Evolutionary Dynamics of Class Structures

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Abstract

We study the emergence and persistence of institutions governing the size of the joint surplus and its distribution between two classes, and we identify conditions under which efficient and/or egalitarian contractual conventions are likely to emerge and to persist. We study transitions between contractual conventions as a perturbed Markov process in which individuals occasionally play idiosyncratically rather than adopting a best response. In contrast to the standard stochastic evolutionary models, we represent idiosyncratic play as intentional rather than uncoordinated individual behavioral errors. In contract to the results of the models with unintentional idiosyncratic play, when class sizes differ, very unequal contracts may be stochastically stable even if they are very inefficient and not risk-dominant. We extend this benchmark model to make the relative sizes of the two classes endogenous, showing that higher barriers to inter-class upward mobility make unequal contracts stochastically stable, and imply higher levels of societal class inequality. We also let the rate of idiosyncratic play vary with the degree of class polarization and group network structure. Finally, we introduce the state as an actor mediating class conflict, and identify the conditions under which it will adopt redistributive strategies. In the penultimate section we suggest extensions addressing the effects of technical change, collective as opposed to individual idiosyncratic play, endogenous barriers to class mobility, variations in state capacities, and the structure of inheritance systems and marital assortment.

JEL codes: D02(Institutions), D3(Distribution), C73 (Stochastic and Dynamic Games; Evolutionary Games)

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1 Introduction

We study the emergence and persistence of conventions governing the size of the joint surplus and its distribution among two classes, and we identify the conditions under which efficient and/or egalitarian institutions will be stochastically stable states. We would like to be able to answer questions such as: Why have institutions that implement highly unequal divisions of the joint surplus been so ubiquitous since the domestication of animals and plants 11 millennia ago? Why do some unequal institutions such as the caste system persist when they seemingly convey no clear efficiency advantages over other feasible social arrangements? Why do societies with limited inter-class mobility often exhibit highly unequal distribution of the joint surplus? Why are some highly unequal institutional arrangements stable over millennia, while others are not?

Ancient, slave, latifundia, sharecropping, feudal, and other class structures often aggregated large numbers of poor workers under the direction of members of a well to do class, who profited from their labor. What accounts for the stability of the earlier institutions in contrast to the formidable challenges to class exploitation mounted by workers in the advanced economies over the past two centuries, and the resulting sharp decline in barriers to class mobility and inequality between classes experienced in most advanced economies in the past 100 years? Other recent contributions to the class dynamics literature have addressed similar questions (Roemer (1982), Matsuyama (2003), Mookherjee and Ray (2002), Axtell, Epstein, and Young (2001), Young and Burke (2001), Banerjee and Newman (1993)).

By class structure we mean the institutions governing the level of the joint surplus and its division between such discrete groups of economic actors as employers and workers, landlords and share croppers, or slave owners and slaves. As these examples suggest, a class structure is not exhaustively described by the distribution of income that it supports, but includes as well the juridical, political, sociological and other aspects of relationships among economic actors, all of which may affect the incentives relevant to both productivity and political behavior.

Are there common structural properties that account for the emergence and persistence of evolutionarily successful institutions governing relations between economic classes? Douglass North (1981) summarized a view that is common among economists: "competition in the face of ubiquitous scarcity dictates that the more efficient institutions will survive and the inefficient ones perish." The theories of property rights due to Harold Demsetz (1966) and firm-specific transaction costs due to Oliver Williamson (1985), Oliver Hart (1995) and others exemplify this presumption of efficient institutional design, as does the political Coase theorem inspired by Donald Wittman (1989) and Gary Becker (1958).

In contrast to these views, Acemoglu, Johnson, and Robinson(2005), Krusell and Rios-Rull(1996), and others model the frequent emergence and persistence of inefficient institutions as the result of conflicting distributional interests among the groups powerful enough to be part of the process by which, in the terms of Acemoglu et al.

"economic institutions are collective choices." In these models inefficiencies arise due to commitment problems facing political actors, but their logic is readily generalized to include other sources such as asymmetric or non-verifiable information and other reasons why bargaining and contracting costs may be significant as in Banerjee, Gertler, and Ghatak(2002), Johansen(1979) and Hirshleifer(2001). This less optimistic view seems more reflective of observed economies, as is suggested by North's comment that "the fact that growth has been more exceptional than stagnation or decline suggests that efficient property rights are unusual in history."(North (1981):6)

Moreover, with two possible exceptions, empirically plausible models of why either competition among entities or individual optimization by powerful actors might implement the efficient design of institutions have not been specified. The first exception is the stochastic evolutionary approach to institutional dynamics, in which equilibrium selection among alternative institutions occurs by idiosyncratic play. These models provide an explanation of the prevalence of fifty-fifty crop shares over long periods and in many economies(Young and Burke 2001).

The second exception is the historical materialism of Karl Marx (1976). He posited that the social revolutions inducing institutional change occur when new knowledge and new technologies cannot be efficiently coordinated by status quo institutions and when social conditions facilitate collective action on the part of those who would benefit from a change. The framework provides important insights on the rise of capitalism in early modern Europe (Brenner (1976), Dobb (1964)) and the demise of Communism (Allen (2003), Roland (1990)).

Whether the efficient design results based either on the Marxian dynamic of class conflict or the idiosyncratic play in the stochastic evolutionary game approach are compelling depends on how well these mechanisms capture the real dynamics of institutional innovation and persistence. Our approach retains the stochastic aspects of the evolutionary approach, but it also draws on the Marxian and other approaches to institutional change that stress intergroup distributional conflict and the role of change agents who deliberately seek to displace the status quo.

We represent institutions as conventions and study transitions among them as a perturbed Markov process in which individuals occasionally play idiosyncratically rather than adopting a best response (Young (1993a), Kandori, Mailath, and Rob (1993)). However, in contrast to the standard stochastic evolutionary models, we represent idiosyncratic play as intentional collective action rather than individual behavioral errors, as in Naidu and Bowles (2005) and Bowles (2004).

As in the standard stochastic evolutionary game theory models, transitions in class structure occur when the number of individuals who reject the terms given by the status quo contractual arrangement is sufficient to induce other best-responding individuals to deviate from the status quo contract as well. In this approach, existing institutions persist, and new institutions emerge in the presence of idiosyncratic play if they have large basins of attraction in an appropriate dynamic. By specifying a historically plausible dynamic we can explore the effects on institutional persistence of such institutional characteristics as efficiency and equality in the production and distribution of the joint surplus.

In the next section we examine a historical case that illustrates the main aspects of the class dynamic we wish to model. Then we introduce a contract game and study the institutional equilibrium selection process when idiosyncratic play is intentional in the sense that deviations from best responses are limited to those which would benefit the individual, were sufficiently many others to do the same. We show that if class sizes and rates of idiosyncratic play are equal, this dynamic reproduces a result analogous to Young's contract theorem (Young 1998), namely that stochastically stable states are both efficient and egalitarian. We then let the sizes of the two classes and their rates of idiosyncratic play differ. The dynamic then selects institutions that favor the less numerous class or the class with the higher rate of idiosyncratic play. If the poorer class is the more numerous (as is typically the case) the equilibria selected need not risk dominant, and may be both unequal and inefficient.

