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# Labor Market Participation and the Business Cycle

Christian Haefke and Michael Reiter



# Labor Market Fluctuations:

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- ⑥ Standard RBC models: only variations in employment; Either match variability in employment *or* consistent with microeconomic evidence.  
Problem: **Employment Variability**

# Labor Market Fluctuations:

- ⑥ Standard RBC models: only variations in employment;
- ⑥ Unemployment:
  - ✓ Indivisible Labor
  - ✓ MP-style matching models

Criticized for getting either the short run fluctuations or the long run response to benefit reforms wrong.

Problem: **previous & Unemployment Variability;**



# Labor Market Fluctuations:

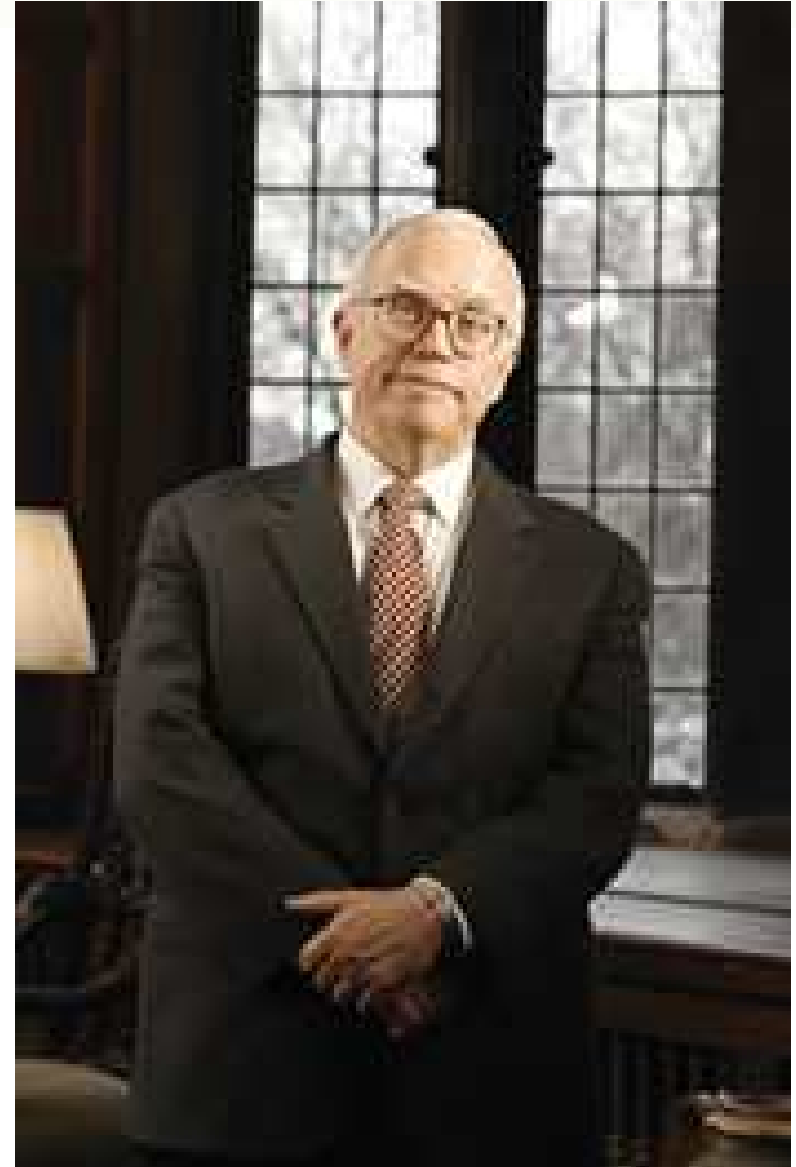
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- ⑥ Standard RBC models: only variations in employment;
  - ⑥ Unemployment:
    - ✓ Indivisible Labor
    - ✓ MP-style matching models
  - ⑥ Labor Market Participation:
    - ✓ Endogenous search intensity;
    - ✓ Endogenous participation decision
- Problem: **previous & Unemployment procyclical;**

# The Extensive Margin:

“[...] any serious model of business-cycle labor market fluctuations must account for **manhour variation at the extensive margin** (employment or labor-force entry decisions) as well as manhour variations at the intensive margin (hours per employee).”

