Input and Output Inventories in General Equilibrium*

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Abstract

We build and estimate a two-sector (goods and services) dynamic general equilibrium model with two types of inventories: finished goods (output) inventories yield utility services while materials (input) inventories facilitate the production of goods. The model, which contains neutral and inventory-specific technology shocks and preference shocks, is estimated by Bayesian methods. The estimated model replicates the volatility and cyclicality of inventory investment and inventory-target ratios. When estimated over subperiods, the results suggest that changes in the volatility of inventory shocks, or in structural parameters associated with inventories, play a minor role in the reduction of the volatility of output.

Keywords: Inventories, business cycles, output volatility, Bayesian estimation

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1. Introduction

Macroeconomists acknowledge an important role for inventories in business cycle fluctuations, but macro models that explain this role successfully have been elusive.[1] Early RBC models, such as Kydland and Prescott (1982), treated inventories as a factor of production. However, Christiano (1988) showed that RBC models with aggregate inventories cannot explain the volatility and procyclicality of inventory investment without a more complex information structure and restrictions on the timing of agents' decisions. Moreover, Christiano and Fitzgerald (1989) concluded, "the study of aggregate phenomena can safely abstract from inventory speculation." Nevertheless, the recent empirical literature continues to affirm the long-held conventional view of inventories and fluctuations. For example, McConnell and Perez-Quiros (2000), among others, argue that structural changes in inventory behavior are an important reason for the decline in the volatility of U.S. GDP since the early 1980s.[2]

We re-examine the role of inventories in business cycle fluctuations by developing and estimating a dynamic stochastic general equilibrium (DSGE) model rich enough to explain the essential elements of inventory behavior. To confront the data properly, the model requires four extensions over existing general equilibrium models with inventories: 1) two sectors, differentiated by whether they hold inventories; 2) inventories disaggregated into two theoretically and empirically distinct types (input and output inventories); 3) several modern DSGE features shown to be necessary to fit the data; and 4) multiple shocks to provide a diverse array of economically interpretable sources of stochastic variation. Because these extensions significantly increase the complexity of the model, we abstract from other potentially important features – variable markups, nominal rigidities, intermediate goods with input-output relationships, and nonconvexities.[3]

The study of inventories in general equilibrium motivates a natural sector decomposition. Because inventories are goods held mostly by firms that produce goods, our model contains a goods-producing sector with inventories and a service-producing sector without them. This inventory-based sector decomposition yields a broader goods sector than in prior studies distinguishing goods from services because it includes industries that distribute goods (wholesale and retail trade plus utilities).[4]

[1] See the surveys by Blinder and Maccini (1991) and Ramey and West (1999). For other estimates of the importance of inventory investment in the decline of GDP during recessions, see Ramey (1989) and Humphreys, Maccini, and Schuh (2001).

[2] See also Blanchard and Simon (2000), Kahn, McConnell and Perez-Quiros (2002), Kahn and McConnell (2003), Irvine and Schuh (2005a), and Herrera and Pesevento (2005).

[3] Papers with models that incorporate variable markups and/or sticky prices include Bils and Kahn (2000); Hornstein and Sartre (2001); Boileau and Letendre (2004); Coen-Pirani (2004); Jung and Yun (2005); and Chang, Hornstein, and Sarte (2006). General equilibrium models with intermediate goods and explicit input-output relationships include Huang and Liu (2001) and Wen (2005a). General equilibrium models with nonconvexities include Fisher and Hornstein (2000) incorporating (S,s) policies for retail inventories, and Khan and Thomas (2006), with (S,s) policies for intermediate goods inventories.

[4] Marquis and Trehan (2005a) define goods as manufacturing, while Lee and Wolpin (2006) use the NIPA defin-

Our model disaggregates inventories into input (materials and work-in-progress) and output (finished goods) stocks as suggested by the stage-of-fabrication approach in Humphreys, Maccini, and Schuh (2001).[5] Wen (2005a) offers an alternative theoretical analysis of stage-of-fabrication input and output inventories in general equilibrium with a supply-chain and intermediate goods. Khan and Thomas (2006) develop a calibrated general equilibrium model with intermediate and final goods producers, allowing for (S,s) inventory policies for intermediate goods, but do no not distinguish among inventory types.

We motivate holdings of the two inventory stocks differently. Input inventories enter as a factor of production, as in earlier models, but only in the goods sector. Output inventories, however, pose a different specification challenge. Most of the inventory literature deals with partial-equilibrium analyses of the inventory-holding problem. Typically, a firm is assumed either to hold output inventories to avoid lost sales or stockouts (Kahn 1987), or to "facilitate" sales (Bils and Kahn 2000). In general equilibrium, however, one must confront the value of output inventories to consumers more explicitly. Like Kahn and McConnell (2003), we assume that output inventories enter the utility function directly, hence their utility services are a proxy for shopping time, variety, or other consumer benefits associated with the underlying retailing service.

Empirically, the data strongly suggest disaggregating aggregate inventories. We define output inventories (F) in general equilibrium as stocks held by retailers for final sale; all other stocks are input inventories (M). By these definitions, input inventories are more volatile and procyclical than output inventories. Perhaps more importantly, the ratios of each inventory type to its steady state target implied by the model exhibit very different cyclical behavior. Input inventories relative to output of goods (M/Y_g) are very countercyclical. However, we find that output inventories relative to consumption of goods (F/C_g) are essentially acyclical.

Our setup also includes several important features now standard in estimated DSGE models, such as adjustment costs on all capital stocks (including inventories) and variable utilization of capital. We also allow for non-zero inventory depreciation (or, equivalently, an inventory holding cost that is proportional to the stock). This is a feature relatively novel to the inventory literature, except in models of inventories with highly perishable goods (Pindyck 1994). We introduce it because it is theoretically plausible and essential to fit the data. The model incorporates six shocks. Two sector-specific technology shocks and one demand-type shock to the discount rate are relatively standard, although we allow for correlation between the technology shocks to cap-

ition (agriculture, mining, construction, and manufacturing). For multi-sector models based on consumption and investment, see Kimball (1994), Greenwood, Hercowitz, and Krusell (1997, 2000), Whelan (2003), and Marquis and Trehan (2005b).

[5] The importance of stage-of-fabrication inventories dates back to Lovell (1961) and Feldstein and Auerbach (1976). More recent structural models of stage-of-fabrication inventories include Husted and Kolintzas (1987), Bivin (1988, 1993), Ramey (1989) and Rossana (1990). Cooper and Haltiwanger (1990) examine the linkages between firms created through inventories playing different roles in production as inputs and outputs. Maccini and Pagan (2007) also adopt the input and output inventory approach, extending the linear-quadratic partial equilibrium framework of Humphreys, Maccini and Schuh.

ture common technological change. A fourth shock captures shifts in preferences between goods and services. Lastly, two inventory-specific shocks introduce a role for unobserved changes in inventory technologies or preferences; such shocks generally have not been introduced explicitly in macroeconomic models.

We estimate the model using a Bayesian approach now common in confronting DSGE models with data. The estimated model fits the data well. Parameter estimates are generally consistent with theory and relatively precise. The estimated model replicates the volatility and procyclicality of inventory investment, and the qualitative differences in observed cyclicality of the two inventory-target ratios. In particular, the model captures the countercyclicality of the input inventory ratio and the relatively acyclicality of the output inventory ratio. We also find that inventory shocks do explain some of the variation in investment and consumption, but little of the variation in aggregate output. Altogether, the results are consistent with the view that inventories are an important part of the propagation mechanism, but not an important source of macroeconomic fluctuations.

Our model and findings are most closely related to Khan and Thomas (2006) who find that a calibrated competitive general equilibrium model with fixed delivery costs and driven by a single technology shock is successful in reproducing the cyclical properties of total inventories. We also match the cyclical properties of inventories in a model that distinguishes inventories by stage of fabrication and does not rely on fixed costs. The multiplicity of shocks in our model provides additional richness in assessing the role of inventories in business cycle fluctuations, and allows us to better capture their volatility. The Bayesian estimation versus calibration methodology constitutes a further difference with Khan and Thomas.

The econometric results shed light on inventory behavior in general equilibrium. Consistent with Christiano (1988), input inventories are complements to capital in production and have a very small weight in production. Instead, the elasticity of substitution between consumption and output inventories is closer to unity. Adjustment costs on fixed capital are large, while adjustment costs on inventory stocks are small and relatively insignificant. In contrast, estimated depreciation rates for inventories are non-negligible, especially for output inventories, which decay at a rate estimated at about 8 percent per quarter. The magnitude of this depreciation rate and its importance in fitting the data motivate further research on understanding inventory depreciation.

Finally, by estimating our model over the sub-periods 1960-1983 and 1984-2004, we can provide the first data-consistent, structural decomposition of the decline in GDP volatility with an estimated general equilibrium model that incorporates a role for input and output inventories. In doing so, we account for the notable changes in the steady-state ratios of inventory-target ratios and relative importance of the service sector. We find that most of the decline in aggregate output volatility is attributable to lower volatility of shocks, which occurred primarily in the goods-sector technology shock.[6] The volatility of the input inventory technology shock also declined, but it accounts for

[6] This result is consistent with other aggregate analyses of the Great Moderation. See the VAR-based analyses of Blanchard and Simon (2001), Stock and Watson (2002), Ahmed, Levin, and Wilson (2004). See also Khan and

only a very small reduction in the volatility of aggregate output or goods output. We also find that structural changes in the parameters account for a small fraction of the reduction in aggregate output volatility. Of these, an increase in the costs of adjusting fixed capital (especially in the goods sector), and an increase in the cost of varying capital utilization (especially in the services sector) are the main contributors to the structural change. The reduced ratio of input inventories to goods output is associated with a decrease in GDP or goods output volatility, but the size of the decrease is small.[7]

The remainder of the paper proceeds as follows. Section 2 describes the model and relates it to the literature in more detail. Section 3 discusses the choices present in applying the two-sector general equilibrium model to data. Specifically, we explain how we disaggregated the economy into goods and services sectors and disaggregated inventories by stages of fabrication. Section 4 presents the estimation methodology. Section 5 presents the estimation results. Section 6 concludes.

2. The model

2.1. Motivating inventories

To motivate the holding of input inventories, we follow the predominant practice in the literature and treat them as a factor of production.[8] This simplified motivation for inventory holding assumes that the stock of input inventories on hand facilitates value added production beyond the usage of materials and intermediate goods.[9] Although it does not explicitly identify the underlying mechanism, the stock of input inventories may facilitate production by minimizing resource costs involved in procuring input materials, guarding against stockouts of input materials that would reduce productivity, allowing batch production, or more generally by making better organization of the production process. As a factor of production, one can think of inventory stocks as a type of capital. As such, inventories have costs of adjustment (likely less than for fixed capital), depreciation (likely greater than fixed capital), and holding costs. If the latter are proportional to the stock, inventory *depreciation* will include physical wastage and the resource cost of holding inventories.

To motivate the holding of output inventories, we take an approach parallel to input inventories

Thomas (2006), Maccini and Pagan (2007), Arias, Hansen, and Ohanian (2006), who use a calibrated RBC model without inventories, and Leduc and Sill (2006), who use an equilibrium model to assess the quantitative importance of monetary policy.

[7] Ramey and Vine (2006), studying the automobile industry, also do not find much evidence of structural change related to inventories. They emphasize structural change in the persistence of exogenous sales and, to a much lesser extent, in employment adjustment costs.

[8] In addition to the early RBC models, see also Ramey (1989) and Feroli (2002).

[9] Humphreys, Maccini, and Schuh (2001) and Maccini and Pagan (2007) argue that it is important to model the delivery and usage of input materials in gross production together with holding of input inventories. However, absent input-output (supply-chain) relationships among firms, a representative-firm approach in a general equilibrium model cannot admit deliveries of raw materials produced by an upstream supplier in an internally, model-consistent fashion. Thus, we make the simplifying assumption that the stock of inventories enters the production function.

by entering the stock of output inventories into the utility function directly, as do Kahn and McConnell (2003). Finished goods on hand or held by retailers are assumed to provide convenience services to the consumer. The idea we are trying to capture is that a large stock of wine bottles at the local wine store increases utility beyond that derived from actually consuming the wine. One way output inventories may yield utility is by economizing on shopping time. This argument is analogous to the one for including money in the utility function, but the argument for inventories is even more compelling.[10] Alternatively, entering output inventories in the utility function is analogous to including the stock of consumer durables in utility, which yields a service flow proportional to the stock.[11] However, the service flow from output inventories is different (convenience, reduction of shopping time, increase in availability of variety) and it pertains to nondurable goods as well.

Our representative-agent approach to holding output inventories abstracts from the decentralized problem of retailing that is common in partial equilibrium analyses of inventories. To address this issue properly, one should model explicitly the relationship between individual consumers and retailers. We leave this important task for future research in the context of a model that also allows for input-output (supply-chain) relationships, which are equally important to the decentralized problem. Here, we just assume the existence of households-owned warehouses and that households withdraw output inventories in such a way that there are no stockout problems. More importantly, we assume that individual utility depends on the stock of privately held inventories (rather than the aggregate inventory stock), so that any externalities or free-riding problems that may arise in that case are ruled out.

2.2. Preferences

The representative consumer chooses consumption of final goods C_g, services C_s, output inventories F, and hours worked in the goods sector L_g and services sector L_s to maximize the following intertemporal utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_{\beta t} \left(\log \left(\gamma \varepsilon_{\gamma t} X_t^{-\phi} + (1 - \gamma \varepsilon_{\gamma t}) \, C_{st}^{-\phi} \right)^{-1/\phi} - \tau \left(L_{gt} + L_{st} \right) \right) \right)$$

where X_t is a CES bundle of goods and output inventories, and is defined as

$$X_t = \left(\alpha \varepsilon_{Ft} C_{gt}^{-\mu} + (1 - \alpha \varepsilon_{Ft}) \, F_{t-1}^{-\mu} \right)^{-1/\mu} \tag{1}$$

where

$$0 < \gamma < 1, \; 0 < \alpha < 1 \text{ and } \mu \geq -1.$$

In this formulation, $1 + \mu$ is the inverse elasticity of substitution between consumption of final goods and output inventories. Similarly, $1 + \phi$ is the inverse elasticity of substitution marginal rate between services and the bundle of goods (consumption-output inventories). Utility is linear in

[10] See, for instance, McCallum and Goodfriend (1987).

[11] See Baxter (1996) and Wen (2005b).

leisure following Hansen (1985) and Rogerson (1988), which assume the economy is populated by a large number of identical households who agree on an efficient contract that allocates individuals either to full time work or to no work at all.

We allow for three unitary mean disturbances to impact on the intertemporal and intratemporal margins of the household. The shock $\varepsilon_{\beta t}$ affects the preference for goods, services and leisure today versus tomorrow. The preference shock $\varepsilon_{\gamma t}$ affects the relative preference between goods and services.[12] Finally, the shock ε_{Ft} affects the relative preference between consumption of goods and output inventories: this shock may also capture the reduced-form impact on utility of changes in output inventory "technology" occurring in the storage of finished goods at retailers. Such technology may include the emergence of megastores like Walmart, internet shopping, and other key retail developments.[13]

2.3. Sectoral technologies

Following Christiano (1988), production in the goods sector is Cobb-Douglas in labor L_{gt} and a CES aggregate of the services from fixed capital and input inventories,

$$Y_{gt} = (A_{gt}L_{gt})^{1-\theta_g} \left(\sigma \left(z_{gt}K_{gt-1}\right)^{-\nu} + (1-\sigma)\left(\varepsilon_{Mt}M_{t-1}\right)^{-\nu}\right)^{-\theta_g/\nu} \tag{2}$$

where

$$0 < \sigma < 1 \text{ and } \nu \geq -1$$

and K_{gt-1} is the end-of-period $t-1$ capital in the goods sector (plant, equipment, and structures), z_{gt} is the time-varying utilization rate of K_{gt-1}, and M_{t-1} is the end-of-period $t-1$ stock of input inventories. In this formulation, $1+\nu$ measures the inverse elasticity of substitution between fixed capital and input inventories. If $\nu > 0$, as Christiano found, then fixed capital and input inventories are complements; if $-1 \leq \nu \leq 0$ they are substitutes.

We allow for two unitary-mean technology disturbances in the goods producing sector: A_{gt} represents a general technology shock, while ε_{Mt} represents shocks that affect the productive efficiency of input inventories, so that $\varepsilon_{Mt}M_{t-1}$ is input inventories in efficiency units. This novel shock is intended to capture the impact on production efficiency of changes in input inventory "technology" in a reduced-form way. Such technology may include new methods of inventory management like just-in-time production, which are characterized by elaborate supply and distribution chains. Irvine and Schuh (2005a) offers evidence that such supply chains may have changed, but the exact

[12] For the model to admit a solution, a necessary condition is that $\gamma\varepsilon_{\gamma t}$ never exceeds unity for each possible realization of $\varepsilon_{\gamma t}$ Even though we assume that log $\varepsilon_{\gamma t}$ has an unbounded support, empirically its standard deviation turns out to be rather small, so that this condition is always satisfied in practice.

