Mediocrity*

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Abstract

We study the initial recruitment of individuals in the political sector. We propose an equilibrium model of political recruitment by two political parties competing in an election. We show that political parties may deliberately choose to recruit only mediocre politicians, in spite of the fact that they could afford to recruit better individuals. Furthermore, we show that this phenomenon is more likely to occur in proportional electoral systems than in majoritarian electoral systems.

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We’d all like to vote for the best man, but he is never a candidate. F. McKinney Hubbard

Our current political system ensures not that the worst will get on top – though they often do – but that the best will never even apply. Paul Jacob

1 Introduction

The quality of politicians has long been an issue of great concern in all democracies. A widespread sentiment summarized by the opening quotes above is that by and large the political class is typically not the best nor the worst a country has to offer. Why is it the case?

Several recent studies have documented that the quality of politicians varies significantly across countries, and that part of this variation is related to differences in the electoral system. For example, Persson, Tabellini, and Trebbi (2006) find that in a sample of 80 democracies, corruption of elected officials is higher in political systems with proportional representation than in majoritarian systems. Gagliarducci, Nannicini, and Naticchioni (2008) find that legislators elected under proportional representation exert less effort than their counterparts elected under plurality rule. Galasso, Landi, Mattoozzi, and Merlo (2009) document that the fraction of legislators without a high school degree is significantly larger in Italy, where proportional representation was used for more than forty years, than in the United States.\(^1\)

In this paper, we provide a novel explanation for these phenomena that hinges on the initial recruitment of individuals in the political sector: we study the effects of different electoral systems on political parties’ incentives to select good politicians and on the quality of elected representatives.

An important premise of our analysis is that in most countries, relatively few individuals start off their political careers by running for a public office. More frequently, they first test their political aspirations by holding positions within party organizations, which represent “breeding grounds” from which the vast majority of elected officials come from. The role of party service, as an essential qualification for pursuing a political career, is

\(^1\)This is not the case in the general population, where the fraction of high school dropouts in the two countries is comparable. See, e.g., Checchi, Ichino, and Rustichini (1999).
especially important in countries with a strong party system, such as, for example, Australia, Germany, Italy, Japan, the Netherlands, Sweden, and the U.K.\textsuperscript{2} In these countries, the individuals who are recruited by political parties determine the quality of the pool of potential electoral candidates.\textsuperscript{3}

Our analysis highlights a fundamental trade-off that each political party faces when deciding who to recruit. On the one hand, parties are long-lasting organizations that need “workers” to continue to operate in the political sector on an ongoing basis. Hence, a party may want to recruit a group of individuals who are willing to “work hard” for the party (e.g., raising funds on behalf of the party or devoting effort to membership drives). On the other hand, the very existence of political parties hinges on the support of the voters, and the parties’ relative fortunes critically depend on their electoral success. Hence, a party may want to recruit politicians who can compete and win in an election. A trade-off emerges since a selection of politicians that work hard for the party does not necessarily maximizes the probability of winning the election. As Besley (2005) suggests in his survey on political selection:

“Candidates are typically chosen by political parties. This fact raises the question of why a party would ever put a bad candidate up for election. One possibility is that if rents are earned by parties as well as successful candidates, and protection of those rents is dependent on selecting bad politicians with little public service motivation, then the party may have an interest in putting up bad candidates. The problem that parties face in making this choice arises from the risk that voters will choose the other party” (page 55).

The end of the quote foreshadows a crucial element that modulates the trade-off that we highlighted above: the competitiveness of the electoral system. Intuitively, the more com-

\textsuperscript{2}Norris and Lovenduski (1995) document that in the 1992 British general election, about 95% of Labour candidates and 90% of Conservative candidates had held a position within the party. Rydon (1986) and Cotta (1979) suggest similar levels of party involvement among members of parliament in Australia and in Italy, respectively. See also Best and Cotta (2000). In other countries, like for example, Canada, Finland, and the U.S., party service is not necessarily a pre-requisite for advancement in political careers. Even in these countries, however, the fraction of party professionals in the political sector has grown considerably over the years. See, e.g., Norris (1997).

\textsuperscript{3}“Competitive democratic elections offer citizens a choice of alternative parties, governments and policies. [...] Which candidates get on the ballot, and therefore who enters legislative office, depends on the prior recruitment process. [...] In most countries recruitment usually occurs within political parties, influenced by party organizations, rules and culture.” Norris (1997) (pp. 1-14).
petitive is the electoral system, the more important is the candidates’ ability in determining the electoral outcome. Our approach uncovers that it is the interaction between parties’ incentives and the competitiveness of elections that shapes parties’ recruitment decisions, and therefore affects the quality of elected politicians.

Before describing the details of our model of political recruitment, it is important to stress that political ability is a rather vague concept, which is very difficult to define, let alone quantify. While there is little doubt that competence, honesty, and integrity should all represent positive traits of a politician, there is no obvious way to define unambiguously what it takes to be a good politician. In this paper, we adopt a fairly general approach and define political ability as the marginal cost of exerting effort in the political sector. We believe that this definition captures several characteristics that jointly define political ability. Furthermore, we assume that political ability is observable by parties. Indeed, people who are potentially interested in becoming politicians typically begin their involvement in politics by engaging in a variety of voluntary, unpaid political activities that are organized and monitored by political parties (e.g., student political organizations, campaign teams, party internships). These activities thus provide opportunities for a political party to observe the political skills of individuals it may be potentially interested in recruiting.

In our model two political parties must recruit new politicians. There are two identical pools of potential recruits, one for each party. Potential recruits are heterogeneous with respect to their marginal cost of exerting effort in the political sector or political ability. A politician’s ability is observable and affects his performance both as a party member and as an electoral candidate. After each party has selected its members (the recruitment phase), the new recruits exert costly effort that benefits the party (the operational phase), and the politician who exerts the highest effort for each party is selected to be the party’s electoral candidate. In the electoral phase, the two candidates then compete by exerting

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4For example, a high ability politician is most probably successful in raising funds on behalf of the party. Also, a high ability politician will effectively contribute in shaping the party’s electoral platform. Furthermore, if nominated as an electoral candidate, a high ability politician will most likely be able to run a successful campaign and attract votes for his party. As Besley (2005) argues: “the idea that potential politicians differ in their competence is no different from a standard assumption in labor market models that individual have specific skills so that they will perform better or worse when matched in certain jobs” (page 48). This line of research has been pursued by Mattozzi and Merlo (2008) in their study of the careers of politicians.
costly effort in the form of campaign activities, which affect the electoral outcome. In a majoritarian (winner-takes-all) system, the candidate who exerts the highest level of effort wins the election. In a proportional system, the probability that each candidate wins the election is proportional to his campaign effort. Each party benefits from the total effort of its members during the operational phase, and also receives an additional benefit if its candidate wins the election. A party member obtains a positive payoff if he is selected by his party as the electoral candidate, and enjoys an additional benefit if he wins the election. We model both the operational phase and the electoral phase as all-pay contests.

The equilibrium of the model determines the ability of the politicians the parties recruit, the effort exerted by the parties' members in the operational phase, the ability and the campaign effort of the electoral candidates, and the ability of the elected politicians. Our main findings are that parties may optimally choose to recruit “mediocre” politicians - not the best nor the worst - in spite of the fact that they could afford to recruit better individuals. We refer to this phenomenon as mediocracy and show that it is more likely to occur in proportional electoral systems than in majoritarian systems.

To illustrate the intuition behind our results, consider first the case in which parties are only concerned about maximizing the total effort by their members in the operational phase (e.g., the case of a safe seat or an uncontested election). Since ability is an important determinant of an individual’s success as a party member, the presence of “superstars” may induce individuals of lesser ability to exert little effort (discouragement effect). Specifically, when the two highest levels of ability in the pool of potential recruits are sufficiently different, the chances that the second highest wins nomination are low if they are both recruited (recall that nomination is the prize of the all-pay contest in the operational phase). The “underdog” will then be discouraged from exerting a high level of effort, inducing the highest ability individual to exert relatively little effort. As a consequence, competition for the nomination is low, which in turn implies that expected total effort will be low. Furthermore, since nomination cannot be shared, only the two highest ability recruits will exert

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5 For a similar approach see, e.g., Snyder (1999) and, more recently, Persico and Sahuguet (2006) and references therein.

