Job Design

and the

Bene¯ts of Private Trade

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Abstract

We reconsider the job design theory of Holmstrom and Milgrom (1991). They maintain that agents have limited attention and consequently tend to get distracted when they perform multiple activities. An agent should only be allowed to pursue outside activities on company time when she is ¯nancially responsible for the principal's output, they conclude. We o®er an alternative model of the job design problem in which it may be optimal to permit outside activities despite weak incentives and costly monitoring. When ¯rms provide employees with a means for acquiring a reputation for high ability, inside and outside activities can become strategic complements and increase overall incentives. Similarly, when employment implies access to corporate resources that employees can utilize for personal bene¯t, the employee increases e®ort in the inside activities in order to retain access to those assets. We show that these synergies obtain in the employment of U.S. faculty members and in the employment of agents in the English East India Company, a historically important ¯rm.

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1 Introduction

The study of multitask principal-agent models owes a great deal to the seminal work of Holmstrom and Milgrom (1991, 1994). By addressing the allocation of agents' e®ort across a range of tasks they expand the standard principal-agent model to analyze a wide array of contractual instruments including incentive pay, asset ownership, and job design. In the area of job design their central ¯nding can be stated simply: \it is optimal to give the agent more freedom to pursue personal business when he is ¯nancially more responsible for his performance" (1991, p. 41). The result is intuitive and rests on the idea that agents have limited attention. When outside (private) activities are allowed and the agent does not have strong incentives to attend to inside (company) activities, he will redirect his attention away from the principal's business and toward his own. The principal must balance the trade-o® between the agent's lower e®ort and the lower wages required of an agent earning private compensation on company time. Holmstrom and Milgrom (HM thereafter) emphasize that the weaker the incentives in place, that is the less responsive the agent's wage is to changes in ¯rm pro¯ts, the greater this reduction in e®ort is. Hence allowing outside activities becomes optimal only when the agent is ¯nancially responsible for inside activities. The absence of performance incentives (which frequently occurs where good performance measures are missing or where monitoring costs are prohibitive) renders outside activities ine±cient.

In contrast, we provide a theoretical discussion of the conditions under which these private activities appear to be desirable even when strong performance incentives cannot be provided. Our basic model involves two-periods and output being produced not only with e®ort (as in HM), but also with ability. This set-up follows in the spirit of Holmstrom's career concern model (Holmstrom, 1982) which emphasizes the role of uncertainty about ability in inducing e®ort even in the absence of explicit incentives. We assume that output is observable but non-veri¯able - implying that contracts can only specify an uncontingent wage and whether outside activities are allowed. Despite not having explicit incentives, future wages will re°ect expectations about ability. The agent will then exert e®ort in the ¯rst period to try to convince the principal and the outside market that she has high ability. In this framework the ¯rm serves as a means by which agents can cultivate their reputation for high ability.

Our basic model assumes that outside activities require an initial investment in period one and yield a pay o® in the second period. We show that when ability is known ex-ante or outside activities do not depend on ability, the two tasks are substitutes and the results in HM obtain. In contrast, the same is not true when ability is uncertain ex-ante and is an input into both inside and outside production.

In that case, the agent has an incentive to increase her e®ort in inside activities when she is allowed to bene¯t from outside activities. Not only will the agent receive the reward of a higher wage from the principal, but returns from the outside market increases when inside production increases, re°ecting the higher expectations about ability. Allowing more freedom to the agent to exploit the returns from a reputation for high ability increases the incentives to acquire that reputation in the ¯rst place. We show that the higher the dependence of the outside activity on ability, the stronger the synergies between the two tasks.

We also explore the case where outside activities require e®ort each period instead of an investment. Since output outside is being produced also in period one, this opens up the possibility of learning about ability from the employee's outside activities, as well. In this case, when the two activities are not technologically linked, allowing agents to trade privately can either increase or decrease e®ort in inside activities. On one hand, the returns to acquiring a reputation increase, just as before. But now, as a result of the additional signal about ability coming from outside, the learning process will not give as much attention to inside production. E®ort inside will then have a smaller impact on reputation. We show, however, that e®ort will be higher (1) the higher the precision of the signal about ability from inside activities and (2) the more outside activities depend on ability. The introduction of learning to the basic model ensures that the comparative statics derived from the basic model are robust and are not merely by-products of the way in which investment connects inside and outside activities.

The model as it is speaks at the role of individual reputation in creating synergies between these two activities. But such reputation e®ects can also arise from the observation of the employment conditions of the agent (whether the principal decides to keep the agent or dispense of her services), rather than individual production. To formalize this idea, we extend the model to allow for the possibility of outsiders not being able to observe the principal's output. When inside production is non-observable, the outside principal faces an asymmetric information problem - reducing the extent to which he will be able to judge ability. In this setting, the inside principal can still signal agent ability by dismissing poor performers. The ability to dismiss thus compensates for the impact of asymmetric information on incentives. Agents will exert e®ort in inside activities in order to reduce the probability of being dismissed. Job retention signals high ability to the outside market and increases the expected payo®s from private trading. Despite asymmetric information, outside activities can again increase inside incentives and lowers the cost of implementing a given level of e®ort.

While we have thus far considered reputational complementarities between inside and outside activities, we want to emphasize that other types of synergies can also prevail which have similar incentive effects. We explore the implications of an agent placing employer-owned assets in the service of outside activities. By "assets" we have in mind those that involve fixed costs and over which employees enjoy substantial residual rights of control. The agent's private use of such company resources, a section of office space for example, does not impose a cost on the employer and does not adversely affect the principal's production level. To the extent that the principal is aware of the private use of company assets, she may actually want to sanction it. By allowing employees to use company assets for private trade, the principal can lower the wages paid and also induce higher effort in inside activities. As before, effort in the inside activities is ensured by the threat of dismissal. We show that when these assets are valuable, the agent will strive for high output in inside activities so as to lower the chances of dismissal and retain access to company assets. This final variation of the basic model underscores the fact that the synergies between private and company transactions can occur in several ways. Reputational and resource complementarities are two such mechanisms. The synergies we describe, however, may in fact arise even when the different tasks are technologically independent.

Our model extends HM's analysis of the job design problem. They emphasize the role of an agent's limited attention in creating a conflict of interest when outside activities are allowed. Their multitasking model has been supported in recent empirical investigations (on the effect of outside activities on contract choice see Slade, 1996; on multitasking and incentives see Cockburn et.al., 1999; on multitasking and the theory of the firm, Baker and Hubbard, 2002). However, we believe that, by explicitly considering the possibility of complementarities between inside and outside tasks, a wider range of employment settings can be explained. Furthermore, HM argue that outside activities should only be allowed when the agent faces good incentives. Indeed, this principle applies more generally: discretion should only be granted when there are strong incentives in place. Our contribution is to point out that discretion might indeed be part of the incentive scheme.

Other authors have studied the problem of job design. Lindbeck and Snower (2000) focus on technological complementarities: intertask learning-by-doing, and intratask learning. Meyer, Olsen and Torsvik (1996) and Olsen and Torsvik (2000) study the interplay between implicit and explicit incentives. The latter focuses on the restrictions on outside activities, and argue that the presence of implicit incentives can reverse the comparative statics in HM. They offer a model where implicit and explicit incentives move in opposite directions, and the former dominate. As a result, when explicit incentives decrease causes total incentives to increase, making it optimal to allow more outside activities.

Recently, Dewatripont et.al. (1999b) have studied a multitask career concern model similar to ours. They explore incentives for government agencies and the effects of the scope of their missions (the number of activities under their responsibility). Consequently they focus upon a model in which all activities are symmetric and only total e®ort matters. How that e®ort is allocated among each activity does not ¯gure prominently in their analysis in contrast to ours. Moreover, future payo®s of the agent (and hence incentives) equal expected ability. As a result, the number of activities does not have a direct e®ect on compensation.[1] In our framework, we depart from both these assumptions, and consider asymmetric activities, with the rewards of the agent being dependent upon the restrictions on private activities. It is precisely the di®erence in how the agent gets compensated in the second period that explains the di®erent predictions we obtain: while Dewatripont et.al. (1999b) show that e®ort is decreasing in the number of activities, our results suggest e®ort might indeed increase when the agent's payo® re°ects her increased responsibilities.

To illustrate the ideas of the model, we discuss in detail two examples. First we look at the familiar example of faculty employment in U.S. higher education. Faculty face a °at wage-incentive schedule over the course of their careers and yet it is common practice to allow professors to complement their scholarship and institutional duties with a wide range of activities outside of the university: consulting to industry and government, public speaking, etc. As our model suggests, to the extent that a strong scholarly reputation opens the opportunity to bene¯t from these outside activities, faculty will not neglect their research output. This complementarity between inside and outside tasks is explicitly discussed in faculty handbooks and in the policy statements of the American Association of University Professors. We also present some evidence consistent with our model from the faculty subsample of the Carnegie Survey of Higher Education 1969. Those faculty members with a stronger academic record (measured by past publications), and those who spend more time doing research earn considerable higher returns on outside activities, even controlling for ability.

Our second illustration, an important historical case, likewise does not accord with the HM result. The East India Company (1600-1858) pioneered English commercial exchange with Asia. It played a pivotal role in the development of the joint-stock, limited liability corporation and in the history of corporate ¯nance.[2] Anderson et. al. (1983) and Carlos and Nicholas (1988) have argued that in the seventeenth and eighteenth century the East India Company was an organizational innovation on par with a modern multinational ¯rm such as General Motors. The Company stationed its employees (called servants) in Asian cities to purchase pepper, textiles, tea, and other commodities for resale in London.

[1]There is an indirect e®ect, since the number of activities a®ects the process of learning about ability.

[2]See for example Harris (2000) Industrializing English Law, Alborn (1999) Conceiving Companies, and Baskin and Miranti (1996) History of Corporate Finance.

Based on our own archival investigations at the India Office Library (London), we have a constructed a database that tracks the careers of every agent sent to Bengal, India in the eighteenth century. The data enable us to comprehensively describe the employment relationship linking Company directors and their overseas servants.

We find that the Company offered servants, separated by more than seven months by sea, low wages and a flat wage structure. The inability to closely monitor the activities of the servants and enforce strong incentives would render, according to HM, any concession on outside activities inefficient. Yet we observe that the Company allowed and actively encouraged the servants to conduct their own trades. The correspondence between the Company director and agents show that the directors recognized and supported the resource complementarities between public and private trade as described by our model. Agents exerted effort in Company trade in order to avoid dismissal, ensure access to firm assets, and increase the returns to private trade.

The rest of the paper is organized as follows. Section 2 provides a formal articulation of the job design problem. We explore the optimality of allowing private trade and then study how our results vary under different assumptions. In section 3 we discuss the basic model in the context of faculty employment in U.S. higher education. Section 4 describes the employment relationship in the English East India Company and how we account for the observed contract design. Section 5 concludes.

2 Job Design: A Formal Account

In this section we provide a formal examination of the job design problem. We model the conditions that would make allowing outside activities, such as private trade, desirable from a job design perspective. We start with the setup of the model, and a simple case that contains the main intuition of the paper. We then extend the model and relax the assumptions of the basic framework.

2.1 The Setup

A risk neutral principal hires a risk neutral agent for two periods, $t = 1, 2$, to conduct public trade.[3] In period t, output will be produced with the following technology: $y_t = e_t + \eta_t + \varepsilon_t$. Output uses the agent's effort and ability, represented by e_t and η_t, respectively, as the basic inputs. But there is an additional random component, ε_t. Ability is uncertain, but both principal and agent share the same (symmetric) information. The prior distribution of ability is normally distributed with mean μ

[3] To avoid any confusion, we will refer to the principal as “he,” and to the agent as “she.”

and variance σ_η^2, i.e. $\eta_t \sim N\left(\bar{\eta}; \sigma_\eta^2\right)$. The noise terms are assumed to be independent and normally distributed, with 0 mean and variance σ_ε^2, i.e. $\varepsilon_t \sim N\left(0; \sigma_\varepsilon^2\right)$. We will denote the precision (the inverse of the variance) of each random variable by h_η and h_ε respectively. The density of a random variable having a normal distribution with zero mean and variance of 1 will be represented by $\phi(\cdot)$, and its corresponding distribution by $\Phi(\cdot)$.

Output is assumed to be observable but non-verifiable. Hence, no explicit incentive scheme will be enforceable. We assume throughout the paper that there is perfect competition. As a result, the agent's wage will reflect her expected productivity. Effort will affect the principal's and outside market's perception of the agent's ability. And consequently effort will also affect her period two compensation from both the principal and outside market. Incentives will then come from career concerns.

Besides the involvement in public (inside) activities, the principal might allow the agent to conduct private (outside) trade. Such activity requires an initial investment (at time $t = 1$), denoted by i, which is observable. And at period 2 it produces a value of $v(i; \eta_2)$. The function $v(i; \eta_2)$ is assumed to be bounded, increasing in both arguments and concave in i. To guarantee an interior solution we assume the derivative with respect to i satisfies $v_i(i; \eta) \to +\infty$ as $i \to 0$. Since ability is uncertain the actual return for the agent from the outside activity is $E[v(i; \eta_2) \mid \text{outsider's information}]$. We implicitly assume here that the agent obtains her rents from outside activities through another agency relationship. The outside principal infers ability from the information available to him and compensates the agent accordingly.

Since output is non-verifiable, the agent will not face any incentive in the final period. It then follows that the optimal choice of effort at time $t = 2$, e_2^*, will be a constant. Consequently, we do not need to consider any explicit cost function for period 2. We will also drop the time subindex for e_1, since only first period effort will be relevant. Total costs for the agent at $t = 1$ will then be $c(e; i)$, which is assumed to be convex in both variables. We will look at two cases. Case 1: When $c(e; i) = c(e + i)$, with $c(\cdot)$ being convex, the agent has limited attention (increasing effort in one task increases the marginal cost of the other task). Following Holmstrom and Milgrom (1991), the agent will work to some extent even in the absence of incentives. For that to be true, we assume the cost function is decreasing up to some $\bar{k} > 0$, i.e. $c'(k) \le 0$, for all $k \le \bar{k}$. Case 2: With a separable cost function, $c(e; i) = c_e(e) + c_i(i)$, the two activities are unrelated.[4] We assume both $c_e(e)$ and $c_i(i)$ are convex, and marginal costs are non-positive at zero: $c_e'(0) \le 0; c_i'(0) \le 0$.

[4] In this case, whether we assume the cost functions are initially decreasing or not does not alter the our results.

The principal has two instruments to provide incentives to the agent. He can set a wage for each period, $\{w_1; w_2\}$. And he can also design the restrictions imposed on the job, which in this case corresponds to whether the agent is allowed to trade freely on her account. Let $\sigma \in \{0; 1\} = 1$ indicate that the agent has been granted that privilege.

To sum up, we outline the timing of the model. The principal offers the agent a contract and, upon acceptance, the agent is hired. The agent works during the first period for a fixed wage (paid ex-ante), and invests in outside activities if they were permitted. Output is then realized. The principal forms an expectation of the effort level of the agent, and together with the realization of output he infers a level of ability. In equilibrium, such conjecture will equal the true level of effort chosen by the agent (which in turn will depend on the effort level expected by the principal). In the second period, the agent may be rehired and paid a fixed wage depending on the reputation she acquired during the first period. The second period employment and compensation conditions, however, will depend on the assumptions regarding the observability of output by outsiders and whether or not contracts are renegotiated after the first period. These possibilities are explored in the following sections.

