lim_{T\to\infty} \Pr\left(\widetilde{s} \neq s^* | s_i = s^*\right) > \alpha$ *for every* $s^* \in \{0,1\}^{p+q}$ *provided* $p + q - i > 1$*.*

Proof of Proposition (4). Without loss of generality, consider the case $s^* = [\mathbf{0}_{p\times 1}, \mathbf{1}_{q\times 1}]'$. Suppose that $p + q - j = 2$. Then:

$$\begin{aligned} Pr\left(\widetilde{s} \neq s^* |\, s_j = s^*\right) &= \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ or } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &= \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right)\right) + \Pr\left(\mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &\quad - \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ and } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &= 2\alpha - \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ and } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &> \alpha \end{aligned}$$

where the last inequality follows since $\Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ and } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) < \alpha$ provided that the joint distribution is non-singular. The proof for cases in which $p+q-j > 2$ is analogous although it is notationally more complicated. ■

7.2 The ICS procedure

We make the following assumption:

Assumption 2.

(a) *For all* $s^* \in \{0,1\}^{p+q}$ *such that* $s^* \neq 0_{1\times(p+q)}$, $\mathcal{T}_T(s) \underset{d}{\Rightarrow} D(s)$ *if* $s = s^*$ *and* $\frac{1}{T}\mathcal{T}_T(s) \underset{d}{\Rightarrow} c(s)$ *if* $|\max(s, s^*)| > |s^*|$, *where* $|s|$ *denotes the number of components in* s *that are different from zero, and* $c(s) > 0$ *is some positive constant.*

(b) *Let* ν_T *be a sequence such that* $\nu_T \to \infty$ *and* $\nu_T = o(T)$.

Remarks. Assumption 2(a) is a slight modification of Assumption 1 and is satisfied by most structural break tests, including Andrews (1993), Andrews and Ploberger (1994), and Nyblom (1989). Basically, it requires that the moment conditions converge in probability to some limiting nonzero value when evaluated when including parameters that have a break: since the parameters will converge to some pseudo-true parameter value different from the true, time-varying parameter, the expected value of such moment conditions will not be zero. Such limiting value will be zero when the moment conditions are evaluated only at stable parameters, otherwise will be a positive number. Note that when $|\max(s, s^*)| > |s^*|$ there will be at least one moment condition that is in expectation different from zero, and given that the test statistic is asymptotically equivalent to a quadratic form of such moment conditions, it will be positive.

Assumption 2(b) defines the properties required for the penalty function. The penalty function is necessary to offset the increase in the value of the test statistic $\mathcal{T}_T(s)$ that typically occurs when testing instabilities on a larger number of parameters even if the additional parameters are stable. For example: a BIC-type penalty involves $\nu_T = \ln(T)$, a Hannan-Quinn-type penalty involves $\nu_T = \varsigma \ln\ln(T)$ for some $\varsigma > 2$. An AIC-type penalty would involve $\nu_T = 2$ but, as well known, AIC-type penalties does not result in a consistent selection criterion and in fact it does not satisfy our Assumption 2(b).

Proposition 5 (Consistency of the ICSeq procedure) *Let Assumption 2 hold, and let* $\widehat{s}_{ICS}$ *be estimated as described by Algorithm (2). Then,*

$$\widehat{s}_{ICS} \underset{p}{\to} s^* \tag{10}$$

Proof of Proposition (5). Because $\nu_T(s)$ is diverging, the tests have size zero, i.e., $\alpha = 0$. Because the test statistics diverge faster than $\nu_T(.)$ under the alternative hypothesis, the tests remain consistent. Therefore (10) follows from (7). ∎