6 According to the Webster’s Third New International Dictionary of the English language, mediocracy is defined as: “rule by the mediocre.”
positive effort. By excluding the potential recruit with the highest ability and selecting a fairly homogeneous group of mediocre politicians the party can increase competition in the operational phase and hence the total effort of its recruits.\(^7\) However, in the presence of electoral competition, while a mediocre selection may increase the total expected effort of party members, it negatively affects the chances of winning the election (competition effect).

The reason why mediocrity is less likely to occur in majoritarian than in proportional elections is due to the relative competitiveness of the two alternative electoral systems. In particular, the winner-takes-all nature of majoritarian elections implies that the continuation value of winning the nomination is positive only for the highest ability politician.\(^8\) On the contrary, the lower competitiveness of proportional elections entails that the continuation value of winning the nomination is strictly positive even for mediocre politicians. Hence, the gains to the party from excluding high ability politicians are larger in proportional elections than in majoritarian elections. In other words, mediocrity is more likely to arise in proportional elections than in majoritarian elections because a less competitive electoral systems leads to a “myopic selection” due to the fact that the party privileges the importance of maximizing the total effort of its members in the operational phase (reduce the discouragement effect) at the expense of its electoral consequences (the competition effect).

Our model is very tractable and allows us to compare the performance of alternative electoral systems in several dimensions. For example a majoritarian system provides the best incentives for exerting campaign effort. Perhaps more interestingly, if we shut down the effects of electoral systems on the initial recruitment decisions - i.e., given the same initial selection of party members - the average quality of elected politicians is remarkably close across electoral systems. Finally, since our reduced form of modeling elections is based on the assumption that voters value campaign effort, voters are always better off with a majoritarian system. On the other hand, this is not always the case for political

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\(^7\) This result is based on the “exclusion principle” for all-pay auctions discovered by Baye, Kovenock, and de Vries (1993).

\(^8\) This follows for the fact that in an all-pay auction with complete information the only player with a strictly positive equilibrium payoff is the one with the highest valuation.
parties. Indeed, given the symmetry of the equilibrium, the ex-ante probability of winning the election for each party is always equal. Furthermore, proportional elections shift competitiveness from the electoral phase to the operational phase. As a result we show that, under some conditions, political parties are better off with a proportional electoral system.

The remainder of the paper is organized as follows. In Section 2, we present the formal model. In Section 3 we analyze a simplified version of the model where elections are uncontested and therefore we can abstract from electoral competition. In Section 4 we introduce electoral competition and present our main results. In Section 5 we review the literature, and we discuss some extensions of the model and conclude in Section 6. Most of the proofs are in the Appendix.

2 The Model

Consider two competing political parties indexed by \( h = \{L, R\} \), and two populations of individuals seeking public office. We ignore inter-party competition in the recruitment phase and assume that each party can select its members from identical pools of recruits. Abusing notation, we use the same index \( h \) for a party and its pool of recruits. Each population \( h \) is composed of \( N \) individuals. Each individual \( i \) of population \( h \) is endowed with a characteristic \( \theta_{ih} \geq 0 \) representing his political ability. We assume that political abilities are strictly ordered, that is \( \theta_{1L} = \theta_{1R} > \theta_{2L} = \theta_{2R} > \cdots > \theta_{NL} = \theta_{NR} \). The individual cost of exerting effort \( e \geq 0 \) in the political sector is equal to \( e/\theta_{ih} \), i.e., the higher is political ability the smaller is the marginal cost of exerting effort. We also assume that political ability is perfectly observable by parties, and that parties serve the role of gatekeepers: individuals can only run for public office if they are members of a party.\(^9\)

\(^9\)In general, inter-party competition for potential politicians seems of secondary importance, as ideological preferences are more likely to draw individuals toward specific parties. In fact, the lack of within-sector competition for sector-specific skills is a distinctive feature of the political sector. We discuss our assumption of two exogenously given political parties in Section 6.

\(^10\)The restrictions applied to candidacy vary a lot across countries with a strong party system, and they sometimes call for additional requirements other than party membership. For example, according to Obler (1974), a potential candidate in the Belgian Socialist Party must: “(1) have been a member at least five years prior to the primary; (2) have made annual minimum purchases from the Socialist co-op; (3) have been a regular subscriber to the party’s newspaper; (4) have sent his children to state rather than Catholic schools; and (5) have his wife and children enrolled in the appropriate women’s and youth organizations.”
The game has three stages. In Stage 0 (the recruitment phase), parties simultaneously select their members at a fixed hiring cost $\nu > 0$ per party member. Let $K_h$ be the set of party $h$ members, where $|K_h| \leq N$. An individual who is not selected by a party earns a payoff of zero.\(^{11}\)

In Stage 1 (the operational phase), party members exert effort $e_{1,i_h}$ (where the first subscript denotes the stage) at a cost equal to $e_{1,i_h}/\theta_{i_h}$. The party member who exerts the highest effort is nominated to be the party’s electoral candidate, which we denote by $i^*_h$ (accordingly, $e_{1,i^*_h}$ denotes the highest effort exerted in the operational phase), and he earns a payoff equal to $\beta \in (0, 1)$. Hence, $\beta$ is the value of being the party’s nominee.\(^{12}\) We define “non active” a party member who chooses not to exert effort in Stage 1 ($e_{1,i_h} = 0$). The operational phase is therefore modeled as an all-pay auction with complete information.

In Stage 2 (the election phase), candidates compete in an election. The electoral outcome is a function of the effort exerted by candidates in the electoral campaign, and the properties of this function depend on the electoral system. Specifically, in a majoritarian electoral system (FPP), $i^*_h$ is elected if and only if $e_{2,i^*_h} > e_{2,j^*_h}$, where $e_{2,i^*_h}(e_{2,j^*_h})$ is Stage 2 effort of party’s $h(-h)$ nominee, and ties are broken randomly. In a proportional electoral system (PR), $i^*_h$ is elected with probability $e_{2,i^*_h}/(e_{2,i^*_h} + e_{2,j^*_h})$.\(^{13}\) The elected politician earns a payoff normalized to 1. The individual cost of campaigning in the election phase is equal to $e_{2,i^*_h}/\theta_{i_h}$.\(^{14}\)

Since behavior is invariant to affine transformations, for convenience we consider an equivalent specification where the effort cost function is the identity function, i.e. $c(e) = e$, and the value of nomination and election equal $\beta \theta_{i_h}$ and $\theta_{i_h}$, respectively. According to this equivalent interpretation, a high ability politician is an individual that values the political job more or has a larger public service motivation.

\(^{11}\)In general the value of the outside option can be itself a function of political ability. See, e.g., Mattozzi and Merlo (2008). Here we abstract from this possibility.

\(^{12}\)In Section 6 we consider the case in which $\beta$ is endogenous, and we discuss our assumption of choosing the electoral candidate through a first-past-the-post mechanism.

\(^{13}\)This reduced form way of modeling elections is common in the literature. See, e.g., Snyder (1999), Grossman and Helpman (1996), Lizzeri and Persico (2001), Persico and Sahuguet (2006) and references therein.

