Submarkets and the Evolution of Market Structure

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We construct a model of industry evolution in which the central force for change is the creation and destruction of submarkets. Firms expand when they are able to exploit new opportunities that arrive in the form of submarkets; they contract and ultimately exit when the submarkets in which they operate are destroyed. This simple framework can transparently explain a wide range of well-known regularities about industry dynamics, most notably the subtle relationships between size, age, growth, and survival. Data on the laser industry, where submarkets are prominent, further illustrate the ability of the model to explain distinctive patterns in the evolution of industries and firms.

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I. Introduction

A remarkable feature of most industries is that at any given moment there is great variation in the size of producers. The bulk of the firms are concentrated at smaller sizes, but there are also a number of larger firms whose size distribution closely resembles the upper tail of the Log-Normal and its cousins, the Pareto and Yule distributions. These patterns have been shown to be consistent with a simple model in which for firms above a minimum efficient size, production is subject to constant returns to scale and firm growth rates are stochastic realizations from a distribution whose mean and variance is independent of firm size. Apart from the constant returns to scale specification, these models do not have much economic content. Consequently, they cannot tell us much about the fundamental drivers of firm growth or about the moments of the firm-size distribution, which are the key determinants of an industry's market structure.

In some ways it has been fortunate that the empirical reality of firm growth has turned out to be more complicated than the early stochastic growth models assumed. Increased availability of confidential establishment data, along with the development of a few comparable panel data sets, in recent years has enabled a more complete understanding of the empirics of firm growth. It has become apparent that among surviving producers, both the mean and variance of firm growth decline with firm size and also with firm age, even after controlling for the other factor. The probability of exit also similarly declines with both firm size and age, but if exiting firms are assigned a -100% growth rate and included in the analysis then mean growth is no longer related to firm age.[1] The size distribution of firms within a single age cohort also evolves with age; its mean and variance rise and the skewness falls as the cohort ages (Cabral and Mata

[1] Dunne, Roberts and Samuelson's [1989] finding of large age effects in the US Census of Manufactures is perhaps the best known set of findings about age. But age effects have been observed in other multi-industry samples constructed from census data (Disney, Haskel and Heden [2000]; Baldwin *et al.* [2000]; Persson [2002-Sweden]) in Dun and Bradstreet data (Evans [1987a, 1987b]), Compustat data (Hall [1987]), and numerous specialized samples (e.g. (Audretsch [1991]; Audretsch and Mahmood [1995]; Baldwin and Gorecki [1991], Mata and Portugal [1994]; Wagner [1994]).

[2003]). These patterns have been challenging to explain, suggesting that they should be revealing about the fundamental determinants of firm growth and market structure. The age effects have posed the greatest challenges—the size effects can be explained by models in which firms experience persistent productivity shocks and production is subject to decreasing returns to scale (cf. Hopenhayn [1992]). But the effects of age on growth and exit suggest there are other factors correlated with age that remain unaccounted for in our analyses.

Remarkably, few theories identify what these missing factors might be. A pioneering exception is Jovanovic's [1982] model of selection, which famously predicts a negative effect of firm age on the variance of growth and, in many datasets, a positive effect of age on survival.[2] His omitted variable is the precision of a firm's beliefs about its quality, which rises with age. More recently, Cooley and Quadrini [2001] have generated size-conditional age effects on growth from a model that combines financial market frictions with persistent shocks to firm productivity. Young firms are assumed to enter the industry as high-productivity firms and, in the presence of financial frictions, high-productivity firms experience more rapid and more volatile growth. The omitted variable in this case is variations in the debt-equity ratio that are correlated with age.[3] While both theories are impressive in their explanatory power, it is also important to recognize their limitations. Jovanovic's model does not predict the effect of age on mean growth and Cooley and Quadrini's model does not predict the effect of age on the probability of exit. Furthermore, neither model addresses why the age effect on mean growth only holds for surviving producers. Both models also require some rather precise assumptions—priors and signals in Jovanovic's model must be Normally distributed (Pakes and Ericson [1998]) and entrants must be more productive than all incumbents in Cooley and Quadrini's model.

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[2] Jovanovic's model predicts that survival is initially declining, and then increasing, with age. However, the initial negative relationship between age and survival holds only for very young firms, and may not be easily observable in census data that span five-year intervals.

[3] Cabral and Mata [2003] also propose a model of financial constraints to explain the evolution of the firm size distribution they observe in data from the Portuguese manufacturing sector.

The main purpose of this paper is to propose another channel through which age affects firm performance that transparently accounts for all the age-size regularities as well as additional related regularities. We start from the self-evident fact that the way we are accustomed to define industries in empirical work (almost invariably by SIC code) suppresses a large amount of heterogeneity in firm activity. Firms defined as belonging to the same industry could, if only we had the appropriate data, be differentiated along numerous dimensions, such as the technology they use, the services they provide, the customer segments they target, or the geographic areas in which they operate. We call these different activities *submarkets*. It is equally self-evident that in many industries, these submarkets have their own dynamics. New opportunities arise, and only firms that succeed in exploiting them benefit from their arrival. Existing submarkets vanish as technologies become obsolete, as geographic areas decline, or as regulations change, and all firms dependent on these submarkets suffer as a consequence. Indeed, firms that are specialized in a single submarket can be expected to vanish when the submarket dies.

We can't imagine that our distinction between the static, homogeneous, industry-as-SIC-code world of empirical analysis, and the dynamic, heterogeneous, collection of submarkets that most industries consist of is either contentious or surprising to most readers. In fact, it has already been the subject of some recent theorizing about industry evolution.[4] Perhaps more surprising is that, in a world of submarkets, age will be found to affect growth and survival any time the econometrician fails to control for the number of submarkets in which each firm is active. We show this in a model in which industries consist of finitely-lived, differentiated submarkets, which we cast as the *sole* driving force behind entry, exit and firm growth. A firm's growth is negatively related, and its survival is positively related, to the number of submarkets in which it is active. At the same time, the number of submarkets in which a firm is active is increasing in age. We show that this simple framework transparently generates *precisely* the effects of age on mean growth, the variance of growth and the evolution of the size distribution that have been

[4] Sutton [1998] used the idea of new growth opportunities appearing in the form of new submarkets in an industry to develop his theory of the firm-size distribution. Mitchell [2000] posited the existence of submarkets utilizing the same knowledge but differing in their competitive challenges to explain firm diversification. Bottazzi *et al.* [2001] find that new submarkets driven by the discovery of new drugs help explain the evolution of the pharmaceutical industry.

observed in the data. We show it also explains a number of other regularities, including properties of the firm-size distribution, mean reversion in firm size over long, but not short, time horizons, and variations in the sizes and diversity of entrants, exiters, and surviving firms.

We hope that our ability to explain all the age-size regularities using a ubiquitous feature of industrial activity will inspire confidence in the importance of submarket phenomena in the process of firm growth. We can also readily point to industries analyzed by others where submarkets have been the key to differential rates of firm growth. For example, in hard disk drives Christensen [1993] implicates new submarkets opened by smaller disk drives as the cause of the leading incumbents repeatedly being displaced by new entrants and other incumbents.[5] In pharmaceuticals, Bottazzi et al. [2001] develop a stylized model of submarket branching to explain variations in firm growth rates. We go beyond these examples to analyze the importance of submarkets in yet another modern industry, the laser industry. We derive a series of distinctive hypotheses from our model that we test using data on the types of lasers produced by entrants into the industry over its first 30 years. Our analysis suggests that submarket creation and destruction played a key role in the entry, exit, and growth of laser producers.

Much as the early stochastic growth models were criticized for their lack of economic content, it will be tempting to criticize our model similarly. Unlike the early stochastic growth models, though, our model identifies the source of firm growth and provides structure on how it is expected to operate. But it is also true that our model is consistent with a lot of different underlying economic mechanisms, as we point out. Indeed, we see this as a virtue, as it shows off the power of submarket phenomena to explain the accumulating regularities. It also demonstrates that the regularities may not be as revealing as we might have hoped about the mechanisms underlying firm growth. With three explanations on the table for the regularities, evidence will be needed about the quantitative importance of each to judge their relevance. The industry examples noted above and our analysis of the laser industry represent our initial foray into this domain. At the same time, the fact that we can explain all the regularities with such a simple model suggests that it might be useful to look elsewhere to come up with further insights into the determinants of firm growth. One place where we suggest looking is in-

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[5] Two subsequent efforts have carefully analyzed entry into the new submarkets (King and Tucci [2002], Chesbrough [2003]).

dustry *irregularities*—patterns that hold only in some industries, particularly patterns regarding market structure, which was the impetus for much of this literature. Our model predicts that the number of firms in a new industry should monotonically rise over time, which is what occurred in the laser industry. But it has also been shown that certain industries, such as autos, tires, and television receivers, experienced extremely sharp shakeouts in the number of producers as they aged despite robust growth in total production (Klepper and Simons [1997]). Our model provides a way to think about what might have been different in these industries regarding submarkets that could account for their shakeouts. This discussion also helps delineate the kinds of industries to which our model is most applicable.

The paper is organized as follows. In Section II we develop the model and derive its implications for the effects of age and size on firm growth and survival. In Section III we report additional predictions of the model and compare them with the evidence. In Section IV we relate the model to evidence from the laser industry. Section V offers some observations on the implications of our analysis.

II. Submarkets and the effects of age and size

An industry is composed of various submarkets. The industry begins at time t=0 when its first submarket is created. Subsequently, submarkets are created according to a homogeneous Poisson point process with mean intensity λ, so that to a first-order approximation there is a probability λdt of a new submarket being created in the interval dt. Each submarket has a random life, z, drawn from the distribution $H(z)$. It is assumed only that $H(z)$ is differentiable almost everywhere with finite mean submarket life, $\mu = \int_0^{\infty} z dH(z)$.[6] There are C potential entrants to each submarket. Each has a probability of θ of entering a new submarket, where θ is the same for all firms and submarkets. Upon entering, the firm's size in the submarket is drawn from the distribution $F(x)$, $F(0)$=0, where F is continuous and strictly increasing. This size remains constant for the duration of the submarket and then goes to zero as soon as the submarket is destroyed.

[6] For some results, we must assume that firm dynamics are Markovian. This requires that $H(z)$ is exponential.

Total firm size at any point in time is therefore the random sum of n draws from the distribution F, where n is the number of currently existing submarkets that the firm entered.

In modeling industry evolution and firm dynamics as an exogenous stochastic process, we do not intend to discount the role of rational firm choice. To the contrary, we assume that the stochastic process is driven by a well-defined maximization problem for each firm. But whatever the details of the maximization problem, it will yield an equilibrium entry rate, θ, and a distribution of firm sizes among entrants, $F(x)$. [7] None of our results depends on the particular value of θ or the form of $F(x)$, and we do not analyze the effects of policy. Hence, the generality afforded by the reduced form serves us well.

Firms may enter and exit a state of zero submarket activity, and in this setting, all firms live forever. To address questions of entry and exit, we therefore adopt the following convenient stylization. Consider two sampling times, t and $t+T$. At time t, we define the age, $s(t)$, of the firm as the length of time that has elapsed since it last entered a state of zero submarket activity. At time $t+T$, we identify a firm as an exit if it is active in no submarkets and it was in one or more submarkets at time t. An entrant is defined symmetrically as a firm active in one or more submarkets at time $t+T$ that was active in no submarkets at time t. This treatment enables us to exploit properties of generalized

[7] To provide one concrete example, assume a new submarket allows firms that successfully exploit it to sell output at a price p. Firms undertake R&D in an attempt to exploit the new opportunity. R&D yields a cost parameter, α, drawn from the distribution $G(a;R)$, which is increasing in R&D expenditure, R. Profits are $\pi(\alpha) = \max\left(0, \max_x \left[px - \alpha c(x) - k\right]\right)$, where k is a fixed cost of production and $c(x)$ is increasing and convex. This technology defines an upper threshold, $\overline{\alpha}$, for the cost draw, above which the firm chooses not to enter. Each firm's optimal level of R&D is given by $R^* = \arg\max\left[-R + \int_0^\infty \int_0^{\overline{\alpha}} \pi(\alpha) dG(\alpha;R) e^{-rz} dH(z)\right]$. Hence, the fraction of firms entering a submarket is $\theta = G(\overline{\alpha};R^*)$. Let $\alpha(x)$ denote the cost draw that yields an optimal output of x. The distribution of firm output is then given by $F(x;R^*) = (1 - G(\overline{\alpha};R^*))^{-1} \int_{\alpha(x)}^{\overline{\alpha}} dG(\alpha;R^*)$. Endogenizing p by defining a submarket in terms of aggregate demand, $D(p)$, or assuming that submarket arrivals are also driven by R&D effort, are straightforward extensions.

equilibrium Poisson point processes to address questions that have usually proved intractable.

