Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations

By YONGSUNG CHANG AND SUN-BIN KIM*

This paper addresses two related issues from the business cycle literature. One is the low correlation between hours and productivity. The second is the large cyclical movement in the wedge derived from the optimality condition for the intratemporal choice of commodity consumption and hours worked. The equilibrium business cycle models (e.g., Robert E. Lucas, Jr., and Leonard A. Rapping 1969; Finn E. Kydland and Edward C. Prescott 1982) impose strong restrictions on movements in consumption, hours, and productivity. According to these models, the economy puts in more work effort and consumes more goods when productivity is high (i.e., when the commodity is cheap relative to leisure). However, the lack of systematic movement among consumption, hours worked, and productivity in aggregate data has resulted in the measurement of a considerable wedge between the marginal rate of substitution and labor productivity—e.g., Robert E. Hall (1997) and Varadarajan V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan (2005). Previous research has offered various explanations for the low correlation between hours and productivity. For example, explanations range from exogenous shocks to the labor-supply schedule, such as the shifts in home production technology in Jess Benhabib, Richard Rogerson, and Randall Wright (1991) to frictions in labor supply, such as the wage rigidities in Jordi Galí, Mark Gertler, and J. David Lopez-Salido (2007).

In this paper, we obtain a low correlation between hours worked and productivity, where the only aggregate disturbance is a (market) technology shock and there is no distortion in the labor market. Our model extends Per Krusell and Anthony Smith's (1998) heterogeneous-agent model with incomplete capital markets (S. Rao Aiyagari 1994) to indivisible labor supply (Richard Rogerson 1988). The interaction between incomplete capital markets and indivisible labor breaks the tight link between employment and wages at the aggregate level. The optimality conditions for the choice of consumption and hours worked hold as inequality at the individual level. Individual optimality conditions do not aggregate nicely. In particular, aggregate employment is not highly correlated with productivity. As a result, we obtain a significant wedge between the marginal rate of substitution and labor productivity.[1] Moreover, the wedge computed from the model-generated aggregate consumption, hours, and productivity exhibits properties similar to those in the wedge measured from the actual aggregate time series data. The wedge is strongly correlated with hours and is almost as volatile as hours worked. Our results caution against viewing the measured wedge as an inefficiency due to the failure of labor-market clearing or as a fundamental driving force behind business cycles.

The paper is organized as follows. Section I briefly discusses the labor-market wedge in the aggregate data. Section II lays out a benchmark model economy in which the capital market is incomplete and labor supply is indivisible. In Section III, we calibrate the model economy and study the cyclical properties of the aggregate variables in the face of productivity shocks. In Section IV, we investigate economies with and without complete capital markets and indivisible labor, in order to distinguish the separate

* Chang: Department of Economics, University of Rochester, Rochester, NY 14627, and Yonsei University (e-mail: ychang14@mail.rochester.edu); Kim: Department of Economics, Korea University, Anam-Dong, Seongbuk-Gu, Seoul, Korea 136-701 (e-mail: sunbink@korea.ac.kr). We would like to thank Mark Bils, Huberto Ennis, Bob King, Per Krusell, the editor, two anonymous referees, and the members of the Research Department at the Federal Reserve Bank of Richmond for their helpful comments. Sun-Bin Kim's research is supported by a Korea University research grant.

[1] Jose Sheinkman and Laurence Weiss (1986) show that capital-market incompleteness can lead to a stochastic term in aggregate preferences. Tomoyuki Nakajima (2005) derives aggregate preference shocks and total factor productivity (TFP) variation in a two-type household model with capacity utilization and government spending shocks.

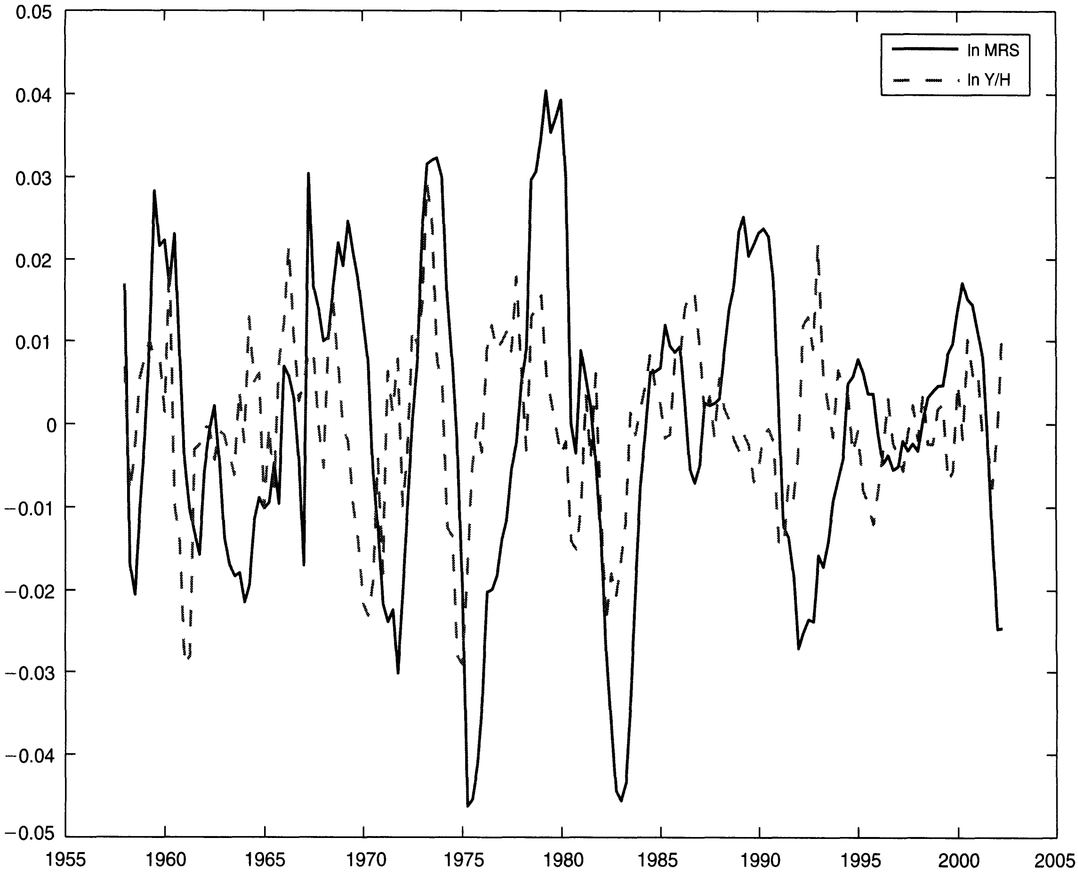

FIGURE 1. CYCLICAL COMPONENTS OF MRS AND LABOR PRODUCTIVITY

Notes: Output and hours worked represent the nonagricultural private sector. Consumption reflects expenditure on nondurable goods and services. The MRS is defined by equation (1).

role of incomplete capital markets and indivisible labor. Section V concludes.

I. Labor-Market Wedge in Aggregate Data

One of the leading research topics in macroeconomics is the identification of the fundamental driving forces behind economic fluctuations. Economists adopt accounting procedures that combine aggregate time-series data with the equilibrium conditions of a prototype model. For optimal allocation of consumption and hours worked, the marginal rate of substitution (MRS) has to equal the marginal product of labor (MPL). To illustrate, suppose that the stand-in household has the following utility function over commodity consumption C_t and hours worked H_t: $U(C_t, H_t) = \ln C_t - B[H^{1+1/\gamma}/(1 + 1/\gamma)]$. The parameter γ represents the (compensated) labor-supply elasticity and B is a constant. Under the assumption that the aggregate production technology is Cobb-Douglas (with the labor-income share denoted by α), at the competitive equilibrium, the MRS should be equal to the MPL:

$$(1) \qquad B\frac{H_t^{1/\gamma}}{C_t^{-1}} = \alpha\frac{Y_t}{H_t}.$$

Figure 1 shows the cyclical components of the MRS (the left-hand side of (1)) and labor productivity (the right-hand side of (1)) for the US economy for 1958:I–2002:II. In computing the MRS, we assume that the aggregate labor-supply elasticity