\(^{14}\)Assuming that the cost of exerting effort is exactly the same across stages is not necessary for our results.
Formally, by letting $e_t = (e_t^h ; e_t^ {K - h})$ denote the effort profile in stage $t = \{1,2\}$, the payoff of individual $i$ in party $h$ in a majoritarian electoral system is equal to

$$
\Pi_{ih}^{PR}(e_1, e_2) = \begin{cases} 
0 & \text{if } i_h \not\in K_h \\
\frac{\theta_{ih} (1 + \beta) - e_{2,ih}}{|Z_h|} - e_{1,ih} & \text{if } e_{1,ih} \geq \max_{j_h \in K_h} \{e_{1,jh}\} \text{ and } e_{2,ih} > e_{2,j_h}^* \\
\frac{\beta - e_{2,ih}}{|Z_h|} - e_{1,ih} & \text{if } e_{1,ih} \geq \max_{j_h \in K_h} \{e_{1,jh}\} \text{ and } e_{2,ih} = e_{2,j_h}^* \\
- e_{1,ih} & \text{if } e_{1,ih} \geq \max_{j_h \in K_h} \{e_{1,jh}\} \text{ and } e_{2,ih} < e_{2,j_h}^* \\
\end{cases}
$$

where $Z_h \equiv \{ j_h \in K_h : e_{1,jh} = \max_{i_h \in K_h} \{e_{1,ih}\}\}$. Similarly, the payoff of individual $i$ in party $h$ in a proportional electoral system is equal to

$$
\Pi_{ih}^{FPP}(e_1, e_2) = \begin{cases} 
0 & \text{if } i_h \not\in K_h \\
\frac{1}{|Z_h|} \left( \frac{e_{2,ih}}{e_{2,ih} + e_{2,j_h}^*} + \beta \right) - e_{2,ih} & \text{if } e_{1,ih} \geq \max_{j_h \in K_h} \{e_{1,jh}\} \\
- e_{1,ih} & \text{otherwise,}
\end{cases}
$$

and if $e_{2,ih} = e_{2,j_h}^* = 0$ each candidate is elected with equal probability.

We assume that party $h$ selects its members in order to maximize the following objective

$$
V^s(e_{2,i_h^*}, e_{2,j_h^*}) + E (\Sigma_{i_h \in K_h} e_{1,ih}) - |K| \nu,
$$

where the last two terms represent expected total effort of party members in the operational phase net of hiring costs, and $V^s(\cdot, \cdot), s \in \{PR, FPP\}$, is the party’s payoff of winning the election. In particular

$$
V^{FPP}(e_{2,i_h^*}, e_{2,j_h^*}) = \begin{cases} 
\gamma & \text{if } e_{2,i_h^*} > e_{2,j_h^*} \\
\frac{\gamma}{2} & \text{if } e_{2,i_h^*} = e_{2,j_h^*} \\
0 & \text{otherwise,}
\end{cases}
$$

$$
V^{PR}(e_{2,i_h^*}, e_{2,j_h^*}) = \gamma \frac{e_{2,i_h^*}}{e_{2,i_h^*} + e_{2,j_h^*}},
$$

where $\gamma \geq 0$ and $V^{PR}(e_{2,i_h^*}, e_{2,j_h^*}) = \gamma/2$ if $e_{2,i_h^*} = e_{2,j_h^*} = 0$. Note that equation (1) captures in the simplest way that two key forces influence a party’s recruitment decisions:
First, an obvious component of political parties’ objective is their desire to win the elections, which is captured by the term $V^*$. This would be the only objective if parties were mere electoral machines. However, political parties are long-lasting organizations that operate in the political sector on an ongoing basis. In this respect, raising funds on behalf of the party or devoting effort to membership drives is crucial also when it’s not election time. Assuming that parties value the (expected) effort of their members in the operational phase captures this latter aspect.\footnote{It is worth mentioning that this is not the only possible interpretation of our assumption. Alternatively one might think of our operational phase as modeling a primary election among party’s members like in Jackson, Mathevet, and Mattes (2007). Interestingly, Hopkin (2001) notes how “the adoption of party primaries is a useful mobilizing strategy and has often been accompanied by membership recruitment drives” (page 348). This suggests another reason why parties may value total effort exerted in the operational phase.}

We characterize the subgame perfect equilibrium of the game where the profile of effort choices in the election phase is a Nash equilibrium of the all-pay contest between candidates, and the profile of effort choices in the operational phase and the selection strategy of the party are optimal given subsequent play. We focus on the case of $\nu$ arbitrarily small. We say that there is “mediocracy”, if parties choose not to select the “best” (with the highest political ability) nor the worst individuals. On the other hand, we say that there is “aristocracy” if parties choose to select the best individuals.\footnote{From the Greek word aristokratiä, literally the government of the best.}

3 Preliminaries: The Case of a Safe Seat

In order to distinguish the various forces at work behind our results, we begin by considering a simplified version of the model where electoral competition is absent: the case of a safe seat or an uncontested election. In this case the recruiting decisions of the two parties are completely independent. Hence, we can focus without loss of generality on a situation in which there is only one party that can recruit individuals and a single population of $N$ individuals seeking office.

Consider, as before, that political ability $\theta_i \geq 0$ is perfectly observable and such that $\theta_1 > \theta_2 > \cdots > \theta_N$. Since election are uncontested, the party’s nominee is elected with
probability one and earns a payoff normalized to $1$. An individual who is not selected to be a party member earns a payoff of zero. Considering the equivalent specification where the effort cost function is the identity function and the payoff from being elected equals $\theta_i$, and letting $e_K$ denote the effort profile, we have that the payoff of individual $i$ is equal to

$$
\Pi_i(e_K) = \begin{cases} 
0 & \text{if } i \not\in K \\
\frac{\theta_i}{|Z|} - e_i & \text{if } e_i \geq \max_{j \in K} \{e_j\} \\
-e_i & \text{otherwise},
\end{cases}
$$

where $Z \equiv \{j \in K : e_j = \max_{i \in K} \{e_i\}\}$ represents the set of party members winning the nomination (ties are resolved with equal probability). The party selects its members in order to maximize their expected total effort net of hiring costs, i.e., party’s payoff is equal to $E(\Sigma_{i \in K} e_i) - |K|\nu$, and we restrict attention to the relevant case of $\nu$ being arbitrarily small.

We assume the following condition throughout the rest of the paper:

**Condition A**

$$
\left(1 + \frac{\theta_2}{\theta_1}\right) \frac{\theta_2}{2} < \left(1 + \frac{\theta_3}{\theta_2}\right) \frac{\theta_3}{2}.
$$

This condition guarantees that there is “enough” heterogeneity between the highest ability politician and the second-highest. Under this assumption the next proposition follows immediately.

**Proposition 1**  If condition A holds then mediocracy is an equilibrium.

**Proof.** Note that the operational phase is equivalent to an all-pay auction with complete information and valuations equal to $\theta_i$. Since valuations are strictly ordered, we can use Theorem 1 and Lemma 1 of Baye, Kovenock, and de Vries (1993), which builds on a previous result by Hillman and Riley (1989), and conclude that expected total effort of party members in equilibrium equals

$$
E(\Sigma_{i \in K} e_i) = \left(1 + \frac{\theta_{\max K + 1}}{\theta_{\max K}}\right) \frac{\theta_{\max K + 1}}{2}, 
$$

\[17\] In the absence of electoral competition, distinguishing between the payoff of winning the nomination and the payoff of winning the election is inconsequential.
where $\theta_{\max_k}$ and $\theta_{\max_{k+1}}$ denote the abilities of the best politician in the party and of the second best politician in the party, respectively. Hence, under Condition A, the party has an incentive not to select the highest-ability individual (i.e., $\theta_1$). Furthermore, in the unique equilibrium, only the two highest-ability politicians selected by the party will be active, i.e., will be choosing positive effort. As a result, the party never selects the worst available individuals.

The intuition for this result is simple. Suppose that Condition A holds, i.e., the distribution of individual characteristics is such that there is only one outstanding potential politician (technically, the ratio of $\theta_2$ and $\theta_1$ is sufficiently smaller than the ratio of $\theta_3$ and $\theta_2$). In the unique equilibrium of the operational phase of the game, the two best politicians (the politicians with the two highest value of $\theta$) randomize over the same interval. However, while the highest ability politician randomizes uniformly over the interval, the second-highest politician’s equilibrium strategy has a mass point on zero effort. In other words, the two best politicians selected by the party will almost mimic each other, but the “underdog” politician will shirk with some positive probability. When the difference in ability between the best party member and the second-best is relatively large, chances that the latter wins the candidacy are relatively low. This implies that the second-best politician will shirk more often in equilibrium. We refer to this as the discouragement effect. As a consequence, since competition within the party will be relatively low, expected total effort will be low as well. By selecting mediocre politicians the party can increase intra-party competition (reduce the discouragement effect), which increases its payoff. This argument is based on the “exclusion principle” discovered by Baye, Kovenock, and de Vries (1993) in the context of all-pay auctions with complete information. In the next section we introduce electoral competition and study how the interaction between intra-party and inter-party competition modulates the discouragement effect and therefore affects the equilibrium selection of politicians.

