# International Stock Market Integration: A Dynamic General Equilibrium Approach

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2003

# Outline of talk

# 1 Introduction

## Objective

- Study the effect of stock market integration on
  1. Stock market returns
  2. Risk premia
  3. Volatilities
  4. Correlation
     - Show how difference in relative country endowment/output size effects these results.
  5. International CAPM for partially integrated markets

## What is stock market integration?

- stock market of a country opened up to investors from other countries

## 1.1 Motivation

- Recent debate on **stock market liberalization** in **emerging markets**
- **Cost of equity capital** – will it decrease, leading to increased foreign investment? Impact on economic growth
- **Equity risk premium**
  - CAPM
    - segmented stock market $\mu_2 - r = \beta_1 \sigma_2^2$
    - integrated stock market $\mu_2' - r' = \beta_1' \sigma_2'^2 + \beta_2' \rho_{12}' \sigma_1' \sigma_2'$
- **Correlation** – does it increase so much that the risk premium increases?

- Can increased investment in a newly liberalized stock market increase **volatility**?

- Recent **empirical work** (Bekaert and Harvey (2000), Henry (2000), Kim and Singal (2000))– studying the effects of stock market liberalization on stock prices

- What does asset pricing theory (two – country version of Lucas exchange economy) have to say about these issues?

## What does the data say about stock market liberalizations ?

- stock returns decline on average by approx 1/3 (Henry, 2000)
- stock return volatilities change very little (Bekaert and Harvey, 2000 and Kim and Singal, 2000)
- correlation increases by .045 on average (Bekaert and Harvey, 2000)
- consumption growth volatility decreases from 3 per cent on average to .75 per cent for emerging markets countries (Bekaert, Harvey and Lundblad, 2002)

## Outline of the model

- Two countries
- Representative investor for each country
- Investors face **constraints**:
  - Different market structures
  - Bond market always open
    * Non-integrated stock markets – **case 1 – base case**
    * One-way stock market integration – **case 2**
    * Two-way stock market integration – **case 3**
    * Partial one-way stock market integration – **case 4**

## Motivating examples

- **India**:
  - Foreign institutions can buy upto 24% of a company
  - Indian residents not allowed to invest abroad, Reserve Bank of India (2002)
- **Japan**
  - Japanese insurance co's may not invest more than 30% of their portfolio in foreign assets, Sercu and Uppal (2001)
- **Israel**
  - Foreign investment ceiling of 20% for pension funds (until April 2002), Jerusalem Post

## What does my theoretical model say about stock market liberalizations ?

- riskless rate rises
- cross-country correlation in stock returns increases

| Country | 1 (closed stock market) | 2 (open stock market) |
|---|---|---|
| Cost of equity capital | higher | lower |
| Equity premium | higher | lower |
| Stock return vol | higher | lower |
| Consumption vol | higher | lower |

Table 1: Summary of Results

## Intuition

- Stock returns
  - **shifts in demand** – changes in returns
- riskless rate
  - less demand for precautionary savings – higher interest rate
- stock return volatility
  - incomplete markets – **agent-specific state-price densities**
  - changes in consumption volatilities – changes in state-price density volatilities – changes in stock return volatilities
- correlation
  - cross-country correlation in state price densities rises – cross-country correlation in stock returns rises

## Related Literature

- Market Integration – Theoretical Work
  - Black (1974), Subrahmanyam (1975), Stapleton and Subrahmanyam (1977), Errunza and Losq (1985), Errunza and Losq (1989), Eun and Jarakiramanan (1986), Sellin and Werner (1993), Obstfeld (1994), Basak (1996), Dumas and Uppal (2001)
- Market Integration – Empirical Work
  - Errunza, Losq and Padmanabhan (1992), Bekaert and Harvey (1995), Kim and Singal (2000), Henry (2000), Bekaert and Harvey (2000), Bekaert, Harvey and Lundblad (2001)
- Asset Pricing Theory – Incomplete Markets
  - Basak and Cuoco (1998), Detemple and Serrat (2002), Basak and Croitoru (2000), Basak and Gallmeyer (2001), Shapiro (2002), Gallmeyer and Hollifield (2002)

# 2 Economy

## Preferences of the two agents

- There are two countries $i \in \{1, 2\}$ each with a representative agent having lifetime expected utility functional

$$E_t \left[ \int_t^T e^{-\beta(s-t)} \log C_i(s)\, ds \right]$$

## Endowment processes

- For each country $i \in \{1, 2\}$ there is an exogenous endowment process described by

$$\frac{de_i(t)}{e_i(t)} = \mu_{e,i}(t)\,dt + \sigma_{e,i}(t)^{\intercal}\,dW(t)$$

- Same type of perishable consumption good in each country

- The aggregate "world" endowment process is defined by

$$y(t) = \sum_{i=1}^{2} e_i(t)$$

## Securities market

- Riskless bond with rate of return $r\left(t\right)$

$$\frac{dB\left(t\right)}{B\left(t\right)}=r\left(t\right)dt$$

- In each country $i\in\{1,2\}$ there is a risky stock with price $S_i\left(t\right)$ with *cumulative* returns described by the process

$$\frac{dS_i\left(t\right)+e_i\left(t\right)dt}{S_i\left(t\right)}=\mu_i\left(t\right)dt+\sigma_i\left(t\right)^{\intercal}dW\left(t\right)$$