2.2 Observable Output

We start with the case of output being observable by the outside market. Because of perfect competition, the agent is paid her expected productivity. This case corresponds to the career concerns model analyzed by Holmstrom (1982). Given the principal's belief about effort, denoted by $\hat{e}$, the agent then receives wages $w_1(\hat{e}) = E[y_1 \mid \hat{e}]$ and $w_2(y_1; \hat{e}) = E[y_2 \mid y_1; \hat{e}]$, where the expectation is taken with respect to the posterior distribution of η and ε_2.[5]

We start with the conditions under which HM's assertion that outside activities crowd out effort inside hold. The agent solves the following problem:

$$\max_{e;i} \{w_1(\hat{e}) - c(e+i) + E_{\varepsilon}[w_2(y_1; \hat{e}) + \sigma \cdot E[v(i; \eta) \mid y_1; \hat{e}]]\} \quad (1)$$

where $E_{\varepsilon}[\cdot]$ denotes the expectation with respect to ε_1.[6]

In the first period, the agent receives a fixed wage and decides the levels of effort and investment. Output is realized and the second period begins. She then receives the second period wage (contingent on first period output), and, if applicable, the returns from the investment in the outside activity.[7]

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[5] Throughout the rest of the paper, $E[\cdot]$ will denote the expectations operator with respect to η and ε_2.

[6] Notice that from an ex-ante perspective, the second period wage is uncertain, since it depends on the realization of ε_1.

[7] We have assumed that principal and outside market share the same conjecture about the effort level, $\hat{e}$. This is without

Effort affects expected second period returns through a higher probability of realizing a high output, and hence inducing high expectations of ability. This yields the following first order conditions with respect to e and i respectively:

$$E\left[\frac{\partial w_2(y_1;\hat{e})}{\partial y_1}\cdot\frac{\partial y_1}{\partial e}+\sigma\cdot\frac{\partial E[v(i;\eta)\mid y_1;\hat{e}]}{\partial y_1}\cdot\frac{\partial y_1}{\partial e}\right]\le c'(e+i^*)\quad\text{with } e^*=0 \text{ if inequality strict}$$

$$E\left[\sigma\cdot\frac{\partial E[v(i;\eta)\mid y_1;\hat{e}]}{\partial i}\right]\le c'(e^*+i)\quad\text{with } i^*=0 \text{ if inequality strict}$$

This condition simply states that marginal return should equal marginal cost in an interior solution. A higher effort level will increase the odds of realizing a high output, and with it, the chances of acquiring a reputation for high ability. The incentives to do so will come from the effects of an increase in first period output on future compensation, which is given by the expression in brackets in the first equation.

We begin by studying the cases where ability does not play a big role. Suppose η_1 and η_2 are independent realizations of the previous distribution, or alternatively, they are known ex-ante. In either case, the first period realization of output provides no additional information about second period ability. Then, both $E[y_2 \mid y_1;\hat{e}]$ and $E[v(i;\eta)\mid y_1;\hat{e}]$ are independent of y_1. The first order condition when no outside activity is allowed is then $c'(e)=0$, and the optimal effort level is $\overline{k}$. However, when the outside activity is allowed, $i^*>0$. Therefore, the condition with respect to e becomes $c'(e+i^*)>0$, and no effort will be put into public trade.

Similarly, when returns from the outside activity are independent of ability (i.e., $v_\eta(i;\eta)=0$) we can write $v(i)$. Then, $E[v(i;\eta)\mid y_1;\hat{e}]=v(i)$ is independent of y_1. The marginal return to effort becomes $\frac{\partial w_2(y_1;\hat{e})}{\partial y_1}\cdot\frac{\partial y_1}{\partial e}$ regardless of whether outside activities are allowed or not. However, when outside activities are allowed the agent will choose a positive investment level and thereby increase marginal costs. This results in the outside activity reducing effort inside. We summarize this in the following proposition:

Proposition 1 Suppose the agent has limited attention. Then, for a fixed belief about effort $\hat{e}$, allowing the outside activity always substitutes effort away from the inside activity in the following cases:

i. ability is known

ii. ability is independent across periods (i.e., $\eta_1;\eta_2$ i.i.d.)

iii. outside activity independent of ability (i.e., $v_\eta(i;\eta)=0$)

loss of generality, since that will certainly be the case in equilibrium.

This proposition parallels HM's multitasking analysis of outside activities. Namely, since the agent has limited attention, increasing the number of tasks she performs outside decreases effort in the tasks that benefit the principal. Further more, $\frac{\partial y_1}{\partial e} = 1$ and $\frac{\partial w_2(y_1;\hat{e})}{\partial y_1}$ will be constant when $\eta_1 = \eta_2 = \eta$, as we will shortly see. Then, when $\frac{\partial E[v(i;\eta)|y_1;\hat{e}]}{\partial y_1} = 0$, the previous first order conditions reduce to the same ones HM find for their static problem. Consequently, analogous results would apply to this framework: as the career concerns incentives get stronger, outside activities become more desirable.

Persistent ability

We now turn to the case where ability plays a more central role. We will assume $\eta_1 = \eta_2 = \eta$, but uncertain ex-ante.[8] Both principal and agent will learn about ability from the first period output. So will outsiders, since they can also observe output. We also ease the restriction that agents have limited attention by using a separable cost function, to isolate the positive effects we emphasize.

The learning process that arises from our assumptions is well known. If we let the conjecture the principal has about effort be $\hat{e}$, the prior about ability will be updated with the signal $y_1 - \hat{e} = \eta + \varepsilon_1$, which is normally distributed with mean η and variance σ_ε^2. The posterior will also be normal, with mean $m = E[\eta \mid y_1;\hat{e}]$ given by:

$$m = \alpha\bar{\eta} + (1 - \alpha)(y_1 - \hat{e}), \text{ where } \alpha = \frac{h_\eta}{h_\eta + h_\varepsilon}$$

The precision of the posterior distribution equals the sum of the precisions of the prior and the signal, $h_\eta + h_\varepsilon$.

The agent still solves the same problem:

$$\max_{e;i} \{w_1(\hat{e}) - c_e(e) - c_i(i) + E_\varepsilon[w_2(y_1;\hat{e}) + \sigma \cdot E[v(i;\eta) \mid y_1;\hat{e}]]\} \tag{2}$$

but with a separable cost function. Denote the solution to this problem by $(e^*(\sigma;\hat{e}); i^*(\sigma;\hat{e}))$.

Expectations now reflect the learning about ability. The second period wage is $w_2(y_1;\hat{e}) = E[y_2 \mid y_1;\hat{e}] = \alpha\bar{\eta} + (1 - \alpha)(y_1 - \hat{e})$. Moreover, compensation from the outside activity is

$$E[v(i;\eta) \mid y_1;\hat{e}] = \int_{-\infty}^{+\infty} v(i;\eta) \cdot \sqrt{h_\eta + h_\varepsilon} \cdot \phi((h_\eta + h_\varepsilon)(\eta - m)) \cdot d\eta$$

Introducing uncertain but persistent ability creates incentives for the agent to work hard to persuade

[8] The results would also obtain when $\eta_1 \neq \eta_2$, to the extent there is some persistence in the process that generates η_2. In other words, learning about η_1 provides information about η_2.

the market she is skillful, since that results in higher future wages. The stronger the career concerns, the stronger the incentives. Furthermore, in this case outside activities can become complementary to and increase effort in inside activities, as the next proposition shows:

Proposition 2 Assume a separable cost function $c(e;i) = c_e(e) + c_i(i)$. Then, for a given belief about effort, $\hat{e}$, the following hold:

i. If $v_\eta(i;\eta) = 0$, outside and inside activities are independent (i.e., e is not affected by outside activities)

ii. If $v_\eta(i;\eta) > 0$, the outside activity is a complement to the inside activity (i.e., the outside activity increases e)

iii. If $v_{i\eta}(i;\eta) > 0$, e and i are strategic complements.

Proof. The first order conditions to the new problem are:

$$E_\varepsilon\left[\frac{\partial w_2(y_1;\hat{e})}{\partial y_1}\cdot\frac{\partial y_1}{\partial e} + \sigma\cdot\frac{\partial E[v(i;\eta)\mid y_1;\hat{e}]}{\partial y_1}\cdot\frac{\partial y_1}{\partial e}\right] = c_e^0(e) \quad (3)$$

$$E_\varepsilon\left[\sigma\cdot\frac{\partial E[v(i;\eta)\mid y_1;\hat{e}]}{\partial i}\right] = c_i^0(i) \quad (4)$$

From the production function, it is clear that $\frac{\partial y_1}{\partial e} = 1$. Furthermore, we can calculate the effect of an increase in first period output on outside activities by:

$$\begin{aligned}\frac{\partial E[v(i;\eta)\mid y_1;\hat{e}]}{\partial y_1} &= \int_{-\infty}^{+\infty} v(i;\eta)\cdot\sqrt{h_\eta+h_\varepsilon}\cdot\frac{\partial\phi((h_\eta+h_\varepsilon)(\eta-m))}{\partial y_1}\cdot d\eta \\ &= -(1-\alpha)(h_\eta+h_\varepsilon)^{3/2}\int_{-\infty}^{+\infty} v(i;\eta)\cdot\phi^0((h_\eta+h_\varepsilon)(\eta-m))\cdot d\eta\end{aligned}$$

If we integrate by parts, we obtain:

$$\begin{aligned}\frac{\partial E[v(i;\eta)\mid y_1;\hat{e}]}{\partial y_1} &= -(1-\alpha)(h_\eta+h_\varepsilon)^{1/3}\left[\begin{array}{l} v(i;\eta)\cdot\phi((h_\eta+h_\varepsilon)(\eta-m))\,|_{-\infty}^{+\infty} - \\ -\int_{-\infty}^{+\infty} v_\eta(i;\eta)\cdot\phi((h_\eta+h_\varepsilon)(\eta-m))\cdot d\eta\end{array}\right] \\ &= (1-\alpha)\int_{-\infty}^{+\infty} v_\eta(i;\eta)\cdot\sqrt{h_\eta+h_\varepsilon}\cdot\phi((h_\eta+h_\varepsilon)(\eta-m))\cdot d\eta\end{aligned}$$

where the last step follows from the assumption that $v(\cdot)$ is bounded and hence $v(i;+\infty)\cdot\phi(+\infty) = v(i;-\infty)\cdot\phi(-\infty) = 0$.

As a result, when $v_\eta(i;\eta) = 0$ we have $\frac{\partial E[v(i;\eta)|y_1;\hat{e}]}{\partial y_1} = 0$. Part (i) of the proposition then follows, since the marginal return and marginal cost of effort e are independent of σ.

When $v_\eta(i;\eta) > 0$, the payoff from outside activities is increasing in first period output, $\frac{\partial E[v(i;\eta)|y_1;\hat{e}]}{\partial y_1} > 0$. Now the marginal return on effort rises when $\sigma = 1$, but the marginal cost remains unchanged. Consequently, effort must increase and we get part (ii).

Finally, when $v_{i\eta}(i;\eta) > 0$ the strategic complementarity follows from the supermodularity of the agent's objective function (when $\sigma = 1$):

$$\begin{aligned} \frac{\partial U}{\partial e \partial i} &= \frac{\partial}{\partial i}\left(\frac{\partial E\left[v(i;\eta) \mid y_1;\hat{e}\right]}{\partial y_1} \cdot \frac{\partial y_1}{\partial e}\right) = \\ &= (1-\alpha)\int_{-\infty}^{+\infty} v_{i\eta}(i;\eta) \cdot \sqrt{h_\eta + h_\varepsilon} \cdot \phi\left((h_\eta + h_\varepsilon)(\eta - m)\right) \cdot d\eta > 0 \end{aligned}$$

This concludes the proof. ■

This proposition expresses the main result of the paper: when there is uncertainty about ability, outside activities can come to complement incentives for effort in inside activities. As long as the rents the agent obtains from outside activities depend on her ability, allowing them provides a stronger incentive to acquire a reputation for high ability, since the agent's second period payoff varies with y_1 to a greater extent. Not only she receives the benefits of a higher wage from the principal, but also outsiders will recognize her value, and compensate her accordingly. Outside activities provide the agent with an opportunity to exploit her reputation further. Hence, increasing ex-ante incentives to acquire such reputation. Moreover, if a larger investment in outside activities is more profitable for more able agents, effort and investment become strategic complements. In other words, when e increases, the likelihood she will gaining a good reputation is improved. This in turn provides more incentives to invest in outside activities. Greater investment in outside activities still further increases the return to acquire a good reputation, enhancing the incentives for e.

In the foregoing analysis we eased the assumption that agents have limited attention. We now reintroduce the additive cost function in order to demonstrate that our comparative statics would remain unchanged when outside activities increase the cost of effort.

Proposition 3 Assume there is limited attention, $c(e;i) = c(e+i)$, and outside activities provide value $v(i;\eta) + \kappa\psi(\eta)$ to the agent, with $\kappa \geq 0$ and $\psi(\eta)$ increasing and bounded. Then, for a given belief about effort, $\hat{e}$, the agent's optimal response $e^*(\hat{e}; \sigma = 1; \kappa)$ is increasing in κ. Therefore, there is a κ^* such that outside activities increase effort if and only if $\kappa \geq \kappa^*$.

Proof. See Appendix. ■

When the agent has limited attention, the outside activity might reduce effort. The more the outside activity depends on ability, however, the higher the variability of the agent's second period payoff as a function of first period output. This responsiveness induces higher effort incentives and makes it more desirable to permit outside activities.

We now turn to analyze the equilibrium. Given the optimal response of the agent to a certain belief of the principal, $e^*(\sigma; \hat{e})$, an equilibrium effort level is characterized by the equation $e = e^*(\sigma; e)$. We will denote by $E^*(\sigma)$ the set of equilibria as a function of the restrictions on outside activities. The equilibrium effort levels have the following property:

Proposition 4 Suppose that $v_\eta(i;\eta) > 0$, and the cost function is separable. Then, there is a unique equilibrium when the outside activity is not allowed, $e^*(\sigma = 0)$. Furthermore, when the outside activity is allowed the effort level at any equilibrium increases: for any $e^* \in E^*(\sigma = 1)$, $e^* > e^*(\sigma = 0)$.

Proof. See Appendix. ■

When outside activities are not allowed, the effort level chosen by the agent is independent of the conjectures about effort. As a result, this will be the unique equilibrium. However, when outside activities are allowed, it is not possible to guarantee a unique equilibrium. When investment and ability are complements, a multiplicity of equilibria might arise. It remains the case, however, that any equilibria will have a higher effort level when the outside activity is allowed than when it is forbidden. Moreover, these increased incentives come at no cost for the principal.

2.3 Learning from outside activities

In the previous section we developed the basic argument using a model in which outside activities required an investment in the first period that paid off in the second. This effectively ruled out the possibility of learning about ability from the output on the outside activity, since that took place at $t = 2$. We address this possibility by analyzing a model in which the outside activity more closely resembles the inside activity. Instead of requiring a period-one investment, the outside activity, like the inside activity, now requires effort each period. This learning framework will demonstrate that the previous findings are not a mere by-product of the particular assumptions relating investments to outside activities.

As before, there are two periods in which ability takes the same value ($\eta_1 = \eta_2$).[9] Output from both activities is non-verifiable, but observable to the principal, agent, and outside market. As before, output in the inside activity takes the linear form $y_t = f(e_t) + \eta + \varepsilon_t$, where now $f(\cdot)$ is a concave function. Outside activities will pay off $v_t = g(i_t) + \gamma(\eta_t + \xi_t)$, with $\xi_t \sim N\left(0; \sigma_\xi^2\right)$, $g(\cdot)$ is concave and $\gamma \geq 0$. By assuming that the parameter γ multiplies both η_t and ξ_t, we make the signal to noise ration independent of γ. This will turn out to be useful for separating the effects of a higher productivity of ability and a more informative signal when doing comparative statics. The random variables $\eta; \varepsilon_t;$ and ξ_t are all independent of each other. We will also assume a separable cost function. Later we will discuss how the results change when limited attention is introduced.