A. *Preliminary results*

The number of active submarkets in the industry changes over time, which causes the number of active submarkets in which firms participate to change over time. Our first task is to establish distributions for the number of active submarkets and for firm participation in submarkets. While we can characterize transitory distributions for any time t, much of our analysis is concerned with market structure in the stationary state. The transitory distributions are therefore given as lemmas in the appendix, while Propositions 1 and 2 provide the corresponding characterization in the stationary state. All proofs are also contained in the appendix.

PROPOSITION 1. *Let $p_k(t)$, k=0,1, 2, 3 . . ., denote the probability that exactly k submarkets are active at time t. The stationary distribution, $p_k = \lim_{t\to\infty} p_k(t)$, exists and is given by the Poisson distribution with mean $\lambda\mu$: $p_k = e^{-\lambda\mu}(\lambda\mu)^k / k!$.*

It is somewhat remarkable, although well known from the theory of queues (e.g. Takács [1958]), that the Poisson distribution holds independently of the form of the distribution $H(z)$. There is a direct counterpart for the distribution of an arbitrary firm's participation in active submarkets. Excluding the industry's first submarket, the number of submarkets in which a firm is active is the sum of n independent Bernoulli trials with probability of success θ, where n is Poisson with mean $\lambda\int_0^t [1-H(z)]\,dz$. This random sum also has a Poisson distribution.

PROPOSITION 2. *Let $v_k(t)$ denote the probability that a firm is active in exactly k submarkets at time t. The stationary distribution, $v_k = \lim_{t\to\infty} v_k(t)$, exists and is given by the Poisson distribution with mean $\theta\lambda\mu$: $v_k = e^{-\theta\lambda\mu}(\theta\lambda\mu)^k / k!$.*

While there will be variation over time in industry and firm output, the model generates stable limiting distributions for the number of industry submarkets and the number of submarkets in which firms are active. The number of active submarkets has an asymptotic mean and variance of $\lambda\mu$, and the number of firms active in at least one submarket has an asymptotic mean of $C(1-v_0)= C(1-e^{-\theta\lambda\mu})$. Both are increasing in the rate at which submarkets are created and in the mean duration of a submarket, while the latter is also increasing in the probability of firm entry and the size of the entry pool.

A firm's size depends not only on the number of active submarkets in which it participates, but also on the sizes at which it enters each one. Firm size, $y = \sum_{i=1}^{n} x_i$, is the sum of a random number, n, of i.i.d. random variables where each term in the sum is a draw from the distribution $F(x)$ and the number of terms is Poisson. It is generally only feasible to characterize the distribution of a random sum in terms of its characteristic function. Let $\phi_x(r)$ denote the characteristic function of the distribution F. Then, conditional on a firm being active in n submarkets, the characteristic function of the sum $y = \sum_{i=1}^{n} x_i$ is simply the n-fold product of $\phi_x(r)$. The characteristic function, $\phi_y(r)$, for the unconditional distribution of firm size is obtained by taking the expectation over n. As we will need to condition on firm age, s, the first task is to obtain a distribution for $n(s)$:

PROPOSITION 3. *Let $w_k(s(t),t)$ denote the probability that a firm of age s at time t, $s(t)$, is active in exactly k submarkets at time t. Then $w_k(s) = \lim_{t\to\infty} w_k(s(t),t)$ exists and is the probability of exactly k events from a Poisson distribution with mean* $\theta\rho(s) = \theta\lambda\int_0^s [1 - H(z)]\, dz$.

In the steady-state, for firms of any age the distribution of the number of submarkets in which they participate is Poisson, but the mean is strictly increasing in age. There is also a corresponding steady-state distribution of the sizes of these firms. The characteristic function of this distribution is derived in the appendix and generates the following moments:

PROPOSITION 4: *In the stationary state, the size distribution of firms of age s has mean* $E(x)\theta\lambda\int_0^s [1-H(z)]dz$, *variance* $E(x^2)\theta\lambda\int_0^s [1-H(z)]dz$, *and coefficient of skewness* $\left(\theta\lambda\int_0^s [1-H(z)]dz\right)^{-1/2} E(x^3)E(x^2)^{-3/2} > 0$.

Regardless of the form of $F(x)$, the distribution is positively skewed and has the same long tail to the right as the Poisson. The older the cohort, the greater the mean and variance and the smaller the skewness of firm size. These predicted properties of the size distribution match precisely those found for Portuguese manufacturing firms by Cabral and Mata [2003], which had been presented as a challenge for theory to explain.

B. *The effects of size and age on survival*

Firms can only exit if their activity first declines to one submarket. Consequently, the more submarkets a firm is in then the lower its chance of exiting in any finite time period. This result is stated as Proposition 5.

PROPOSITION 5. *For any* $t, T \in (0,\infty)$, *the probability of exit by time* $(t+T)$ *is strictly decreasing in* $n(t)$.

The relationship between age, size and survival is complicated by the fact that, for arbitrary $H(z)$, history matters, and the age of the firm is bound up with submarket age and the hazard of submarket destruction in possibly complicated ways.[8] If $H(z)$ is assumed to be exponential, however, the stochastic process is Markovian. The probability of submarket destruction is then independent of the age of submarkets, and $n(t)$ is a sufficient statistic for the probability of exit. Proposition 3 implies that $n(t)$ is positively related to a firm's age, $s(t)$. Since firm size equals the product of $n(t)$ and the average

[8] Firm age is correlated with the ages of the submarkets in which the firm is active (Lemma 3 in the appendix proves that submarket age is increasing in firm age in the sense of first-order stochastic dominance). This in turn influences the probabilities of submarket destruction and hence the probability of firm exit.

size of the firm in each of its submarkets, it is also positively related to $n(t)$. Firm age and firm size are related to $n(t)$ in different ways, and thus both will be positively related to $n(t)$ even conditional on the other. Therefore, it follows that:

PROPOSITION 6. *If $H(z)$ is exponential, for any $t, T \in (0, \infty)$ the probability of exit by time $(t+T)$ is a strictly decreasing function of $y(t)$ and $s(t)$.*

Proposition 6 implies that the probability of exit will decline with firm size holding firm age constant and will decline with firm age holding firm size constant, consistent with the evidence.

C. *The effects of size and age on firm growth*

In this subsection, we derive propositions that address the relationships between firm growth rates and firm age and size. It is useful to restate the evidence on growth, as it is quite specific:

G-1: Conditional on firm survival, average firm growth declines with firm size holding firm age constant and declines with firm age holding firm size constant (Dunne, Roberts and Samuelson [1989], Evans [1987], Hall [1987]).

G-2: Assigning a fractional growth rate of –1 to exiting firms, the inverse growth-size relationship holds for all firms but it is weaker than for surviving firms, while the inverse growth-age relationship does not hold for all firms [Dunne, Roberts and Samuelson [1989]).

G-3: The variance of firm growth declines with firm size holding firm age constant (Dunne, Roberts and Samuelson [1989], Hymer and Pashigian [1962], Mansfield [1962], Stanley, et. al. [1996], Sutton [2000]) and declines with firm age holding firm size constant (Dunne, Roberts and Samuelson [1989]).

The empirical evidence on the relationship between mean firm growth and firm age and size depends on whether one conditions on firm survival. We begin with the model's predictions for the unconditional case, G-2:

Proposition 7. *Among all firms, mean firm growth is strictly decreasing in firm size conditional on firm age, but ambiguously related to firm age conditional on firm size.*

The ambiguity in the effects of age is the result of the fact that, for arbitrary $H(z)$, the hazard of submarket destruction may rise or fall with submarket age. In the Markovian case, the hazard is constant, so that for $H(z)$ exponential we have the following result:

Corollary. *Suppose H(z) is exponential. Among all firms, mean firm growth is strictly decreasing in firm size conditional on age, but unrelated to firm age conditional on size.*

The net change in a firm's size in any time period equals its output in submarkets created during the time period minus its output in submarkets destroyed during the time period. All firms have the same expected increment to size due to the creation of new submarkets. Consequently, the expected proportional increase in size due to the creation of new submarkets is a decreasing function of firm size and is independent of firm age. Assuming $H(z)$ is exponential, the expected decrement to firm size due to the destruction of submarkets is proportional to the firm's size at the start of the period. Hence, the expected proportional decline in a firm's size due to the destruction of submarkets is independent of its size and its age. Therefore, the expected growth rate of firms is a decreasing function of firm size and is independent of firm age.

Regularity G-1 can be explained as follows. Conditioning on survival strengthens the inverse relationship between expected firm growth and firm size. Smaller firms have a greater probability of exit, which depresses their overall growth rate. Consequently, conditioning on survival increases the growth rate of smaller firms by more than larger firms, which enhances the negative effect of firm size on mean growth. Conditioning on firm survival also allows for firm age to affect firm growth through its influence on the probability of exit. With younger firms having higher exit rates, conditioning on firm survival increases the growth rates of younger firms more than older ones, inducing a negative relationship between expected firm growth and firm age. This is stated formally in the following proposition:

PROPOSITION 8. *Assume H(z) is exponential. Conditional on firm survival, mean firm growth is strictly decreasing in firm size conditional on firm age and strictly decreasing in firm age conditional on firm size. The negative effect of size on mean growth is stronger when conditioning on survival than when not conditioning.*

Regularity G-3, concerning the variance of firm growth rates, can be explained as follows. Submarket destruction and creation are independent. Consequently, the variance of growth for a firm of size $y(t)$ active in $n(t)$ submarkets can be written as the sum of the variance of changes in firm size caused by entry into new submarkets and changes caused by destruction of some or all of the existing $n(t)$ submarkets. We show in the appendix that the variance of the growth rate contributed by entry into new submarkets is decreasing in firm size but independent of $n(t)$. We also show, for the exponential case, that the variance of the proportional loss in output due to the destruction of submarkets is decreasing, not only in $y(t)$, but also in $n(t)$. Firms with more submarkets are able to pool their losses due to submarket destruction over more submarkets, which will lower the variance of their lost output. In view of the relationship already established between firm age and submarket participation, it follows that:

PROPOSITION 9. *Assume H(z) is exponential. Among all firms, the variance of firm growth is strictly decreasing in firm size conditional on firm age and strictly decreasing in firm age conditional on firm size.*

III. Additional Predictions

Some additional empirical regularities arise as immediate consequences of the structure of the model. For example, the average size of entrants and exiting firms, both of which are active in just a single submarket, is less than that of surviving incumbents, consistent with the evidence in Cable and Schwalbach [1991]. Other predictions of the model are not so immediate. In this section we report some of them and relate them to empirical evidence accumulated over the last couple of decades.

A. *Entry, exit and the size distribution of firms*

The mean entry and exit rates are, to first order, both equal to $\theta\lambda dt e^{-\theta\lambda\mu}/(1-e^{-\theta\lambda\mu})$. Hence, consistent with the evidence (Cable and Schwalbach [1991], Dunne, Roberts and Samuelson [1988], Evans and Siegfried [1994]), the model predicts that industries characterized by higher entry rates also have higher exit rates, although not necessarily contemporaneously.

For younger industries yet to attain the steady state, the implications of the model concerning entry, exit, and the number of firms are summarized in the following proposition.