# 3 Comparing equilibria

## Case 1 – Base case

- Stock markets are not integrated, while bond markets are integrated
- Agent one can hold only stock one
- Agent two can hold only stock two

## Case 2 – One-way stock market integration

- bond markets are integrated
- Agent one can hold both stocks
- Agent two can only hold stock two

# 4 Methodology

- Markets are **incomplete**
- Under **incomplete** markets
  - Consumption-portfolio policy – Cvitanic and Karatzas (1992)
  - Equilibrium valuation
    - ∗ construct a representative agent with stochastic weights – Cuoco and He (1994)
      - · Must solve a forward-backward stochastic differential equation to find prices and the stochastic weight

## "World" representative investor

- State-dependent world representative investor

$$U\left(y\left(t\right);\lambda\left(t\right)\right)=E\int_{t}^{T}e^{-\beta(s-t)}u\left(y\left(s\right),\lambda\left(s\right)\right)ds\ ,$$

where

$$u\left(y\left(t\right),\lambda\left(t\right)\right)=\log\frac{y\left(t\right)}{1+\lambda\left(t\right)}+\lambda\left(t\right)\log\frac{\lambda\left(t\right)y\left(t\right)}{1+\lambda\left(t\right)}$$

- stochastic weight, $\lambda\left(t\right)$
- cross-sectional wealth distribution

$$x\left(t\right)=\frac{X_1\left(t\right)}{X_1\left(t\right)+X_2\left(t\right)}=\frac{1}{1+\lambda\left(t\right)}$$

- stochastic weight and cross-sectional wealth distribution are equivalent

## Equilibrium prices

- Forward-backward stochastic differential equation system
  - **Backward** sde for price of stock 1

$$S_1(t) = y(t)\, x(t)\, E_t \int_t^T e^{-\beta(s-t)} \frac{e_1(s)}{y(s)\, x(s)} ds$$

  - **Forward** sde for cross-sectional wealth distribution

$$\begin{aligned} \frac{dx(t)}{x(t)} &= \mu_x(t)\, dt + \sigma_x(t)^{\intercal} dW(t),\; x(0) = x_0 \\ \sigma_x(t) &= F(S_1(t), \sigma_1(t)) \\ \mu_x(t) &= G(S_1(t), \sigma_1(t)) \end{aligned}$$

- Market clearing—price of stock two is given by

$$S_2(t) = \frac{1 - e^{-\beta(T-t)}}{\beta} y(t) - S_1(t)$$

## Solution idea

– Solve for $S_1(t)$ in terms of $x(t)$ and exogenous variables

$$S_1(t) = f(x(t))$$

– Solve for $x(t)$ in terms of price of stock 1, $S_1(t)$ and its volatility, $\sigma_1(t)$

$$x(t) = g(S_1(t), \sigma_1(t))$$

– Volatility is just a derivative of the stock price

$$\sigma_1(t) = \mathcal{D}S_1(t)$$

– Therefore obtain the cross-sectional wealth distribution in terms of exogenous parameters only

$$x(t) = g(f(x(t)), \mathcal{D}f(x(t)))$$

## PDE Approach

– Backwards stochastic differential equation for price of stock 1

$$S_1(t) = y(t)\,x(t)\,E_t \int_t^T e^{-\beta(s-t)} \frac{e_1(s)}{y(s)\,x(s)} ds$$

– Use Markovian property of problem

– Transform backward sde into a quasilinear pde with generator $\mathcal{L}$

$$\begin{aligned} \frac{\partial S_1(t)}{\partial t} + \mathcal{L} S_1(t) + h(t) &= 0 \\ S(T) &= 0 \end{aligned}$$

– Solve pde – obtain price of stock 1 in terms of cross-sectional wealth distribution and exogenous parameters

$$S_1(t) = f(x(t))$$

## Solving the pde – use continuation methods

– Key idea: when endowment growth rates are perfectly correlated, $\rho = 1$, can solve pde exactly

$$S_{10}(t) = \frac{1 - e^{-\beta(T-t)}}{\beta} e_1(t)$$

– Expand around this solution

$$\begin{aligned} S(t) &= S_{10}(t) + \epsilon S_{11}(t) + \epsilon^2 S_{12}(t) + O\left(\epsilon^3\right) \\ \epsilon &= \sqrt{\frac{1-\rho}{2}} \end{aligned}$$

## Error Analysis

$\beta = 0.02, \sigma_e = .01, t = 0, T = 5., e_1(0) = e_2(0), x(0) = 1/2$

- $S_{12}(t)$ is $10^{-4}$ times smaller than $S_{10}(t)$
- $S_{14}(t)$ is $10^{-4}$ times smaller than $S_{12}(t)$
- Substituting the second order expansion for the stock price into the pde gives at most $10^{-6}$
- Substituting the fourth order expansion for the stock price into the pde gives at most $10^{-9}$

- Errors are small

# 5 Results

Effects of opening stock market two to external investment from country one on asset prices

Effects of difference in country output per capita on these results

– $N(t) = e_1(t)/e_2(t)$

**Background example**

– large developed country (region) – country (region) one, e.g N America + EU

– small emerging market country – country two – liberalized its stock market, e.g. India.