James Heckman: “Comments on the Ashenfelter and Kydland Papers” *Carnegie Rochester Series on Public Policy*, 21, 1984, 209–224.





# Why Labor Market Participation?

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- ⑥ *Unemployment Variability*: Transitions to inactivity quantitatively as important as transitions to unemployment;
- ⑥ *Employment Variability*: Transitions from inactivity second most important determinant (after job separations);

Shimer, R (2005): “Assessing the Ins and Outs ...”

# Why Labor Market Participation?

- ⑥ *Unemployment Variability:*
- ⑥ *Employment Variability:*
- ⑥ A large part of the population is out of the labor force:
  - ✓ US, total population: participation rate 65%
  - ✓ stdev log participation: 0.4-0.5%,  
i.e. one quarter of GDP's.
  - ✓ corr (participation, GDP): 0.6
  - ✓ Sample Period: 1/1976 – 12/2001
  - ✓ Data is HP-filtered ( $\lambda = 100000$ ) and seasonally adjusted.

$$\text{Participation rate} = \frac{\text{No. Employed} + \text{No. Unemployed}}{\text{Population}} > 16$$



# What we know so far:

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- ⑥ The slope of the reservation wage distribution affects the labor supply elasticity (Ben-Porath 1973);
- ⑥ Unemployment benefits affect the participation decision (Garibaldi, Wasmer, 2005);
- ⑥ A basic matching model with participation cannot replicate core stylized facts (Shimer, 2004; Veracierto 2004), in particular unemployment rates are procyclical.





# Our Key Results

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- ⑥ we get variability of the participation rate right (cross sectional density of home productivity)
- ⑥ Implied elasticity of labor supply on extensive margin consistent with micro evidence.
- ⑥ Employment variability substantially improved compared to the model without participation.
- ⑥ Strongly **countercyclical** unemployment rate!
  - ✓ Labor supply elasticity
  - ✓ Time aggregation
- ⑥ Additional mechanism is necessary to match unemployment variability.

# The Model

## Per Period Payoff

at home

$h$

- ⑥ Per period payoff  $h$  is:
  - ✓ idiosyncratic;
  - ✓ with cross-sectional distribution  $F(h)$ ;

# The Model

## Per Period Payoff

at home

$h$

**search actively**

$b(h) + \text{Chance of finding job}$

- ⑥ Per period payoff  $h$  is:
  - ✓ idiosyncratic;
  - ✓ with cross-sectional distribution  $F(h)$ ;
- ⑥ Only active searchers receive job offers;

# The Model

Flow Values:

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at home:  $rV^h = h$

searching:  $rV^u = b(h) + \lambda (V^e - V^u)$

---

not employed:  $rV^n = \max_{\{H,U\}} \{rV^h; rV^u\}$

# The Model

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**Reservation Strategy** with critical value  $h_c$ :

stay at home  $h > h_c$

participate  $h \leq h_c$

# The Model

The flow value of non-employment becomes:

$$rV^n(h) = \begin{cases} h & \text{for } h > h_c \\ b(h) + \lambda^w (V^e(h) - V^n(h)) & \text{for } h \leq h_c \end{cases}$$

# The Model

The flow value of non-employment becomes:

$$rV^n(h) = \eta (\bar{V}^n - V^n(h)) + \begin{cases} h & \text{for } h > h_c \\ b(h) + \lambda^w (V^e(h) - V^n(h)) & \text{for } h \leq h_c \end{cases}$$

When *not employed*, agents receive a **new draw of home productivity** with probability  $\eta$ :

- ⑥ life cycle considerations;
- ⑥ events in family;
- ⑥ helps rationalize large gross flows.