We then study the evolution of class sizes resulting from inter-generational mobility across class boundaries. We model barriers to upward class mobility of the type studied by Sokoloff and Engerman (2000) and show that for a given cost of mobility, there exists a unique equilibrium distribution of class membership and distribution of the joint surplus between the two classes. By limiting the size of the well off class, barriers to class mobility support higher levels of equilibrium inequality. This is true for two reasons: the stochastically stable contract is more unequal, and the endogenously determined class sizes allow the richer class to engage in contracts with a larger number of the poor. We then explicitly model the idiosyncratic play process by making explicit the information available to members of each class. In the penultimate section we introduce governmental policies of redistribution, and also let the rate of idiosyncratic play vary with the degree of class polarization at each state. We suggest that the changing information structure of classes, and heightened polarization of incomes in early capitalism may have provided conditions favorable to the emergence of a redistributive state. In the penultimate section we suggest extensions of the basic model to take account of the effects of technical progress, correlated idiosyncratic play as a result of collective action, endogenous barriers to upwards class mobility, differences in state fiscal and other capacities, and distinct systems of inheritance and marital assortment.

2 Class Conflict and Institutional Transitions

We combine the decentralized individual-based dynamic of evolutionary game theory with the group distributional conflict approach common to political economy because we think that in many historically important cases both aspects were important. Among these cases is the transitions to democratic rule in South Africa.

The labor market aspects of South African apartheid were a convention regulating the patterns of racial inequality which had existed throughout most of South Africa's recorded history and had been formalized in the early 20th century and especially in the aftermath of World War II. For white business owners, the convention might be expressed: Offer only menial jobs at low wages to black workers. For black workers the convention was: Offer one's labor at low wages, do not demand access to skilled employment. These actions represented mutual best responses: As long as (almost) all white employers adhered to their side of the convention, the black workers' best response was to adhere to their aspect of the convention, and conversely.

The power of apartheid labor market conventions is suggested by the fact that real wages of black gold miners did not rise between 1910 and 1970, despite periodic labor shortages on the mines and a many-fold increase in productivity (Wilson (1972)). But a series of strikes beginning in the early 1970's and burgeoning after the mid 1980's with the organization of the Congress of South African Trade Unions (COSATU) signaled a rejection of apartheid by increasing numbers of black workers. The refusal of Soweto students to attend classes taught in Afrikaans and the ensuing 1976 uprising returned civil disobedience to levels not experienced since the anti-pass law demonstrations a decade and a half. The acceleration of urban protests loosely coordinated by the United Democratic Front (UDF), contributed to what came to be termed the "ungovernability" of the country and its businesses. Figure 1 depicts these trends.

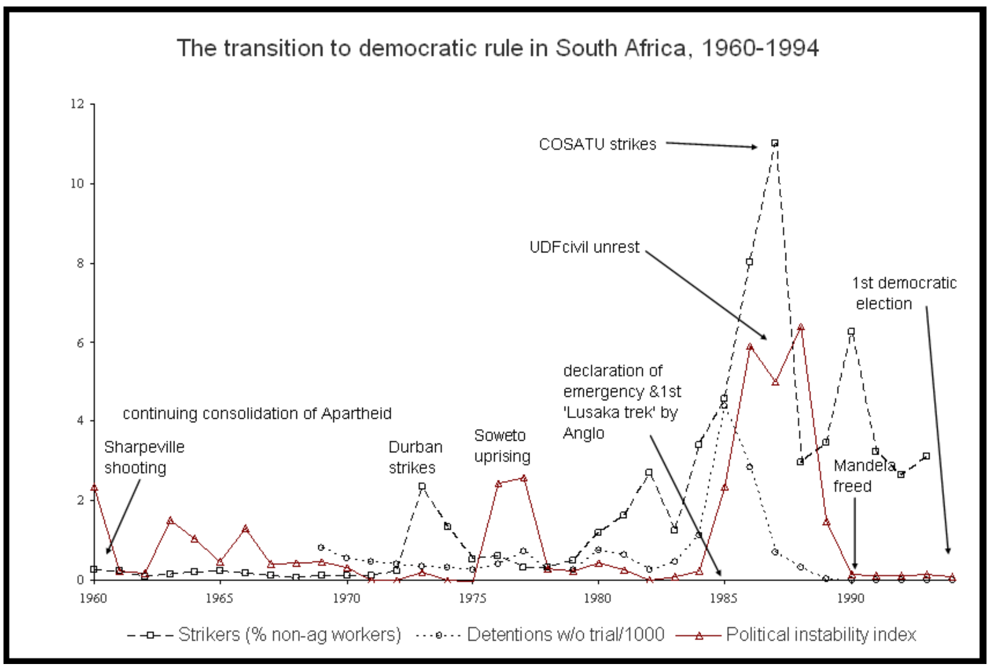


Figure 1: Political and economic disturbances in South Africa, 1960-1994 (Sources: Strikers: Statistics South Africa; Detentions: Institute of Race Relations, Yearbooks; Political Instability: Fedderke, De Kadt, and Luiz (2001))

Many business leaders concluded that adherence to the old convention was no longer a best response, leading them to alter their labor relations, raising real wages and promoting black workers. An executive of the Anglo American Corporation, South Africas largest, commented: "...in the business community we were extremely concerned about the long-run ability to do business..." (Wood (2000): 171) Starting in the mid 1980s, the Corporation developed new policies for 'managing political uncertainty' and to address worker grievances, including granting workers a half day off to celebrate the Soweto uprising. In September 1985, Anglo American's Gavin Relly led several business leaders on a clandestine trek to Lusaka to seek common ground with African National Congress leaders in exile. In 1986 the Federated Chamber of Industries issued a business charter with this explanation: "the business community has accepted that far reaching political reforms have to [be] introduced to normalize the environment in which they do business." FCI (1990). An official of the Chamber of Mines described the situation in 1987

> The political situation in the country was really dismal and we knew that we were going to have one mother of a wage negotiation. And that the issue wasnt what level of increases we negotiated; the issue was do we survive or not? Will there, after this negotiation, still be such a thing as managerial prerogative. Who controls the mines, really? That was what it would boil down to. (Wood (2000): 169)

Reflecting on the 1987 strike in the gold mines, a business executive said:

> ..the most important thing that both sides learned is that you mustn't underestimate the bargaining power of your opponent and his ability to hurt you. .. On our side, gone was the thought that if they strike it will be for only four days (Wood (2000):172).

In addition to conceding many of their black employees' workplace demands, business pressure for political reforms mounted, joined by reform advocates from the governments intelligence services, churches and others. Late in 1989, four years after the state of emergency had been declared in response to the strike wave and urban unrest, F. W. de Klerk replaced the intransigent P. W. Botha as state president. In 1990 he lifted the ban on the African National Congress, the South African Communist Party and other anti-apartheid organizations, and released Nelson Mandela from prison. Mandela was elected president in South Africa's first democratic election in 1994.

Note the following about this process. First, the concessions of best responding businesses to the idiosyncratically playing black workers occurred well before and constituted one of the causes of the eventual abandonment of apartheid by the National Party. The redistribution of economic bargaining power thus predated and contributed to the redistribution of political resources. Second, even at the peak of the 1980s strike wave, the number of black workers rejecting the status quo transaction never exceeded 11 per cent of the non-agricultural labor force. Third, while trade unions, 'civics' (community organizations), and other groups were involved in the rent strikes, student stay aways, and strikes against employers, the rejection of apartheid was highly decentralized and only loosely coordinated prior to the unbanning of the ANC in 1990.