## Riskless rate

The riskless rate is **increased** by

$$\frac{(1-\rho)^2}{4}\frac{N(t)}{(1+N(t))^2}\left(\beta+4\sigma_e^2-\frac{4e^{-\beta(T-t)}}{1-e^{-\beta(T-t)}}\beta(T-t)\sigma_e^2\right)\frac{\sigma_e^2}{\beta}$$

– improved risk-sharing lowers precautionary savings motive of investors

– Setting $N(t) > 1$ diminishes this effect

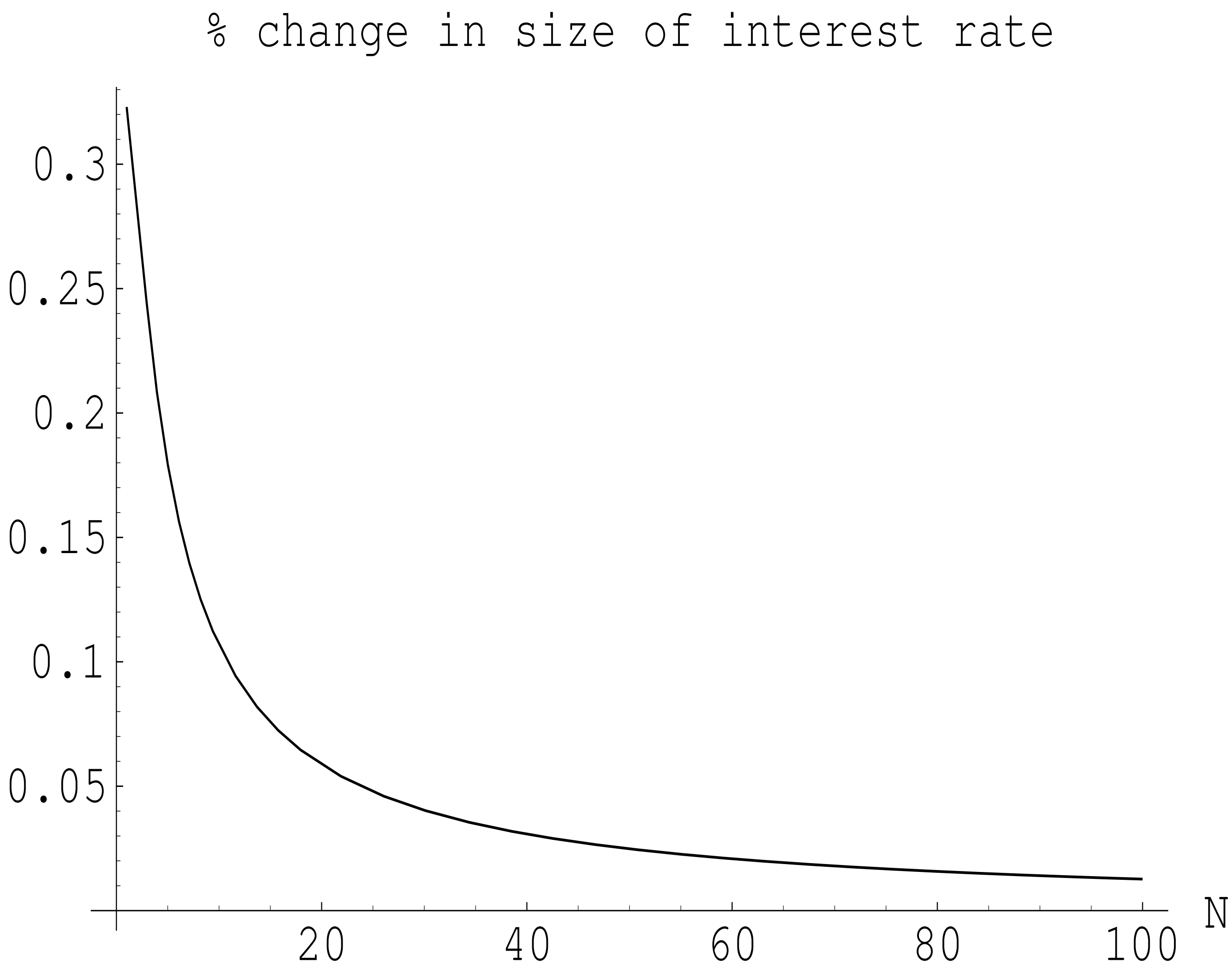



Figure 1: riskless rate

## Returns

Conditional expected returns on stock one are **increased** by

$$(1-\rho)\,\sigma_e^2 \frac{2N(t)-(1+N(t))\,x(t)}{(1+N(t))\,x(t)}$$

Conditional expected returns on stock two are **decreased** by

$$-(1-\rho)\,\sigma_e^2 N(t)\,\frac{2N(t)-(1+N(t))\,x(t)}{(1+N(t))\,x(t)}$$

- Portfolio diversification – "demand effects"
  - $*$ increase in agent one's demand for stock two
  - $*$ decrease in agent one's demand for stock one

## Risk premia

The risk premium on stock one is **increased** by

$$(1-\rho)\frac{1}{x(t)}\frac{2N(t)-(2N(t)+1)x(t)}{1+N(t)}\sigma_e^2$$

This increase is diminished as $N(t)$ increases.