4 Electoral Competition

A mediocre selection of politicians negatively affects the chances of winning a contested election since a high ability candidate will improve the electoral prospects of his political
party. Hence, when we consider inter-party electoral competition a *competition effect* that goes in the opposite direction with respect to the discouragement effect (due to intra-party competition) emerges. Alternative electoral systems, which influence the way campaign effort maps into the probability of winning the election, will have an effect on both electoral candidates’ optimal strategy and, in turn, on political selection.

We first characterize the subgame perfect equilibrium of the game and, to simplify the analysis, we assume the following condition throughout the rest of the paper:

**Condition B** For all \( k > 3 \)

\[
\left( 1 + \frac{\theta_3}{\theta_2} \right) \frac{\theta_3}{2} > \left( 1 + \frac{\theta_k}{\theta_{k-1}} \right) \frac{\theta_k}{2}.
\]

This condition guarantees that in the recruitment phase of the game the optimal selection for each party is either the two highest ability individuals (\( \theta_1 \) and \( \theta_2 \)) or the second and the third-highest ability individuals (\( \theta_2 \) and \( \theta_3 \)).\(^{18}\) Under Condition A and Condition B we obtain the next result.

**Theorem 1** Consider the electoral system \( s = \{FPP, PR\} \). There exists a threshold \( \bar{\gamma}^s \) such that mediocracy is an equilibrium if and only if \( \gamma < \bar{\gamma}^s \).

Theorem 1 completely characterizes the equilibrium of the model. The proof of this result, which can be found in the appendix, boils down to construct the subgame perfect equilibrium for each electoral system. In equilibrium both parties will either select the two highest ability individuals (aristocracy) or the second and the third-highest ability individuals (mediocracy). The reason why the existence of mediocracy depends on the value of \( \gamma \) is rather intuitive. When \( \gamma \) is small, parties care relatively more about the expected total effort of their members in the operational phase than about winning the election. Hence, the discouragement effect is more important than the competition effect. In this case a mediocre selection provides the best incentives for all party members to exert effort in the operational phase. On the other hand, as \( \gamma \) becomes large enough, the payoff from winning elections is so big that having mediocre but hard working members is not

\(^{18}\)This immediately follows from equation (2) in the proof of Proposition 1. Notice that, while it simplifies our analysis, Condition B it is not necessary for our results.
optimal from the party’s perspective, since a mediocre candidate will most probably run an unsuccessful campaign.

Given the result of Theorem 1, it is worth investigating what the effects of changing incentives in the operational phase are (i.e., varying the value $\beta$ of winning the nomination) on the likelihood that mediocracy arises in equilibrium. An increase in $\beta$ has two opposite effects on $\bar{\gamma}^s$: it decreases parties’ gains in the recruitment phase from excluding the highest ability individual (the discouragement effect is less severe), which leads to a decrease in $\bar{\gamma}^s$; but it also increases the probability of winning the election following a downward deviation in the recruitment phase (the competition effect is weaker), which leads to an increase in $\bar{\gamma}^s$. The former effect is due to intra-party competition and it is very intuitive: an increase in the value of winning the nomination increases intra-party competition and hence reduces the discouragement effect. The latter effect is more subtle and pertains to the interaction between intra-party and inter-party competition.

Suppose that party $L$ is selecting the two highest ability individuals as its members. The incentives for party $R$ to do the same rather than opt for a mediocre selection are given by the consequences of such a choice on its expected probability of winning the election. In particular, the electoral incentives are stronger the higher is the probability that party $L$’s nomination process will lead to the candidacy of the highest ability individual. Since the nomination is awarded through a winner-takes-all contest, and in equilibrium the two party members with the highest values of $\theta$ will randomize continuously on an interval, an increase in the value of winning the nomination leads the less able politician in party $L$ to act more aggressively. Hence, it is more likely that the less able politician becomes party $L$ electoral candidate. But this benefits party $R$ since its chances of winning election with a mediocre selection actually increase (i.e., the competition effect is watered down). For a distribution of types that most favors mediocrity in equilibrium, this latter effect is the dominant one. Indeed, when $\theta_3$ approaches $\theta_2$, $\bar{\gamma}^s$ is increasing in $\beta$, that is the higher is the value of winning the nomination, the higher is the likelihood that mediocrity is an equilibrium.\footnote{Note that it is easier to satisfy conditions A and B when $\theta_3$ approaches $\theta_2$. In the appendix, at the end of the proof of Theorem 1, we discuss the case in which $\theta_3$ is exactly equal to $\theta_2$.}

At this point, a natural question to ask is whether a positive value of winning the party
nomination, i.e. $\beta > 0$, is indeed a necessary condition for mediocracy. Perhaps interestingly, this is true only in the case of majoritarian elections. Indeed, when $\beta$ approaches zero $\bar{\gamma}_{FPP}$ vanishes. On the contrary, there exist type profiles such that $\bar{\gamma}_{PR}$ is always bounded away from zero for all $\beta$.\(^{20}\) Hence, we have the following corollary to Theorem 1:

**Corollary 1**

- **Necessary and sufficient conditions for mediocracy to be an equilibrium in majoritarian elections are that 1) politicians are sufficiently valuable for the party even if they do not win elections, and that 2) politicians are rewarded even if they do not win elections.**

- **A necessary condition for mediocracy to be an equilibrium in proportional elections is that politicians are sufficiently valuable for the party even if they do not win elections. Furthermore, there exist type profiles such that this condition is also sufficient.**

To get the intuition for this result let us focus on majoritarian elections and note that the winner-takes-all nature of this electoral system makes the equilibrium continuation value of being an electoral candidate very steep (in fact discontinuous) in $\theta$. Indeed, when $\beta$ vanishes, and nomination has almost no value per-se, the equilibrium continuation value of being party $h$ candidate is strictly positive if and only if $\theta_{i_h^*} > \theta_{j_{-h}^*}$ (i.e. party $h$ candidate has a strictly higher ability that his opponent in the general election), and it is equal to zero otherwise.\(^{21}\) Hence, there is no gain from working hard as a party member in the operational phase unless there is a positive chance of i) becoming the electoral candidate and ii) facing a “weak” (low $\theta$) challenger in the general election. As a result, if elections are majoritarian, the party cannot react to the discouragement effect if nomination has no value, and it gains nothing from selecting mediocre individuals irrespective of the value of $\gamma$. On the other hand, since in proportional elections the equilibrium continuation value of being an electoral candidate is always positive, increasing and smooth in $\theta$, a mediocre

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\(^{20}\)The proof of this result is part of the proof of Theorem 1 in the appendix.

\(^{21}\)Recall that in the unique equilibrium of a two-bidders all-pay auction with valuations $\theta_1 > \theta_2$, the expected equilibrium payoff of bidder 1 is equal to $\theta_1 - \theta_2$, while the second bidder completely dissipates his rents.
selection can be effective in counteracting the discouragement effect for all values of $\beta$. In the proof of Theorem 1 we show that this is indeed the case when $\theta_3$ is close to $\theta_2$.

As Corollary 1 suggests, the conditions for mediocrity to be an equilibrium are more demanding in the case of a majoritarian electoral system than in a proportional one. We next investigate whether electoral systems can be ranked in terms of their performance in selecting high ability politicians. This ranking is particularly relevant when political talent is relatively scarce as in the case of $\theta_3$ approaching $\theta_2$. In this case the second and third best political talents are similar and there is only one outstanding politician. It turns out that $\theta_3$ approaching $\theta_2$ is a sufficient condition to rank electoral systems independently of the level of $\beta$. We state this result in the next proposition.

**Proposition 2** When $\theta_3$ approaches $\theta_2$ mediocrity is more likely to arise in proportional elections than in majoritarian elections, i.e., $\bar{\gamma}_{FPP} < \bar{\gamma}_{PR}$.