3 Institutional Equilibrium Selection

Suppose we have a large population of agents of size N, and that γ is the fraction of population that are of class A, ignoring integer problems. Each period, agents from class A are randomly matched with agents from class B and play the following contract game, with A as the row player illustrated in table 1. We consider A the non-elite agents, or poor, and B the elite, or wealthy agents. Each side proproses a contract (termed 0 or 1) governing the distribution of the surplus (e.g. union recognition, crop-shares, or land tenure norms). If they fail to coordinate on a contract, they both get 0, reflecting the fact that agents are bargaining over a discrete institution, agreement on which is necessary for the production of a surplus, rather than simply over a divisible surplus.

	0	1
0	$\sigma\rho, (1-\sigma)\rho$	0,0
1	0,0	1,1

Table 1: Payoffs in the Contract Game

Assume that $\sigma\rho < 1 < (1-\sigma)\rho$, reflecting the assumption that the poor agents do worse in an unequal arrangement. Notice that this game has two strict Nash equilibria $(1, 1)$ and $(0, 0)$. Agents are myopic, and play a best response to the distribution of play in the previous period. This will define a large state-space. If we suppose that γ is fixed, then we can represent the state-space by α, β, where α and β denote the fraction of class A and class B playing 1.

The implied Markov process induced by the best-response dynamic has two recurrent classes, which correspond to the strict Nash equilibria of the contract game. The advantage of the stochastic evolutionary approach is that, by introducing small perturbations ϵ, a single recurrent class, and therefore a Nash equilibrium, can be selected. The perturbations correspond to non-best-response behavior, which we term "deviant" or "idiosyncratic". We have in mind rejections of the terms of the status quo contract by either side, such as lockouts, union decertification campaigns, private enclosures of common lands, strikes, slave revolts, and urban food riots. (McAdam, Tarrow and Tilly, 2001)

Suppose also that the status quo convention is (0,0) namely the convention that favors the better off Bs. If sufficently many As demand contract 1 rather than the status quo contract 0, best responding Bs will switch to offering contract 1. The minimum number of As deviating from the status quo to induce a switch from contract 0 to contract 1, R_{01}, is termed the resistance for a transition from 0 to 1 is given by (1). The corresponding resistance for a B-induced transiton from the 1 contract to the 0 contract is given by (2). Without loss of generality, we normalize all the resistances by N, so that they are approximated by:

$$R_{01} = \gamma \frac{(1-\sigma)\rho}{1+(1-\sigma)\rho} \tag{1}$$

$$R_{10} = (1-\gamma) \frac{1}{1+\sigma\rho} \tag{2}$$

If the rates of idiosyncratic play do not differ between the classes, the population will spend most of the time at the convention whose displacement requires more deviations from the status quo. This is the stochastically stable state, given by i such that $R_{ij} > R_{ji}$. In this case the expected waiting time before a transition out of i to j will exceed that of the reverse transition, so that the population will spend more than half of the time near the convention given by i.

These resistances differ from those in the standard perturbed Markov process models in which the resistances that drive transitions are identified by letting ϵ go to zero so that transitions are induced by the idiosyncratic play of that group for which the least number are required to induce the best responders in the other group to switch strategies(Kandori, Mailath, and Rob 1993, Young 1993, Binmore, Samuelson and Young 2003). By contrast, the above resistances are the least number of idiosyncratic plays by those would gain that is required to induce a transition. In the contract game, it is always the case that fewer errors are required by members of the group that stands to lose from the transition. The reason is that inducing a switch by those who would benefit from a transition requires fewer idiosyncratic players on the other side than it takes to induce a switch by those who would be worse off should the transition take place. Thus the resistances that drive the two processes are always different: the minimal fraction of a class necessary to induce a transition in the standard perturbed Markov process model is always less than one half, while in ours it is always greater than one half.

Thus the long run behavior of the system when ϵ is non-vanishing can by summarized by τ_0, the expected fraction of time that contract 0 will be the most common contract. To determine τ_0 we calculate the expected waiting time (number of periods) before non best response play by the As induces a transition from contract 0 to contract 1. This is the inverse of the probability, μ_0, that in any period a transition from out of contract 0 will be induced. To determine this probability we count the subsets of As sufficiently numerous to induce a transition, then determine the probability (given ϵ) that each subset will be drawn; then sum these probabilities to get the probability that any transition-inducing event occurs, μ_0:

$$\mu_0 = \sum_{j \geq R_{01}N}^{\gamma N} \binom{\gamma N}{j} \epsilon^j (1-\epsilon)^{\gamma N - j} \tag{3}$$

An analogous calculation gives us the probability of a transition from 1:

$$\mu_1 = \sum_{j \geq R_{10} N}^{(1-\gamma)N} \binom{(1-\gamma)N}{j} \epsilon^j (1-\epsilon)^{\gamma N - j} \tag{4}$$

The long-term behavior of the system is summarized by the relative times spent in each state:

$$\tau_0 = \frac{1}{1 + \frac{\mu_0}{\mu_1}} \tag{5}$$

$$\tau_1 = \frac{1}{1 + \frac{\mu_1}{\mu_0}} \tag{6}$$

In the two contract case we say that contract j is selected if $\tau_j > \frac{1}{2}$ and that parameter changes favor contract j if they increase τ_j. We restrict ourselves to the two-contract case for tractability as well as ease of exposition; some of our results extend to many contracts, presented in Naidu and Bowles(2005).

4 An Institutional Equality-Efficiency Trade-Off

Setting $R_{01} = R_{10}$ from (1) and (2) gives the characteristics of unequal contracts such that the population would spend half of the time at the unequal and half at the egalitarian contract, that is $\tau_0 = \tau_1 = \frac{1}{2}$.

$$\gamma \frac{(1-\sigma)\rho}{1+(1-\sigma)\rho} = (1-\gamma)\frac{1}{1+\sigma\rho} \iff \frac{1-\gamma}{\gamma} = \frac{(1-\sigma)\rho + \rho^2(1-\sigma)\sigma}{1+(1-\sigma)\rho} \tag{7}$$

It is simple to check that if $\gamma = 1/2$, the stochastically stable equilibrium is risk-dominant. In the 2x2 contract game, this will be the contract that maximizes the product of the payoffs of the two classes, namely $\rho^2(1-\sigma)\sigma$ for convention 0 and 1 for convention 1. Thus, if $\rho^2(1-\sigma)\sigma > 1$ then $R_{01} > R_{10}$, and 0 will be selected, and the reverse inequality implies that 1 is selected.

Proposition 4.1. *For the contract set above and the dynamic process with transition resistances R_{01} and R_{10}, and $\gamma > \frac{1}{2}$, we have*
1: If $\rho > 2$, then there exists a $\sigma^* < \frac{1}{2}$ such that for $\frac{1}{2} > \sigma > \sigma^*$, contract 0 is selected, and if $0 < \sigma < \sigma^*$, contract 1 is selected.
2: $\frac{d\sigma^*}{d\rho} < 0$
3: If $\rho < 2$ then contract 1 is selected.
4: $\frac{d\sigma*}{d\gamma} < 0$

Proof. 1 follows from equation (1) and (2), since $\sigma = 0$ ensures that $R_{01} < R_{10}$ and $\sigma = \frac{1}{2}$ ensures that $R_{01} > R_{10}$.
2 and 4 follow from totally differentiating equation (3), setting the result equal to 0, and solving for $\frac{d\sigma}{d\rho}$ and $\frac{d\sigma*}{d\gamma}$.
3 follows from noting that $\rho < 2$ implies $R_{01} < R_{10}$.