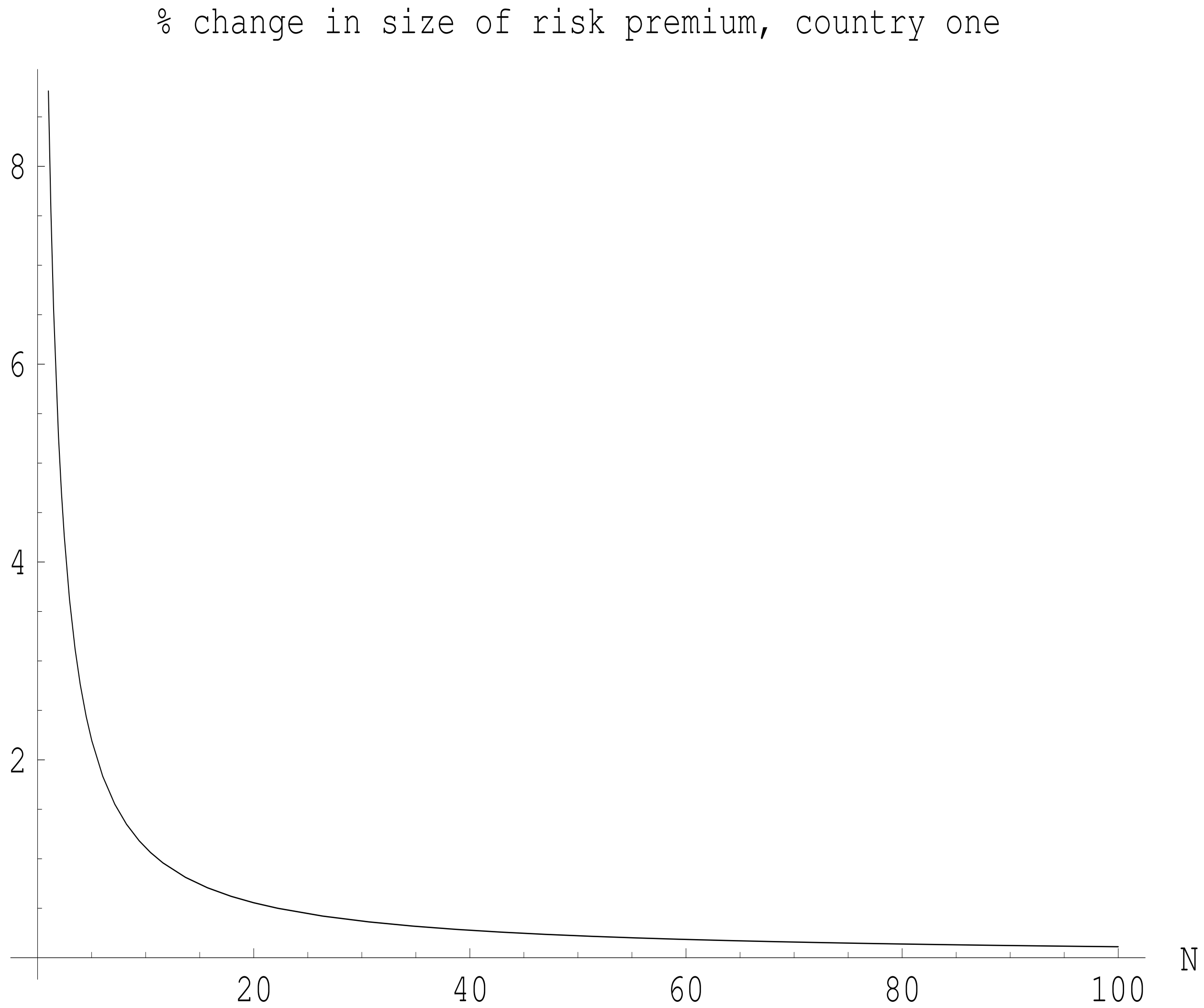
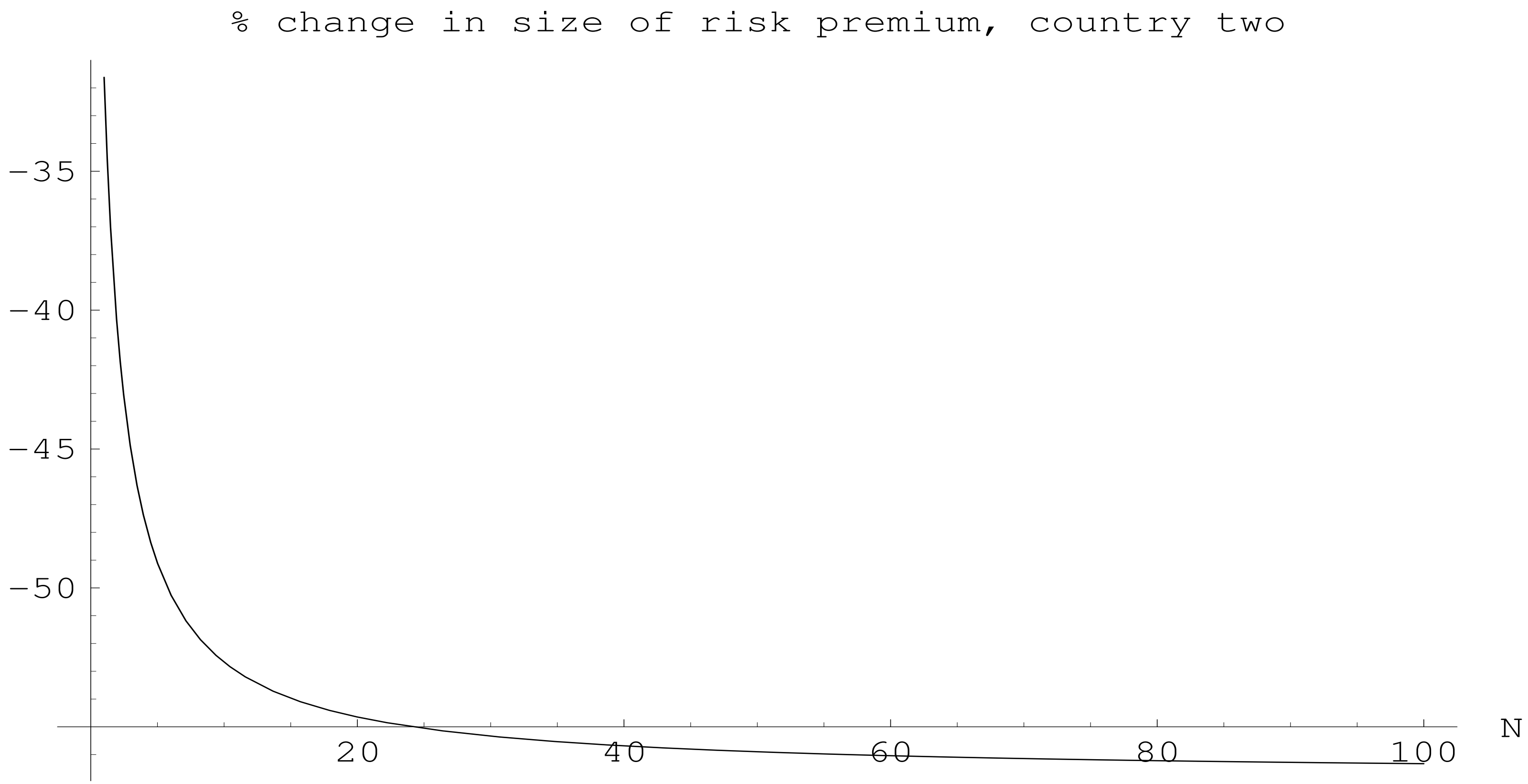