8 Appendix B - Detailed description of the New-Keynesian model

Ireland's (2007) model is log-linearized around a steady state where consumption, output, and the marginal utility of consumption grow at the rate of technological process (a random walk with drift). The model is as follows. Let $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{e}_t$, $\widehat{z}_t$, $\widehat{a}_t$, $\widehat{v}_t$, $\widehat{\lambda}_t$ $\widehat{\pi}_t^*$ denote the deviation of output, inflation, the cost-push shock, technology, the preference shock, the transitory monetary policy shock, the marginal utility of consumption, and the time-varying inflation target from the their steady state levels, and the following hold:

$$\widehat{g}_t^y = \widehat{y}_t - \widehat{y}_{t-1} + \widehat{z}_t,$$
$$\widehat{g}_t^\pi = \widehat{\pi}_t - \widehat{\pi}_{t-1} + \widehat{\pi}_t^*,$$
$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \sigma_a \varepsilon_{at},$$
$$\widehat{e}_t = \rho_e \widehat{e}_{t-1} + \sigma_e \varepsilon_{et},$$
$$\widehat{z}_t = \sigma_z \varepsilon_{zt},$$
$$\widehat{v}_t = \rho_v \widehat{v}_{t-1} + \sigma_v \varepsilon_{vt}$$

The model builds on a series of parameters: z (the steady state level of technology), β (the discount factor), γ (the habit formation), α (the parameter measuring the extent to which price setting is backward or forward looking: $\alpha = 0$ means purely forward looking), ψ (a function of the magnitude of the adjustment cost and of the long-run level of the cost-push shock), ρ_π (the Fed's inflation aversion), ρ_{gy} (the Fed's aversion to the output gap), σ_π (the standard deviation of the shock to the inflation target), δ_e (the reaction of the time-varying inflation target to the shock to the time varying elasticity of demand for each intermediate good), δ_z (the reaction of the time-varying inflation target to the temporary shock to aggregate technology). The parameters β and z are calibrated prior to estimation; Ireland (2007) also calibrates ψ.

The core of the model is formed by the following equilibrium conditions:

(1) the IS curve:

$$(z-\gamma)(z-\beta\gamma)\widehat{\lambda}_t = \gamma z \widehat{y}_{t-1} - (z^2 + \beta\gamma^2)\widehat{y}_t + \beta\gamma z E_t \widehat{y}_{t+1} + (z-\gamma)(z-\beta\gamma\rho_a)\widehat{a}_t - \gamma z \widehat{z}_t$$

(2) the Euler equation:

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \widehat{r}_t - E_t \widehat{\pi}_{t+1}$$

(3) the Phillips curve:

$$(1+\beta\alpha)\widehat{\pi}_t = \alpha\widehat{\pi}_{t-1} + \beta E_t \widehat{\pi}_{t+1} + \psi\left(\widehat{a}_t - \widehat{\lambda}_t\right) - \widehat{e}_t - \alpha\widehat{\pi}_t^*$$

(4) the Monetary Policy reaction function:

$$\widehat{r}_t - \widehat{r}_{t-1} = \rho_\pi \widehat{\pi}_t + \rho_{gy} \widehat{g}_t^y - \widehat{\pi}_t^* + \widehat{v}_t$$
$$\widehat{\pi}_t^* = \sigma_\pi \varepsilon_{\pi t} - \delta_e \varepsilon_{et} - \delta_z \varepsilon_{zt}$$

Ireland (2007) considers three specifications:

(i) the endogenous inflation target case (all parameters are estimated freely);

(ii) the exogenous inflation target case ($\delta_e = \delta_z = 0$);

(iii) the backward looking price setting ($\alpha = 1$).

9 Tables and Figures

Table 1. Actual Coverage Probabilities

Sample Size	*DGP*	*Naïve Procedure*	*ESS Procedure*	*ICS Procedure (Schwarz)*	*ICS Procedure (Hannan-Quinn)*	*ICS Procedure (Akaike)*
250	1	0.462	0.905	1.000	1.000	0.972
250	2	0.526	0.861	0.103	0.609	0.873
250	3	0.508	0.773	0.000	0.028	0.673
500	1	0.498	0.932	1.000	1.000	0.981
500	2	0.580	0.943	0.751	0.992	0.976
500	3	0.560	0.928	0.000	0.602	0.969
1000	1	0.508	0.938	1.000	1.000	0.984
1000	2	0.594	0.953	1.000	1.000	0.979
1000	3	0.591	0.932	0.533	1.000	0.972

Note. The table reports Monte Carlo actual coverage probabilities of our ESS and ICS procedures relative to the naïve procedure for different sample sizes and the various DGPs described in section 3.