We start first by looking at the case where outside activities are not allowed. Again, m represents the updated beliefs about ability at the end of period 1: $m = E[\eta \mid y_1]$. The updating process takes the following form:

$$m = \alpha\bar{\eta} + (1 - \alpha)\left(y_1 - f(\hat{e})\right), \text{ where } \alpha = \frac{h_\eta}{h_\eta + h_\varepsilon}$$

$\hat{e}$ represents the conjecture of the principal and outside market about the agent's effort level.

Since second period effort is nil, m will also be the expected output, and hence the wage the agent receives. First period effort will affect that wage through an increase in first period output (which in turn increases the expectation of ability m). She then maximizes:

$$\max_{e;i} \{w_1 - c_e(e) + m(e)\}$$

which yields the first order condition:

$$(1 - \alpha) f'(e) = c_e'(e)$$

To compare the equilibrium with the case with outside activities, we will denote these variables by $\alpha(\sigma = 0)$ and $e^*(\sigma = 0)$. From this, it is easy to see that $(1 - \alpha^*(\sigma = 0))$, and as a result $e^*(\sigma = 0)$, are increasing in h_ε and decreasing in h_η. The higher the precision of the signal coming from first period output (h_ε), the more weight the update of beliefs will put on it. Effort, then, will have a bigger impact on future wages, increasing incentives. The opposite will be true for h_η.

[9] All the results in this section extend to the steady state equilibrium of the model with an infinite number of periods as in Holmstrom (1982).

We now turn to study the case where both activities are allowed. In this case, there is more information available to update beliefs about ability. This new information can be combined to provide a signal with which to update beliefs:

$$z_1 = E[\eta \mid y_1; v_1] = \frac{h_\varepsilon (y_1 - e_t) + h_\nu \left(\frac{v_1 - i_t}{\gamma} \right)}{h_\varepsilon + h_\nu}$$

the precision of the signal being $h_\varepsilon + h_\nu$. The updating process becomes:

$$m = \alpha \bar{\eta} + (1 - \alpha) z_1 = \frac{h_\eta \bar{\eta} + h_\varepsilon (y_1 - \hat{e}_1) + h_\nu \left(\frac{v_1 - \hat{i}_1}{\gamma} \right)}{h_\eta + h_\varepsilon + h_\nu}, \text{ where } \alpha = \frac{h_\eta}{h_\eta + h_\varepsilon + h_\nu}$$

Therefore, having the two activities is equivalent to observing a single output, z_1, whose precision is the sum of the two. Hence the same analysis we had above carries through for z_1. The agent's problem becomes:

$$\max_{e;i} \{w_1 - c_e(e) - c_i(i) + (1 + \gamma) m(e)\}$$

The first order conditions that determine effort in the inside activity and in private trade now become:

$$\begin{aligned} (1 + \gamma) \left(\frac{h_\varepsilon}{h_\eta + h_\varepsilon + h_\nu} \right) f'(e) &= c_e'(e) \\ \left(\frac{1 + \gamma}{\gamma} \right) \left(\frac{h_\nu}{h_\eta + h_\varepsilon + h_\nu} \right) g'(i) &= c_i'(i) \end{aligned}$$

The following result characterizes the conditions under which effort increases with the outside activity:

Proposition 5 Assume a separable cost function. Then, $e^*(\sigma = 1) \geq e^*(\sigma = 0)$ if and only if $\gamma (h_\eta + h_\varepsilon) \geq h_\nu$.

Proof. Since the cost function is separable, we only need to find the condition for the returns to e to be higher with outside activities. That is: $(1 + \gamma) \left(\frac{h_\varepsilon}{h_\eta + h_\varepsilon + h_\nu} \right) \geq \frac{h_\varepsilon}{h_\eta + h_\varepsilon}$. It is immediate to see this is satisfied when $\gamma (h_\eta + h_\varepsilon) \geq h_\nu$. ■

This result shows that under certain conditions, outside activities increase effort inside also in this framework. However, this case opens the possibility for outside activities crowding out effort, even when the marginal cost of effort is not increased by the larger number of tasks. The reason being that output from the extra activity provides a further signal about ability and hence less weight is put on output produced inside during the updating process. This is apparent from the fact that $\frac{h_\varepsilon}{h_\eta+h_\varepsilon+h_\xi} < \frac{h_\varepsilon}{h_\eta+h_\varepsilon}$. Effort e thus does not have as high an impact on the perception of ability. The positive effect remains the same as before: the extra activity provides more opportunities for profiting from a reputation for high ability, since $(1+\gamma) > 1$. Whenever outside activities provide a poor signal about ability (low precision h_ξ), or are greatly affected by ability (high γ), outside activities increase incentives for effort e.

Finally we briefly describe how the results change when we introduce limited attention in the model. The first order conditions when the cost function takes the form $c(e;i) = c(e+i)$ become:

$$(1+\gamma)\left(\frac{h_\varepsilon}{h_\eta+h_\varepsilon+h_\xi}\right) f'(e) \leq c'(e+i) \text{ with } e^* = 0 \text{ if inequality strict}$$
$$\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{h_\xi}{h_\eta+h_\varepsilon+h_\xi}\right) g'(i) \leq c'(e+i) \text{ with } i^* = 0 \text{ if inequality strict}$$

The following result makes clear the same intuition of the separable cost case still goes through here:

Proposition 6 $e^*(\sigma=1)$ is increasing in both h_ε and γ. $i^*(\sigma=1)$ is increasing in h_ξ but decreasing in γ. Moreover, $\gamma(h_\eta+h_\varepsilon) \geq h_\xi$ is a necessary condition for $e^*(\sigma=1) \geq e^*(\sigma=0)$, but it is not always sufficient.

Proof. The results follow from the supermodularity of the agent's objective function in $(e; -i; \gamma; h_\varepsilon; -h_\xi)$. ■

If outside activities reveal little information about ability (they have low precision) or if outside activities depend on ability to a great extent (γ large), then outside activities increase effort in the inside activity. When h_ξ is small, outside activities are not a very useful measure of ability.[10] Therefore they receive little weight in the updating process, reducing the incentives for i. Also when γ is high, effort in outside activities has a small effect on the update of beliefs ($1/\gamma$). This translates into an increase in total wages next period on the order of $\left(\frac{1+\gamma}{\gamma}\right)$, whereas effort in inside activities obtain a return proportional to $(1+\gamma)$.

The outside activity might still be beneficial for incentives. However, since limited attention introduces yet an additional negative effect, the condition required might become stronger. Indeed, that will be the case whenever $i^* > 0$, since this strictly increases the marginal cost of e. But even when $i^* = 0$, $\gamma(h_\eta+h_\varepsilon) \geq h_\xi$ might not be sufficient. To see this consider the case where $f(e) = e; g(i) = i$.

[10] This is not to say that ability is unimportant for outside activities, since that is determined by the parameter γ.

It is immediate to see that, because of the constant returns to scale technology, the agent will only put effort in the most productive activity.[11] The condition that guarantees that $e^* > 0$ is $\gamma h_\varepsilon > h_\xi$. When this is satisfied, $i^* = 0$ and e^* solves the equation $(1+\gamma)\left(\frac{h_\varepsilon}{h_\eta + h_\varepsilon + h_\xi}\right) = c'(e)$. Moreover, the condition $\gamma h_\varepsilon > h_\xi$ is stronger than that for the separable case, $\gamma(h_\eta + h_\varepsilon) \geq h_\xi$, and guarantees that $(1+\gamma)\left(\frac{h_\varepsilon}{h_\eta + h_\varepsilon + h_\xi}\right) > \frac{h_\varepsilon}{h_\eta + h_\varepsilon}$. Therefore, we obtain that $e^*(\sigma = 1) \geq e^*(\sigma = 0)$ if and only if $\gamma h_\varepsilon \geq h_\xi$.

2.4 Non-observable Output

In this section we consider the case where outsiders cannot observe the first period output in inside activities. The reason for that is twofold. First, we check the robustness of our results to alternative assumptions. But most importantly, new insights arise in this case. Now, the reputational effects will not come from individual output, but rather from the employment condition (whether the agent is dismissed after the first period or not).

The non-verifiability of output creates a problem for the transmission of information to the market when outsiders cannot observe the first period output. Without any information, they would have to expect an average ability level from the agent. In principle, when outsiders do not observe output they cannot adjust their compensation (on outside activities) to the new information generated. Moreover this information asymmetry translates into a de facto departure from perfect competition. Job terminations can compensate for informational imbalances, however. When dismissals are correlated with first period output, they provide a mechanism to transfer (at least partially) information to the markets. By applying dismissals, we extend the results of the basic model to the case where outsiders are uninformed about first period performance. Since some degree of information still goes to the market, an analogous result to proposition 2 will hold.

We will consider long-term contracts specifying a wage profile $\{w_1; w_2\}$ and the restrictions on outside activities, $\sigma \in \{0, 1\}$. Finally, the principal can decide to dismiss the agent after the first period, if he finds it appropriate. If termination of the relationship occurs, principal and agent enjoy a last-period payoff of 0 and $\underline{u} \geq 0$, respectively. The agent will be able to obtain a wage equal to her expected productivity in the market. Moreover, the agent can still obtain the rents from outside activities if they were allowed in the first place (i.e., $\sigma = 1$), even if there is termination of the relationship after the first period. This results in an outside option (for period 2) of $\underline{u} = E[y_2 \mid \text{outsider's information}] + \sigma \cdot E[v(i;\eta) \mid \text{outsider's information}]$.

[11] When both activities are equally productive, the agent in indifferent about the allocation of effort between the two tasks. Only total effort is uniquely determined in this case.

We consider two polar cases here. First, the initial contract cannot be renegotiated ex-post (after the realization of the first period output). Alternatively, principal and agent might not be able to commit not to renegotiate when mutually beneficial. In the latter case, the principal will be assumed to have all the bargaining power.

Throughout this section we will assume that with some positive probability, τ, there will be an exogenous termination. In case that occurs, both principal and agent will exercise their outside option in the second period. This will be sufficient to guarantee that some dismissals will occur after the first period, as the following lemma shows.

Lemma 7 Suppose that a fraction $\tau > 0$ (small) of agents leave the principal for exogenous reasons. Then, there is no equilibrium in which no dismissal takes place (i.e., the agent gets laid-off after the first period with positive probability).[12]

Proof. Suppose the agent never gets dismissed. Then, whenever an agent leaves the principal, the market will believe it is for exogenous reasons, and will assume an average level of ability. Similarly, when the agent stays with the principal for a second period, the market learns nothing about ability. Hence, the posterior distribution of ability for the outside market will equal the prior in both cases. The market wage in the second period for someone leaving the principal will then be $\overline{w}_2 = E[y_2]$, and output from outside activities will be $E[v(i;\eta)]$. However, if the agent stays inside, output is expected to be $E[y_2 \mid y_1; \hat{e}]$, which is smaller than $E[y_2]$ for y_1 low enough. And outside activities would still pay $E[v(i;\eta)]$. It is easy to see the principal would not be willing to match the offer from the outside market, and termination will occur after a low realization of the first period output. ■

As a result, we will only look at the case where dismissals occur with positive probability. Outsiders will hold beliefs about ability that will depend on the employment conditions of the agent (i.e., whether he works for the principal or not at $t = 2$). Since the principal's expectations about future output are increasing in y_1, it is immediate to see that for a fixed belief about ability the outsiders have, the optimal rule is of the following form:

$$\begin{cases} y_1 < \underline{y} \Rightarrow \text{agent dismissed} \\ y_1 \geq \underline{y} \Rightarrow \text{continuation} \end{cases}$$

[12] If we assume the principal can hire another agent in the second period from the outside market, the same result would go through even when markets are not prefectly competitive. To see this, let $\overline{w}_2$ be the market wage when nobody gets dismissed in the first period, and the markets learn nothing about first period performance. Then, if the agent leaves, she gets $\overline{w}_2 (+E_\eta[v(i;\eta)])$, and the principal would hire another agent, making a profit of $E_\theta[y_2] - \overline{w}_2$. Total surplus is then the same as for the competitive case. Hence, termination is optimal, as shown in the proof.

Outsiders will then expect such policy. Therefore, we can summarize the principal's dismissal policy by $\underline{y}$, and the market's conjecture about such policy by $\hat{\underline{y}}$. In equilibrium, $\underline{y} = \hat{\underline{y}}$.

Principal's posterior beliefs about ability will take the usual form. However, outsiders only observe the outcome of the binary signal consisting of the employment condition: $s \in \{in, out\}$. Denote the density of the posterior distribution by $f\left(\eta \mid \hat{\underline{y}}, \hat{e}, s\right)$. Then, the updating of the prior normal distribution takes the following form:

$$
\begin{aligned}
f\left(\eta \mid \hat{\underline{y}}, \hat{e}, in\right) &= \frac{\Pr\left(in \mid \eta; \hat{\underline{y}}, \hat{e}\right) \cdot \frac{1}{\sigma_\eta} \phi\left(\frac{\eta - \mu}{\sigma_\eta}\right)}{\Pr\left(in \mid \hat{\underline{y}}, \hat{e}\right)} \\
f\left(\eta \mid \hat{\underline{y}}, \hat{e}, out\right) &= \frac{\Pr\left(out \mid \eta; \hat{\underline{y}}, \hat{e}\right) \cdot \frac{1}{\sigma_\eta} \phi\left(\frac{\eta - \mu}{\sigma_\eta}\right)}{\Pr\left(out \mid \hat{\underline{y}}, \hat{e}\right)}
\end{aligned} \tag{5}
$$

Given the conjectures about the effort of the agent, and the policy of the principal, $\Pr\left(in \mid \eta; \hat{\underline{y}}, \hat{e}\right)$ is the probability the agent is employed at $t = 2$ provided she is of known ability η. This will happen when the noise term ε_1 is large enough. In particular, $\Pr\left(in \mid \eta; \hat{\underline{y}}, \hat{e}\right) = (1 - \tau) \Pr_\varepsilon\left\{\varepsilon_1 \geq \hat{\underline{y}} - \hat{e} - \eta\right\} = (1 - \tau)\left(1 - \Phi\left(\frac{\hat{\underline{y}} - \hat{e} - \eta}{\sigma_\varepsilon}\right)\right)$. $\Pr\left(in \mid \hat{\underline{y}}, \hat{e}\right)$ is the prior probability of the agent being employed at $t = 2$. This will now happen when the sum of the realizations of ability and noise are large enough: $\Pr\left(in \mid \hat{\underline{y}}, \hat{e}\right) = (1 - \tau) \Pr_{\varepsilon,\eta}\left\{\eta + \varepsilon_1 \geq \hat{\underline{y}} - \hat{e}\right\} = (1 - \tau)\left(1 - \Phi\left(\frac{\hat{\underline{y}} - \hat{e} - \mu}{\sigma_{\varepsilon+\eta}}\right)\right)$, where $\sigma_{\varepsilon+\eta} = \sqrt{\sigma_\eta^2 + \sigma_\varepsilon^2}$. Then, $\Pr\left(out \mid \eta; \hat{\underline{y}}, \hat{e}\right) = \tau + (1 - \tau)\Phi\left(\frac{\hat{\underline{y}} - \hat{e} - \eta}{\sigma_\varepsilon}\right)$ and $\Pr\left(out \mid \hat{\underline{y}}, \hat{e}\right) = \tau + (1 - \tau)\Phi\left(\frac{\hat{\underline{y}} - \hat{e} - \mu}{\sigma_{\varepsilon+\eta}}\right)$ are the complementary probabilities.