Figure 2: Risk premium in country 1

The risk premium on stock two is **decreased** by

$$-\left(1-\rho\right)\frac{N\left(t\right)}{1+N\left(t\right)}\sigma_e^2$$

This decrease is magnified as $N(t)$ increases.


% change in size of risk premium, country two
-35
-40
-45
-50
20
40
60
80
100
N

– increase in “demand effects” outweighs decrease in “precautionary savings effects”

## Stock return volatility decomposition

- conditional stock return volatility $=$ conditional state-price density volatility $+$ other terms

$$\sigma_i(t) = \theta_i(t) + \text{other terms}$$

- conditional state-price density volatility $=$ conditional consumption growth volatility

conditional consumption growth volatility $\rightarrow$ conditional stock return volatility

- other terms = average effect of current exogenous endowment shocks on **future spd variability** with **fixed wealth distribution** +

  average effect of current exogenous endowment shocks on **future spd variability via wealth distribution** +

  future **variability in dividend growth**

- Link between stock return volatility and state-price density volatility

$$\sigma_i(t) = \theta_i(t) + \frac{E_t \int_t^T \xi_i(s)\, e_i(s)\, \frac{\xi_{i,y}(s)}{\xi_i(s)} \mathcal{D}_t y(s)^{\mathsf{T}}\, ds}{E_t \int_t^T \xi_i(s)\, e_i(s)\, ds} + \frac{E_t \int_t^T \xi_i(s)\, e_i(s)\, \frac{\xi_{i,x}(s)}{\xi_i(s)} \mathcal{D}_t x(s)^{\mathsf{T}}\, ds}{E_t \int_t^T \xi_i(s)\, e_i(s)\, ds}$$
$$+ \frac{E_t \int_t^T \xi_i(s)\, e_i(s)\, \frac{\mathcal{D}_t e_i(s)^{\mathsf{T}}}{e_i(s)} ds}{E_t \int_t^T \xi_i(s)\, e_i(s)\, ds}$$

## State-price density variances

The state-price density variance in country one is increased by

$$(1-\rho)\,\frac{2N(t)}{(1+N(t))^2}\left[N(t)\,\frac{1-x(t)^2}{x(t)^2}-1\right]\sigma_e^2$$

if $x(t)^2 < N(t)\,/\,(1+N(t))$.

