State of Technology, Innovation and Finance

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Abstract

This paper investigates how the growth rate of a sector can be influenced not only by the initial state of technology but also by the financing of innovation. I find that the relation between the state of technology and the growth rate is not continuous.As the distance of a sector to the technological frontier narrows, the alternative to innovation (imitation) becomes relatively more profitable. This implies that more imitation is undertaken, both if it is financed with debt or with equity. As the technological gap narrows too much (that is for sectors at a low distance to the technology frontier), financing for innovation is available only for unskilled entrepreneurs, due to adverse selection effects. It turns out that in this region of close distance to the frontier, the growth rate is very low due to the no participation of skilled entrepreneurs to innovation projects. In this way the financing of innovation may explain the low growth rate of economies which have only small technology gaps with respect to the world frontier.

JEL Codes: G10, O00, O31

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PRELIMINARY AND INCOMPLETE

1 Introduction

In recent years there has been a wide debate about the financing of innovation. Indeed, often firms have ideas but do not have funds, therefore they have to be financed externally. The major problem of external funding of R&D is that it is greatly affected by information asymmetries. In this sense, a wide literature has focused on various institutional factors: financial development, specialization of investors, the level of legal investor protection and contract incompleteness.[1] However, these factors require a different set of

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[1] For financial development, see for example the theoretical contribution of De la Fuente and Marin (1996), while an important empirical work is Rajan and Zingales (1998). For the legal investor protection

conditions that often depend on the countries and the sectors under consideration (Gompers and Lerner, 2001; Black and Gilson, 1998).

One aspect that has received little attention is how the state of technology (s.o.t. henceforth) affects the relation between innovation and finance. With the term s.o.t. we mean whether an entrepreneur operates in a sector that in her country is more or less distant to the world technology frontier. This variable has two interesting characteristics when related to innovation. First, the s.o.t. determines whether entrepreneurs have profitable alternatives to innovation. In fact, in sectors *at* the world technology frontier entrepreneurs may only undertake projects that shift out the current frontier (*innovation*). Instead, in sectors *inside* the frontier entrepreneurs may *also* undertake projects that improve upon the local sector technology, but does not reach the world frontier (*imitation*).[2] Second, technology is also a product of institutional factors (like financial development, legal investor protection, etc...), and so it can be used as a *proxy* for the institutional level of an economy.

We argue that the financing of innovation has two important implications on the productivity growth rate depending on the s.o.t.. First, whether a sector *inside* or *at* the world technology frontier implies that entrepreneurs incentives change. In fact, if entrepreneurs operate inside the frontier they may undertake innovation or imitation; at the frontier they may only undertake innovation. This implies that since the projects relative payoffs are different in one or the other case, also the financial frictions are more or less important. Second, we also argue that the way financial frictions affect productivity growth changes in sectors more or less distant from the frontier. In particular, when a sector is inside but close to the frontier, we show that financial frictions do not allow skilled entrepreneurs to be financed for innovation. In this last case, productivity growth would be lower and therefore a form of financial development is needed. In particular, we provide two examples of financial development as possible extensions, so that productivity growth may be increased: partial creditor protection and investor activism in the equity contract. These examples are to be intended as possible extensions; in fact, the model is flexible enough to allow different changes to the basic financial contracts.

These results are particularly useful to explain why in the last decade the productivity growth rate of the most advanced European economies has been very low with respect to the US, especially in sectors at high innovation potential. In fact, as reported by Gordon (2004), after decades of catching up, the divergence between the two economic blocks started in the 90's when the productivity gap with the US was limited: "*The growth rate in output per hour over 1995-2003 in Europe was just half that in the United States, and this annual growth shortfall caused the level of European productivity to fall back from 94 per cent of the United States level to 85 per cent.*"[3] With regard to this problem, an increasing number of studies have focused on the role of various factors that might positively facilitate the financing of innovation activity in Europe: the development of venture capital markets, legal investor protection reforms, dynastic management, labour

literature see the seminal papers of La Porta et al. (1997 and 1998). For contract incompleteness, see Aghion and Bolton (1992). For the specialization of investors see Casamatta (2003). A recent comprehensive review of the literature is Levine (2004).

[2] This activity usually consits of adoption of technology from the world frontier.

[3] Gordon, 2004, page 1.

market reforms, etc...[4] With respect to this literature, we take the view that the financing of innovation plays an important role in explaining low productivity growth. Moreover, we add to the current literature the idea that financial frictions have a negative impact on investment and productivity growth only at advanced stages of the s.o.t..

The idea why at advanced levels of the s.o.t. productivity growth is slowed down can be explained as follows: even if financial contracts are designed so as to reduce the problems of information asymmetry, entrepreneurs convenience to invest in innovation (relative to imitation) changes if they are more or less distant to the world technology frontier. When they are very distant, they find very convenient to undertake innovation. Instead, at short distance levels entrepreneurs do not undertake it because expected returns from innovation are lower than those of imitation: in this way, technology catch up with the frontier is hampered in sectors with small technology lags.

In order to understand the mechanism behind the distance to the frontier and the financing of innovation, we provide the following example. Consider that entrepreneurs in technology laggard countries have to decide what type of investment (innovation or imitation) to undertake. In this respect, they compare the expected profits from the two activities but also take into consideration whether asymmetric information with the investor generates a cost of capital that is too high to undertake innovation. In our case, information asymmetry is due to the different skills of the entrepreneurs (high or low skills). The investor offers two types of contracts: the first excludes highly skilled entrepreneurs from innovation (due to adverse selection): debt; the other alleviates the problems of asymmetric information: equity. The interesting result is that when the technology gap with the frontier is small, innovation may not be financed with equity. In fact, since expected returns from innovation are lower than those of imitation, entrepreneurs demand a too high share (for innovation) with respect to what investors are able to give up. Then, when the technology gap is small, the equity contract cannot be signed and highly skilled entrepreneurs do not undertake innovation. In this case it is appropriate to improve the financial contracts. Two examples of financial development are provided. In the first case, the debt contract is improved with partial creditor protection against default. In the second example, the equity contract is improved by allowing the investor to exert an active role on the R&D process of the firm. It is shown that investor activism can enhance growth more than creditor protection.

As stated above, we consider that the investor may offer two financial contracts: debt and equity. Debt is considered because it is the most natural and diffused financial contract and our model predicts a form of credit rationing that is consistent with a wide literature on the financing of innovation (Guiso, 1997). Equity is also very diffused and plays a considerable role in the financing of innovation (Black and Gilson, 1998; Kaplan and Stromberg, 2003). Indeed, equity in our model is such that it just alleviates the problems of asymmetric information. In particular, this feature of the equity contract reflects the reality of financial development where information asymmetries are only imperfectly solved. Moreover, considering only these two financial contracts is sufficient to reflect

[4] Hege + al. (2003), Bottazzi and Da Rin (2002) for empirical analysis of Venture Capital in Europe; for dynastic management see Panunzi, Buckhart, Shleifer (2003) and also Gennaioli Caselli (2020202) at macroeconomic level. See Gordon (2004) for a review.

the main characteristics that financial contracts for innovation have Aghion and Bolton (1992).[5]

In this respect, this paper derives endogenously two stylized facts:

1. *High skills entrepreneurs may face credit constraints to innovation (*Guiso, 1998*);*

2. *High skills entrepreneurs may be financed using equity (*Levine, 2004*).*

A more important implication of this model is that given the equilibria for a sector *inside* the frontier, it is possible to relate these results to a *distance to the frontier* index. The main results are:

1. *Credit constraints are stronger in sectors close to the technology frontier;*

2. *The productivity growth rate of a sector is lower in the region where only debt is feasible.*[6]

3. *Debt and equity are both feasible at the frontier.*

The intuition behind point one is due to the impossibility of writing equity contracts. In fact, with equity entrepreneurs' desired share of profits increases as we move closer to the world frontier, because the alternative activity to innovation (imitation) becomes relatively more convenient than innovation, so entrepreneurs must be given a higher share of profits to undertake innovation. There is a threshold level where the desired share is 100%: after this point (i.e.: moving closer to the world frontier), investors refuse to sign an equity contract, as their residual share would be zero. Therefore, only the debt contract remains, but debt causes credit constraints to innovation due to adverse selection.

With regard to point two, it follows from point one. Since only debt is feasible in the region close to the technology frontier, due to adverse selection only low skilled entrepreneurs undertake innovation. As a consequence, the productivity growth rate is lower (see figure 11).

Finally, the intuition behind point three is that since at the frontier thee is not imitation, both contracts do not depend on the relative evaluation of profits of innovation Vs imitation, so both debt and equity can be signed.

Therefore, the model shows that if there are no obstacles to innovation other than financial frictions, productivity growth may be slowed down. Low growth in the model is driven by adverse selection and it happens only at advanced stages of the s.o.t. In this way, we find that financial development plays a crucial role for productivity growth at advanced stages of development.

1.1 Related Literature

This paper is related to three strands of literature. First, the paper is related to a recent contribution by Aghion, Acemoglu, Zilibotti (2002). They use a specific assumption on

[5]For more recent empirical surveys on Europe, see also Aghion, Bond, Klemm, Marinescu (2003) for UK firms and Audretsch and Lehmann (2004) for German firms.

[6]Productivity is also referred to TFP in throughout the work.

the form of the TFP change of a sector, which depends of the relative skills of managers to imitation or innovation projects: therefore, the form of the TFP is exogenous.[7] In this paper there are three main similarities with that work: first, firms inside the frontier have a double technology choice (innovation/imitation); second, there are imperfections in the financial markets; third and most important, the growth rate can be lower at advanced stages of development if institutional changes are not adopted.

Differently from them, in this paper the growth rate is not derived imposing any particular shape of a sector TFP, which instead is simply stated in the most general possible way. Moreover, while they show that countries may adopt institutions that impede technology transfer and slow growth, here more specifically growth is slowed down if financial development is not adopted.

This paper is also related to the wide literature on the financing of innovation. It is beyond the scope of this paper to present an exhaustive review of the literature here, but we report few important contributions. The early literature has focused on credit rationing to high tech firms by banks; for example Guiso (1998) has argued that banks ration credit to innovative firms while preferring to finance traditional activities, since they know more of their businesses than about high tech firms. A huge literature has looked at the role of specialized investors (Gompers and Lerner, 2001) like venture capitalists. Differently from this literature, this work does not explore in depth the role of specialized investors, but simply considers the links between the s.o.t. and the financial contracts to innovation: debt and equity. Indeed, considering just these two contracts, is not in contrast with the venture capital literature. In fact, as reported by Kaplan and Stromberg (2003), these two contracts have the main characteristics of the complex contracts the venture capitalists sign with the entrepreneurs.[8] In addition to this, the work is also different from all the literature on financial development and growth in the context of innovation-based growth models, as for example in De la Fuente and Marin (2002) and Aghion et al. (2005). while in all these works financial development has a positive impact on growth at all stages of technology, our work is different because it stresses that the financial contracts need to be improved only at late stages of development.

Third, it is related to the literature of agency problems in external finance. In this sense, Bernanke and Gertler (1989) and Aghion *et al. (1999)* have assumed that the access to external finance is limited by a multiple of the entrepreneur's own wealth. Differently from them, no such mechanism is assumed here, since usually entrepreneurs have no collateral at all for innovation activities.[9] Moreover, Bencivenga and Smith (1993) showed that credit rationing reduces growth due to adverse selection problems; while they limited their analysis to credit contracts, here we extend the sources of external finance also to equity contracts.

The paper is structured as follows: in section 2, we set up the basic model. In section 3, we consider financing with debt for firm in a sector *inside* the frontier; in section 4,

[7] In particular, in AAZ (2004), the form of the TFP is such that it grows more rapidly with "large non selective investments " in more backward economies, while in advanced economies, it grows more if "selective projects" are chosen.

[8] See also Hall (2001) and Aghion and Bolton (1992).

[9] In the Appendix an extension of the model replicates the main results of the paper when the entrepreneur invests her own wealth.

we model the same sector *at* the frontier with debt. Then, in section 5 there is firms in a sector *inside* the frontier with equity financing and in section 6 the model with equity is extended to innovation in sectors *at* the frontier. Section 7 shows how the distance to the frontier is associated with each form of financing (zero values) and section 8 contains the consequences of these choices on the growth rates and some policy recommendations. Section 9 concludes.

2 The model

Consider a closed economy where there are three types of agents: entrepreneurs, specialized workers, investors. Entrepreneurs lead individual firms for the production of intermediate or final goods. Specialized workers are researchers working in the intermediate sector firms. Investors have funds to finance entrepreneurs projects. In the economy there are V intermediate productive sectors $(1, 2,.....v,....,V)$. These inputs are used to produce a unique final good (Y) according to the following production function:

$$Y_t = \frac{1}{\alpha}\sum_{v=1}^{V} A(v)_t^{1-\alpha} k(v)_t^{\alpha} \tag{1}$$

where $A(v)_t$ is the TFP of firms operating in sector v (it also represents the quality level of that good), $k(v)_t$ is the amount of input v used in the final good production and α $(\in (0, 1))$ is the intensity of capital parameter.[10] Following standard quality improving models, we assume that the final good is produced competitively, while every intermediate good v is produced by the firm that wins the patent race in every period.[11]

The timing is as follows: each period is divided into two consecutive subperiods. At the beginning of the first subperiod all entrepreneurs engage in a patent race undertaking R&D projects (innovation/imitation): the first successful entrepreneur obtains a patent that grants her the right of producing the intermediate good in the second subperiod as a monopolist. After the second subperiod, a new period starts with another patent race. Moreover, the timing comes at regular intervals for simplicity and is shown in figure 1:

INSERT FIGURE 1 HERE

At the beginning of the patent race period, in each of the intermediate sectors the previous period TFP $[A(v)_{t-1}]$ is available to any firm.[12] For each sector a patent race starts in order to improve the quality of each good. The improvement in quality may come form two different activities: imitation and/or innovation.

[10] So production of final goods depends only on capital inputs.