Since only those who obtain a high output get to the second period, we would expect to see a "better" distribution of ability among these agents. In the appendix we formalize this intuition. We show that the posterior distribution that arises after $s = in$ first-order stochastically dominates the posterior distribution after $s = out$. Therefore, the expectation of any increasing function of η will be higher under the former than the latter. Moreover, the posterior distribution that arises after $s = in$ with conjecture $\hat{\underline{y}}$ first-order stochastically dominates the posterior distribution when the conjecture is $\hat{\underline{y}}'$, as long as $\hat{\underline{y}} \geq \hat{\underline{y}}'$. In other words, as the dismissal policy becomes more stringent a better pool of agents is selected. The same is true when the signal is $s = out$ (i.e., increasing the dismissal threshold improves the distribution of agents that get fired, in a first-order stochastic dominance sense). This will prove useful to derive the results that follow.

No renegotiation case

After these preliminaries, we can now start studying the case where no renegotiation is possible.

Since the second period wage cannot be altered, the dismissal policy is simple: there is continuation if and only if $w_2 \leq E[y_2 \mid y_1; \hat{e}] = \alpha\beta + (1-\alpha)(y_1 - \hat{e})$. The cut-off will satisfy:

$$w_2 = \alpha\beta + (1-\alpha)\left(\underline{y} - \hat{e}\right) \Rightarrow \underline{y} = \frac{w_2 - \alpha\beta}{1-\alpha} + \hat{e}$$

Notice this cut-off is independent of what happens with the outside activity, and of the outsider's conjecture about the dismissal policy. It is uniquely determined by the second period wage, and the effort conjecture. Hence, this will also be the equilibrium dismissal policy (after imposing $\underline{y} = \hat{\underline{y}}$). In particular, the equilibrium dismissal policy will only depend on w_2 and $\hat{e}$, but not on whether outside activities are allowed.

The outside option of the agent is $\underline{u} = E\left[y_2 \mid \underline{y}; \hat{e}, out\right] + \sigma \cdot E\left[v(i;\eta) \mid \underline{y}; \hat{e}, out\right]$. The probability she gets fired and exercises the outside option is $\Pr\left\{y_1 < \underline{y}\right\} = \Pr_{\varepsilon;\eta}\left\{\eta + \varepsilon_1 \geq \underline{y} - e\right\} = \Phi\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)$. The objective problem now becomes:

$$\max_{e;i} \left\{ w_1(\hat{e}) - c_e(e) - c_i(i) + \underline{u} + (1-\tau)\left[1 - \Phi\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)\right] \cdot [\Delta w_2 + \sigma \cdot \Delta v] \right\} \tag{6}$$

where $\Delta w_2 = w_2 - E\left[y_2 \mid \underline{y}; \hat{e}, out\right]$, and $\Delta v = E\left[v(i;\eta) \mid \underline{y}; \hat{e}, in\right] - E\left[v(i;\eta) \mid \underline{y}; \hat{e}, out\right]$.

The first order conditions for this problem with respect to e and i are:

$$(1-\tau) \cdot \sigma_{\varepsilon+\eta}^{-1} \cdot \phi\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right) \cdot [\Delta w_2 + \sigma \cdot \Delta v] = c'_e(e) \tag{7}$$

$$\sigma \cdot E\left[v_i(i;\eta) \mid \underline{y}; \hat{e}, out\right] + (1-\tau)\left[1 - \Phi\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)\right] \cdot \sigma \cdot \Delta v_i = c'_i(i) \tag{8}$$

where $\Delta v_i = E\left[v_i(i;\eta) \mid \underline{y}; \hat{e}, in\right] - E\left[v_i(i;\eta) \mid \underline{y}; \hat{e}, out\right]$.[13]

When $v_\eta(i;\eta) = 0$, outside activities do not depend on ability, and as a result, $\Delta v = 0$. It follows from the first equation that effort is independent of σ. However, when $v_\eta(i;\eta) > 0$, the first-order stochastic dominance of the distribution after $s = in$ over the one after $s = out$ implies that $\Delta v > 0$. Compensation from outside activities is then higher when the agent remains working for the principal in the second period. And incentives for effort increase, just as in the basic set-up.

Renegotiation case.

We now turn to the case where renegotiation is possible whenever both parties agree to it. Just as in

[13] We assume throughout that the principal sets a wage such that $\Delta w_2 + \sigma \cdot \Delta v \geq 0$. Otherwise, the agent would leave after the first period for the outside market.

the previous case, if $w_2 \leq E[y_2 \mid y_1; \hat{\theta}]$ there will be continuation. Moreover, since both parties agree to transact for the second period, there will be no renegotiation.[14] However, when $w_2 > E[y_2 \mid y_1; \hat{\theta}]$ there will not necessarily be a dismissal. We will denote the new dismissal cut-off by $\underline{\underline{y}}$, and the corresponding conjecture made by outsiders by $\hat{\underline{\underline{y}}}$. To the extent the surplus from continuation, $E[y_2 \mid y_1; \hat{\theta}] + \sigma \cdot E\left[v(i;\eta) \mid \hat{\underline{\underline{y}}}; \hat{\theta}; in\right]$, is larger than the outside option, $E\left[y_2 \mid \hat{\underline{\underline{y}}}; \hat{\theta}; out\right] + \sigma \cdot E\left[v(i;\eta) \mid \hat{\underline{\underline{y}}}; \hat{\theta}; out\right]$, there will be renegotiation. The cut-off for the dismissal policy will now be lower than the one for the case without renegotiation. It will be determined by the point at which both principal and agent are indifferent between continuation and separation. At that point, the surplus will vanish:

$$E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}\right] + \sigma \cdot E\left[v(i;\eta) \mid \hat{\underline{\underline{y}}}; \hat{\theta}; in\right] = E\left[y_2 \mid \hat{\underline{\underline{y}}}; \hat{\theta}; out\right] + \sigma \cdot E\left[v(i;\eta) \mid \hat{\underline{\underline{y}}}; \hat{\theta}; out\right]$$

and the renegotiated wage will be $w_2^R = E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}\right]$. This will make the agent indifferent between accepting the renegotiated contract and going to the outside market.

In equilibrium, the outsider's conjecture about the dismissal policy is correct, $\hat{\underline{\underline{y}}} = \underline{\underline{y}}$. The equilibrium cut-off will come from the solution to the following equation

$$E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}\right] + \sigma \cdot E\left[v(i;\eta) \mid \underline{\underline{y}}; \hat{\theta}; in\right] = E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}; out\right] + \sigma \cdot E\left[v(i;\eta) \mid \underline{\underline{y}}; \hat{\theta}; out\right] \tag{9}$$

The left-hand side of the equation is increasing in $\underline{\underline{y}}$, since both $E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}\right] = \alpha\mu + (1-\alpha)\left(\underline{\underline{y}} - \hat{\theta}\right)$ and $E\left[v(i;\eta) \mid \underline{\underline{y}}; \hat{\theta}; in\right]$ are. Similarly, the right-hand side is also increasing in $\underline{\underline{y}}$. However, in the appendix we show there is a single value of $\underline{\underline{y}}$ that satisfies the previous equation. As a result, the two curves will intersect only once, and the equilibrium dismissal policy is unique for each possible value of σ. Notice, however, that the value of $\underline{\underline{y}}$ will depend on the initial investment of the agent and whether the outside activity is allowed, $\underline{\underline{y}}(i;\sigma)$. Moreover, since i and σ increase the left-hand side more than the right-hand side, they provide more incentives to renegotiate. As a result, $\underline{\underline{y}}(i;\sigma)$ is decreasing in both arguments. To simplify notation, we will drop the arguments $(i;\sigma)$, except when necessary.

When $y_1 \geq \underline{y}$ no renegotiation will take place and the agent obtains $w_2 + \sigma \cdot E\left[v(i;\eta) \mid \underline{y}; \hat{\theta}; in\right]$. When $y_1 \in [\underline{\underline{y}}; \overline{y})$ renegotiation will occur and when $y_1 < \underline{\underline{y}}$ the agent gets dismissed. From the assumption that the principal has full bargaining power, the agent gets $E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}\right] + \sigma \cdot E\left[v(i;\eta) \mid \underline{\underline{y}}; \hat{\theta}; in\right]$ in these last two cases. The difference between both these payoffs is then $w_2 - E\left[y_2 \mid \underline{\underline{y}}; \hat{\theta}\right]$.

[14] We do not allow for renegotiations to take place when their only purpose is to redistribute rents from one party to the other. In such a case, one of them gets hurt, and would rather continue under the terms of the previous contract. Renegotiation will only take place when both parties benefit from it.

The objective function of the agent now becomes:

$$\max_{e;i} \left\{ w_1(\hat{e}) - c_e(e) - c_i(i) + \underline{u} + (1-\tau)\left[1 - \Phi\left(\frac{\underline{y} - \beta - e}{\sigma\varepsilon + \eta}\right)\right] \cdot [\Delta w_2 + \sigma \cdot \Delta v] \right\} \qquad (10)$$

where $\Delta w_2 + \sigma \cdot \Delta v = w_2 - E\left[y_2 \mid \underline{y}; \hat{e}\right] = w_2 - \left[\alpha\beta + (1-\alpha)\left(\underline{y} - \hat{e}\right)\right]$. It implicitly depends on i and σ through $\underline{y}$. It follows that $\Delta w_2 + \sigma \cdot \Delta v$, and hence incentives for effort, are increasing in σ (since $\underline{y}$ is decreasing in the same argument). Outside activities again increase effort in inside activities.

We summarize the results for both cases in the following proposition:

Proposition 8 Assume a separable cost function $c(e; i) = c_e(e) + c_i(i)$. Then for a fixed wage w_2 and belief about effort, $\hat{e}$, the following hold both when renegotiation is possible and when it is not:

i. If $v_\eta(i; \eta) = 0$, outside and inside activities are independent (i.e., e is not affected by outside activities)

ii. If $v_\eta(i; \eta) > 0$, the outside activity is a complement to the inside activity (i.e., the outside activity increases e)

iii. If $v_{i\eta}(i; \eta) > 0$, then e and i are strategic complements.

Proof. See Appendix. ■

As in the observable case, we can also consider the equilibrium effort level. When outside activities are not allowed, the equilibrium is unique, despite the fact that now effort will not be independent of the conjecture about effort. Furthermore, any equilibrium effort when the outside activity is allowed is still increased.

Proposition 9 Suppose that $v_\eta(i; \eta) > 0$, and the cost function is separable and sufficiently convex.[15] Then, for a fixed wage w_2 there is a unique equilibrium when outside activities are not allowed, $e^*(w_2; \sigma = 0)$. Furthermore, when outside activities are allowed the effort level at any equilibrium, , is increased: for any $e^* \in E^*(w_2; \sigma = 1)$, $e^* > e^*(w_2; \sigma = 0)$.

Proof. See Appendix. ■

[15] We make this assumption more precise in the appendix. It is required to guarantee the solution to the agent's problem is continuous in $\hat{e}$, and therefore, an equilibrium effort level exists.

A low realization of the first period output signals low ability. The principal then has an incentive to dismiss those who perform poorly. This motivates the agent to exert effort in order to avoid dismissal, and obtain the second period wage. Furthermore, when outside activities are allowed, there is yet another good reason to avoid dismissal. Such outcome would be viewed very unfavorably by the market. Not only would the agent lose the second period wage, but also the prospects of obtaining high returns on outside activities would vanish. Gaining a good reputation with the principal therefore has spill-over effects on outside activities. For a fixed second period wage, then, incentives are increased. Or alternatively, the cost of implementing a particular effort level (in terms of second period wage) falls when outside activities are allowed.

2.5 Use of Company Resources

So far we have stressed the complementarities that arise between inside and outside activities when reputation effects are important. In general, incentives will increase whenever the compensation received on the outside activity varies with the performance on the inside activity. Reputation is just one such mechanism that generates this effects. But certainly not the only one. In this section we show how access to company resources can have similar effects to the model based on reputational capital. The ability to use company resources for private purposes is often regarded as an informal perquisite rather than a formal incentive instrument. The use of one's office phone or the company car for one's personal business may be the most commonplace examples. Less obvious are examples in which the asset is specific to the firm and becomes unavailable upon job termination. Customized software and equipment, access to clients, corporate reputational capital and other resources may all be formally available to an employee provided she remains attached to the firm. When these resources are costly to obtain in the market, access to them can provide a strong inducement to inside work effort.

We will assume here that production in outside activities requires effort every period and is independent of ability. Alternatively, the profitability of those activities depends on the use of an asset owned by the principal. Formally, the value generated by outside activities equals $v(i; a)$, where $a \in \{A; \emptyset\}$. Since the principal owns the asset, $a = A$ only when working for him. The use of the asset enhances the value of outside activities, resulting in $v(i; A) > v(i; \emptyset)$ for all i.

We will denote by $v^*(a) = \max_i \{v(i; a) - c_i(i)\}$ the value of the outside activity in a single period. Let $\underline{\pi} = v^*(A) - v^*(\emptyset)$ represent the value for the agent of the use of the principal's asset. The principal has the option of hiring another agent for period 2 in the outside market. Because of perfect competition, in the absence of any asset, this would result in a zero profit for the principal. However, any prospective agent would accept a reduction in wage up to $\underline{\pi}$ to work for the principal. This will then be the second period outside option of the principal.

We again can consider two cases. If no renegotiation can take place, the agent gets to continue until $t = 2$ if and only if expected second period profits exceed $\underline{\pi}$: $E[y_2 \mid y_1; \hat{e}] - w_2 \geq \sigma \cdot \underline{\pi}$. The dismissal policy will be determined by the equation $E\left[y_2 \mid \underline{y}; \hat{e}\right] = w_2 + \sigma \cdot \underline{\pi}$. Upon dismissal, the agent will receive $\underline{u} = E\left[y_2 \mid \underline{y}; \hat{e}; out\right] + \sigma \cdot v^*(\perp)$. The problem of the agent becomes:

$$\max_{e;i} \left\{ w_1(\hat{e}) + v^*(A) - c_e(e) + \underline{u} + (1-\tau)\left[1 - \Phi\left(\frac{\underline{y} - \beta - e}{\sigma_\varepsilon + \eta}\right)\right] \cdot [\Delta w_2 + \sigma \cdot \Delta v] \right\} \qquad (11)$$

where $\Delta w_2 = w_2 - E\left[y_2 \mid \underline{y}; \hat{e}; out\right]$, and $\Delta v = v^*(A) - v^*(\perp) = \underline{\pi} > 0$.

Since $\Delta v > 0$, it follows that allowing the outside activity increases incentives. Higher effort increases the probability of being rehired in the second period. This will result in an increase in wage Δw_2 with respect to the outside market and continued use of the principal's asset. Being dismissed would result in a loss of $\underline{\pi}$ on those activities.