PROPOSITION 10. *Assume $H(z)$ is exponential and $\theta < (\lambda\mu - 1)/\lambda\mu$. Then*: i) *The number of active firms increases with industry age*; ii) *The rate of entry decreases with industry age*; iii) *The rate of exit may increase or decrease with industry age and need not vary monotonically with age.*

The condition $\theta < (\lambda\mu - 1)/\lambda\mu$ places an upper bound on the expected importance of the industry's first submarket. If relatively few firms enter the first submarket, the number of firms is expected to increase over time toward a limiting steady-state value. If $\theta > (\lambda\mu - 1)/\lambda\mu$, the mean number of firms initially falls, but it still must eventually rise and approach its limiting steady state from below. Consequently, the number of potential entrants (i.e., the number of firms with zero output) is expected to (eventually) decline over time, and hence the ratio of entrants to the number of firms is expected to (eventually) decline. The number of submarkets in which firms participate is also expected to rise over time. However, the expected proportion of firms active in just one submarket (i.e., the firms that face a nonnegligible probability of exit in the subsequent interval *dt*) need not rise monotonically with time, so industries may experience exit rates that are non-monotonic with respect to industry age.

The moments of the steady-state firm-size distribution predicted by the model can be conveniently obtained by letting $s \to \infty$ in the moments reported in Proposition 3:

PROPOSITION 11: *The stationary steady-state firm-size distribution has mean* $\theta\lambda\mu E(x)$, *variance* $\theta\lambda\mu E(x^2)$, *and coefficient of skewness* $(\theta\lambda\mu)^{-1/2}E(x^3)E(x^2)^{-3/2}>0$.

Firms are larger on average, and the variance of firm size is greater, when firm entry into a new submarket is more likely, when new submarkets arrive more frequently, and when submarkets survive longer. If parameters take on similar values in the same industry in different countries, then in each country industries will have a similar ordering in terms of measures related to the moments, such as the k-firm concentration ratio. There is an extensive literature on industry size distributions. Although in most industries there is considerable entry, exit, and mobility of firms, industry firm-size distributions appear to be stable over time, with summary measures such as the k-firm concentration ratio taking on similar values in the same industry in different countries (e.g., Bain [1966], Pryor [1972]).

The distributions are generally positively skewed with a long upper tail (e.g., Ijiri and Simon [1977]). No single distribution fits all industries (Curry and George [1983], Schmalensee [1989]), but the upper tail of most industry firm-size distributions resembles a family of statistical distributions, including the Log-normal, Pareto, and Yule. Using a geometric distribution to approximate industry firm-size distributions, Sutton [1998] shows that the k-firm concentration ratio will satisfy a lower bound that is closely satisfied in manufacturing industries. The particular form of the firm-size distribution implied by the model can in principle be obtained by specifying F. Among the tractable cases, some familiar distributions emerge. If F has the discrete logarithmic distribution, then y is distributed as negative binomial (Lüders [1934], Quenouille [1949]). The Yule distribution associated with the work of Simon [1955] and colleagues can be obtained by compounding the geometric distribution with a mixing distribution $F(p)=p^{w-1}$.[9]

A distinctive feature of our model is that the industry firm-size distribution will differ from the distribution of market shares within submarkets, which cannot be a predic-

[9] Of course, many firm-size distributions can be obtained from the model, and the same size distribution can be obtained from conceptually distinct models (Boswell and Patil [1970]), so not too much should be made of the model's ability to generate any particular one.

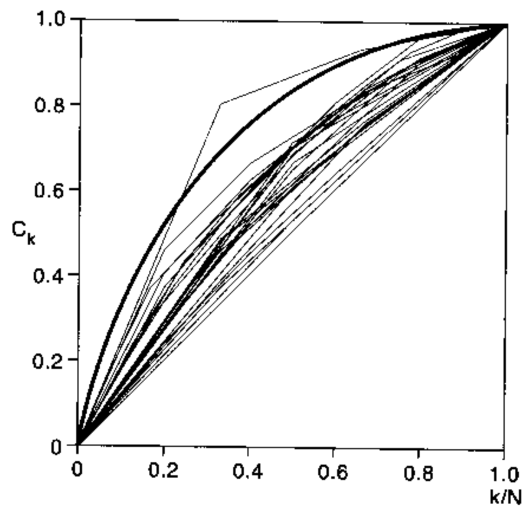

FIGURE 1. Lorenz curves for US cement industry. Graph contains Lorenz curves for each state, plus national curve (bold). Source: Sutton [1998, p. 310].

tion of models that assume a homogeneous industrial structure. Specifically, our model predicts that the industry distribution will always exhibit greater concentration than the within-submarket distribution. Figure 1 reports some revealing evidence from the US cement industry. Because of high transportation costs, Sutton [1998] argues that state boundaries provide a useful approximation to submarket boundaries. Figure 1 illustrates clearly that the national size distribution is much more concentrated than almost all the size distributions at the state level. Sutton further shows that the difference between the national and state size distributions is explained by the fact that larger firms operate across multiple submarkets, as would be expected based on our model.

B. *Mean reversion in firm size*

In any model combining a stationary firm size distribution with a persistent churning of firms, size must eventually be mean reverting. However, there is no requirement that mean reversion holds at all time horizons. Indeed, the evidence indicates that while firm size is mean reverting over long time horizons, it is not mean reverting over short horizons (Baldwin [1995, ch. 5], Bailey, Hulten, and Campbell [1992]). Our model predicts this.

PROPOSITION 12. *In the stationary state, there exists a* $T^*(y(t))$ *such that* $E[y(t+T^*)-y(t)] >[<] 0$ as $y(t) <[>] E(y)$, *where* $E(y)= \theta\lambda\mu E(x)$.

Intuitively, larger firms participate in more submarkets on average and have a larger average output per submarket than smaller firms. Consequently, larger firms have more output to lose from the destruction of submarkets. In contrast, all firms have the same expected increment to size due to the creation of new submarkets. Consequently, firm growth is mean reverting.

C. *Firm diversification*

Mitchell [2000] has noted two regularities that appear to characterize how diversity is related to firm size and age. First, firms tend to begin specialized and expand their scope as they age. Our model readily explains this. Firms enter in one submarket, and in the stationary state the expected number of submarkets in which a firm participates increases (at a decreasing rate) with its age.[10] Second, diversification is positively related to firm size, so that within industries the largest firms have the broadest product portfolios. In our model, this result follows readily from the fact that larger firms on average have more submarkets. The appendix provides a formal proof, along with some related predictions:

PROPOSITION 13. (i) *Larger firms are on average more diversified than smaller firms.* (ii) *The more diversified a firm, the greater the probability that it is a leader in one or more*

[10] The claim can be verified formally on differentiating (twice) the mean of the distribution given in Proposition 3 with respect to firm age, *s*.

submarkets. (iii) *The larger the firm, the greater the probability that it is a leader in one or more submarkets.*[11]

IV. The US Laser Industry

We have shown that the model is capable of explaining a wide array of regularities pertaining to industry dynamics. We noted in the introduction that many of these regularities could also be explained by the models of Jovanovic [1982] and Cooley and Quadrini [2001]. To provide further support for the importance of submarket phenomena, we use data from the laser industry pertaining to submarkets to test distinctive implications of our model. Some of these implications have already been derived and additional ones will be established. The only way these implications could be addressed in Jovanovic and Cooley and Quadrini's models is if submarkets were differentiated and firm productivities were allowed to differ at the submarket level. Even then, it is not clear whether these models could account for the distinctive implications of our model. We make no pretense at comprehensively testing our model. Indeed, it is sufficiently stylized that no doubt some of its implications would fail exacting tests. Our goal is more modest. We want to show that in the laser industry, where submarkets have been prominent, distinctive predictions of our model are supported.

Submarkets can occur at many levels, but in lasers it is useful to think of them as corresponding to particular applications serviced by specific lasers. Lasers differ regarding their power and the wavelength of light they emit, which constrains the applications

[11] We may note here some corollaries to Proposition 13. First, it is also true that diversified firms are more likely to be one of the smallest firms in one or more submarkets. In fact, if we learn that a firm fell into *any* particular range of the size distribution in a submarket, Bayes' rule would cause us to raise our expectation of the number of active submarkets. This suggests that part (ii) of the proposition is not all that interesting (or more relevant, people's attempts to exploit part (ii) as evidence of economies of scope/knowledge spillovers is not useful). Second, it is important to note that we *cannot* say the same about larger firms being more likely to be one of the smallest in one or more submarkets because of the effects of $F(x)$ on size.

for which they can be used. For example, very different lasers are required for scanning, eye surgery, and the working of metals. Consequently, the laser market can be thought of as being composed of submarkets with little if any relationship on the demand side, consistent with our model.

We don't have data on each submarket, but we have data compiled by Klepper and Sleeper [2004] on nine broad categories of lasers that differ in terms of their lasing material. Using abbreviations and putting dates of introduction in parentheses, they include: six gases, Helium Neon—HeNe (1961), Carbon Dioxide—CO_2 (1966), Argon Ion—Ion (1966), Helium Cadmium—HeCd (1970), Excimer—Exc (1978), and a catchall category of Other Gas Lasers—GasOth (1963); solid state crystal lasers—SState (1961); semiconductor lasers—Semic (1963); and chemical dye lasers—Dye (1968). Each of these broad laser types comes in different varieties that service different users and tend to be produced by different firms, so that generally multiple submarkets exist within each laser type. For example, different firms tend to produce short-wave semiconductor lasers used in CD players and laser printers than long-wave semiconductor lasers used in fiber optics. Similarly, different firms tend to produce low-power CO_2 lasers used in certain types of surgery than high-power CO_2 lasers used in the working of certain materials.

Over time, the number of laser submarkets has mushroomed. Some, such as the use of semiconductor lasers in CD players and in computer applications, were driven by the invention of the CD player and the advent of the personal computer. Others, such as the development of hand-held scanners, were created by the miniaturization of the HeNe laser, which was the work-horse of stationary scanning applications. Submarkets have also been destroyed. In part, this has occurred by the improvement of nonlaser approaches to certain applications, such as surgery and printing. Perhaps more important is what might be termed creative destruction, which is the displacement of one type of laser by another, often within the same broad category. For example, the first material that was made to lase was a Ruby crystal, and initially Ruby lasers were used in many applications. Subsequently, Nd:Yag lasers, which are also solid state lasers, displaced Ruby in many applications.

We have annual data from Klepper and Sleeper [2004] on the types of lasers produced by each of 464 entrants into the U.S. commercial laser industry from its inception

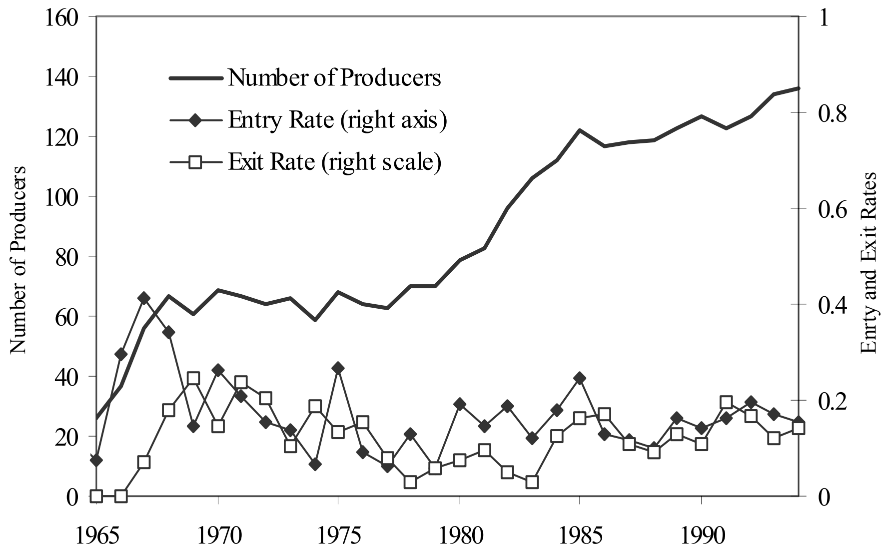

FIGURE 2. Number of Producers, Entry and Exit Rates in the Laser Industry, 1965-1994.

in 1961 through 1994. Firms tended to specialize by laser type, reflecting the limited number of submarkets in which they participated. Over their lifetime, 55% of the firms produced only one type of laser, 20% two types, 23% 3-6 types, and 2% 7-9 types. While the nine categories of lasers are typically broader than submarkets, most of the predictions of the model pertain to laser types as well as submarkets. We do not have output data, so we restrict our focus to the predictions concerning firm scope, entry, and exit.