□

Proposition 4.1 demonstrates what we term the "institutional efficiency equality tradeoff"(Bowles 2004). When the classes are of equal size, and σ is strictly less than one half, the unequal convention (0,0) will be stochastically stable only if the joint surplus that it generates is correspondingly greater than the egalitarian contract. This expression reproduces the intuition underlying Young's contract theorem.

The reason why the unequal convention is disfavored in this dynamic is transparent from figure 1, which gives the resistances for moving to and from a benchmark contract with payoffs, 1,1, when the other contract has payoffs $\sigma\rho$ and $(1-\sigma)\rho$. Notice that if the As get nothing in the unequal contract ($\sigma = 0$), even if all of the B's (that is $1-\gamma$) idiosyncratically offer their preferred contract, it is only a weak best response for the A's to concede, as they would thereby gain nothing. By contrast, idiosyncratic play by fewer than all of the A's, namely, $\gamma\frac{\rho}{1+\rho}$, is sufficient to induce a transition in the other direction, that is, out of the $\sigma = 0$ contract. As can be seen, as the unequal contract becomes more unequal both resistances increase, but R_{10} rises faster than R_{01}, so movement from the equal to the unequal convention become relatively more difficult. This is the reason why highly unequal contracts are not stochastically stable.

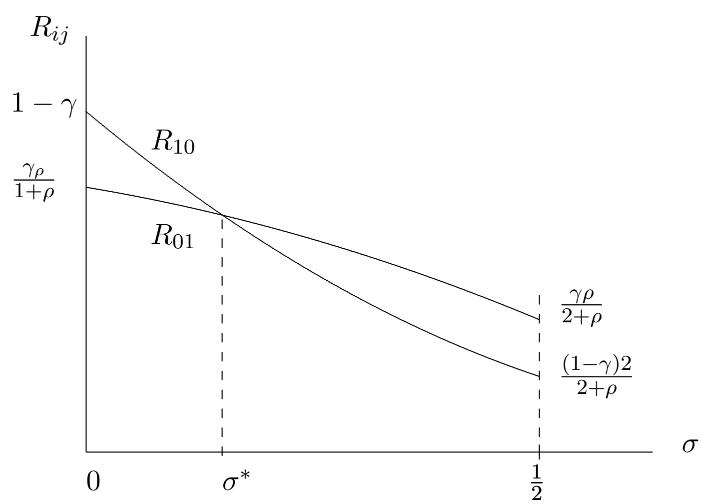


Figure 1. Resistances for a transition to the equal and unequal contract as a function of the degree of inequality in the latter. If the As share in the unequal contract, $\sigma > \sigma^*$ the unequal convention is selected; for shares less than σ^* the equal contract is selected.

However, if class sizes differ and if the As are more numerous, the evolutionary dynamic need not favor the egalitarian contract. As as is clear from (7) and figure 1, for any given level of σ and ρ, there exists some level of γ such that if the relative size of the A class exceeds this critical level, the unequal contract will be selected. Note that this runs counter to the standard results about equilibrium selection in 2x2 games, where the risk-dominant equilibrium is selected. In this model, even if $\rho^2\sigma(1-\sigma) < 1$, the unequal contract can be selected as the stochastically stable equilibrium if γ is sufficiently large.

Thus the equilibrium selection process favors smaller classes. The reason is not the incentive-based logic stressed by the political science literature on collective action inspired by Olson (1965); nor is it related to the fact that excess supply of a factor of production may disadvantage its 'owners' in markets. Rather the advantage of small size arrises, simply because smaller groups are more likely to experience realizations of idiosyncratic play large enough to induce a transition as long as the rate of idioscyncratic play is less than the critical fraction of idiosyncratic players required to induce a transition.

5 Endogenous Class Sizes

The assumption that class sizes are given may now be relaxed. Suppose that being a member of the B class requires that one's parents have joint income not less than some minimum amount, which, for simplicity, constitutes the next generations inheritance. This impediment to class mobility arises because class membership requires that one undertake a project with a minimum size, as in Legros and Newman (1996), for example owning capital goods sufficient to employ an economically viable team of workers. In this case, inheritance of the asset is required because members of the less well off class are credit constrained. In the absence of credit constraints, the minimum asset requirement could also reflect educational, life style, and other impediments to vertical mobility. Those who inherit less than this amount are members of the A class. In the resulting model, then, the stochastically stable contract and the relative sizes of the two classes will be jointly determined. We will make the simplifying assumption that the population dynamic is rapid relative to the best-response dynamic, so that we only need to analyze population steady-states.

Changes in the sizes of the two classes will occur due to class mobility that occurs when an offspring of a B parent has insufficient wealth to retain its upper class status, or when a child of an A has sufficient wealth to change class membership. How often this occurs will depend on four things: the degree of class assortment in parenting, the inheritance rules in force (primogeniture or equal inheritance, for example), the minimal inheritance required for membership in a given class, and the payoffs of the two parents. We assume equal inheritance to the two offspring of each couple and abstract from marital assortment as it will not affect the resulting equilibria. We assume that when parents belong to the same class, the two offspring retain the parents' class membership, the payoffs of two B's always being sufficient for both offspring to remain B's and the payoffs to two A's never being sufficient to allow their two offspring to become

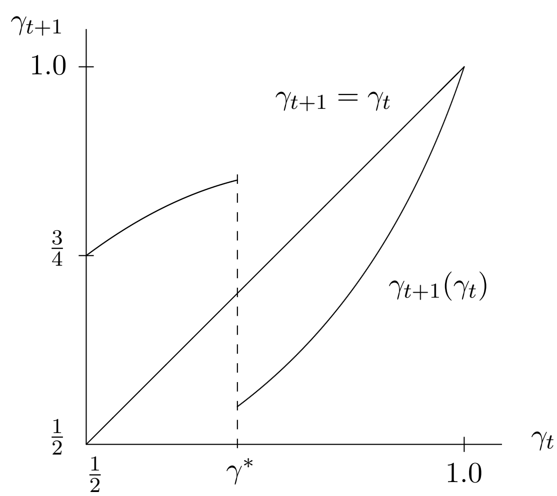
B's.

The mean income of the cross-class couple is given by the mean income of the A parent, $\alpha\beta + (1-\alpha)(1-\beta)\rho\sigma$, plus the mean income of the B parent. Thus :

$$y_c(\gamma) := \alpha_t\beta_t + (1-\alpha_t)(1-\beta_t)\rho\sigma + (\beta_t\alpha_t + (1-\beta_t)(1-\alpha_t)\rho(1-\sigma))\frac{\gamma_t}{1-\gamma_t} \qquad (8)$$

where the last term is the income of the B parent (who gains a mean payoff of $\beta_t\alpha_t + (1-\beta_t)(1-\alpha_t)\rho(1-\sigma)$ in each of $\frac{\gamma}{1-\gamma}$ interactions with members of the A class.) The amount required for entry to the B-class is $\overline{\pi}$, so children of the cross-class couple will become members of the B class if $y_c - 2\overline{\pi} > 0$.