Table 2. Empirical results for the VAR

Panel A. Reduced form VAR		Panel B. Structural VAR	
Parameter	Individual p-value	Parameter	Individual p-value
k_{11}	1	k_{11}	1
a_{11}	1	a_{11}	1
a_{12}	1	a_{12}	1
a_{13}	1	a_{13}	1
k_{22}	1	k_{22}	1
a_{21}	1	a_{21}	1
a_{22}	1	a_{22}	1
a_{23}	1	a_{23}	1
k_{33}	1	k_{33}	1
a_{31}	1	a_{31}	1
a_{32}	1	a_{32}	1
a_{33}	0.70	a_{33}	0.70
ω_{11}	0	σ_{11}	0.71
ω_{12}	1	σ_{12}	1
ω_{13}	1	σ_{13}	1
ω_{22}	0.04	σ_{22}	0
ω_{23}	1	σ_{23}	1
ω_{33}	0.04	σ_{33}	1
Joint test – all param:	0	Joint test – all param:	0
Joint test – all a, k:	0.11	Joint test – all a, k:	0.11
Joint test – all ω:	0	Joint test – all σ:	0
Set of stable parameters (95% probability level): $\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \omega_{12}, \omega_{13}, \omega_{23}\}$		Set of stable parameters (95% probability level): $\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{23}, \sigma_{33}\}$	

Note to table 2. The table reports p-values of Andrews' (1993) QLR test on individual parameters for both the reduced form VAR ((5), in panel A), and the structural VAR ((6), in panel B). The VAR contains GDP growth, inflation, and the nominal interest rate. The table also reports p-values of joint tests on subsets of parameters and the set of stable parameters obtained by our procedure (1). ω_{ij} denotes the i-j-th element of the vech of the covariance matrix in the reduced form VAR and σ_{ij} denotes the i-j-th element of the vech of the variance in Cholesky factor. Subscripts are as follows: $i = 1$ denotes the real interest rate, $i = 2$ denotes GDP, $i = 3$ denotes the inflation.

Table 3. Structural VAR param. estimates

Parameter	Full sample	Pre-1985	Post-1985	Parameter	Full sample	Pre-1985	Post-1985
k_{11}	0	0.00	-0.00	σ_{11}	0.19	0.24	0.10
a_{11}	0.91	0.89	0.95	σ_{12}	0.22	0.26	0.17
a_{12}	0.05	0.04	0.10	σ_{13}	0.04	0.06	-0.02
a_{13}	0.10	0.11	0.13	σ_{22}	0.08	0.94	0.50
k_{22}	0	0.01	0.00	σ_{23}	-0.06	-0.06	-0.07
a_{21}	-0.25	-0.25	-0.18	σ_{33}	0.28	0.32	0.18
a_{22}	0.20	0.19	0.12				
a_{23}	-0.10	-0.18	-0.03				
k_{33}	0	0.00	0.00				
a_{31}	0.06	0.05	0.09				
a_{32}	0.01	-0.01	0.08				
a_{33}	0.84	0.82	0.55				

Note to Table 3. The table reports estimated parameter values of the Structural VAR over the full sample (column labeled “Full sample”), as well as before and after the break (labeled “Pre-1985” and “Post-1985”, respectively). Subscripts are as follows: $i = 1$ denotes inflation, $i = 2$ denotes GDP, $i = 3$ denotes the interest rate. Variances are multiplied by 100.

Table 4 (Panel A). Empirical results for Ireland's (2007) model (ψ estimated).