# The Employment Relationship

Value of being employed:

$$rV^e(h) = w(h) + \chi (\bar{V}^n - V^e(h))$$

Value of a filled job:

$$rJ(h) = z - w(h) + \chi (0 - J(h))$$

Free entry condition:

$$\frac{1}{q(\theta)}\phi = \int_{\underline{h}}^{h_c} J(h)dF(h) \frac{1}{F(h_c)}$$



# Wages

- ⑥ Generalized Nash bargaining to split match surplus; Worker's bargaining share,  $\alpha$ .

$$\alpha J(h) = (1 - \alpha) (V^e(h) - V^n(h))$$

- ⑥ By assumption all workers are equal *on* the job.
- ⑥ Threatpoints:

- ✓ Heterogeneous; (tractable in steady state);

- ✓ Homogeneous: Trick: Reset home productivity

- when bargain breaks down (counterfactual implications);

- when matched (**ResetH**).

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- ⑥ but workers redraw anyway upon separation.

# Two Useful Assumptions

**ConstB** All agents receive the same amount of flow utility, i.e.  
 $b(h) \equiv b_0$ .

- ✓ Makes steady state wages independent of *current* home productivity.
- ✓ Simplifies steady state job creation condition to

$$J = \frac{\phi}{q(\theta)}.$$

# Two Useful Assumptions

**ConstB** All agents receive the same amount of flow utility, i.e.  
 $b(h) \equiv b_0$ .

**EtaChi** The probability,  $\eta$ , of a new draw of home-productivity is equal to the separation probability,  $\chi$ .

- ✓ employed and non-employed receive new home-productivity with equal probability;
- ✓ Together with **ConstB** this effectively separates the participation decision from wage determination.

# Equilibrium

- ⑥ Wage curve and Job creation condition  
⇒ wage, labor market tightness;
- ⑥ Flow conditions (Beveridge Curve)  
⇒ unemployment;
- ⑥ Indifference between staying at home and participating  $V^h(h_c) = V^u(h_c) \Rightarrow h_c$   
⇒ Participation Rate

# Wages and Tightness

Under assumption *ConstB* job creation condition and wage curve are given by

$$\text{JCC: } w = z - (r + \chi) \frac{\phi}{q(\theta)}$$

$$\begin{aligned} \text{WC: } w = & \frac{r + \chi}{r + \chi + \alpha\lambda} (1 - \alpha)b + \frac{r + \chi + \lambda}{r + \chi + \alpha\lambda} \alpha z \\ & + (\eta - \chi) \frac{1}{r + \chi + \alpha\lambda} \int_{h_c}^{\infty} (h - h_c) dF(h) \end{aligned}$$

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Furthermore, under assumption *EtaChi* the wage curve is also independent of the home-sector, i.e. the reservation home productivity  $h_c$ .



# Participation and (Un)employment

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In the textbook model without participation:



# Participation and (Un)employment

Assume  $\eta = \chi$ , then

$$u = \frac{\chi}{\chi + \lambda} \pi$$

$$e = \frac{\lambda}{\chi + \lambda} \pi$$

$\pi$  : Participation Rate

$$\pi = F(h_c)$$

# Participation and (Un)employment

Assume  $EtaChi$ , then

$$u = \frac{\chi}{\chi + \lambda} \pi \quad \pi : \text{Participation Rate}$$

$$e = \frac{\lambda}{\chi + \lambda} \pi \quad \pi = F(h_c)$$

- ⑥  $Ela_{e,z} = Ela_{es,z} + Ela_{\pi,z}$   
Participation **magnifies employment** fluctuations;
- ⑥  $Ela_{u,z} = Ela_{us,z} + Ela_{\pi,z}$   
Participation **dampens unemployment** fluctuations;

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Participation **dampens unemployment** fluctuations;  
If  $f(h_c)$  too large, unemployment procyclical.