The main force driving this result is that a majoritarian system is fundamentally more competitive than a proportional system, because of its winner-takes-all nature. This implies that a politician’s continuation value of winning the nomination is flatter in proportional elections, and the gains to the party from excluding high ability politicians are larger. To understand why this is the case, consider a downward deviation of say party $h$ in the recruitment phase. A deviation toward a mediocre selection has two consequences: First, it increases intra-party competition for nomination and therefore it reduces the discouragement effect. This represents the benefit from the deviation. Second, it reduces the probability of winning the general election, which is the cost of deviating. The latter is higher in majoritarian than in proportional elections since the probability of winning the general election with a mediocre selection is lower in a majoritarian electoral system than in a proportional one. On the other hand, comparing the benefit of deviating across electoral systems is less immediate.

The benefit of deviating depends itself on two intertwined components: i) the homogeneity of party $h$ members after the deviation, and ii) how big is the continuation value of being the electoral candidate for the “worst” party member, which is related to his likelihood of winning the general election. While the first component affects the level of
competition in the operational phase (the size of the discouragement effect), the second determines an upper bound on individual effort within the party. When $\theta_3$ is close to $\theta_2$ the first component is similar across electoral systems. On the contrary, the maximal effort exerted by politicians in the operational phase is higher in proportional elections. The reason for this is that the continuation value of being the electoral candidate (net of $\beta$) for every party member but the very best is always zero in majoritarian elections while it is strictly positive in proportional elections. Hence, the party has a stronger incentive to select mediocre politicians in proportional elections than in majoritarian elections which implies that $\bar{\gamma}_{FPP}$ must be smaller than $\bar{\gamma}_{PR}$.

Figure 1 represents the equilibrium selection of politicians in the space $(\beta, \gamma)$ for given value of $\theta_2/\theta_1$, and the arrows describe the effect of an increase in $\theta_2/\theta_1$ on the boundaries of the regions. If we interpret the two parameters $(\beta, \gamma)$ of our model as capturing the parties’ weight between phases/objectives ($\gamma$) and the politicians’ weight between phases/objectives ($\beta$), Figure 1 provides several intuitive insights. First, the likelihood of mediocracy being an equilibrium increases when party service is more important than electoral success (as one moves southwest in Figure 1). Second, for fixed $\gamma$ and $\beta$ a proportional electoral system, by weakening the link between political ability and electoral performance, “endogenously” shifts parties’ focus from inter-party competition to intra-party competition and it therefore...
makes a worst selection of politicians more likely. Finally, the less the best politician stands out with respect to the next best alternative ($\theta_2/\theta_1$ increases), the more likely is to have a mediocre selection of politicians in equilibrium.

While Proposition 2 focuses on the relative performance of alternative electoral system in selecting the highest ability individual, the next proposition compares the relative performance of alternative electoral systems in electing the highest ability politician (i.e., a type $\theta_1$).

**Proposition 3** Let $\gamma > \bar{\gamma}^{PR}$. There exists $h^*(\beta) \in [0, 1)$ such that the probability of electing the highest ability politician is higher in majoritarian elections than in proportional elections if $\theta_2/\theta_1 > h^*(\beta)$. Further, there exists $\beta^* \in (0, 1)$ such that $h^*(\beta) > 0$ if $\beta < \beta^*$.

When parties select the best available politicians in both electoral systems (i.e., $\gamma > \bar{\gamma}^{PR}$), Proposition 3 establishes that the highest ability politician is elected more often in a majoritarian system than in a proportional system if either the value of winning the party’s nomination is large or when the distribution of political talent is such that “there is no superstar”, i.e. $\theta_2/\theta_1$ is relatively large. Furthermore, it can be shown numerically that the sufficient conditions of Proposition 3 are also necessary. Hence, while we have established that parties select better politicians under a majoritarian system, the comparison between the two systems is less clear when we focus on their relative performance in electing the highest ability politician given the same initial selection of individuals. We will refer to this latter feature as the relative “electing performance” of alternative systems.

When $\beta$ is small, contrary to a proportional system, a majoritarian systems elects the highest ability politician when it is less needed, i.e. when the difference between the two best politicians is small and therefore the next best alternative is relatively close to the best available option.$^{22}$ This suggests that it may be useful to compare alternative electoral systems according to the average quality of elected politicians (maintaining fixed the initial selection of party members). Somewhat interestingly, while the two systems cannot be

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$^{22}$The reason why a proportional system performs better than a majoritarian system in electing the best politician when $\theta_2/\theta_1$ is relatively small is due to the fact that the unique equilibrium of the operational phase is in mixed strategies. In particular, when the underdog politician is much weaker than the best one, he has to exert zero effort with higher probability in a proportional system than in a majoritarian system to preserve indifference.
ranked, the average quality of the elected politician is remarkably close across systems as Figure 2 shows.

![Figure 2: Average quality of elected politician in aristocracy.](image)

Letting $Z_s$ be the probability of electing a type $\theta_1$ in electoral system $s$, Figure 2 plots the ratio $Z_{FPP}/Z_{PR}$ as a function of $\theta_2/\theta_1$ for two values of $\beta = \{\beta_1, \beta_2\}$. In particular, $\beta_1 = \arg \min \{\min_{\theta_2/\theta_1} (Z_{FPP}/Z_{PR})\}$ and $\beta_2 = \arg \max \{\max_{\theta_2/\theta_1} (Z_{FPP}/Z_{PR})\}$. In words, the ratio $Z_{FPP}/Z_{PR}$ cannot be smaller than the minimum of the lower curve depicted in Figure 2 (i.e., $Z_{FPP}/Z_{PR}$ evaluated at $\beta = \beta_1$), and it cannot be larger than the maximum of the higher curve (i.e., $Z_{FPP}/Z_{PR}$ evaluated at $\beta = \beta_2$). Given the range of values on the vertical axes, it is rather apparent that the two electoral systems are quite similar in their electing performance. While a proportional system is more likely to elect the best politician for relatively low values of the ratio $\theta_2/\theta_1$, the difference between the two systems when a proportional election is performing better is negligible. Indeed, for given selection of politicians, moving from a majoritarian to a proportional system may increase the probability of electing the best politician by less than half a percentage point at most. Further, when $\beta$ is sufficiently large $Z_{FPP}$ is larger than $Z_{PR}$ for all values of $\theta_2/\theta_1$. On the other hand, moving from a proportional to a majoritarian system may increase the probability of electing the best politician by less than seven percentage points at most.

\[\text{Figure 2: Average quality of elected politician in aristocracy.}\]

$Z_{FPP}/Z_{PR}$

Letting $Z_s$ be the probability of electing a type $\theta_1$ in electoral system $s$, Figure 2 plots the ratio $Z_{FPP}/Z_{PR}$ as a function of $\theta_2/\theta_1$ for two values of $\beta = \{\beta_1, \beta_2\}$. In particular, $\beta_1 = \arg \min \{\min_{\theta_2/\theta_1} (Z_{FPP}/Z_{PR})\}$ and $\beta_2 = \arg \max \{\max_{\theta_2/\theta_1} (Z_{FPP}/Z_{PR})\}$. In words, the ratio $Z_{FPP}/Z_{PR}$ cannot be smaller than the minimum of the lower curve depicted in Figure 2 (i.e., $Z_{FPP}/Z_{PR}$ evaluated at $\beta = \beta_1$), and it cannot be larger than the maximum of the higher curve (i.e., $Z_{FPP}/Z_{PR}$ evaluated at $\beta = \beta_2$). Given the range of values on the vertical axes, it is rather apparent that the two electoral systems are quite similar in their electing performance. While a proportional system is more likely to elect the best politician for relatively low values of the ratio $\theta_2/\theta_1$, the difference between the two systems when a proportional election is performing better is negligible. Indeed, for given selection of politicians, moving from a majoritarian to a proportional system may increase the probability of electing the best politician by less than half a percentage point at most. Further, when $\beta$ is sufficiently large $Z_{FPP}$ is larger than $Z_{PR}$ for all values of $\theta_2/\theta_1$. On the other hand, moving from a proportional to a majoritarian system may increase the probability of electing the best politician by less than seven percentage points at most.

\[\text{Figure 2: Average quality of elected politician in aristocracy.}\]

$Z_{FPP}/Z_{PR}$
Bringing together the results of propositions 2 and 3, our model highlights the importance of taking into account the effects of different electoral systems on the initial recruitment of politicians and, in this respect, tilts the comparison between electoral systems in favor of majoritarian elections. We conclude the analysis by assessing which system provides the best incentives to exert effort in the general election.