By way of background, in Figure 2 we present the annual number of producers, entrants, and exits for the period 1965 to 1994.[12] The patterns are broadly consistent with the model. The number of firms increased over time, consistent with Proposition 10. The entry rate was high initially and then declined, which is also consistent with Proposition 10. Note the particularly low entry rate for most of the 1970s. This corresponds to a period in which the laser was jokingly characterized as a solution looking for a problem,

[12] The main data source used to compile the list of producers began in 1965, and the only data we have for before 1965 pertains to the firms listed as producers in 1965. Consequently, we restrict Figure 3 to the period 1965 to 1994.

reflecting the slow rate at which new submarkets were created. The exit rate patterns are less distinctive, and Proposition 10 makes no predictions about the trend in the exit rate over time.[13]

The model predicts that for the average firm, the number of submarkets in which it participates should increase with age. In Table 1, we present data on the average number of laser types produced by all firms from ages 1 to 34. Among the 464 firms, Table 1 indicates that at age 1 they produced an average of 1.36 lasers. Not all of these firms survived past age 1, and Table 1 lists the number of survivors to each age and the average number of laser types produced by the survivors at each age. The average number of laser types increased monotonically through age 10, and after age 10 the trend continues upward through age 31, which was attained by only two firms. Thus, consistent with Proposition 3, the average number of laser types produced by all firms increased with age. We also report in Table 1 the average number of laser types produced by firms that never exited. It is conceivable that the number of laser types produced by all firms increased with age due to the exit of firms that produced a smaller number of laser types. The patterns for the non-exiters are similar to all firms, though, suggesting that the rise in laser types with age for all firms is not due to exiters being markedly different from continuing firms.

While the evidence in Table 1 is consistent with the predictions of the model, the patterns may also be induced through other mechanisms, such as learning bringing about scope economies (c.f. Mitchell [2000]). We can derive yet more exacting predictions that can help distinguish such mechanisms from our explanation for the patterns. A majority of the laser firms ultimately exited. The model predicts these firms will produce only one type of laser just before exiting. Moreover, it predicts that the expected number of laser types they produce will monotonically decline with age, which is the opposite of the prediction for all laser firms. More precisely, if observations on all firms that ultimately exit are ordered by the number of years prior

[13] Regressions of entry and exit rates on a linear time trend and lagged entry or exit return a significant decline in the entry rate (p=0.09) and no trend in the exit rate (p=0.73).

TABLE 1

Average Number of Laser Types of Firms at Each Age

(*Number of firms in parentheses*)

AGE	ALL FIRMS	NON-EXITING FIRMS	AGE	ALL FIRMS	NON-EXITING FIRMS
1	1.36 (464)	1.31 (136)	18	2.10 (31)	2.12 (16)
2	1.43 (336)	1.44 (115)	19	2.14 (28)	2.19 (16)
3	1.51 (268)	1.48 (97)	20	2.12 (25)	2.21 (14)
4	1.62 (227)	1.57 (81)	21	2.17 (23)	2.36 (14)
5	1.66 (197)	1.54 (70)	22	2.19 (21)	2.33 (12)
6	1.78 (166)	1.60 (65)	23	2.47 (17)	2.58 (12)
7	1.78 (144)	1.67 (57)	24	2.57 (14)	2.60 (10)
8	1.85 (127)	1.77 (52)	25	2.90 (10)	3.12 (8)
9	1.86 (107)	1.84 (49)	26	2.89 (9)	3.14 (7)
10	1.91 (90)	1.98 (43)	27	3.00 (8)	3.33 (6)
11	1.89 (73)	2.00 (35)	28	3.80 (5)	5.33 (3)
12	1.87 (67)	2.00 (32)	29	3.00 (4)	4.50 (2)
13	1.86 (59)	1.94 (31)	30	3.33 (3)	4.00 (2)
14	1.96 (47)	2.08 (24)	31	4.00 (2)	4.00 (2)
15	2.23 (39)	2.40 (20)	32	3.00 (2)	3.00 (2)
16	2.22 (32)	2.25 (16)	33	3.00 (2)	3.00 (2)
17	2.16 (31)	2.19 (16)	34	2.50 (2)	2.50 (2)

to exit, the model implies that the mean number of submarkets in which firms participate is a monotonically increasing function of the number of years prior to exit. We prove this for the Markovian case in the appendix and state the result formally here:

PROPOSITION 14. *Assume $H(z)$ is exponential. Let $E[n(t-T)\,|\,n(t)=0]$ denote the expected number of submarkets in which a firm was active at time t–T, given that it is inactive at time t. In the stationary equilibrium, $E[n(t-T)\,|\,n(t)=0]$ increases with T at a decreasing rate.*

To our knowledge, no one has studied how the scope of exiting firms changes with age up to the time of exit, but we can examine this for lasers. Table 2 orders each year-firm observation by the number of years yet to elapse before the firm exits.[14] Proposition 14 is strongly supported. The trend in the average number of laser types is steadily upward from one year prior to exit through 25 years prior to exit, after which there were only two firms (i.e., only two firms that exited survived 25 years).

TABLE 2

Average Number of Laser Types of Exiting Firms in Each Year Prior to Exit

(*Number of firms in parentheses*)

YEARS BEFORE EXIT	ALL EXITING FIRMS	YEARS BEFORE EXIT	ALL EXITING FIRMS
1	1.45 (328)	16	2.06 (16)
2	1.48 (222)	17	1.87 (15)
3	1.69 (171)	18	2.00 (15)
4	1.74 (146)	19	2.08 (12)
5	1.74 (127)	20	2.18 (11)
6	1.65 (101)	21	2.33 (9)
7	1.66 (87)	22	2.33 (9)
8	1.76 (75)	23	3.00 (5)
9	1.70 (57)	24	3.00 (4)
10	1.64 (47)	25	3.50 (2)
11	1.58 (38)	26	1.50 (2)
12	1.80 (35)	27	1.00 (2)
13	1.82 (28)	28	1.00 (2)
14	1.78 (23)	29	1.00 (2)
15	1.79 (19)	30	1.00 (1)

The basis for this pattern is that the probability of exit decreases with the number of types of lasers produced, as reflected in Proposition 5. We can also evaluate this prediction directly. Table 3 presents the fraction of firms exiting in all years as a function of the number of laser types produced, where the number of observations indicates the number of firm-years over which each fraction was computed (e.g., across all years there

[14] Obviously this can only be done retrospectively for the 328 firms (out of 464) that actually exit during the years we have observed.

were 1,503 instances of firms producing one laser). The exit rate is highest for firms producing only one laser type and then declines sharply as the number of laser types produced increases. Apart from the slightly higher exit rate of firms producing four versus three laser types, the patterns completely conform with Proposition 5.[15]

TABLE 3

Probability of Exit by Number of Laser Types Produced

NUMBER OF LASER TYPES	EXIT RATE	NO. OF OBSERVATIONS
1	0.15	1503
2	0.13	589
3	0.08	285
4	0.09	92
5	0.03	31
6	0.00	21
7	0.00	18
8	0.00	5

The model associates all entry and exit, along with growth and decline, with the creation and destruction of submarkets. Klepper and Sleeper [2004] analyze entry in lasers by spinoffs, which are firms founded by employees of laser firms. Their illustrative examples support the idea that spinoff entrants initially serviced new submarkets, ones that were not surprisingly related to the submarkets their parents serviced. More generally, the model implies that entry and exit should be clustered at the submarket level. Entry results from the creation of a new submarket, just as exit can occur only as the result of the destruction of a submarket. Therefore, all entry at a given moment in time must occur by firms entering the same submarket, and all exit must occur by firms participating in the same submarket. However, the extent to which we see entry and exit

[15] This small departure may be due to three suspicious firms that were listed for only one year as producing four lasers. Only six other firms were listed in their first year as producing four lasers. The three firms listed for only one year may not have been manufacturers, with the error in listing them caught in the next year. If they are eliminated, the patterns would conform precisely with Proposition 5.

clustered within submarkets depends on the time-interval considered. The longer the interval, the less entry and exit will be concentrated in one submarket.[16]

To test this, the lasers produced by each firm in their first year and last year were computed for the 25-year period 1970-1994, which encompasses the period when all but the Excimer laser was consistently produced. Accordingly, the Excimer laser was excluded from the analysis. The years were grouped into five five-year periods, from 1970-1974 to 1990-1994, and for each period the number of firms producing each of the eight lasers other than Excimer in their entry and exit year was computed. Table 4 presents the figures for the entrants and Table 5 for exiting firms.[17]

Consider first entry. Table 4 indicates that four of the eight laser types, CO_2, HeNe, SState, and Semic, attracted considerably more entrants than the other four laser types. These were also the lasers that were the most frequently produced by all firms, not just entrants. This would be expected if these laser types had the most submarkets. More importantly, our model predicts that there should be more clustering of laser types in the five-year intervals than in the 25-year interval spanning the five five-year periods. Let p_{it} denote the probability that an entrant in period t produces laser i in period t, and let p_i denote the probability that an entrant over all periods produces laser i when it enters. If clustering occurs, then the p_{it} should differ from p_i for all i and t. We can test this hypothesis against the null hypothesis of $p_{it}=p_i$ for all i and t using the Chi-squared statistic for contingency tables. The Chi-squared statistic, which has 28 degrees of freedom, exceeds 88, which is significant at the .001 level. Thus, clustering of entry by laser types is strongly supported. The entries in Table 4 in bold are the ones that exceed their expected number by at least 1.65 standard deviations.[18] Using the normal approximation

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[16] Let $n_1(t_1)$, $n_2(t_2)$, ... , $n_k(t_k)$, $t < t_1 <$ . . . $< t_k < t+T$, denote the number of entrants into all k submarkets that arrived at times $t_i \in [t, t+T]$. Then, the fraction of firms entering submarket i, $m_i=n_i/(n_1+ n_{2+}$ . . . .$+n_k)$, is strictly decreasing in k. As k is increasing in T, the claim follows.

[17] When firms produced more than one laser type in their entry or exit year, each was listed in Tables 4 and 5, but for simplicity this was ignored in the implementation of the statistical tests.

[18] For laser i, the expected number in period t was computed as f_iN_t, where f_i equals the total number of times laser i was produced in a firm's first year divided by the total number of lasers

to the binomial, the probability of the actual number exceeding this threshold is .05 (the Chi-squared statistic takes into account interdependencies in these probabilities). The laser types that attracted disproportionate numbers of entrants in each period varied considerably over the five periods. In the first period, it was HeNe lasers, in the second Dye lasers, in the third CO_2 and GasOth lasers, in the fourth period SState lasers, and in the fifth period SState and Semic lasers.