[13] We also experimented with including ε_{Ft} only in front of inventories. In this case ε_{Ft} could be interpreted more clearly as a technology shock to inventory management of final goods that allow the consumer to reach a higher level of utility for a given level of inventories. Similar stochastic specifications have been used in money-in-utility models. In that context, shocks to real money balances in the CES aggregate represent improvements in the transaction technology related to ATM, credit cards, etc. See, for instance, Ireland (1997).

mechanisms underlying these methods have not been incorporated much in macroeconomic models. Information and computing technology (ICT) may also play an important and related role, as argued by Kahn, McConnell, and Perez-Quiros (2002). We introduce ε_{Mt} as a useful shorthand to capture variations in input inventory management.[14] The evolution in input inventories management techniques can also be reflected in the weight of input inventories in the CES aggregate, $1-\sigma$, in the parameter governing the elasticity of substitution, ν, or, more in general, in the ratio between the stock of input inventories and goods output.

Production in the services sector is Cobb-Douglas in labor L_{st} and capital services only:

$$Y_{st} = \left(A_{st}L_{st}\right)^{1-\theta_s}\left(z_{st}K_{st-1}\right)^{\theta_s} \tag{3}$$

where K_{st-1} is the end-of-period $t-1$ capital in the service sector (plant, equipment, and structures) and z_{st} is the time-varying utilization rate of K_{st-1}. The empirical fact that service-producing firms do not hold inventories motivates the different specification of the services production technology. We also allow for one unitary-mean general technology disturbance, A_{st}, in the services sector.

2.4. Aggregate economy

Output from the goods sector provides consumption goods, new fixed investment in both sectors, and investment in output and input inventories. Output from the services sector provides services. Thus, the resource constraints for the goods and service sectors of the economy are respectively:

$$\begin{aligned} Y_{gt} &= C_{gt} + K_{gt} - \left(1-\delta_K\left(z_{gt}\right)\right)K_{gt-1} + K_{st} - \left(1-\delta_K\left(z_{st}\right)\right)K_{st-1} + F_t - \left(1-\delta_F\right)F_{t-1} \\ &+ M_t - \left(1-\delta_M\right)M_{t-1} + \xi_{Kg}\left(K_{gt},K_{gt-1}\right) + \xi_{Ks}\left(K_{st},K_{st-1}\right) + \xi_F\left(F_t,F_{t-1}\right) + \xi_M\left(M_t,M_{t-1}\right) \end{aligned} \tag{4}$$

and

$$Y_{st} = C_{st}. \tag{5}$$

The capital depreciation rates in both sectors, $\delta_{Kg}\left(z_{gt}\right)$ and $\delta_{Ks}\left(z_{gt}\right)$ are increasing functions of the respective utilization rates. The inventory depreciation rates, δ_F and δ_M, are fixed and possibly capture inventory holding costs as well. We also allow for standard, convex adjustment costs in K_{gt}, K_{st}, F_t and M_t, $\xi_{Kg}\left(K_{gt},K_{gt-1}\right)$, $\xi_{Ks}\left(K_{st},K_{st-1}\right)$, $\xi_F\left(F_t,F_{t-1}\right)$, $\xi_M\left(M_t,M_{t-1}\right)$.

Gross aggregate output (GDP) is the sum of output from the two sectors. The relative size of each sector is determined on average by the preference weight γ, which relates the goods bundle to services in the utility function.

[14] The information structure in Christiano (1988), as well as Kahn and McConnell (2003), is more complex. Part of the shocks are unobserved when some of the decisions are made. For instance Christiano (1988) assumes that hours and capital investment decisions are made on the basis of a noisy information on the shocks, while inventories and consumption decisions are made after the shock is revealed. Although important, this difference is not central.

2.5. Optimality conditions

The two welfare theorems apply, so we solve the model as a social planner's problem. The first-order conditions for the planner's problem are standard and summarized here:

$$\frac{\partial u_t}{\partial C_{gt}} = \lambda_t \tag{6}$$

$$\frac{\partial u_t}{\partial Y_{st}} = \omega_t \tag{7}$$

$$-\frac{\partial u_t}{\partial L_{gt}} = \lambda_t \frac{\partial Y_{gt}}{\partial L_{gt}} \tag{8}$$

$$-\frac{\partial u_t}{\partial L_{st}} = \omega_t \frac{\partial Y_{st}}{\partial L_{st}} \tag{9}$$

$$\lambda_t \left(1 + \frac{\partial \xi_{Kst}}{\partial K_{st}}\right) = \beta E_t \left(\lambda_{t+1} \left(1 - \delta_{Kst+1} - \frac{\partial \xi_{Kst+1}}{\partial K_{st}}\right) + \omega_{t+1} \frac{\partial Y_{st+1}}{\partial K_{st}}\right) \tag{10}$$

$$\lambda_t \left(1 + \frac{\partial \xi_{Kgt}}{\partial K_{gt}}\right) = \beta E_t \left(\lambda_{t+1} \left(1 - \delta_{Kgt+1} - \frac{\partial \xi_{Kgt+1}}{\partial K_{gt}} + \frac{\partial Y_{gt+1}}{\partial K_{gt}}\right)\right) \tag{11}$$

$$\lambda_t \left(1 + \frac{\partial \xi_{Ft}}{\partial F_t}\right) = \beta E_t \left(\frac{\partial u_{t+1}}{\partial F_t} + \lambda_{t+1} \left(1 - \delta_F - \frac{\partial \xi_{Ft+1}}{\partial F_t}\right)\right) \tag{12}$$

$$\lambda_t \left(1 + \frac{\partial \xi_{Mt}}{\partial M_t}\right) = \beta E_t \left(\lambda_{t+1} \left(1 - \delta_M - \frac{\partial \xi_{Mt+1}}{\partial M_t} + \frac{\partial Y_{gt+1}}{\partial M_t}\right)\right) \tag{13}$$

$$\frac{\partial \delta_{Kgt}}{\partial z_{Kgt}} K_{gt-1} = \frac{\partial Y_{gt}}{\partial z_{Kgt}} \tag{14}$$

$$\frac{\partial \delta_{Kst}}{\partial z_{Kst}} K_{st-1} = \frac{\partial Y_{st}}{\partial z_{Kst}}. \tag{15}$$

The first two equations equate the marginal utility of consumption of goods and services to their respective shadow costs λ_t and ω_t. Equations 8 and 9 are the optimality conditions between consumption of goods and services and leisure. Equations 10 to 13 are the intertemporal optimality condition that govern the choice of , K_{st}, K_{gt}, F_t, and M_t. The last two equations set the marginal benefit from capital utilization equal to its marginal cost.

2.6. Driving processes

The model is closed by assumptions about the stochastic behavior of the preference and technology shocks. Shocks $\varepsilon_{\beta t}$, $\varepsilon_{\gamma t}, \varepsilon_{Ft}, \varepsilon_{Mt}, A_{gt}, A_{st}$ follow AR(1) stationary processes in logs:

$$\ln(\varepsilon_{\gamma t}) = \rho_\gamma \ln(\varepsilon_{\gamma t-1}) + \left(1 - \rho_\gamma^2\right)^{1/2} u_{\gamma t} \tag{16}$$

$$\ln(\varepsilon_{\beta t}) = \rho_B \ln(\varepsilon_{\beta t-1}) + \left(1 - \rho_\beta^2\right)^{1/2} u_{\beta t} \tag{17}$$

$$\ln(\varepsilon_{Ft}) = \rho_F \ln(\varepsilon_{Ft-1}) + \left(1 - \rho_F^2\right)^{1/2} u_{Ft} \tag{18}$$

$$\ln(\varepsilon_{Mt}) = \rho_M \ln(\varepsilon_{Mt-1}) + \left(1 - \rho_M^2\right)^{1/2} u_{Mt} \tag{19}$$

$$\ln(A_{gt}) = \rho_g \ln(A_{gt-1}) + \left(1 - \rho_g^2\right)^{1/2} u_{gt} \tag{20}$$

$$\ln(A_{st}) = \rho_s \ln(A_{st-1}) + \left(1 - \rho_s^2\right)^{1/2} u_{st}. \tag{21}$$

The innovations $u_{\beta t}$, $u_{\gamma t}$, u_{Ft}, u_{Mt}, u_{gt} and u_{st} are serially uncorrelated with zero means and standard deviations given by $\sigma_{\beta t}$, $\sigma_{\gamma t}$, σ_{Ft}, σ_{Mt}, σ_{gt} and σ_{st}. In addition, we allow for mutual correlation between the two technology innovations u_{gt} and u_{st}.

2.7. Functional form assumptions

Adjustment costs are quadratic and given by the following expressions:

$$\xi_{\Xi t} = \frac{\psi_\Xi}{2\delta_\Xi}\left(\frac{\Xi_t - \Xi_{t-1}}{\Xi_{t-1}}\right)^2 \Xi_{t-1} \tag{22}$$

for $\Xi_t = (K_{gt}, K_{st}, M_t, F_t)$. In this formulation, the marginal adjustment cost is zero in steady state, and it is straightforward to show that the elasticity of capital (investment) with respect to shadow price is δ_Ξ/ψ_Ξ $(1/\psi_\Xi)$.

For the utilization function, we choose a parameterization such that marginal cost of utilization equals the marginal product of capital in steady state (so that the steady state depreciation rate is independent of the curvature of the utilization function). In particular, the time t depreciation rate of K_{it}, defined as δ_{Kit} (with $i = g, s$) is given by:

$$\delta_{Kit} = \delta_{Ki} + \frac{1}{2} b_{Ki}\zeta_{Ki} z_{Kit}^2 + b_{Ki}\left(1-\zeta_{Ki}\right) z_{Kit} + b_{Ki}\left(\frac{\zeta_{Ki}}{2} - 1\right). \tag{23}$$

In this formulation, $\zeta_{Ki} > 0$ determines the curvature of the capital utilization function, where $b_{Ki} = 1/\beta - (1-\delta_{Ki})$ is a normalization that guarantees that steady state utilization is unity.[15]

2.8. The steady state

In absence of shocks, the model equations imply that variables converge to a constant in the steady state. The first-order conditions for fixed capital in the goods sector and input inventories imply that the steady state ratios in the goods sector of capital-to-output, $k_g = K_g/Y_g$, and input-inventories-to-output, $m = M_g/Y_g$, can be written as:

$$k_g = \frac{\theta_g \sigma \beta}{1-\beta\left(1-\delta_{Kg}\right)} \frac{1}{\sigma + (1-\sigma)\left(\frac{\sigma}{1-\sigma}\frac{1-\beta(1-\delta_M)}{1-\beta\left(1-\delta_{Kg}\right)}\right)^{\frac{\nu}{1+\nu}}} \tag{24}$$

$$m = \frac{\theta_g \beta(1-\sigma)}{1-\beta\left(1-\delta_M\right)} \frac{1}{(1-\sigma) + \sigma\left(\frac{\sigma}{1-\sigma}\frac{1-\beta(1-\delta_M)}{1-\beta\left(1-\delta_{Kg}\right)}\right)^{-\frac{\nu}{1+\nu}}}. \tag{25}$$

These conditions state that the capital-to-output ratio and input-inventory-to-output ratio are increasing in their relative weights in production, σ and $1-\sigma$ respectively. At the same time, factor intensities depend on the degree of substitutability. When the ratios in large parentheses are larger than one, which is true because input inventories are much smaller than the capital stock, then capital is decreasing in ν and input inventories are increasing in ν.

[15] See for instance Christiano (2004).

The optimality conditions for consumption of goods and final output inventories imply in the steady state that:

$$c_g = \left(\frac{\alpha}{1-\alpha} \frac{1-\beta(1-\delta_F)}{\beta} \right)^{\frac{1}{1+\mu}} f \tag{26}$$

where $c_g = C_g/Y_g$ and $f = F/Y_g$. The ratio of consumption to output inventories is increasing in α, while it is decreasing (increasing) in μ when the term in parentheses is larger (smaller) than one. Using the linear homogeneity of the CES aggregators and the first-order conditions for K_s, C_g, and C_s we derive the following expression for k_s:

$$k_s = \left(\frac{\lambda}{\omega}\right)^{\frac{-\phi}{1+\phi}} \left(\frac{1-\gamma}{\alpha\gamma}\right)^{\frac{1}{1+\phi}} \frac{\theta_s \beta}{1-\beta(1-\delta_{Ks})} \left(\alpha + (1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{\mu-\phi}{\mu(1+\phi)}} c_g. \tag{27}$$

This equation says that capital in the services sector is higher when the relative price of goods in terms of services (λ/ω) is low, when the weight to services in utility $1-\gamma$ is high, or when the production function for services is capital intensive (θ_s is high). Using the first-order conditions for labor and the linear homogeneity of the production functions, the relative price of goods can be written as:

$$\frac{\lambda}{\omega} = \left(\frac{(1-\theta_s)\left(\frac{\theta_s\beta}{1-\beta(1-\delta_{Ks})}\right)^{\frac{\theta_s}{1-\theta_s}}}{(1-\theta_g)\left(\sigma k_g^{-\nu} + (1-\sigma)\, m_g^{-\nu}\right)^{\frac{-\theta_g}{\nu(1-\theta_g)}}} \right)^{1-\theta_s}. \tag{28}$$

Finally, the market-clearing condition for the goods sector is:

$$c_g + \delta_F f + \delta_{Kg} k_g + \delta_{Ks} k_s + \delta_M m = 1. \tag{29}$$

Equations 24 to 29 above can be jointly solved for f, c_g, k_s, k_g, m and λ/ω, for given parameter values. Finally the first-order conditions for L_g, L_s, C_g, and Y_s, together with the production functions, can be solved (nonlinearly) for $L_g, L_s, Y_g, Y_s, \omega$, and λ. Details of all the derivations are in the appendix. The model's optimality conditions, together with the market-clearing conditions and laws of motion for the shocks, can be used to obtain a log-linear approximation around the steady state for the decision rules of the model variables given initial conditions and realizations of the shocks. For a given set of values of the model's structural parameters, the solution takes the form of a state-space econometric model that links the behavior of the endogenous variables to a vector of partly unobservable state variables that includes the six autoregressive shocks. In our econometric application, we use observations on deviation from steady state of the output of goods and services, of the stock of input inventories and output inventories, of the relative price of goods, and of total fixed investment to estimate the model parameters and the properties of the shocks. Before doing so, an important task is to map the model variables into their data counterparts.

3. Data

3.1. Sector and inventory definitions

To obtain model-consistent sectors, we divide the economy according to the inventory-holding behavior of industries. The goods sector includes all seven industries that hold empirically measured inventories: agriculture, mining, utilities, construction, manufacturing, and wholesale and retail trade. All other private industries that do not hold inventories are classified as services.

Table 1 shows our sectoral classification, along with 2000 output shares. Model-consistent (inventory-based) sectors are quite different from the now-conventional view of the U.S. economy as service-dominated. The model goods sector is about twice as large as services (59 percent versus 28 percent), and accounts for about two-thirds of private-sector output. However, because the model-consistent goods sector accounts for a much larger portion of international trade, the sectors are much closer in size when we abstract from net exports in the next subsection.

Table 1 also illustrates how the model-consistent (inventory-based) sector definitions differ from standard national income and product account (NIPA) definitions and why the goods sector is so much more important. We include NIPA structures (construction) and roughly one-quarter of NIPA services (utilities and wholesale and retail trade) in the goods sector because these industries hold inventories. Including construction in the goods sector is uncontroversial because, in the NIPA two-sector classification scheme, the "goods-producing" definition does so. However, moving a sizable portion of the NIPA service sector to the goods sector calls for some justification. We include utilities and the trade industries as part of the goods sector because the "services" they provide involve transporting goods from one location to another. We view this distribution of goods as reasonable to subsume in the overall production process of a representative goods firm, but recognize that separate treatment of production and distribution may have advantages.[16]

We divide total NIPA inventories into input (M) and output (F) stocks following the stage-of-fabrication perspective used by Humphreys, Maccini, and Schuh (2001) for manufacturing stocks. Table 2 shows the inventory definitions by industry and type, along with 2000 inventory shares. The industries are listed approximately in stage-of-fabrication order, with the most raw materials first and the most finished goods last.[17] We assume that, to a first approximation, the output and inventories of each industry is an input to the next industry along this supply chain. Prior research has focused mainly on manufacturing and thus generally has not grappled with the task of classifying total inventories in general equilibrium. We define retail inventories as output inventories because they are the most finished goods in the supply chain, accounting for one-fourth of the total. Thus, our general equilibrium input inventories account for about three-fourths of the total compared with about two-thirds in manufacturing. Also, our general equilibrium stage-of-fabrication stocks

[16] The reclassification of utilities as goods production is consistent with other energy production being already included in the goods sector. Petroleum refining is in manufacturing, part of the standard NIPA goods sector.