**Proposition 4** *The expected total campaign effort of electoral candidates is always greater in majoritarian elections than in proportional elections.*

It is immediate to check that the ranking of Proposition 4 extends to expected average campaign effort and the intuition for these results comes from the uniformly steeper incentives provided by majoritarian elections.

While studying the endogenous choice of electoral systems is behind the scope of this paper, it is nevertheless worth touching upon the welfare consequences of alternative elections in the context of our model. The parties’ welfare is unambiguously defined by equation (1). Furthermore, our reduced form approach of modeling elections is based on the assumption that voters’ welfare is monotonic in the electoral campaign effort. Since majoritarian elections provide the best incentives to exert campaign effort, voters are better off with a majoritarian electoral system. This is not always the case for parties. Indeed, each party has a fair (1/2) expected chance of winning the election in equilibrium, independently of the electoral system. Moreover, we can show that there exist an $h^{**}$ such that the expected equilibrium effort of party members in the operational phase is higher when elections are majoritarian if and only if $\theta_2/\theta_1 > h^{**}(\beta)$.$^{24}$ The reason for this result is that when $\theta_2/\theta_1$ is relatively large the equilibrium continuation value of winning the nomination as a function of $\theta$ is flatter in proportional than in majoritarian elections. This is due to the fact that in a proportional electoral system the election outcome is less sensible to campaign effort. Hence, party members are more homogeneous in terms of their equilibrium continuation value of winning nomination when a proportional system is adopted in the election phase, which implies a smaller discouragement effect. When instead $\theta_2/\theta_1$ is very small, this is not true anymore.$^{25}$ However, in this latter case, the electoral system has very little effect

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$^{24}$The proof of this result is very similar to the proof of Proposition 4 and it is therefore omitted.

$^{25}$See footnote 22.
on the equilibrium effort of party members as Figure 3 shows. Indeed, it can be shown numerically that the ratio between expected equilibrium effort of party members in majoritarian and in proportional elections is bounded above by 1.05. In conclusion, while voters are always better off with a majoritarian electoral system, political parties can be better off with a proportional electoral system.

Figure 3: Ratio between expected equilibrium effort of party members in FPP and in PR for $\beta = 0.01$(bottom curve), $\beta = 0.5$(middle curve), $\beta = 1$(top curve).

5 Literature Review

Our paper is related to the literature on the endogenous selection of politicians (see, e.g., the survey by Besley (2005)). Within this literature, Acemoglu, Egorov, and Sonin (2009) study the dynamic selection of governments under alternative political institutions (i.e., democratic vs non-democratic societies) and show that any deviation from perfect democracy may lead to an incompetent government in office being a stable and persistent outcome because of the dynamics of government formation. Caselli and Morelli (2004), Mattozzi and Merlo (2008) and Messner and Polborn (2004) focus on majoritarian systems, provide alternative explanations for the selection of bad politicians, and analyze the relationship between the salary of elected officials and their quality. Finally, Caillaud and Tirole (2002), Carrillo and Mariotti (2001), Castanheira, Crutzen, and Sahuguet (2008), Jackson, Mathevet, and
Mattes (2007) and Snyder and Ting (2002) study the internal organization of parties and the selection of electoral candidates within parties. None of these contributions, however, studies the issue of political recruitment, and the effect of alternative electoral systems on political parties’ recruitment.

Our work also relates to the theoretical literature on all-pay contest. In particular we build on results of Baye, Kovenock, and de Vries (1993), Baye, Kovenock, and de Vries (1996), and Hillman and Riley (1989) that study complete information environment. In the context of this literature an interesting recent paper by Kaplan and Aner (2008) study two stages political contests with private entry costs. They analyze a primary election where there is an entry stage and a campaigning stage and show that low ability contestants (those with a higher marginal cost of exerting effort) may enter more often than high ability contestants. Like in our paper, the campaigning stage of their model (which corresponds to our operational phase) is modeled as an all-pay auction. Contrary to our paper, the party does not choose contestants, i.e., there is no recruitment since individuals can choose whether or not to participate in the contest at a (private) cost and, more importantly, there is no electoral competition.

6 Concluding Remarks

In this paper, we propose a novel approach to study the effects of alternative electoral systems on the endogenous quality of politicians. Our model is very tractable and it can be extended in several directions. Here we briefly discuss the consequences of some simplifying assumptions.

First, in the model we assume that $\beta$, i.e. the value for being nominated as the party candidate, is exogenous. It might be interesting to ask whether and how our conclusions change if $\beta$ were endogenous. For example $\beta$ can be optimally chosen by parties, and we may think that different electoral systems will naturally lead to different optimal $\beta^*_s$. Clearly, if parties can increase $\beta$ at no cost, i.e. $c(\beta) = 0$, they will do so and $\beta^*_PR = \beta^*_FPP = 1$. In this case our results about the superiority of majoritarian elections both in terms of selection and election of good politicians are reinforced.\textsuperscript{26} If instead $c(\beta)$ is increasing and

\textsuperscript{26}This is apparent from Figure 1 and Proposition 3.
convex, it can be shown that there exists a threshold $t$ such that $\beta_{PR}^* > \beta_{FPP}^*$ if and only if $\theta_2/\theta_1 > t$. The most relevant case is when $\theta_2/\theta_1 < t$ and the optimal value for being nominated as the party candidate is higher in majoritarian elections. In this case, the relative performance of alternative electoral systems may also depend on the convexity of $c(\beta)$. Preliminary analysis suggests that this additional component, however, has a very limited impact on our results.

Our common-value environment departs from the standard downsian approach since we abstract from policy preferences. Introducing policy preferences in our model can be done in a relatively straightforward way and it does not affect our conclusions. For example, assume that the two political parties have observable policy positions $\{x_L, x_R\} \in [-y, y]^2$ that can be perfectly represented by the elected politicians and such that $x_L = -x_R$ to preserve the symmetry of the model. Further, assume that both a policy component and campaign effort enter in voters’ utility in an additive fashion, and that voters’ (ideological) preferences are distributed symmetrically in the interval $[-y, y]$. It is easy to see that in this model the key mechanism leading to mediocracy and our results on the comparison between alternative electoral systems will be preserved. This is also true if political parties choose their policy positions. The additional predictions of this extended model concern the policy outcome, which will be more or less polarized depending on whether the parties are assumed to be policy or office motivated and on the specific extensive form of the game.

In the model we assume that parties’ nominations are awarded through a winner-takes-all mechanism as opposed to say a contest with weights proportional to the effort exerted in the operational phase. Even though endogenizing the choice of the nomination process is behind the scope of this paper, it is easy to find conditions under which the auction mechanism generates a higher total expected effort in the operational phase than the contest mechanism.

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27 In the opposite case, when $\beta_{PR}^* > \beta_{FPP}^*$, our ranking of electoral systems in terms of both selection and election of high ability politicians is preserved.

28 For example, in the case of quadratic costs, when $\theta_2/\theta_1 < t$ and hence $\beta_{PR}^* < \beta_{FPP}^*$, the ratio $\beta_{FPP}^*/\beta_{PR}^*$ is approximately equal to 1 and therefore treating $\beta$ as exogenous has no consequences for our results.

Finally, to maintain the model simple, we focus on two exogenously given political parties. In the case of majoritarian electoral systems both theory and empirical evidence suggest that this assumption is largely plausible.\textsuperscript{30} This is not necessarily the case for proportional electoral systems. However, for any number of parties in proportional elections, the marginal impact of individual campaign effort on the probability of winning the election will always be bounded. On the contrary, the winner-takes-all nature of majoritarian elections entails that an increase in campaign effort just above the competitors level will lead to a discrete jump in the probability of winning. This suggests that the probability of electing the best contestant in the general election will be higher in majoritarian elections than in proportional elections for any number of candidates.\textsuperscript{31} While extending our model to the case of more than two parties represents an interesting avenue for future research, there is no obvious reason indicating that with more than two parties high ability politicians will be more valuable in proportional than in majoritarian elections.