# Standard Steady State Results

Comparative Statics (for *EtaChi* and *ConstB*):

$$\frac{d\theta}{\theta} = \frac{r + \chi + \alpha\lambda^w}{\vartheta(r + \chi) + \alpha\lambda^w} \frac{dz - db_0}{z - b_0}$$

- ⑥ The variability of labor market tightness,  $\theta$ , depends on the match surplus  $z - b_0$ .
- ⑥ Endogenous participation does not affect the response of labor market tightness to variations in productivity.
- ⑥ The wage, labor market tightness, and the threshold value of homeproductivity are independent of the redraw distribution,  $F(h)$ .

# Participation and Productivity

*Proposition 2:* In steady state the general equilibrium response of the participation threshold satisfies:

$$\frac{dh^c}{dz} = \frac{\alpha\lambda^w}{\vartheta(r + \chi) + \alpha\lambda^w} > 0$$

*Proposition 3:* The change in the participation rate is proportional to the cross sectional density of home productivities at the participation threshold:

$$\frac{dP}{dz} = f(h^c) \frac{dh^c}{dz} > 0$$

# Some Limiting Cases

⑥ *Frictionless limit:  $\phi \rightarrow 0 \Rightarrow$*

✓  $\lambda \rightarrow \infty$

✓  $w \rightarrow z$

⑥ *Constant Participation*

if no mass around the participation threshold.  
(Recall  $h_c$  independent of  $F(h)$ .)



# Summary: Theoretical Findings

- ⑥ Standard Pissarides model and standard (frictionless) RBC model are limiting cases of our model.
- ⑥ The participation threshold varies approximately 1:1 with aggregate productivity  $\Rightarrow$  variation in participation is largely determined by the cross-sectional distribution of home productivity.
- ⑥ Endogenous participation has little (or no) effect on labor market tightness and its dynamics.



# Calibration

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- ⑥ EtaChi: Frequency of home-productivity redraw ( $\eta$ ) equal to exogenous rate of match separation ( $\chi$ );





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 $b = 0.615$  to match semi-elasticity of 1.3;



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steady state participation rate of 65.35%.



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 $b = 0.615$  to match semi-elasticity of 1.3;
- ⑥ Median home productivity  $\bar{h}$ :  
steady state participation rate of 65.35%.
- ⑥ Spread of cross sectional density of home productivity ( $\sigma$ ):  
Match U.S. wage distribution;