The change in γ from one generation to the next is as follows. Letting γ_{t+1} represent the fraction of As in the next generation, it will equal the fraction of As this generation plus the children of cross-class marriages that became As minus the children of cross-class marriages that became Bs. Each cross-class couple produces either 2 B children or 2 A children, subtracting or adding one member of the A class (since one of the parents is of each class). In the absence of class assortment in marriage, the number of cross-class couples is $\gamma(1-\gamma)$. Thus we can write:



textbfFigure 2. Endogenous determination of class size. For $\gamma < \gamma^*$ the cross-class couple has insufficient income for their children to become Bs; for $\gamma > \gamma^*$ both of the cross class couple's children become Bs.

$$\gamma_{t+1} = \gamma_t + \gamma_t(1-\gamma_t)sgn(2\overline{\pi} - y_c(\gamma_t)) \qquad (9)$$

To show that such an equilibrium exists, is unique, and is stable, it is necessary and sufficient that $\gamma_{t+1}(\frac{1}{2}) > \frac{1}{2}$ and that for some $\gamma \in (\frac{1}{2}, 1)$, $\gamma_{t+1}(\gamma) < \gamma$. Recall that the income of the cross-class couple is increasing in γ. For a given σ, the existence and stability condition will obtain if the expected income of the poorest possible cross-class couple (that is when $\gamma = \frac{1}{2}$) is insufficient to allow upward mobility for their offspring, while the expected income of the richest possible cross-class couple ensures upward mobility. The first condition obtains if $\overline{\pi} > \rho/2$ because with $\gamma = \frac{1}{2}$, the cross-class couple has an income of just ρ in the unequal contract. To see when the second condition obtains, note that the richest possible cross-class couple, in the unequal contract, is that in which $\gamma = \frac{(N-1)}{N}$ so that the lone B in the population is married to an A, and interacts economically with every A member of the population including the spouse. The income of this couple, summing the income of the B and the A is $(N-1)(1-\sigma)\rho + \sigma\rho$. Both children of this couple will become Bs, then, if this income exceeds $2\overline{\pi}$ or $\overline{\pi} < \frac{(N-1)\rho - \sigma(N-2)}{2}$.

The second condition is most stringent, in the unequal contract, when $\sigma = 1/2$ and is then equivalent to $\overline{\pi} < \rho N/4$. Thus there will exist a stable population distribution between the two classes if the cost of vertical mobility is more than half the average income and less than a quarter of the total income of the society. These two conditions ensure that the equilibrium A-class size γ^* is that which equates $2\overline{\pi}$ to $y_c(\gamma)$, and because y_c is monotonic in γ, this equilibrium is unique. Thus finding the value of γ that equates $2\overline{\pi}$ to the income of the cross-class couple for the two contracts we have:

$$\gamma_0^* = \frac{2\overline{\pi} - \sigma\rho}{(1-\sigma)\rho + 2\overline{\pi} - \sigma\rho} \tag{10}$$

$$\gamma_1^* = 1 - 1/2\overline{\pi} \tag{11}$$

As expected, both are increasing in $\overline{\pi}$. This generates a joint discrete dynamic system on the space α, β, γ, that, without perturbations, has two recurrent classes, given by $(1, 1, \gamma_1^*)$, which we will call 1 when there is no ambiguity, and $(0, 0, \gamma_0^*)$, which we will similarly call 0. It is straightforward to compute the (approx.) resistances for the transistions between these two recurrent states. Since the relative populations are different in each equilibrium, and the population that is playing idiosyncratically is different in each equilibrium, we obtain the following resistances:

$$R_{01} = \gamma_0^* \frac{(1-\sigma)\rho}{1 + (1-\sigma)\rho} \tag{12}$$

$$R_{10} = (1 - \gamma_1^*) \frac{1}{1 + \sigma\rho} \tag{13}$$

Thus we are able to explore the effects of exogenous changes in $\overline{\pi}$,σ ,and ρ on the stochastically stable contract, the equilibrium class sizes, and hence on the income inequality between members of the two classes. Intuitively we would expect that as the barrier to mobility increased (higher $\overline{\pi}$) the poor class would be more numerous in equilibrium and the population would spend a larger fraction of the time at the unequal contract. The result of these two consequences of an increase in $\overline{\pi}$, one would expect, would be to increase the income difference between the two classes.

Proposition 5.1 shows that these intuitions are correct.

Proposition 5.1. *Given ρ, σ, there exists a value of $\overline{\pi}^* = \overline{\pi}^*(\sigma, \rho)$ such that, for all $\overline{\pi} > \overline{\pi}^*$, we have $(0, 0, \gamma_0^*)$ as the stochastically stable state.*

Proof. Select $\overline{\pi}^*$ such that $R_{01} = R_{10}$, so that it is equally easy to leave $(1, 1, \gamma_1^*)$ as $(0, 0, \gamma_0^*)$. Using (10) and (11), the resulting $\overline{\pi}^*$ solves:

$$(1-\gamma_1^*)(\overline{\pi})\frac{1}{1+\sigma\rho} = \gamma_0^*(\overline{\pi})\frac{(1-\sigma)\rho}{1+(1-\sigma)\rho} \iff \frac{1}{2\overline{\pi}}\times\frac{1}{1+\sigma\rho} = \frac{2\overline{\pi}-\sigma\rho}{(1-\sigma)\rho+2\overline{\pi}-\sigma\rho}\times\frac{(1-\sigma)\rho}{1+(1-\sigma)\rho}$$

Now note that $\frac{2\overline{\pi}-\sigma\rho}{(1-\sigma)\rho+2\overline{\pi}-\sigma\rho}$ is increasing in $\overline{\pi}$, which implies that $\frac{(1-\sigma)\rho+2\overline{\pi}-\sigma\rho}{(2\overline{\pi})(2\overline{\pi}-\sigma\rho})$ is decreasing in $\overline{\pi}$ and asymptotes to 0 as $\overline{\pi}$ gets large. Therefore, there is a $\overline{\pi}^*$ such that, $\forall\overline{\pi} > \overline{\pi}^*$ we have $R_{10} < R_{01}$, which implies that 0 is stochastically stable.

□

Proposition 5.1 says that greater barriers to vertical class mobility stabilize the more unequal contract, an implication of which is that the risk dominant contract will not be selected if the cost of vertical class mobility is sufficiently high. Note (from 10, given $2\overline{\pi} > \rho$) that $d\gamma_0^*/d\sigma > 0$ so more unequal contracts are associated with reduced endogenous size of the poorer class. The reason is that the income of the cross class couple increases with the inequality of the contract (because the B member of the couple is married to a single A but interacts economically with more than one A) so for a given $\overline{\pi}$ upward mobility will be greater the more unequal division of the surplus. Given that smaller class size facilitates transitions out of contracts that are disadvantageous to a class, this provides an additional reason why unequal contracts may not be stochastically stable.

An increase in ρ (from 10) lowers γ_0^* as it increases the income of the cross-class couple, thereby facilitating mobility out of the A-class, reducing the equilibrium size of that class and hence favoring them. But the effect on equilibrium selection is ambiguous, as an increase in the productivity of the unequal contract (with no change in the equal contract) will increase the fraction of idiosyncratically playing As necessary to induce the best responding Bs to abandon the unequal contract and at the same time reduce the number of idiosyncratically playing B's required to induce a shift from 1 to 0. However, a proportional increase in the productivity of both contracts, for example, scaling up the payoff matrix in Table 1 by some $\omega > 1$, does not affect the fraction of each class whose idiosyncratic play is sufficient to induce a transition. In this case the only effect is, assuming $\overline{\pi}$ fixed as above, to reduce the equilibrium size of the A-class in both contracts, favoring the As and unambiguously increasing the fraction of time spent at the more equal convention.

6 Modern and Pre-Modern Class Structures

Over the course of history, institutions that have implemented unequal outcomes have differed dramatically in ways not captured thus far by our evolutionary contract game. As a result we might expect that contracts with identical values of σ and ρ would experience quite different dynamics. The distributional consequences of a class structure are thus necessary but not sufficient to the explanation of its historical trajectory.