Models:	Endog. infl. target			Exog. infl. target	
	Individual	Recursive		Individual	Recursive
Parameter:	p-value	p-value		p-value	p-value
ψ	0	0	ρ_π	0	0
ρ_π	0	0	ψ	0	0
σ_v	0	0	σ_v	0	0
α	0	0	α	0	0
ρ_e	0	0	ρ_e	0	0
σ_a	0	0	σ_a	0	0
ρ_{gy}	0	0	ρ_{gy}	0	0
δ_e	0	0	σ_π	0	0
δ_z	0	0	ρ_v	0	0
ρ_v	0	0	ρ_a	0	0
σ_z	0	0	γ	0.16	1
ρ_a	0	0	σ_e	0.82	1
γ	0.44	0.57	σ_z	0.92	1
σ_e	0.68	0.77	- -		
σ_π	0.70	0.80			
	Set of stable parameters (95% probability level): $\mathcal{S} = \{\gamma, \sigma_e, \sigma_\pi\}$			Set of stable parameters (95% probability level): $\mathcal{S} = \{\gamma, \sigma_e, \sigma_z\}$	

Table 4 (Panel B). Empirical results for Ireland's (2007) model (ψ calibrated).

Models:	Endog. infl. target			Exog. infl. target	
	Individual	Recursive		Individual	Recursive
Parameter:	p-value	p-value		p-value	p-value
ρ_{gy}	0	0	σ_a	0	0
δ_e	0	0	ρ_v	0	0
ρ_e	0	0	ρ_e	0	0
ρ_v	0	0	ρ_{gy}	0	0
σ_z	0	0	σ_π	0	0
σ_v	0	0	γ	0	0
σ_π	0	0	σ_z	0	0
γ	0	0	σ_v	0	0
σ_e	0	0	ρ_π	0	0
ρ_π	0	0	σ_e	0.72	0.31
ρ_a	0.04	0	ρ_a	0.80	1
σ_a	0.015	0	α	1	1
δ_z	0.365	0.013	- -		
α	0.999	1	- -		
	Set of stable parameters (95% probability level): $\mathcal{S} = \{\alpha\}$			Set of stable parameters (95% probability level): $\mathcal{S} = \{\alpha, \rho_a, \sigma_e\}$	

Note to table 4. The table reports p-values of Andrews' (1993) QLR test on individual parameters. It also reports both recursive p-values and the set of stable parameters obtained by our recursive procedure (1). The tests are implemented in a Wald form, using standard errors obtained by bootstrap with 1,000 replications.

Table 5 (Panel A). Parameter Estimates – ψ estimated

Parameters	1959:Q1-1983:Q4	1984:Q1-2004:Q1
γ	0.281	0.281
	(0.001)	(0.001)
α	0.001	0.000
	(0.000)	(0.000)
ρ_π	0.675	0.926
	(0.065)	(0.083)
ρ_{gy}	0.129	0.319
	(0.046)	(0.083)
ρ_a	0.928	0.888
	(0.004)	(0.056)
ρ_e	0.020	0.000
	(0.000)	(0.003)
ρ_v	0.189	0.001
	(0.001)	(0.000)
σ_a	0.030	0.028
	(0.013)	(0.000)
σ_e	0.001	0.001
	(0.000)	(0.000)
σ_z	0.015	0.012
	(0.002)	(0.000)
σ_v	0.002	0.002
	(0.000)	(0.002)
σ_π	0.001	0.001
	(0.000)	(0.000)
δ_e	0.001	0.000
	(0.001)	(0.000)
δ_z	0.001	0.000
	(0.000)	(0.001)
ψ	0.093	0.090
	(0.000)	(0.000)