[17] According to the U.S. Census of Construction, inventories in the construction industry are materials and do not include unsold finished buildings.

are heavily oriented toward work-in-process (54 percent), whereas the three types of inventories are roughly equal sizes in manufacturing.[18]

3.2. Data construction

Based on the sector and inventory definitions, we use standard NIPA data and identities to construct the following model-consistent data for use in our econometric work:[19]

$$\begin{aligned} Y_g &= C_g + I_g + I_s + \Delta F + \Delta M \\ Y_s &= C_s \\ Y &= Y_g + Y_s. \end{aligned}$$

Because the inventory-based sector definitions differ from the NIPA definitions, the consumption, investment, and inventory data require three types of adjustments to obtain model-consistent variables. First, consumption of energy services is output produced by the utilities industry, so we redefine it as goods consumption. Second, we use non-NIPA investment-by-industry data to obtain measures of investment in each sector. Finally, we splice inventory data from two industrial classification schemes (SIC and NAICS) to obtain consistent time series data for our sample. See Appendix B for full details of the data adjustments and construction.

Despite these adjustments, efforts to construct private-sector output for a closed economy model without government yield an imperfect approximation. Excluding international trade causes exports of goods and services to be excluded from sector output but imports of goods and services are implicitly attributed to domestic sector output. One does not want to omit imports of capital goods, however, because investment purchased from abroad and installed domestically should be included in the capital accumulation equations. Relatedly, some inventory holdings probably are associated with international trade. Finally, government spending probably has at least an indirect influence on the actual consumption and investment. In sum, it is not generally possible to construct exact data for a private-sector, closed-economy model and our attempt yields only a best-effort approximation.

[18] The definition of stage-of-fabrication inventory stocks in general equilibrium is not well-guided by theory or data. One could make a reasonable theoretical case for manufacturing finished goods and wholesale stocks as output inventories too. However, no strong empirical justification exists for any particular alternative definition. For instance, wholesales inventories include construction material supplies and manufacturing output inventories contain goods that do not enter the consumer's utility function. Moreover, each industry's inventory investment exhibits different cyclical and trend characteristics, and the correlation of inventory investment between industries is low. This heterogeneity suggests that further disaggregation of the goods sector may be necessary to properly model the inventory components.

[19] For simplicity, we suppress the details of chain-weighting from equations in the text. When constructing the actual real chain-weighted data, we use the Tornquist index that weights growth rates of subcategories by their nominal shares, as recommended by Whelan (2002). The Tornquist index is a good approximation to the BEA's Fisher ideal chain index, which is a geometric average of a Paasche and a Laspayres fixed-weight index.

3.3. Output and investment

Figure 1 depicts the model-consistent output and investment data by sector. The left column contains output shares (Y_g/Y and Y_s/Y) and the right column contains investment-to-output ratios (I_g/Y and I_s/Y), both in nominal and real terms. Nominal output clearly has been shifting away from goods and toward services, though the shares have stabilized in recent years. In contrast, real output shares have been relatively stable over the full sample, each sector accounting for roughly half of real aggregate output, with goods slightly larger (roughly 50-55 percent). In nominal terms, gross investment relative to aggregate income has been rising in the service sector and falling in the goods sector. In real terms, investment-to-income in goods is roughly stable, even slightly decreasing, but it is clearly rising in services.

These nominal and real output trends differ from those previously noted in the two-sector literature. Sectoral decompositions based on consumption versus investment (Greenwood, Hercowitz, and Krusell 1997, 2000; Marquis and Trehan 2005b) or durable goods versus nondurable goods and services (Whelan 2003) tend to find that nominal shares are roughly stationary. In contrast, they find that real shares exhibit trends because technological change is faster in the investment and durable goods sectors and the relative prices of capital goods are declining. Marquis and Trehan (2005a) find the same is true for the manufacturing sector alone. However, the inventory-holding goods sector contains durable goods, nondurable goods, and several services industries. Because these industries exhibit heterogeneous trends, their combination yields substantially different aggregate real and nominal sectoral trends.

Although these trend differentials are interesting and important, we abstract from them for two reasons. First, our focus is on inventory investment and business cycle fluctuations, hence on deviations from trends. Second, to address the issue of sectoral differences in technical change it may well be necessary to divide the economy with inventories into more than just two sectors. Given the lack of success in incorporating inventories into RBC models thus far, we leave more detailed sectoral analyses for future research. Instead, we detrend all data used in the econometric work following the common procedure in the inventory literature.[20]

Regarding business cycle properties, goods output is much more variable than services output. Fluctuations in output of the inventory-based goods sector accounts for 76 percent of the variance of detrended private, domestic real output. By comparison, the growth rate of real goods output in

[20] A trend is removed from the variables in logs, using the band-pass filter of Baxter and King (1999) that isolates frequencies between 3 and 32 quarters. Linear quadratic detrending and first-differencing are also common in the literature, but these techniques tend to yield similar cyclical properties of the detrended data. However, Wen (2005c) shows that the cyclical properties of detrended inventory investment are sensitive to the frequency. Business cycle frequencies like ours yield procyclical inventory investment whereas higher frequencies (2-3 quarters) yield countercyclical inventory investment. After detrending, the data are restored to natural units by adding the detrended log residual to the mean of the log data then anti-logging this sum. Working with natural units is important for ratios, which are central to steady state calculations. All detrended data used in estimation are divided by steady state (average) goods output, but this scaling is not crucial in any sense.

the narrower, more volatile three-sector NIPA definition (which includes only agriculture, mining, and manufacturing industries) accounts for 89 percent of the variance of real GDP growth (Irvine and Schuh 2005b).

3.4. Input and output inventories

Figure 2 plots data on input and output inventories in general equilibrium that portray the central inventory facts we wish to explain with our model. The left column plots actual data and the right column plots detrended data; all data are in real $2000 chain-weighted terms.

The upper left panel of Figure 2 shows that actual inventory-target ratios defined by the steady state of our model, F/C_g and M/Y_g, exhibit opposing trends and trend breaks, which we do not attempt to explain with our model.[21] Input inventories have been declining relative to their target (M/Y_g) throughout our sample, with a notably faster rate of decline since the early 1980s. This reduction in input inventories probably resulted at least partly from improved inventory management techniques and better information flows. And because this trend break occurs about the same time that aggregate output volatility declined, a connection between these two events is a natural hypothesis.

In contrast, output (retail) inventories have been rising relative to their target (F/C_g). Little attention has been devoted to explaining this phenomenon and its implications for the aggregate economy. However, by separating inventories into input and output components, we highlight the need to understand the economic factors behind the trend increase in output inventories. The output inventory-target ratio leveled off in the 1990s, much later than the break for the input inventory-target ratio, a fact that also warrants further investigation.

Distinctly different trends in inventory-target ratios provides one motivation for disaggregation of inventories in general equilibrium. Secular trends in these ratios have been analyzed carefully by Kahn and McConnell (2002, 2003, and 2005), Ramey and Vine (2004), and Irvine (2005), but this research has not produced consensus explanations for the kinds of low frequency movements seen in Figure 2. Because our primary focus is on the role of inventories in business cycle fluctuations, we abstract from the secular trends in inventory-target ratios by detrending the data.

The key business cycle fact to emphasize regarding inventory-target ratios is their very different cyclical properties (right panels, Figure 2). On average, the output inventory ratio is roughly acyclical (correlation with goods output of 0.10), as can be seen by the lack of consistent movement during recessions (shaded regions). Although the output inventory ratio shot up during the 1973-75 recession, it has not done so during other recessions. In contrast, the input inventory

[21] Although M/Y is consistent with tradition in the inventory literature, such as Lovell (1961) and Feldstein and Auerbach (1976), F/C differs from the traditional inventory-to-sales ratio specified by microeconomic models of the firm. In the two-sector general equilibrium model, the "sales" measure most analogous to inventory literature is final goods sales, $S_g = C_g + I$. Empirically, however, the choice of scale variable for inventories does not alter the qualitative properties of inventory-target ratios.

ratio is very countercyclical (correlation with goods output of -0.89), as can be seen by its consistent increase during recessions. Thus, the stylized fact of countercyclical inventory-target ratios emphasized by Bils and Kahn (2000) for manufacturing output inventories is not evident for all inventories, suggesting that theories of inventory behavior must be comprehensive enough to explain the heterogenous behavior for different types of stocks.[22]

Finally, input inventory investment is nearly four times (3.8) more variable than output inventory investment in real terms, when scaled by goods output (lower left panel, Figure 2). This relative volatility is comparable to the analogous variance ratio observed within manufacturing (Blinder and Maccini 1991). However, the relative volatility of the two scaled stocks has declined dramatically since 1983 (ratio of 4.6 before 1984 to 2.5 since 1983), because the volatility of scaled input inventory investment has fallen while the volatility of scaled output inventory investment has remained constant. Both types of detrended inventory investment are procyclical over the full sample, but input inventories investment is more procyclical than output inventory investment (correlation with goods output is 0.64 for input, 0.42 for output). Interestingly, their cross-correlation is only 0.26. The procyclicality of detrended output inventory investment decreased from 0.47 in the early pre-1984 sample to 0.28 in the latter sample, but the cyclical correlation of detrended input inventory investment has remained relatively stable.

Distinctly different cyclical properties of input and output inventory investment provide additional motivation for disaggregating inventories in general equilibrium.[23] Theoretical models that allow different inventory target stock adjustment and volatility across stocks are likely to have an advantage in explaining and understanding aggregate inventory behavior over the business cycle.

4. Model Estimation

4.1. Overview

In our econometric work we use observations on the following variables: (1) output of the goods sector; (2) output of the service sector; (3) the stock of input inventories and (4) output inventories; (5) the relative price of goods to services; (6) total fixed investment. We estimate the model for the full sample from 1960:1 to 2004:4. We also estimate the model for the two sub-periods 1960:1-1983:4 and 1984:1-2004:4. The breakpoint corresponds to point estimates of the beginning of the Great Moderation, as indicated in McConnell and Perez-Quiros (2000). We plot our series in Figure 3.

We use Bayesian techniques to estimate the structural parameters.[24] For given values of the parameters, the solution to our linearized model takes the form of a state-space econometric model,

[22] Note that manufacturing output inventories are included in our definition of input inventories.

[23] The nature of cyclicality in our data may be sensitive to the frequency of the detrending procedure, as suggested by Wen (2005c). Because we use the same detrending procedure for input and output inventories, the cyclical differences between them are robust within a frequency. However, it may be useful to investigate whether the differences in cyclicality of input and output inventories varies across frequency in future research.

[24] For the solution and the estimation of the model, we use the Dynare toolkit developed by Michel Juillard.

and the Kalman filter permits to evaluate the likelihood of the observables as follows

$$L\left(\{x_t\}_{t=1}^{T}|\Upsilon\right)$$

where Υ is the vector collecting all the model parameters and x_t is the vector of observable. We combine the information in the data with prior information on the model parameters to construct the posterior density function:

$$p\left(\Upsilon|\,\{x_t\}_{t=1}^{T}\right) \propto L\left(\{x_t\}_{t=1}^{T}|\Upsilon\right)\Pi\left(\Upsilon\right).$$

We first calculate the posterior mode of the parameters using a numerical optimization procedure. We then generate 250,000 draws from the posterior mode using the Metropolis-Hastings algorithm to obtain the posterior distribution. The mean of the posterior is used to compute impulse response functions, variance decompositions and moments of the estimated model.

4.2. Prior Distributions

We keep some parameter fixed during our estimation exercise. More specifically, we set the discount factor at 0.99, implying an annual interest rate of 4 percent. We also calibrate the depreciation rates for fixed capital and we set $\delta_{Kg} = \delta_{Ks} = 0.02$, a conventional choice in the literature. Once these values are set, 29 remaining parameters need to be estimated. We partition them into three groups:

1. The autocorrelation $(\rho_g, \rho_s, \rho_B, \rho_\gamma, \rho_F, \rho_M)$ and standard deviation of the innovation disturbances (σ_g, σ_s, σ_B, σ_γ, σ_F, σ_M) and the correlation coefficient between the innovations in the goods and the service sector technologies ($\sigma_{g,s}$).

2. The adjustment cost parameters (ψ_{Kg}, ψ_{Ks}, ψ_F, and ψ_M) and the parameters characterizing the curvature of the fixed capital utilization functions (ζ_{Kg}, ζ_{Ks}).

3. The inventory depreciation rates (δ_M and δ_F), the elasticities of substitution (ν, ϕ, μ), the labor shares (θ_g, θ_s), the weight of services in utility (γ), the weight of input inventories in the CES capital aggregator (σ), and the weight (α) on consumption in the goods-bundle aggregator. This last group of parameters affect not only the dynamics of the model but also the steady state values of fixed capital and input and output inventory stocks relative to output, as well as the relative size of the service versus the goods sector. For our sample (and for the two sub-samples), the average values of these ratios (F/Y_g, M/Y_g, K_g/Y_g, K_s/Y_g, $(\omega/\lambda)\,Y_s/Y_g$) are reported in Table 3. Observe that, for each combination of δ_M, δ_F, ν, ϕ, μ, it is possible to determine a unique set of values for θ_g, θ_s, γ, σ, and α that are consistent with these five ratios of Table 3 (see appendix A for details).[25] Accordingly, in the estimation of the model, for each value of δ_M, δ_F, ν, ϕ and μ, we set θ_g, θ_s, γ, σ, and α to the values

[25] The formulas for these parameters are in equations 29 to 33 in the Appendix.

that match the ratios.[26] Intuitively, we let the likelihood function use information on the behavior around the steady state of our observables to determine values for the depreciation rates, δ_F and δ_M, and the elasticity of substitution in the CES aggregates in the production and utility functions, ν, ϕ, μ, without compromising the ability of the model to be consistent with the first moments of the data. This procedure also allows to account for the changes in the ratios over the sample period. In fact, when we estimate the model separately over the dates 1960:1-1983:4 and 1984:1-2004:4, we use the average values of the relevant ratios in each period.

Our priors are summarized in the first three columns of Table 4. For the parameters measuring adjustment costs ψ, we specify a beta prior over $\frac{\psi}{1+\psi}$, with a mean equal to 0.5 and a large standard deviation: this value corresponds to a prior mean for the elasticity of investment to its shadow price of unity. For the curvature of the utilization function, we choose a beta prior over $\frac{\zeta}{1+\zeta}$with mean equal to 0.5. For the elasticity of substitution between services and the goods bundle, between consumption and inventories and between input inventories and capital, we select prior values centered around two thirds (that is, $1 + \mu = 1.5$). In other words, our prior goes slightly in favor of complementarity.

The literature and the national income and product accounts (NIPA) offer little guidance for the choice of the inventory depreciation rates, δ_F and δ_M. An assumption in line with the procedures used in NIPA would be that inventories do not depreciate. Yet inventories are subject to various forms of "shrinkage", such as obsolescence, perishability, wear and tear, theft, and breakage, in addition to holding costs, so that the depreciation parameter may well be larger than the one for fixed capital. For instance, on a quarterly basis, Ramey (1989) reports inventory holding and storage costs of 4 percent per quarter and Khan and Thomas (2006) set these costs at 3 percent per quarter. We choose to be conservative and set a tight prior mean for the depreciation rates equal to 0.02.

The autoregressive coefficients of the exogenous shocks have beta prior distributions (as in Smets and Wouters, 2003) centered at 0.75. The unconditional standard deviations of the shocks are assigned diffuse inverse gamma prior. This distribution guarantees positive variance with a large domain. The correlation between u_{gt} and u_{st} is assumed to be normal and is centered around 0.50. The choices of the mean of the prior distribution for the standard deviation of the technology and preference shocks are in the ballpark of the findings in the literature.[27] Preliminary estimation attempts also suggested that a higher standard deviation should be chosen for the input inventory shock.

[26] Christiano (1988) follows the same strategy: in his model with inventories in the production function, he chooses σ (in our notation) to maximize the likelihood function and ν to match the steady state rental rate of inventories in the data.

[27] See for instance Ireland (2004) and Smets and Wouters (2003).

5. Estimation results

5.1. Full sample

Parameter estimates. We begin our discussion with the estimates over the full sample. Table 4 reports the mean, 5th and 95th percentile of the posterior distribution of the parameters obtained through the Metropolis-Hastings algorithm.[28]

In line with the prior, all shocks are estimated to be quite persistent, with autoregressive parameters ranging from 0.82 to 0.94. The unconditional standard deviation of the shocks ranges from 0.0032 (for the output inventory shock) to 0.0944 (for the input inventory shock): the quantitative relevance of each shock will be discussed below in the variance decomposition exercise. Perhaps unsurprisingly, we find that the standard deviation of the technology shock in the goods sector (0.0175) is higher than that of the service sector (0.0142).