When we introduce the possibility of renegotiation, the agent will only get dismissed when the surplus from continuation becomes negative. As in the previous section, there is no renegotiation when $y_1 \geq \underline{y}$. For $y_1 \in [\underline{\underline{y}}; \underline{y})$ renegotiation will take place, and after $y_1 < \underline{\underline{y}}$ the agent gets dismissed. $\underline{y}$ is determined by the same equation for the no renegotiation case: $E\left[y_2 \mid \underline{y}; \hat{e}\right] = w_2 + \underline{\pi}$. The cut-off for a dismissal is such that both principal and agent are indifferent between continuation and termination: $E\left[y_2 \mid \underline{\underline{y}}; \hat{e}\right] - w_2^R = \sigma \cdot \underline{\pi}$ and $w_2^R + \sigma \cdot v^*(a) = E\left[y_2 \mid \underline{\underline{y}}; \hat{e}; out\right] + \sigma \cdot v^*(\perp)$. Combining both equations, it follows that $E\left[y_2 \mid \underline{\underline{y}}; \hat{e}\right] = E\left[y_2 \mid \underline{\underline{y}}; \hat{e}; out\right]$ characterizes $\underline{\underline{y}}$. The agent's outside option becomes $\underline{u} = E\left[y_2 \mid \underline{\underline{y}}; \hat{e}; out\right] + \sigma \cdot v^*(\perp)$. The agent still solves the same problem (11), only that now $\Delta w_2 + \sigma \cdot \Delta v = w_2 - w_2^R(\sigma) = w_2 - E\left[y_2 \mid \underline{\underline{y}}; \hat{e}\right] + \sigma \cdot \underline{\pi}$. Again, allowing the outside activity increases the variance in total second-period compensation by $\underline{\pi}$. Following dismissal (or renegotiation), the agent not only looses the wage w_2, but also the rents from outside activities that emerge when using the principal's asset.

In both the no renegotiation and renegotiation cases, for a fixed second-period wage incentives rise after allowing outside activities. The cost of providing incentives falls when outside activities are permitted.

When the agent's compensation from outside activities depends on the performance on inside activities, synergies arise among the different tasks. We tried to show that this can occur for several reasons. We have stressed the importance of reputation building and the use of company resources as two channels through which these incentives appear.

3 Job design: contemporary examples

In this section we describe several modern examples, with a special attention to incentives in academe. They suggest that an analysis based solely on conflicts of interest might not give us a full understanding of job design problems. Incentive effects of outside activities are also present.

3.1 U.S. Higher Education

Our model gives some new insights on the role of outside activities in the provision of incentives to faculty in U.S. higher education. It is a common practice to allow college and university professors to engage in remunerative outside activities such as outside teaching, sponsored research, consulting, public speaking, and other projects related to her field of expertise. That colleges and universities are actively interested in the allocation of effort between inside and outside activities is evident in policies governing faculty consulting, privately sponsored research, conflicts of interests, and the like. Some universities require department chairs to be notified of and to approve all faculty participation in outside activities. Others insist that a share of external grants received go to the university. Such policies testify it is of course possible that some non-university activities may place competing demands on a faculty member's attention, as HM suggest.

At the same time, the reputational mechanism proposed in our model also seems to play an important role. Success in employer-centered tasks, for example a solid record of scholarly achievement, signals high ability to outside employers and may have a corresponding positive impact on private, non-university, options. Therefore, the availability of such outside options may encourage greater effort in public activities (which may in turn lead to greater outside rewards). Such synergies between a faculty member's inside and outside options are also explicitly acknowledged in most universities' faculty handbooks.

In order to examine these predictions in some more detail, we analyze the faculty subsample of the Carnegie Survey of Higher Education. This survey was conducted in 1969, to gauge the opinions of academics, and their influence on policy. We use this data because, despite being over 30 years old, it provides detailed individual information on, among other variables, income from outside consulting, as well as the allocation of effort among different tasks for 20,008 respondents. Table 2 describes the data we used, after dropping those observations with missing values in the variables of interest.

On average, about 60% of faculty members do some consulting, the great majority of them spending less than 10% of their time. Among the di®erent groups, business applied (containing engineering, business and agriculture) and social scientists are among those who do most consulting (66.22% and 61.15% respectively). In the same table we also see the source of income from di®erent outside activities. Teaching is the major source for most of the groups (except for business applied), followed by consulting and research salaries.

If we look at the link between academic and outside activities, we do see in table 3 that it is indeed very strongly signi¯cant. In it, we regress the income from outside activities (as a fraction of the basic university salary) on a measure of research e®ort. We include controls for the institutional salary and the amount of time spent doing consulting and private practice. And we also use several proxies for ability, with an index of the quality of the university of teaching appointment, and controlling for the university where the highest degree was obtained and the rank. We ¯nd that those faculty members who spend more time on research indeed receive a larger compensation from outside activities, as the theory would suggest.

In tables 4 we disaggregate these results by type of major source of outside income. The result is robust for most types of outside activities. The only exception comes from those academics whose primary outside activity consists of private practice. For them we see that e®ort in research comes at a cost of a lower outside income. But for the rest of categories, the reverse is true. Both academic reputation and e®ort in research are important determinants of income from outside activities.

Finally, in table 5 we show the relationship between e®ort in research, consulting and private practice. We can see that those who spend more time on consulting also exert more e®ort in research activities, whereas the time spending on a private practice is negatively related (although not statistically signi¯cant when the controls are included in the regression) to research e®ort. Overall, these results are consistent with our theory, suggesting that the particular synergies we emphasize do play an important role.

3.2 Other examples

Other professions also place a similar emphasis than academe on the trade-o® between con°icts of interests and reputational synergies when dealing with outside activities. That is the case in journalism or among librarians, for instance. The Association of Colleges and Research Libraries writes, with respect to rare book, manuscript, and special collections librarians: \Certain types of outside employment, including teaching, lecturing, writing, and consulting, can be of bene¯t to both the institution and the employee by stimulating professional development. Consequently, special collections librarians should be encouraged in these activities."

There have been advocates calling for the use of a two-track system for high technology companies, specially in research and development (see O'Kelley, 1978). These are jobs that require the services of highly quali¯ed technicians and engineers. And in such a system, agents would pursue a career inside and outside the company, much in the spirit of our model. On one hand, as part of the organization, they are able to contribute to the ¯rm. But at the same time, by being able to participate in outside professional activities, they are more motivated by being able to obtain outside recognition, and reap the rewards of a strong professional reputation.

Many professional athletes, also, have started restaurant chains, starred in motion pictures, written autobiographies and pursued numerous activities while being active players. Even though they certainly face very strong incentives to continue improving performance, being a star athlete has obvious positive externalities for their private business. It seems natural, then, to see these outside activities as an important factor driving their motivation, much in the spirit of our model.

4 Job design: an historical account

The case of the English East India Company should have been an exemplar of the principle \(¯nancial) responsibility and authority should go hand in hand" (Holmstrom and Milgrom 1991, p. 41). As in the case of faculty employment, we ¯nd instead low explicit incentives to exert e®ort in inside activities plus the provision to trade privately. Our discussion refers mainly to operations in the ¯rst half of the eighteenth century.

4.1 Overview of the employment relationship

Our analysis focuses on roughly the ¯rst half of the eighteenth century, 1700-1757, when the Directors (located in London) appointed 318 employees or servants to Bengal, the Company's principle input market.[16] In India the servants engaged in a variety of transactions directed at the provision of the \investment" and management of Company assets.[17] Examples of such transactions included contracting with indigenous merchants for future delivery of cotton piecegoods, minting and disbursing the

[16] For a more detailed account of the employment relationship in the EIC and a full description of the archival data see Hejeebu (2002). Chaudhuri (1978) describes the organization of the Company and its trading system. Marshall (1976) documents the private trade of servants in India.

[17] The Company referred to its "investment" as the goods that were purchased in India and later sold in London. Because the English produced few goods demanded in India, they traded bullion for Bengali textiles, raw silk, and saltpetre.

Company's treasure, repairing and maintaining the Company's buildings and grounds, and negotiating with the Bengal governor or his representatives. The challenge facing the Directors was to prevail upon the servants to undertake those tasks in a cost minimizing manner.

Two main factors undermined the Directors' ability to do so: the distance separating the two parties and the mortality risks faced by the servants. The outbound voyage took from 7 months to a year. Total transportation time by sea, therefore, was between one-half and two years. This was the length of time required for goods and information to reach the Directors. Such lags implied signi¯cant bottlenecks in information °ows. News came infrequently and could not easily be corroborated by independent sources. The close accounting of transactions in Bengal were produced by senior agents outside the purview of other agents stationed elsewhere in India, such as Madras and Bombay. The agents' tight control over information reaching directors implied that the truth-value of reported costs of transactions in Bengal was established in London at least seven months later by persons who had no direct experience in India.

Consequently it is not surprising that the formal contracts, called \covenants of indenture," were incomplete: they did not clearly specify expectations of the employee in a variety of states. Despite their great length, restrictive tone, and careful wording of legal rights and responsibilities, the contracts reveal that the Directors could neither anticipate nor operationalize a best response for the wide variety of market transactions the servants would encounter. Both the initial contracts and the ongoing correspondence left a great of decision-making in the hands of the servants in Bengal. For example, in a letter to the Bengal President and Council, dated 11 February 1731/32, the Directors write, \Having thus far acquainted you with our thoughts by ordering some things positively and leaving others in a manner to your own discretion, we expect as to the ¯rst a strict compliance and to the last your reasons why you di®er from us in opinion" (para. 126). The Directors referred to their procedural rules as \positive orders." Although they expected close adherence to the \positive orders" they provided the following caveat \If hereafter any unforeseen accident happens of the like kind, to render a strict compliance of our orders impracticable, we shall never be displeased, if upon mature considerations you take such measures as will make the disappointment most easy to us, in respect to our pro¯ts here, which you must be tolerable good judges of" (para. 77).[18] Both the initial contracts and the ongoing correspondence left a great of decision-making in the hands of the servants.

[18] The 42-paged letter of 11 February 1731/2 belongs to a set of twenty years of correspondence examined for this project. Typically the early years of each decade in the period were chosen. For each year there was at least one General Letter to the President and Council. The letter of 11 February 1731 was written shortly after dismissing President John Deane for persistent returns of low quality goods. The letter reveals the Directors' frustration with Deane's successor and illustrates the range and depth of information problems.

The second exogenous constraint on wage incentives was the high degree of mortality of Europeans in Asia. Using life table analysis, Hejeebu (1998) reports that for the cohort of employees entering the service between 1700-1724 the chance of dying within the first five years was 31%. The chance in the next years was 38%. For the cohort of servants entering between 1725 and 1749, the probability of dying in active employment was 26% in the first five years and 44% in the second five years. The median length of service was just 9 years. The high mortality rates most likely biased the interests of the servants towards pursuing their own short-term objectives, rather than the long-term interest of the Company.

The incentive scheme in the English East India Company specified in the actual contracts involved a fixed annual salary, room and board, promotion based on strict seniority, and the privilege of trading within Asia.[19] Overseas employees were grouped into four ranks of increasing responsibility: writer, factor, junior merchant, and senior merchant. A writer earned $5 per year. After 5 years he was promoted to the rank of factor and received $15 per year. After 3 years he qualified for promotion to junior servant at $30 per year. After another 3 years he could become senior merchant earning $40 per year. These salaries were paltry by the standards of the other chartered companies and by the standards of middle class England in the eighteenth century (Carlos 1994, 324; Carlos and Nicholas 1993, 245; Williamson 1982, 48; Langford 1989, 63; Grassby 1995, 258-262). Williamson (p.48) estimated that in 1737, barristers and solicitors earned annually $ 178 and bank, commercial, or law clerks $68. Langford (p.63) suggests that middle class membership would require an annual family income of at least $40 per annum, obtainable in the Company's service only after 11 years. The odds were against an agent surviving that long.

Instead of demanding "financial responsibility" the directors permitted private trading. Servants were allowed to trade freely within Asia but were forbidden from trading between Asia and Europe. Private trade injected an entrepreneurial element in the servants' and thus the Company's activities. As Watson (1980, 77) describes, the Company came to believe that "the advantages of a thriving private trade in the Indian settlements lay in the attractions for population, which in turn led to better defensive postures and increased revenues through civil levies, quit rents, ground rents, and customs duties." In addition to permitting private trade, the Directors after 1714 provided a mechanism for remitting money home: bills of exchange. The bill simply transferred money from employees in Calcutta to beneficiaries in London. In a letter to the Directors in 1716 Joseph Collet explained that servants found the Company

[19] Hejeebu (2002) provides a more extensive discussion of the contractual issues in the Company plus a full description of the archival data.

bills desirable \for sake of security and speedy payment and because it would be a service to provide the Company's cash" (E/4/2 p.90). Thus the Company's bills of exchange freed the servants from the di±culty of ¯nding a secure means to transfer funds. At the same the purchase of Company bills eased its cash °ow problems in India.

Based on aggregate annual °ows of remittances and the number of persons making remittances each year, Hejeebu (2002) con¯rms the opinion of many historians: the value of outside activities greatly exceeded inside remuneration. By individually tracking the roughly 1500 bills of exchange issued on the Company in Bengal between 1747 and 1756, she estimates that the English community annually remitted $297 on average per person. It should be noted that the amounts remitted represent a fraction of actual earnings. They correspond to net savings and do not include consumption expenditure or investments kept in India.

The Director's most e®ective punishment against servants' violating contractual norms was dismissal. Among the 318 persons who entered the Bengal service from 1700 to 1756, 40 (12.6%) were dismissed, suspended, or transferred. Of those 40, 29 dismissals occurred before 1757 and the remaining 11 occurred after our period. Of the 29 persons appointed and dismissed before 1757, we ¯nd that the great bulk (18 of 29) occurred at the highest levels of responsibility, the senior servants. If we look at the full set of 40 dismissals, again we ¯nd that they occurred mainly (29 of 40) at the top, upon the senior managers. Yet it was the senior servants who had the most to lose from having their employment relationship severed. The senior servants had the greatest access to and authority over Company resources. They formalized contracts with Company suppliers, negotiated treaties with local o±cials, and determined the exact distribution of Company resources within the general guidelines speci¯ed by the directors. At the same time it was the senior servants who had invested most heavily in private trade. By the time a servant reached the highest rank, he would have nurtured long-term relationships with Indian creditors, brokers, and suppliers. He would have developed a mature understanding of local trading conventions, currencies, and weights and acquired some facility in the local languages. By the time a person reached a senior rank, he could reap the dividends of his speci¯c investments in the both private and Company trades.

The case of Richard Becher (1744-1758) is illustrative. Becher arrived in Bengal in 1744 at the rank of factor. Over the course of his 15 year career, he purchased a total of 19 bills valuing $8,781 against the Company in London. However he made no remittances in the ¯rst 8 years. In his 11th year, 1754, he sent 3 bills totalling $607, in the next year 6 bills worth $926. In his last year, Becher, now a member of the Calcutta Council and serving as the Company's accountant, sent 4 bills worth $ 6,648. The close details of Becher's pattern of remittances, and many others like it, reveal that for servants in the ¯eld the value of the employment relation increased with seniority. This in turn made the threat of dismissal a serious way to discipline powerful veterans. By failing to exert himself in the Company's business, a servant jeopardized his membership in the Company plus all the private bene¯ts membership might bring.

4.2 Job design in the English East India Company

The East India Company case resonants with our analytical account of the job design problem emphasizing synergies between public and private trade. In particular the East India Company illustrates the resource complementarities that can arise when company assets can be pro¯tably used in private activities. In this setting Company employment provides a means to obtain speci¯c resources. E®ort in inside activity is induced through the instrument of dismissal. Servants sought to avoid job terminations and retain the bene¯ts from o±ce.