# The Wage - Homeproductivity Calibration

kommt noch

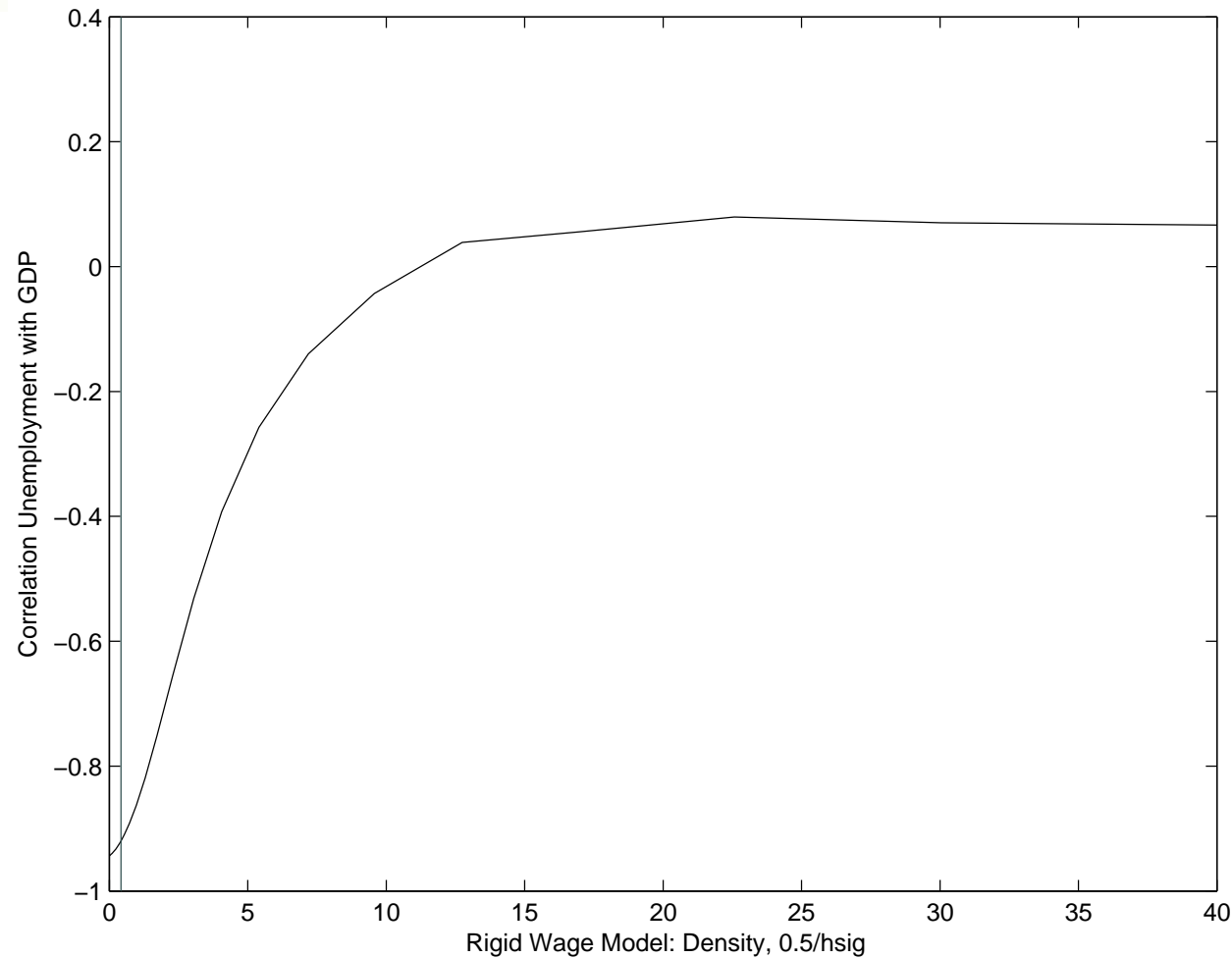
# Micro Studies: LS Elasticities

Name (Year)	Dataset (Years)	Sample	Dependent Variable	Model Approach	Labor Supply Elasticity
Ashenfelter (1978)	NC-Iowa R.I.M.				husband: 0.2 wife: 0.9
van Soest et al (JoEMX, 2002)	Dutch SEP (1995)	mar f	positive desired hours worked	structural	Own Wage: 0.6–0.75
Kimmel & Kniesner (JME, 1998)	SIPP (1984)	sin/mar m/f	Employment vs Nonemployment	FE probit	men: 0.65 – 1.08 women: 1.85–2.41
Chang & Kim (mimeo, 2005)	PSID 79-92 CPS 68-01	mar HH	reservation wage distribution	Calibrate model	men: 0.84–0.96 women: 1.36–1.71