The estimated elasticity of substitution between M and K (the inverse of $1+\nu$) equals $1/3.33 = 0.30$. The elasticity of substitution between F and C_g (the inverse of $1+\mu$) equals 0.94, and it is not significantly different from unity. Similarly, the elasticity of substitution between services and the CES aggregator for consumption of goods and output inventories (the inverse of $1+\phi$) is close to one.[29]

The estimates of the inventory adjustment cost parameters ψ_F and ψ_M are close to zero, indicating very small costs of adjusting input and output inventories, while the bigger values of ψ_{Kg} and ψ_{Ks} indicate larger adjustment costs for fixed capital. At the posterior mean, the estimated values imply an elasticity to the user cost of investment that is equal to 1.12 in the goods sector, and equal to 2.96 in the service sector.[30] This confirms that input inventories and fixed capital are indeed distinguished by different degrees of adjustment costs.

Another important difference between inventories and fixed capital emerges when we look at the estimates of the depreciation rates for inventories. We find small but non-zero depreciation rates for M (2.0 percent), larger depreciation rates (7.8 percent) for F. Finally, estimates of the convexity of the utilization function suggest that the marginal cost of capital utilization (in terms of increased depreciation) is more sensitive to changes in utilization rate in the goods sector than

[28] As is well known (see for instance Canova, 2007), an important issue concerns the convergence of the simulated draws from the posterior distribution of the parameters. We fine tune our estimation algorithm in order to obtain acceptance rates between 30 and 40 percent, and we check for convergence using the cumulative sum of the draws statistics. Although convergence typically obtains within 50,000 iterations, we set the number of draws to 250,000 and calculate the statistics of our estimated model based on the last 75 percent of the draws.

[29] Interestingly, while the estimates of $1+\mu$ and $1+\phi$ are both close to unity, the elasticity of substitution between goods and services is very tightly estimated: its 95 percent interval lies between 1.03 and 1.09. We conjecture that the precise estimate is due to the inclusion of the relative price of goods versus services in the set of variables that we use to estimate the model.

[30] To a first order, one can interpret the inverse of ψ as the elasticity of each type of investment to its shadow price. Our numbers are in line with microeconometric findings based on estimates of investment equations: see for instance Chirinko (1993).

in the service sector. As a result, variable capital utilization is more important in the service sector than in goods sector.

As we mentioned in the previous section, we do not directly estimate the parameters measuring the relative shares in utility and production of consumption, capital and inventories. Rather, given the estimated parameters, we calculate the values of σ, α, θ_g, θ_s, and γ (Table 5) that are consistent with the sample means of F/Y_g, M/Y_g, K_g/Y_g, K_s/Y_g, and $(\omega/\lambda)\, Y_s/Y_g$ (Table 3).

Impulse Responses and Variance Decompositions. Figure 4 presents the impulse responses of our variables to the estimated shocks. In Table 6, we report the (asymptotic) variance decompositions. Both in the Figure and in the Table, we choose an orthogonalization scheme that orders the goods technology shock before the services technology shock. As a result, any variation due to the correlation between the goods and the services shock is attributed to the goods technology disturbance in the Figures and in the Tables.

The first row of Figure 4 plots the responses to a one standard deviation, favorable goods technology shock. Note that his shock leads to an increase in A_g of 0.87 percent, and to an increase in A_s (given our ordering scheme) by 0.37 percent. This disturbance is key in generating comovement of quantities in our model, and accounts for a large fraction of the fluctuations in real economic activity. In response to the shock, consumption, business investment and inventory investment all rise.[31] The goods shock spills over to the service sector (over and above the effect caused by the correlation of the shocks) because it facilitates the production of fixed capital that is then used in the service sector. This shock also accounts for a non-negligible fraction of the fluctuation of investment in both types of inventories (around 12 percent for output inventories and 16 percent for input inventories).

The second row shows the responses to a discount factor shock: this shock moves consumption and investment in opposite directions, and creates negative comovement across the output of the two sectors. It also contributes to the fluctuation of input inventories (16 percent of the total variance).

The third row shows the responses to a shock that shifts preferences away from final inventories towards goods consumption. The workings of this disturbance have the classic implications of a demand shock. Consumption of goods increases; inventories of finished goods fall; following the increase in demand, output of goods increases with a modest lag, while output of services is only marginally affected (because the estimated elasticity of substitution implies an approximate separability in utility between goods and services). This shock accounts for a large share (about 80%) of the fluctuations in output inventory investment.

[31] We scale the response of inventory investment by steady-state goods output, so that the vertical axis can be interpreted as the percent growth contribution of inventories to the goods output response. To facilitate comparison across all investment categories, we also scale the response of business investment by goods GDP. Since the ratio of business investment to GDP is about 30 percent, the percentage response of investment is larger than that of output and consumption.

The fourth row shows the response to a shock that shifts preferences away from services towards goods. While this shock accounts only for a small fraction of GDP fluctuations (basically, it reflects shifts in the composition of demand), it accounts for a quarter of the variance of output in the service sector and about half of the total variance of sectoral hours, since it causes reallocation of labor from one sector to the other (not reported in the table).

The fifth row plots the response to a shock to the efficiency of input inventories. This shock captures a large fraction (about 67 percent) of the variation in input inventory investment. Higher efficiency of input inventories reduces their usage, increases the demand for fixed capital, and raises consumption (immediately) and output (with a slight delay). The input inventory shock accounts for 6 percent of the variance of investment in fixed capital and for about 2.5 percent of the variance of goods output. The rest of the variance of input inventories is explained by the general preference shock and by the general technology shock in the goods sector.

Finally, the last row captures the responses to a productivity shock in the service sector. While this shock is obviously important in explaining output in the service sector, the effects of shocks in this sector transmit only marginally to the rest of the economy, since the service sector does not produce capital.[32]

Because the literature has often looked at the business cycle properties of the inventory-target ratios, we report in Figure 5 the impulse responses of total GDP and the inventory target ratios to the three disturbances - goods technology shock, output inventory shock, and input inventory shock - that cause most of the variation in GDP and inventories respectively. In response to the goods technology shock, the input inventory target ratio is strongly countercyclical, as in the data. Input inventory investment rises, but, because business capital is costly to adjust, the stock of input inventories – which is complementary to business capital – does not rise substantially, so that its ratio to GDP falls. The output inventory-target ratio is virtually acyclical (as in the data), since the household prefers to keep a relatively constant target of output inventories to consumption. The second and third row of Figure 5 show the responses to the inventory specific shocks. These shocks are key in reproducing the volatility of inventory investment observed in the data, but they mostly affect the inventory target ratios through their effects on the numerators, without large effects on output or consumption. In other words, they allow fitting better the volatility of inventory investment, but their presence does not alter the cyclical properties of the inventory target ratios, which are mostly driven by the aggregate productivity shocks.

Comparison between model and the data. Figure 6 offers a visual check of the ability of the model to reproduce key features of the data. We compare the empirical impulse responses and the model responses, which were obtained from the reduced form of the model by ordering and

[32] The logic of this result can be interpreted using an analogy to Kimball's (1994) consumption-technology neutrality result: with separable preferences over goods and services (as implied by our estimated model), technology shocks that affects the consumption-producing sector alone (in our model, the service sector) have no impact on employment or capital accumulation at all.

orthogonalizing the model shocks as in the VAR. In Table 9, we focus instead on some unconditional correlations in the data and we compare them with those generated by our estimated model.[33]

The message that emerges both from Figure 6 and Table 9 is that our setup does remarkably well in accounting not only for the variance of the key model variables, but also for their responses to various shocks, and for a large set of correlations that are found in the data. In particular, our model accounts at the same time for the volatility and procyclicality of inventory investment.[34] More specifically, it captures well the greater volatility of input inventory investment and its higher degree of procyclicality compared to output inventory investment. This is true whether we look at the correlation between inventory investment and goods output or between the changes of inventory investment and the change in GDP. Moreover, it can reproduce the countercyclicality of the input inventory/target ratio (although not its magnitude), and the relative acyclicality of the output inventory/target ratio. Finally, the model does an excellent job in fitting the relative volatilities of all types of investment.

Why? To better gain insights on how our model achieves this result, it is useful to think of a reference model that treats all types of capital symmetrically with respect to adjustment costs and assumes a zero inventory depreciation rate. Recall that Christiano's (1988) RBC model with inventories as a factor of production has no adjustment costs and assumes a zero depreciation rate for aggregate inventories. When these assumptions are imposed, our model predicts (1) extremely high volatility of business fixed investment; and (2) very small volatility in inventories, aside from the ones generated by their own innovations. Figure 7 provides a quantitative flavor, showing the impulse responses to a positive technology shock in the goods sector when we assume either zero inventory depreciation or no adjustment costs for K, M and F. With the estimated depreciation rates but without adjustment costs (starred lines), the response of output is larger than the one found in the data (thanks to the much larger response of business investment), and is twice as large on impact compared to our estimated model. However, inventories are less volatile relative to output, since business investment is much more responsive to changes in productivity. Such a model would generate a counterfactually high volatility of business fixed investment relative to GDP. With the estimated adjustment costs but with zero inventory depreciation (circled lines), the response of business investment relative to output would be as in our baseline estimated model, but inventories would move too little relative to the data.

Put differently, positive depreciation rates for inventories (potentially capturing holding costs as well) and their relatively small adjustment costs are two key model ingredients to fit the relative

[33] The impulse responses are based on a 6-variable VAR with a constant and two lags and are based on the ordering shown in the Figure.

[34] In Christiano (1988) it was necessary to rely on a more complex information structure in order to account for these two features of the data. He assumes that at the time hours and capital decisions are made, firms observe the shocks with noise. Inventory and consumption decisions are, instead, made with full knowledge of the shocks. When there is no signal extraction problem his model can generate enough inventory investment variability, but at the cost of a negative correlation between the change in inventory investment and output growth.

volatilities of all forms of investment. If inventories did not depreciate, they would tend to be very smooth and persistent over the cycle. Output inventories would be smooth for standard consumption smoothing reasons. Input inventories would not be very volatile because capital, in the absence of adjustment costs, would react more quickly in response to general productivity shocks, since these shocks have a larger effect on the marginal return to fixed capital. This occurs because a productivity shock has the same effect percentagewise on the marginal return to fixed capital and inventories. If the depreciation rate on inventories is much smaller (equal to zero) and fixed capital must be compensated for the higher depreciation rate with a higher return, the absolute effect of a shock on the marginal return to capital is much greater in absolute value.[35] As a result, capital would respond to productivity shocks more than input inventories.

5.2. Sub-samples.

We have re-estimated the model (keeping the same priors) over the sub-periods 1960:1-1983:4 and 1984:1-2004:4. In this exercise, we allow σ, α, θ_g, θ_s, and γ to differ across sub-samples in order to match the different sample means for the share of services in the economy, and the investment and the inventory ratios relative to goods output (reported in Table 3). This exercise allows us to investigate what lies at the root of the decrease of output volatility and what role, if any, inventories have played in this regard.

Parameter estimates. We present our results in Table 7. With few exceptions, the estimates for the full sample lie roughly in between those for the two sub-samples. Three results are worth emphasizing regarding the *structure* of the economy. First, the depreciation rate for F is smaller in the second part of the sample (it goes from 6 percent to 4 percent). Second, the utilization function for capital in the service sector becomes more convex (the value of ζ_{Ks} rises). Third, fixed capital becomes somewhat more costly to adjust.

It is difficult to provide exhaustive explanations for changes in these *deep* parameters of our model. A possible reason for the lower estimate of the depreciation rate δ_F might be a change in the inventory mix or better management of inventories in general. It is not clear how to interpret the higher adjustment costs for fixed capital, although they might reflect an increased weight of innovative investment in the second sub-period and the associated greater costs in terms of learning and disruption, or they might capture higher firm (or sectoral) specificity of capital goods.

We also find important and significant changes in the parameters measuring the stochastic processes for technology and preferences. The most important result is that the standard deviation of the technology shock in the goods sector and the input inventory shock experience the largest fall. The decrease in the importance of the input inventory shock is consistent with the idea that the new methods of inventory management have made it easier to control the level of input inventories in efficiency units. The fall in the variance of the technology shock to the goods sector

[35] See Christiano (1988).

is consistent with the idea the decline in GDP volatility observed in the data by many researchers is due to a change in the nature of the shocks, in particular of those affecting technology (Stock and Watson (2002)). We also find that the correlation between the technology shocks in the two sectors decreases substantially, from 75% to 51%.[36]

Interestingly, while we find evidence of a fall in the variance of technology/supply-type shock, we also find that the innovation variances of disturbances coming from the preference/demand side of the economy have all increased across sub-periods. This is true for the output inventory shock, for the discount factor shock, and for the goods versus services shock.

Correlations and Variance Decompositions. Table 9 (last four columns) shows that, across the two sub-periods, the model can reproduce the decline in the volatility of most macroeconomic aggregates. There are no sizeable changes in the volatility of input or output inventory investment, as there are not in the data. The model captures well the reduced procyclicality of output inventory investment after 1983.

Table 8 shows how movements in inventories in the second period depend more on their own innovations in relative terms. As for the other variables, a larger chunk of the volatility in economic activity (as summarized by the GDP measure) appears due to demand-preference shocks: in the second part of the sample, the share of GDP variance that can be accounted for by non-technological shocks rises from 13 percent to 31 percent. Technology shocks in the goods sector play a smaller role in accounting for the variance of input and output inventory investment. Finally, there is a reduction in the fraction of the variance of fixed investment explained by the input inventory shock (from 12 percent to 7 percent).

5.3. The role of inventories in the Great Moderation

A natural question to ask is to what extent the reduced volatility of economic activity in the post-1984 sample is due to a reduction in the volatility of the shocks (the "good luck" hypothesis) or to a change in the structure of the economy.

We begin by observing that our estimated model captures well the reduction in volatility across the two sub-periods. In our data, the standard deviation of detrended GDP drops by 0.82 percentage points between the 1960-1983 and the 1984-2004 period, from 2.16 percentage points to 1.35. As shown by the last four columns of Table 9, our sub-samples estimates capture the volatility decline almost perfectly, showing a reduction in GDP standard deviation by 0.86 percentage points. With this in mind, we ask what are the factors contributing in the model to the reduction in volatility. We partition the elements that can independently affect the implied volatility of the model variables in three sets.

[36] This result parallels the decrease in the covariance both between major sectors of the economy (Irvine and Schuh 2005b) and between industries within the goods (inventory-holding) sector (Irvine and Schuh 2005a).

1. Parameters determined using the steady state of the model (steady state parameters for short). Recall that when we estimate the model across sub-samples we choose values of $\alpha, \gamma, \sigma, \theta_g$ and θ_s that allow matching exactly the average values of inventories and business investment to output ratio and the share of services over GDP in each sub-period.

2. Parameters measuring the unconditional standard deviation of shocks ($\sigma_{\beta t}$, $\sigma_{\gamma t}$, σ_{Ft}, σ_{Mt}, σ_{gt} and σ_{st}).

3. Parameters affecting the dynamics of the model around the steady state and estimated without using information on the steady state ratios. This set of parameters includes the autocorrelation of the shocks as well as the inventory depreciation rates, the elasticities of substitution, the adjustment cost and the capital utilization parameters.

In Table 10 we break down the contribution of the three sets of parameters described above to the reduction in volatility. Using the estimates obtained for the 1960-1983 sample as a reference point, we change one estimated parameter at the time and set its value to its estimated value for the 1984-2004 period. In this way, we can measure the independent contribution of each parameter to the change in volatility.[37]

The main result that emerges from 10 is that the largest fraction of the reduction in volatility of GDP comes from the reduction in the volatility of the underlying shocks, especially of the technology shock in the goods sector. By themselves, the smaller shocks can explain a reduction in the volatility of GDP by 0.70 percentage points, compared to an estimated total decline of 0.82 percentage points. Most of the residual component is caused by the larger capital adjustment costs and capacity utilization costs, and by the increased share of services in the economy. Interestingly, while the rise of the service economy explains a non-negligible part of the reduction in volatility, the net effect of changes in the steady state parameters is smaller; in particular, the larger ratios of aggregate capital to goods output offset the effects of a larger service share: this happens because a larger investment share makes output inherently more volatile.

What about the role of inventories in the Great Moderation? The smaller volatility of input inventory shocks accounts for about 0.01 or 0.02 percentage points in the reduction in volatility, depending on whether we look at total GDP or at the output of the goods sector. The reduced input inventories to goods output ratio accounts for approximately 0.03 percentage points (0.08) of the decrease in the volatility of GDP (goods output).

We conclude that the reduced innovation variances account for the largest fraction of the reduction in aggregate volatility, and that structural changes have contributed by a smaller amount, mostly working through a reduction in the volatility of business fixed investment. The reduction in the variance of inventory shocks or in the parameters associated with inventories have played a minor role in the reduction in aggregate volatility.[38]

[37] Of course, because the model decision rules are nonlinear in the model structural parameters, it is possible that this exercise biases the actual contribution of a variable to the reduction in volatility.