[11] Note that all the analysis does not consider the economy as a whole, but sector by sector. Considering each sector separately is simply due to the fact that we do not impose any a priori rule of aggregation across sectors. Each sector productivity depends only on its own technology, so I ignore cross sectors spill - overs.

[12] With some abuse of terminology, quality, knowledge and TFP are used indifferently.

A successful imitation brings about a limited increase in the sector TFP, while with a successful innovation the increase in TFP is highest.[13] So, the sector TFP ($A(v)_t$) evolves as follows:

$$\begin{array}{ll} A_t(v) = \gamma \bar{A}_{t-1}(v) & \text{if innovation succeeds} \\ A_t(v) = \gamma A_{t-1}(v) & \text{if only imitation succeeds} \end{array}$$

where $A_t(v)$ is the local sector technology level and $\bar{A}_{t-1}(v)$ is the world sector technology frontier (see figure 2).

INSERT FIGURE 2 HERE

This expression says that if at time t an innovation project succeeds, the "quality" improvement is of a proportion γ (with $\gamma > 1$) of the previous period world technology frontier: $\gamma \bar{A}_{t-1}(v)$; while if all innovators fail, the "quality" improvement is smaller ($\gamma A_{t-1}(v)$) if there is a successful imitation.

As entrepreneurs improve upon the old technology, this becomes immediately obsolete: in this sense innovations are drastic. However, we assume for simplicity that innovation does not displace imitation. For example, if one entrepreneur is successful at innovation and anoother at imitation, the innovation good does not drive imitation out of the market. While this assumption might be at odds with the standard on quality improving models (Grossman and Helpman, 1991)[14], it can be justified if we consider for example that for each sector v, there are two goods, which are imperfectly substitutable: one has a high productivity growth potential (innovation) and the other has a lower productivity growth potential (imitation). A similar approach has been thouroughly discussed by *Aghion + al.* (2001).

We assume that imitation does not require any specific skill, while innovation requires a certain degree of skill or "intelligence." With respect to this, entrepreneurs are assumed to be of two types: those with low entrepreneurial skills (L-types), who correspondingly have a lower probability of success, and those with high entrepreneurial skills (H-types), who correspondingly have a higher probability of success. The difference in skills matters for the financing of innovation, as the type is private information of the entrepreneur.

Finally, for each sector there is a finite amount of entrepreneurs X, of which H are of H-types and the L are of L-types: $X = H + L$.

[13] It could be argued that if the initial level of knowledge of a sector is very much inside the frontier, then it is very hard to reach it and to go beyond it with a single project. Some literature has put forth this point arguing about the disadvantage of backwardness (Papageorgiou, 2002). However, another part of the literature has also argued about the advantage of backwardness (Gerschenkron, 1962). While the point is debated among economists, we take the view that there is no restriction or other impediment to adopt foreign technology. There are two reasons for this choice. First this allows to compare innovation projects in sectors at the frontier and inside the frontier more easily. Second, we just want to focus on financial frictions at the R&D stage. For a similar approach see Howitt (2000), and Aghion et al. (2002).

[14] Grossman and Helpman (1991, chap. 4) make the case of non drastic innovations. In this case, they argue that limit pricing by the technology leader drives the follower out of the market. See also Barro and Sala-i-Martin (2004).

3 Debt inside the frontier

3.1 Imitation

Imitation will improve a sector TFP according to the following law of motion:[15]

$$A_t = \begin{cases} \gamma A_{t-1} & \text{with prob. } zN_t^{\beta} \\ 0 & \text{otherwise} \end{cases}$$

where N_t is the amount of funds invested in imitation. The probability of success increases at a decreasing constant rate (β, where $0 < \beta < 1$). The constant z guarantees that the probability is bounded: $zN_t^{\beta} < 1$. The expression says that imitation, if successful, will bring about an increase in TFP as big as a γ factor ($\gamma > 1$) with respect to last period technology level (A_{t-1}).

If all costs are financed by external finance (debt) at the gross interest rate R_t, and because imitation is independent of the type of entrepreneur (H or L), expected total profits of imitation ($E\left(\Pi_M\right)_t$) are

$$E\left(\Pi_M\right)_t = \; zN_t^{\beta}\left[\delta(\chi)A_t - R_t N_t\right] \tag{2}$$

That is to say that with a probability zN_t^{β}, the entrepreneur is successful at imitation; in this case she enjoys profits from imitation $\delta(\chi)A_t$,[16] and is able to repay the debt, for the amount of funds borrowed N_t at the gross interest rate R_t. We therefore assume that R&D output from imitation (but also from innovation) is fully verifiable.

3.2 Innovation inside the frontier

If an entrepreneur is successful with innovation, she will then be a monopolist in this sector at least until the end of the current period; on the other hand, in case of a failure, she gains nothing and defaults on her debt. She will then maximize the expected profits of innovation ($E(\Pi^i)_t$)

$$E(\Pi^i)_t = zN_t^{\beta}\frac{p_i}{X}\left[\delta(\chi)\bar{A}_t - R_t N_t\right] \qquad \text{for } i = H, L \tag{3}$$

Similarly to imitation, expected profits from innovation are: a) financed externally by an investor at the interest rate R_t; b) increasing in the amount of production profits: these are represented by $\delta(\chi)\bar{A}_t$(see Appendix A for details). Differently, from imitation, innovation depends on: c) a probability of success which depends on an intrinsic parameter (p_i), which in turn depends on the type: it is lower for L-types than for H-types: $0 < p_L < p_H < 1$; d) the actual number of entrepreneurs participating in the patent race for innovation (i, for $i = H, L$), that is, as more participants in the race reduce the individual

[15] From now on, we drop the (v) notation for the sector TFP, where it creates no confusion.

[16] In particular, $\delta(\chi) = (\chi - 1)\,\chi^{\frac{1}{\alpha-1}}$, where χ is the price of the intermediate good and α is the intensity of capital parameter (see Appendix A details).

probability of success ($\frac{p_i}{X}$).[17]

As stated above, the two types of entrepreneurs differ in skills. This represents any kind of skills that makes it easier for some entrepreneurs to do innovation and less easy for others. For example, this could be due to organizational or deeper knowledge of the sector of some entrepreneurs with respect to others; obviously; it does not refer to the skills of specialized workers who are homogeneous with regard to this characteristic.[18]

3.3 Entrepreneurs indifference curves

Notice, first, that each entrepreneur would like to maximize expected profits from innovation.[19] However, in order to increase the probability of success either for innovation and for imitation, each entrepreneur increases the investment in R&D (N_t) in order to increase the probability of success (see the term N_t^β in 2 and in 3). By doing so they squeeze expected profits as much as possible. To determine how much expected profits are squeezed, notice that they will prefer to undertake innovation inasmuch as expected profits from this activity are higher than those of the alternative activity (imitation). That is to say that the participation constraint to innovation

$$zN_t^\beta \frac{p_i}{X}\left[\delta(\chi)\bar{A}_t - R_tN_t\right] \geq zN_t^\beta\left[\delta(\chi)A_t - R_tN_t\right] \qquad \text{for} \quad i = H, L \tag{4}$$

is always binding for both types.

Solving (4) requires imposing that in equilibrium, because of competition in the patent race, profits fall until the marginal entrepreneur is indifferent between the two activities. In this way, we can derive the indifference curves of both types of entrepreneurs:

$$N_i^E : N_t = \frac{\delta(\chi)\bar{A}_t}{R_t}\left[\frac{1-\frac{X}{p_i}a_t}{1-\frac{X}{p_i}}\right] \qquad \text{for} \quad i = H, L \tag{5}$$

where $a_t = \frac{A_t}{\bar{A}_t}$ is an inverse index of distance to the frontier[20]. The two curves are shown in figure 1.

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[17] The probability of success here is modelled with two implicit assumptions: first, each type of entrepreneur uses a similar research strategy; second, there cannot be two winners at the same time. For a similar approach, see Zeira (2003).

[18] Notice that we have specified the debt contract excluding the possibility that the entrepreneur invests its own funds in the project. However, this assumption is not a shortcoming of the model; in fact, as it is shown in Appendix B, the same results hold even in the case that the entrepreneur invests her own wealth.

[19] The maximization plan should be:

$$\max_{\{N_t,R_t\}} zN_t^\beta \frac{p_L}{x}\left[\delta(\chi)\bar{A}_t - R_tN_t\right]$$
$$\text{s.t. } zN_t^\beta \frac{p_L}{x}\left[\delta(\chi)\bar{A}_t - R_tN_t\right] \geq zN_t^\beta\left[\delta(\chi)A_t - R_tN_t\right]$$
$$\text{s.t.} N_t \geq 0$$

[20] Notice that when $a_t = 1$ the sector is at the frontier, while if $a_t = \frac{1}{2}$ the sector is at half distance and so on.

INSERT FIGURE 3 HERE

With regard to the indifference curves, three observation are due. First, consider that since entrepreneurs in the R&D race are competitive and there is free entry, their expected profits would be driven to zero. This does not happen because as stated in the participation constraint (4), expected profits fall until they reach those of imitation. At this level there is a locus of combinations of N_t and R_t which describes the indifference curve for each type.

Second, notice that both curves represent the highest amount of funds that H and L-type firms are willing to use in order to undertake innovation for any interest rate R_t. That is to say that for each entrepreneur's type the innovation area is the area south-west of the respective indifference curve. The intuition is that since firms compete each other in order to be the first to patent, they raise the amount of funds invested (N_t) in order to increase the probability of success (see the N_t^{β} component) as much as possible. Above the indifference curve, the costs are so high that it is more convenient (in expected values) to undertake the alternative: imitation.

Finally, the position of the two curves can be explained as follows. Consider the case in which both high and low skill entrepreneurs are charged the same interest rate (say R). Then, since L-types are less efficient (lower probability of success: $p_L < p_H$), for this interest rate they will need to use a higher amount of funds in order to undertake innovation instead of imitation. Indeed, when comparing the two indifference curves as in figure 1, one can check that for every level of funds demanded by both types (N_t) the indifference curve of L-types will always lie above that of H-types.[21]

3.4 Investors indifference curves

With regard to investors, the following assumptions are made: *investors are perfectly competitive, there is free entry and they are risk neutral.* Moreover, we assume that there is no collateral for innovation activities: this is not a strong assumption inasmuch as innovative firms mostly rely on the human capital of its researchers (specialized workers in this framework).[22]

The investor knows that the entrepreneur (of types i) will repay the debt for the full amount at the agreed interest only in case of success of the project, which happens with probability $zN_t^{\beta}\frac{p_i}{X}$, otherwise she will get nothing. Therefore, the expected total returns $E[TR_f^i]_t$ of the investor (f) is

$$E[TR_f^i]_t = \begin{cases} R_t N_t & \text{with prob. } zN_t^{\beta}\frac{p_i}{X} \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i = H, L$$

[21] Notice that the derivative of N_i^E with respect to p_i is negative: $\frac{\partial N_i^d}{\partial p_i} = \frac{\delta(\chi)\bar{A}_t}{R_t} \frac{\left(1-\frac{X}{p_i}\right)\frac{X}{p_i^2} - \left(1-\frac{X}{p_i}a_t\right)\frac{X}{p_i^2}}{\left(1-\frac{X}{p_i}\right)^2} < 0$ for $a_t < 1$.

[22] For the role of collateral, see Besanko and Thakor (1987) where collateral is a sortice device to avoid problems asymmetric information. Stiglitz and Weiss (1981) have an opposing view on the role of collateral. However, see Appendix B where the debt contract is extended to the case where the entrepreneur invests her own wealth.

while the expected profit $E[\Pi^i_f]_t$ is

$$E[\Pi^i_f]_t = \ zN_t^{\beta}\frac{p_i}{X}R_tN_t - N_t \qquad\qquad \text{for } i=H,L^{23} \tag{6}$$

Due to free entry among investors, her expected profits will be driven to minimum, that in this case is the net return of a storage technology: r_0N_t (r_0 is the interest rate of the riskless asset).

$$E\left(\Pi^i_f\right)_t = zN_t^{\beta}\frac{p_i}{X}R_tN_t - N_t \ = r_0N_t \qquad \text{for } i=H,L \tag{7}$$

This implies immediately that the indifference profit curve of an investor financing an innovation project to each type of entrepreneur (N_L^F) is:

$$N_i^F : N_t = \left(\frac{R_0}{R}\frac{X}{zp_i}\right)^{\frac{1}{\beta}} \qquad \text{for } i=H,L \tag{8}$$

where is: $R_0 = 1 + r_0$. Note that L-types are charged a higher interest rate because they have a lower probability of success than H-types.

3.5 Equilibrium inside the frontier with debt

Assume for the moment that information about the entrepreneurs types is symmetric. In this case we can derive the symmetric equilibrium outcomes. From the indifference curves of each type (5) and from the investors zero profit curves (8), we can derive the equilibrium amount of funds and interest rate of both types:

$$N^*_{d,L} = \left[\frac{1-\frac{X}{p_L}a_t}{1-\frac{X}{p_L}}\frac{z}{R_0}\frac{p_L}{X}\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{9}$$

$$N^*_{d,H} = \left[\frac{1-\frac{X}{p_H}a_t}{1-\frac{X}{p_H}}\frac{z}{R_0}\frac{p_H}{X}\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{10}$$

$$R^*_{d,i} = \left\{\frac{z}{R_0}\frac{p_i}{X}\frac{1}{\left[\frac{1-\frac{X}{p_i}a_t}{1-\frac{X}{p_i}}\delta\left(\chi\right)\bar{A}_t\right]^{\beta}}\right\}^{\frac{1}{1-\beta}} \qquad \text{for } i=H,L$$

Notice that solving for $\bar{A}_t$ and a_t, both $N^*_{d,L}$ and $N^*_{d,H}$ depend positively on A_t. That is to say that as the local technology improves, more and more funds are necessary to finance R&D innovative projects. Put it in another way, this is consistent with the assumptions of Aghion et al. (2005) on the capital requirement of innovation projects.[24] Moreover, the

[23] With regard to imitation we can proceed in a similar way. Then, expected profits of an investor financing an imitation project ($E[\Pi^M_f]_t$) are: $E[\Pi^M_f]_t = \widehat{\Omega}R_tN_t - N_t$

[24] This feature of the model that is often called "fishing out" effect, is common to various quality improving models See Eaton and Kortum (2005). This fishing out has no direct implications on the model,

equilibrium interest rate charged to L-types is higher ($R^*_{d,L} > R^*_{d,H}$), while H-types can borrow more in equilibrium ($N^*_{d,H} > N^*_{d,L}$) because they have a higher expected income.