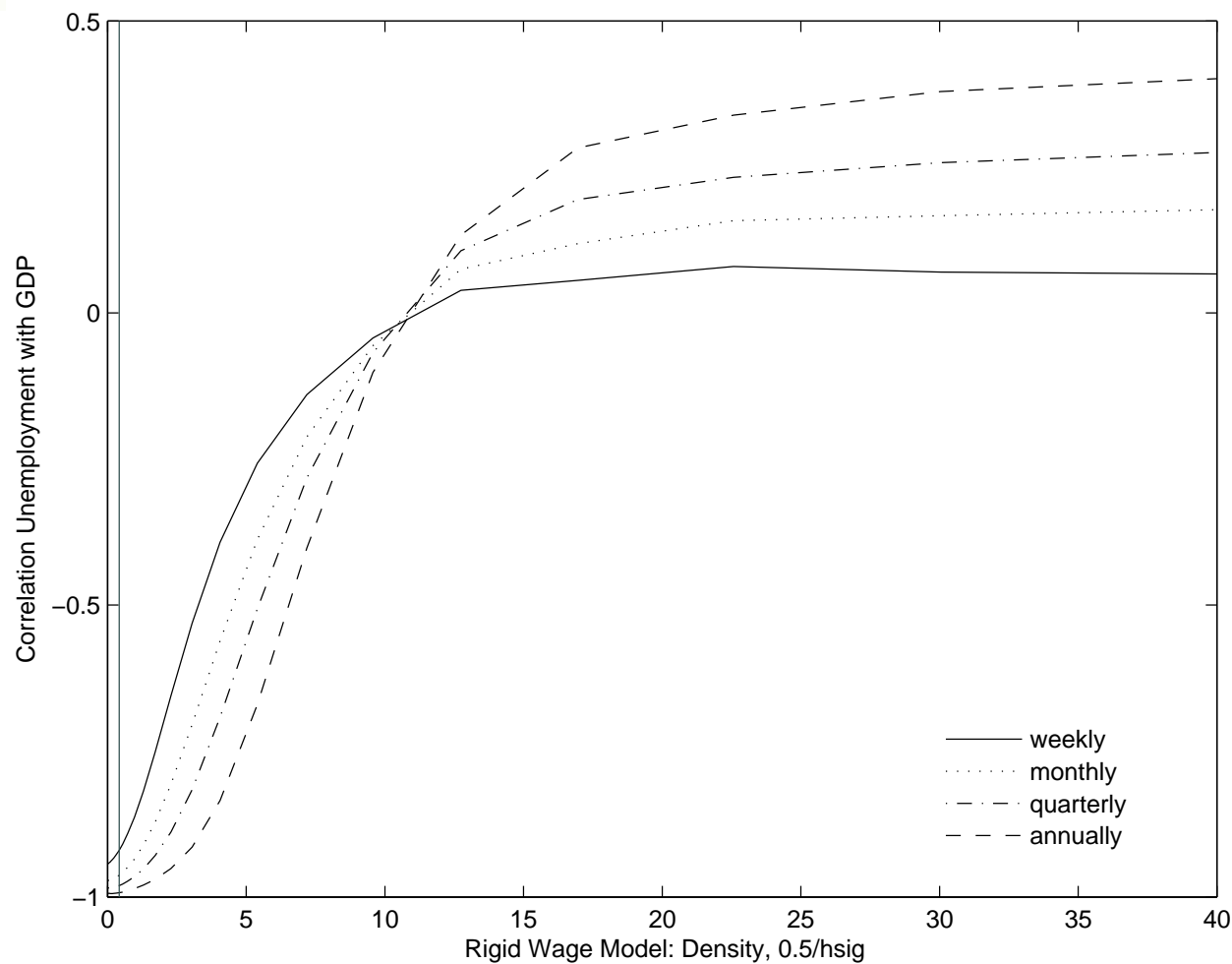
Estimated labor supply elasticities range from 0.2 – 2.4.

Our calibration implies 0.6.