[38] This conclusion is consistent with Khan and Thomas (2006), who consider how aggregate volatility changes in a general equilibrium model following a decrease in fixed ordering costs. It is also consistent with the one reached

6. Conclusions

Perhaps the most important lesson learned from this paper is that it is possible to specify a dynamic general equilibrium model with inventories that provides reasonable and interesting estimates of the parameters characterizing the behavior of input and output inventories. Investment in each of the two types of inventories plays a different logical role in the model and is characterized empirically by a different degree of volatility and procyclicality. The model can replicate the observed volatility and cyclicality of both input and output inventory investment and in particular the fact that input inventory investment is more volatile and procyclical than output inventory investment. Moreover, it can reproduce the strong countercyclicality of the input inventory/target ratio, and the relative acyclicality of the output inventory/target ratio. This finding represents a substantial step forward relative to previous attempts at modeling inventories in DSGE models, and it provides a new framework for analyzing the role of inventories in business cycle fluctuations.

When estimated across sub-periods (with 1984 as the breaking point), the model captures the general reduction in volatility of economic aggregates and of inventories in particular. Moreover, it captures the decrease in the procyclicality of output inventory investment. However, the reduction in the volatility of inventory shocks accounts for only a small portion of the decrease in the volatility of output. The latter is mostly explained by a fall in the estimated variance of the technology shock in the good sector. The volatility of the input inventory technology shock also declined dramatically, but this accounts only for a small reduction in the volatility of output. Nevertheless, this relatively aggregate framework identifies several dimensions along which the structure of the economy changed and contributed substantially to lower output volatility. Some of these dimensions are related to inventory behavior, but, at best, they have only played a minor role in accounting for the reduced volatility of output.

These conclusions are based on a two-sector general equilibrium model with some novel features, in addition to others that have now become standard, such as adjustment costs and variable capital utilization. The main new elements are the distinction between goods and services producing sectors according to their inventory-holding behavior and the distinction between input and output inventories. Non-zero inventory depreciation, which in the model provides an incentive to adjust inventories more in response to shocks is another novel feature that is empirically important.

Despite the additional complexity, the nature of our model still precludes examination of certain aspects of inventory behavior that are potentially quite important to understanding business cycle fluctuations. To keep the general equilibrium model sufficiently tractable in this exercise, we abstracted from at least three key issues. First, we abstracted from a richer examination of the stage-of-fabrication structure within the goods sector. For example, simplifying inventories into only two types abstracts from supply and distribution chains that now pervade the input-output

by Maccini and Pagan (2007) for the NIPA goods sector in a model that takes sales and factor prices as exogenous. They focus on the effects of changes in the parameters affecting the ratios of finished goods inventories to sales and raw materials to output.

structure of goods production and appear to play a role in the propagation and amplification of shocks. Introducing formal relationships among heterogeneous firms along these chains also will motivate a more careful examination of the manner in which input inventories facilitate production by distinguishing among the separate decisions by firms to order, use, and stock material inputs.

A second important issue is that our model is silent on how markup variations and nominal features matter for inventory behavior or business cycle fluctuations. Some inventory research examines how markup variation or interest rate policies influence inventory behavior.[39] However, this work has not incorporated the stage-of-fabrication inventory distinction we have advanced here in a general equilibrium setting. Relatedly, we have sidestepped in our model a central issue in the inventory literature, i.e. the motivation for firms' holding of finished goods (output inventories). By focusing on the value of output inventories to households through utility and concentrating on the planner's solution, we have not taken up a more detailed examination of the deep determinants of a retailer's decision to hold output inventories in a market environment. Tackling these important issues requires substantive extensions of the model in this paper. We plan to address them in future work, and we hope others will too.

[39] See footnote 3 for detailed references on this issue and on supply and distribution chains.

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Appendix A: Modeling Details

The model equations

We summarize below the model equations. The stochastic processes for the shocks are described in the main text.

$$Y_{gt} = C_{gt} + K_{gt} - \left(1 - \delta_{K_t^g}\right) K_{-1}^g + K_{st} - \left(1 - \delta_{K_t^s}\right) K_{-1}^s + F_t - (1 - \delta_F) F_{-1} + M_t - (1 - \delta_M) M_{-1} \tag{1}$$

$$\varepsilon_{\beta t} (1 - \gamma \varepsilon_{\gamma t}) \frac{W_t^{\phi}}{Y_{st}^{1+\phi}} = \omega_t \tag{2}$$

$$\varepsilon_{\beta t} \varepsilon_{\gamma t} \varepsilon_{Ft} \gamma \alpha \frac{W_t^{\phi} X_t^{\mu - \phi}}{C_{gt}^{1+\mu}} = \lambda_t \tag{3}$$

$$\tau \varepsilon_{\beta t} = \lambda_t (1 - \theta_g) \frac{Y_{gt}}{L_{gt}} \tag{4}$$

$$\tau \varepsilon_{\beta t} = \omega_t (1 - \theta_s) \frac{Y_{st}}{L_{st}} \tag{5}$$

$$\lambda_t \left(1 + \frac{\psi_{Kg}}{\delta_{Kg}} \frac{\Delta K_{gt}}{K_{t-1}}\right) = \beta \lambda_{t+1} \left(1 + \frac{\psi_{Kg}}{2\delta_{Kg}} \frac{K_{gt+1}^2 - K_{gt}^2}{K_{gt}^2} - \delta_{Kgt+1} + \theta_g \sigma \frac{Y_{gt+1} H_{t+1}^{\nu}}{z_{Kgt+1}^{\nu} K_{gt}^{1+\nu}}\right) \tag{6}$$

$$\lambda_t \left(1 + \frac{\psi_{Ks}}{\delta_{Ks}} \frac{\Delta K_{st}}{K_{st-1}}\right) = \beta \left(\lambda_{t+1} \left(1 + \frac{\psi_{Ks}}{2\delta_{Ks}} \frac{K_{st+1}^2 - K_{st}^2}{K_{st}^2} - \delta_{Kst+1}\right) + \omega_{t+1} \theta_s \frac{Y_{st+1}}{K_{st}}\right) \tag{7}$$

$$\lambda_t \left(1 + \frac{\psi_M}{\delta_M} \frac{\Delta M_t}{M_{t-1}}\right) = \beta \lambda_{t+1} \left(1 + \frac{\psi_M}{2\delta_M} \frac{M_{t+1}^2 - M_t^2}{M_t^2} - \delta_M + \theta_g (1 - \sigma) \frac{Y_{gt+1} H_{t+1}^{\nu}}{\varepsilon_{Mt+1}^{\nu} M_t^{1+\nu}}\right) \tag{8}$$

$$\lambda_t \left(1 + \frac{\psi_F}{\delta_F} \frac{\Delta F_t}{F_{t-1}}\right) = \beta \left(\varepsilon_{\beta t+1} \varepsilon_{\gamma t+1} \gamma (1 - \alpha \varepsilon_{Ft+1}) \frac{W_{t+1}^{\phi} X_{t+1}^{\mu - \phi}}{F_t^{1+\mu}} + \lambda_{t+1} \left(1 + \frac{\psi_F}{2\delta_F} \frac{F_{t+1}^2 - F_t^2}{F_t^2} - \delta_F\right)\right) \tag{9}$$

$$\theta_g \frac{\sigma Y_{gt} H_t^{\nu}}{z_{Kgt}^{1+\nu} K_{gt}^{\nu}} = b_{Kg} \left(\zeta_{Kg} z_{Kgt} + 1 - \zeta_{Kg}\right) K_{gt-1} \tag{10}$$

$$\omega_t \theta_s \frac{Y_{st}}{z_{Kst}} = b_{Ks} (\zeta_{Ks} z_{Kst} + 1 - \zeta_{Ks}) K_{st-1} \lambda_t \tag{11}$$

$$\delta_{Kgt} = \delta_{Kg} + b_{Kg} \zeta_{Kg} z_{Kgt}^2 / 2 + b_{Kg} \left(1 - \zeta_{Kg}\right) z_{Kgt} + b_{Kg} \left(\zeta_{Kg}/2 - 1\right) \tag{12}$$

$$\delta_{Kst} = \delta_{Ks} + b_{Ks} \zeta_{Ks} z_{Kst}^2 / 2 + b_{Ks} (1 - \zeta_{Ks}) z_{Kst} + b_{Ks} (\zeta_{Ks}/2 - 1) \tag{13}$$

$$\xi_{Kgt} = \frac{\psi_{K_g}}{2\delta_{Kg}} \left(\frac{K_{gt} - K_{gt-1}}{K_{gt-1}}\right)^2 K_{gt-1} \tag{14}$$

$$\xi_{Kst} = \frac{\psi_{K_s}}{2\delta_{K_s}} \left(\frac{K_{st} - K_{st-1}}{K_{st-1}}\right)^2 K_{st-1} \tag{15}$$

$$\xi_{Mt} = \frac{\psi_M}{2\delta_M} \left(\frac{M_t - M_{t-1}}{M_{t-1}}\right)^2 M_{t-1} \tag{16}$$

$$\xi_{Ft} = \frac{\psi_F}{2\delta_F} \left(\frac{F_t - F_{t-1}}{F_{t-1}}\right)^2 F_{t-1} \tag{17}$$

$$Y_{gt} = (A_{gt} L_{gt})^{1-\theta_g} H_t^{\theta_g} \tag{18}$$

$$H_t = \left(\sigma (z_{gt} K_{gt-1})^{-\nu} + (1 - \sigma) (\varepsilon_{Mt} M_{t-1})^{-\nu}\right)^{-1/\nu} \tag{19}$$

$$X_t = \left(\alpha \varepsilon_{Ft}^{-\mu} C_t^{-\mu} + (1 - \alpha) F_{t-1}^{-\mu}\right)^{-1/\mu} \tag{20}$$

$$W_t = \left(\gamma \varepsilon_{\gamma t} X_t^{-\phi} + (1 - \gamma \varepsilon_{\gamma t}) \varepsilon_{Ft}^{-\phi} Y_{st}^{-\phi}\right)^{-1/\phi} \tag{21}$$

$$Y_{st} = (A_{st} L_{st})^{1-\theta_s} (z_{st} K_{st-1})^{\theta_s}. \tag{22}$$

where AC in the first equation denotes the total adjustment cost.

The steady state

After some algebra, we can show that the main ratios that describe the steady state of our model are described by the following equations:

$$c_g = \left(\frac{\alpha}{1-\alpha} \frac{1-\beta(1-\delta_F)}{\beta} \right)^{\frac{1}{1+\mu}} f \tag{23}$$

$$k_s = \left(\frac{\omega}{\lambda}\right)^{\frac{\phi}{1+\phi}} \left(\frac{1-\gamma}{\alpha\gamma}\right)^{\frac{1}{1+\phi}} \frac{\theta_s \beta}{1-\beta(1-\delta_{Ks})} \left(\alpha + (1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{\mu-\phi}{\mu(1+\phi)}} c_g \tag{24}$$

$$k_g = \frac{\theta_g \sigma \beta}{1-\beta(1-\delta_{Kg})} \frac{1}{\sigma + (1-\sigma)\left(\frac{\sigma}{1-\sigma}\frac{1-\beta(1-\delta_M)}{1-\beta\left(1-\delta_{Kg}\right)}\right)^{\frac{\nu}{1+\nu}}} \tag{25}$$

$$m = \frac{\theta_g \beta (1-\sigma)}{1-\beta(1-\delta_M)} \frac{1}{(1-\sigma) + \sigma\left(\frac{\sigma}{1-\sigma}\frac{1-\beta(1-\delta_M)}{1-\beta\left(1-\delta_{Kg}\right)}\right)^{-\frac{\nu}{1+\nu}}} \tag{26}$$

$$\frac{\lambda}{\omega} = \left(\frac{(1-\theta_s)\left(\frac{\theta_s \beta}{1-\beta(1-\delta_{Ks})}\right)^{\frac{\theta_s}{1-\theta_s}}}{(1-\theta_g)\left(\sigma k_g^{-\nu} + (1-\sigma) m_g^{-\nu}\right)^{\frac{-\theta_g}{\nu(1-\theta_g)}}} \right)^{1-\theta_s} \tag{27}$$

$$c_g + \delta_F f + \delta_{Kg} k_g + \delta_{Ks} k_s + \delta_M m = 1. \tag{28}$$

Given values of the model parameters, equations 23 to 28 can be solved for f, c_g, k_s, k_g, m and the ω/λ ratio.

Matching steady-state ratios through choices of $\alpha, \theta_g, \theta_s, \sigma, \gamma$

For each estimated value of ν, ϕ, μ, δ_F and δ_M, and given calibrated values for β, δ_{Kg} and δ_{Ks}, our estimation procedure aims at exactly matching the following steady state ratios that we take to be the average values obtained from the data (denoted with a bar):

$$\begin{aligned}
\overline{f} &= \text{output inventories over goods output} \\
\overline{m} &= \text{input inventories over goods output} \\
\overline{k_g} &= \text{capital stock in goods industries over goods output} \\
\overline{k_s} &= \text{capital stock in service industries over goods output} \\
\overline{y_s'} &= \text{services output over goods output.}
\end{aligned}$$

where $y_s' = \frac{\omega Y_s}{\lambda Y_g} = \frac{\omega}{\lambda} y_s$ measures services output in units of goods output.

Given $\beta, \delta_{Ks}, \delta_{Ks}, \phi, \mu, \nu, \delta_F, \delta_M$,simple algebra shows that there is a unique set of values of $\alpha, \theta_g, \theta_s, \sigma, \gamma$ that satisfies the five ratios above. These values are obtained as follows. Given the $\left(\overline{k_g}/\overline{m}\right)$ ratios, we obtain:

$$\sigma = \frac{\left(\overline{k_g}/\overline{m}\right)^{1+\nu} \frac{1-\beta\left(1-\delta_{K_g}\right)}{1-\beta(1-\delta_M)}}{1 + \left(\overline{k_g}/\overline{m}\right)^{1+\nu} \frac{1-\beta\left(1-\delta_{Kg}\right)}{1-\beta(1-\delta_M)}}. \tag{29}$$

>From the $\overline{c}_g/\overline{f}$ ratio, we derive:

$$\alpha = \frac{\left(\overline{c_g}/\overline{f}\right)^{1+\mu} \frac{\beta}{1-\beta(1-\delta_F)}}{1 + \left(\overline{c_g}/\overline{f}\right)^{1+\mu} \frac{\beta}{1-\beta(1-\delta_F)}}. \tag{30}$$

The formula for k_g/y_g can be used to derive:

$$\theta_g = \frac{\left(\sigma + (1-\sigma)\left(\overline{k_g}/\overline{m}\right)^{\nu}\right)(1-\beta(1-\delta_{Kg}))}{\sigma\beta} \overline{k_g}. \tag{31}$$

We finally need to choose γ and θ_s to match the values of k_s and y_s'. From the formula for k_s/y_s' , we derive an expression for θ_s:

$$\theta_s = \frac{1-\beta(1-\delta_{Ks})}{\beta} \frac{\overline{k_s}}{\overline{y_s'}}. \tag{32}$$

Last, we need to obtain γ. Using the expressions for k_s/f and k_s/y_s' above, we obtain:

$$\gamma = \frac{\left(\overline{c_g}/\overline{y_s'}\right)^{1+\phi}(\omega/\lambda)^{\phi}\, c_g^{\mu-\phi}\left(\alpha\overline{c}_g^{-\mu}+(1-\alpha)\,\overline{f}^{-\mu}\right)^{\frac{\mu-\phi}{\mu}}}{\alpha+\left(\overline{c_g}/\overline{y_s'}\right)^{1+\phi}(\omega/\lambda)^{\phi}\, c_g^{\mu-\phi}\left(\alpha\overline{c}_g^{-\mu}+(1-\alpha)\,\overline{f}^{-\mu}\right)^{\frac{\mu-\phi}{\mu}}} \tag{33}$$

where ω/λ is given by the expression 27 above.

Summarizing: (1) : for given observed values in the data of $k_g,\ k_s,\ m,\ c_g,\ f,\ \ y_s'$; and (2) for any possible combination of $(\beta, \delta_{Ks}, \delta_{Ks}, \phi, \mu, \nu, \delta_F, \delta_M)$, the values of $\alpha, \theta_g, \theta_s, \sigma, \gamma$ that satisfy expressions 29 to 33 are consistent with the the steady state values of the ratios $k_g,\ k_s,\ m,\ c_g,\ f,$and y_s'.