This reasoning originates with Marx's distinction between the industrial proletariat, whose agglomerated conditions of work facilitated coordinated collective identity and action, on the one hand, and the pre-capitalist agricultural and urban *lumpen-proletariat*, whose dispersed conditions of work did not (Marx (1963)). The difference between the two, according to Marx, was not that the former is exploited more intensively than the latter, but that the labor markets characteristic of modern capitalist economy facilitate the emergence of a culture of solidarity among workers. Marx used the expression "abstract labor" to capture the lack of transaction-specific assets and footloose nature of employees in classical capitalism. By contrast, earlier class systems, according to Ernest Gellner(1983), were characterized by "laterally separated petty communities of the lay members of society" speaking different dialects or even languages, presided over by a culturally and linguistically homogeneous class. Class relations in such societies often took the form of patron-client relationships that endured over generations with little mobility of the clients among the patrons (Fafchamps (1992), Platteau (1995), Blau (1964)).

The patron client relationship will support a very different dynamic from the relationship of employee to employer in the modern labor market. The reason is that these two institutions affect the information available to agents when they adopt best responses. Suppose that when adopting a best response the members of the two classes do not know the entire distribution of play in the previous period. Thus As and Bs know the play of a fraction of the opposing class given by k_A and k_B. Pre-capitalist agrarian class structures, in Gellner's view, entailed $k_A < k_B$, for the upper class communicated readily amongst themselves and therefore had information about the recent play of a large segment of the less well-off class. The geographical, cultural and linguistic isolation of the As, by contrast, militated against information sharing beyond ones local community.

The advantage enjoyed by the B's is not that a given B-patron may engage the A-clients of other B's. Rather, by drawing information from a larger sample of A's, the B's less noisy signal of the distribution of play reduces the likelihood that their myopic best response will overreact to the chance occurrence of a high level of idiosyncratic play among their particular A-clients.

Assuming for simplicity that γ_1, γ_0 are given, the resistances R_{01} and R_{10} are:

$$R_{10} = k_A(1-\gamma_1^*)\frac{1}{1+\sigma\rho} \tag{14}$$

$$R_{01} = k_B\gamma_0^*\frac{(1-\sigma)\rho}{1+(1-\sigma)\rho} \tag{15}$$

The two 'scope of vision' parameters in the resistances mean that more idiosyncratic players are required to induce a concession by the best responding members of the population that has more information. If k_A is small, then it only takes a few idiosyncratic plays by Bs to convince the best responding As to concede to the unequal contract. As is evident from (14) and (15) an increase in k_A is equivalent to a increase in the size of the B population and conversely for an increase in k_B.

Proposition 6.1. *For given $\gamma_0, \gamma_1, \rho > 2, \sigma < \frac{1}{2}$, there is a $\frac{k_A^*}{k_B^*}$, such that for all $\frac{k_A}{k_B} < \frac{k_A^*}{k_B^*}$, the unequal contract is stochastically stable.*

Proof. Obvious from comparing the resistances.

□

The "institutional efficiency-equality tradeoff" obtains in this extended model, but a highly unequal contract may nonetheless be selected in this dynamic as long as the information structure associated with it is characterized by a sufficiently low value of $\frac{k_A}{k_B}$.

The model thus suggests a possible reason for the trend in many countries over the past 2 centuries towards a reduction in the relative incomes of the well off(Piketty 2005). The geographic, industrial, and occupational mobility characteristic of modern labor markets (coupled with the spread of literacy and greater ease of communication) made workers less responsive to the demands of a small number of local employers, as they both knew about and could imagine taking advantage of the offers of employers outside their local area. The effect would be to raise k_A and thus to destabilize highly unequal contracts. The global integration of national economies may reverse this process, recreating something akin to Gellner's view of pre-capitalist class structures with a culturally unified, cosmopolitan B-class, and a nationally and culturally, parochial segmented A class (Bowles and Pagano 2006).

7 Polarization and Redistributive Politics

Here we introduce a government that seeks to stabilize the unequal class structure by adopting ameliorative policies of redistribution. Class polarization, as studied by Duclos, Esteban, and Ray (2004), may enhance the frequency of deviant play by the less well off group by providing additional motives and opportunities to challenge the status quo contract (Scott, 1976, Moore, 1978, Wood, 2003). To capture this insight we make the rate of idiosyncratic play state dependent, and study the response of a far-sighted state seeking on behalf of the myopic Bs to deter a transition to the egalitarian state.

Bergin and Lipman(1996) show that, if one allows ϵ to vary arbitrarily as an error function of the state, θ, then one can choose a function that selects any recurrent class of the unperturbed process as the stochastically stable state. But what error functions are empirically plausible? We would like to capture the idea that idiosyncratic play will be greater in highly unequal societies. To do this simply, we modify our preceding model, letting a state-dependent idiosyncratic play rate be given by:

$$\epsilon(\theta) = \epsilon^{\frac{1}{1+\lambda(\pi^B(\theta)-\pi^A(\theta))}} \tag{16}$$

Where $\lambda > 0$ captures the extent to which inequality increases idiosyncratic play, and $\pi^B(\theta), \pi^A(\theta)$ are the payoffs to the members of the two classes in state θ. Redistributive policies funded by a tax on B income at rate t will thus reduce the rate of idiosyncratic play in the unequal contract. Sociological conditions favoring subordinate responses to inequality as well as religious or other cultural settings that make economic inequality illegitimate will increase λ. In the equal contract, the B class idiosyncratically plays at rate ϵ, since $\pi^B(1) = \pi^A(1) = 1$, and in the unequal contract, the A class plays at a rate $\epsilon^{\phi(t;\lambda)}$, where $\phi(t;\lambda) := frac11 + \lambda((1-2\sigma)\rho - 2t(1-\sigma)\rho)$, which is clearly increasing in σ. This implies the next proposition.

Proposition 7.1. *Given any ρ, γ, and σ such that $\rho^2\sigma(1-\sigma) > 1$, so that contract 0 is risk dominant, but $\epsilon(\theta)$ is as above. Then if $t = 0$, there exists some λ^* such that for any $\lambda > \lambda^*$ the equal contract is selected.*

Suppose, now, that the government implements the policy preferences of the p percentile of the income distribution (following Benabou, 2000), choosing taxes and transfers to maximize the expected income of the members of the class to which this percentile belongs. We restrict attention to the case when $p > \gamma$; the state acts as the custodian of the long-term interest of the Bs.

The state chooses a level of redistribution at the unequal contract. Each B pays a tax on the surplus it receives from each worker. Each worker receives an equal share of the total taxes collected. Thus, in each transaction, B members receive $(1-t)(1-\sigma)\rho$, while A members receive $\sigma\rho + t(1-\sigma)\rho\frac{(1-\gamma)}{\gamma}\frac{\gamma}{1-\gamma}$, since total taxes collected are $t(1-\sigma)(\rho)(1-\gamma)\frac{\gamma}{1-\gamma}$. So, for a given tax rate, the class difference in income is $(1-2\sigma)\rho - 2t(1-\sigma)\rho$, thus the rate of idiosyncratic play in the unequal contract is $\epsilon^{\frac{1}{1+\lambda((1-2\sigma)\rho-2t(1-\sigma)\rho)}}$. As the tax rate rises, the rate of idiosyncratic play by the A class falls in the unequal state.