The Company allowed overseas agents to use its legal assets (which it called \purchased privileges"), its physical assets, and its reputational capital in ways that were privately bene¯cial. By legal assets we refer to the Company's trading privileges, negotiated with the Mughal court in Delhi and applicable over the region of Bengal. The most notable of such privileges was the ¯rman of 1717. Awarded by the Mughal Emperor Faruksiyar, the ¯rman or imperial grant gave the Company the right to trade in Bengal free of customs and in-land transit tax in exchange for an annual payment of Rs. 3,000.[20] Even though the agreements were not originally intended to apply to the private trade of Company employees, the servants insisted that the Company's privileges applied to all Englishmen in Bengal whether in their public activities or in their private activities. After 1717, Company servants began issuing passes called dastaks that exempted the holder from paying customs and in-land taxes. The directors in London were indeed aware that their servants were creatively using the Company's trading privileges and on several occasions warned against it. Yet their sentiments never amounted action: no explicit prohibition of it was ever made in the contract and no individual was ever dismissed on the basis of private appropriation of legal assets. The directors allowed it to continue. The Company also provided increased security from its army and its forti¯ed settlements. For example the Company often provided armed escorts for vessels carrying goods from inland stations such as Patna to the headquarters at Calcutta. Those convoys were also said to carry a signi¯cant portion of private cargo and thereby subsidize the private traders involved.

[20] This corresponds to about $375 on a volume of purchases averaging over $300,000 per year between 1717 and 1760 for textiles alone (Chaudhuri 1978, 544-545).

Finally, servants also made private use of the Company's reputation of being a long-lived institution regularly infused with shipments of bullion from Europe, what gave them access to local merchant networks. Indian merchants and money-lenders were willing to trade with the English immigrants because they believed that the Company would guarantee the servant's private transactions. Thus despite the high mortality of Englishmen in India, merchants were willing to extend credit to employees because they believed that the Company would settle the balance in the event of the servant's death. Such local access to corporate resources gave Company servants competitive advantages over other traders and gave them strong incentive to prevent job dismissal.

The directors understood and sanctioned these privileges of o±ce. They were well aware that allowing private trade could bene¯t the Company as well. As mentioned, the Company took deliberate steps such as allowing remittances on bills of exchange. This increased the amount of local funds available for the Company trade and thereby lowered the cost of ¯nancing operations from London. Our model suggests that by allowing outside activities, employers bene¯t by being able to o®er lower wages and still sustain an increased likelihood of higher inside production. If we consider the Company's transition from a regime of restricted or forbidden private trade (pre-1680) to one of complete freedom of private trade, we do indeed observe a decline in wages coupled with an expansion of both Company and private trade. Before 1680 salaries were higher and the volume of trade smaller. In 1670 Edward Littleton entered the Bengal service at the lowest rank (then called factor or agent) and was paid at the rate of $25 per annum (Despatch Book, vol. 87, p. 391). In the era of private trade by contrast he would have entered at the rank of writer and would have been paid $5 per annum. Furthermore the transition to complete freedom of private trade was met with a sustained increase in the volume of the Company's purchases in Bengal and with a rising °ow of remittances from Company servants (Chaudhuri 1978, Appendices C16, C22 and Marshall 1976, 229-241).

4.3 Other historical examples

It is also worth contrasting the East India Company to its great rival in Asia, the Dutch East India Company or VOC. As opposed to the East India Company, the VOC did not initially allow private trade but paid substantially higher salaries. They shared the same technology and market environment. Yet it pursued a di®erent strategy in which the company participated as a corporate body in Asian waters. The VOC established a centralized bureaucracy in Batavia (modern Jakarta, Indonesia) and directed its operations, including personnel policies, from this Asia-based headquarters. Salaries at each rank within the VOC were higher than salaries in the corresponding ranks within the East India Company.

At the lowest rank of \assistant," the VOC paid nearly six times higher than the East India Company's writer. However as one moved up the ranks the gap narrowed dramatically. The VOC's Directeur van Bengalen in Chinsura was paid $216 per annum whereas his English counterpart, the President of the Council in Calcutta, was paid $200 per annum between 1738-1760. That the VOC's policy ultimately failed to provide su±cient incentives for senior employees is evident by the fact that in 1743-4 the VOC shifted to the English policy of permitting private trade.

5 Conclusion

In this paper we try to expand the multitasking model to accommodate the possibility of synergies between di®erent activities in job design problems. We investigate two mechanisms by which outside activities complement incentives for inside activities, even though there is no technological linkage between them. Complementarities might arise when employment sustains an agent's reputation for high ability or when employment provides access to resources that can enhance the returns from outside activities. The presence of such synergies determine the desirability of private trade.

Our analysis and interpretation of the job design problem expands the range of employment settings that can be explained by existing models. Our model is amenable to situations in which e®ort inside detracts from e®ort outside as in Holmstrom and Milgrom (1991). It also amenable to cases in which e®ort outside the ¯rm can induce greater e®ort within the ¯rm. Our ¯ndings are moreover robust to changes in the assumptions about the observability of output, to how inside and outside activities are technologically linked, and to whether or not the contract can be renegotiated. The reputational and resource synergies we investigate arise in a variety of cases such as the employment of faculty by US colleges and universities and the employment of overseas agents by the English East India Company. Our model thus o®ers a new approach that reconciles historical and contemporary jobs with the current understanding of job design.

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6 Appendix: proofs

6.1 Observable Output

Proof (Proposition 3). Lets $U(e;i;\kappa) = w_1(\hat\theta) - c(e+i) + E[w_2(y_1;\hat\theta) + \sigma \cdot E[v(i;\eta) + \kappa\psi(\eta) \mid y_1;\hat\theta]]$ be the objective function of the agent as a function of $(e;i;\kappa)$. Consider the change of variables $(e;i;\kappa) \to (e;s;\kappa)$, where $s = e+i$. Then, the transformed function $\tilde U(e;s;\kappa) = U(e;s-e;\kappa)$ is supermodular in $(e;s;\kappa)$ when $\sigma = 1$, since

$$\frac{\partial^2 \tilde U}{\partial e \partial \kappa} = E_\varepsilon\left[\frac{\partial E[\phi(\eta) \mid y_1;\hat\theta]}{\partial y_1} \cdot \frac{\partial y_1}{\partial e}\right] = E_\varepsilon\left[(1-\alpha)\int_{-\infty}^{+\infty} \psi'(\eta) \cdot \sqrt{h_\eta + h_\varepsilon} \cdot \phi((h_\eta + h_\varepsilon)(\eta - m)) \cdot d\eta\right] > 0$$

$$\frac{\partial^2 \tilde U}{\partial s \partial \kappa} = 0$$

$$\frac{\partial^2 \tilde U}{\partial e \partial s} = E_\varepsilon\left[\frac{\partial E[v_i(s-e;\eta) \mid y_1;\hat\theta]}{\partial y_1} \cdot \frac{\partial y_1}{\partial e} - E[v_{ii}(s-e;\eta) \mid y_1;\hat\theta]\right]$$

$$= E_\varepsilon\left[\begin{array}{c}(1-\alpha)\int_{-\infty}^{+\infty} v_{i\eta}(s-e;\eta) \cdot \sqrt{h_\eta + h_\varepsilon} \cdot \phi((h_\eta + h_\varepsilon)(\eta - m)) \cdot d\eta \\ -\int_{-\infty}^{+\infty} v_{ii}(s-e;\eta) \cdot \sqrt{h_\eta + h_\varepsilon} \cdot \phi((h_\eta + h_\varepsilon)(\eta - m)) \cdot d\eta\end{array}\right] > 0$$

where the first and last lines follow from the integration by parts of the derivative of the expectations with respect to y_1, just as in the proof of proposition 2. It then follows from the supermodularity of $\tilde U$ that e^* and s^* are both increasing in κ. The last part of the proposition is immediate from this. ■

Proof (Proposition 4). In case the outside activity is not allowed, the marginal return on effort is $(1-\alpha)$, which is independent of $\hat\theta$. As a result, this is the unique equilibrium $e^*(\sigma = 0)$. When the outside activity is allowed, notice that for any belief $\hat\theta$ the agent chooses a positive level of effort, $e^*(\hat\theta;\sigma) > 0$. This is true even when $\hat\theta = 0$. Moreover, from Proposition 2, $e^*(\hat\theta;\sigma = 1) > e^*(\hat\theta;\sigma = 0) = e^*(\sigma = 0)$ for any $\hat\theta \geq 0$. Therefore, any equilibrium when $\sigma = 1$ satisfies $e^*(\sigma = 1) > e^*(\sigma = 0)$. ■

6.2 Non-observable Output

We work towards the proof of propositions 8 and 9 in several steps. We will start with some initial results about comparisons of distributions which will be used later on.

Consider two continuous distributions, F and G, on the same support $X = (-\infty;+\infty)$, with densities $f(x)$ and $g(x)$ respectively. To compare two distributions we will use the concept of first-order stochastic dominance. We say that F first-order stochastically dominates G if $F(x) \leq G(x)$ for every $x \in X$.

Lemma 10 Suppose that $f(x) \leq g(x)$ if and only if $x \leq \bar{x}$. Then, F first-order stochastically dominates G.

Proof. The distributions can be expressed as $F(x) = \int_{-\infty}^{x} f(x)\,dx$, and $G(x) = \int_{-\infty}^{x} g(x)\,dx$. It follows immediately that $F(x) \leq G(x)$ for any $x \leq \bar{x}$, since $f(x) \leq g(x)$ throughout this range. Suppose now that $F(x^*) > G(x^*)$ for some $x^* > \bar{x}$. Then, we have

$$F(+\infty) = \int_{-\infty}^{x^*} f(x)\,dx + \int_{x^*}^{+\infty} f(x)\,dx = F(x^*) + \int_{x^*}^{+\infty} f(x)\,dx > G(x^*) + \int_{x^*}^{+\infty} g(x)\,dx = G(+\infty)$$

where the inequality follows from the assumption that $F(x^*) > G(x^*)$ and $f(x) \geq g(x)$ for all $x \geq x^* > \bar{x}$. This is a contradiction, however, since $F(+\infty) = G(+\infty) = 1$. Therefore, the inequalities cannot reverse, and the statement is satisfied. ■

Remark 1. From the conditions of the lemma, an even stronger conclusion obtains. It is easy to see that since $f(x) > g(x)$ if $x > \bar{x}$, not only $F(x) \leq G(x)$, but $F(x) < G(x)$ for all x. As a result, the expectation of any strictly increasing function of x will be strictly higher under F than under G.

For the purpose of checking if the previous condition is satisfied, we will make use of the following result:

Lemma 11 Let $f(x)$ and $g(x)$ be two bounded and differentiable functions such that $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} g(x)$. Assume their derivatives are continuous, bounded and satisfy $f'(x) \leq g'(x)$ if and only if $x \leq \bar{x}$. Then, there exists an $\bar{x}'$ such that $f(x) \leq g(x)$ if and only if $x \leq \bar{x}'$.

Proof. We can write $f(x) = \lim_{x \to -\infty} f(x) + \int_{-\infty}^{x} f'(x)\,dx$ and $g(x) = \lim_{x \to -\infty} g(x) + \int_{-\infty}^{x} g'(x)\,dx$. Then, $f(x) - g(x) = \int_{-\infty}^{x} [f'(x) - g'(x)]\,dx$. It suffices to show that $f(x) - g(x) \leq 0$ if and only if $x \leq \bar{x}'$.

Since $f'(x) - g'(x) \leq 0$ if and only if $x \leq \bar{x}$, it follows that $f(x) - g(x) \leq 0$ for any $x \leq \bar{x}$. Suppose now that $f(x^*) - g(x^*) > 0$ for some $x^* > \bar{x}$. Then, for any $x \geq x^*$ we have $f(x) - g(x) = f(x^*) - g(x^*) + \int_{x^*}^{x} [f'(x) - g'(x)]\,dx \geq f(x^*) - g(x^*) > 0$. After $f(x) - g(x)$ goes above 0 it can only increase. As a result, $f(x) > g(x)$ after some point $\bar{x}'$. ■

We can now start studying the properties of the posterior distribution of the outside market given by (5). It will be useful to distinguish between the following posterior distributions:

$$f\left[\eta \mid \underline{\hat y}, \hat e, \text{out}\right] = \frac{\tau+(1-\tau)\Phi\left(\frac{\underline{\hat y}-\hat e-\eta}{\sigma_\varepsilon}\right)}{\tau+(1-\tau)\Phi\left(\frac{\underline{\hat y}-\hat e-\mu}{\sigma_\varepsilon+\sigma_\eta}\right)} \cdot \frac{1}{\sigma_\eta}\phi\left(\frac{\eta-\mu}{\sigma_\eta}\right)$$

$$f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}\right] = \frac{\Phi\left(\frac{\underline{\hat y}-\hat e-\eta}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{\underline{\hat y}-\hat e-\mu}{\sigma_\varepsilon+\sigma_\eta}\right)} \cdot \frac{1}{\sigma_\eta}\phi\left(\frac{\eta-\mu}{\sigma_\eta}\right)$$

The first density corresponds to the posterior after observing $s = \text{out}$. The second density, on the other hand, corresponds to the posterior after observing $s = \text{out}$, but knowing that separation did not occur for exogenous reasons.

The following lemma compares the posterior distributions after the different signals $s = \text{in}, \text{out}$.

Lemma 12 The posterior $f\left[\eta \mid \underline{\hat y}, \hat e, \text{in}\right]$ first-order stochastically dominates both $f\left[\eta \mid \underline{\hat y}, \hat e, \text{out}\right]$ and $f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}\right]$.

Proof. >From lemma 12 it suffices to show that $f\left[\eta \mid \underline{\hat y}, \hat e, \text{in}\right] \le f\left[\eta \mid \underline{\hat y}, \hat e, \text{out}\right]$ if and only if $\eta \le \hat\eta$. Moreover, since both densities are multiplied by $\frac{1}{\sigma_\eta}\phi\left(\frac{\eta-\mu}{\sigma_\eta}\right)$ the condition simplifies to checking that

$$\frac{1-\Phi\left(\frac{\underline{\hat y}-\hat e-\eta}{\sigma_\varepsilon}\right)}{1-\Phi\left(\frac{\underline{\hat y}-\hat e-\mu}{\sigma_\varepsilon+\sigma_\eta}\right)} \le \frac{\tau+(1-\tau)\Phi\left(\frac{\underline{\hat y}-\hat e-\eta}{\sigma_\varepsilon}\right)}{\tau+(1-\tau)\Phi\left(\frac{\underline{\hat y}-\hat e-\mu}{\sigma_\varepsilon+\sigma_\eta}\right)} \text{ if and only if } \eta \le \hat\eta$$

But this must hold since the left-hand side is increasing in η whereas the right-hand side is decreasing. The same argument applies for $f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}\right]$. ■

The distribution of ability after observing the agent remaining working for the principal is always better than after there has been termination (both when exogenous terminations can occur, and when they do not).

Now we turn to compare the distributions of the posterior when the dismissal policy changes.

Lemma 13 Consider $\underline{\hat y} \ge \underline{\hat y}'$. Then, $f\left[\eta \mid \underline{\hat y}, \hat e, \text{in}\right]$ first-order stochastically dominates $f\left[\eta \mid \underline{\hat y}', \hat e, \text{in}\right]$, and $f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}\right]$ first-order stochastically dominates $f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}'\right]$.