Calculating steady state hours, output and prices

The optimal labor supply schedules satisfy

$$\tau = \lambda\,(1-\theta_g)\,\frac{Y_g}{L_g} \tag{34}$$

$$\tau = \omega\,(1-\theta_s)\,\frac{Y_s}{L_s}. \tag{35}$$

A closed form solution can be calculated only when $\phi = 0$. Otherwise, we need to use the following expressions. From the first order conditions for C_g and Y_S, we can obtain, after some algebra, the following formula. Hence:

$$\lambda Y_g = \frac{\gamma\alpha}{c_g}\left(\gamma\left(\alpha+(1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)+(1-\gamma)\left(\alpha+(1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{\mu-\phi}{\mu(1+\phi)}}\left(\frac{\omega}{\lambda}\frac{\alpha\gamma}{1-\gamma}\right)^{\frac{\phi}{1+\phi}}\right)^{-1}. \tag{36}$$

By the same token, we find that:

$$\omega Y_s = (1-\gamma)\left(\gamma\left(\alpha+(1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{1+\mu}{\mu}\frac{\phi}{1+\phi}}\left(\frac{\omega}{\lambda}\frac{\alpha\gamma}{1-\gamma}\right)^{\frac{-\phi}{1+\phi}}+(1-\gamma)\right)^{-1}. \tag{37}$$

From the production functions, we know that:

$$Y_g = L_g\left(\sigma k_g^{-\nu}+(1-\sigma)\,m_g^{-\nu}\right)^{-\frac{\theta_g}{\nu(1-\theta_g)}} \tag{38}$$

$$Y_s = L_s\left(\frac{K_s}{Y_s}\right)^{\frac{\theta_s}{1-\theta_s}}. \tag{39}$$

Equations 34 through 39 can be then be solved for $L_g, L_s, Y_g, Y_s, \omega, \lambda$ using a non-linear equation algorithm.

Appendix B: Data

Most of our data come from the national income and product accounts (NIPA) produced by the U.S. Commerce Department, Bureau of Economic Analysis (BEA), and obtained from Haver Analytics, Inc. All NIPA data are quarterly. The real data are in chain-weighted \$2000. Table B.1 lists the variable names, Haver mnemonics, and variable descriptions. Our model and data exclude net exports and government spending.

Although the formulas in this appendix suppress the notational details associated with the proper manipulation of chain-weighted real data, we use the appropriate Tornquist approximation for chain-weighted data recommended by Whelan (2002) in constructing the actual data.

The NIPA data classify output by sectors called goods (g), structures (t), and services (s):

$$Y = Y^g + Y^t + Y^s.$$

In contrast, inventory investment, ΔV, is classified by industry (goods inventories include the agriculture, mining, and manufacturing industries; structures inventories include the construction industry; and the services sector includes utilities and trade). Thus, the NIPA output and inventory data do not correspond to the inventory-based sectors of our model definitions of goods and services.

To obtain model-consistent data, we can think of the task as one of condensing the three NIPA sectors into two by redefining and combining the NIPA sector variables as follows. First, write the components of aggregate output as

$$Y = (C^g + I^g + \Delta V^g) + \left(I^t + \Delta V^t\right) + (C^{sg} + C^{ss} + I^s + \Delta V^s)\ .$$

There is no household consumption of structures (C^t) because the construction of structures is purely investment, which is installed in each sector. Household consumption of services, $C^s = C^{sg} + C^{ss}$, includes two components, distinguished by a second superscript indicating the appropriate model sector to which the services consumption data should belong. Thus, C^{sg} represents the consumption of services from industries that distribute goods (utilities and trade) that we wish to redefine as goods consumption. Also, C^{ss} includes the service flow from housing.

Given these definitions, we can then write model-consistent goods output as

$$Y^g = (C^g + C^{sg}) + \left(I^g + I^t + I^s\right) + \left(\Delta V^g + \Delta V^t + \Delta V^s\right)$$

and model-consistent services output as

$$Y^s = C^{ss}.$$

The remainder of this appendix explains how each of the relevant variables is defined and constructed.

Consumption

NIPA consumption data are classified by the **type of good** consumed by households,

$$C = C^{gn} + C^{gd} + C^s,$$

where goods includes nondurables (gn) and durables (gd), and services (s) includes the service flow from housing. Theoretically, it would be preferable to construct the service flow from other consumer durable goods besides housing, rather than use actual expenditures, but this is not done in the NIPA data (except auto leasing, which is implicitly a service yield). Because we are ultimately trying to explain the volatility, and change in volatility, of actual GDP data, we use the raw NIPA data instead.

Using the NIPA consumption data to construct model-consistent consumption data, we must reclassify the portion of data on consumption of services from goods-producing industries (C^{sg}) into consumption of goods (C^g). Because the NIPA consumption data do not treat energy (such as electricity) as a good, we must define consumption of household energy (e) services as model-consistent goods consumption: $C^{sg} = C^{sge}$. Because energy is output of the utilities industry, which holds inventories, we assume that all types of energy are measurable goods distributed to consumers. In this regard, electric and natural gas utilities are similar to wholesale and retail trade, which distribute finished goods from their producers to their final consumers. Thus, the model-consistent data (denoted by a tilde, $\tilde{}$) for services consumption are

Variable	Mnemonic	Description
C	C	Consumption
C^{gn}	CN	Consumption of nondurable goods
C^{gd}	CD	Consumption of durable goods
C^{s}	CS	Consumption of services
C^{se}	CSE	Consumption of energy services
I	F	Fixed investment (including residential)
V^{ga}	SF	Farm inventories
V^{gm}	SNM	Manufacturing inventories
V^{sw}	SNW	Wholesale trade inventories
V^{sr}	SNR	Retail trade inventories
V^{o}_{SIC}	SNO2	Other inventories, SIC (fixed-weight $1996)
V^{MUC}	SNB	Mining, utilities, and construction inventories
V^{o}_{NAICS}	SNT	Other inventories, NAICS
V^{CW}	RES513	Inventory chain-weight residual
P	JC	Consumption chain-weight price index
P^{s}	JCS	Consumption of services chain-weight price index
P^{se}	JCSE	Consumption of energy services chain-weight price index

Table B.1: Variable Names and Data Definitions

Note: These Haver mnemonics are for the nominal data; the real data have an 'H' at the end and, unless otherwise noted, are in chain-weighted $2000.

$$\widetilde{C^s} = C^s - C^{sg}$$

and the model-consistent data for goods consumption are

$$\widetilde{C^g} = C^{gn} + C^{gd} + C^{sg}.$$

Because the underlying NIPA data are based on the type of good consumed, C^{gn} and C^{gd} already contain the output of the retail trade industry and any output of the wholesale trade industry that is consumption (a final sale, as opposed to an intermediate input into retail trade or back into manufacturing).[40]

Investment

Because capital is a good (by definition), it is straightforward to define investment as output of the goods sector. However, because our model has two distinct sectors that each accumulate a sector-specific capital stock (i.e., it has two capital accumulation equations), the model requires that we construct investment data classified by the **type of firm or industry (sector)** in which the capital is installed. Although the NIPA data do not include investment classified by sector of installation, the BEA does offer annual data on investment by industry, which we use to divide total investment into sector-specific investment components.[41]

We define the goods sector to include the seven inventory-holding industries: agriculture, mining, utilities, construction, manufacturing, wholesale trade, and retail trade. Using the BEA data on investment by industry (denoted by a caret, ^) for these industries, we define the share (s) of goods investment as

$$\widehat{s} = \left(\hat{I}^g / \hat{I}\right) \ .$$

[40] One way to think of the different types of "goods" is in terms of their depreciation rates: $0 < \delta^{sh} < \delta^{d} < \delta^{n} < \delta^{so} = 1$, where superscript sh denotes housing services and so denotes other services (that is, not a flow from a durable stock).

[41] These data can be obtained from http://bea.gov/bea/dn/FA2004/Index.asp.

These annual share data are interpolated to a quarterly frequency. Then the goods investment data are

$$\widetilde{I^g} = \widehat{s}I \ ,$$

where I is total fixed investment, and services investment data are

$$\widetilde{I^s} = (1 - \widehat{s})\, I = I - \widetilde{I^g} \ .$$

and applied to the actual quarterly data on total fixed investment.

Inventories

According to NIPA definitions, each output sector is associated with at least one inventory-holding industry,

$$\begin{aligned} V^g &= V^{ga} + V^{gn} + V^{gm} \\ V^t &= V^{tc} \\ V^s &= V^{su} + V^{sw} + V^{sr} \end{aligned}$$

with industries defined as agriculture (a), mining (n), manufacturing (m), construction (c), utilities (u), wholesale trade (w), and retail trade (r). Thus, to construct model-consistent inventories, we redefine the goods sector as the holder of all inventories,

$$\widetilde{V^g} = V^g + V^t + V^s \ ;$$

the services sector holds no inventories ($\widetilde{V^s} = 0$) by assumption.[42]

We further divide total inventories into two types,

$$\widetilde{V^g} = M + F \ ,$$

where M denotes input and F denotes output. Theory provides no clear definition of input and output inventories in general equilibrium. We view goods as being produced and distributed along a supply chain, so one logical definition of output inventories for our model is the last link of the chain, the retail industry:

$$\widetilde{M} = V^{sr} \ .$$

In this case, input inventories are

$$\widetilde{F} = V^{ga} + V^{gn} + V^{gm} + V^{tc} + V^{su} + V^{sw} \ .$$

In general, all non-retail inventory stocks can be considered inputs into production along the supply chain. According to the Census of Construction, V^{tc} certainly is input inventory (raw materials) and does not include unsold finished structures. In actuality, some fraction of the remaining stocks may be sold directly to consumers, and hence should be classified as output inventories, but we assume this fraction is small.

To obtain long time series of inventory data, we combine non-farm stocks constructed under two different industry classifications: SIC (1947-1997) and NAICS (1987-present).[43] At this high level of industry definition, the manufacturing, wholesale, and retail inventory data are generally consistent across industry classification schemes, so we splice them without further manipulation. The inventories for all remaining industries ($*$), however, are defined as follows:

$$\begin{aligned} V^*_{SIC} &= V^o_{SIC} \\ V^*_{NAICS} &= V^{MUC} + V^o_{NAICS} + V^{CW} \end{aligned}$$

where o denotes "other" industries in each classification system; MUC denotes mining, utilities, and construction; and CW denotes the chain-weight residual for real data (real data on an SIC basis are in fixed-weight \$1996, and thus have no residual). In splicing the data, we use the SIC stocks through 1997 and then use the growth rates of the NAICS from 1997 on to extend the SIC data.

[42] The NIPA make this same assumption, equation output and final sales in the structures and services sectors and associated all inventory investment with the goods sector.

[43] Farm, or agricultural, inventory stocks on a consistent industry classification are already available for the full sample period (1947-present).

Consumption Prices

The prices goods and services consumption are constructed analogously to the quantities of consumption. Let w^{se} be the nominal expenditure weight for energy services and $w^{\tilde{s}} = (1 - w^{se})$ be the nominal expenditure weight for model-consistent (non-energy) services. Then the model-consistent price of services consumption is (again we have calculated the appropriate Tornquist index on the data):

$$\widetilde{P^s} = (1/w^{\tilde{s}})\,[P^s - w^{se} P^{se}] \ .$$

Likewise, let $w^{\tilde{s}}$ be the nominal expenditure weight for model-consistent services and $w^g = (1 - w^{\tilde{s}})$ be the nominal expenditure weight for model-consistent goods. Then the model-consistent price of goods consumption is

$$\widetilde{P^g} = (1/w^g)\left[P - w^{\tilde{s}}\widetilde{P^s}\right] \ .$$

And the ratio of consumption prices,

$$\frac{\widetilde{P^g}}{\widetilde{P^s}} = \frac{\lambda}{\omega} \ ,$$

equals the ratio of Lagrange multipliers from the model's first-order conditions.

Model Sectors	NIPA Sectors and Industries (NAICS)		2000 GDP Share
	Sector	Industry	(in percent)
Goods (59.3%) [67.6% of private sector]	Goods (35.1%)	Agriculture	1.0
		Mining	1.2
		Manufacturing	14.5
	Structures (9.6%)	Construction	4.4
	Services (55.3%)	Utilities	1.9
		Wholesale Trade	6.0
		Retail Trade	6.7
Services (28.4%) [32.4% of private sector]		Transportation	3.1
		Information	4.7
		FIREL	19.7
		Services	11.6
		Education & Health	6.9
		Leisure	3.6
Public (excluded) (12.3%)		Government	12.3

Table 1: Sector Definitions and Output Shares
NOTE: FIREL denotes Finance, Insurance, Real Estate, and Leasing.

Model Inventories		NIPA Inventories (NAICS)	2000 Share
Input & Output	Stage-of-Fabrication	Industry	(in percent)
Input (73.4%)	Raw Materials (18.9%)	Agriculture	8.6
		Mining, utilities, construction (MUC)	2.9
		Mining	n.a.
		Utilities	n.a.
		Construction	n.a.
		Other	7.4
	Work-in-process (54.5%)	Manufacturing	31.1
		Materials and supplies	11.0
		Work-in-process	8.9
		Finished goods	11.1
		Wholesale trade	23.4
Output (26.6%)	Finished goods (26.6%)	Retail trade	26.6

Table 2: Inventory Stock Definitions and Shares

	Full sample	1960-1983	1984-2004
F/Y_g	0.29	0.34	0.32
M/Y_g	1.34	1.01	1.12
K_g/Y_g	7.73	6.54	6.89
K_s/Y_g	6.74	9.32	8.36
Y_s'/Y_g	0.53	0.81	0.75

Table 3: Target Steady State Ratios

Notes: Target steady state ratios of the model. Output is expressed in quarterly units. The last row is the ratio of nominal output of services over nominal output of the goods sector. The capital output ratios are calculated from the investment to output ratios assuming depreciation rates of $\delta_{Kg} = 0.02$ and $\delta_{Ks} = 0.02$.

	Prior			Full Sample		
	Mean	Distribution	St.dev.	Mean	5%	95%
δ_F	0.020	beta	0.01	0.0784	0.0505	0.1088
δ_M	0.020	beta	0.01	0.0204	0.0114	0.0316
$1+\mu$	1.500	norm	0.5	1.0691	0.6290	1.5285
$1+\nu$	1.500	norm	0.5	3.3277	2.8768	3.8174
$1+\phi$	1.500	norm	0.5	1.0525	1.0290	1.0915
$\psi_F/(1+\psi_F)$	0.500	beta	0.2	0.0242	0.0143	0.0367
$\psi_{Kg}/(1+\psi_{Kg})$	0.500	beta	0.2	0.4722	0.2422	0.8103
$\psi_{Ks}/(1+\psi_{Ks})$	0.500	beta	0.2	0.2523	0.1608	0.4180
$\psi_M/(1+\psi_M)$	0.500	beta	0.2	0.0183	0.0101	0.0289
$\zeta_{Kg}/(1+\zeta_{Kg})$	0.500	beta	0.2	0.9133	0.8130	0.9785
$\zeta_{Ks}/(1+\zeta_{Ks})$	0.500	beta	0.2	0.5586	0.3446	0.7653
ρ_g	0.750	beta	0.1	0.8674	0.8287	0.9028
ρ_B	0.750	beta	0.1	0.8839	0.8428	0.9203
ρ_F	0.750	beta	0.1	0.8838	0.7803	0.9576
ρ_γ	0.750	beta	0.1	0.8173	0.7556	0.8761
ρ_M	0.750	beta	0.1	0.9381	0.9104	0.9619
ρ_s	0.750	beta	0.1	0.9343	0.9021	0.9611
σ_g	0.02	invg	Inf	0.0175	0.0149	0.0205
σ_B	0.02	invg	Inf	0.0187	0.0142	0.0250
σ_F	0.01	invg	Inf	0.0032	0.0025	0.0039
σ_γ	0.01	invg	Inf	0.0056	0.0047	0.0066
σ_M	0.10	invg	Inf	0.0944	0.0740	0.1193
σ_s	0.01	invg	Inf	0.0142	0.0114	0.0178
$\sigma_{g,s}$	0.50	norm	0.25	0.7115	0.6352	0.7771

Table 4: Prior Distributions and Parameter Estimates, Full Sample

	Full sample	1960-1983	1984-2004
α	0.9625	0.9716	0.9776
γ	0.4939	0.5562	0.4837
σ	0.9976	0.9958	0.9969
θ_g	0.2305	0.2602	0.2217
θ_s	0.3191	0.3653	0.3354

Table 5: Values of the Share Parameters implied by the estimation results

	Full Sample					
	σ_g	σ_β	σ_F	σ_γ	σ_M	σ_s
$Y_g/\overline{Y}_g$	78.9	16.1	1.5	1.0	2.5	0.0
$Y_s/\overline{Y}_s$	58.3	4.0	0.0	11.5	0.4	25.7
$I/\overline{Y}_g$	51.1	42.1	0.1	0.7	6.0	0.0
$\Delta F/\overline{Y}_g$	11.9	4.2	81.7	0.2	2.1	0.0
$\Delta M/\overline{Y}_g$	16.5	16.0	0.2	0.0	67.3	0.0
$C_g/\overline{C}_g$	77.5	3.1	4.9	9.8	4.7	0.0
$gdp/\overline{gdp}$	84.3	11.3	0.4	0.1	1.2	2.8

Table 6: Variance Decompositions of the model, full sample
Notes: For each variable, the columns indicate the fraction of total variance explained by each shock.