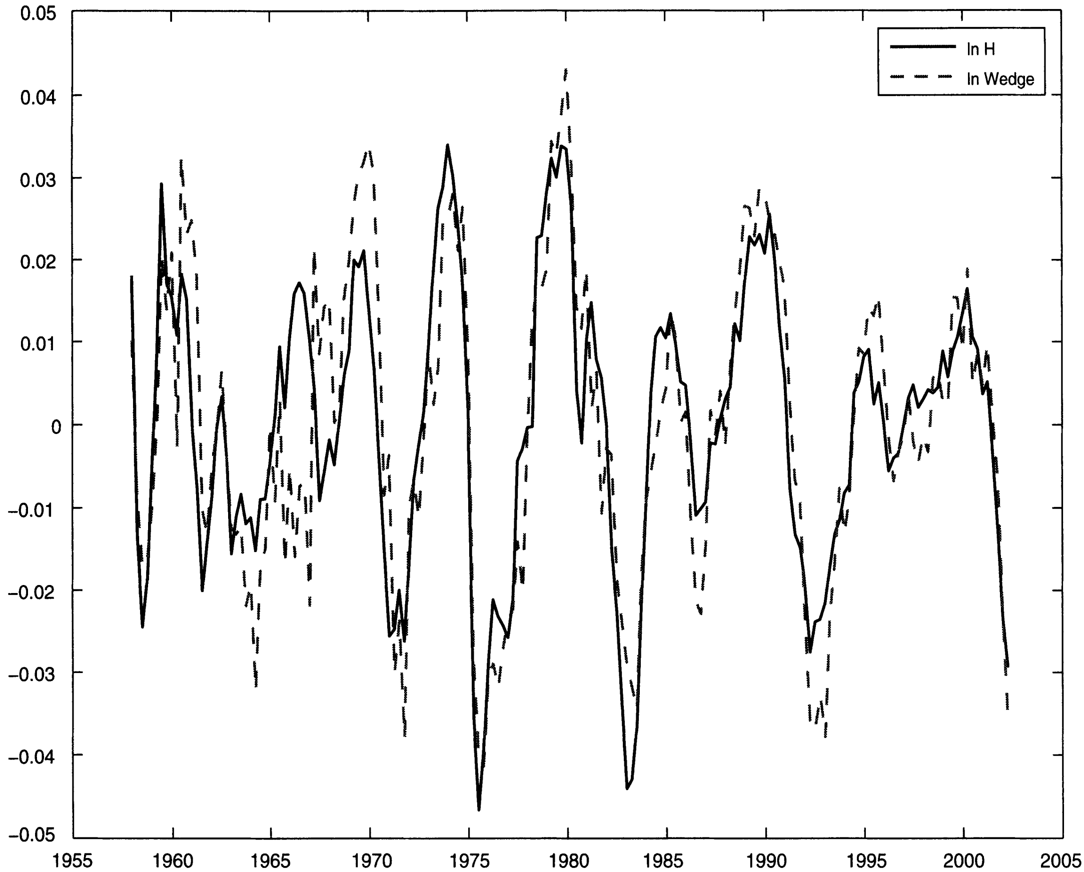

FIGURE 2. CYCLICAL COMPONENTS OF HOURS AND LABOR-MARKET WEDGE FOR THE UNITED STATES

Note: The wedge is computed from equation (2) with the aggregate labor-supply elasticity (γ) of 1.5.

γ is 1.5.[2] Output and hours worked are based on the private business sector. Consumption reflects expenditures on nondurable goods and services. Both the MRS and labor productivity are logged and detrended using a Hordrick-Prescott filter. The MRS is more volatile than hours and, more importantly, often moves in the opposite direction to productivity, suggesting a serious departure from the competitive equilibrium.

We now define the labor-market wedge as the gap between the MRS and labor productivity:

$$\text{(2)} \quad \ln \text{Wedge}_t = \ln MRS_t - \ln \frac{Y_t}{H_t} + constant.$$

Figure 2 shows the time series of this wedge. The wedge is highly correlated with hours worked, and its volatility is the same order of magnitude as hours worked. The aggregate labor-supply elasticity of 1.5 is higher than a typical estimate in the micro data, which is usually less than 0.5 (e.g., Thomas MaCurdy 1981). If we assume an inelastic labor supply (a smaller value of γ), we obtain a bigger wedge as the MRS becomes more volatile.[3] Conversely, using an elastic labor supply (a bigger value of γ) tends to produce a smaller wedge. Nevertheless, there is no choice of γ that eliminates the wedge completely. In essence, the wedge arises because hours worked are not highly correlated with productivity—the correlation coefficient between the two time series is virtually zero (0.08).

[2] The choice of this value will be explained in Section IVB.

[3] For example, Hall (1997) uses γ=1/1.7. We have also computed the wedge based on the real wage (instead of

The existing literature offers various interpretations for this wedge. They range from exogenous shocks to the labor-supply schedule, e.g., preference shifts in Hall (1997) and Allison Holland and Andrew Scott (1998); changes in home production technology in Benhabib, Rogerson, and Wright (1991); shifts in government spending in Lawrence Christiano and Martin Eichenbaum (1992); and changes in labor-income taxes in Casey B. Mulligan (2002), to various frictions in the labor market, e.g., wage rigidity in Galí, Gertler, and Lopez-Salido (2007); households' market power in labor supply in Diego Comin and Gertler (2003); search frictions in Hall (1997); and labor unions and suspension of antitrust policy in Harold L. Cole and Lee. E. Ohanian (2002) and Chari, Kehoe, and McGrattan (2005). In the next section, we present a model economy in which the labor-market wedge arises endogenously, despite there being neither exogenous shocks to the labor supply nor distortions to the allocation of hours and consumption.

II. The Model

The model economy is a simplified version of Chang and Kim (2006) which extends Krusell and Smith's (1998) heterogeneous-agent model with incomplete capital markets to indivisible labor supply. There is a continuum (measure one) of workers who have identical preferences but different productivity. A worker has separable preferences over consumption, c_t, and hours worked, h_t: $\ln c_t - B[h_t^{1+1/\gamma}/(1+1/\gamma)]$. Workers trade claims for physical capital, a_t, which yields the rate of return, r_t. The capital markets are incomplete. Physical capital is the only asset available to workers, and workers face a borrowing constraint: $a_t \geq \bar{a}$ for all t (Aiyagari 1994). The labor supply is indivisible (Rogerson 1988).[4] If employed, a worker supplies $\bar{h}$ units of labor and earns $w_t x_t \bar{h}$, where w_t is the wage rate per effective unit of labor x_t, which varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. Individual productivity x_t represents idiosyncratic risks that agents face in our model economy and is the only source of heterogeneity.

The representative firm produces output according to a constant returns-to-scale Cobb-Douglas technology in capital, K_t (which depreciates at rate δ each period), and effective units of labor, $L_t(=\int h_t x_t d\mu)$, where μ is the distribution of workers:[5]

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha}.$$

The aggregate productivity λ_t evolves with a transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$.[6]

The value function for an employed worker, denoted by V^E, is:

$$(3)\quad V^E(a,x;\lambda,\mu) = \max_{a' \in \mathcal{A}} \Big\{ \ln c - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma} + \beta E\big[\max\{V^E(a',x';\lambda',\mu'), V^N(a',x';\lambda',\mu')\}|x,\lambda\big]\Big\},$$

subject to

$$c = w(\lambda,\mu)x\bar{h} + (1 + r(\lambda,\mu))a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbb{T}(\lambda,\mu),$$

where $\mathbb{T}$ denotes a transition operator that defines the law of motion for the distribution of

labor productivity) and the main conclusion of our analysis does not change. We prefer using labor productivity, since the standard argument that wages are not allocational suggests that the implications for wages are not fundamental.

[4] In general, the labor-supply decision operates on both the extensive and intensive margins. However, it is rare for workers to be allowed to choose completely flexible work schedules or to supply a small number of hours. Furthermore, it is well known that the variation in the number of employees is the dominant source of fluctuations in total hours worked (e.g., James J. Heckman 1984).

[5] This implicitly assumes that workers are perfect substitutes for each other. While this assumption abstracts from reality, it greatly simplifies the labor-market equilibrium.

[6] In this model economy, a productivity shock is the only aggregate disturbance. This does not necessarily reflect our view on the source of business cycles. Since we would like to show that the wedge contains a significant specification error, rather than true shifts in labor supply, we intentionally exclude shocks that may shift the labor-supply schedule itself (e.g., shifts in home production technology, government spending, or the income tax rate) from the present analysis.

workers $\mu(a, x)$.[7] The value function for a nonemployed worker, denoted by $V^N(a, x; \lambda, \mu)$, is defined similarly with $h = 0$. Then, the labor-supply decision is characterized by

$$V(a,x;\lambda,\mu) = \max_{h \in \{0, \bar{h}\}} \{V^E(a,x;\lambda,\mu),\ V^N(a,x;\lambda,\mu)\}.$$

Equilibrium consists of a set of value functions, $\{V^E(a, x; \lambda, \mu),\ V^N(a, x; \lambda, \mu),\ V(a, x; \lambda, \mu)\}$, a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a, x; \lambda, \mu),\ a'(a, x; \lambda, \mu),\ h(a, x; \lambda, \mu)\}$, aggregate inputs, $\{K(\lambda,\mu), L(\lambda,\mu)\}$, factor prices, $\{w(\lambda, \mu), r(\lambda, \mu)\}$, and a law of motion for the distribution $\mu' = \mathbb{T}(\lambda, \mu)$ such that:

- Individuals optimize: given $w(\lambda, \mu)$ and $r(\lambda, \mu)$, the individual decision rules $c(a, x; \lambda, \mu)$, $a'(a, x; \lambda, \mu)$, and $h(a, x; \lambda, \mu)$ solve $V^E(a, x; \lambda, \mu)$, $V^N(a, x; \lambda, \mu)$, and $V(a, x; \lambda, \mu)$.