Proof. From lemma 12 it suffices to show that $f\left[\eta \mid \underline{\hat y}, \hat e, \text{in}\right] \le f\left[\eta \mid \underline{\hat y}', \hat e, \text{in}\right]$ if and only if $\eta \le \hat\eta$ and $f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}'\right] \le f\left[\eta \mid \hat e, y_1 \le \underline{\hat y}\right]$ if and only if $\eta \le \hat\eta'$. Moreover, since all the densities are multiplied by $\frac{1}{\sigma_\theta}\phi\left(\frac{\theta-\bar\theta}{\sigma_\theta}\right)$ the condition simplifies to checking that

$$\lambda(\theta) = \frac{1-\Phi\left(\frac{\underline{\mu}-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)}{1-\Phi\left(\frac{\underline{\mu}-\bar{\mu}-\bar\theta}{\sigma_{\varepsilon+\theta}}\right)} \le \lambda'(\theta) = \frac{1-\Phi\left(\frac{\underline{\mu}'-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)}{1-\Phi\left(\frac{\underline{\mu}'-\bar{\mu}-\bar\theta}{\sigma_{\varepsilon+\theta}}\right)} \text{ if and only if } \theta \le \hat\theta$$

$$\gamma(\theta) = \frac{\Phi\left(\frac{\underline{\mu}-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{\underline{\mu}-\bar{\mu}-\bar\theta}{\sigma_{\varepsilon+\theta}}\right)} \le \gamma'(\theta) = \frac{\Phi\left(\frac{\underline{\mu}'-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{\underline{\mu}'-\bar{\mu}-\bar\theta}{\sigma_{\varepsilon+\theta}}\right)} \text{ if and only if } \theta \le \hat\theta'$$

To check these conditions we will look at the derivatives with respect to θ. The denominators do not depend on θ, and hence can be treated as constants. Then, $\lambda_\theta(\theta) = K\cdot\phi\left(\frac{\underline{\mu}-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)$ and $\lambda'_\theta(\theta) = K'\cdot\phi\left(\frac{\underline{\mu}'-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)$. Taking logarithms on these expressions, we have that $\lambda_\theta(\theta) \le \lambda'_\theta(\theta)$ if and only if $\log(K/K') - \frac{1}{2}\left(\frac{\underline{\mu}-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)^2 \le -\frac{1}{2}\left(\frac{\underline{\mu}'-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)^2$. Rearranging we obtain $2\log(K/K') \le \left(\frac{\underline{\mu}-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)^2 - \left(\frac{\underline{\mu}'-\bar{\mu}-\theta}{\sigma_\varepsilon}\right)^2 = \sigma_\varepsilon^{-2}\left(\underline{\mu}'+\underline{\mu}'-2\bar{\mu}-2\theta\right)\left(\underline{\mu}-\underline{\mu}'\right)$. Therefore, $\lambda_\theta(\theta) \le \lambda'_\theta(\theta)$ if and only if $\theta \le -\frac{\sigma_\varepsilon^2\log(K/K')}{(\underline{\mu}-\underline{\mu}')} - \bar{\mu} + \frac{1}{2}\left(\underline{\mu}+\underline{\mu}'\right)$. The first-order stochastic dominance then follows from lemma 13. The proof of the second case follows the same steps. ∎

When the threshold for the dismissal policy increases (meaning more agents will get dismissed), both the pool of agents that manage to continue and those who get dismissed is improved. This result is intuitive: when the bar is raised, less people exceeds it. As a consequence, only the best agents get selected to continue. But at the same time, more of the good agents will fail, improving also the pool of dismissed agents.

Remark 2. The same observation made in remark 1 is still valid for both lemmas 13 and 14. Namely that the expectation of an increasing function of ability is strictly higher under $s = in$ than under $s = out$, and higher for a bigger $\underline{\mu}$ under $s = in$, and $s = out$ with no exogenous termination.

In order to compute the equilibrium, it will be useful to have an expression for the outsiders' expectation of ability after they observe a termination. We develop this next. From the posterior distribution we have:

$$E\left[\eta \mid \underline{y}; b; out\right] = \int_{-\infty}^{+\infty} \eta \cdot \frac{\tau + (1-\tau)\Phi\left(\frac{\underline{y}-b-\eta}{\sigma_{\varepsilon}}\right)}{\tau + (1-\tau)\Phi\left(\frac{\underline{y}-b-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)} \cdot \frac{1}{\sigma_{\eta}}\phi\left(\frac{\eta-\bar{\eta}}{\sigma_{\eta}}\right) d\eta \tag{12}$$

$$= \mu\left(\underline{y}\right)\bar{\eta} + \left(1-\mu\left(\underline{y}\right)\right) \cdot \int_{-\infty}^{+\infty} \eta \cdot \frac{\Phi\left(\frac{\underline{y}-b-\eta}{\sigma_{\varepsilon}}\right)}{\Phi\left(\frac{\underline{y}-b-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)} \cdot \frac{1}{\sigma_{\eta}}\phi\left(\frac{\eta-\bar{\eta}}{\sigma_{\eta}}\right) d\eta$$

$$= \mu\left(\underline{y}\right)\bar{\eta} + \left(1-\mu\left(\underline{y}\right)\right) \cdot E\left[\eta \mid b; y_1 < \underline{y}\right]$$

where $\mu\left(\underline{y}\right) = \frac{\tau}{\tau + (1-\tau)\Phi\left(\frac{\underline{y}-b-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}$. The last integral term in the second line represents the expectation of ability provided there is no exogenous termination. The result is then a convex combination between the prior expectation of ability $\bar{\eta}$ (which corresponds to the case there is an exogenous termination and outsiders learn nothing) and the posterior in the case termination occurred after a dismissal, $E\left[\eta \mid b; y_1 < \underline{y}\right]$. The weights reflect the relatively likelihood of each case. We will now derive an expression for the last term.

By the law of iterated expectations, we can write:

$$\begin{aligned} E\left[\eta \mid b; y_1 < \underline{y}\right] &= E\left[E\left[\eta \mid b; y_1 < \underline{y}; y_1 = y\right] \mid b; y < \underline{y}\right] \\ &= E\left[E\left[\eta \mid b; y_1 = y\right] \mid b; y < \underline{y}\right] \end{aligned}$$

The interior expectation corresponds to the posterior expectation of ability provided a first period output of y has been observed. This expression takes the same linear form we obtained for the observable output case: $\alpha\bar{\eta} + (1-\alpha)(y - b)$. If we substitute for it, we obtain:

$$\begin{aligned} &= E\left[\alpha\bar{\eta} + (1-\alpha)(y-b) \mid b; y < \underline{y}\right] \\ &= \alpha\bar{\eta} + (1-\alpha) \cdot E\left[y - b \mid b; y < \underline{y}\right] \\ &= \alpha\bar{\eta} + (1-\alpha) \cdot E\left[\eta + \varepsilon \mid \eta + \varepsilon < \underline{y} - b\right] \end{aligned}$$

This last term corresponds to the truncated expectation of $\eta + \varepsilon$ given that $\eta + \varepsilon < \underline{y} - b$. Since $\eta + \varepsilon$

is normally distributed, this expectation displays a familiar form:

$$= \alpha\bar{\eta} + (1-\alpha)\cdot\left[\bar{\eta} - \sigma_{\varepsilon+\eta}^{-1}\cdot\frac{\phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}{\Phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}\right]$$

And, hence, the posterior expectation of ability when no exogenous termination occurs becomes:

$$E\left[\eta \mid \hat{b}, y_1 < \underline{y}\right] = \bar{\eta} - (1-\alpha)\cdot\sigma_{\varepsilon+\eta}^{-1}\cdot\frac{\phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}{\Phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}$$

Finally, substituting back into (12) we obtain:

$$\begin{aligned} E\left[\eta \mid \underline{y}; \hat{b}; out\right] &= \bar{\eta} - \left(1-\mu\left(\underline{y}\right)\right)\cdot(1-\alpha)\cdot\sigma_{\varepsilon+\eta}\cdot\frac{\phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}{\Phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)} \\ &= \bar{\eta} - (1-\alpha)\cdot\sigma_{\varepsilon+\eta}\cdot\frac{(1-\tau)\phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)}{\tau+(1-\tau)\Phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right)} \qquad (13)\end{aligned}$$

Now we are ready to study the equilibrium dismissal policy for the renegotiation case. The following result states its existence and uniqueness:

Proposition 14 Suppose that renegotiation is possible. Then, for each value $\sigma \in \{0; 1\}$, there exists a unique equilibrium dismissal policy satisfying the equation $E\left[y_2 \mid \underline{y}; \hat{b}\right] + \sigma\cdot E\left[v(i;\eta) \mid \underline{y}; \hat{b}; in\right] = E\left[y_2 \mid \underline{y}; \hat{b}; out\right] + \sigma\cdot E\left[v(i;\eta) \mid \underline{y}; \hat{b}; out\right]$.

Proof. First notice that $E\left[y_2 \mid \underline{y}; \hat{b}\right] = \alpha\bar{\eta} + (1-\alpha)\left(\underline{y} - \hat{b}\right)$ takes values in $(-\infty; +\infty)$, as a function of $\underline{y}$. Moreover, $E\left[y_2 \mid \underline{y}; \hat{b}; out\right]$ is bounded, since $\phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right) \in [0; \phi(0)]$ and $\tau + (1-\tau)\Phi\left(\frac{\underline{y}-\hat{b}-\bar{\eta}}{\sigma_{\varepsilon+\eta}}\right) \in [\tau; 1]$ is bounded away from 0. Furthermore, $E\left[v(i;\eta) \mid \underline{y}; \hat{b}; in\right]$ and $E\left[v(i;\eta) \mid \underline{y}; \hat{b}; out\right]$, are both bounded, since so it is $v(i;\eta)$. Then, the existence of an equilibrium when $\sigma = 0$ and $\sigma = 1$ follows immediately from continuity.

When $\sigma = 0$ the equilibrium condition translates into

$$\alpha\eta + (1-\alpha)\left(\underline{y} - \bar{\theta}\right) = \eta - (1-\alpha)\cdot\sigma_\varepsilon+\eta\cdot\frac{(1-\tau)\phi\left(\frac{\underline{y}-\bar{\theta}-\eta}{\sigma_\varepsilon+\eta}\right)}{\tau+(1-\tau)\Phi\left(\frac{\underline{y}-\bar{\theta}-\eta}{\sigma_\varepsilon+\eta}\right)}$$

$$\left(\frac{\underline{y}-\bar{\theta}-\eta}{\sigma_\varepsilon+\eta}\right) = \frac{-(1-\tau)\phi\left(\frac{\underline{y}-\bar{\theta}-\eta}{\sigma_\varepsilon+\eta}\right)}{\tau+(1-\tau)\Phi\left(\frac{\underline{y}-\bar{\theta}-\eta}{\sigma_\varepsilon+\eta}\right)}$$

To simplify notation, we will denote $z = \left(\frac{\underline{y}-\bar{\theta}-\eta}{\sigma_\varepsilon+\eta}\right)$ from now on. The equation simplifies to $z = \frac{-(1-\tau)\phi(z)}{\tau+(1-\tau)\Phi(z)}$. Rearranging, $z\tau = -(1-\tau)[\phi(z)+z\Phi(z)]$. The left-hand side is increasing in z, and takes values from $-\infty$ to $+\infty$. The derivative of the right-hand side is $-(1-\tau)\left[\phi'(z)+\Phi(z)+z\phi(z)\right] = -(1-\tau)\Phi(z) < 0$, since the density of a normal distribution satisfies $\phi'(z) = -z\phi(z)$. Hence, the two lines must intersect only once, and the equilibrium is unique.

When $\sigma = 1$ the equilibrium condition becomes

$$z + \frac{(1-\tau)\phi(z)}{\tau+(1-\tau)\Phi(z)} + \frac{\sigma}{(1-\tau)\sigma_\varepsilon+\eta}\cdot\left(E\left[v(i;\eta)\mid\underline{y};\bar{\theta},in\right] - E\left[v(i;\eta)\mid\underline{y};\bar{\theta},out\right]\right) = 0$$

or equivalently, multiplying by $\tau+(1-\tau)\Phi(z)$:

$$z\tau + (1-\tau)[\phi(z)+z\Phi(z)] + \frac{\tau+(1-\tau)\Phi(z)}{(1-\tau)\sigma_\varepsilon+\eta}\cdot\left(E\left[v(i;\eta)\mid\underline{y};\bar{\theta},in\right] - E\left[v(i;\eta)\mid\underline{y};\bar{\theta},out\right]\right) = 0$$

Similarly to the decomposition in (12), we can express $E\left[v(i;\eta)\mid\underline{y};\bar{\theta},out\right]$ as

$$E\left[v(i;\eta)\mid\underline{y};\bar{\theta},out\right] = \mu\left(\underline{y}\right)E[v(i;\eta)] + \left(1-\mu\left(\underline{y}\right)\right)\cdot E\left[v(i;\eta)\mid\bar{\theta},y_1<\underline{y}\right]$$

where $\mu\left(\underline{y}\right) = \frac{\tau}{\tau+(1-\tau)\Phi(z)}$. As a result, we get

$$\begin{aligned}
&[\tau+(1-\tau)\Phi(z)]\cdot\left(E\left[v(i;\eta)\mid\underline{y};\bar{\theta},in\right] - E\left[v(i;\eta)\mid\underline{y};\bar{\theta},out\right]\right) && (14)\\
=\ &[\tau+(1-\tau)\Phi(z)]\cdot E\left[v(i;\eta)\mid\underline{y};\bar{\theta},in\right] - \tau E[v(i;\eta)] - (1-\tau)\Phi(z)\cdot E\left[v(i;\eta)\mid\bar{\theta},y_1<\underline{y}\right]\\
=\ &\tau\left(E\left[v(i;\eta)\mid\underline{y};\bar{\theta},in\right] - E[v(i;\eta)]\right) - (1-\tau)\Phi(z)\cdot\left(E\left[v(i;\eta)\mid\underline{y};\bar{\theta},in\right] - E\left[v(i;\eta)\mid\bar{\theta},y_1<\underline{y}\right]\right)
\end{aligned}$$

Consider now the following identity[21]:

$$E\left[v(i;\eta)\right] = \left[1-\Phi(\imath)\right]\cdot E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right] + \Phi(\imath)\cdot E\left[v(i;\eta)\,|\,\hat{e};y_1<\underline{y}\right]$$

It states that the prior expectation of outside output is a convex combination of the posterior expectations after first period output is above and below $\underline{y}$. And the weights reflect the probability of each event occurring. Rearranging we obtain:

$$E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right] - E\left[v(i;\eta)\right] = \Phi(\imath)\cdot\left(E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right] - E\left[v(i;\eta)\,|\,\hat{e};y_1<\underline{y}\right]\right)$$

Substituting this back into (14) gives us

$$\left[\tau+(1-\tau)\Phi(\imath)\right]\cdot\left(E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right] - E\left[v(i;\eta)\,|\,\underline{y};\hat{e};out\right]\right) = E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right] - E\left[v(i;\eta)\right]$$

Finally, the equilibrium equation simplifies to

$$\imath\tau+(1-\tau)\left[\phi(\imath)+\imath\Phi(\imath)\right]+\frac{1}{(1-\tau)^{\sigma\varepsilon+\eta}}\cdot\left(E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right]-E\left[v(i;\eta)\right]\right)=0 \qquad (15)$$

Now it is immediate to see the left-hand side is strictly increasing in $\underline{y}$, since $\imath$, $[\phi(\imath)+\imath\Phi(\imath)]$ and $E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right]$ are. Hence, the equilibrium must be unique also when $\sigma=1$. ■

Now we know the equilibrium dismissal policy must be unique, we are ready to see some of its properties. This is done in the next proposition.

Proposition 15 The dismissal policy $\underline{y}$ is strictly decreasing in σ whenever $v_\eta(i;\eta)>0$. Moreover, if $v_{i\eta}(i;\eta)>0$, then $\underline{y}$ is strictly decreasing in i when $\sigma=1$.