Therefore, if we had symmetric information, the two equilibria would be in points A and B as shown in figure 3.[25]

INSERT FIGURE 3 HERE

3.6 Asymmetric information equilibrium

We now introduce asymmetric information. In order to derive the asymmetric equilibrium outcomes, we start from the symmetric equilibria derived above. When information is asymmetric, the equilibria imply that only L-types are financed for innovation, due to adverse selection.

In particular, we have one important adverse selection result for a sector inside the frontier:

Proposition 1 *If an intermediate sector v is inside the frontier, there is a separating equilibrium for innovation, with H-types undertaking imitation and L-types undertaking innovation. This equilibrium is revealing for L-types, and it is at the point $[N_L^{*\prime}, r_L^{*\prime}]$.*

Proposition 1 says that if the investor, due to asymmetric information, were to propose any pooling contract (like in point D in figure 3), this is accepted by both types. However, the interest rate is too high for H-types to undertake innovation, but not L-types. So, when the investor proposes a pooling contract, H-types undertake imitation, while L-types undertake innovation. Then, since each type is revealed, a pooling contract cannot be an equilibrium. This implies that the probability of success for L-types changes (from $\frac{p_L}{X}$ to $\frac{p_L}{L}$) and so the equilibrium contract (for innovation) becomes the following:

$$N'^*_{d,L} = \left[\frac{1-\frac{L}{p_L}a_t}{1-\frac{L}{p_L}}\frac{z}{R_0}\frac{p_L}{L}\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{11}$$

$$R^*_{d,L} = \left\{\frac{z}{R_0}\frac{p_L}{L}\frac{1}{\left[\frac{1-\frac{L}{p_L}a_t}{1-\frac{L}{p_L}}\delta\left(\chi\right)\bar{A}_t\right]^{\beta}}\right\}^{\frac{1}{1-\beta}} \tag{12}$$

To see why we have this result, consider that N_L^E is always higher than N_H^E (see figure 3), because since L-types have lower skills, they have to pay a higher interest rate for any amount of funds. It follows that when the type is private information, there is an incentive for L-types to demand the same amount as H-types so that they may pay a lower interest rate. In terms of figure 3, consider for example that if H-types demand an amount of funds N_H, then L-types can demand the same amount. This implies that since the investor does

as we do not consider general equilibrium implications. Finally, note that this feature is common to all the equilibrium amount of funds: inside/at the frontier, with debt/equities.

[25] For simplicity, the curves are linear in the graphs, where this does not create confusion.

not know the real type of each entrepreneur, she will offer a pooling contract at point D, with an average interest rate of those of the two types [R_D]. In our case, this average rate is given by the average of the two zero profit curves N_H^Z and N_L^Z. So, if there is a total of X entrepreneurs, the average interest rate at $N_t = N_0$ is a $E(R_t^*|_{N_t=N_H})$ given by:

$$E(R_t|_{N_t=N_H}) = \left(LR_L^* + HR_H^*\right)/X \tag{13}$$

Obviously, this rate is lower than any rate that L-types would be charged if their type were known. So they would accept this rate. On the other hand, since R_D is higher than the equilibrium interest rate of H-types, these would be paying a rate higher than what they can accept in order to invest in innovation. This implies that H-types are out of the innovation market and so undertake imitation. L-types, instead, would accept this lower interest rate. However, since their type is revealed, the equilibrium contract for them is given by (11) and (12).

Note that this adverse selection result is only similar to standard cases of credit rationing in the literature of financing with asymmetric information, as in the paper of Stiglitz and Weiss (1981). In that paper, rationing *may* happen when the investor finds that the project is too risky, even if entrepreneurs are willing to pay high interest rates. Differently, here H-types are not willing to accept a higher rate for innovation, so at this higher rate they undertake imitation. Since high skills entrepreneurs do not undertake the socially more convenient activity (innovation), we have a form of adverse selection result.

4 Debt at the frontier

If the sector is *at* the frontier, then quality improvements can only be obtained by innovation, since there is no possibility of imitation. So, being at the frontier or inside of it implies a different technological choice, which also alters the structure of incentives for the entrepreneurs.

This implies that since there is no imitation, the participation constraint for investors and for entrepreneurs is with respect to an alternative activity that we suppose to be a storage technology. Then, in order to derive the equilibrium for both types, we need first to derive the indifference curves in this different economic environment.

4.1 Entrepreneurs indifference curves

In order to derive the entrepreneurs indifference curves, first consider that entrepreneurs expected profits:[26]

$$E\left(\bar{\Pi}^i\right)_t = zN_t^\beta \frac{p_i}{X}\left[\delta(\chi)\bar{A}_t - R_tN_t\right] \qquad \text{for } i = H, L \tag{14}$$

Due to competition among entrepreneurs, their expected profits would be driven to the minimum possible amount: each entrepreneur will undertake innovation if the expected profits from innovation are higher than those of an alternative activity, which in this case

[26] $\bar{A}_t$ represents the v sector TFP at the frontier (v is omitted, see footnote above)

it is not imitation, but simply the storage technology with a net return of r_0 for every unit of money invested. The participation constraint for an L-type becomes:

$$zN_t^{\beta}\tfrac{p_L}{X}\left[\delta(\chi)\bar{A}_t - R_tN_t\right] \geq r_0N_t$$

From which we can derive the entrepreneurs indifference curves (see Appendix C for details):

$$\bar{N}_i^{E} : R_t = \frac{\delta(\chi)\bar{A}_t}{N_t} - \frac{X}{zp_i}\frac{r_0}{N_t^{\beta}} \quad \text{for } i = H, L \tag{15}$$

Differently from the curves derived for a sector inside the frontier, in this case the indifference curves are higher for H-types: $\bar{N}_H^{E}$is higher than $\bar{N}_L^{E}$(see Appendix C). The reason of this inverted order is due to the fact that now the alternative activity, the storage technology, grows linearly in the amount of funds N_t. So, for example, consider that at a given interest rate, since H-types produce more expected income, they are indifferent between the two activities only when they demand more funds than L-types.

4.2 Investors indifference curves and symmetric information equilibrium

In order to derive the curves of investors, consider that the expected profit of an investor at the frontier financing entrepreneur is

$$E[\bar{\Pi}_f^{i}]_t = zN_t^{\beta}\frac{p_i}{X}R_tN_t - N_t \qquad \text{for } i = H, L$$

imposing the equality of expected profits with the storage technology:

$$E[\bar{\Pi}_f^{i}]_t = zN_t^{\beta}\tfrac{p_i}{X}R_tN_t - N_t = r_0N_t \qquad \text{for } i = H, L$$

we get (as in the case of a sector inside the frontier) for L and H-types the same relations as inside the frontier:

$$\bar{N}_i^{F} : N_t = \left(\frac{R_0}{R}\frac{X}{zp_i}\right)^{\frac{1}{\beta}} \quad \text{for } i = H, L \tag{16}$$

In turn, the symmetric information equilibrium values of the amount of funds and interest rates for both types are derived from (15) and (16); they are:

$$\bar{N}_{d,i}^{*} = \left(\frac{z\frac{p_i}{X}\delta(\chi)\bar{A}_t}{1+2r_0}\right)^{\frac{1}{1-\beta}} \quad \text{for } i = H, L \tag{17}$$

$$\bar{R}_{d,L}^{*} : \quad R_t = R_0\left(\frac{X}{zp_i}\right)^{\frac{1}{1-\beta}}\left(\frac{1+2r_0}{\delta(\chi)\bar{A}_t}\right)^{\frac{\beta}{1-\beta}} \tag{18}$$

Notice, again that $\bar{N}_{d,H}^{*} > \bar{N}_{d,L}^{*}$and $\bar{R}_{d,L}^{*} > \bar{R}_{d,H}^{*}$.

Notice that differently from a sector inside the frontier, the interest rates of H-types is lower, while the amount of funds is higher. They are shown in figure 4.

INSERT FIGURE 4 HERE

4.3 Asymmetric information equilibrium

As in the previous section, we now proceed to see how the model works under asymmetric information. In particular, we will show the following proposition:

Proposition 2 *If an intermediate sector v is at the frontier, there is a pooling equilibrium for innovation, with only H-types undertaking innovation and L-types undertake imitation. This equilibrium is revealing and it is at the point* $\left[\bar{N}^{*\prime}_{d,L}, \bar{R}^{*\prime}_{d,H}\right]$.

Proposition 2 states that the equilibrium debt contract at the frontier excludes L-types from the patent race. In this sense, the equilibrium is revealing for H-types, and the equilibrium debt contract offered by the investor for innovation is given by:

$$\bar{N}^{*\prime}_{d,H} = \bar{N}^{*}_{d,i} = \left(\frac{z \frac{p_H}{H} \delta(\chi) \bar{A}_t}{1 + 2r_0} \right)^{\frac{1}{1-\beta}}$$

$$\bar{R}^{*\prime}_{d,H} : \quad R_0 \left(\frac{H}{z p_H} \right)^{\frac{1}{1-\beta}} \left(\frac{1 + 2r_0}{\delta(\chi) \bar{A}_t} \right)^{\frac{\beta}{1-\beta}}$$

Notice that in figure 4 the respective equilibrium values (of symmetric information) for H and L-types are at points A and B, whose values are given by (17) and (18). Also in this case it turns out that there is an incentive for one of the two groups to hide its type. In fact, consider what happens if an L-type entrepreneur were to hide her type, and consequently demand the same amount of H-types ($\bar{N}_H$). However, the investor does not offer an average rate as at point D, this is because this rate is too high for both types. However, the investor may offer a contract at point B. At this point, L-types would be charged a higher interest rate than at their point A for innovation, and so they undertake imitation. H-types, in turn, would be charged exactly the interest rate of perfect information, so they accept it. Then, point B is a pooling equilibrium contract, because both types accept it, but only H-types undertake innovation.

We can now summarize the results obtained so far with debt. Asymmetric information has different consequences on innovation whether a sector is inside the frontier or at the frontier:

a) in the sector inside the frontier, it drives H-types out of the innovation market and they undertake imitation, while innovation is undertaken by L-types;

b) in a sector at the frontier it is the reverse: H-types undertake innovation while L-types invest in the storage technology.

5 Equity inside the frontier

In the previous section we have found that the debt contract always creates equilibria such that just one type undertakes innovation. We now proceed to investigate what happens if the contract specifies the cost of financing in terms of a share of expected profits. We call this the equity contract.

If innovation projects are financed with equity, it means that the entrepreneur will have to give up part of firm ownership to the investor. In turn, the investor will get a share of profits in exchange for the investment. In the following subsection we will focus on equity without any form of monitoring in order to show that this is sufficient to remove some of the inefficiencies that we have with debt. In section 8.1.2 we introduce a form of investor monitoring with the equity contract.

5.1 Entrepreneurs indifference curves

Consider an entrepreneur undertaking imitation financed with equity by an external investor. Let θ be the share of innovation profits of the entrepreneur for imitation and it is exogenous (with $\theta < \frac{1}{2}$). Then, adapting (2), her expected profits from imitation are

$$E\left(\Pi_{eq}^{M}\right)_t = zN_t^{\beta}\theta\delta\left(\chi\right)A_t$$

that is to say that with a probability zN_t^{β}, the entrepreneur is successful with imitation, and gets a share of profits: $\theta\delta\left(\chi\right)A_t$. Expected profits from an innovation project financed with equity are:

$$E(\Pi_{eq}^{i})_t = zN_t^{\beta}\frac{p_i}{X}\eta\delta\left(\chi\right)\bar{A}_t \qquad \text{for } i = H, L$$

where η is the variable describing the share of profits of the entrepreneur: $0 < \eta < 1$.

In order to derive the indifference curve in this case, consider that an entrepreneur is willing to undertake innovation if the expected profits are higher than expected profits coming from imitation; so, for example, for L-types, we have $E(\Pi_{eq}^{i})_t \geq E\left(\Pi_{eq}^{M}\right)_t$. Imposing this constraint implies:

$$zN_t^{\beta}\tfrac{p_L}{X}\eta\delta\left(\chi\right)\bar{A}_t \geq zN_t^{\beta}\theta\delta\left(\chi\right)A_t$$

from which we derive the following:

$$\eta_L^E = \frac{X}{p_L}\theta a_t \qquad \text{and} \qquad \eta_H^E = \frac{X}{p_H}\theta a_t \tag{19}$$

Both of the indifference curves represent the minimum share of profits desired by the entrepreneurs such that she undertakes innovation. Note that while the order $\eta_L^E > \eta_H^E$ is not immediately intuitive, it can explained by the following reasoning. These values are the desired shares that entrepreneurs have before they meet the investor. In particular, since the skills of H-types are higher, their reservation utility is for a lower share of profits; this implies that they can accept to undertake the more difficult activity (innovation) for a lower share of profits than what L-types can do. Finally, note that both the desired shares do not depend on N_t (see figure 5).

5.2 Investors indifference curves and symmetric equilibrium

On the supply side, the investor financing innovation gets the following expected profits:

$$E(\Pi^i_{eq,f})_t = zN_t^{\beta}\frac{p_i}{X}(1-\eta_i)\,\delta(\chi)\,\bar{A}_t - N_t \quad \text{for} \quad i = H, L$$

Imposing the equality relation with the storage technology

$$zN_t^{\beta}\tfrac{p_i}{X}(1-\eta_i)\,\delta(\chi)\,\bar{A}_t - N_t = r_0 N_t \quad \text{for} \quad i = H, L$$

we have the indifference curves for the investors:

$$N^F_{i,eq} : N_t = \left[(1-\eta)\frac{p_i}{X}\frac{z\delta(\chi)\,\bar{A}_t}{R_0}\right]^{\frac{1}{1-\beta}} \qquad \text{for} \quad i = H, L \tag{20}$$

Both the zero profit curves are downward sloping with respect to η (see figure 6). The reason is that the higher is the initial outlay of the investor, the higher her reward has to be (share of profits: $1-\eta$) in order for the investor to finance innovation. Moreover, notice that the areas where innovation is financed is south-east with respect to the each zero profit curve.