\footnote{There is a large theoretical literature providing a formalization of the well-known Duverger’s law, namely that majoritarian elections lead to a two-party system. See, e.g., Iaryczower and Mattozzi (2008) and references therein.}

\footnote{For example, it can be shown that the probability that the best contestant wins a proportional election when he is facing two competitors is always bounded above by his probability of winning when he is facing only one competitor.}
Appendix

Proof of Theorem 1

Proof. We first analyze the subgame perfect equilibrium of the game with a FPP electoral system. We proceed by backward induction. First, note that election phase of the game is an all-pay auction between the two nominees with valuations $\theta^*_i$ and $\theta^*_j$, respectively. Without loss of generality, assume that $\theta^*_i \geq \theta^*_j$. Using well-known equilibrium properties of all-pay auctions, we have that the equilibrium is unique. Furthermore, we have two possible situations:

1. If $\theta^*_i = \theta^*_j$, the equilibrium is symmetric and both candidates randomize continuously on $[0, \theta^*_i]$. Their expected payoff is zero.

2. If $\theta^*_i > \theta^*_j$, candidate $i$ randomizes continuously on $[0, \theta^*_j]$, and earns an expected equilibrium payoff of $\left(\theta^*_i - \theta^*_j\right)$. Candidate $j$ randomizes continuously on $(0, \theta^*_j]$, he places an atom of size at $(\theta^*_i - \theta^*_j)/\theta^*_i$ at zero, and earns a payoff of zero.

We now move to the operational phase of the game and define by $\theta_{\text{max}_h}$ and $\text{max}_h$, the highest quality among politicians selected in party $h$ and the identity of the highest quality politician selected in party $h$, respectively. In order to save notation let $\theta_{\text{max}_h} \equiv \theta_{\text{max}}$ and $\text{max}_h \equiv \text{max}_h$. We consider two cases:

Case 1: $\theta_{\text{max}_L} = \theta_{\text{max}_R}$

Consider the following strategy profile: in each party $h$ the highest quality politician randomizes continuously on $[0, \beta \theta_{\text{max}_h} + 1]$. The second highest quality politician randomizes continuously on $(0, \beta \theta_{\text{max}_h} + 1]$ and places an atom of size $\alpha_h$ at zero. All other politicians are not active. Note that, if politicians in party $L$ follow this profile, the expected value of participating in the election for party $R$’s politicians is zero (net of the nomination prize) for all potential candidates with less than highest quality, and it is equal to

$$(\theta_{\text{max}_R} - \theta_{\text{max}_L}) \left(1 - \frac{\alpha_L}{2}\right)$$
for the highest quality politician ($\theta_{max_R}$). By defining

$$v_{1R} \equiv \beta \theta_{max_R} + (\theta_{max_R} - \theta_{max_L+1}) \left(\frac{1 - \alpha_L}{2}\right) > \beta \theta_{max_R+1}$$

and

$$v_{jR} \equiv \beta \theta_{max_R+j-1} \text{ for all } j = \{2, \ldots, |K_R|\},$$

it follows that the strategy profile described above is the unique best response for party $R$’s politicians since they are playing an all-pay auction with complete information and valuations $v_{jR}, j = \{1, \ldots, |K_R|\}$ defined above. Finally, we can pin down the unique value of $\alpha_h$ by using the fact that the highest quality candidate must be indifferent within his mixed-strategy support, and that his expected payoff must equal $v_{1h} - v_{2h}$. This implies that if a politician with quality $\theta_{max_h+1}$ exerts effort $e$ according to the distribution function $F_{max_h+1}$, it must be that $v_{1h} F_{max_h+1}(e) - e = v_{1h} - v_{2h}$ for all $e \in [0, \beta \theta_{max_h+1}]$. Hence, by solving

$$F_{max_h+1}(0) = 1 - \frac{v_{2h}}{v_{1h}(\alpha_h)} = \alpha_h,$$

and letting $z = \theta_{max_h+1}/\theta_{max_h}$, we obtain that

$$\alpha_h = 1 - \frac{\sqrt{\beta^2 + 2 \beta z (1 - z)} - \beta}{1 - z}, \quad (3)$$

which is decreasing in $\beta$ and $z$.

Case 2: $\theta_{max_L} > \theta_{max_R}$

For simplicity we focus on the case where $\theta_{max_L+1} = \theta_{max_R}$. Other cases can be analyzed in a similar way. Consider the following strategy profile: In party $R$ the highest quality politician randomizes continuously on $[0, \beta \theta_{max_R+1}]$. The second highest quality politician randomizes continuously on $(0, \beta \theta_{max_R+1}]$ and places an atom of size $\alpha'_{R}$ at zero. In party $L$ the highest quality politician randomizes continuously on $[0, x]$, where

$$x = \beta \theta_{max_L+1} + (\theta_{max_L+1} - \theta_{max_L+2}) \left(\frac{1 - \alpha'_{L}}{2}\right).$$

The second highest quality politician randomizes continuously on $(0, x]$ and places an atom of size $\alpha'_{L}$ at zero. All other politicians are not active. Note that, if politicians in party $L$ follow the candidate profile, the expected value of participating in the election for all party
$R$’s politicians is zero (net of the nomination prize), which implies that by redefining $v'_{j_R} \equiv \beta \theta_{\max_R+j}$ for all $j = \{1, \ldots, |\mathcal{K}_R|\}$, their strategy profile is optimal. On the other hand, if politicians in party $R$ follow the candidate profile, the expected value of participating in the election for party $L$’s politicians is zero (net of the nomination prize) for all potential candidates with less than second highest quality, and it is equal to

$$(\theta_{\max_L} - \theta_{\max_R}) \left( \frac{1 + \alpha'_{R}}{2} \right) + (\theta_{\max_L} - \theta_{\max_R+1}) \left( \frac{1 - \alpha'_{R}}{2} \right) =$$

$$(\theta_{\max_L} - \theta_{\max_R+1}) \left( \frac{1 + \alpha'_{R}}{2} \right) + (\theta_{\max_L} - \theta_{\max_R+2}) \left( \frac{1 - \alpha'_{R}}{2} \right)$$

for the highest quality politician, and equal to

$$(\theta_{\max_{L+1}} - \theta_{\max_R}) \left( \frac{1 - \alpha'_{R}}{2} \right) = (\theta_{\max_{L+1}} - \theta_{\max_{L+2}}) \left( \frac{1 - \alpha'_{R}}{2} \right)$$

for the second highest quality politician. By redefining

$$v'_{1_L} = \beta \theta_{\max_L} + (\theta_{\max_{L}} - \theta_{\max_{L+1}}) \left( \frac{1 + \alpha'_{R}}{2} \right) + (\theta_{\max_{L}} - \theta_{\max_{L+2}}) \left( \frac{1 - \alpha'_{R}}{2} \right),$$

$$v'_{2_L} = \beta \theta_{\max_{L+1}} + (\theta_{\max_{L+1}} - \theta_{\max_{L+2}}) \left( \frac{1 - \alpha'_{R}}{2} \right)$$

and

$$v'_{j_L} = \beta \theta_{\max_{L+j-1}} \text{ for all } j = \{3, \ldots, |\mathcal{K}_R|\},$$

and letting

$$\alpha'_L = 1 - \frac{v'_{2_L}}{v'_{1_L}} = 1 - \frac{\beta \theta_{\max_{L+1}} + (\theta_{\max_{L+1}} - \theta_{\max_{L+2}}) \left( \frac{1 - \alpha'_{R}}{2} \right)}{\beta \theta_{\max_{L}} + (\theta_{\max_{L}} - \theta_{\max_{L+1}}) \left( \frac{1 + \alpha'_{R}}{2} \right) + (\theta_{\max_{L}} - \theta_{\max_{L+2}}) \left( \frac{1 - \alpha'_{R}}{2} \right)},$$

and

$$\alpha'_R = 1 - \frac{\theta_{\max_{R+1}}}{\theta_{\max_{R}}},$$

it follows that the strategy profile described above is the unique best response for party $L$’s politicians.