- The representative firm maximizes profits: for all (λ, μ),

$$w(\lambda,\mu) = F_1(L(\lambda,\mu),K(\lambda,\mu),\lambda),$$

$$r(\lambda,\mu) = F_2(L(\lambda,\mu),K(\lambda,\mu),\lambda) - \delta.$$

- The goods market clears: for all (λ,μ),

$$\int \{a'(a,x;\lambda,\mu) + c(a,x;\lambda,\mu)\}\, d\mu$$

$$= F(L(\lambda, \mu),\ K(\lambda, \mu), \lambda) + (1-\delta)K.$$

- Factor markets clear: for all (λ, μ),

$$L(\lambda,\mu) = \int xh(a,x;\lambda,\mu)\, d\mu,$$

$$K(\lambda,\mu) = \int a\, d\mu.$$

- Individual and aggregate behaviors are consistent: for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$,

$$\mu'(A^0, X^0)$$

$$= \int_{A^0, X^0} \left\{ \int_{\mathcal{A},\mathcal{X}} \mathbb{1}_{a'=a'(a,x;\lambda,\mu)}\, d\pi_x(x'|x)\, d\mu \right\} da'dx'.$$

III. Quantitative Analysis

A. *Calibration*

We briefly explain the choice of the model parameters. The unit of time is a business quarter. We assume that individual productivity x follows an AR(1) process: $\ln x' = \rho_x \ln x + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. We estimate ρ_x and σ_x by estimating the AR(1) process of wages from the Panel Study of Income Dynamics (PSID) for 1979–1992. We control for time effects by annual dummies and individual fixed effects by sex, age, schooling, age^2, schooling^2, and age $\times$ schooling. We then convert the annual estimates to quarterly values. The quarterly values we obtain are $\rho_x = 0.929$ and $\sigma_x = 0.227$.[8] The other parameters are in accordance with the business cycle analysis and empirical labor-supply literature. A working individual spends one-third of discretionary time: $\bar{h} = 1/3$. The intertemporal elasticity of hours at the individual level, γ, is 0.4. The labor-income share, α, is 0.64, and the depreciation rate, δ, is 2.5 percent. We search for the weight parameter in the disutility from working, B, such that the steady-state employment rate is 60 percent, the average of the Current Population Survey (CPS) for 1964:I–2003:IV. The discount factor β is chosen so that the quarterly rate of return to capital is 1 percent. The aggregate productivity shock, λ_t, follows an AR(1) process: $\ln \lambda' = \rho_\lambda \ln \lambda + \varepsilon_\lambda$, where $\varepsilon_\lambda \sim N(0, \sigma_\lambda^2)$. We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$ following Kydland and Prescott (1982). Table 1 summarizes the parameter values of the model economy.

B. *Cross-Sectional Distributions for Earnings, Wealth, and Reservation Wages*

Since we investigate the aggregation issue, it is desirable for the model economy to possess a reasonable amount of heterogeneity. We compare cross-sectional earnings and wealth—two important observable dimensions of heterogeneity in

[7] Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a,x)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{X}$.

[8] We estimate the AR(1) process of the wage residual using Heckman's (1979) maximum-likelihood estimation procedure, correcting for a sample selection bias because productivities (wages) of workers who did not work are not reported. See Chang and Kim (2006) for details.

TABLE 1—PARAMETERS OF THE BENCHMARK MODEL ECONOMY

Parameter description	
$\alpha = 0.64$	Labor share in production function
$\beta = 0.98267$	Discount factor
$\gamma = 0.4$	Individual labor-supply elasticity with divisible labor
$B = 166.3$	Utility parameter
$\bar{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.929$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.227$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

the labor market—found in the model and in the data.

Table 2 summarizes both the PSID and the model's detailed information on wealth and earnings. As we control for the observed fixed effects in estimating individual productivity, we will compare our model to the PSID statistics after conditioning on educational attainment and age. The category "PSID primary households" denotes households whose head is a high-school graduate and whose age is between 35 and 55 as of 1983 (1984 survey). Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. For each quintile group of wealth distribution, we calculate the wealth share, the ratio of group average to economy-wide average, and the earnings share.

In both the data and the model, the poorest 20 percent of families in terms of wealth distribution were found to own virtually nothing. The PSID found that households in the second, third, fourth, and fifth quintiles own 7.07, 13.01, 21.10, and 57.76 percent of total wealth, respectively, while, according to the model, they own 3.27, 12.21, 26.05, and 60.93 percent, respectively.[9] The average wealth of those in the second, third, fourth, and fifth quintiles is, respectively, 0.36, 0.64, 1.06, and 2.97 times larger than that of a typical household, according to the PSID. These ratios are 0.16, 0.61, 1.30, and 3.08 according to our model. Households in the second, third, fourth, and fifth quintiles of wealth distribution earn, respectively, 14.67, 20.08, 25.07, and 25.86 percent of total earnings, according to the PSID. The corresponding groups earn 17.87, 20.50, 22.65, and 25.46 percent, respectively, in the model. We argue that the model economy presented in this paper possesses a reasonable degree of heterogeneity, thus making it possible to study the effects of aggregation in the labor market.

In our model, labor-market participation is determined by market opportunity (wage) and wealth (asset holdings). We plot the steady-state reservation wage schedule in Figure 3. Panel A graphs the reservation wage for all asset levels and panel B for assets less than $200,000. At a given asset level, workers with a wage (productivity) above the line choose to work. The reservation wage increases as the asset level increases. To illustrate, we adjust the units such that the mean asset of the model matches the average asset of the comparison group (household head is a high-school graduate and is between 35 and 55 years of age) in the 1984 PSID survey, $102,744; thus, the values are in 1983 dollars.[10] Consider a worker whose assets are $61,563, the median of the wealth distribution from the model. According to the model, he is indifferent about working and not working at quarterly earnings of $6,927. Another worker whose assets are equivalent to the average asset holding of the economy, $102,744 (which belongs to the sixty-third

[9] One should note that the unconditional wealth distribution is much more skewed than that of "primary households." For example, according to the unconditional wealth distribution (i.e., all households in the 1984 PSID), the first to fifth quintiles own, respectively, –0.52, 0.50, 5.06, 18.74, and 76.22 of total wealth.

[10] The mean asset in our model is 11.59 units. The reservation wages in the vertical axis of Figure 3 reflect quarterly earnings (the reservation wage rate multiplied by $\bar{h}$).

TABLE 2—CHARACTERISTICS OF WEALTH DISTRIBUTION

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID–primary households						
Share of wealth	1.03	7.07	13.01	21.10	57.76	100
Group average/population average	0.05	0.36	0.64	1.06	2.97	1
Share of earnings	14.29	14.67	20.08	25.07	25.86	100
Participation rate	0.86	0.84	0.83	0.87	0.79	1
Benchmark model						
Share of wealth	−2.46	3.27	12.21	26.05	60.93	100
Group average/population average	−0.12	0.16	0.61	1.30	3.08	1
Share of earnings	13.52	17.87	20.50	22.65	25.46	100
Participation rate	0.86	0.63	0.56	0.50	0.43	1

Notes: The PSID statistics reflect the family wealth and earnings in the 1984 survey. The statistics of "primary households" are those for household heads whose education was 12 years and whose age is between 35 and 55. The participation rate is based on individual employment status (household heads and spouse) for the same group.