Table 5 (Panel B). Parameter Estimates – ψ calibrated

Parameters	1959:Q1-1983:Q4	1984:Q1-2004:Q1
γ	0.297	0.297
	(0.001)	(0.001)
α	0.000	0.000
	(0.000)	(0.000)
ρ_π	0.731	0.961
	(0.221)	(0.195)
ρ_{gy}	0.120	0.305
	(0.043)	(0.053)
ρ_a	0.924	0.882
	(0.003)	(0.003)
ρ_e	0.208	0.016
	(0.003)	(0.001)
ρ_v	0.201	0.000
	(0.001)	(0.000)
σ_a	0.026	0.027
	(0.009)	(0.010)
σ_e	0.001	0.001
	(0.001)	(0.001)
σ_z	0.016	0.012
	(0.002)	(0.002)
σ_v	0.002	0.002
	(0.001)	(0.000)
σ_π	0.001	0.001
	(0.000)	(0.000)
δ_e	0.001	0.000
	(0.001)	(0.001)
δ_z	0.001	0.000
	(0.000)	(0.000)

Note to Table 5. The table reports parameter estimates for Ireland's (2007) endogenous inflation target model in the two sub-samples, as well as their standard errors in parenthesis. ψ is estimated. Panel A reports results for the case in which ψ is estimated, and panel B reports results for the case in which ψ is calibrated.

Table 6 (Panel A). Standard Deviations of Macroeconomic Shocks. ψ estimated.

	1959:Q1-1983:Q4	1984:Q1-2004:Q1	s_2/s_1	Relative Contribution to GDP Variance Reduction
Inflation Target*	0.122	0.085	0.696	-0.275
Technology Shock*	1.529	1.154	0.755	0.749
Preference Shock	8.083	6.106	0.755	-0.228
Cost-Push Shock	0.093	0.093	1.000	0.000
Transitory Monetary Shock	0.237	0.246	1.038	0.047

Table 6 (Panel B). Standard Deviations of Macroeconomic Shocks. ψ calibrated.

	1959:Q1-1983:Q4	1984:Q1-2004:Q1	s_2/s_1	Relative Contribution to GDP Variance Reduction
Inflation Target*	0.131	0.085	0.650	-0.217
Technology Shock*	1.572	1.173	0.746	0.803
Preference Shock	6.832	5.691	0.833	-0.236
Cost-Push Shock	0.073	0.071	0.978	-0.002
Transitory Monetary Shock	0.241	0.244	1.011	0.051

Notes to Table 6. The standard deviations are multiplied by 100. For the processes with asterisk (*), the variance of the disturbance term, not the variance of the process, is reported because they are unit-root processes. Results are qualitatively very similar in the case of a break in 1980:Q1. Panel A reports results for the case in which ψ is estimated, and panel B reports results for the case in which ψ is calibrated.

Table 7. VAR with the same data as Ireland (2007)

Panel A. Reduced form VAR		**Panel B. Structural VAR**	
Parameter	Individual p-value	Parameter	Individual p-value
k_{11}	1	k_{11}	1
a_{11}	1	a_{11}	1
a_{12}	1	a_{12}	1
a_{13}	1	a_{13}	1
k_{22}	1	k_{22}	1
a_{21}	1	a_{21}	1
a_{22}	1	a_{22}	1
a_{23}	1	a_{23}	1
k_{33}	1	k_{33}	1
a_{31}	1	a_{31}	1
a_{32}	1	a_{32}	1
a_{33}	1	a_{33}	1
ω_{11}	0.139	σ_{11}	1
ω_{12}	1	σ_{12}	1
ω_{13}	1	σ_{13}	1
ω_{22}	0.062	σ_{22}	0
ω_{23}	1	σ_{23}	1
ω_{33}	0.054	σ_{33}	1
Joint test – all param:	0	Joint test – all param:	0
Joint test – all a, k:	1	Joint test – all a, k:	1
Joint test – all ω:	0	Joint test – all σ:	0
Set of stable parameters (95% probability level): $\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \omega_{12}, \omega_{13}, \omega_{23}\}$		Set of stable parameters (95% probability level): $\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{33}\}$	

Note to Table 7. The table reports p-values of Andrews' (1993) QLR test on individual parameters for both the reduced form VAR ((5), in panel A), and the structural VAR ((6), in panel B). The VAR contains GDP growth, inflation growth, and the real interest rate. The table also reports p-values of joint tests on subsets of parameters and the set of stable parameters obtained by our procedure (1). The break is dated 1985:2. ω_{ij} denotes the i-j-th element of the vech of the covariance matrix in the reduced form VAR and σ_{ij} denotes the i-j-th element of the vech of the variance in Cholesky factor. Subscripts are as follows: $i = 1$ denotes the real interest rate, $i = 2$ denotes GDP, $i = 3$ denotes the inflation.