Recall that $\tau_0 = \frac{1}{1+\mu_0(t)/\mu_1(t)}$. We need to explore the effect of the tax on μ_0 and μ_1. We can write the probability of exit, taking into account the effect of inequality on idiosyncratic play as:

$$\mu_0(t) = \sum_{j \geq R_{01}(t)N}^{\gamma N} \binom{\gamma N}{j} \epsilon^{j\phi(t;\lambda)}(1-\epsilon^{\phi(t;\lambda)})^{\gamma N - j} \tag{17}$$

which, for ϵ small, implies that we only have to concern ourselves with the minimum number of agents necessary to induce a transition. Thus (19) is on the order of $\epsilon^{R_{01}(t)\frac{1}{1+\lambda(((1-2\sigma)\rho-2t(1-\sigma)\rho)}}$. Note that $\mu_1(t)$ is the same as expression (4), save for the fact that R_{10} is now a function of t, since there is no change in the rate of idiosyncratic play in the equal contract.

The tax has three effects on τ_0 and they do not all have the same sign. As intended, it reduces the rate of idiosyncratic play by the As, making a transition from 0 less likely. But it has two consequences working against this effect. By reducing the difference in the B's payoffs in the two contracts it reduces R_{01}, the number of idiosyncratically responding As required to induce a transition out of the unequal contract. For analogous reasons, it also reduces R_{10}, the number of idiosyncratically responding Bs required to induce a transition out of the equal contract. However, we can show:

Lemma 7.2. *If λ is sufficiently large, $\frac{d\tau_0}{dt} > 0$.*

Proof. see Appendix. □

The maximum problem for the state must weigh the costs of the tax on the per period income of the Bs against the effect of the effect of reduced income polarization on the probability of a transition out of the unequal to the equal state μ_0. The expected (undiscounted) income of the B class is given by

$$W_B = (1-t)(1-\sigma)\rho\tau_0 + (1-\tau_0) \tag{18}$$

with the transparent first order condition

$$-(1-\sigma)\rho\tau_0 + \frac{d\tau_0(t)}{dt}((1-t)(1-\sigma)\rho - 1) = 0 \tag{19}$$

The first term is the marginal cost to the B-class of the tax, and if $\frac{d\tau_0}{dt} > 0$ and $t < 1 - \frac{1}{(1-\sigma)\rho}$ the second is the marginal benefit. A tax rate in excess of $1 - \frac{1}{(1-\sigma)\rho}$ makes the egalitarian contract preferable to the Bs so they would not benefit from a such tax if it did perpetuate the unequal contract.

We proceed by eliminating 0 and $\bar{t}$ as solutions and appealing to well-known results about continuous functions on closed intervals. Clearly the tax rate cannot be $\bar{t}$, as then all the B's are indifferent between the two contracts. They can clearly do better with no tax rate at all. But $t = 0$ cannot be optimal either, for sufficiently large λ. The reason is that that in (19) increasing λ lowers τ_0, reducing the "marginal cost" of the tax, and raises $\frac{d\tau_0}{dt}$, raising the marginal benefit.

Proposition 7.3. *For sufficiently large λ, there exists a positive interior tax rate $t^* \in (0, \bar{t})$ where $\bar{t} = 1 - \frac{1}{\rho(1-\sigma)}$*

Proof. see Appendix. □

It follows readily that $\frac{dt^*}{d\lambda} > 0$ because as we have just seen from differentiating the first order condition (19) with respect to λ, the marginal costs of the tax are declining in λ and the marginal benefits are rising. Thus as sociological or ideological conditions become more hostile to inequality one would expect far sighted governments acting on behalf of the long term interests of the B class to subject the well to do to redistributive taxation.

8 Extensions

Our model of institutional equilibrium selection captures some key aspects of historically observed class dynamics. Among these are the ways that smaller group size may enhance bargaining power, the relationship between barriers to class mobility and the long run degree of income inequality, the importance of class differences in network structure and information, and the effects of state-sponsored ameliorative redistribution on the evolution of class structures. The model is readily extended to take account of other aspects of class dynamics.

Technological progress. A correspondence between technologies and class structures is suggested by Marx's famous aphorism: "The hand-mill gives you society with the feudal lord; the steam-mill, society with the industrial capitalist." (Marx (1959 [1847])). While his account is overly simple, there does seem to be a rough correspondence between technologies and social structures as is suggested by the major institutional shifts associated with the advent of agriculture (Boyd and Richerson, 2001) and the later emergence of industrial production or the common association of slavery or latifundia agriculture with some crops and yeoman farming with others (Ortiz 1963). Thus class structures may differ with respect to both the size of the joint surplus ρ and the cost of entering the well off class ($\overline{\pi}$). Sharecropping, for example, is commonly practiced in low productivity agricultural pursuits in which, due to, e.g., the absence of economies of scale, the cost of becoming a (small) landowner is quite minimal. Slave economies commonly comprised high productivity activities with high entry costs to the favored class (due to economies of scale in processing and marketing and the cost of acquiring personal freedom). By contrast to the class structures that it replaced, early capitalism was characterized by high levels of both productivity and cost of entry to the favored class. Our model would then predict an initial period of sharply rising inequality between employers and workers. The absence of a sustained increase in real wages during the first century of the industrial revolution in both Europe and Japan is consistent with this pattern (Allen, 2005).

Collective action. The only corporate actor we have considered thus far is the state; but some individual members of each class may choose to act in unison, either best responding or playing idiosyncratically. The members of a trade union may decide to work under the current contact, or to refuse to do so. Where members of such organizations may commit themselves to acting in unison, class dynamics are affected in two ways. First the effective size of the class is reduced to the number of autonomously acting entities, a number less than the number of individuals in the class. The effect is increase the fraction of time spent governed by the class' favored contract. The second effect is to alter the rate of idiosyncratic play, the sign of the effect depending on the structure of the social interactions among the subgroups with each class. Models of social networks in which one's behavior is influenced by (but not identical to) one's neighbors provide a framework to take account of the fact that corporate bodies such as trade unions and business associations are rarely able to perfectly enforce action in unison (Young (2002), Durlauf (2001)).

Endogenous barriers to class mobility. Suppose that $\overline{\pi}$ is the cost of capital necessary to hire the average number of As employed by a B, or $\kappa\gamma/(1-\gamma)$ here κ is the capital required to hire a single worker. Now as γ increases, the cross-class couple, as before becomes richer, but the minimal amount required for their two children to become Bs also rises. As a result $2\overline{\pi}(\gamma) - y_c(\gamma)$ need not be monotonically declining in γ, so more than one change in $sgn(2\overline{\pi}(\gamma) - y_c(\gamma))$ may occur over the interval $\gamma \in (\frac{1}{2}, 1)$. Thus, there may be multiple endogenously determined class sizes for a given contract, the same σ and ρ supporting a highly unequal society in which each B profits from interacting with many As and a more egalitarian one in which Bs interact with few As.

The cost of upward class mobility may also be endogenous due to the actions of the state. We have modeled state redistribution in the interest of perpetuating the status quo (unequal) class structure by attenuating the associated income differences. Similar policies have been widely adopted to reduce $\overline{\pi}$, the (private) cost of upward class mobility. Included are public education, meritocratic promotion rules and policies to relax the credit constraints facing the less well off. Notice, however, that these policies may have ambiguous affects. They could succeed in raising τ_0 if they legitimated the income differences associated with the unequal contract, lowering λ and hence reducing the responsiveness of the As idiosyncratic play to class disparities in income. But by reducing the equilibrium size of the A-class and the payoffs to the Bs in the unequal contract, these policies reduce R_{01} thereby facilitating transitions out of the unequal state.