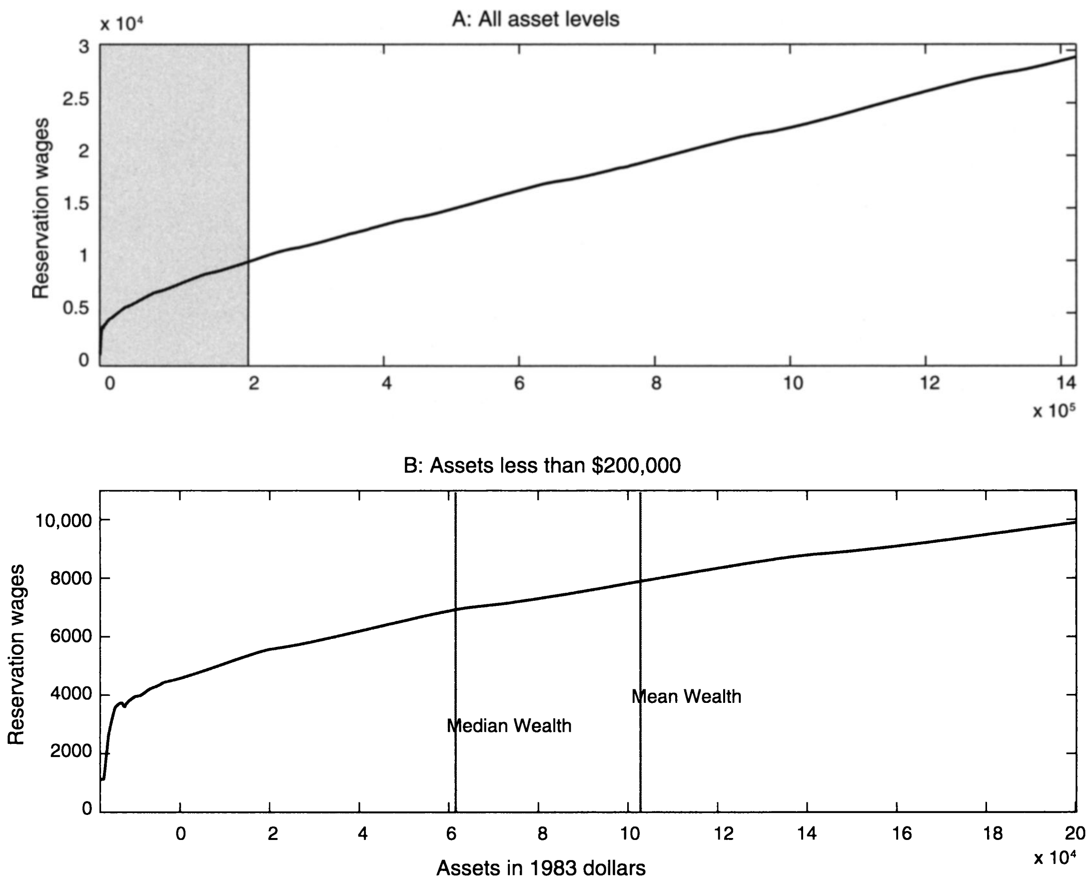

FIGURE 3. RESERVATION WAGES FROM THE BENCHMARK MODEL

Note: The graph denotes the reservation wages from the benchmark model. Wages (quarterly earnings) and assets are in 1983 dollars.

TABLE 3—VOLATILITIES OF AGGREGATE VARIABLES

Variable	US data	Model
σ_Y	2.06	1.28
σ_C/σ_Y	0.45	0.39
σ_I/σ_Y	2.41	3.06
σ_H/σ_Y	0.82	0.76
σ_L/σ_Y	—	0.50
$\sigma_{Y/H}/\sigma_Y$	0.50	0.50
$\sigma_H/\sigma_{Y/H}$	1.64	1.72
σ_{MRS}/σ_Y	0.90	0.83
$\sigma_{\text{wedge}}/\sigma_Y$	0.92	0.76

Notes: All variables are logged and detrended by the H-P filter. The volatility of output is measured by its standard deviation and that of all other variables are measured by the standard deviations relative to output. The variable L denotes the effective unit of hours. The MRS and wedge are defined, respectively, by equations (1) and (2).

percentile of the wealth distribution in our model and to the sixty-ninth percentile in the PSID), is indifferent about working at $7,890 per quarter. The model predicts a fairly strong wealth effect on participation. The labor-market participation rates are 0.86, 0.63, 0.56, 0.50, and 0.43, respectively, from the first to fifth quintiles. According to the PSID, however, the wealth effect seems much weaker. The labor-market participation rates are 0.86, 0.84, 0.83, 0.87, and 0.79, respectively, from the first to fifth asset quintiles.

C. *Cyclical Properties of the Model*

To study the business cycle properties of the model, we solve the equilibrium of the model using the "bounded rationality" method developed by Krusell and Smith (1998): agents make use of a finite set of moments of μ in forecasting aggregate prices. The detailed description of our computation procedure is given in the Appendix. As in Krusell and Smith (1998), we achieve a fairly precise forecast using the first moment of μ only (i.e., the mean asset). We also find that the results do not change significantly when we allow for the second moment of assets in forecasting functions (see Table A in the Appendix for the comparison of these results).

Table 3 shows the volatility of the key aggregate variables of our model economy. In the face of aggregate productivity shocks whose stochastic process resembles that of TFP in the United States, the model output exhibits a volatility of 1.28, slightly less than two-thirds of actual output volatility. This is not very different from the findings of the standard representative-agent models (e.g., Kydland and Prescott 1982). Other statistics are also similar to those found in the standard models: consumption is about 40 percent as volatile as output, and investment is about three times as volatile as output.

A distinguishing feature of our model lies in the labor-market fluctuations. The volatility of hours relative to output is 0.76 (0.82 in the data), and the volatility of labor productivity relative to output is the same as that in the data (0.50). The relative volatility of hours to productivity is 1.72, very close to that in the data (1.64). In our model, the aggregate labor supply is quite elastic, despite the fact that individual intertemporal substitution elasticity for hours is assumed to be 0.4. As Chang and Kim (2006) show, in a model economy like this, the aggregate labor supply elasticity depends on the shape of the reservation wage distribution. In our model, based on the steady-state reservation wage distribution, the elasticity of the participation rate with respect to the reservation wage is 1.5 at the steady-state employment rate of 60 percent. The composition effect also increases the volatility of aggregate hours relative to average productivity. On average, less-productive workers participate in the labor market during expansions and exit during contractions. This makes the measured hours more volatile than the hours in effective units and the average wage less volatile than individual wages. When we measure hours worked in effective units, they are half as volatile as output (0.50).

Table 4 shows the cyclicality of key aggregate variables. The correlations between output, consumption, investment, and labor productivity are higher than those in the data, a feature common in standard real business cycle (RBC) models. The correlation of hours with output is 0.84, close to that in the data (0.86). A surprising aspect of the model is that hours worked and labor productivity exhibit a fairly low correlation (0.23) in our model—it is 0.08 in the data—despite the fact that the only driving force in the simulation is the aggregate productivity shock. This is a striking result because the failure to generate a low correlation between hours and labor productivity is known to be one of the most salient shortcomings of the RBC models. In our model, the interaction between

TABLE 4—CYCLICALITY OF AGGREGATE VARIABLES

Variable	Data	Model
Corr(Y, C)	0.69	0.84
Corr(Y, I)	0.90	0.98
Corr(Y, H)	0.86	0.87
Corr(Y, L)	—	0.92
Corr(Y, Y/H)	0.57	0.68
Corr(H, Y/H)	0.08	0.23
Corr(Y/H, MRS)	0.25	0.43
Corr(Y, wedge)	0.55	0.56
Corr(H, wedge)	0.85	0.87

Note: See the note in Table 3 for description of the variables.

incomplete capital markets and indivisible labor breaks the tight link between employment and wages at the aggregate level.[11] Because of indivisible labor, the optimality conditions for the choice of consumption and hours worked hold as inequality at the individual level. Owing to the partial insurance of idiosyncratic risks, individual optimality conditions do not aggregate nicely. Moreover, the labor-supply curve is time-varying in the face of aggregate productivity shifts as the reservation wage distribution (wealth distribution) evolves over time.[12]

D. *Labor-Market Wedge from the Model*

From the perspective of an optimizing agent in a competitive market, a lack of systematic movement among consumption, employment, and productivity is manifested by measurement of a stochastic wedge between the MRS and productivity. When we apply a fictitious representative agent's optimality condition to the model-generated aggregate time series, we also find a time-varying wedge.

Figure 4 shows total hours worked and the wedge from the model-generated aggregate time series under the assumption that the aggregate labor-supply elasticity is 1.5. Similar to the measured wedge from the actual data, the wedge is as volatile as hours and is highly correlated with total hours. The standard deviation of the wedge relative to output ($\sigma_{\text{wedge}}/\sigma_Y$) is 0.76 (0.92 in the data). Given that the output volatility of the model is about two-thirds of that in the data, the wedge from the model is about half as volatile as the one in the data. The correlation between the wedge and total hours worked (Corr(H, wedge)) is 0.87 (0.85 in the data). Despite there being no inherent preference shifts or distortions, the wedge arises endogenously because of imperfect aggregation and a time-varying reservation-wage distribution. In computing the wedge, we use the aggregate labor-supply elasticity of 1.5, the same value we used to compute the wedge in the actual data. A bigger (smaller) value of aggregate labor-supply elasticity produces a smaller (bigger) wedge. Nevertheless, in our model (as well as in the actual data), there is no choice of γ that eliminates the wedge completely. The wedge arises not only because productivity is not as volatile as the marginal rate of substitution but also because they are not correlated with each other. For example, even with $\gamma = 10$, the measured wedge from the model is still about half as volatile as output ($\sigma_{\text{wedge}}/\sigma_Y = 0.56$) and highly correlated with hours worked (Corr(H, wedge)) = 0.53).[13]

IV. Role of Incomplete Markets and Indivisible Labor

The interaction between incomplete markets and indivisible labor results in a wedge between the MRS and productivity. To investigate the marginal contributions of each, we consider three additional model economies. For comparison, we refer to the benchmark economy as HII, which stands for "heterogeneity-incomplete markets-indivisible labor."