TABLE 4

Lasers Produced by Entrants in First Year

PERIOD	CO_2	DYE	GASOTH	HECD	HENE	ION	SSTATE	SEMIC	ALL TYPES
1970-74	10	7	10	2	**23**	7	13	5	77
1975-79	3	**10**	9	3	15	6	6	14	66
1980-84	**28**	7	**14**	3	13	7	19	10	101
1985-89	20	6	5	1	14	8	**36**	21	111
1990-94	14	13	6	2	12	8	**44**	**37**	136
ALL YEARS	75	43	44	11	77	36	118	87	491

The exit patterns in Table 5 also exhibit statistically significant clustering. The Chi-squared statistic equals 45.2, which is significant at the .025 level. The bold entries, which are computed analogously to those in Table 4, indicate the lasers that had unusually high exit in each of the periods. Comparing the laser types with high entry and those with high exit in each period, there is considerable overlap. In the first, third, and fifth periods the laser type with exit exceeding the .05 cutoff also had entry exceeding the .05 cutoff. Among the other laser types in the entry and exit tables exceeding the .05 cutoff, their counterpart in the other table also tended to be above its expected value, though not enough to exceed the .05 threshold. This is suggestive of creative destruction, with the occurrence of new submarkets within a laser type destroying other submarkets

produced in their first year by entrants over the 25-year period, and N_t is the the total number of lasers produced by entrants in period t. The standard deviation was computed as $[f_i(1-f_i)N_t]^{1/2}$.

within the laser type. While this was not included in our model, it is not inconsistent with the essence of the model.

TABLE 5

Lasers Produced by Exiting Firms in Last Year

PERIOD	CO_2	DYE	GASOTH	HECD	HENE	ION	SSTATE	SEMIC	ALL TYPES
1970-74	13	6	8	0	**25**	9	16	10	87
1975-79	2	3	3	0	**12**	2	6	7	35
1980-84	13	7	**10**	1	9	7	12	13	72
1985-89	**25**	6	8	1	12	10	27	8	97
1990-94	18	14	5	1	14	6	**35**	17	110
ALL YEARS	71	36	34	3	72	34	96	55	401

There are further analyses that could be conducted to test even more detailed predictions of the model, but the ones we featured get at the essence of the model. The average firm increased its scope as it aged, while exiting firms decreased their scope as they aged. More diversified firms were less likely to exit. Both entry and exit were clustered over time in different laser types. These patterns are indicative of the importance of submarkets.

V. Conclusions

In empirical analysis we are accustomed to treating industries as homogeneous, although we are well aware that the way data are reported from industrial censuses typically lumps together a lot of diverse submarkets. Unfortunately, this diversity is not merely an easily-corrected artifact of reporting conventions. If submarkets arrive and depart in unpredictable fashion, there is little chance that we can devise practical procedures to purge broad industrial data of the diversity of firm activity. Instead, we need to devise models that acknowledge the diversity.

We developed a model in which firms' fortunes are determined by the success they have in exploiting the opportunities presented as new submarkets arise, and the time that elapses before those opportunities vanish. We showed that a remarkably sparse model having submarkets as the driving force can explain a large number of empirical regularities that for some time have been the subject of intense theorizing. Most notable is the ability of our model to generate predictions about the effects of age on growth and survival. The model predicts that (i) conditional on size, firm survival is increasing in firm age; (ii) conditional on size and survival, firm growth is decreasing in age; (iii) conditioning on size but not on survival, growth does not depend on age; (iv) conditional on size, the variance of firm growth is declining in firm age; and (v) the mean and variance of firm size rise with the age of a cohort, but skewness declines. These predictions match precisely with empirical observation.

Jovanovic's [1982] selection model and Cooley and Quadrini's [2001] model of financial market frictions with persistent firm shocks have previously generated some of these age effects. Our model and theirs are not mutually exclusive. There is ample evidence that firms have different abilities, and selection is an important determinant of which firms exit.[19] Financial market frictions also seem to be necessary to explain why age is related to various financial characteristics of the firms. However, Jovanovic's conclusion that the ability of his model to generate age effects shows that selection matters, and Cooley and Quadrini's conclusion that financial factors are important for the properties of the growth of firms, both seem to be premature given that we can generate age effects that appear more consistent with the empirical regularities using a simple model in which there is neither selection nor financial friction.

One possible interpretation of our analysis, therefore, is that once the dynamics of intra-industry diversity is accounted for, there may be little left in the empirical regularities to discriminate between competing models of industrial dynamics. This is probably going too far. Our analysis has shown that the concept of submarkets can generate *qualitative* behavior consistent with the empirical regularities, but no claim is made about the concept's quantitative power. Thus, any of the alternative mechanisms in the previous literature could be quantitatively important. What is needed, then, is a new direction of

[19] In our own work (Klepper [2002] and Thompson [2005]) we have provided direct evidence for selection.

empirical effort aimed at constructing data sets that may enable us to assess the relative importance of alternative sources of age effects.

Much of the early literature on firm-size distributions and growth was directed toward understanding how industry output got divided up among competing firms and why market structure differed across industries. If we want to learn more about the determinants of market structure, it may also be instructive to turn our empirical focus away from purely statistical patterns in the data. Indeed, a fruitful approach might be to explore empirical *irregularities*, both across industries and over time. For example, the laser industry experienced a steady growth in the number of producers over time. However, a number of prominent industries have experienced sharp shakeouts in the number of producers as they evolved despite robust growth in output. Our model cannot account for this. While this helps delineate the limits of our model, it may also be revealing about the determinants of growth in ways that statistical regularities cannot reveal.

Indeed, we suspect that the experience of industries that have undergone shakeouts may actually support the importance of submarket phenomena for firm entry, exit, and growth. The histories of autos, tires, and televisions, each of which experienced a pronounced shakeout in the number of producers, are instructive (cf. Klepper and Simons [1997]). At first, many different types of autos and tires were produced and firms tended to specialize in the product varieties they produced. Geographic markets may also have been segmented. But subsequently, major product innovations slowed in both products, limiting the creation of new submarkets and opportunities for entry. Producers soon expanded nationally and strong economies of scope across different product varieties led firms to diversify widely, limiting the protection afforded by different submarkets. Both developments may explain why in both industries entry eventually dried up and the number of firms subsequently declined for many years. Meaningful submarkets in television were more limited from the outset of the industry than in autos and tires and the only major innovation that occurred in televisions was color tv, which was successfully introduced over 15 years after the start of the industry. This may help explain why entry was concentrated in the first five years of the industry and the number of producers subsequently declined sharply.

Another industry irregularity that may have something to do with submarkets is abrupt turnover in the leaders of the industry, sometimes after long stability in the leading firms. For example, in the tire industry Goodyear, Goodrich, U.S. Rubber (Uniroyal), and Firestone dominated the U.S. market for over 60 years until the advent of the radial tire. Radial tires were initially purchased by small car owners in Europe, and European firms pioneered their development. Later radials took over much of the U.S. market, at which point all the U.S. firms lost a great deal of market share and all but Goodyear sold out to foreign producers (French [1991]). In the disk drive industry, the leaders turned over three times after the introduction of successively smaller disk drives that initially catered to new buyers of mini-computers, personal computers, and laptops respectively. Unexpectedly, these smaller drives were improved sufficiently to take over much of the market, causing the decline of the leaders of the industry, who were slow to produce the new, smaller drives (Christensen [1993]). In both instances, the development of a new submarket that turned out to have much wider appeal than originally anticipated precipitated a major change in the industry's leaders.

These examples suggest that submarket phenomena may prove to be an attractive way to explain industry irregularities, but we have a long way to go before this can be convincingly established. Our model embodies a much simpler account of submarkets than these examples require. Both the probability of entry into a new submarket and the size draw upon entry are independent of size and age. Furthermore, all entry into submarkets occurs immediately, and there is no exit from a submarket until it is destroyed. Some of these assumptions can be relaxed without undermining the distinctive implications of the model, but there is a limit to how far this can go.[20] So we don't expect our

[20] For example, we can allow the probability of entry into a new submarket to increase with the number of submarkets in which the firm participates, up to the point where it is proportional to the number of submarkets in which the firm participates, and still retain the distinctive predictions of the model. In the proportional case, the expected increment to size from new submarkets will be proportional to the number of submarkets in which a firm participates, but its expected percentage growth from new submarkets will be inversely related to its average size per submarket. Consequently, expected firm growth would still decline with firm size and also with firm age conditioning on survival.

model to be equally important in all industries. The laser industry is virtually a prototype for where we expect the model to be especially applicable—a steady creation of new submarkets, limited participation of firms in different submarkets, and no one submarket that dominates the industry.

For industries with similar characteristics, our model may usefully serve as an empirical framework for further research into market structure, in the spirit of Ericson and Pakes [1995]. We provided an example of how one particular technological structure, in which the rate of entry, θ, into new submarkets depends on R&D effort, would give rise in equilibrium to the stochastic processes we have modeled. This structure might be useful for asking questions about such issues as the consequences of R&D policies. For example, a subsidy to R&D raises θ, and our analysis shows that this in turn increases the mean and variance of firm size while decreasing skewness; it reduces firm failure rates; and it decreases the rate of mean reversion in firm size. This example is purely illustrative, because the structure of the model is intentionally very general. It can encompass a wide variety of technological structures and modes of competition that give rise to the within-submarket distribution of market shares, and it can allow for a number of ways in which new opportunities arrive and are exploited by firms. In future work we intend to build upon this framework to explore the consequences of different technological environments, market structures and policies. We hope others see value in doing so as well.

Appendix: Proofs

LEMMA 1: *Let $p_k(t)$, k=0,1, 2, 3 . . ., denote the probability that exactly k submarkets are active at time t. Then,*

$$p_k(t) = \begin{cases} H(t)\pi_k(\rho(t)), & k = 0, \\ \big(1 - H(t)\big)\pi_{k-1}(\rho(t)) + H(t)\pi_k(\rho(t)), & k = 1, 2, 3, \quad . \quad . \quad ., \end{cases}$$

where $\pi_k(\rho(t))$ is the probability of exactly k events from a Poisson distribution with mean $\rho(t) = \lambda \int_0^t \big[1 - H(z)\big]\,dz$.

PROOF. The proof is based on an exposition in Takács [1958, pp. 67-8]. Assume that $N(t)$ submarkets, *in addition to the first,* have been created at times τ_i, $0 < \tau_1 < \tau_2 < \ ... \ \tau_{N(t)} \leq t$. The probability that the i^{th} market is still active at t is given by $1 - H(t - \tau_i)$. For Poisson events, τ_i is uniformly distributed on $(0, t]$, so the unconditional probability that submarket i is active is

$$\Pr\{\text{submarket } i \text{ is active at } t\} = t^{-1}\int_0^t \left(1 - H(v)\right) dv\,.$$

It then follows that, conditional on there being $N(t)$ submarkets that arrived after t=0, the number of active submarkets apart from the first, $n^*(t)$, is binomial:

$$\Pr\left\{n^*(t) = k \mid N(t)\right\} = \binom{N(t)}{k}\left[\frac{1}{t}\int_0^t \left(1 - H(v)\right) dv\right]^k \left[\frac{1}{t}\int_0^t H(v)dv\right]^{N(t)-k}.$$

Now, as $N(t)$ is Poisson with parameter λt, the unconditional distribution can be written as

$$\pi_k(t) = \sum_{N=k}^{\infty}\binom{N}{k}\left(\frac{e^{-\lambda t}(\lambda t)^N}{N!}\right)\left[\frac{1}{t}\int_0^t \left(1 - H(v)\right) dv\right]^k \left[\frac{1}{t}\int_0^t H(v)dv\right]^{N-k}$$

$$= \frac{e^{-\lambda t}}{k!}\left[\int_0^t \left(1 - H(v)\right) dv\right]^k \sum_{N=k}^{\infty}\frac{\lambda^N}{(N-k)!}\left[\int_0^t H(v)dv\right]^{N-k}.$$

A change of variables, $z=N-k$, gives

$$\pi_k(t) = \frac{e^{-\lambda t}}{k!}\left[\lambda\int_0^t \left(1 - H(v)\right) dv\right]^k \sum_{z=0}^{\infty}\frac{1}{z!}\left[\lambda\int_0^t H(v)dv\right]^z$$

$$= \frac{1}{k!}\left[\lambda\int_0^t \left(1 - H(v)\right) dv\right]^k \exp\left\{-\lambda t + \lambda\int_0^t H(v)dv\right\}$$

$$= \frac{1}{k!}\left[\lambda\int_0^t \left(1 - H(v)\right) dv\right]^k \exp\left\{-\lambda\int_0^t \left(1 - H(v)\right) dv\right\},$$

where the second line makes use of the series expansion $e^x = \sum_{z=0}^{\infty} x^z / z!$. Inspection of the last line shows that the distribution of active submarkets, except for the first, is Poisson with parameter $\rho(t) = \lambda \int_0^t [1 - H(v)]\, dv$. Finally, the probability that the first submarket is still active is simply 1–$H(t)$. It then follows that the density of $n(t)$, the number of active markets including the first, is given by Lemma 1.