Proof. From equation (15) we can rewrite the equilibrium equation for a general σ as:

$$\imath\tau+(1-\tau)\left[\phi(\imath)+\imath\Phi(\imath)\right]+\frac{\sigma}{(1-\tau)^{\sigma\varepsilon+\eta}}\cdot\left(E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right]-E\left[v(i;\eta)\right]\right)=0$$

where the left hand side is increasing in $\underline{y}$. The equilibrium is determined by the unique value of $\underline{y}$ at which the left-hand side crosses 0. Since the posterior distribution after $y_1\geq\underline{y}$ first-order stochastically dominates the prior distribution, we have $\left(E\left[v(i;\eta)\,|\,\underline{y};\hat{e};in\right]-E\left[v(i;\eta)\right]\right)>0$. Making $\sigma=1$,

[21] This identity can be easily verified manipulating the integral expressions for the expectations.

then, shifts the left-hand side upwards. As a result, the crossing point must decrease. Similarly, when $v_{i\eta}(i,\eta) > 0$, $\left(E\left[v_i(i,\eta) \mid \underline{\underline{y}}, \hat{e}, in\right] - E\left[v_i(i,\eta)\right]\right) > 0$. An increase in i has then the same effect as σ. ■

We now have all the ingredients to show the results in proposition 8.

Proof (Proposition 8). In both the renegotiation and no renegotiation cases the agent solves the following problem:

$$\max_{e,i} \left\{ w_1(\hat{e}) - c_e(e) - c_i(i) + \underline{u} + (1-\tau)\left[1 - \Phi\left(\frac{\underline{y} - \mu - e}{\sigma_{\varepsilon+\eta}}\right)\right] \cdot \left[\Delta w_2 + \sigma \cdot \Delta v\right] \right\}$$

Consider first the no renegotiation case. Here, $\Delta w_2 = w_2 - E\left[y_2 \mid \underline{y}, \hat{e}, out\right]$, and $\Delta v = E\left[v(i,\eta) \mid \underline{y}, \hat{e}, in\right] - E\left[v(i,\eta) \mid \underline{y}, \hat{e}, out\right]$. If $v_\eta(i,\eta) = 0$, we can write $v(i,\eta) = v(i)$, and if follows that $\Delta v = 0$. The objective function (and hence optimal effort) is then independent of σ. From lemma 13, $\Delta v > 0$ whenever $v_\eta(i,\eta) > 0$, and hence, incentives increase. Finally, when $v_{i\eta}(i,\eta) > 0$, $\Delta v_i = E\left[v_i(i,\eta) \mid \underline{y}, \hat{e}, in\right] - E\left[v_i(i,\eta) \mid \underline{y}, \hat{e}, out\right] > 0$, again from lemma 13. The objective function is then supermodular in (e, i), since

$$\frac{\partial U}{\partial e \partial i} = (1-\tau) \cdot \sigma_{\varepsilon+\eta}^{-1} \cdot \phi\left(\frac{\underline{y} - \mu - e}{\sigma_{\varepsilon+\eta}}\right) \cdot \frac{\partial \Delta v}{\partial i} > 0$$

When renegotiation is possible, then $\Delta w_2 + \sigma \cdot \Delta v = w_2 - \left[\alpha\mu + (1-\alpha)\left(\underline{\underline{y}} - \hat{e}\right)\right]$. From the previous proposition, it follows that $\underline{\underline{y}}$ is independent of σ if $v_\eta(i,\eta) = 0$. Effort is then unaffected by the policy on the outside activity. When $v_\eta(i,\eta) > 0$, $\underline{\underline{y}}$ is decreasing in σ, and hence incentives increase, since so it does $\Delta w_2 + \sigma \cdot \Delta v$. Finally, the objective function is supermodular in (e, i) if $v_{i\eta}(i,\eta) > 0$, since

$$\frac{\partial U}{\partial e \partial i} = (1-\tau) \cdot \sigma_{\varepsilon+\eta}^{-1} \cdot \phi\left(\frac{\underline{y} - \mu - e}{\sigma_{\varepsilon+\eta}}\right) \cdot \left(-(1-\alpha)\frac{\partial \underline{\underline{y}}}{\partial i}\right) > 0$$

This concludes the proof for both cases. ■

The proof of proposition 9 now follows.

Proof (Proposition 9). Suppose that $\sigma = 0$. Then, the first order condition with respect to effort is

$$(1-\tau) \cdot \sigma_{\varepsilon+\eta}^{-1} \cdot \phi\left(\frac{\underline{y} - \mu - e}{\sigma_{\varepsilon+\eta}}\right) \cdot \Delta w_2 = c_e'(e)$$

Since $c_e'(0) \leq 0$, the solution to the agent's problem will always be interior and satisfy this equation.

Moreover, if the cost functions is sufficiently convex, the solution will be unique. To see this, consider the second derivative of the objective function: $-(1-\tau)\cdot\sigma_{\varepsilon+\eta}^{-2}\cdot\phi'\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)\cdot\Delta w_2 - c_e''(e)$. The first term is bounded, both when renegotiation is allowed, and when it is not. To see this, notice that $\underline{y}=\frac{w_2-\alpha\beta}{1-\alpha}+\hat{e}$ is a linear function of w_2. With no renegotiation, $\Delta w_2 = w_2 - E\left[y_2 \mid \underline{y};\hat{e},out\right]$, where $E\left[y_2 \mid \underline{y};\hat{e},out\right]$ is bounded. When renegotiation is allowed, $\Delta w_2 = w_2 - \left[\alpha\beta+(1-\alpha)\left(\underline{\underline{y}}-\hat{e}\right)\right]$, where $\underline{\underline{y}}$ is independent of w_2. Then, for any w_2, $-(1-\tau)\cdot\sigma_{\varepsilon+\eta}^{-2}\cdot\phi'\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)\cdot\Delta w_2$ is bounded, since so they are all the terms of the form $K\cdot\phi'\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)$ and $K'\cdot w_2\cdot\phi'\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)$, for K constant. Denote this bound by $M>0$. Then, the existence of a unique solution to the first order condition is guaranteed when $c_e''(e)>M$. In that case, we can obtain the sensitivity of effort to the conjecture of effort by differentiating the equation above:

$$\frac{de^*}{d\hat{e}}=\frac{-(1-\tau)\cdot\sigma_{\varepsilon+\eta}^{-2}\cdot\phi'\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)\cdot\Delta w_2\cdot\frac{d\underline{y}}{d\hat{e}}}{c_e''(e)-(1-\tau)\cdot\sigma_{\varepsilon+\eta}^{-2}\cdot\phi'\left(\frac{\underline{y}-\beta-e}{\sigma_{\varepsilon+\eta}}\right)\cdot\Delta w_2\cdot\frac{d\underline{y}}{d\hat{e}}}<1$$

The denominator is always positive by the condition $c_e''(e)>M$. The derivative is then no larger than one, since the denominator is larger than the numerator. As a result, the equation $\hat{e}=e^*(\hat{e};\sigma=0)$ will have a single solution. For the comparison of the equilibrium effort with the case of $\sigma=1$, the proof follows the same steps as in proposition 4. ■

Table 1. Definitions

Variable Name	Definition (Survey Question)
Dependent Variables	
Income from Outside Activities	In recent years, roughly how much have you earned over and avobe your basic salary: 1 = 0%, 2 = under 10%, 3 = 10-19%, 4 = 20-29%, 5 = 30-39%, 6 = 40-49%, 7 = 50% and over.
Effort in Research	How often, on average, do you spend 4 hours uninterruptedly on professionally reading, writing or research: 1 = once a year or less, 2 = a few times a year, 3 = about once a month, 4 = two or three times a month, 5 = once a week or more.
Independent Variables	
Time on Consulting	In a normal week, what proportion of your work time is devoted to consulting with or without pay: 1 = 0%, 2 = 1-10%, 3 = 11-20%, 4 = 21-40%, 5 = 41-60%, 6 = 61-80%, 7 = 81-100%.
Time on Private Practice	In a normal week, what proportion of your work time is devoted to outside professional practice: 1 = 0%, 2 = 1-10%, 3 = 11-20%, 4 = 21-40%, 5 = 41-60%, 6 = 61-80%, 7 = 81-100%.
Control Variables	
Experience	How long have you been employed (beyond the level of teaching or research assistant) in colleges or universities: 1 = One year or less, 2 = 2-3 years, 3 = 4-6 years, 4 = 7-9 years, 5 = 10-14 years, 6 = 15-19 years, 7 = 20-29 years, 8 = 30 or more years.
Salary	What is your basic institutional salary, before tax, and deductions, for the current accademic year: 1 = below $7,000, 2 = $7,000 - $9,999, 3 = $10,000 - $11,999, 4 = $12,000 - $13,999, 5 = $14,000 - $16,999, 6 = $17,000 - $19,999, 7 = $20,000 - $24,999, 8 = $25,000 - $29,999, 9 = $30,000 and over.
12 Months	Is your basic institutional salary based on: 1 = 9/10 months, 2 = 11/12 months.
Rank	What is your present rank: 1 = professor, 2 = associate professor, 3 = assistant professor, 4 = instructor, 5 = lecturer, 6 = no rank designated, 7 = other.
Quality of Institution	Index of school quality: 3 = lowest quality rating, 27 = highest quality rating.
Highest Degree	On the following list of large American universities, mark where you obtained your highest degree.
Field	From the following list, mark your department of teaching appointment.

Table 2. Outside activities of faculty members.

Fraction (%) of respondents

	Social Sciences	Humanities	Natural Sciences	Business Applied	Total
Time spent doing consulting					
0%	38.85	56.46	54.57	33.78	41.47
1-10%	43.85	31.51	36.54	46.96	41.85
11-20%	13.05	8.67	6.7	15.24	11.98
21-40%	3.16	2.59	1.47	2.95	3.47
41-60%	0.69	0.59	0.51	0.68	0.85
61-80%	0.25	0.14	0.19	0.34	0.3
81-100%	0.15	0.03	0.03	0.04	0.08
Time spent on outside professional practice					
0%	75.98	76.61	84.45	67.98	71.46
1-10%	16.41	15.34	11.84	23.5	18.63
11-20%	4.5	4.25	2.16	5.14	5.09
21-40%	1.78	2.07	0.48	1.33	2.21
41-60%	0.3	1	0.29	0.6	1.15
61-80%	0.54	0.48	0.37	0.6	0.71
81-100%	0.49	0.24	0.4	0.86	0.74
Income from outside activities (as fraction of basic salary)					
0%	14.19	26.54	21.34	14.81	20.49
Under 10%	26.4	33.97	26.01	23.5	29.72
10-19%	24.22	21.77	21.21	19.65	20.85
20-29%	18.78	9.95	18.51	16.99	14.06
30-39%	7.41	2.87	7.26	10.06	6.15
40-49%	2.42	1.45	1.97	4.67	2.36
50% and over	6.57	3.46	3.71	10.34	6.37
Largest source of suplementary earnings					
Summer Teaching	25.16	33.34	20.17	15.67	23.55
Teaching Elsewhere	4.35	5.15	2.88	4.37	4.25
Consulting	14.78	3.14	11.76	25.21	12.5
Private Practice	3.36	2.87	0.56	3.77	4.76
Royalties	5.54	6.6	3.6	2.87	4.17
Speeches and Lectures	3.81	4.22	3.33	1.93	3.75
Research Salaries	17.3	4.01	24.01	12.8	11.05
Other	10.83	13.13	11.68	17.98	14.54
None	14.88	27.54	22.01	15.41	21.43
Observations	2023	2894	3749	2336	16680

Table 3. Fraction of Income From Outside Activities

OLS regressions of Income from Outside Activities on Effort in Research and other controls.

	(1)	(2)	(3)	(4)	(5)	(6)
Effort in Research	0.045	0.061	0.054	0.047	0.052	0.030
	(4.41)**	(6.35)**	(5.70)**	(5.04)**	(5.55)**	(3.19)**
Time on Consulting		0.214	0.188	0.207	0.207	0.214
		(13.04)**	(11.90)**	(13.36)**	(13.39)**	(14.06)**
Time on Private Practice		0.450	0.468	0.462	0.459	0.456
		(26.77)**	(27.78)**	(27.22)**	(27.13)**	(27.45)**
Salary				0.012	0.006	-0.020
				(1.03)	(0.56)	(1.74)*
12 Months				-0.766	-0.765	-0.766
				(27.66)**	(27.68)**	(27.83)**
Experience					0.042	0.046
					(5.33)**	(5.77)**
Quality of Institution						0.034
						(13.31)**
Controls:						
Rank	No	No	Yes	Yes	Yes	Yes
Field	No	No	Yes	Yes	Yes	Yes
Highest Degree	No	No	No	No	No	Yes
Observations	16680	16680	16680	16680	16680	16680
R-squared	0.00	0.10	0.20	0.24	0.24	0.26

Robust t statistics in parentheses

* significant at 5%; ** significant at 1%

Table 4. Fraction of Income From Outside by type of Activity

OLS regressions of Income from Outside Activities on Effort in Research and controls. In each regression, the sample is restricted to those faculty members whose stated largest source of supplementary earnings (from outside activities) corresonds to the category in the column heading.

	Summer Teaching	Teaching Elsewhere	Consulting	Private Practice	Royalties	Speeches and Lectures	Research Salaries
Effort in Research	0.027	0.127	0.058	-0.117	0.168	0.032	0.061
	(2.06)*	(3.00)**	(2.22)*	(2.49)**	(3.15)**	(1.14)	(2.23)*
Time on Consulting	0.078	0.053	0.483	0.232	0.398	0.048	0.252
	(3.57)**	(0.81)	(12.14)**	(3.74)**	(4.43)**	(1.17)	(5.60)**
Time on Private Practice	0.020	0.273	0.171	0.486	0.488	0.435	0.293
	(0.85)	(4.63)**	(3.82)**	(11.38)**	(4.89)**	(5.15)**	(4.73)**
Salary	0.029	-0.122	-0.046	-0.179	0.020	0.009	-0.004
	(1.59)	(2.65)**	(1.53)	(4.31)**	(0.33)	(0.28)	(0.13)
12 Months	-0.366	-0.470	-1.076	-0.039	-0.823	-0.341	-0.518
	(7.93)**	(4.03)**	(12.99)**	(0.25)	(5.78)**	(2.91)**	(4.64)**
Experience	-0.012	0.000	-0.021	0.084	0.077	0.003	-0.055
	(1.06)	(0.01)	(1.01)	(2.02)*	(1.73)*	(0.12)	(2.50)**
Quality of Institution	0.025	0.020	0.042	0.023	0.037	0.028	0.022
	(6.89)**	(1.88)*	(5.12)**	(2.06)*	(2.31)*	(3.05)**	(3.10)**
Controls:							
Rank	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Field	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Highest Degree	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3928	709	2085	794	696	626	1843
R-squared	0.14	0.34	0.41	0.51	0.41	0.43	0.24

Robust t statistics in parentheses

* significant at 5%; ** significant at 1%

Table 5. Research Effort as a Function of Time Doing Consulting and Private Practice

OLS regressions of Effort in Research on Time on Consulting and Private Practice and other controls.

	(1)	(2)	(3)	(4)	(5)
Time on Consulting	-0.010	0.032	0.025	0.024	0.035
	(0.85)	(2.64)**	(2.07)*	(2.03)*	(2.98)**
Time on Private Practice	-0.048	-0.023	-0.007	-0.001	-0.000
	(4.52)**	(2.12)*	(0.66)	(0.11)	(0.01)
Salary			0.091	0.099	0.062
			(10.32)**	(11.26)**	(7.04)**
12 Months			-0.139	-0.139	-0.142
			(5.79)**	(5.83)**	(6.04)**
Experience				-0.075	-0.076
				(11.57)**	(11.81)**
Quality of Institution					0.034
					(13.79)**
Controls:					
Rank	No	Yes	Yes	Yes	Yes
Field	No	Yes	Yes	Yes	Yes
Highest Degree	No	No	No	No	Yes
Observations	16680	16680	16680	16680	16680
R-squared	0.00	0.06	0.07	0.08	0.11

Robust t statistics in parentheses
* significant at 5%; ** significant at 1%