A. *Alternative Model Specifications*

Heterogeneity + Complete Market + Indivisible Labor.—The second model we consider

[11] Jang-Ok Cho and Rogerson (1988) also obtain a negative productivity-hours correlation from the heterogeneity of productivity in a two-member household model.

[12] Francois Gourio and Pierre-Alexander Noual (2006) report that the implied Frisch elasticity of aggregate labor supply, estimated from the National Longitudinal Survey of Youth (NLSY) data for 1979–1992, exhibits significant variations over time.

[13] According to Sungbae An, Chang, and Kim (2007), the GMM estimation of the static first-order condition using aggregate hours, consumption, and wages based on this model economy often yields a negative labor-supply elasticity—a nonconcave utility—similar to the finding in N. Gregory Mankiw, Julio J. Rotemberg, and Lawrence H. Summers (1985) based on the actual aggregate data.

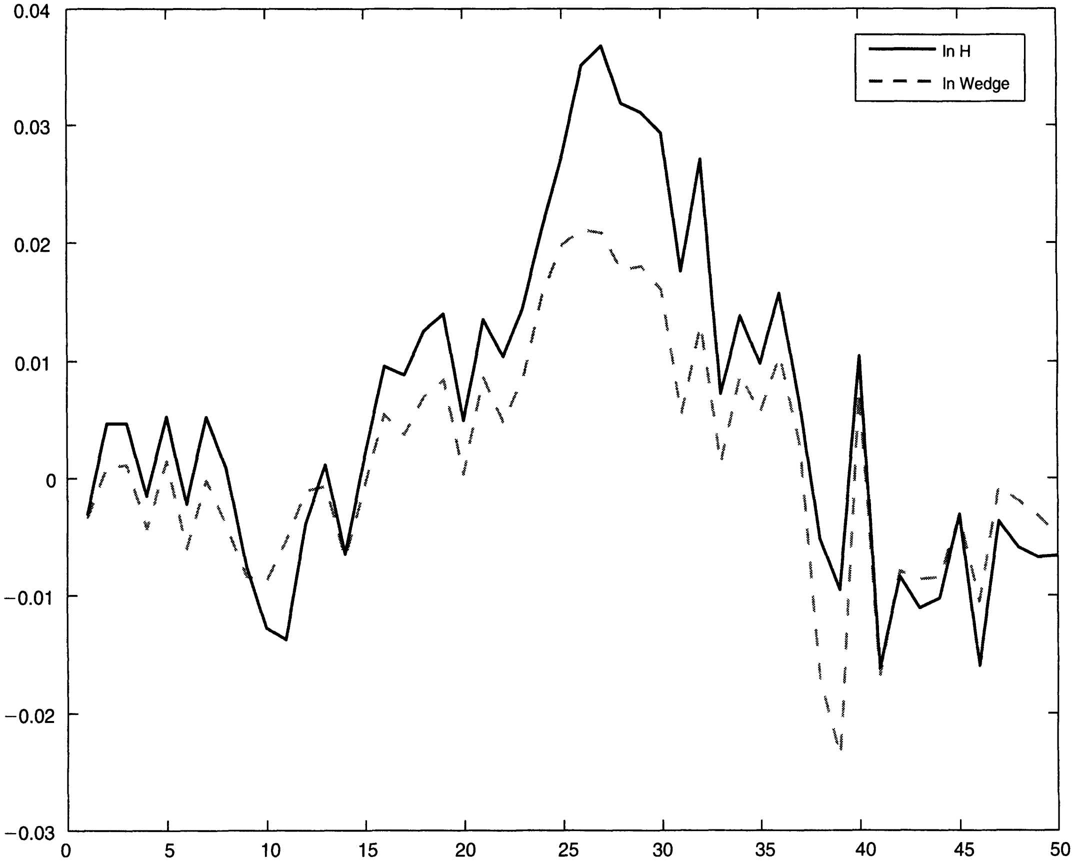

FIGURE 4. TOTAL HOURS AND LABOR-MARKET WEDGE FROM THE BENCHMARK MODEL

Note: The wedge is computed from equation (2) with the aggregate labor-supply elasticity (γ) of 1.5.

allows for complete capital markets but maintains indivisible labor: heterogeneity-complete markets-indivisible labor (HCI). This is similar to Cho (1995), which incorporates ex post heterogeneity into a standard real business cycle framework.[14] Thanks to perfect risk sharing, agents enjoy the same level of consumption regardless of their employment status, productivity, or asset holdings.[15]

The equilibrium of this economy is identical to the allocation made by a social planner who maximizes the equally weighted utility of the population. The planner chooses the sequence of consumption $\{C_t\}_{t=0}^{\infty}$ and the cut-off productivity $\{x_t^*\}_{t=0}^{\infty}$ for labor-market participation. To ensure an efficient allocation, the planner assigns workers who have a comparative advantage in the market (more productive workers) to work. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor.

The planner's value function in the complete market, denoted by $V^C(K,\lambda)$, and the decision rules for consumption, $C(K,\lambda)$, and cut-off

[14] The difference between Cho's model and ours is the cross-sectional distribution of productivity. Cho uses a uniform distribution, whereas we use a log-normal distribution, closer to the cross-sectional income distribution in the data.

[15] The distribution of workers is no longer a state variable in the individual optimization problem. Moreover, because of the ergodicity of the stochastic process for idiosyncratic productivity, the cross-sectional distribution of workers is always stationary.

productivity, $x^*(K,\lambda)$, satisfy the following Bellman equation:

$$V^C(K,\lambda) = \max_{C,x^*}\left\{\ln C - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma} \times \int_{x_t^*}^{\infty}\phi(x)\,dx + \beta E\left[V^C(K',\lambda')|\lambda\right]\right\}$$

subject to

$$K' = F(K,L,\lambda) + (1-\delta)K - C,$$

where $L = \bar{h}\int_{x^*}^{\infty} x\phi(x)dx$ is the aggregate effective unit of labor, and $\phi(x)$ is the productivity distribution of workers. The planner chooses the cut-off productivity x^* so that:

$$\frac{1}{C}F_L(K,L,\lambda)\,\bar{h}x^*\phi(x^*) = B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\phi(x^*). \tag{4}$$

The left-hand side is the (society's) utility gain from assigning the marginal worker to production. There are $\phi(x^*)$ number of workers with productivity x^* in the economy. Each of them supplies $\bar{h}x^*$ units of effective labor, and the marginal product of labor is F_L. The right-hand side represents the disutility incurred by these workers. The key point here is that, under complete markets, the first-order condition for the choice between hours and consumption is *exactly* defined in terms of effective units of labor and wages at the aggregate level. Thus, the wedge reflects the "measurement error" in aggregate wages and hours. As we show in Section IVB, the wedge would be zero with an appropriate choice of aggregate labor-supply elasticity.

Heterogeneity + Incomplete Market + Divisible Labor.—The third model economy we consider allows for a divisible labor supply, but capital markets are incomplete: "heterogeneity-incomplete markets-divisible labor (HID). This is essentially the same specification as in Krusell and Smith (1998). The equilibrium of this economy can be defined similarly to that of the benchmark model with the worker's value function with divisible labor, $V^D(a,x;\lambda,\mu)$:

$$V^D(a,x;\lambda,\mu) = \max_{a'\in\mathcal{A},h\in(0,1)}\left\{\ln c - B\frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta E\left[V^D(a',x';\lambda',\mu')|x,\lambda\right]\right\}$$

subject to

$$c = w(\lambda,\mu)xh + (1+r(\lambda,\mu))a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbb{T}(\lambda,\mu).$$

Representative-Agent Model.—The last model we consider is the "representative-agent" (RA) model. The value function of the representative agent, $V^R(K,\lambda)$, is:

$$V^R(K;\lambda) = \max_{C,H}\left\{\ln C - B\frac{H^{1+1/\gamma}}{1+1/\gamma} + \beta E\left[V^R(a',x';\lambda',\mu')|x,\lambda\right]\right\}$$

subject to

$$K' = F(K,H,\lambda) + (1-\delta)K - C.$$

B. *Comparison of Four Model Economies*

Except for β and B, the same parameter values are used across all models. In the RA model, β is 0.99 and B is chosen so that the steady-state hours worked are the same as the aggregate hours in the benchmark economy, which is 0.2 $(= \bar{h} \times 60$ percent). For HCI, β is 0.99 and B is chosen to be consistent with 60 percent employment along with $\bar{h} = 1/3$.[16] For HID, β and B are jointly searched to be consistent with average hours of 0.2 and an interest rate of 1 percent in a steady state. The equilibrium of the HCI economy is solved by Albert Marcet and Guido Lorenzoni's (1999) parameterized expectation

[16] Specifically, we find the steady-state cutoff productivity, x^*, from the sixtieth percentile of the cross-sectional productivity distribution, $\phi(x) : \int_{x^*}^{\infty}(x)\phi(x)\,dx = 0.6$. Then, we find B that satisfies the labor-supply equation, (4).

algorithm, while the equilibrium of HID is solved by Krusell and Smith's (1998) "bounded rationality" method, and the equilibrium of RA is solved by a value function iteration.