PROOF OF PROPOSITION 1. As $t \to \infty$ the first submarket vanishes with probability 1, so the asymptotic distribution of the number of all active submarkets is Poisson. It therefore suffices to show that $\int_0^\infty (1 - H(v))\, dv = \mu$:

$$\int_0^\infty (1 - H(v))dv = \lim_{t\to\infty} \left[t - \int_0^t H(v)dv \right]$$

$$= \lim_{t\to\infty} t[1 - H(t)] + \lim_{t\to\infty} \int_0^t vdH(v)$$

$$= \int_0^\infty vdH(v) ,$$

where the second line was obtained from an integration by parts, and the third upon noting that a finite mean requires that $H(t) \to 1$ at a rate $O(t)$.

LEMMA 2. *Let $v_k(t)$ denote the probability that a firm is active in exactly k submarkets at time t. Then,*

$$v_k(t) = \begin{cases} \big(\theta H(t) + (1-\theta)\big)\pi_k(\theta\rho(t)), & k = 0, \\ \theta\big(1 - H(t)\big)\pi_{k-1}(\theta\rho(t)) + \big(\theta H(t) + (1-\theta)\big)\pi_k(\theta\rho(t)), & k = 1, 2, 3, \quad . \quad . \quad ., \end{cases}$$

where $\pi_k(\theta\rho(t))$ is the probability of exactly k events from a Poisson distribution with mean $\theta\rho(t) = \theta\lambda \int_0^t [1 - H(z)]\, dz$.

PROOF: Excluding the industry's first submarket, the number of submarkets in which a firm is active is the sum of n Bernoulli trials with probability of success θ, where n is Poisson with mean $\rho(t)$. The distribution of the random sum is therefore

$$v_k(t) = \sum_{n=k}^{\infty} \binom{n}{k} \frac{e^{-\rho(t)}(\rho(t))^n \theta^k (1-\theta)^{n-k}}{n!} ,$$

which, following similar rearrangements used to prove Lemma 1, can be written as

$$v_k(t) = \frac{e^{-\theta\rho(t)}(\theta\rho(t))^k}{k!}.$$

Thus, the random sum has a Poisson distribution with mean $\theta\rho(t)$. Adding to this the probability, $\theta(1-H(t))$, that the firm is active at time t in the industry's first submarket arrives at the stated result.

PROOF OF PROPOSITION 2. As $t \to \infty$, the first submarket vanishes with probability one. Hence, Proposition 2 follows from directly Lemma 2 directly upon letting $t \to \infty$.

PROOF OF PROPOSITION 3. As $t \to \infty$, the first submarket vanishes with probability one. Recall that firm age is defined as the time that has elapsed since the firm last entered a state of zero submarket activity. Since exit is due to submarket destruction and submarket destruction is independent of subsequent arrivals, firm age is independent of the subsequent arrival of submarkets. The process of submarket arrivals is therefore an equilibrium Poisson process in the sense of Cox and Isham [1980:8]. The remainder of the proof consequently follows the steps for Lemma 2, replacing t by s and ignoring the first submarket.

PROOF OF PROPOSITION 4. The characteristic function, $\phi_y(r)$, for the unconditional distribution of size for a firm of age s is given by

$$\phi_y(r;s) = E\left[\phi_x(r)^n \mid s\right]$$

$$= \sum_{k=0}^{\infty} \frac{e^{-\theta\lambda\int_0^s[1-H(z)]dz}(\theta\lambda\int_0^s[1-H(z)]dz)^k \phi_x(r)^k}{k!}$$

$$= \exp\left\{\theta\lambda\int_0^s[1-H(z)]dz\left(\phi_x(r)-1\right)\right\},$$

where the second line makes use of the distribution given in Proposition 3, and the third line makes use of the series expansion $e^x = \sum_{z-0}^{\infty} x^z / z!$. The moments can be obtained from the characteristic function in the usual manner.

PROOF OF PROPOSITION 5. Let $G_n(\tau \mid z_1, z_2, \ldots, z_n)$ denote the distribution of the first passage time, τ, to a state of zero active markets for a firm with n submarkets of ages z_i. Now add one submarket of age z_{n+1}. By construction, the first passage distribution is given by

$$G_{n+1}(\tau \mid z_{0,} z_1, z_2, \ldots, z_n, z_{n+1}) = \frac{H(z_{n+1}+\tau) - H(z_{n+1})}{1 - H(z_{n+1})} G_n(\tau \mid z_1, z_2, \ldots, z_n)$$

$$< G_n(\tau \mid z_1, z_2, \ldots, z_n)\,,$$

which completes the proof.

PROOF OF PROPOSITION 6. Given Proposition 5, we need only establish that n is increasing in y and s. Size, age, and submarket participation are related by

$$y = n\overline{x}\ ,\ n \text{ and } \overline{x} \text{ independent,}$$

$$s = f(n) + \varepsilon\ ,\ E(\varepsilon) = 0\ ,\quad f^{/}(n) > 0\ ,\ n \text{ and } \varepsilon \text{ independent.}$$

Consider the regression equation,

$$n = \beta_0 + \beta_1 y + \beta_2 s + u\ .$$

The coefficients β_1 and β_2 are defined by

$$\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} = \begin{pmatrix} V(y) & Cov(y,s) \\ Cov(y,s) & V(s)\end{pmatrix}^{-1} \begin{pmatrix} Cov(n,y) \\ Cov(n,s)\end{pmatrix}, \tag{A.1}$$

where

$$Cov(n,y) = E\Big[(n - E(n))\cdot n\overline{x}\Big]$$

$$= E(x)\Big(E(n^2) - E(n)^2\Big) = E(x)V(n) > 0,$$

$$Cov(n,s) = E\Big[(n - E(n))\cdot(f(n)+\varepsilon)\Big]$$

$$= E(nf(n)) - E(n)E(f(n)) > 0,$$

$$Cov(y,s) = E\Big[(n\overline{x} - E(n)E(x))\cdot(f(n)+\varepsilon)\Big]$$

$$= E(x)Cov(n,s) > 0,$$

and

$$V(y) = E(y^2) - E(y)^2 = E(n^2\cdot\overline{x}^2) - (E(n)E(x))^2$$

$$= \Big[E(n^2) - E(n)^2\Big]E(x)^2 + E(n^2)\Big[E(\overline{x}^2) - E(x)^2\Big]$$

$$= V(n)E(x)^2 + E(n^2)V(\overline{x})$$

$$>0.$$

Substituting these expressions into (A.1) yields, after some rearrangement,

$$\beta_1 = \frac{E(x)\Big(V(s)\cdot V(n) - Cov(n,s)^2\Big)}{V(s)\cdot V(y) - Cov(y,s)^2} > 0\,,$$

$$\beta_2 = \frac{Cov(n,s)\,E(n^2)\,V(\overline{x})}{V(s)\cdot V(y) - Cov(y,s)^2} > 0\,.$$

To prove Proposition 7, we will need the following lemma:

LEMMA 3. *Consider a firm of age s in the stationary state and active in at least one submarket. The distribution of the ages of the submarkets in which the firm is active satisfies*

$$G(z;s) = \left[\int_0^s e^{-\int_0^v \tilde{h}(t)dt} dv\right]^{-1}\left[\int_0^z e^{-\int_0^v \tilde{h}(t)dt} dv\right],$$

where $\tilde{h}(z) = H^{/}(z)/(1 - H(z))$ *is the exit hazard for a submarket of age z.* $G(z;s)$ *has the following properties*: i) *It is strictly increasing and concave over its domain with* $G(0;s) = 0$ *and* $G(s;s) = 1$; ii) (*stochastic dominance*) *For any* $s^{/} > s^{//}$ *and* $z \in (0, s^{//}]$, $G(z;s^{/}) < G(z;s^{//})$.

PROOF. All submarkets entered by the firm have births uniformly distributed on the interval [0, s]. The probability that a submarket entered when the firm was age τ is still active when the firm is age s is $(1 - H(s-\tau))$. Thus, for any $\{\tau, \tau^{/}\}$, the probability that a market entered at age τ divided by the probability that a market entered at age $\tau^{/}$ is still active is $(1 - H(s-\tau))/(1 - H(s-\tau^{/}))$. Let $G(z;s)$ denote the distribution of ages of active markets for a firm of age s, and let $g(z;s)$ denote the corresponding density. The density $g(z;s)$ must satisfy the relationship

$$\frac{g(z+\Delta z;s)}{g(z;s)} = \frac{1 - H(z+\Delta z)}{1 - H(z)}\,,$$

which can be written as

$$g(z+\Delta z;s)-g(z;s)=-\frac{1}{1-H(z)}\left[H(z+\Delta z)-H(z)\right]g(z;s)\,.$$

Divide through by Δz and let $\Delta z\to 0$:

$$g^{/}(z;s)=-\frac{H^{/}(z)g(z;s)}{1-H(z)}=-\tilde{h}(z)g(z;s)\,.$$

The solution to this differential equation is

$$g(z;s)=c(s)e^{-\int_0^z\tilde{h}(v)dv}\,,$$

for some constant, $c(s)$, to be determined. Integrating over z yields

$$G(z;s)=c(s)\int_0^z e^{-\int_0^v\tilde{h}(t)dt}dv\,.$$

Noting that $G(s;s)=1$, the constant satisfies

$$c(s)^{-1}=\int_0^s e^{-\int_0^v\tilde{h}(t)dt}dv\,,$$

so $G(z;s)$ is as given in the lemma. The properties of $G(z;s)$ can be verified by direct calculation. Note that for the special case of the exponential distribution for $H(z)$, $H(z)=1-e^{-z/\mu}$, we obtain

$$G(z;s)=\frac{1-e^{-z/\mu}}{1-e^{-s/\mu}}\,,$$

which is simply $H(z)$ with domain truncated at s. This completes the proof of the lemma.

Proof of Proposition 7. Write output as $y(s)=\overline{x}_n(s)n(s)$, where $\overline{x}_n(s)$ is the average size of a firm of age s in each of its $n(s)$ submarkets. Over the subsequent interval of length T, the expected number of new submarkets gained that remain active is, from Lemma 1,

$$E\left[\text{number of new submarkets active after interval } T\mid s\right]=\theta\lambda\int_0^T[1-H(v)]dv\,.$$

The firm has an expected size in each of these submarkets of $E(x)$. Now, given a submarket of age z, the probability that it vanishes in the subsequent interval T is $[1-H(z)]^{-1}\int_z^{z+T}dH(v)$. Consequently, taking expectations over all possible ages using the distribution of the ages of currently active submarkets in Lemma 3, we have

$E\left[\text{number of lost submarkets after interval } T \mid s\right] =$

$$n(s)\left[\int_0^s e^{-\int_0^v \tilde{h}(t)dt} dv\right]^{-1} \int_0^s \left[e^{-\int_0^z \tilde{h}(v)dv} \left[1-H(z)\right]^{-1} \int_z^{z+T} dH(v)\right] dz \,.$$

Each lost submarket has an expected size of $\overline{x}_n(t)$. Hence, we can express the expected growth rate of the firm in terms of the difference between expected gains and losses:

$$g_y(t,t+T;y,s) = \frac{E(y(t+T \mid s)) - y(t,s)}{y(t,s)}$$

$$= \frac{E(x)\theta\lambda}{y(t,s)} \int_0^T [1-H(v)]dv$$

$$-\left[\int_0^s e^{-\int_0^v \tilde{h}(t)dt} dv\right]^{-1} \int_0^s \left[e^{-\int_0^z \tilde{h}(v)dv} \left[1-H(z)\right]^{-1} \int_z^{z+T} dH(v)\right] dz \,,$$

from which we have

$$\left.\frac{dg_y(t,t+T;y,s)}{dy(t,s)}\right|_{s \text{ constant}} = -\frac{E(x)\theta\lambda}{y(t,s)^2} \int_0^T [1-H(v)]dv < 0 \,,$$

while $dg_y(t,t+T;y,s) / ds(t) \,|_{y \text{ constant}}$ is a somewhat more complicated expression with ambiguous sign. For exponentially distributed submarket lives, using $H(z) = 1 - e^{-z/\mu}$ and evaluating the integrals yields

$$g_y(t,t+T;y,s) = \left(\frac{\theta\lambda\mu E(x)}{y} - 1\right)\left(1 - e^{-T/\mu}\right),$$

which is the basis for the corollary to Proposition 7.