The state-price density variance in country two is decreased by

$$-(1-\rho)\,\frac{2N(t)}{1+N(t)}\sigma_e^2$$

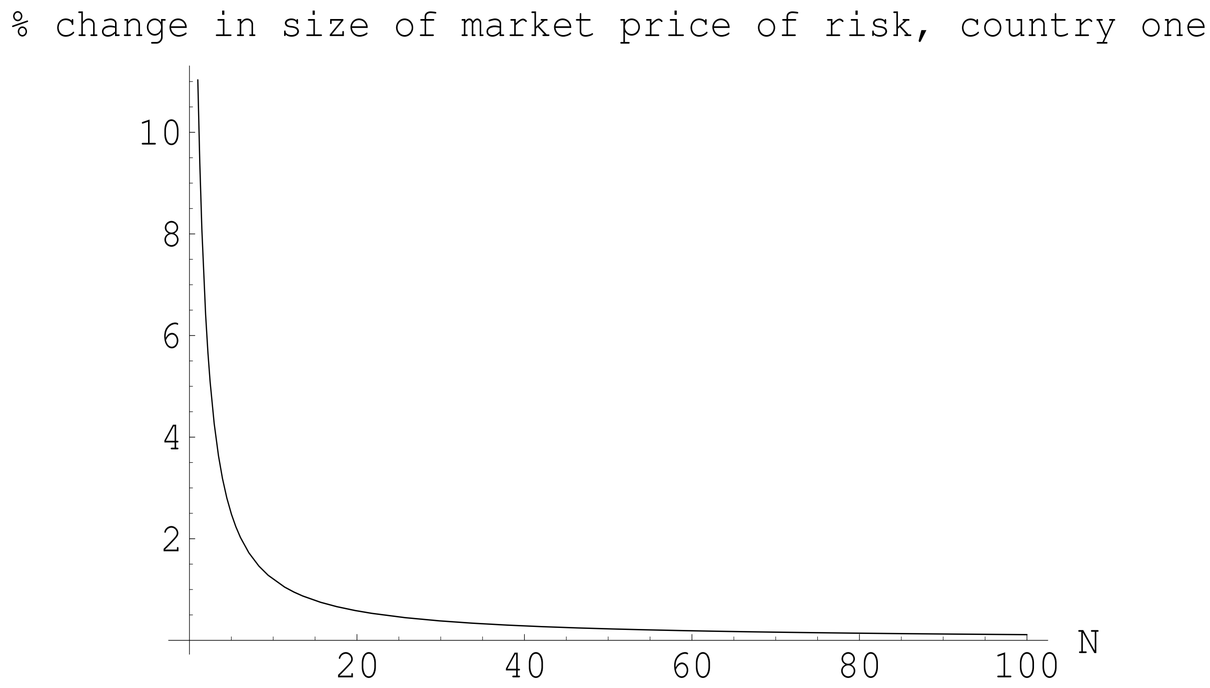



Figure 4: Market price of risk in country 1

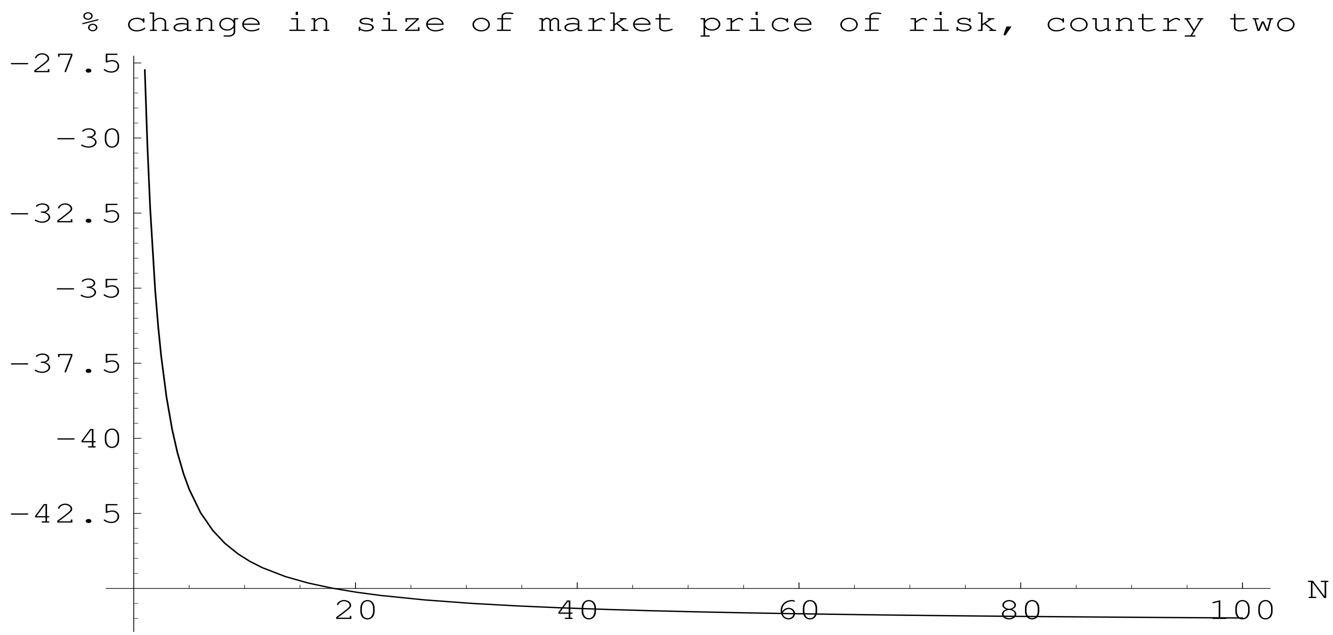



Figure 5: Market price of risk in country 2

## Stock return variances

The conditional variance of returns on stock one is **increased** by

$$(1-\rho)^2 \frac{4N(t)^2}{(1+N(t))^3} \frac{1-x(t)}{x(t)^2} \left(1 - \frac{\beta(T-t)e^{-\beta(T-t)}}{1-e^{-\beta(T-t)}}\right) \frac{\sigma_e^4}{\beta}$$