One must assume an aggregate labor-supply elasticity to compute the MRS. For the divisible-labor economies (RA and HID), the natural choice is 0.4, which is the same as the individual elasticity. However, when the labor supply is indivisible (HII and HCI), the aggregate labor-supply elasticity can depart from the individual elasticity. We compute the wedge for indivisible-labor economies assuming that the aggregate elasticity is 1.5 for the following reasons. First, according to the steady-state reservation wage distribution, the elasticity of the participation rate with respect to the reservation wage is 1.5 at the steady-state employment rate of 60 percent.[17] Second, this value is close to the empirical estimates of aggregate Frisch elasticity (e.g., Francois Gourio and Pierre-Alexander Noual 2006). Third, it provides a direct comparison with the wedge computed from the actual data in Section I.

For the complete market model (HCI), as discussed later in this section, an appropriate choice of γ can eliminate the wedge completely. Thus, $\gamma = 1.5$ is only for convenience.

Figure 5 shows the sample paths (percentage deviations from the steady states) of the wedges from four model economies. These sample paths are comparable to each other because all model economies are subject to an identical sequence of aggregate productivity shocks. As expected, there is no wedge in the RA model. The wedges of the HID and HCI are not large enough to account for the wedge in the data. The volatilities of the wedge relative to output are 0.09 for both HID and HCI, which are, respectively, only one-tenth of that found in the data (0.92).

As the HID model shows, capital-market incompleteness alone does not generate a wedge comparable to what we observe in the data. With divisible labor, in response to aggregate productivity shocks, hours are highly correlated across households, allowing for a fairly precise aggregation. On the other hand, with indivisible labor, the intratemporal optimality condition for the choice between commodity consumption and leisure does not hold with equality for most households. Individual choices are at the corner, and the aggregation of inequalities does not lead us to meaningful aggregate relationships among hours, consumption, and productivity.

The HCI model shows that indivisible labor alone cannot account for the wedge we observe in the data either. Under complete capital markets the consumption-hours choice holds exactly in *effective units* at the aggregate level—recall equation (4). Thus, in the complete market model, the wedge reflects the "specification error" in the aggregate labor-supply elasticity (or "compositional bias" in hours and wages). In other words, with an appropriate choice of aggregate elasticity, one can eliminate the wedge completely. In fact, if we assume an aggregate labor-supply elasticity of 2.09 in HCI, there is virtually no wedge.[18]

It is well known that low-wage and less-skilled workers enter the labor market during expansions and exit during recessions, making aggregate hours more volatile than the effective unit of hours (Hansen 1993), and making the aggregate wages less volatile than individual wages (Mark Bils 1985; Gary Solon, Robert Barsky, and Jonathan A. Parker 1994).[19] However, the compositional bias has an impact mostly on the volatilities, not on the correlations. In both the model and the data, the wedge arises because of low correlation between employment and productivity. From the data (as well as our benchmark model), we find the wedges regardless of the value for the aggregate labor-supply elasticity. Moreover, the aggregate labor-supply curve is no longer an invariant parameter in

[17] In an environment similar to the one in this paper, Chang and Kim (2006) obtained an aggregate labor-supply elasticity of around one. Since we control for observed fixed effects here, we obtain smaller values for persistence and variance of idiosyncratic productivity. This makes the reservation-wage distribution less dispersed and results in a slightly higher aggregate labor-supply elasticity.

[18] The aggregate elasticity in the complete market economy, such as HCI, depends on the ratio of the marginal density relative to the cumulative distribution of x at the participation cutoff point x^*. For example, in Gary D. Hansen (1985) where agents are identical, the cross-sectional distribution of x is degenerate and the aggregate elasticity becomes an infinity.

[19] Bils (1985) and Solon, Barsky, and Parker (1994), based on the individual panel data, find that aggregate wages are less cyclical than individual wages. Hansen (1993) computes the effective unit of hours based on the worker characteristics provided by the CPS. He finds that while the effective unit of hours adjusted for quality exhibits a greater

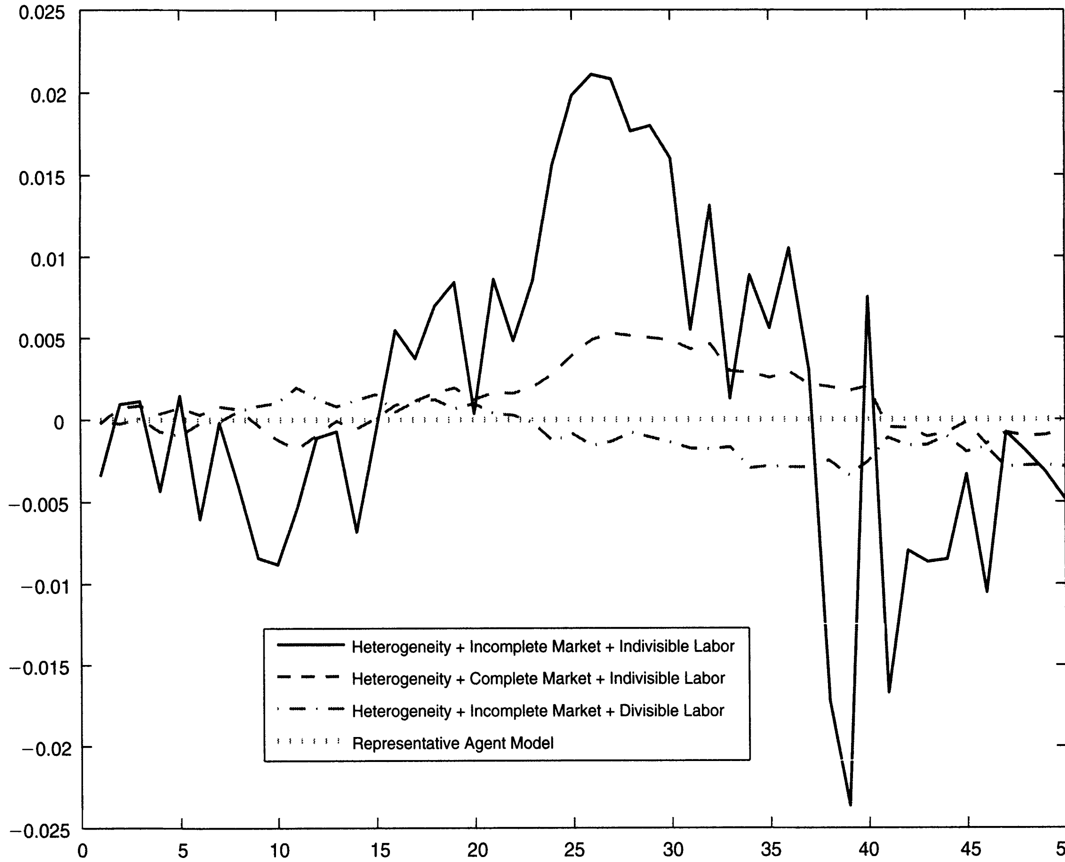

FIGURE 5. LABOR-MARKET WEDGES FROM THE MODELS

Note: The wedge is computed from equation (2). We use the aggregate labor-supply elasticity (γ) of 0.4 and 1.5, respectively, for divisible- and indivisible-labor models.

our benchmark model, since the shape of the reservation-wage distribution varies over time.