Proof of Proposition 8. To analyze the growth of surviving firms, let $g_y(t,t+T;y,s \mid n(t+T) > 0)$ denote the mean growth rate of non-exiting firms, and let $\Pr\left\{n(t+T) = 0 \mid y(t), s(t)\right\}$ denote the probability of exiting. Noting that exiting firms have growth rates of –1, we can write the unconditional mean growth rate as

$$g_y(t,t+T;y,s) = \left(1 - \Pr\left\{n(t+T)=0 \mid y(t),s(t)\right\}\right) g_y(t,t+T;y,s \mid n(t+T)>0)$$

$$+ \Pr\left\{n(t+T)=0 \mid y(t),s(t)\right\}(-1),$$

so that,

$$g_y(t,t+T;y,s \mid n(t+T)>0) = \frac{g_y(t,t+T;y,s) + \Pr\left\{n(t+T)=0 \mid y(t),s(t)\right\}}{1 - \Pr\left\{n(t+T)=0 \mid y(t),s(t)\right\}}$$

In view of the ambiguous effect of age in the case of arbitrary $H(z)$, we consider the exponential case, so that

$$g_y(t,t+T;y,s \mid n(t+T)>0) = \frac{\left(\frac{\theta\lambda\mu E(x)}{y} - 1\right)\left(1 - e^{-T/\mu}\right) + \Pr\left\{n(t+T)=0 \mid y(t),s(t)\right\}}{1 - \Pr\left\{n(t+T)=0 \mid y(t),s(t)\right\}}.$$

Given that the probability of firm exit is decreasing in firm size and age when *H(z)* is exponential, it follows that mean firm growth and firm size and age for surviving firms must be related according to Proposition 8.

PROOF OF PROPOSITION 9. Submarket destruction and creation are independent. Consequently, the variance of growth for a firm of size $y(t)$ active in $n(t)$ submarkets can be written as the sum of the variance of changes in firm size caused by entry into new submarkets and changes caused by destruction of some or all of the existing $n(t)$ submarkets. The increment in size contributed by new arrivals over an interval of length T is equivalent in distribution to the size of a firm of age T. Thus, substituting T for s in the variance given in Proposition 3 and dividing by the square of current size gives the variance of the growth rate contributed by the gains over T, $\left[E(x^2)\theta\lambda\int_0^T \left[1 - H(z)\right]dz\right]/y^2$. This is decreasing in y but independent of n.

We show next that the contribution of existing submarkets is decreasing in y for fixed n and decreasing in n for fixed y. Consider a firm of size y active in n submarkets with sizes x_1, x_2, . . . x_n, such that $\sum_{i=1}^{n} x_i = y$. The destruction of each of these submarkets is independent and, assuming an exponential distribution, occurs in the interval of length T with probability $e^{-T/\mu}$. Hence, the contribution to the variance of output of each current submarket is, from the variance of a Bernoulli trial, $e^{-T/\mu}(1 - e^{-T/\mu})x_i^2$. The contribution to the variance of the growth rate of all existing submarkets is therefore

$$\frac{e^{-T/\mu}\left(1-e^{-T/\mu}\right)}{y^2} = e^{-T/\mu}\left(1-e^{-T/\mu}\right)\frac{\sum_{i=1}^n x_i^2}{\left(\sum_{i=1}^n x_i\right)^2}.$$

To establish the proposition, it is therefore necessary to show that the ratio $\sum_{i=1}^n x_i^2 \Big/ \left(\sum_{i=1}^n x_i\right)^2$ is (i) decreasing in y for given n, and (ii) decreasing in n for given y.

To establish part (i), differentiate the ratio with respect to an arbitrary x_i:

$$\frac{d}{dx_i}\left(\frac{\sum_{i=1}^n x_i^2}{\left(\sum_{i=1}^n x_i\right)^2}\right) = \frac{\left[x_i\sum_{i=1}^n x_i - \sum_{i=1}^n x_i^2\right]}{\left(\sum_{i=1}^n x_i\right)^3}, \tag{A.2}$$

As the denominator is positive, the sign of the derivative depends only on the sign of the numerator. Now, the numerator can be expressed as

$$x_i\sum_{i=1}^n x_i - \sum_{i=1}^n x_i^2 = \sum_{j\neq i, i=1}^n E\left(x_i - x_j\right)x_j .$$

Conditioning on $\sum_{i=1}^n x_i$ (which enables us to abstract from the denominator in [A.2]), and averaging, we obtain

$$E\left[x_i\sum_{i=1}^n x_i - \sum_{i=1}^n x_i^2\right] = \sum_{j\neq i, i=1}^n E\left(x_i - x_j\right)x_j$$

$$= \sum_{j\neq i, i=1}^n E\left(x_i x_j\right) - E\left(x_j^2\right).$$

Now x_i and x_j, $i \neq j$, are independent. Therefore,

$$E\left[x_i\sum_{i=1}^n x_i - \sum_{i=1}^n x_i^2\right] = (n-1)\left[E\left(x \mid \sum_{i=1}^n x_i\right)^2 - E\left(x^2 \mid \sum_{i=1}^n x_i\right)\right]$$

$$= -(n-1)\,\mathrm{Var}\left(x \mid \sum_{i=1}^n x_i\right) < 0 .$$

Hence, as the firm's size is increased infinitesimally with all the increase in one submarket, $\sum_{i=1}^{n} x_i^2 \Big/ \left(\sum_{i=1}^{n} x_i\right)^2$ declines. This analysis can be repeated for each existing submarket, which completes part (i) of the proof.

For part (ii), increase the number of existing submarkets by one, adjusting the size of submarkets from x_i to $x_i^{/}$ such that $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n+1} x_i^{/}$. We will now establish that on average

$$\frac{\sum_{i=1}^{n} x_i^2}{\left(\sum_{i=1}^{n} x_i\right)^2} > \frac{\sum_{i=1}^{n+1} \left(x_i^{/}\right)^2}{\left(\sum_{i=1}^{n+1} x_i^{/}\right)^2},$$

which will complete the proof. The denominators on both sides of the inequality are the same, so we can restrict attention again to the numerators:

$$\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n+1} \left(x_i^{/}\right)^2 = \sum_{i=1}^{n} \left(x_i^2 - \left(x_i^{/}\right)^2\right) - \left(x_{n+1}^{/}\right)^2 \tag{A.3}$$

$$= \sum_{i=1}^{n} \left(x_i^2 - \left(x_i^{/}\right)^2\right) - \left(\sum_{i=1}^{n} x_i - x_i^{/}\right)^2 .$$

Some tedious algebra yields

$$\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n+1} \left(x_i^{/}\right)^2 = 2\left(\sum_{i=1}^{n} x_i x_i^{/} - \sum_{i=1}^{n} \left(x_i^{/}\right)^2\right) + \sum_{i=1, i \neq j}^{n} \sum_{i=1}^{n} x_i \left(x_j^{/} - x_j\right) + \sum_{i=1, i \neq j}^{n} \sum_{i=1}^{n} \left(x_i - x_i^{/}\right) x_j^{/} .$$

Assume that the $(n+1)^{\text{th}}$ submarket is constructed by subtracting a random amount from each of the existing n submarkets such that for i=1,2, . . ,n, $E(x_i^{/}) = E(x_{n+1}^{/})$. Then, the expected fraction of each submarket lost to the $n+1^{\text{th}}$ is $1/(n+1)$, so each submarket has expected size $(n/(n+1))E\left(x_i \mid \sum_{i=1}^{n} x_i\right) = n\overline{x}/(n+1)$, where $\overline{x} = n^{-1}\sum_{i=1}^{n} x_i$. The new submarket has expected size $(n+1)^{-1}\sum_{i=1}^{n} x_i = n\overline{x}/(n+1)$. Moreover, the amount subtracted from each submarket is independent of the amount subtracted from the others. Thus,

$$E\left(\sum_{i=1}^{n} x_i x_i^{/} - \sum_{i=1}^{n} \left(x_i^{/}\right)^2\right) = \sum_{i=1}^{n} \frac{n}{n+1} E\left(x_i^2\right) - \sum_{i=1}^{n} \frac{n^2}{(n+1)^2} E\left(x_i^2\right)$$

$$= \frac{n^2}{(n+1)} E\left(x_i^2 \mid \sum_{i=1}^{n} x_i\right),$$

$$E\left(\sum_{i=1,i\neq j}^{n}\sum_{i=1}^{n}x_i\left(x_j^{/}-x_j\right)\right)=-\frac{n(n-1)}{n+1}E\left(x_i\mid\sum_{i=1}^{n}x_i\right)^2,$$

and

$$E\left(\sum_{i=1,i\neq j}^{n}\sum_{i=1}^{n}\left(x_i-x_i^{/}\right)x_j^{/}\right)=\frac{n^2(n-1)}{(n+1)^2}E\left(x_i\mid\sum_{i=1}^{n}x_i\right).$$

Substituting these expressions into (A.3) and rearranging yields

$$E\left(\sum_{i=1}^{n}x_i^2-\sum_{i=1}^{n+1}\left(x_i^{/}\right)^2\right)=\frac{2n^2}{(n+1)^2}E\left(x_i^2\mid\sum_{i=1}^{n}x_i\right)-\frac{n(n-1)}{(n+1)^2}E\left(x_i\mid\sum_{i=1}^{n}x_i\right)^2$$

$$>\frac{2n^2}{(n+1)^2}\left[E\left(x_i^2\mid\sum_{i=1}^{n}x_i\right)-E\left(x_i\mid\sum_{i=1}^{n}x_i\right)^2\right]$$

$$=\frac{2n^2}{(n+1)^2}\operatorname{Var}\left(x_i\mid\sum_{i=1}^{n}x_i\right)>0,$$

which completes the proof.

Proof of Proposition 10. i) From Lemma 2, the expected number of active firms is

$$E(N(t))=C\left[1-\left(\theta(1-e^{-t/\mu})+(1-\theta)\right)\exp\left\{-\theta\lambda\mu(1-e^{-t/\mu})\right\}\right]. \tag{A.4}$$

(A.4) is strictly increasing in t iff $\theta<(\lambda\mu-1)/\lambda\mu$. If $\theta>(\lambda\mu-1)/\lambda\mu$, the expected number of active firms initially falls, but then must eventually rise as t increases. ii) The expected number of firms entering the industry in an interval dt is equal to the expected number of inactive firms times the expected rate of entry to submarkets, $\theta\lambda dt$. Under the assumption that $\theta<(\lambda\mu-1)/\lambda\mu$, this is clearly decreasing with t. As the number of active firms increases with t, then it is clear that entry as a proportion of active firms also declines with industry age. For $\theta>(\lambda\mu-1)/\lambda\mu$, the expected entry rate initially falls, but then must eventually rise as t increases. ii) The expected number of firms exiting in an interval dt is proportional to the number of firms active in exactly one market. This expected number is given by

$$C\left[\theta\left((1-e^{-t/\mu})+(1-\theta)\right)\exp\left\{-\theta\lambda\mu(1-e^{-t/\mu})\right\}\theta\lambda\mu(1-e^{-t/\mu})\right]$$

$$+C\left[\theta e^{-t/\mu}\exp\left\{-\theta\lambda\mu(1-e^{-t/\mu})\right\}\right]. \tag{A.5}$$

The first term gives the probability that a firm is in exactly one market after the first times the probability that either the first submarket has vanished or the firm did not enter the first submarket. The second term is the probability of being in zero submarkets after the first times the probability of still being active in the first submarket. Taking the ratio (A.5)/(A.4), it is easy to produce numerical examples in which, even for $\theta < (\lambda\mu - 1)/\lambda\mu$, the expected rate of exit need not change monotonically with respect to time, and may approach its asymptotic limit, $\theta\lambda\mu e^{-\theta\lambda\mu}/(1-e^{-\theta\lambda\mu})$, from above or below.