	Prior			1960:1-1983:4			1984:1-2004:4		
				Mean	5%	95%	Mean	5%	95%
δ_F	0.020	beta	0.01	0.0604	0.0351	0.0883	0.0426	0.0207	0.0688
δ_M	0.020	beta	0.01	0.0187	0.0105	0.0285	0.0224	0.0124	0.0362
$1+\mu$	1.500	norm	0.5	0.9783	0.5163	1.4586	1.1940	0.5364	1.8500
$1+\nu$	1.500	norm	0.5	3.0918	2.6475	3.5884	3.1426	2.8101	3.6231
$1+\phi$	1.500	norm	0.5	0.9626	0.9309	0.9855	1.1809	1.0910	1.2863
$\psi_F/(1+\psi_F)$	0.500	beta	0.2	0.0214	0.0113	0.0347	0.0397	0.0214	0.0610
$\psi_{Kg}/\left(1+\psi_{Kg}\right)$	0.500	beta	0.2	0.4199	0.2149	0.8257	0.4956	0.2852	0.8160
$\psi_{Ks}/(1+\psi_{Ks})$	0.500	beta	0.2	0.3335	0.1530	0.5936	0.3631	0.2165	0.5396
$\psi_M/(1+\psi_M)$	0.500	beta	0.2	0.0278	0.0145	0.0440	0.0161	0.0073	0.0278
$\zeta_{Kg}/\left(1+\zeta_{Kg}\right)$	0.500	beta	0.2	0.8734	0.7396	0.9653	0.8680	0.7153	0.9673
$\zeta_{Ks}/(1+\zeta_{Ks})$	0.500	beta	0.2	0.3047	0.1168	0.5120	0.8002	0.5960	0.9463
ρ_g	0.750	beta	0.1	0.8496	0.8035	0.8943	0.9034	0.8551	0.9461
ρ_B	0.750	beta	0.1	0.8780	0.8270	0.9210	0.9004	0.8577	0.9379
ρ_F	0.750	beta	0.1	0.9244	0.8306	0.9774	0.9121	0.8439	0.9655
ρ_γ	0.750	beta	0.1	0.8314	0.7505	0.9059	0.8181	0.7315	0.8974
ρ_M	0.750	beta	0.1	0.9547	0.9337	0.9714	0.9707	0.9551	0.9835
ρ_s	0.750	beta	0.1	0.9341	0.8956	0.9677	0.9460	0.9123	0.9725
σ_g	0.02	invg	Inf	0.0224	0.0188	0.0269	0.0142	0.0111	0.0183
σ_B	0.02	invg	Inf	0.0207	0.0150	0.0279	0.0235	0.0157	0.0351
σ_F	0.01	invg	Inf	0.0035	0.0026	0.0046	0.0047	0.0033	0.0065
σ_γ	0.01	invg	Inf	0.0053	0.0042	0.0069	0.0056	0.0044	0.0074
σ_M	0.10	invg	Inf	0.1340	0.1016	0.1769	0.1071	0.0816	0.1418
σ_s	0.01	invg	Inf	0.0159	0.0123	0.0215	0.0145	0.0111	0.0193
$\sigma_{g,s}$	0.50	norm	0.25	0.7565	0.6592	0.8345	0.5144	0.3572	0.6497

Table 7: Parameter Estimates, Subsamples

1960-1983						
	σ_g	σ_β	σ_F	σ_γ	σ_M	σ_s
$Y_g/\overline{Y}_g$	82.0	11.3	1.4	0.5	4.8	0.0
$Y_s/\overline{Y}_s$	62.1	2.4	0.0	15.9	1.1	18.6
$I/\overline{Y}_g$	55.9	30.8	0.1	1.0	12.2	0.0
$\Delta F/\overline{Y}_g$	14.1	5.1	79.1	0.1	1.7	0.0
$\Delta M/\overline{Y}_g$	20.3	14.6	0.2	0.0	64.9	0.0
$C_g/\overline{C}_g$	77.0	3.3	4.2	6.8	8.7	0.0
$gdp/\overline{gdp}$	87.3	8.3	0.4	0.2	2.6	1.2

1984-2004						
	σ_g	σ_β	σ_F	σ_γ	σ_M	σ_s
$Y_g/\overline{Y}_g$	62.7	25.8	4.0	1.7	5.8	0.0
$Y_s/\overline{Y}_s$	38.6	7.4	0.1	9.3	0.7	43.9
$I/\overline{Y}_g$	32.0	60.6	0.1	0.3	7.0	0.0
$\Delta F/\overline{Y}_g$	2.3	3.4	93.9	0.0	0.4	0.0
$\Delta M/\overline{Y}_g$	9.4	24.5	0.2	0.0	65.9	0.0
$C_g/\overline{C}_g$	68.9	3.6	12.2	9.4	5.7	0.2
$gdp/\overline{gdp}$	69.3	20.7	1.2	0.0	2.7	6.1

Table 8: Variance Decompositions, Subsamples

	Full sample		**1960-1983**		**1984-2004**	
Standard deviation	**Model**	**Data**	**Model**	**Data**	**Model**	**Data**
$gdp/\overline{gdp}$	0.0167	0.0183	0.0216	0.0216	0.0134	0.0135
$Y_g/\overline{Y_g}$	0.0228	0.0297	0.0281	0.0349	0.0186	0.0223
$Y_s/\overline{Y_s}$	0.0132	0.0067	0.0156	0.0076	0.0117	0.0056
$I/\overline{Y_g}$	0.0116	0.0139	0.0135	0.0161	0.0092	0.0108
$\Delta F/\overline{Y_g}$	0.0045	0.0037	0.0051	0.0039	0.0055	0.0032
$\Delta M/\overline{Y_g}$	0.0076	0.0068	0.009	0.0068	0.0068	0.0066
Correlations						
$Y_g/\overline{Y_g}, \Delta F/\overline{Y_g}$	0.3183	0.4175	0.3396	0.4698	0.2627	0.2798
$Y_g/\overline{Y_g}, \Delta M/\overline{Y_g}$	0.4517	0.6404	0.427	0.6574	0.4122	0.6199
$\Delta gdp/\overline{gdp}, \Delta^2 F/\overline{Y_g}$	0.4441	0.4163	0.5017	0.4661	0.3408	0.3702
$\Delta gdp/\overline{gdp}, \Delta^2 M/\overline{Y_g}$	0.5557	0.5324	0.5669	0.5544	0.6016	0.4546
$Y_g/\overline{Y}_g, F_g/C_g$	0.1133	0.104	0.0736	0.1067	-0.0328	0.0348
$Y_g/\overline{Y}, M_g/Y_g$	-0.3015	-0.8923	-0.3465	-0.9205	-0.2298	-0.8049
$Y_g/\overline{Y_g}, Y_s/\overline{Y_s}$	0.5573	0.6849	0.6215	0.7566	0.4675	0.5066

Table 9: Estimated Model Properties: Correlations and Standard Deviations

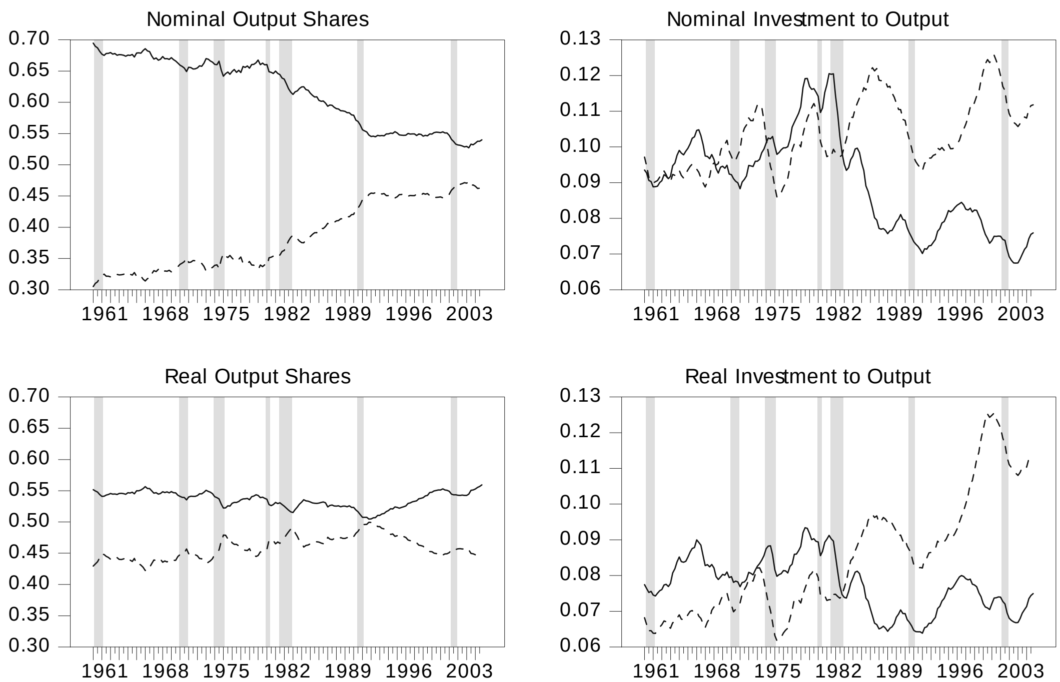
	Value		Contribution to change ($\times 100$)			
Parameter	**1960-1983**	**1984-2004**	$\sigma\,(gdp)$	$\sigma\,(Y_g)$	$\sigma\,(Y_s)$	$\sigma\,(I/Y_{gss})$
δ_F	0.0604	0.0426	0.00	0.00	0.00	0.00
δ_M	0.0187	0.0224	0.00	0.02	0.00	0.02
$1+\mu$	0.9783	1.194	0.00	0.00	0.00	0.00
$1+\nu$	3.0918	3.1426	0.00	0.00	0.00	0.00
$1+\phi$	0.9626	1.1809	0.02	-0.04	0.04	-0.12
$\psi_F/\left(1+\psi_F\right)$	0.0214	0.0397	-0.01	-0.02	0.00	0.00
$\psi_{Kg}/\left(1+\psi_{Kg}\right)$	0.4199	0.4956	-0.06	-0.09	0.00	-0.11
$\psi_{Ks}/\left(1+\psi_{Ks}\right)$	0.3335	0.3631	-0.02	-0.03	-0.01	-0.05
$\psi_M/\left(1+\psi_M\right)$	0.0278	0.0161	0.04	0.07	0.00	0.00
$\zeta_{Kg}/\left(1+\zeta_{Kg}\right)$	0.8734	0.868	0.01	0.01	0.00	0.01
$\zeta_{Ks}/\left(1+\zeta_{Ks}\right)$	0.3047	0.8002	-0.12	-0.09	-0.16	-0.12
All estimated parameters			**-0.13**	**-0.17**	**-0.14**	**-0.36**
F/Y_g	0.2881	0.3397	0.01	0.02	0.00	0.00
M/Y_g	1.3396	1.0148	-0.03	-0.08	0.00	-0.03
$\left(K_g+K_s\right)/Y_g$	14.47	15.86	0.17	0.25	-0.02	0.22
Y_s'/Y_g	0.5331	0.8061	-0.15	0.00	-0.03	0.00
All steady state parameters			**-0.03**	**0.20**	**-0.06**	**0.20**
σ_F	0.0035	0.0047	0.00	0.02	0.00	0.00
σ_M	0.1340	0.1071	-0.01	-0.02	0.00	-0.03
σ_g	0.0224	0.0142	-0.59	-0.81	-0.13	-0.25
All shocks			**-0.70**	**-0.77**	**-0.27**	**-0.16**
All parameters and shocks			**-0.82**	**-0.94**	**-0.39**	**-0.42**

Table 10: Accounting for the decline in volatility

Note: Columns 2 and 3 indicate the estimated value of the parameter in the first column in each sub-sample. In the last four columns we take the period 1960-1983 as the baseline period and change each parameter to its 1984-2004 value to account for its contribution to the reduction in volatility. The columns indicate, for each variable, the change in the standard deviation (times 100) due to the change in that parameter. Two important caveats are that: (1) standard deviations are not additive; (2) the effects of each model parameter are not independent from the values of other model parameters. For this reason, the values in each column do not add up to the last value in the column.

Figures

Figure 1
Output and Investment Data, by Sector.



Note: Solid lines: Goods sector; Dashed lines: Service Sector.