The conditional variance of returns on stock two is **decreased** by

$$-(1-\rho)^2 \frac{2N(t)}{(1+N(t))} \left(1 - \frac{2}{1+N(t)} \frac{N(t)}{x(t)} \left(1 + \frac{N(t)}{x(t)}\right)\right) \left(1 - \frac{\beta(T-t)e^{-\beta(T-t)}}{1-e^{-\beta(T-t)}}\right) \frac{\sigma_e^4}{\beta}$$

- agent one's optimal consumption growth becomes more volatile – volatility of country one state-price density rises
- agent two's optimal consumption growth becomes less volatile – volatility of country two state-price density falls

This table shows how state-price density volatility and stock return volatility in country two change when stock market two opens, for various $N(t)$, assuming that $x(t) = N(t)/(1 + N(t))$. The following set of parameter values is used: $\beta = .01$, $\sigma_e = .03$, $\rho = 0.4$, $\tau \to \infty$.

| | % change in $\|\theta_2\|$ | % change in $V_2$ |
|---|---|---|
| N=1 | -26.6% | -7.0% |
| N=10 | -41.7% | -7.0% |
| N=100 | -43.4% | -7.0% |
| N=1000 | -43.5% | -7.0% |

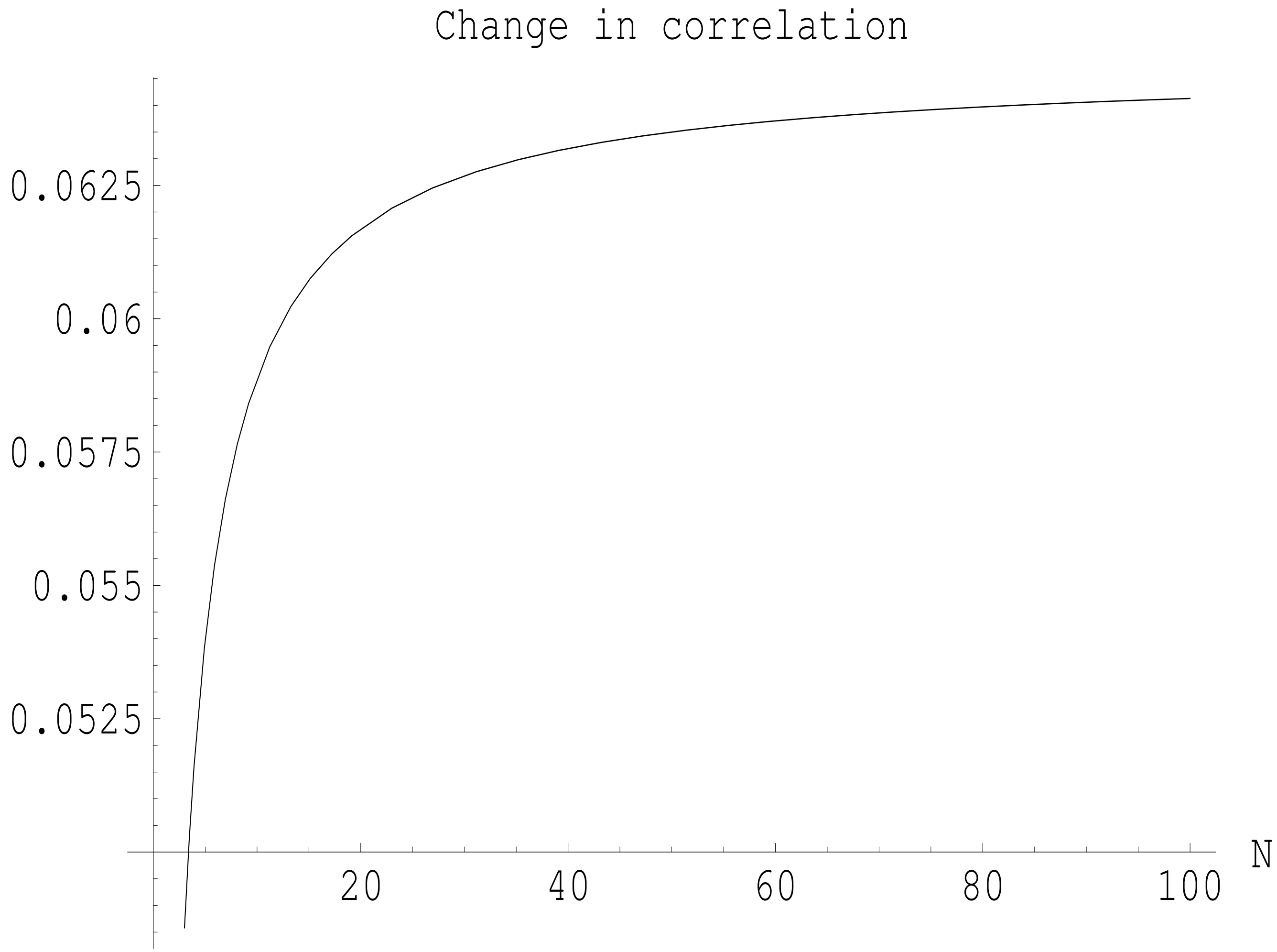
Table 2: Changes in state-price density and stock return volatility in country two