From the indifference curves of the investor and of the entrepreneur derived above we have the following symmetric information equilibrium values for each type:

$$N^*_{i,eq} = \left[\left(1-\frac{X}{p_i}\theta a_t\right)\frac{\delta(\chi)\,\bar{A}_t}{R_0}\frac{p_i}{X}\right]^{\frac{1}{1-\beta}} \qquad \text{for } i = H, L \tag{21}$$

$$\eta^*_i = \frac{X}{p_i}\theta a_t \qquad \text{for } i = H, L$$

with $\eta^*_L > \eta^*_H$, and $N^*_{H,eq} > N^*_{L,eq}$, as shown in the graph below

INSERT FIGURE 5 HERE

5.3 Pooling equilibrium

Consider now the symmetric information equilibrium points A and B in figure 5.

In case of asymmetric information, we can derive the following result:

Proposition 3 *In a sector inside the frontier, if innovation is financed with equity, there is a pooling equilibrium such that H-types and L-types undertake innovation.*

To get the intuition, consider that with asymmetric information, points A and B are not in equilibrium.

In fact, if information is asymmetric, H-types, might demand the same amount of funds of L-types ($N^*_{L,eq}$), in order to grab a higher share of profits than what they could get at their reservation level (point B). In this case, the investor might offer a pooling contract in point D because she is unable to discern the true type, where she receives the same share

$(1-\eta_D)$ from both types . At point D, L-types would be better off because for the same amount of funds they gain an additional share of profits $(\eta_D - \eta_A)$ and H-types would benefit even more because their share of profits increases by the difference $(\eta_D - \eta_B)$. It follows that the pooling contract in D is an equilibrium contract where H-types are better off than if they were to accept the separating contract in B. In particular, the contract D is specified by the pair:

$$N^*_{i,eq} = \left[\left(1 - \frac{X}{p_L}\theta a_t\right)\frac{\delta(\chi)\bar{A}_t}{R_0}\frac{p_L}{X}\right]^{\frac{1}{1-\beta}}$$

$$\eta_L^E = \eta_H^E = \frac{X}{\widehat{p}}\theta a_t$$

Then, the main result with the equity contract inside the frontier is that both types participate to the patent race for the innovation.

6 Equity at the frontier

6.1 Entrepreneurs indifference curves

In the sector at the frontier, expected profits of the entrepreneurs are the following

$$E(\bar{\Pi}^i_{eq})_t = zN_t^\beta \bar{\eta}_i \frac{p_i}{X}\delta(\chi)\bar{A}_t \qquad \text{for } i = H, L$$

where $\bar{\eta}_i$is the variable for the share of profits of the entrepreneur innovating at the frontier. At the frontier, since there is no imitation, competition drives expected profits to the lower bound determined by the net return on the storage technology. Therefore, the innovation participation constraint is given by $E(\bar{\Pi}^i_{eq})_t \geq r_0 N_t$ and from this, the indifference curve of the entrepreneur (for each type) is derived:

$$\bar{\eta}_i^E = \frac{X}{zp_i}N_t^{1-\beta}\frac{r_0}{\delta(\chi)\bar{A}_t} \qquad \text{for} \quad i = H, L \tag{22}$$

Differently from the case inside the frontier, the desired share of profits increases with the funds invested. This is due to the fact that the two technologies grow at different speeds with respect to the amount of funds. In fact, while the storage technology return increases proportionally to N_t, the innovation technology increases less than proportionally. So, in order for an entrepreneur to undertake innovation, it needs that her desired share of profits increases as more funds are invested. Moreover, notice that again $\bar{\eta}_L^d > \bar{\eta}_H^E$; the reason for this ordering is the same as in the case inside the frontier.

6.2 Investors zero profit curves and symmetric equilibrium

In order to derive the investors zero profit curves, we need to consider the expected profits:

$$E(\bar{\Pi}^{i}_{eq,f})_t = zN_t^{\beta}\frac{p_i}{X}\left(1-\bar{\eta}_i\right)\delta\left(\chi\right)\bar{A}_t - N_t \qquad \text{for } i = H, L$$

Also here, imposing the equality condition with the storage technology, gives (derived in a way similar to the case of equity inside the frontier, in Appendix C):

$$\bar{N}^{F}_{L,eq} : N_t = \left[z\frac{p_L}{X}\left(1-\bar{\eta}_L\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{23}$$

$$\bar{N}^{F}_{H,eq} : N_t = \left[z\frac{p_H}{X}\left(1-\bar{\eta}_H\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{24}$$

Deriving the symmetric information equilibrium from previous expressions (22) and (23) and (24), the symmetric information equilibrium amount of funds would be:

$$\bar{N}^{*}_{i,eq} = \left[\frac{zp_i}{X}\frac{\delta\left(\chi\right)\bar{A}_t}{1+r_0}\right]^{1-\beta} \qquad \text{for } i = H, L \tag{25}$$

and the equilibrium share of profits is given by

$$\bar{\eta}^{*}_{L} = \bar{\eta}^{*}_{H} = \frac{r_0}{1+r_0} \tag{26}$$

Notice that differently form the case inside the frontier, they are exactly the same, but this is just a technical feature of the model.

6.3 Equilibrium

Consider now the separating equilibrium in points A and B as reported in figure 6.

INSERT FIGURE 6 HERE

Differently from the case inside the frontier, the problem of asymmetric information does not cause adverse selection. The reason is that the equilibrium share of profits of H and L types (26) are the same. So, the separating contract at the points A and B (in figure 7) is an equilibrium contract, because no entrepreneur has an incentive to deviate by hiding her type.

Proposition 4 *In a sector at the frontier, if innovation is financed with equity, there is a separating equilibrium such that both types are financed for innovation.*

It is due to remark that this is a particular result which is simply due to the technical details of the model. Despite it might look a shortcoming of the model, we argue that this is not the case for two reasons. First, since we are mainly interested to see the financing of innovation in a sector inside the frontier, what is relevant for the model is that financing at the frontier is available for both types with at least one of the two contracts. Second, it is often argued that problems of information asymmetry are lower in more developed countries (*citation*) and our result is not at odds with this view.

More in general, we can sum up all the results obtained so far:

a) When the sector is *inside* the frontier, in equilibrium

a1) *there is adverse selection with the debt contract, so that only L-types are financed for innovation;*

a2) *there is a pooling with the equity contract, so that both types are financed for innovation;*

b) When the sector is *at* the frontier, in equilibrium

b1) *only H-types undertake innovation, if this is financed with debt;*

b2) *both types undertake innovation, if it is financed with equity.*

7 Zero Values

Until now we have found the equilibrium contracts for each situation (inside/at the frontier) and contract (debt/equity) careless of when these contracts are actually implemented. It turns out that in sectors with initial technology inside the frontier not all contracts are always feasible, and the choice crucially depends on the distance to the frontier index a_t.

To understand this point, in this section we derive the so called *zero values*, that is the values of the index of distance to the frontier such that the equilibrium amount of funds either with debt and with equity (for sectors inside the frontier) is non negative. This allows to determine which contract will be used according to the initial s.o.t. (or distance to the frontier).

7.1 Debt zero values

Consider the case of debt firstly.

a) The equilibrium amount for funds of L-types (9) is given by the following condition: $N^*_{d,L} \geq 0 \implies \left[\frac{1-\frac{X}{p_L}a_t}{1-\frac{X}{p_L}} \frac{z}{R_0} \frac{p_L}{X} \delta\left(\chi\right) \bar{A}_t \right]^{\frac{1}{1-\beta}} \geq 0$. Solving for a_t, the previous relation implies that $a_t \geq a_{d,L} = \frac{p_L}{X}$.[27]

b) Similarly, for H-types, the condition (given by 10) is $N^*_{d,H} \geq 0$ requires that $a_t \geq a_{d,H} = \frac{p_H}{X}$.

Notice that from points a) and b) we have that debt is allowed as long as $a_t > a_{d,i}$ (for $i = H, L$), which implies that if a_t is below one or both the thresholds, the debt contract will not be applied. This is because as we move farther from the frontier (that is as a_t is very low) innovation becomes relatively more convenient; so entrepreneurs are indifferent between the two activities for a lower amount of funds invested. At $a_{d,L/H}$ the cost reduction reaches the minimum amount: zero.[28]

[27] Note that in the expression $N^*_{d,L} = \left(\frac{p_L}{X}\right) \left[\frac{1-\frac{X}{p_L}a_t}{1-\frac{X}{p_L}} \right] \delta\left(\chi\right) \bar{A}_t$, the sign of the denominator $[1 - \frac{X}{p_L}]$ is negative . Therefore, the positivity condition $N^*_{d,L} \geq 0$ requires that: $1 - \frac{L}{p_L}a_t \leq 0$.

[28] Another way to see this is to consider that when the production profit of the alternative activity (imitation) $[\delta\left(\chi\right) A_t]$ is so low with respect to one of innovation $[\delta\left(\chi\right) \bar{A}_t]$, entrepreneurs raise costs (from the origin to north east: see figure 3) just a little, the inferior technology becomes immediately more convenient (in expected values).

In other words, as we move farther from the frontier (A_t decreases relatively to $\bar{A}_t$) imitation becomes relatively less convenient for any pair of costs (N_t, R_t) to innovation; so innovators may decrease their costs more (inward shift of N_i^E) to reach the indifference locus of points.

7.2 Equity zero values

With regard to equity, in a way similar to debt, we can derive the threshold values for the equilibrium amount of funds.

c) For L-types (from 21) is positive for $a_t \leq a_{eq,L} = \frac{p_L}{X\theta}$.

d) For H-types (from 21) it is positive for $a_t \leq a_{eq,H} = \frac{p_H}{X\theta}$.

Notice in this case that both the threshold values are determined by the conflicting interests of the two shareholders: the entrepreneur and the investor. In fact, in the equity contract the entrepreneur bears no costs, but only a share of profits. Then, as we move closer to the frontier, since imitation profits $[\delta(\chi) A_t]$ become relatively more convenient with respect to those of innovation, the entrepreneur will be willing to undertake innovation only if her share ($\eta_i^E = \frac{X}{p_i}\theta a_t$) increases. In turn this implies that the investor's share of profits $(1 - \eta_i^E)$ decreases with the distance to the frontier index a_t. So, when the sector gets too close to the frontier, the investor's share is squeezed until it reaches zero at $a_{eq,i}$ (for $i = H, L$). This is why there is an upper limit for both $N^*_{eq,L}$ and $N^*_{eq,H}$.

7.3 Zero Values Regions

Putting the results of the previous subsections together, we have the following ranking:

$$0 < a_{d,L} < a_{eq,L} < a_{d,H} < a_{eq,H} < 1$$

as shown in figure 7.[29]

INSERT FIGURE 7 HERE

From figure 7 it is evident that there are 6 regions, 5 *inside* the frontier and the sixth is the region *at* the frontier. In order to better understand this ranking, following figure 8, it is possible to see that for example for $a_t < a_{eq,L}$, L-types entrepreneurs may demand funds with equity, or that for $a_t > a_{d,H}$ H-type entrepreneurs may demand funds with a debt contract. Finally, notice that for $a_t = 1$ both debt and equity are available choices for H and L-types (region VI in figure 8).

INSERT FIGURE 8 HERE

Given the regions outlined above, we are now able to determine which contracts will be used according to the distance to the frontier (see figures 8 and 9). In order to understand this, consider that in region I, we have $a_t < a_{d,L}, a_{d,H}$, that is the distance to the frontier is too high (a_t too small) for financing with debt of both types, but it is also $a_t < a_{eq,L}, a_{eq,H}$,

[29] Note that the ranking sets $a_{H,d} < a_{L,eq}$ which holds for $\theta > \frac{p_L}{p_H}$. The reults of the model would not change if this inequality does not hold.

which means that a_t is short enough to allow equity financing to both types. So, in region I innovation is financed with equity and, as such there is a pooling equilibrium as stated in proposition 4. In region II, the distance to the frontier is still sufficiently short for financing with equity ($a_t < a_{eq,L}, a_{eq,H}$), but it is also sufficient for debt financing of L-types only ($a_t > a_{d,L}$ but $a_t < a_{d,H}$). Then, L-types may choose debt or equity, while H-types can only choose equity. This implies that both types will be financed in region II.[30] Similarly, the reasoning can be done for regions III, IV, V and VI. The results are summarized in figure 9.

INSERT FIGURE 9 HERE

Notice that all of the regions are with both types undertaking innovation, except for region V, where L-types are financed for innovation while H-types are financed for imitation. This happens after the threshold value inside the frontier $a_t = a_{eq,H}$, as shown in figure 10.

INSERT FIGURE 10 HERE

Dividing the distance span as in figure 10 is a convenient way to understand how the productivity growth rate changes for a sector close to the frontier (region V), as it will be clear in the next section.