Figure 2
Input and Output Inventory Data

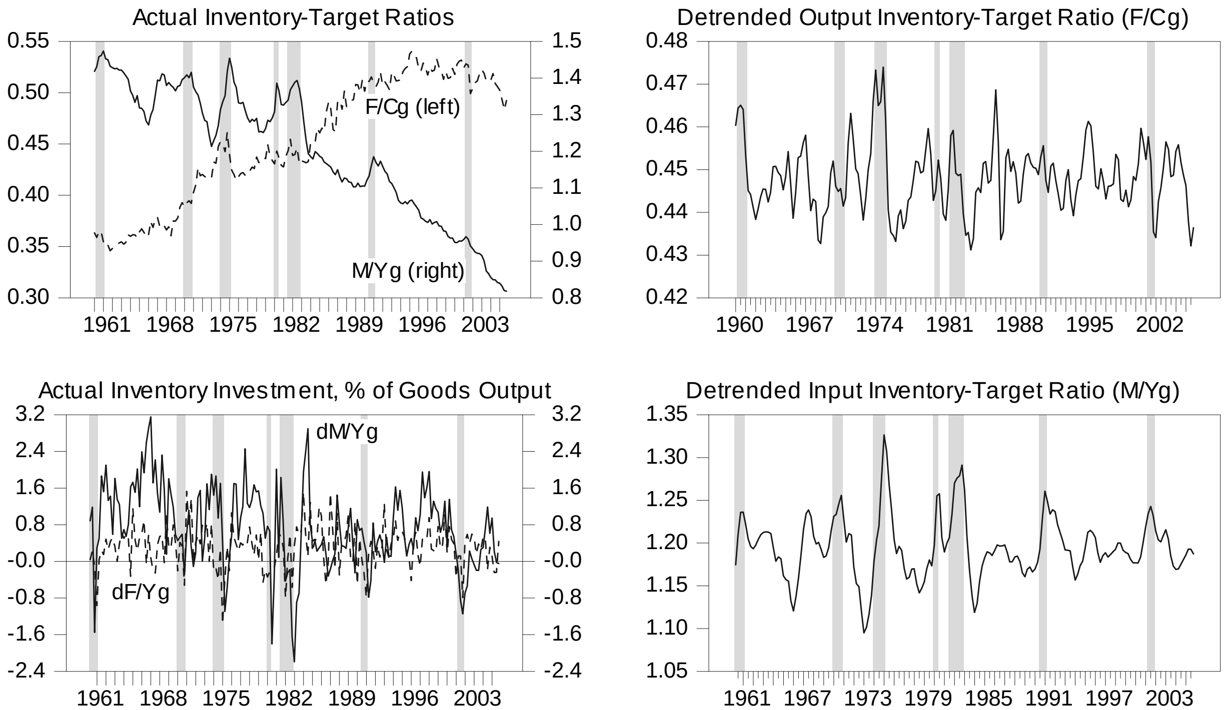
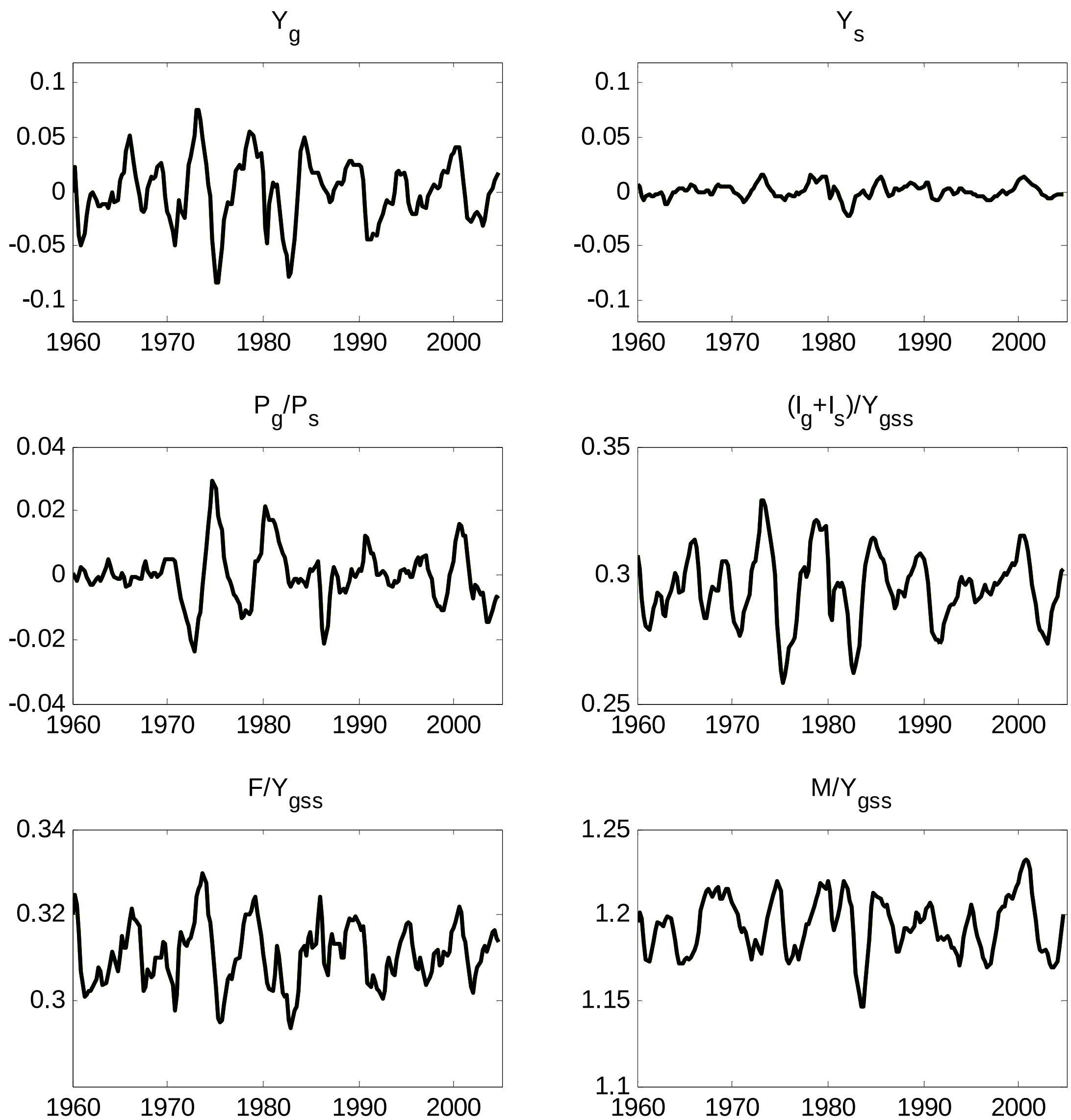
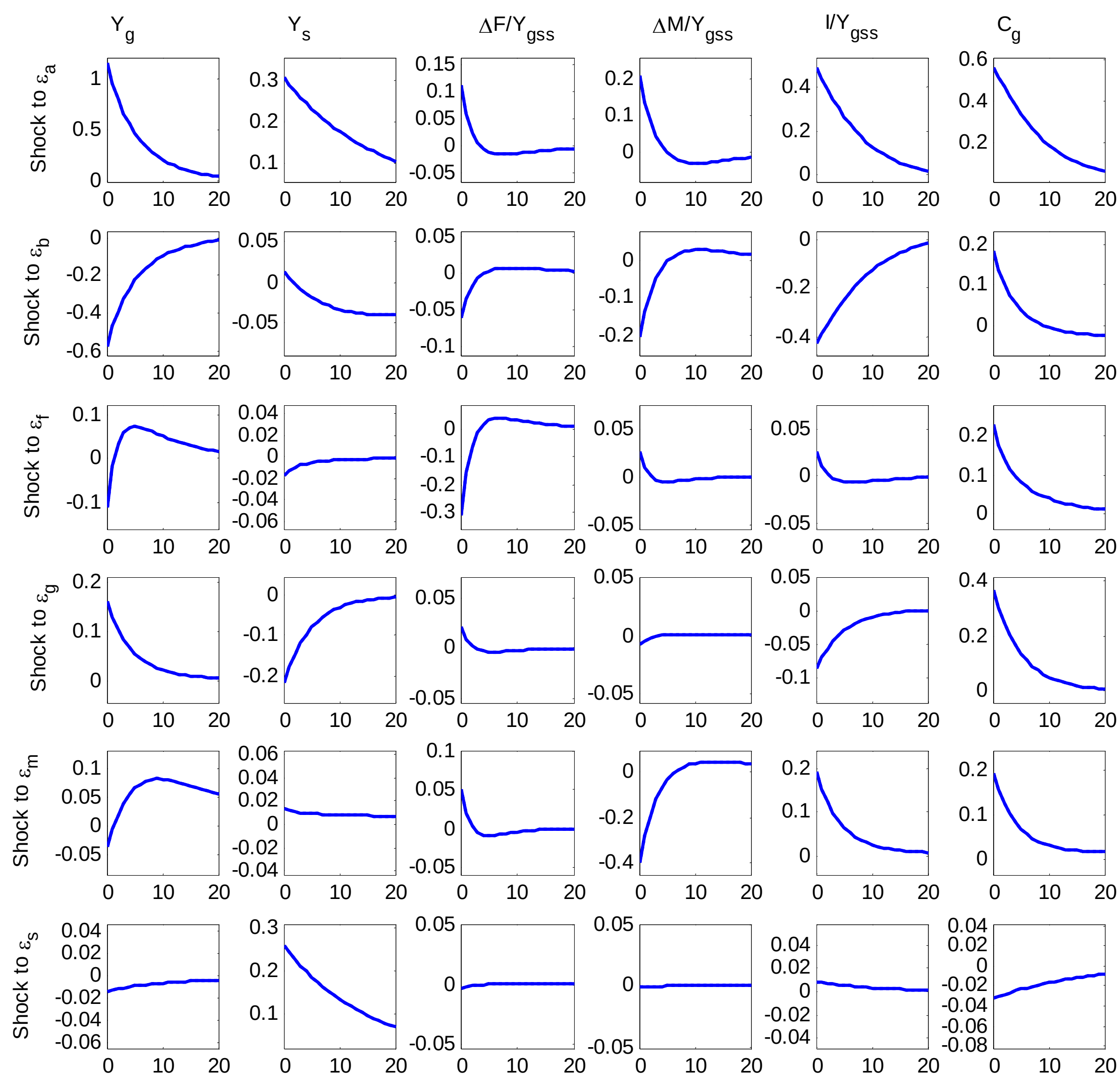


Figure 3
Variables used in estimation



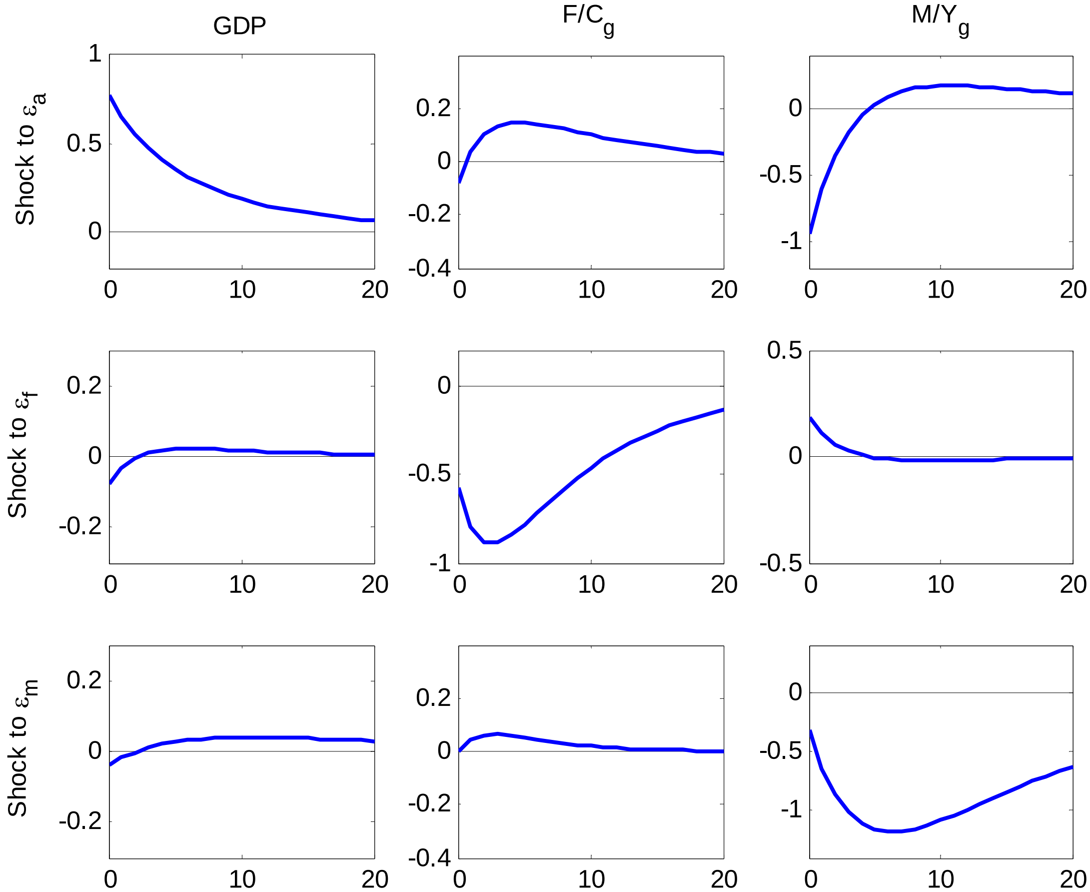
Note: The ss subscript indicates the average value of the variable over the sample period.

Figure 4
Impulse Responses of the estimated model.
Sectoral output, inventory and business investment, and consumption.



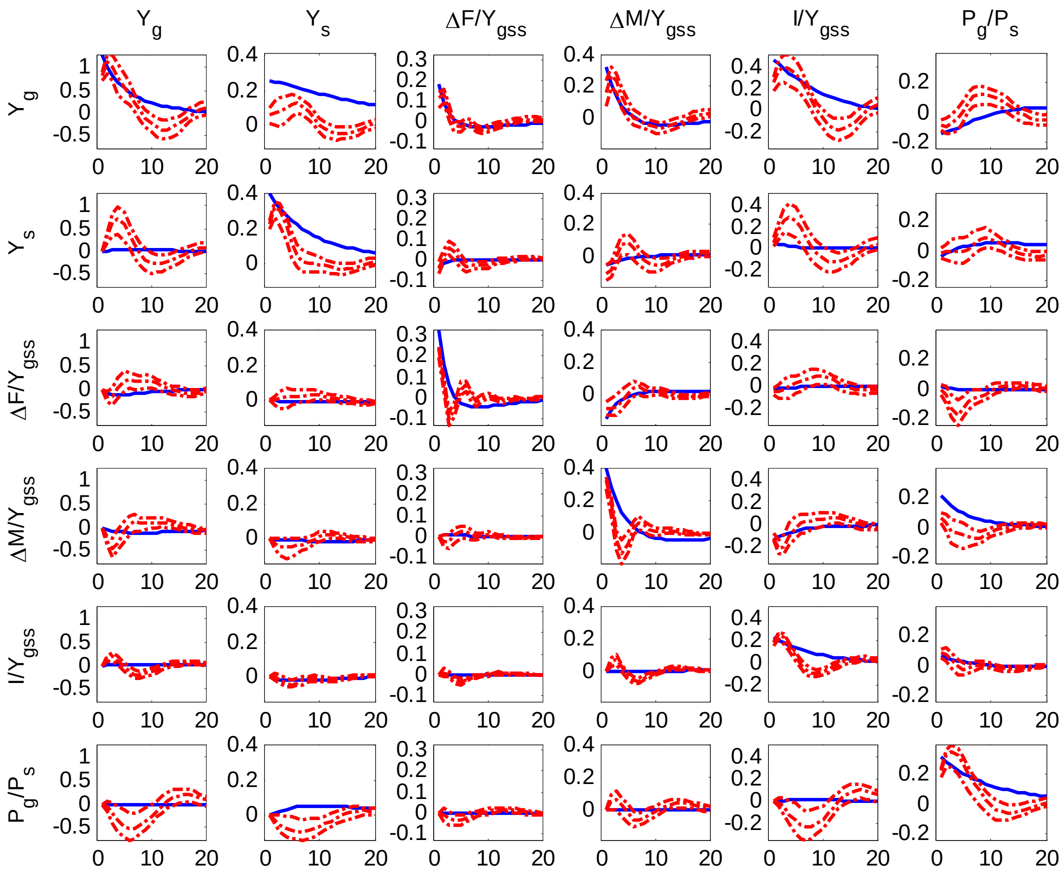
Note: Each row shows the impulse responses to an estimated one standard deviation shock. Ordinate: Time Horizon. Coordinate: Deviation from Baseline, in percent.
Inventory investment and business investment are scaled by goods output, so that their impulse responses measure the growth contribution to goods output.

Figure 5
Impulse Responses of the estimated model to selected shocks.
Total GDP and inventory to target ratios.



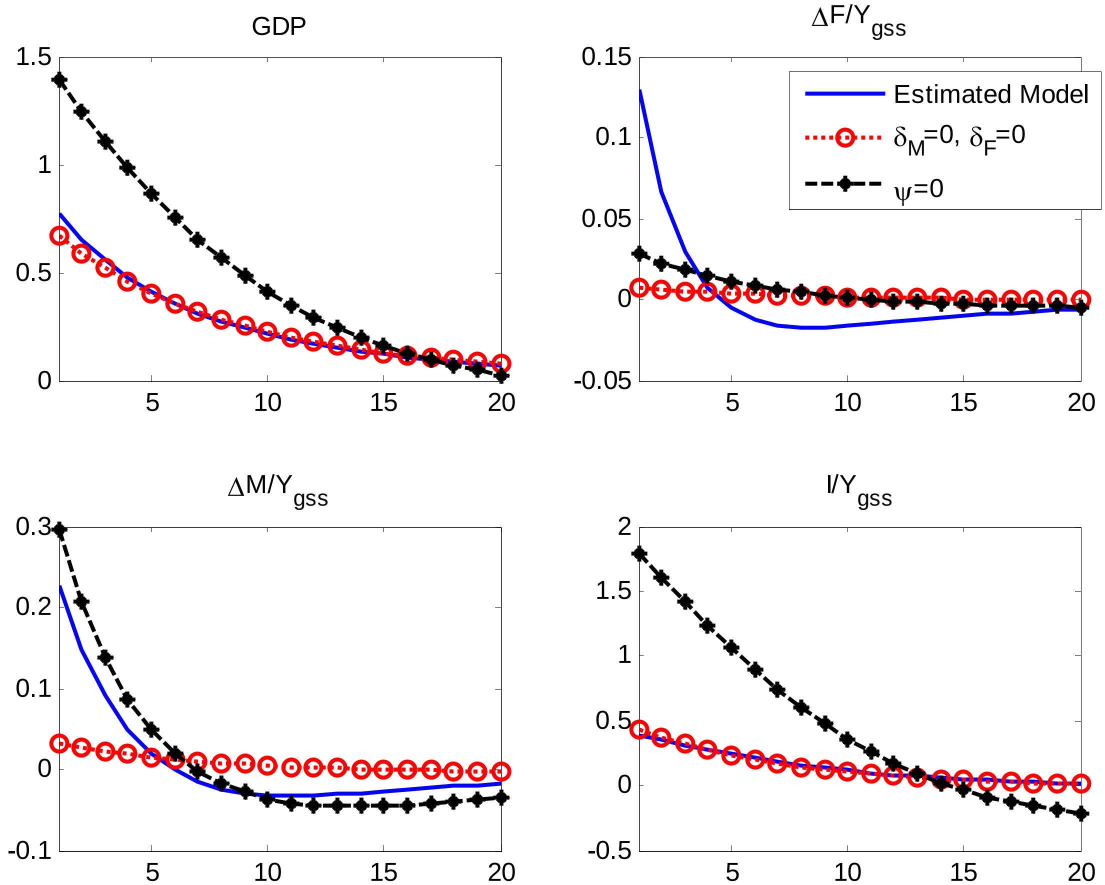
Note: Each row shows the impulse responses to an estimated one standard deviation shock. Ordinate: Time Horizon. Coordinate: Deviation from Baseline, in percent.

Figure 6
Orthogonalized Impulse Responses of the estimated model, comparison with VAR.



Note: Impulse responses comparisons, VAR based on actual data (dashed lines, with two lags and 95% bootstrapped confidence bands) and model. Each row represents one shock. Both sets of impulse responses have been orthogonalized in the same order. Shocks are one standard deviation. Coordinate: Percentage Deviation from baseline.

Figure 7
Impulse responses to a favorable technology shock in the goods sector



Note: Responses to an estimated one standard deviation technology shock in the goods sector. Ordinate: Horizon in quarters. Coordinate: Percentage Deviation from baseline.