Table 8. Structural VAR param. estimates

Parameter	Estimate	Parameter	Estimate
k_{11}	0.0006	σ_{11}	0.0032
a_{11}	0.8558	σ_{12}	0.0020
a_{12}	0.0258	σ_{13}	-0.0022
a_{13}	0.2371	σ_{22}	0.0082
k_{22}	0.0040	σ_{23}	0.0002
a_{21}	-0.1214	σ_{33}	0.0017
a_{22}	0.2734		
a_{23}	0.1033		
k_{33}	-0.0003		
a_{31}	0.0473		
a_{32}	0.0241		
a_{33}	-0.2630		

Note to Table 8. The table reports parameter values of the structural VAR estimated over the full sample. The VAR contains GDP growth, inflation growth, and the real interest rate. Subscripts are as follows: $i = 1$ denotes inflation, $i = 2$ denotes GDP, $i = 3$ denotes the interest rate.

Figure 1 (Panel A). SVAR impulse responses before the Great Moderation

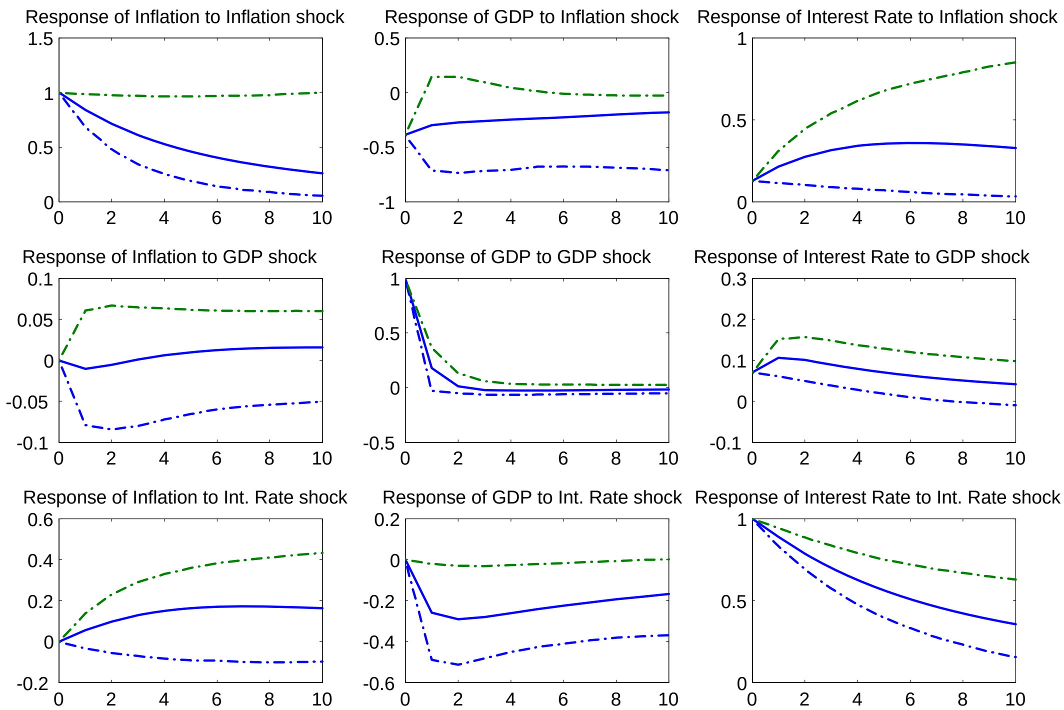


Figure 1 (Panel B). SVAR impulse responses after the Great Moderation

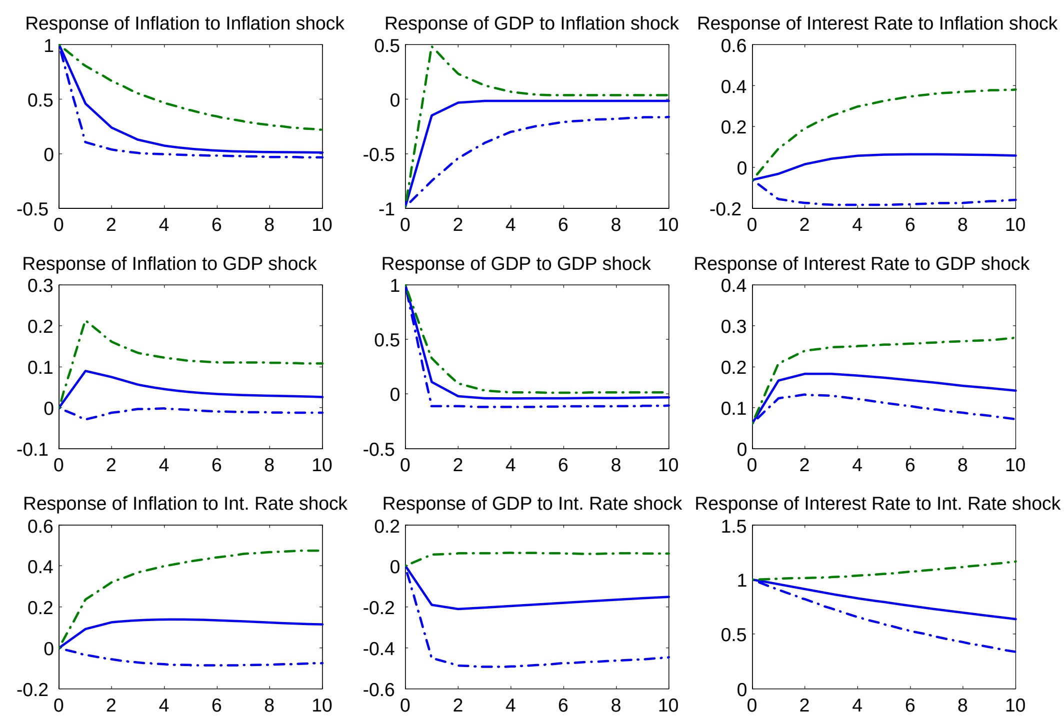


Figure 2 (Panel A). Impulse responses in Ireland's model before 1984

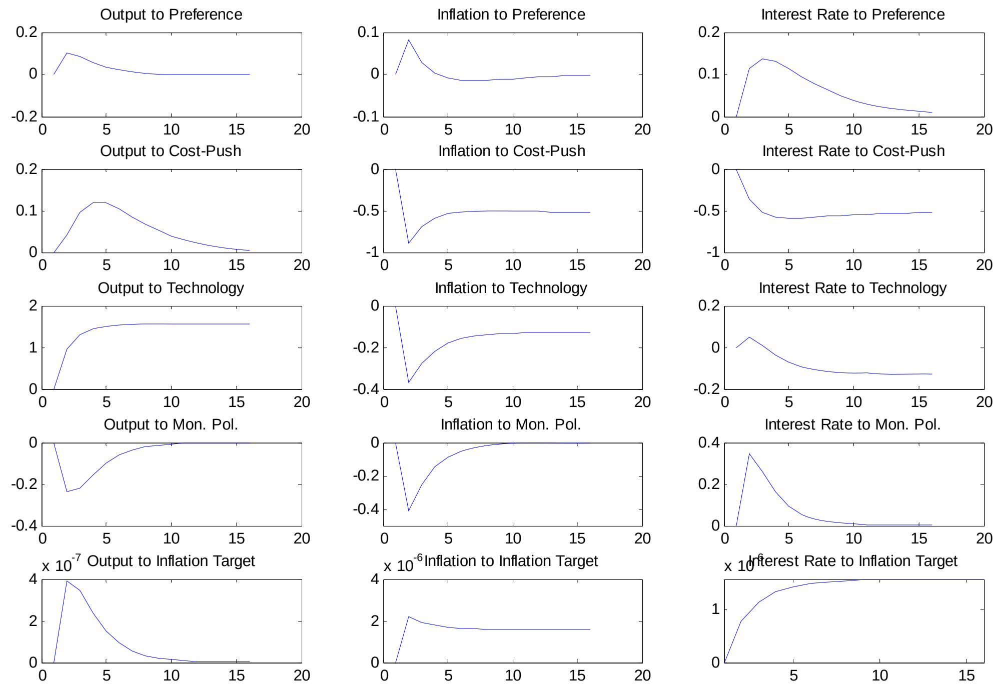
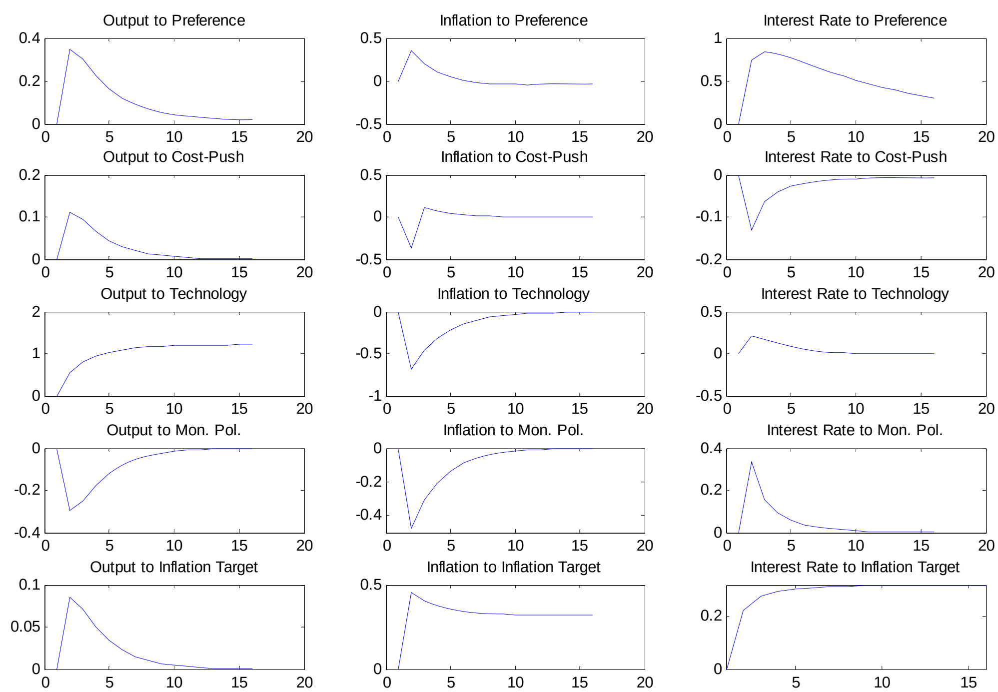


Figure 2 (Panel B). Impulse responses in Ireland's model after 1984



Notes to the figures.

Note to Figure 1. The figure reports impulse responses from the structural VAR (6). The VAR contains GDP growth, inflation and the nominal interest rate. The lag length is chosen by BIC and equals one. Panel A shows impulse responses before the estimated break, and Panel B shows impulse responses after the estimated break.

Note to Figure 2. The figure reports impulse responses from Ireland's (2007) unconstrained model with the endogenous inflation target and ψ calibrated. Each panel shows the percentage-point response of one of the model's variables to a one-standard deviation shock. The inflation and interest rates are expressed in annualized terms. Panel A shows impulse responses before the estimated break, and Panel B shows impulse responses after the estimated break.