Finally, recall that at the frontier only H-types have access in equilibrium to debt finance at the frontier (proposition 2), while both types have access to equity finance (proposition 4). So, while L-types may only be financed through equity, H-types may choose between equity and debt and in any case both types would be financed. From this reasoning it is possible to state that:

Proposition 5 *In regions I-IV, both types of entrepreneurs are financed for innovation, using debt or equity depending on the region. The same applies in region VI. However, in region V only H-types are financed for innovation and they are financed with debt.*

8 Growth Rates

From the findings of the previous section, we can derive the growth path of a sector along all of the regions over the distance to the frontier index a_t. First of all, notice that given the form of the final output production function (1), for a generic sector, the productivity growth rate (g) is the same as the output growth rate, where g is given by

$$g = \frac{E(A_t) - A_{t-1}}{A_{t-1}} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

where $E(A_t)$ is the expected value of TFP in the period t. $E(A_t)$ is different according to the regions. In fact, in regions I to IV we have that both types are financed for innova-

[30] H-types have no choice but equities, while L-types can choose. So, if they choose debt they are recognized as L-types and not rationed, while if they choose equity they are not rationed as well (see proposition 6).

tion, so no-one undertakes imitation; therefore, the expected improvement in technology is:

$$E\left(A_t\right)_{I-IV} = \begin{cases} \gamma \bar{A}_{t-1} & \text{if Innovation succeeds with prob.} \quad z\overset{\frown}{N}^{\beta} \overset{\frown}{p} \\ A_{t-1} & \text{if Innovation fails with prob. } 1 - z\overset{\frown}{N}^{\beta} \overset{\frown}{p} \end{cases}$$

the growth rate is

$$g_{I-IV} = \frac{E(A_t)-A_{t-1}}{A_{t-1}} = \frac{\gamma z\overset{\frown}{N}^{\beta} \overset{\frown}{p} \bar{A}_{t-1}}{A_{t-1}} + \frac{A_{t-1}}{A_{t-1}} + \frac{z\overset{\frown}{N}^{\beta} \overset{\frown}{p} A_{t-1}}{A_{t-1}} - 1 \quad \text{or (call it } g_1\text{)}$$

$$g_1 = z\overset{\frown}{N}^{\beta} \overset{\frown}{p} \left[\frac{\gamma}{a_{t-1}} - 1 \right] \tag{27}$$

where $\overset{\frown}{N}_t$ is given by the equilibrium amount of funds according to the type of contract used (see Appendix D for details).

In region V, only L-types undertake innovation, while H-types undertake imitation. Since we are interested to the highest possible increase of productivity, we consider only the productivity growth rate given by the highest technology: imitation. Then, we have that:

$$E\left(A_t\right)_{V} = \begin{cases} \gamma \bar{A}_{t-1} & \text{if Innovation succeeds with prob.} \quad zN_L^{\beta} \frac{p_L}{L} \\ A_{t-1} & \text{if Innovation fails with prob. } 1 - zN_L^{\beta} \frac{p_L}{L} \end{cases}$$

and the growth rate is

$$g_V = \frac{E(A_t)-A_{t-1}}{A_{t-1}} = \frac{\gamma z N_L^{\beta} \overset{\frown}{p} \bar{A}_{t-1}}{A_{t-1}} + \frac{A_{t-1}}{A_{t-1}} + \frac{z N_L^{\beta} \overset{\frown}{p} A_{t-1}}{A_{t-1}} - 1$$

$$g_2 = zN_L^{\beta} \frac{p_L}{L} \left[\frac{\gamma}{a_{t-1}} - 1 \right] \tag{28}$$

where $N_L$is given by (11).

In region VI (at the frontier), both types are financed for innovation and so no-one invests in the riskless bond:

$$E\left(A_t\right)_{VI} = \begin{cases} \gamma \bar{A}_{t-1} & \text{if Innovation succeeds with prob.} \overset{\frown}{p} \; zN_t^{\beta} \\ \bar{A}_{t-1} & \text{if Innovation fails with prob. } 1 - \overset{\frown}{p} zN_t^{\beta} \end{cases}$$

and the growth rate is

$$g_3 = (\gamma - 1)\, zN_t^{\beta} \overset{\frown}{p} \tag{29}$$

where again is given by the equilibrium amount of funds according to the type of contract used.

Notice that both g_1 and g_2 are decreasing in the distance to the frontier ($g_a < 0$), that is as the sector is closer to the frontier index the lower the expected growth rate. This result depends on assuming that innovation is possible also at early stages of development.

In order to have a better understanding of how the growth rate changes, consider that even if g_1 and g_2 are both decreasing as the distance to the frontier is reduced, g_2 is much lower than g_1 (see proof in Appendix D): $g_1 > g_2$. Moreover, it can be shown (see Appendix D for details) that for realistic values of γ and p_L, it is $g_3 > g_2$. So, the projection of the growth path on the inverse index of distance to the frontier a_t can be seen in figure 11.

INSERT FIGURE 11 HERE

This figure tells us that the growth rate is ever decreasing with the distance to the frontier, as for g_1. As mentioned above, the rate is decreasing because as the initial state of technology increases the percentage productivity jump $(E(A_t) - A_{t-1})$ is lower and lower: indeed, g_1 is decreasing in a_{t-1}.

Most important, note that there is a downward jump of the growth rate at $a_{t-1} = a_{eq,H}$. The reason is due to the effect on g_2 of adverse selection (only L-types are financed for innovation).[31] So, the probability of success of innovation in g_2 is lower (p_L) than the respective probability in g_1 ($\breve{p}$: where both types are financed). Note also that this effect could be counterbalanced by an opposite effect on g due to the contribution of imitation to productivity growth. In fact, consider the case innovation fails: A_{t-1}doesn't change in regions I-IV (see $E(A_t)_{I-IV}$), because no-one undertakes imitation; instead, in region V A_{t-1} is increased by a factor γ, that is if imitation (undertaken by H-types) is successful. However, this effect is of an order of magnitude inferior to the previous effect of adverse selection. The reason is that innovation led growth is ever higher, even when only L-types undertake it, than the imitation push to productivity growth.

Finally, the path of g changes again at the frontier. In fact, at this point, due to the absence of the alternative of imitation, the structure of incentives changes, and the growth rate (g_3) *may* increase again.

Therefore, we have that while the growth rate is ever decreasing as a sector moves closer to the frontier, there is an intermediate region (region V) where g is still decreasing, but is subject to a shortfall. In this region the growth rate (g_2) is much lower due to adverse selection than it would be otherwise (see dotted line in figure 12). This growth loss is due to the impossibility to sign equity contract between the parties: since the alternative to innovation (imitation) is very profitable, the entrepreneur pretends a very high share of profits in order to undertake innovation. Conversely, the residual share for the investor becomes null, and so the equity contract cannot be signed. So, the reason of the low growth in region V depends on the effects of the excessively high profitability of imitation through the financing effects (no equity and adverse selection with debt).

8.1 Growth and Financial development

In this subsection we consider how the inefficiency of slow growth derived above can be "removed" by appropriately designing the financial contracts. in particular, instead of deriving a general theory, we provide only two examples that are built following a by now

[31] See proposition 7 above.

rich literature on corporate finance and innovation (Hall, 2002). In the first example we improve the credit contract by introducing a partial or full creditor protection. We show that creditor protection increases the amount of funds invested in equilibrium, and so also the expected productivity growth rate of a sector; however, this contract still excludes H-types from undertaking innovation. In the second example, we introduce a form of monitoring by the investor in the equity contract. We show that the region where equity cannot be signed and growth is lower (region V in figure 11) can be reduced by setting appropriate incentives entrepreneurs.

The result of this effort will lead to the general conclusion that financial development is growth enhancing only at intermediate stages of development, which is in line with the most recent findings on this topic by Aghion et al. (2005).

8.1.1 Creditors protection

Suppose that a fraction (λ) of the investment N_t can be surely recovered by the creditor, through for example the intervention of a court. So λ represents the efficiency of the court in recovering funds in case of default of the borrower or in more general terms, it can be interpreted as a parameter of legal investor protection.

In this case, entrepreneur's expected profits from innovation inside the frontier are

$$E(\Pi^i_{d,\lambda})_t = zN_t^{\beta}\frac{p_i}{X}\left[\delta(\chi)\bar{A}_t - (1-\lambda)\,R_tN_t\right] - \lambda R_tN_t \quad \text{for} \;\; i = H, L$$

that is to say that entrepreneurs will repay for sure a fraction λ of the amount due to the creditor (R_tN_t). The residual amount $(1-\lambda)\,R_tN_t$ is subject to risk of default. In this case, the indifference relation with imitation is

$$zN_t^{\beta}\tfrac{p_i}{X}\left[\delta(\chi)\bar{A}_t - (1-\lambda)\,R_tN_t\right] - \lambda R_tN_t \geq zN_t^{\beta}\left[\delta(\chi)A_t - (1-\lambda)\,R_tN_t\right] - \lambda R_tN_t \qquad \text{for} \;\; i = H, L$$

where creditor protection is extended also to imitation. It is easy to solve for the indifference relation:

$$N^E_{\lambda,i} : N_t = \frac{\delta(\chi)\bar{A}_t}{R_t\,(1-\lambda)}\left[\frac{1-\frac{X}{p_i}a_t}{1-\frac{X}{p_i}}\right] \qquad \text{for} \qquad i = H, L$$

Two observation are due to arrive to satisfying conclusions. First, these curves are exactly the same of those derived in section 3 except for a term $(1-\lambda)$; however, the downward slope of $N^E_{\lambda,i}$ with respect to the interest rate R_t and the other properties are not changed. So, they have the same properties of those derived earlier. Second, for each type of entrepreneur, the indifference curve with creditor protection $\left[N^E_{\lambda,i}\right]$ lies above the curve without creditor protection $\left[N^E_i\right]$ for a factor λ. This implies that higher investor protection λ increases the amount of funds for which entrepreneurs are willing to undertake innovation at any level of the interest rate: the indifference curves shift upward (see figure 3).

Investors are (again) indifferent between financing innovation and a storage technology:

$$zN_t^{\beta}\frac{p_i}{X}(1-\lambda)R_tN_t+\lambda R_tN_t-N_t \geq R_0N_t-N_t \qquad \text{for} \;\; i=H,L$$

and the indifference curves are:

$$N^Z_{\lambda,i}: N_t=\left[\frac{1-\rho-\lambda}{(1-\lambda)\,z\frac{p_i}{X}}\right]^{\frac{1}{\beta}} \qquad \text{for} \qquad i=H,L$$

where $\rho=1-\frac{R_0}{R_t}$.[32]
The symmetric information equilibrium values are:

$$N^*_{\lambda,d,i}=\frac{\left[z\frac{p_i}{X}k\delta(\chi)\bar{A}_t\right]^{1-\frac{1}{\beta}}}{\left(\frac{z(1-\lambda)}{1-\rho-\lambda}\frac{p_i}{X}\right)^{\frac{1}{\beta}}} \qquad \text{for} \qquad i=H,L \tag{30}$$

$$R^*_{\lambda,d,i}=\frac{z\frac{p_i}{X}k\delta(\chi)\bar{A}_t}{1-\rho-\lambda} \quad \text{for} \qquad i=H,L$$

where $k=\frac{1-\frac{X}{p_L}a_t}{1-\frac{X}{p_L}}$, and $\frac{\partial R^*_{\lambda,d,i}}{\partial p_i}<0$.

From these considerations, two main conclusions can be drawn. First, since L-types indifference curve lies above that of H-types, there is still a credit constraint for H-types on innovation. Second, because each $N^E_{\lambda,i}$ increases with λ, there is more investment in innovation, and so also the probability of success ($zN_t^{\beta}\frac{p_i}{X}$) is increased, so enhancing the expected growth rate.

8.1.2 Active investors

The results of creditor protection may not be completely satisfying. In fact, if on the one hand expected growth is enhanced because there is more investment in R&D, on the other hand this implies that innovation has become more expensive in general, because more resources have to devoted to innovation. Moreover, H-types are still excluded from innovation. From a welfare point of view, it would be preferable to have a mechanism such that it does not increase the amounts of funds invested but that includes H-types in the innovation patent race. In this section we explore this possibility, by assuming that the equity contract implies an active role of the investor in the innovation process. This role may consist in monitoring, advising, mentoring, or in one word having a role in the governance of the firm and it is in line with much of the literature on the financing of innovation.[33]

We assume that investor activism implies an additional cost for the entrepreneur but not for the investor. The rationale of this assumption for the entrepreneur is that if she

[32] For $N^Z_{\lambda,i}$ to be non negative, we need to impose $1-\rho-\lambda>0$. Moreover, notice that in this expression, $N^Z_{\lambda,i}$ is negatively sloped with respect to R_t, as in the initial case in section 3. In fact, imposing that $R_0=(1-\rho)R_t$, and inserting the value of $\rho=1-\frac{R_0}{R_t}$ into the expression above, we see that the slope is negative.

[33] See for example Bottazzi, Da Rin and Hellmann (2004), Hellmann and Puri (2000), Kaplan and Strömberg (2003), Lerner (1995).

loses some control over the firm, this can be interpreted as a loss in monetary terms. For the investor, more governance activism usually implies a cost but also an additional benefit, as in models of double sided moral hazard; however, for notational simplicity we do not model this, assuming that more governance enhances the value of the project so that it just offsets the costs of exercising it.

Entrepreneurs 'expected profits in this case are:

$$E(\Pi^i_{eq,\psi})_t = zN_t^\beta \frac{p_i}{X}\eta\delta(\chi)\bar{A}_t - \psi a_t N_t \quad \text{for } \ i = H, L$$

where ψa_t ($\psi > 0$) is the cost of investor's activism, and it is proportional to the local technology level of a sector. Entrepreneurs indifference relation becomes

$$E(\Pi^i_{eq,\psi})_t = zN_t^\beta \tfrac{p_i}{X}\eta\delta(\chi)\bar{A}_t - \psi a_t N_t \geq zN_t^\beta\theta\delta(\chi)A_t$$

The indifference relation in this case becomes:

$$\eta_i^E = a_t \frac{X}{p_i}\left[\frac{\psi N_t^{1-\beta}}{z\delta(\chi)} + \theta\right] \qquad \text{for } \ i = H, L$$

In turn, investors indifference curves are unchanged with respect to before:

$$N^F_{i,eq,\psi} : N_t = \left[(1-\eta)\frac{p_i}{X}\frac{z\delta(\chi)\bar{A}_t}{R_0}\right]^{\frac{1}{1-\beta}} \qquad \text{for } \ i = H, L$$

One can show that the equilibrium values of symmetric equilibrium in this case is given by:

$$\eta^*_{i,\psi} : \eta_i = \frac{A_t(p_i\psi + XR_0)}{(R_0 + \psi A_t)p_i} \quad \text{for } \ i = H, L$$

$$N^*_{i,eq,\psi} : N_t = \left[z\left(\frac{p_i}{X} - \theta A_t\right)\frac{\delta(\chi)\bar{A}_t}{r + \psi A_t}\right]^{\frac{1}{1-\beta}} \qquad \text{for } \ i = H, L$$

with $N^*_{H,eq,\psi} > N^*_{L,eq,\psi}$ and $\eta^*_{H,\psi} < \eta^*_{L,\psi}$ exactly as in the case of no investor activism. Notice also that all the other properties we had without investor activism apply also here. Therefore, one can conclude that innovation can be financed with this type of equity contract as long as $N^*_{i,eq,\psi} \geq 0$, which happens for $\theta \leq \frac{p_i}{XA_t}$. Therefore, by reducing appropriately the entrepreneur's share of profits from imitation, this modified equity contract becomes feasible also when the s.o.t. is close to the technology frontier. In this way it also resolves the low productivity growth in region V.