# Cyclicality of Unemployment rate



# Cyclicality of Unemployment rate





# Data and Benchmark

<b>Data</b>	Mean	$\sigma_x$	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	$\rho_{-1}$	$\rho_{x,\text{GDP}}^q$	$\rho_{x,\text{GDP}}^m$
Participation rate	65.354	0.50	0.231	0.877	0.549	
Employment rate	61.205	1.43	0.660	0.955	0.896	
Unemployment	4.150	14.26	6.566	0.947	-0.918	
Real GDP	6888.580	2.17	1.000	0.932	1.000	
<b>Benchmark Model</b>						
Participation rate	65.365	0.358	0.227	0.953	0.911	0.887
Employment rate	61.188	0.428	0.271	0.959	0.896	0.855
Unemployment	4.177	1.084	0.703	0.611	-0.498	-0.286
Real GDP	61.210	1.569	1.000	0.913	1.000	1.000

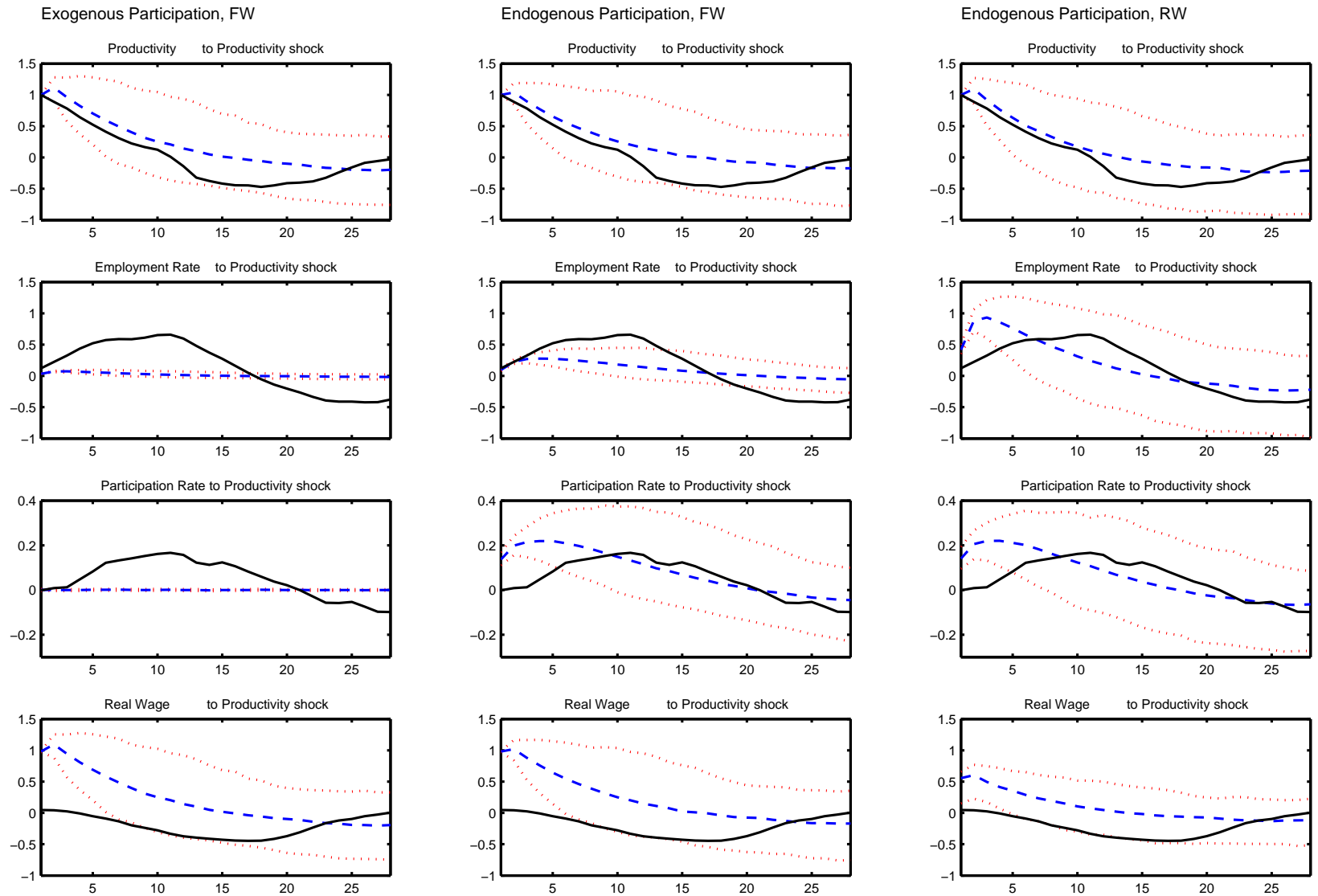
# Employment Variability

	Mean	$\sigma_x$	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	$\rho_{-1}$	$\rho_{x,\text{GDP}}^q$
Data	61.205	1.43	<b>0.660</b>	0.955	0.896
No Participation	61.145	0.085	<b>0.066</b>	0.924	0.965
Flexible, EW	61.188	0.428	<b>0.271</b>	0.959	0.896
Flexible, DW					
$b(h) = b_0$	61.218	0.447	<b>0.279</b>	0.958	0.904
$b(h) = h - 0.2$	61.153	0.407	<b>0.263</b>	0.965	0.848

# Employment Variability

	Mean	$\sigma_x$	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	$\rho_{-1}$	$\rho_{x,\text{GDP}}^q$
Data	61.205	1.43	<b>0.660</b>	0.955	0.896
No Participation	61.145	0.085	<b>0.066</b>	0.924	0.965
Flexible, EW	61.188	0.428	<b>0.271</b>	0.959	0.896
<b>Rigid, EW</b>	61.063	1.227	<b>0.513</b>	0.936	0.981
Rigid, DW: $b(h) = h - 0.2$	61.089	1.148	<b>0.496</b>	0.936	0.980

# Impulse Responses



# Unemployment Variability

	Mean	$\sigma_x$	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	$\rho_{-1}$	$\rho_{x,\text{GDP}}^q$