V. Conclusion

The cyclical behavior of aggregate consumption, hours worked, and productivity is hard to reconcile with the equilibrium outcome of the representative agent with standard preferences. The fact that hours worked are not strongly correlated with labor productivity has been considered one of the most salient shortcomings of the equilibrium business cycle theory. We demonstrate that a heterogeneous-agent economy with incomplete capital markets and indivisible labor can generate a low employment-productivity correlation. When we apply the optimality condition implied by the representative agent to the model-generated aggregate time series, we find a time-varying wedge between the marginal rate of substitution and labor productivity, despite the fact that our model has neither distortion nor exogenous labor-supply shocks. Our results caution against viewing the measured wedge as a failure of labor-market clearing or as a fundamental driving force behind aggregate fluctuations.

volatility, such an adjustment does not significantly change the cyclical property of hours. In practice, estimating quality-adjusted hours is not easy since observed characteristics account for only a fraction of a worker's productivity. For example, the R^2 of the cross-sectional wage regression is usually well below 0.4.

Appendix A: Computational Procedures for Steady-State Equilibrium

The distribution of workers, $\mu(x, a)$, is invariant in the steady state, as are factor prices. In finding the invariant, μ, we use the algorithm suggested by José-Víctor Ríos-Rull (1999). We search for the discount factor β that clears the capital market, given the quarterly rate of return of 1 percent. Computing the steady-state equilibrium amounts to finding the value functions, the associated decision rules, and the time-invariant measure of workers. Details are as follows:

1. First, we choose the grid points for asset holdings (a) and idiosyncratic productivity (x). The number of grids is denoted by N_a, and N_x. We use $N_a = 1{,}163$ and $N_x = 17$. The asset holding a_i is in the range of $[-2{,}250]$, where the average asset holding is 11.6. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate the savings decisions of workers with lower assets. For example, at the asset range close to the borrowing constraint, the grid points are as fine as 0.02, which is approximately 2.5 percent of the average labor income. At the high end, the asset grid increases by 0.4, which corresponds to 42 percent of the average labor income. For idiosyncratic productivity, we construct a grid vector of length N_x of which elements, $\ln x_j$'s, are equally spaced on the interval $[-3\sigma_x/\sqrt{1-\rho_x^2}, 3\sigma_x/\sqrt{1-\rho_x^2}]$.
Then, we approximate the transition matrix of the idiosyncratic productivity using George Tauchen's (1986) algorithm.

2. Given β, we solve the individual value functions V^E, V^N, and V at each grid point of the individual states. In this step, we also obtain the optimal decision rules for asset holding $a'(a_i, x_j)$ and labor supply $h(a_i, x_j)$. This step involves the following procedure:

(a) Initialize value functions $V_0^E(a_i, x_j)$ and $V_0^N(a_i, x_j)$ for all $i = 1, \ldots, N_a$ and $j = 1, \ldots, N_x$.

(b) Update value functions by evaluating the discretized versions

$$V_1^E(a_i, x_j) = \max\Big\{u(w\bar{h}x_j + (1+r)a_i - a', \bar{h}),$$

$$+ \beta \sum_{j'=1}^{N_x} V_0(a', x_{j'})\pi_x(x_{j'}|x_j))\Big\},$$

where $\pi_x(x_{j'}|x_j)$ is the transition probabilities of x, which is approximated using Tauchen's algorithm. V_1^N is computed in a similar way. Then, update $V_1(a_i, x_j)$ as follows:

$$V_1(a_i, x_j) = \max\Big\{V_1^E(a_i, x_j), V_1^N(a_i, x_j)\Big\}.$$

(c) If V_1 and V_0 are close enough for all grid points, then we have found the value functions. Otherwise, set $V_0^E = V_1^E$ (likewise for V^N), and go back to step 2(b).

3. Using $a'(a_i, x_j)$, $\pi_x(x_{j'}|x_j)$ obtained from step 2, we obtain time-invariant measures $u^*(a_i, x_j)$ as follows:

(a) Initialize the measure $\mu_0(a_i, x_j)$.

(b) Update the measure by evaluating the discretized version of (5):

$$\mu_1(a_{i'}, x_{j'}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \mathbb{1}_{a_{i'}=a'(a_i,x_j)}\, \mu_0(a_i, x_j)\pi_x(x_{j'}|x_j).$$

(c) If μ_1 and μ_0 are close enough for all grid points, then we have found the time-invariant measure. Otherwise, replace μ_0 with μ_1 and go back to step 3(b).

(4) We calculate the real interest rate as a function of β, i.e., $r(\beta) = \alpha(K(\beta)/(L(\beta))^{1-\alpha} - \delta$, where $K(\beta) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} a_i\mu^*(a_i, x_j)$ and $L(\beta)\sum_{i=1}^{N_a} \sum_{j=1}^{N_x}\{x_j h(a_i, x_j)\}\mu^*(a_i, x_j)$. Other aggregate variables of interest are calculated using μ^* and decision rules. If $r(\beta)$ is close enough to the assumed value of the real interest rate, we have found the steady state. Otherwise, we choose a new β and go back to step 2.

Appendix B: Computational Procedures for Equilibrium with Aggregate Fluctuations

Approximating the equilibrium in the presence of aggregate fluctuations requires us to include the measure of workers and the aggregate productivity shock in the list of state variables and to keep track of the evolution of the measure μ over time. Since μ is an infinite dimensional object, it is almost impossible to implement these tasks as they are. We follow the procedure suggested by Krusell and Smith (1998): agents are assumed to make use of its first moment of assets (i.e., the mean asset $K = E\mu[a]$) only in predicting the law of motion for μ. We are implicitly assuming that the average productivity of workers $E\mu[x]$ is constant due to the law of large numbers. Therefore, computing the equilibrium with aggregate fluctuations amounts to finding the value functions, decision rules, and law of motion for the aggregate capital within the class of log-linear functions in K and λ.

1. In addition to the grids for individual state variables specified above, we choose 11 equally spaced grid points for the aggregate capital, K, in the range of $[0.9K^*, 1.1K^*]$, where K^* denotes the steady-state aggregate capital. In our numerous simulations, the capital stock has never reached the upper or lower bound. For aggregate productivity, we choose nine grid points of $\log \lambda$ that are equally spaced on the interval of $\left[-3\sigma_\lambda/\sqrt{1-\rho_\lambda^2}, 3\sigma_\lambda/\sqrt{1-\rho_\lambda^2}\right]$. The transition probability matrix of the aggregate productivity is approximated by Tauchen's algorithm as for the idiosyncratic productivity shocks.

2. Let the parametric law of motion for the aggregate capital take a log linear function in K and λ:

$$\ln K_{t+1} = \kappa_0^0 = \kappa_1^0 \ln K_t + \kappa_2^0 \ln\lambda_t. \tag{B1}$$

In order for individuals to make their decisions on savings and labor supply, they have to know (or forecast) the interest rate and wage rate for an effective unit of labor. While the factor prices depend on aggregate capital and labor, aggregate labor input is not known to individuals at the moment when they make decisions. Thus, individuals need to predict the factor prices. These forecasts of factor prices, in turn, must be consistent with the outcome of individual actions and the factor market-clearing conditions. We also assume that individuals forecast the market wage and the interest rate using a log-linear function of K and λ:

$$\ln w_t = b_0^0 = b_1^0 \ln K_t + b_2^0 \ln\lambda_t. \tag{B2}$$

$$\ln(r_t + \delta) = d_0^0 + d_1^0 \ln K_t + d_2^0 \ln\lambda_t. \tag{B3}$$

3. We chose the initial values for the coefficients κ^0's, b^0's, and d^0's. Good initial values may come from a representative-agent model.

4. Given the law of motion for the aggregate capital and the forecast functions for factor prices, we solve the individual optimization problem. This step is analogous to step 2 in the steady-state computation:

(a) We have to solve for the value functions and the decision rules over a bigger state space. Now the state variables are (a, x, K, λ).

(b) Computation of the conditional expectation involves the evaluation of the value functions not on the grid points along the K dimension, since K' forecasted by (B1) need not be a grid point. We polynomially interpolate the value functions along the K dimension when necessary.

5. Using $a'(a_i, x_j, K_l, \lambda_m)$, $h(a_i, x_j, K_l, \lambda_m)$, $\pi_x(x_{j'}|x_j)$, and $\pi_\lambda(\lambda_{m'}|\lambda_m)$, we simulate the saving and labor supply decisions of 200,000 individuals for 3,500 periods. We then generate a set of aggregate time series data $\{K_t, w_t, r_t\}$ by aggregating these individuals' decisions each period. We discard the first 500 observations in order to reduce the effect of initial condition.

6. We obtain new values for coefficients κ^1's, b^1's, and d^1's by the OLS from the simulated data. If κ^1's, b^1's, and d^1's are close enough to κ^0's, b^0's, and d^0's, respectively, we have found the law of motion. Otherwise, we update coefficients by setting $\kappa^0 = \kappa^1$'s, $b^0 = b^1$'s, and $d^0 = d^1$'s, and go back to step 4.

The estimated law of motion for capital and forecast functions and their accuracy, measured by R^2 for the prediction equations, are as follows.