9 Conclusions

In this paper, we have analyzed the interaction between the initial state of technology and the financing of innovation to explain why a sector that is close to the technology frontier may have a low growth rate. In other terms, this is equivalent to understanding if there is an advantage or disadvantage of backwardness, taking the financial system and other factors as given. In order to answer this question we have focused on the link between the initial state of technology of a sector and type of contract for financing an innovation project. This is to say, studying the incentives for both the investor and the entrepreneur when a sector is at the frontier (so that it can only innovate) or inside the frontier (so it can also imitate).

The first finding of this paper is that for a sector inside the frontier there is credit rationing if innovation is financed with debt, but not with equity. In fact, in the case of financing with debt, there is always an incentive for low skilled entrepreneurs to hide their type; this creates adverse selection so that the high skilled are driven out of the innovation patent race. Instead, if innovation is financed with equity, then there is always a pooling equilibrium such that both types of entrepreneurs are financed.

Second, when these results are related to the distance to the frontier parameter, it emerges that by using a combination of debt and equity both types of entrepreneurs are financed for innovation. This happens only in sectors which are either very *distant* or *at* the frontier. Instead there is an intermediate distance region where only debt is available and due to adverse selection only low skilled entrepreneurs are financed for innovation. This implies that the growth rate in this region is much lower than it could be if also equity could be used. Equity cannot be used because the entrepreneurs pretend a too high share of profits for innovation.

It turns out that this analysis conducted insofar may contribute to explain the low growth of European countries in the last two decades. In fact, while European countries have just a small productivity gap with respect to the US (which can be considered as the world technology leader), there are still many sectors where the investment levels on innovation are low or the returns on innovation projects are lower than in the US. Moreover, it is widely known that the equity markets are much more developed in the US than in Europe.

At a more general level, it is evident that innovation and growth do not depend only on factors (like institutions and/or financial development) which can be summarized in the productivity (state of technology) level, but also on how much the world technology can be transferred to backward countries. In fact, if world's best technology may spread from leader countries to technology laggards without restrictions, the question is if countries benefit from being backward or not. While some literature has focused on productivity differences or financial development, there is no consensus about the so called "advantage of backwardness."[34] If we accept the view that laggard countries have an advantage of backwardness, then we should see long run convergence in the growth rate to frontier countries; and the reverse (divergence) if we allow for a disadvantage of backwardness. However, the empirical evidence seems quite mixed: while on one hand there are some countries catching up very quickly to the richest ones, on the other hand, some very rich countries seem to grow at different rates.[35] In particular while very backward countries

[34] In the sense that there is an advantage if "*the further a country falls behind the world's technology leaders the easier it is for that country to progress technologically simply by implementing new technologies that have been discovered elsewhere.*Aghion Howitt, Foulkes (2003), page 1. The original idea of advantage of backwardness is due to Gensckenkon (1962).

[35] See for contrasting views Barro, Sala-i-Martin (2004), chap. 8 and Howitt (2000).

like those in South East Asia have grown at very high rates, European countries have suffered very low growth rates in the last two decades.

Finally, while the contribution of this paper is to highlight at which stages of development there is a risk of low growth, the analysis conducted insofar does not suggest policy measures intended to stimulate innovation. With regard to appropriate policy measures, we leave the study of these to future research.

References

[1] Acemoglu D., Aghion P., Zilibotti F., "Distance to frontier, Selection, and economic growth", NBER WP, no. 9066, Jul 2002.

[2] Aghion, Bond, Klemm, Marinescu, I., "Technology and the financial structure: are innovative firms different?", mimeo, 2003.

[3] Aghion P., Bolton P., "An incomplete contracts approach to financial contracting" The Review of Economic Studies, vol. 59, no. 3, Jul 1992.

[4] Aghion, P., Harris, C., Howitt, P., Vickers, J., "Competition, Imitation and Growth with step-by-step Innovation", Review of Economic Studies, vol. 68, no. 3, Jul. 2001.

[5] Aghion, P., Howitt, P:, "A model of growth through creative destruction", Econometrica, vol. 60, 1992.

[6] Aghion, P., Howitt, P., "Growth with Quality improving Innovations: an integrated framework", chap. 2 in Handbook of Economic Growth, 2004, forthcoming.

[7] Aghion, P., Howitt, P., Foulkes, Meyer, D., "The effect of financial development on Convergence: Theory and Evidence", Quarterly ournal of Economics, Feb. 2005.

[8] Akerlof, M., "Lemons market: quality uncertainty and the market mechanism" Quarterly Journal of Economics, 1970.

[9] Audretsch, D.B., Lehmann, E.E., "Financing high- tech growth: the role of debt and equity" in Max Planck Institute for Research into Economic Systems, Group for Entrepreneurship, Growth and Public Policy Papers on Entrepreneurship, Growth and Public Policy, no. 19, 2004.

[10] Barro, R.J. and Sala-i-Martin, X., Economic growth, MIT Press, 2004.

[11] Bencivenga, V.R., and Smith, B.D., "Some consequences of credit rationing in an endogenous growth model", Journal of Economic Dynamics and Control, 1993, pp. 97-122.

[12] Bernanke, B. and Gertler, M., "Agency Costs, Net Worth and business fluctuations", American Economic Review, vol. 79, 1989, pp. 14-31.

[13] Besanko, D. and Thakor, A., "Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets", in International Economic Review, vol. 28, no. 3, 1987, pp.671-89.

[14] Black, B. and Gilson, R., "Venture capital and the structure of Capital markets: banks versus stock markets", Journal of Financial Economics, vol. 47, 1998, pp. 243-277.

[15] Bottazzi, L., Da Rin, M., Hellmann, T., "Active Financial Intermediation: Evidence on the Role of Organizational Specialization and Venture Capital,' RICAFE Working Paper n.12, 2004.

[16] Burkart, M., Panunzi, F., Shleifer,. A., "Family Firms", Journal of Finance, vol. 58 n. 4, , 2003, pp. 2173-2207.

[17] Casamatta, C., "Financing and advising: optimal financial contracts with venture capitalists", Journal of Finance, vol. 58, no. 5, 2003. pp. 2059 - 2086.

[18] Caselli, F., Gennaioli, N., "Dynastic Management", mimeo, Harvard Un., 2003.

[19] De la Fuente, A. and Marin, J.M., "Innovation, Bank monitoring and endogenous financial development", Journal of Monetary Economics, vol. 38, 1996, pp. 269-301.

[20] Eaton, J., Kortum, S., "Technology, Trade and Growth: a unified framework", European Economic Review: Papers and Proceedings, forthcoming.

[21] Gerschenkron, A., Economic Backwardness in Historical Perspective, Harvard University Press, Cambridge MA, 1962.

[22] Gompers P., Lerner, J., "The Venture Capital Cycle", MIT Press, 1999.

[23] Gompers, P., Lerner, J., "The Venture Capital Revolution", Journal of Economic Perspectives, vol. 15, no. 2, 2001.

[24] Gordon, R.J."Why was Europe left at the station when America's productivity locomotive departed?", NBERWP no. 10661, Aug 2004.

[25] Grossman, G.M., Helpman, E., "Innovation and Growth in the Global Economy", Cambridge, MA, MIT Press, 1991.

[26] Guiso, L., "High tech Firms and credit rationing", CEPR DP, no. 1696, Oct 1997.

[27] Hege, U., Palomino, F., Schwienbacher, A., "Determinants of Venture Capital Performance: Europe and the United States ", Ricafe Working paper, no.1, Nov 2003 (http://www.lse.ac.uk/collections/RICAFE)

[28] Hall, B.H., "The Financing of Research and Development" Oxford Review of Economic Policy , vol. 18, Jan 2002, pp. 35 – 51.

[29] Hellmann, T. Puri, M., "The Interaction between Product Market and Financing Strategy: The Role of Venture Capital," Review of Financial Studies, vol. 13 no. 4, 2000, pp. 959—84.

[30] Holmstrom, B., Tirole, J., "Financial Intermediation, Loanable Funds and the Real Sector", Quarterly Journal of Economics, vol. 112, Nov. 1997, pp. 663-691.

[31] Howitt, P., Endogenous Growth and Cross-Country Income Differences, American Economic Review, vol. 90, no. 4, 2000.

[32] Kaplan, S. and Stromberg, P., "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts", in Review of Economic Studies, no.2, vol. 70, Apr 2003.

[33] La Porta, R., F. Lopez-de-Silanes, A. Shleifer, R. Vishny, "Legal determinants of external Finance", Journal of Finance, vol. 52, 1997, pp. 1131-50.

[34] La Porta, R., F. Lopez-de-Silanes, A. Shleifer, R. Vishny, "Law and Finance", Journal of Political Economy, vol. 106, 1998, pp. 1113-55.

[35] Lerner, J., "Venture Capitalists and the Oversight of Private Firms", Journal of Finance, vol. 50 no. 1, 1995, pp. 301—18.

[36] Levine, R., "Finance and Growth: Theory, evidence, and mechanisms", NBER WP no. 10766, Sep 2004.

[37] Papageorgiou, C., "Technology adoption, Human capital and Growth theory", in Review of Development Economics, vol. 6 no. 3, 2002.

[38] Rajan, R.G., Zingales, L., "Financial dependence and Growth", American Economic Review, no. 3, Jun 1998.

[39] Stiglitz, J.E., Weiss, A., "Credit Rationing in Markets with Imperfect Information", American Economic Review, vol. 71, no. 3, Jun 1981.

[40] Zeira, J., "Innovation, Patent races and Endogenous growth", CEPR DP, no. 3974, 2003.

APPENDIX
A. Production Profits

In order to better understand the expression form of production profits π_t, consider that in order to produce k units of the intermediate v, it is necessary to employ one unit of the final good; so production profits are:

$$\pi_t(v) = P_t(v)k_t(v) - k_t(v) = [p_t(v) - 1]k_t(v)$$

where $p_t(v)$ is the price of the intermediate good. Now, since the final good is produced competitively, the price of the intermediate good can be set to a constant (χ) that is equal to the marginal product with respect to that input. Then, in the case of innovation the price is:

$$P_t(v) = \chi = \frac{\vartheta Y_t}{\vartheta k(v)_t} = \bar{A}_t(v)^{1-\alpha} k(v)_t^{\alpha-1} \tag{31}$$

while in the case of imitation it is:

$$P_t(v) = \chi = \frac{\vartheta Y_t}{\vartheta k(v)_t} = A_t(v)^{1-\alpha} k(v)_t^{\alpha-1} \tag{32}$$

So (following only the innovation case for simplicity), the optimal demand for the input is:

$$k(v)_t = \chi^{\frac{1}{\alpha-1}} \bar{A}_t(v) \tag{33}$$

and production profits are:

$$\pi_t(v) = (\chi - 1)\, \chi^{\frac{1}{\alpha-1}} \bar{A}_t(v)$$

or if we set $\delta(\chi) = (\chi - 1)\, \chi^{\frac{1}{\alpha-1}}$, they become:

$$\pi_t(I) = \delta(\chi) \bar{A}_t(v) \tag{34}$$

where the TFP here is $\bar{A}_t(v) = \gamma A_{t-1}(v)$ if innovation is successful; in the case of imitation, production profits become:

$$\pi_t(M) = \delta(\chi) \bar{A}_t(v)$$

where the TFP here is $A_t(v) = \gamma A_{t-1}(v)$.**QED**

B. Debt with entrepreneur's wealth

The debt contract for innovation with the entrepreneur participating with her own wealth is defined as follows. Let Entrepreneur's wealth be a fraction λ of the total amount of funds due for the project. Then, the participation for a debt contract inside the frontier becomes

$$zN_t^{\beta} \tfrac{p_i}{X} \left[\delta(\chi)\bar{A}_t - (1-\lambda)\, R_t N_t\right] - \lambda R_t N_t \geq zN_t^{\beta} \left[\delta(\chi) A_t - R_t N_t\right]$$

Define the following equality $S_0 z N_t^{\beta} = N_t$, where S_0 is an object that makes the two sides equal. Then the participation constraint can be rewritten as (imposing the equality):

$$zN_t^{\beta} \tfrac{p_i}{X} \left[\delta(\chi)\bar{A}_t - (1-\lambda)\, R_t N_t\right] - \lambda R_t S_0 z N_t^{\beta} = zN_t^{\beta} \left[\delta(\chi) A_t - R_t N_t\right]$$

solving this expression becomes

$$N_i^E : R_t = \frac{\delta(\chi)\bar{A}_t \left(1 - \frac{X}{p_i} a_t\right)}{\left(1 - \lambda - \frac{X}{p_i}\right) N_t + \frac{X}{p_i}\frac{\lambda}{z} N_t^{1-\beta}} \quad \text{for } i = H, L$$

It can be shown that for this expression: $\frac{\partial R_t}{\partial p_i} < 0$, so that $N_H^E < N_L^E$ and $\frac{\partial R_t}{\partial N_t} < 0$.[36]

With regard to investors, it is immediate to see that their indifference curves are exactly the same as without collateral. In fact, rewrite the indifference relation for the investor as follows

$$zN_t^{\beta}\tfrac{p_i}{X}R_t\left(1-\lambda\right)N_t-\left(1-\lambda\right)N_t=r_0\left(1-\lambda\right)N_t$$