Data	4.150	14.26	<b>6.566</b>	0.947	-0.918
No Participation	4.173	1.239	<b>0.968</b>	0.924	-0.965
Flexible, EW	4.177	1.084	<b>0.703</b>	0.611	-0.498
Flexible, DW					
$b(h) = b_0$	4.135	1.159	<b>0.736</b>	0.692	-0.636
$b(h) = h - 0.2$	4.155	1.085	<b>0.724</b>	0.591	0.359

# Unemployment Variability

	Mean	$\sigma_x$	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	$\rho_{-1}$	$\rho_{x,\text{GDP}}^q$
Data	4.150	14.26	<b>6.566</b>	0.947	-0.918
No Participation	4.173	1.239	<b>0.968</b>	0.924	-0.965
Flexible, EW	4.177	1.084	<b>0.703</b>	0.611	-0.498
<b>Rigid, EW</b>	4.262	12.393	<b>5.234</b>	0.920	-0.981
Rigid, DW $b(h) = h - 0.2$	4.213	11.174	<b>4.868</b>	0.918	-0.982



# Key Findings

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- ⑥ **Countercyclical Unemployment Rate:**  
Time Aggregation, Labor Supply Elasticity;
- ⑥ **Employment Fluctuations:**  
Participation Margin
- ⑥ **Unemployment Fluctuations:**  
Wage Rigidity

# Summary: Numerical Results

- ⑥ Implied elasticity of labor supply consistent with micro evidence;
- ⑥ Improve employment variability;
- ⑥ Strongly counter-cyclical unemployment rate;
- ⑥ Some wage rigidity: → unemployment fluctuates much more than output.
- ⑥ Results are robust to relaxing assumptions *EtaChi*, *ConstB*.



# Conclusions

Two ingredients ...

- ⑥ Right degree of heterogeneity
- ⑥ Continuous time rather than quarterly simulations

...help improving

- ⑥ on the matching literature in terms of variability of macro aggregates
- ⑥ on the RBC literature by making the labor supply elasticity consistent with micro evidence.

# Robustness