State capacities. Rather than assuming costless tax and transfer policies it would be more plausible historically to posit that some fraction (β, for 'leaky bucket') of the revenues collected is not transferred to the A class but is claimed by tax farmers and administrators. States may also affect the off diagonal payoffs, so that the likely cost of idiosyncratic play is not merely a contract foregone, but may include direct costs imposed on deviants.

Tensions between bargaining power and political power. We have seen (proposition 4.1) that a reduction in the size of the A-class confers advantages in bargaining to the As. But with the extension of suffrage to male (and eventually all) workers, the political power of the A-class is likely to increase with its size. Once we incorporate into the model a state whose managers are selected in competitive elections with an inclusive electorate, a tension emerges between the requisites for success in private contractual bargaining and success in the public determination of the tax and other policies that shape these bargains. This is another reason for the common observation that trade unions benefit by being exclusive while their affiliated political parties benefit by being inclusive.

Inheritance systems, marital assortment, and demographic structure. For simplicity we abstracted from marital sorting, income pooling within couples, and sex differences and we assumed equal inheritance to each of two members of the next generation for both classes. Taking account of each produces interesting variants of the above dynamic. For example, as is clear from (9) marital assortment reduces the number of cross-class couples (to less than $\gamma(1-\gamma)$ but does not affect the equilibrium class sizes. However, fewer cross-class marriages could affect class dynamics if income pooling in couples and the response of idiosyncratic play to income polarization are taken into account. The reason is that with income pooling the average difference between the income of the As and Bs is increasing in the degree of marital assortment. Another example is provided by primogeniture. If the richer class produces more than two surviving offspring (as they surely did since the emergence of classes until the 20th century, see e.g. Betzig 1987 and Stone 1977) then primogeniture, by restricting the size of the B-class, will support higher levels of inequality.

9 Conclusion

Our hybrid model encompassing aspects of both the political economy conflict approach and evolutionary equilibrium selection provides some limited support for the efficient institutions view. *Ceteris paribus* institutions generating larger joint surpluses are selected. But it also shows that highly unequal and inefficient institutions may in an evolutionary sense out compete more egalitarian and efficient institutions if the barriers to upward class mobility are sufficiently great. In contrast to the complementary models developed by Acemoglu, Johnson and Robinson, and others, commitment problems play no role in explaining inefficient institutions in our approach. Rather an inefficient class structure may persist when mobility into the class that benefits from the unequal contract is sufficiently restricted.

Returning to the questions with which we began, we suggest that this model may illuminate such paradoxes as the millennia-long stability of high levels of class inequality in ancient civilizations (Trigger (2003), Yoffe (2005)) and the greater instability of extreme class inequality since the emergence of capitalism(Hobsbawm and Rude (1967), Hobsbawm (1964)). It also appears to capture dynamic of popular unrest and elite response during the French Revolution (Markoff , Soboul, 1964, and Rude 1959) and the U.S. civil rights movement (MacAdam and Yu, 2002). We think that the model may also provide insights concerning the emergence of some early welfare states (e.g Bismarkian Germany, Mares (2004)) and systems of public education (e.g. United States, Katz (1968)), and in addition to the case of the end of apartheid, the emergence of democracy in Brazil (Seidman 1990), the legalization and recognition of trade unions in the U.S.(Freeman 1998) and the end of Communist rule in Poland (Ekiert and

Kubik, 1999). But the historical work required to say if we are right remains to be done.

10 Appendix

10.1 Proof of Lemma 7.2

Proof.

$$\frac{d\mu_0(t)}{dt} = ln(\epsilon)\epsilon^{R_{01}(t)\phi(t;\lambda)}(\frac{dR_{01}(t)}{dt}\phi(t;\lambda) + R_{01}(t)\frac{d\phi(t;\lambda)(t)}{dt}) \tag{20}$$

$$\frac{d\mu_1(t)}{dt} = ln(\epsilon)\epsilon^{R_{10}(t)}\frac{dR_{10}(t)}{dt} = ln(\epsilon)\epsilon^{R_{10}(t)}\frac{-(1-\sigma)\rho}{(1+\sigma\rho+t(1-\sigma)\rho)^2} > 0 \tag{21}$$

Since $\phi(t,\lambda)$ becomes small as λ becomes large, and $\frac{d\phi(t,\lambda)}{dt}$ is increasing and bounded above by $\frac{2(1-\sigma)\rho)}{(1-2\sigma)\rho-2t(1-\sigma)\rho} > 0$ in λ. Then it is clear that for λ sufficiently large, (20) is negative. Thus we can establish that:

$$\frac{\frac{d\mu_0(t)}{dt}}{\frac{d\mu_1(t)}{dt}} < 0 < \frac{\mu_0(t)}{\mu_1(t)} \implies \frac{d\frac{\mu_0(t)}{\mu_1(t)}}{dt} < 0 \tag{22}$$

Since the first condition clearly holds, as μ_0 and μ_1 are both greater than 0, we have $\frac{d\tau_0}{dt} > 0$.

□

10.2 Proof of Proposition 7.3

Proof. If we differentiate (18) with respect to λ we get:

$$-(1-\sigma)\rho\frac{d\tau_0}{d\lambda} + \frac{d^2\tau_0}{dtd\lambda}((1-t)(1-\sigma)\rho - 1) \tag{23}$$

And we can readily see that:

$$\frac{d\tau}{d\lambda} = -\frac{1}{(1+\frac{\mu_0}{\mu_1})^2}\frac{d\mu_0}{d\lambda}\frac{1}{\mu_1} \tag{24}$$

$$\frac{d^2\tau}{dtd\lambda} = \frac{2}{(1+\frac{\mu_0}{\mu_1})^3}\frac{\frac{d\mu_0}{d\lambda}}{\mu_1}\frac{d\frac{\mu_0}{\mu_1}}{dt} - \frac{d^2\frac{\mu_0}{\mu_1}}{dtd\lambda}\frac{1}{(1+\frac{\mu_0}{\mu_1})^2} \tag{25}$$

So, noting that:

$$\frac{d\mu_0}{d\lambda} = ln(\epsilon)\epsilon^{\phi(t;\lambda)R_{01}(t)}R_{01}(t)\frac{\phi(t;\lambda)}{d\lambda} > 0 \tag{26}$$

$$\frac{d^2\mu_0}{dtd\lambda} = ln(\epsilon)(ln(\epsilon)\epsilon^{\phi(t;\lambda)R_{01}(t)}R_{01}(t)\frac{\phi(t;\lambda)}{d\lambda}(\frac{dR_{01}(t)}{dt}\phi(t;\lambda) + R_{01}(t)\frac{d\phi(t;\lambda)(t)}{dt})$$
$$+\epsilon^{\phi(t;\lambda)R_{01}(t)}(\frac{dR_{01}(t)}{dt}\frac{d\phi(t;\lambda)}{d\lambda} + R_{01}(t)\frac{d^2\phi(t;\lambda)(t)}{dtd\lambda}) > 0$$

The last inequality follows since ϕ is decreasing in λ, and $\frac{d\phi}{dt}$ is increasing in λ. Thus we get that the derivative is increasing at 0. Thus, since we have shown that the government's value function is not maximized at 0 or $\bar{t}$, there must exist an interior maximum t^*.

□

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