Deleting $(1-\lambda)$ on both sides of this expression we get back exactly to the previous case described in the text. Therefore, all the analysis conducted with debt inside the frontier holds also in the case the entrepreneur participates with its own wealth. **QED**

C. Debt at the frontier

In this section, I derive the entrepreneurs indifference curve. The derivations of L-types only are shown. From the participation constraint: $E\left(\bar{\Pi}^{i}\right) \geq r_0N_t \quad$ for $i=H,L$, one can derive the indifference curve of entrepreneurs willing to undertake innovation. In fact, imposing that the constraint is binding and written in full detail (see (4)), we have:

$$zN_t^{\beta}\tfrac{p_L}{X}\left[\delta(\chi)\bar{A}_t-R_tN_t\right]=r_0N_t$$

Define the following equality $Q_0N_t^{\beta}=r_0N_t$, where Q_0 is an object that makes the two sides equal. Then the participation constraint can be rewritten as:

$$zN_t^{\beta}\tfrac{p_L}{X}\left[\delta(\chi)\bar{A}_t-R_tN_t\right]=Q_0N_t^{\beta}$$

or $\quad \delta(\chi)\bar{A}_t-R_tN_t=\frac{X}{zp_L}Q_0 \quad \Longrightarrow \quad R_t=\frac{\delta(\chi)\bar{A}_t}{N_t}-\frac{X}{zp_L}\frac{Q_0}{N_t}$

but since $Q_0=r_0N_t^{1-\beta}$, we have

$$R_t=\frac{\delta(\chi)\bar{A}_t}{N_t}-\frac{X}{zp_L}\frac{r_0}{N_t^{\beta}}$$

which is exactly (15). **QED**

D. Growth Rates

In order to determine if $g_2 < g_1$, consider that in region IV innovation is undertaken respectively by L-types with debt and by H-types with equity. Then, since both undertake innovation, the amount of funds used by each type in this region is given by the sector expected growth rate is

[36] Note that if the derivative is positive we have that innovation inside the frontier financed with debt is only undertaken by H-types, so maximizing the expected growth rate. This happens for $\lambda > \frac{1-\frac{X}{p_i}}{1-\frac{(1-\beta)}{zN_t^{\beta}\frac{p_i}{X}}}$ (where a sufficient condition for this inequality to hold is that $(1-\beta) < \frac{p_i}{X}zN_t^{\beta}$), which is consistent with the view that the project value is maximized if the entrepreneur participates with at least a fraction of her own wealth, like for example Holmstrom and Tirole (1997).

$$g_1 = z\overset{\frown}{N}_t^{\beta}\overset{\frown}{p}\left[\frac{\gamma}{a_{t-1}} - 1\right]$$

which is a combination of (11) and (21) appropriately modified (replace X to L or H):

$$\overset{\frown}{N}_{d,L} = \left[\frac{1 - \frac{X}{\overset{\frown}{p}}a_t}{1 - \frac{X}{\overset{\frown}{p}}}\frac{z}{R_0}\overset{\frown}{p}\delta(\chi)\bar{A}_t\right]^{\frac{1}{1-\beta}}$$

On the other hand, in region V only L-types undertake innovation, therefore the expected growth rate is:

$$g_2 = zN_t^{\beta}\frac{p_L}{L}\left[\frac{\gamma}{a_{t-1}} - 1\right]$$

where the amount of funds is given by:

$$N_{d,L}^{*\prime} = \left[\frac{1 - \frac{L}{p_L}a_t}{1 - \frac{L}{p_L}}\frac{z}{R_0}\frac{p_L}{X}\delta(\chi)\bar{A}_t\right]^{\frac{1}{1-\beta}}$$

In order to determine the change of the productivity growth rate between region IV and region V, it is sufficient to compare the amount of funds for each case. That is to say that the ratio $\frac{\overset{\frown}{N}_{d,L}}{N_{d,L}^{*\prime}}$ must be greater than one. In fact, the ratio is:

$$\frac{\overset{\frown}{N}_{d,L}}{N_{d,L}^{*\prime}} = \frac{\frac{\overset{\frown}{p}-aX}{\overset{\frown}{p}-X}\overset{\frown}{p}}{\frac{p_L-aL}{p_L-L}\frac{p_L}{L}} \Longrightarrow \frac{\overset{\frown}{N}_{d,L}}{N_{d,L}^{*\prime}} = \frac{\overset{\frown}{p}-aX}{\overset{\frown}{p}-X}\frac{p_L-L}{p_L-aL}\frac{\overset{\frown}{p}}{p_L}L$$

Since each ratio is greater than one, $g_1 > g_2$. **QED**

Figure 1: Timing Line:

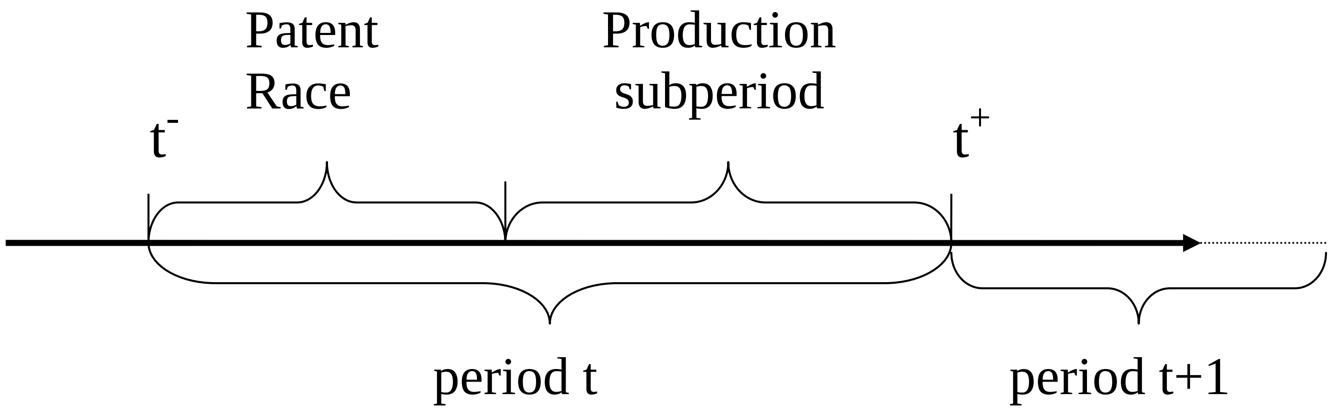
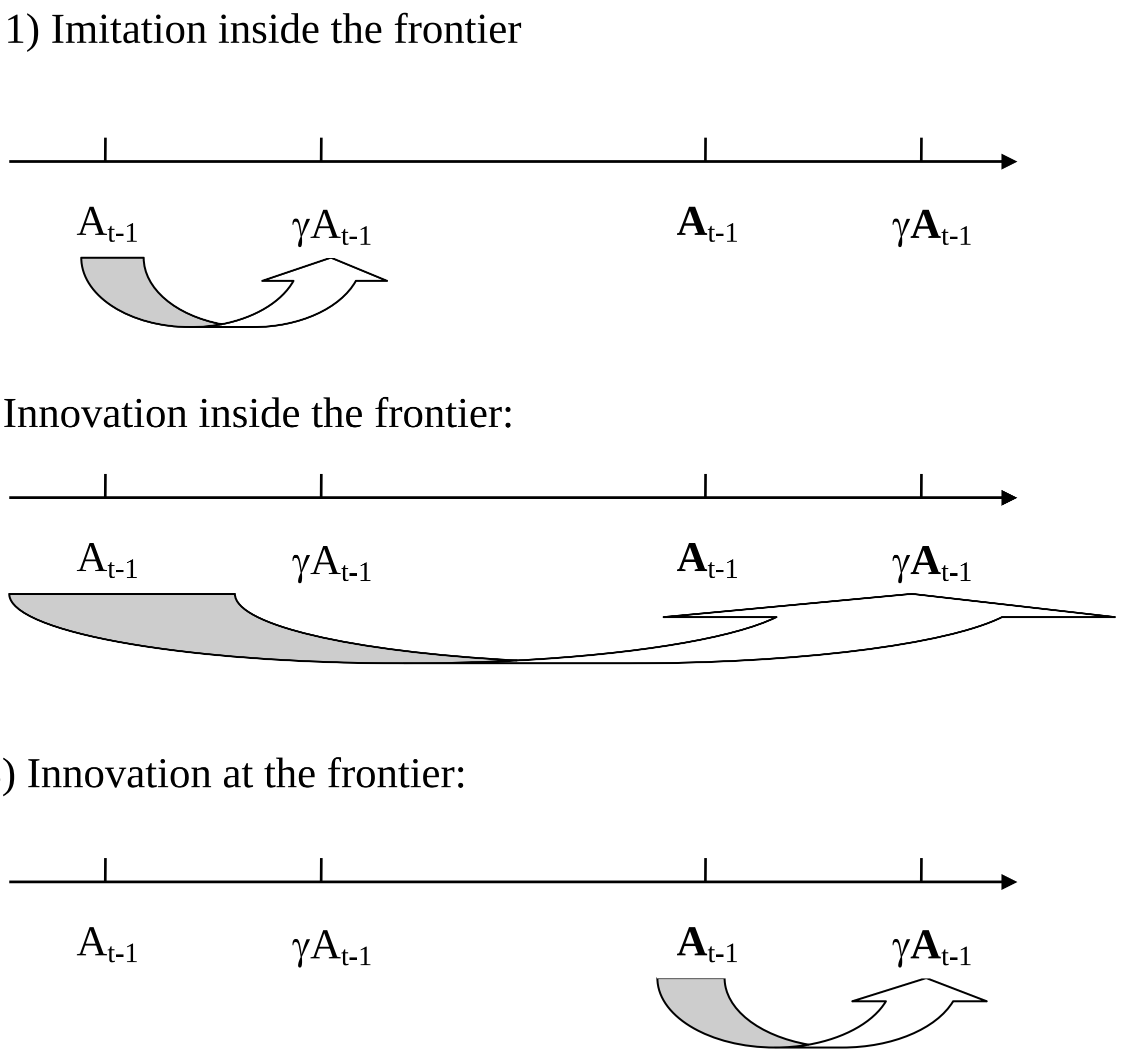


Figure 2: Technology Improvement:



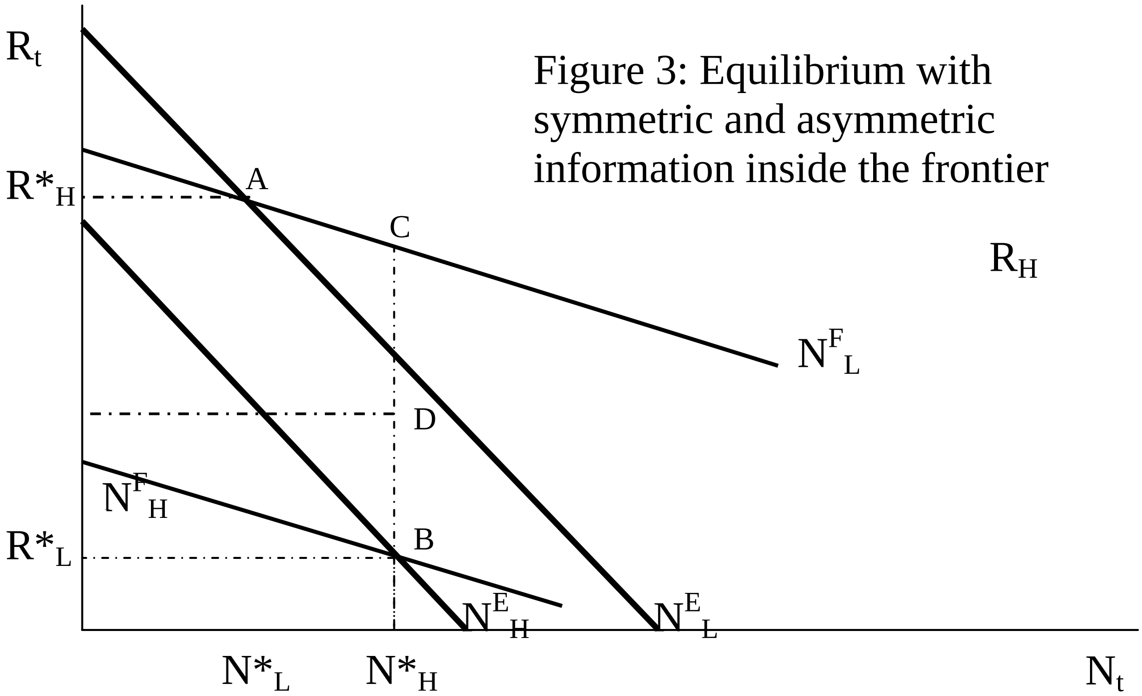


Figure 3: Equilibrium with symmetric and asymmetric information inside the frontier