PROOF OF PROPOSITION 12. Define $Y^{+}(T)$ as the expected number of newly created submarkets the firm enters between t and T that are still active at time T, and define $Y^{-}(n(t),T)$ as the expected number of submarkets existing at time t that are destroyed by time $t+T$. The variables $Y^{+}(T)$ and $Y^{-}(n(t),T)$ can be written as

$$Y^{+}(T) = \theta\lambda\int_0^T [1-H(v)]dv\,,$$

and

$$Y^{-}(n(t),T) = n(t)\int_0^\infty \left[\frac{1}{1-H(t)}\int_t^{t+T} dH(v)\right]dH(t)\,.$$

Thus, $E[n(t+T)-n(t)] > [<]0$ as $Y^{+}(T) - Y^{-}(n(t),T) > [<]0$. It is easy to show that $Y^{+}(T)$ and $Y^{-}(n(t),T)$ are continuous, monotonically increasing functions of T, with $Y^{+}(0)=Y^{-}(n(t),0)=0$, $\lim_{T\to\infty} Y^{+}(T) = \theta\lambda\mu = E(n)$ and $\lim_{T\to\infty} Y^{-}(n(t),T) = n(t)$. Figure A.1 plots $Y^{+}(T)$ along with $Y^{-}(n(t),T)$ for two values of $n(t)$. Clearly, for each $n(t)$ there exists a $T^{*}(n(t))$ such that if $n(t)>[<]E[n]$, then $E[n(t+T)-n(t)] < [>]0$ for all $T \geq T^{*}(n(t))$.

The extension to include random draws from $F(x)$ is straightforward. A firm of above [below] average size in each of its submarkets can expect future submarkets in which it participates to be smaller [larger] on average. Mean reversion in the number of submarkets consequently implies mean reversion in firm size.

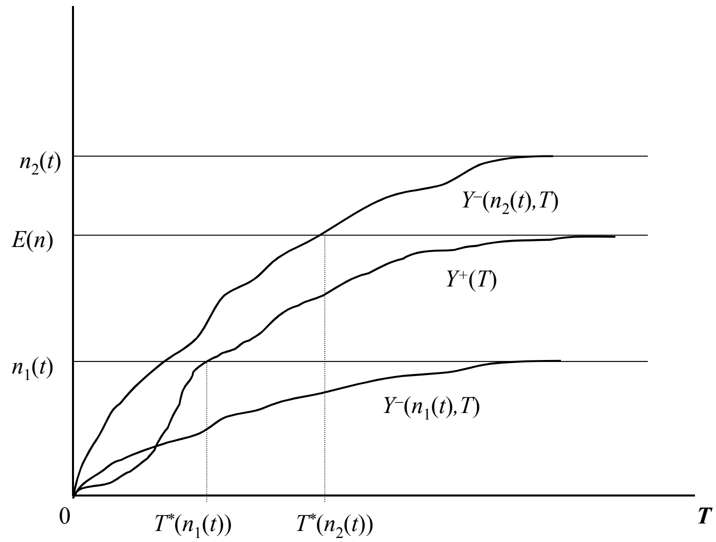

Figure A.1

PROOF OF PROPOSITION 13. Consider a firm active in n submarkets. Let x_i, i=1, 2, . . .,n denote the firm's size in each of the n submarkets. (i) Let $\overline{x}_n$ denote the average size of the firm in each submarket. By definition, $n = y / \overline{x}_n$ and so $E(n \mid y) = yE(1 / \overline{x}_n)$. As $x_i \geq 0 \quad \forall i$, then $E(1 / \overline{x}_n) > 0$ and $dE(n \mid y) / dy > 0$. (ii) Define z_i=$F(x_i)$. z_i has a uniform distribution on [0,1] so $\Pr\{z_i \leq c\} = c$, $\forall c \in [0,1]$. It then follows that the probability that the firm is among the top $100c$ percent in at least one submarket is $\Pr\left\{z_i > c \text{ for some } i\right\} = 1 - \Pr\left\{y_i \leq c \ \forall i\right\} = 1 - c^n$, which is strictly increasing in n. Part (iii) is a straightforward consequence of parts (i) and (ii).

PROOF OF PROPOSITION 14. The proof proceeds by obtaining an explicit expression for $\Pr\left\{n(t+s) = 0 \mid n(t) = k\right\}$ and applying Bayes' rule to obtain $\Pr\left\{n(t) = k \mid n(t+s) = 0\right\}$. An expression for $E(n(t) \mid n(t+s) = 0)$ is then derived and shown to be increasing in s at a decreasing rate.

Consider the stationary equilibrium, and assume that $H(x) = 1 - e^{-x/\mu}$, in which case the process for $n(t)$ reduces to an immigration-death model in which the immigration rate is $\theta\lambda$, and the death rate is $\mu^{-1}n(t)$. Now, without loss of generality, re-index time so that t=0. The immigration-death process must satisfy:

$$v_n(t + \Delta t \mid n_0) = \theta\lambda v_{n-1}(t \mid n_0)\Delta t + \left[1 - \left(\theta\lambda - \mu^{-1}n(t)\right)\Delta t\right] v_n(t \mid n_0)$$

$$+\mu^{-1}\left(n(t)+1\right)v_{n+1}(t \mid n_0)\Delta t + o(\Delta t)\,, \tag{A.6}$$

where $v_n(t+\Delta t \mid n_0)$ denotes the probability that a firm is active in n submarkets at time $t+\Delta t$ given that it was active in n_0 submarkets at time 0, and where $v_{-1}(t \mid n_0) \equiv 0$. In an immigration-death process, there are only three paths by which one can arrive at n at time $t+\Delta t$. First, one might increase n by one unit having been in state n–1 at time t; this happens with probability $\theta\lambda v_{n-1}(t|n_0)\Delta t+o(\Delta t)$. Second, one might have already been in state n, and there was no change in the state during the interval; this possibility occurs with probability $[1-\theta\lambda-\mu^{-1}n(t)]v_n(t|n_0)\Delta t+o(\Delta t)$. Third, one could arrive in state n by losing one submarket, which occurs with probability $\mu^{-1}(n(t)+1)v_{n+1}(t|n_0)\Delta t+o(\Delta t)$.

Subtracting $v_n(t|n_0)$ from both sides of (A.6), dividing through by Δt, and taking limits as $\Delta t \to 0$, we obtain

$$v_n^{/}(t \mid n_o) = \theta\lambda v_{n-1}(t \mid n_0) - \left(\theta\lambda + \mu^{-1}n(t)\right)v_n(t \mid n_0) + \mu^{-1}\left(n(t)+1\right)v_{n+1}(t \mid n_0)\,, \tag{A.7}$$

for n=0,1,2, ...

The first task is to obtain an explicit expression for $v_0(t|n_0)$, which requires that we solve (A.7). This is most conveniently done through the probability generating function, defined as

$$v^*(s,t \mid n_0) = \sum_{n=0}^{\infty} v_n(t \mid n_0)s^n\ . \tag{A.8}$$

Differentiating (A.8), and substituting (A.7) yields

$$\frac{\partial v^*(s,t \mid n_0)}{\partial t} = \sum_{n=0}^{\infty} v_n^{/}(t \mid n_0)s^n$$

$$= -\theta\lambda v_0(t \mid n_0) + \mu^{-1}v_1(t \mid n_0)$$

$$+\sum_{n=1}^{\infty} s^n\left(\theta\lambda v_{n-1}(t \mid n_0) - \left(\theta\lambda + \mu^{-1}n(t)\right)v_n(t \mid n_0) + \mu^{-1}\left(n(t)+1\right)v_{n+1}(t \mid n_0)\right)$$

$$= \theta\lambda\left[-v_0(t \mid n_0) + s\sum_{n=1}^{\infty} s^{n-1}v_{n-1}(t \mid n_0) - \sum_{n=1}^{\infty} s^n v_n(t \mid n_0)\right]$$

$$+\mu^{-1}\left[v_1(t \mid n_0) + \sum_{n=1}^{\infty}(n+1)s^n v_{n+1}(t \mid n_0) - s\sum_{n=1}^{\infty} ns^{n-1}v_n(t \mid n_0)\right]$$

$$= \theta\lambda\left[s\sum_{n=0}^{\infty} s^n v_n(t \mid n_0) - \sum_{n=0}^{\infty} s^n v_n(t \mid n_0)\right]$$

$$+\mu^{-1}\left[\sum_{n=0}^{\infty}(n+1)s^n v_{n+1}(t \mid n_0) - s\sum_{n=1}^{\infty} n s^{n-1} v_n(t \mid n_0)\right]$$

$$= (s-1)\theta\lambda v^*(s,t \mid n_0) + (1-s)\mu^{-1}\frac{\partial v^*(s,t \mid n_0)}{\partial s}.$$

There is an initial condition associated with this partial differential equation. Given the initial state $n(0)=n_0$, it follows that $v_n(0|n_0)=0$ for all $n \neq n_0$, while $v_n(0|n_0)=1$ for $n=n_0$. This simplifies to $v^*(s,0 \mid n_0) = s^{n_0}$. Saaty [1961:99-101] uses the method of Lagrange to solve this initial value problem, the details of which are omitted here:

$$v^*(s,t \mid n_0) = \exp\left\{\theta\lambda\mu(s-1)\left(1-e^{-t/\mu}\right)\right\}\left(1-(1-s)e^{-t/\mu}\right)^{n_0}, \tag{A.9}$$

from which the probabilities can in principle be derived using

$$v_n(t \mid n_0) = \frac{1}{n!}\frac{\partial^n v^*(s,t)}{\partial s^n}\bigg|_{s=0} \tag{A.10}$$

As n rises, the expressions implied by (A.10) rapidly become unwieldy. However, our interest is in $v_0(t|n_0)$, which is simply expressed:

$$v_0(t \mid n_0) = \exp\left\{-\theta\lambda\mu\left(1-e^{-t/\mu}\right)\right\}\left(1-e^{-t/\mu}\right)^{n_0}. \tag{A.11}$$

The final task is to obtain $\Pr\{n(t-T)=k \mid n(t)=0\}$ by applying Bayes' rule to (A.11):

$$\Pr\{n(t-T)=k \mid n(t)=0\} = \frac{\Pr\{n(t-T)=k\}\Pr\{n(t)=0 \mid n(t-T)=k\}}{\Pr\{n(t)=0\}}$$

$$= \frac{\dfrac{e^{-\theta\lambda\mu}(\theta\lambda\mu)^k}{k!}\exp\left\{-\theta\lambda\mu\left(1-e^{-T/\mu}\right)\right\}\left(1-e^{-T/\mu}\right)^k}{e^{-\theta\lambda\mu}}$$

$$= \frac{\exp\left\{\theta\lambda\mu\left(1-e^{-T/\mu}\right)\right\}\left(\theta\lambda\mu\left(1-e^{-T/\mu}\right)\right)^k}{k!},$$

which is Poisson with mean $\theta\lambda\mu\left(1-e^{-T/\mu}\right)$. It then follows that

$$\frac{dE\left[n(t-T) \mid n(t)=0\right]}{dT} = \theta\lambda e^{-T/\mu} > 0\,,$$

and

$$\frac{d^2E\left[n(t-T) \mid n(t)=0\right]}{dT^2} = -\frac{\theta\lambda e^{-T/\mu}}{\mu} < 0\,,$$

as claimed in the proposition.

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