- Law of motion for aggregate capital in equation (B1):

$$\text{(B4)} \qquad \ln K_{t+1} = 0.1133 + 0.9537 \ln K_t + 0.0997 \ln\lambda_t, \quad R^2 = 0.999937.$$

- Wage rate in equation (B2):

$$\text{(B5)} \qquad \ln w_t = -.02370 + 0.4494 \ln K_t + 0.7997 \ln\lambda_t, \quad R^2 = 0.997669.$$

- Interest rate in equation (B3):

$$\text{(B6)} \qquad \ln(r_t + \delta) = -1.3936 - 0.7989 \ln K_t + 1.3559 \ln\lambda_t, \quad R^2 = 0.988726.$$

The law of motion for aggregate capital provides the highest accuracy. The wage function is more accurate than the interest rate function. Overall, forecast functions are fairly precise as R^2's are close to one. As the agents make decisions based on the forecast prices, the actual employment may not be necessarily consistent with the predicted prices. We also used the method suggested in Ríos-Rull (1999) in which labor-market clearing is imposed as an extra step. The result with a two-step process was very similar to the one reported here, since the forecast prices approximate the actual prices very closely.

Appendix C: Labor-Market Wedge and Forecast Errors

The capital-market incompleteness (partial insurance of idiosyncratic risks) forces us to compute the equilibrium of the benchmark model by the approximation method of Krusell and Smith (1998), which assumes that agents use the limited number of moments of the asset distribution (μ) in forecasting aggregate prices. In particular, we assume that the agents forecast K', w, and r by the log-linear function of the mean asset ($K = E_\mu[a]$) and aggregate productivity (λ) in equations (B1)–(B3). As in Krusell and Smith (1998), these forecasts are highly precise, since R^2's are close to one. According to Table A below, the standard deviations of the forecast errors are (0.02456, 0.13621, and 0.24217 percent, respectively, for K', w, and r), much smaller than that of the labor-market wedge (1.23)—i.e., by a order of magnitude. Given that the wedge is a gap between the MRS and labor productivity (which is in the same unit as the wage rate), the measured wedge cannot be completely accounted for by forecast errors. While the wedge is nonlinear in nature (aggregation and indivisibility), we ask whether the measurement of the wedge changes if we allow for higher-order moments of the distribution in the forecasting functions. We compute the equilibria of the benchmark economy assuming that the agents make use of the second moment ($E_\mu[a^2]$) as well as the first moment of the asset distribution in forecasting functions. Table A shows that adding the second moment of asset distribution to the forecast functions improves the measure of fit (in terms of the R^2 and the standard deviation of the forecast error) in all forecasting functions, but only marginally. As a result, the labor-market wedge remains virtually the same as the linear approximation case. For example, the standard deviation of the wedge decreases by only 0.03 percent (from 1.23452 to 1.234074) with the addition of the second moment in all forecasting functions.

TABLE A—ACCURACY OF FORECASTS

Moments of $\mu(a)$ used	$E_\mu[a]$	$E_\mu[a]$ and $E_\mu[a^2]$
R^2 of forecasts		
K'	0.9999372	0.9999397
w	0.9976695	0.9976698
r	0.9887260	0.9887277
S.D. of forecast error (%)		
K'	0.024564	0.024118
w	0.136219	0.136210
r	0.242171	0.242153
σ_{wedge}	1.234520	1.234074

REFERENCES

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Savings." *Quarterly Journal of Economics*, 109(3): 659–84.

An, Sungbae, Yongsung Chang, and Sun-Bin Kim. 2007. "Estimating Preferences Using Aggregate Data." Unpublished.

Benhabib, Jess, Richard Rogerson, and Randall Wright. 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy*, 99(6): 1166–87.

Bils, Mark. 1985. "Real Wages over the Business Cycle: Evidence from Panel Data." *Journal of Political Economy*, 93(4): 666–89.

Bils, Mark, and Peter J. Klenow. 1998. "Using Consumer Theory to Test Competing Business Cycle Models." *Journal of Political Economy*, 106(2): 233–61.

Chang, Yongsung, and Sun-Bin Kim. 2006. "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy." *International Economic Review*, 47(1): 1–27.

Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R. McGrattan. 2005. "Business Cycle Accounting." Federal Reserve Bank of Minneapolis Staff Report 328.

Cho, Jang-Ok. 1995. "Ex-post Heterogeneity and the Business Cycle." *Journal of Economic Dynamics and Control*, 19(3): 533–51.

Cho, Jang-Ok, and Richard Rogerson. 1988. "Family Labor Supply and Aggregate Fluctuations." *Journal of Monetary Economics*, 21(2–3): 233–45.

Christiano, Lawrence J., and Martin Eichenbaum. 1992. "Current Real-Business Cycle Theories and Aggregate Labor-Market Fluctuations." *American Economic Review*, 82(3): 430–50.

Cole, Harold L., and Lee E. Ohanian. 2002. "The U.S. and U.K. Great Depressions through the Lens of Neoclassical Growth Theory." *American Economic Review*, 92(2): 28–32.

Comin, Diego, and Mark Gertler. 2003. "Medium Term Business Cycles." National Bureau of Economic Research Working Paper 10003.

Gali, Jordí, Mark Gertler, and J. David Lopez-Salido. 2007. "Markups, Gaps, and the Welfare Costs of Business Fluctuations." *Review of Economics and Statistics*, 89(1): 44–59.

Gourio, Francois, and Pierre-Alexander Noual. 2006. "The Marginal Worker and the Aggregate Elasticity of Labor Supply." Unpublished.

Hall, Robert E. 1987. "Productivity and Business Cycles." *Carnegie-Rochester Conference Series on Public Policy*, 27: 421–44.

Hall, Robert E. 1997. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics*, 15(1): S223–50.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, 16(3): 309–27.

Hansen, Gary D. 1993. "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked." *Journal of Applied Econometrics*, 8(1): 71–80.

Heckman, James J. 1979. "Sample Selection Bias as a Specification Error." *Econometrica*, 47(1): 153–61.

Heckman, James J. 1984. "Comments on the Ashenfelter and Kydland Papers [Macroeconomic Analyses and Microeconomic Analyses of Labor Supply] [Labor-Force Heterogeneity and the Business Cycle]." *Carnegie-Rochester Conference Series on Public Policy*, 21: 209–24.

Holland, Allison, and Andrew Scott. 1998, "The Determinants of UK Business Cycles." *Economic Journal*, 108(449): 1067–92.

Krusell, Per, and Anthony A. Smith, Jr. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy*, 106(5): 867–96.

Krusell, Per, and Anthony A. Smith, Jr. 2006. "Quantitative Macroeconomic Models with Heterogeneous Agents." Unpublished.

Kydland, Finn, and Edward Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica*, 50(6): 1345–70.

Lucas, Robert E., Jr., and Leonard A. Rapping. 1969. "Real Wages, Employment, and Inflation." *Journal of Political Economy*, 77(5): 721–54.

MaCurdy, Thomas. 1981. "An Empirical Model of Labor Supply in a Life-Cycle Setting." *Journal of Political Economy*, 89(6): 1059–85.

Mankiw, N. Gregory, Julio J. Rotemberg, and Lawrence H. Summers. 1985. "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, 100(1): 225–51.

Marcet, Albert, and Guido Lorenzoni. 1999. "The Parameterized Expectations Approach: Some Practical Issues." In *Computational Methods for the Study of Dynamic Economies*, ed. Ramon Marimon and Andrew Scott, 143–71. New York: Oxford University Press.

Mulligan, Casey B. 2001. "Aggregate Implications of Indivisible Labor." National Bureau of Economic Research Working Paper 8159.

Mulligan, Casey B. 2002. "A Century of Labor-Leisure Distortions." National Bureau of Economic Research Working Paper 8774.

Nakajima, Tomoyuki. 2005. "A Business Cycle Model with Variable Capacity Utilization and Demand Disturbances." *European Economic Review*, 49(5): 1331–60.

Ríos-Rull, José-Víctor. 1999. "Computation of Equilibria in Heterogeneous-Agents Models." In *Computational Methods for the Study of Dynamic Economies*, ed. Ramon Marimon and Andrew Scott, 238–64. New York: Oxford University Press.

Rogerson, Richard. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics*, 21(1): 3–16.

Sheinkman, Jose A., and Laurence Weiss. 1986. "Borrowing Constraints and Aggregate Economic Activity." *Econometrica*, 54(1): 23–45.

Solon, Gary, Robert Barsky, and Jonathan A. Parker. 1994. "Measuring the Cyclicality of Real Wages: How Important Is Compositional Bias?" *Quarterly Journal of Economics*, 109(1): 1–25.

Tauchen, George. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters*, 20(2): 177–81.