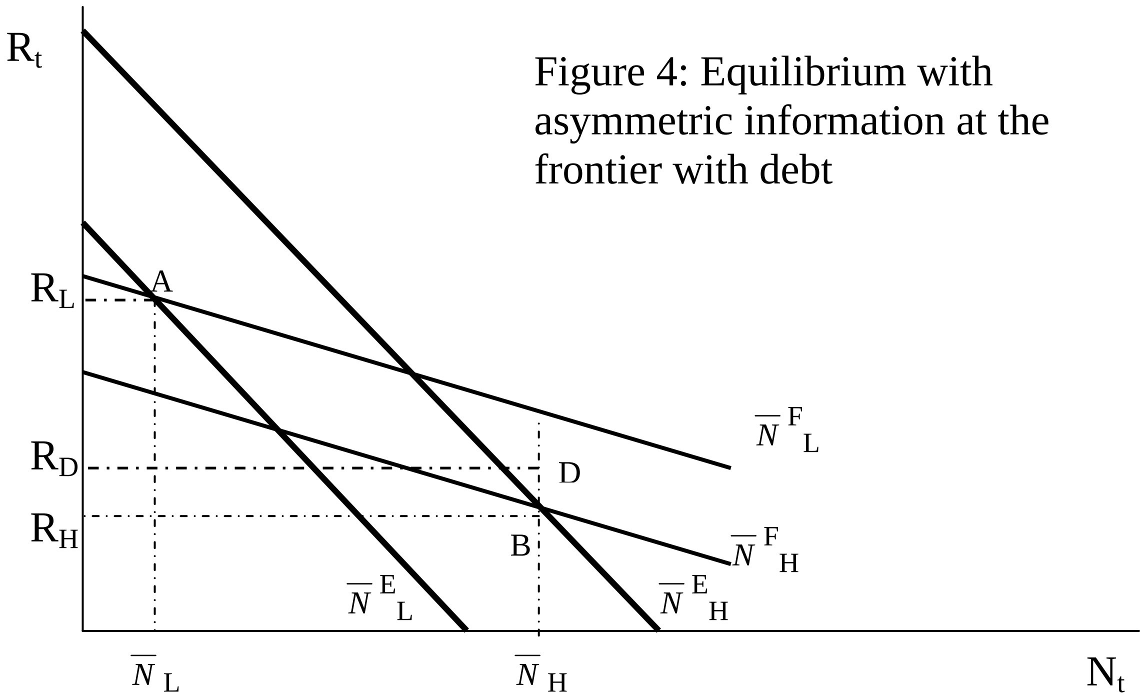


Figure 4: Equilibrium with asymmetric information at the frontier with debt

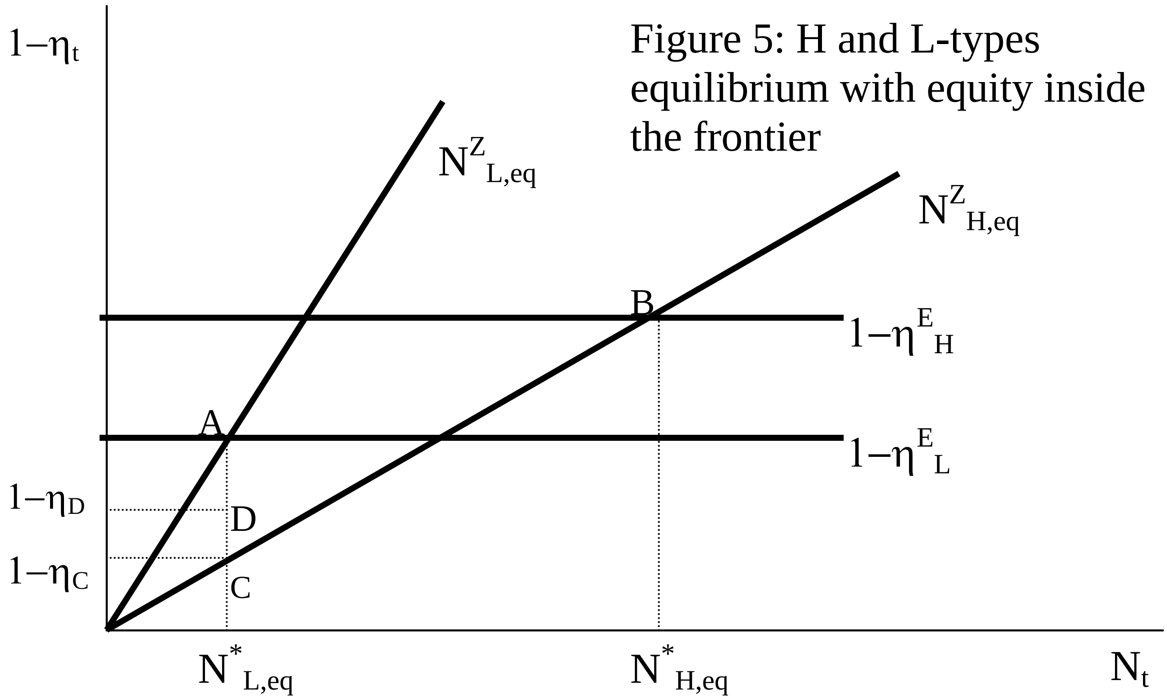


Figure 5: H and L-types equilibrium with equity inside the frontier

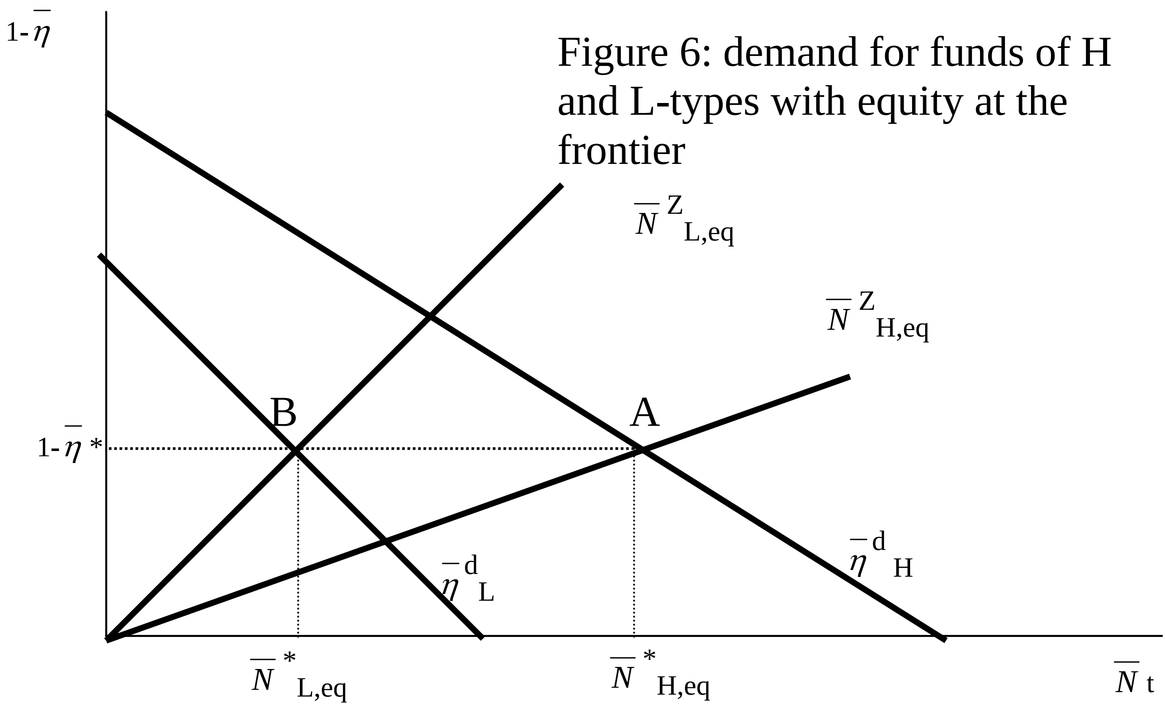


Figure 6: demand for funds of H and L-types with equity at the frontier

Figure 7: regions of zero values of debt and equity

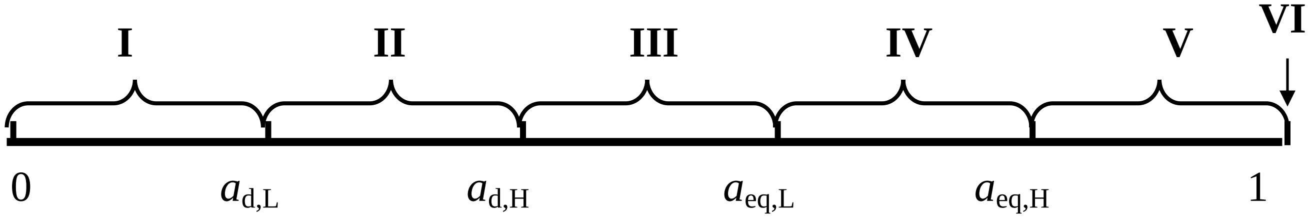


Figure 8: regions of zero values of debt and equity

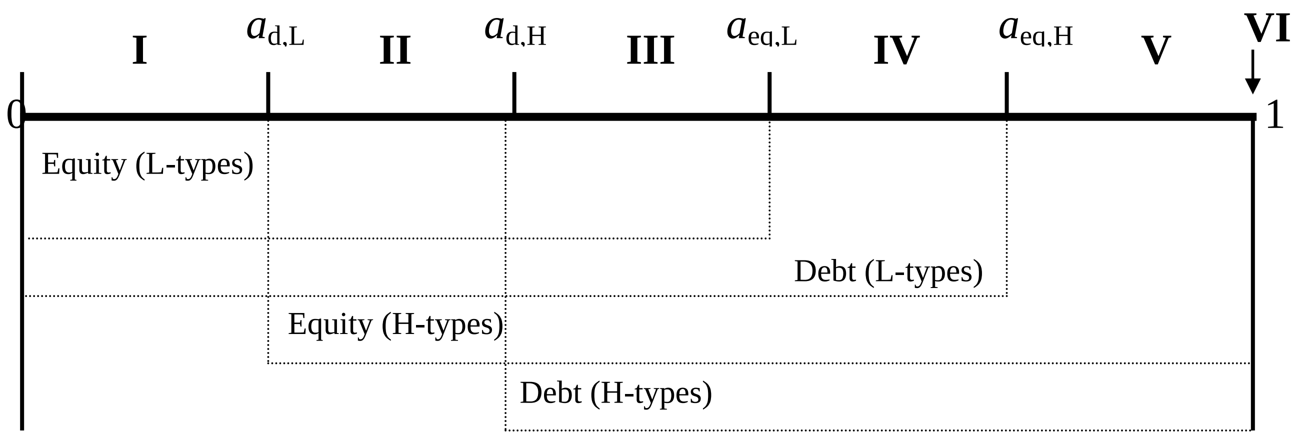


Figure 9: regions of zero values of debt and equity

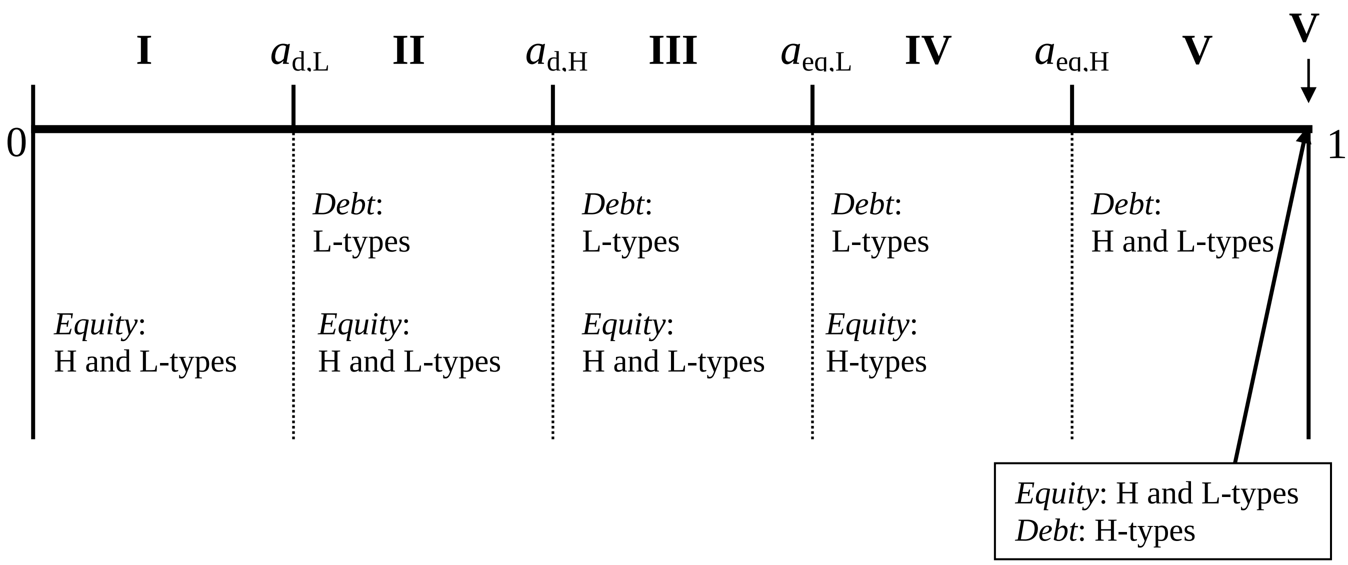


Figure 10: regions of debt and equity

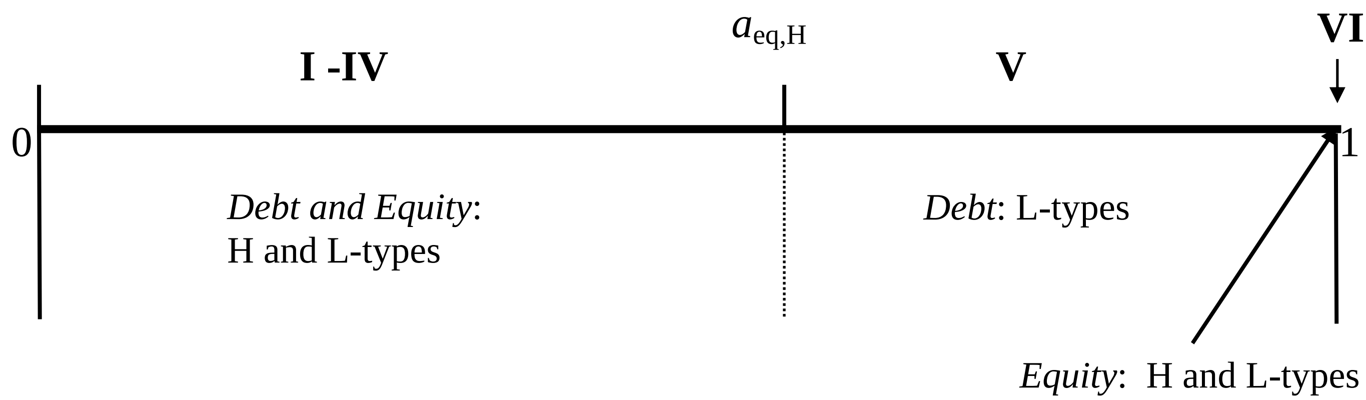


Figure 11: growth rate along the distance to the frontier