## Correlation

The cross-country correlation in stock returns is **increased** by

$$(1-\rho)^2 \frac{2N(t)}{(1+N(t))^2} \frac{[2N(t)-x(t)(1+N(t))]}{x(t)} \left( \frac{1-[1+\beta(T-t)]e^{-\beta(T-t)}}{1-e^{-\beta(T-t)}} \right) \frac{\sigma_e^2}{\beta}$$

- cross-country correlation in state-price densities rises
- increase is magnified by increasing $N(t)$


Change in correlation
0.0625
0.06
0.0575
0.055
0.0525
20
40
60
80
100
N

This table shows how equilibrium variables in country two change when stock market two opens, for various $N(t)$, assuming that $x(t) = N(t)/(1 + N(t))$. The following set of parameter values is used: $\beta = .01$, $\sigma_e = .03$, $\rho = 0.4$, $\tau \to \infty$.

| | % change in risk premium | % change in $\|\theta_2\|$ | % change in $V_2$ | change in $\rho_{12}$ |
|---|---|---|---|---|
| N=1 | -31.7% | -26.6% | -7.0% | 0.03 |
| N=10 | -52.7% | -41.7% | -7.0% | 0.06 |
| N=100 | -56.3% | -43.4% | -7.0% | 0.06 |
| N=1000 | -56.7% | -43.5% | -7.0% | 0.06 |

Table 3: Changes in equilibrium variables in country two

## Summary of Intuition

- **Case One** $\rightarrow$ **Case Two**: risk-sharing improves, but demand for stock one decreases and demand for stock two increases
  - demand effects shift stock prices and hence returns
    - stock price one decreases, stock price two increases
  - improved risk-sharing – overall demand for bond falls, interest rate rises
  - demand effects outweigh risk-sharing effects
    - risk premium in country one rises
    - risk premium in country two falls
  - changes in stock return volatility driven by changes in state-price density volatility

## Other types of integration considered

- both stock markets opened
- stock market two partially opened
  - Interpretation: exogenously impose home-bias on investor one

## CAPM – both agents constrained

- agent $i$ can invest upto $I_i$ of its wealth in stock market $j$
-

$$E_{t-1}\left[R_{i,t} - r_t\right] = \left(\frac{s_{i,t-1}}{x_{i,t-1}} - A_{i,t-1}\right) Var_{t-1}\left(R_{i,t}\right) + A_{j,t-1} Cov_{t-1}\left(R_{i,t}, R_{j,t}\right)$$

where

$$A_{k,t} = y_t x_{k,t} I_k$$

-
- $R_{i,t}$ returns in country i
- $s_{i,t}$ proportion of aggregate financial wealth invested in stock i by investor i

  $x_{i,t}$ proportion of aggregate financial wealth held by investor i
- an equilibrium CAPM
- different level of stock market integration for each country
- identifies coefficients of variance and covariance terms in terms of relative stock market wealth, financial wealth and investor constraints

## 5.1 Comparison with Bekaert and Harvey(1995)

Bekaert and Harvey(1995)

$$\begin{aligned} E_{t-1}\left[R_{i,t} - r_t\right] = & \left(1 - I_{i,t-1}\right)\lambda_{i,t-1} Var_{t-1}\left(R_{i,t}\right) \\ & + I_{i,t-1}\lambda_{t-1} Cov_{t-1}\left(R_{i,t}, R_{w,t}\right) \end{aligned}$$

This paper

$$\begin{aligned} E_{t-1}\left[R_{i,t} - r_t\right] = & \left(\frac{s_{i,t-1}}{x_{i,t-1}} - A_{i,t-1}\right) Var_{t-1}\left(R_{i,t}\right) \\ & + A_{j,t-1} Cov_{t-1}\left(R_{i,t}, R_{j,t}\right) \end{aligned}$$

- heterogeneity in investor constraints, $I_1 \neq I_2$
- show explicitly how international diversification effects betas of CAPM

# 6 Conclusions

- Analyzed effects of stock market integration on asset prices
- Main findings:
  - Riskless rate is raised
  - Stock returns, risk premium fall in liberalizing country
  - Stock return volatility decomposition
  - Stock return volatility and consumption volatility fall in liberalizing country
  - Correlation rises
  - Effects of relative country output-size on these results – good match with data
- Technical contribution : solved for eqm with incomplete markets and 2 risky assets

  - Could be used to investigate how higher ambiguity about the drift of stock returns in foreign markets effects prices

- CAPM for partially integrated markets
  - useful for computing cost of capital for projects in emerging markets
- New intuition
  - Understanding volatility and correlation in terms of state-price densities
- Implications for empirical studies of stock market volatility
  - should control for consumption volatility