In order to show that this is the unique equilibrium of the operational phase, suppose that party $R$’s members play any strategy $\sigma_j : \theta_j \to \Delta[0, b_j], j = \{\max_R, \cdots, |\mathcal{K}_R|\}$, where
\( \Delta[0, b_j] \) denotes a probability distribution on the interval \([0, b_j]\) and \(b_j < B < \infty\). The profile \(\sigma = (\sigma_{max_R}, \cdots, \sigma_{|K_R|})\) generates a probability of winning party \(R\)'s nomination \(q_j(\sigma) \in [0, 1]\) for \(j = \{1, \cdots, |K_R|\}\) such that \(\sum_j q_j(\sigma) = 1\) and, if \(max_R > 1\), \(q_j(\sigma) = 0\) for \(j = \{1, \cdots, max_R\}\). The expected value of winning the nomination in party \(L\) is therefore

\[
\hat{v}_j = \beta \theta_{max_L + j - 1} + \sum_{s = max_L + j - 1}^{K_R} \theta_s (\theta_{max_L + j - 1} - \theta_s),
\]

for \(j = \{1, \cdots, |K_L|\}\). Furthermore,

\[
\hat{v}_j - \hat{v}_{j+1} = \left( \beta + \sum_{s = max_L + j}^{K_R} q_s(\sigma) \right) (\theta_{max_L + j - 1} - \theta_{max_L + j}) > 0.
\]

Hence, for any strategy profile \(\sigma = (\sigma_{max_R}, \cdots, \sigma_{|K_R|})\) of party \(R\)'s members, the operational phase of the game for party \(L\)'s members is an all-pay auction with complete information and strictly ordered expected valuations \(\hat{v}_j\) defined above, which has a unique equilibrium.

We now move to the recruitment phase of the game and show that there exists a \(\hat{\gamma}^{FPP}\) such that a necessary and sufficient condition to have a mediocrity equilibrium is \(\gamma < \hat{\gamma}^{FPP}\). In order to show this, suppose that we want to support a symmetric selection profile where aristocracy arises in equilibrium, i.e., each party in the recruitment phase selects only \(\{\theta_{1h}, \theta_{2h}\}\), \(h = \{R, L\}\). Note that condition B guarantees that the selection that maximizes expected total effort in each party is either \(\{\theta_{2h}, \theta_{3h}\}\) or \(\{\theta_{1h}, \theta_{2h}\}\). Since the probability of winning the election decreases by selecting worst politicians, it follows that it is enough to check that a party does not want to deviate to a selection \(\{\theta_{2h}, \theta_{3h}\}\).

The expected payoff of each party \(h\) in an aristocracy equilibrium is

\[
\frac{\gamma}{2} + \left( 1 + \frac{v_{2h}}{v_{1h}} \right) \frac{v_{2h}}{2}
\]

where

\[
v_{1h} = \beta \theta_{1h} + (\theta_{1h} - \theta_{2h}) \left( \frac{1 - \alpha}{2} \right) \quad \text{and} \quad v_{2h} = \beta \theta_{2h},
\]

and, using (3) and suppressing the party index,

\[
\alpha = 1 - \frac{\sqrt{\beta^2 \theta_1^2 + 2\beta \theta_2 (\theta_1 - \theta_2)} - \beta \theta_1}{\theta_1 - \theta_2}.
\]
By deviating to \{\theta_{2h}, \theta_{3h}\} (without loss of generality let \(h\) be the deviating party), party \(h\)'s payoff is

\[ \gamma P_h + \left(1 + \frac{v_{2h}}{v_{1h}}\right) \frac{v_{3h}}{2}, \]

where \(v_{ih} = \beta \theta_{ih}\), and \(P_h < 1/2\) is the probability that party \(h\) wins the election. Hence, a necessary and sufficient condition for party \(h\) not to deviate is

\[ \gamma > \gamma^{FPP} \equiv \left(1 + \frac{v_{3h}}{v_{2h}}\right) v_{3h} - \left(1 + \frac{v_{2h}}{v_{1h}}\right) v_{2h}, \]

(4)

Furthermore, by defining

\[ \rho_1 = Pr(e_{1,2h} < e_{1,3h}) = Pr(e_{2,2-h} < e_{2,3h}) = \frac{1}{2} \frac{\theta_3}{\theta_2}, \]

\[ \rho_2 = Pr(e_{1,1-h} < e_{1,2-h}) = \frac{1}{2} \frac{2\theta_2}{\theta_1} + \frac{\frac{\theta_2}{\theta_1} - \frac{\theta_3}{\theta_1}}{2(1 - \frac{\theta_2}{\theta_1}) + \frac{\theta_3}{\theta_1}} \frac{\theta_3}{\theta_2}, \]

\[ \rho_3 = Pr(e_{2,1-h} < e_{2,2h}) = \frac{1}{2} \frac{\theta_2}{\theta_1} \quad \text{and} \quad Pr(e_{2,1-h} < e_{2,3h}) = \frac{1}{2} \frac{\theta_3}{\theta_1} = 2\rho_1\rho_3, \]

where Condition A implies that \(\rho_2 < \rho_3 < \rho_1\), we obtain that \(P_h\) equals

\[ P_h = (1 - \rho_1 (1 - 2\rho_1)) \left(\rho_2 \frac{1}{2} + (1 - \rho_2) \rho_3\right) \in (\rho_3, \rho_1), \]

(5)

which is increasing in \(\beta\) since \(\rho_2\) is increasing in \(\beta\) and \(\rho_3 < 1/2\). Further, it is immediate to see that \(P_h > \rho_3\), while condition A and tedious algebra delivers that \(P_h\) is increasing in \(\theta_3\) and that \(P_h < \rho_1\). In a similar fashion it can be shown that a necessary and sufficient condition to support a symmetric selection profile where each party in the recruitment phase selects only \(\{\theta_{2h}, \theta_{3h}\}\), \(h = \{R, L\}\) is \(\gamma < \gamma^{FPP}\).

Since \(P_h < 1/2\), the denominator of (4) is always positive. Further, since the numerator vanishes as \(\beta\) approaches zero, we have that \(\lim_{\beta \to 0} \gamma^{FPP} = 0\). When \(\gamma\) vanishes, mediocrity arises if and only if

\[ \left(1 + \frac{\theta_3}{\theta_2}\right) \theta_3 > \left(1 + \frac{v_2}{v_1}\right) \theta_2, \]

and condition A is a sufficient condition for the above inequality to hold since \(v_2/v_1 < \theta_2/\theta_1\).
We now analyze the subgame perfect equilibrium of the game with a PR electoral system. Consider first the election phase of the game in a PR electoral system. In this case, in the unique equilibrium, the nominees will choose
\[
\hat{e}_{2,i} = \frac{\theta_{i_h}^2 \theta_{j_{-h}}}{(\theta_{i_h} + \theta_{j_{-h}})^2} \quad \text{and} \quad \hat{e}_{2,j_{-h}} = \frac{\theta_{j_{-h}}^2 \theta_{i_h}}{(\theta_{i_h} + \theta_{j_{-h}})^2}.
\]
Furthermore, \(i_h^*\) and \(j_{-h}^*\) will earn payoffs \(\theta_{i_h}^3 / \left(\theta_{i_h} + \theta_{j_{-h}}\right)^2\) and \(\theta_{j_{-h}}^3 / \left(\theta_{i_h} + \theta_{j_{-h}}\right)^2\), respectively.

We now move to the operational phase of the game. Consider the following strategy profile: in each party the highest quality politician randomizes continuously on \([0, w_{\text{max}h+1}]\). The second highest quality politician randomizes continuously on \((0, w_{\text{max}h+1}]\) and places an atom of size \(\delta_h\) at zero. All other politicians are not active. Note that, if politicians in party \(-h\) follow this profile, the expected value of participating in the election for a party \(h\) politician with quality \(\theta_{i_h}\) is
\[
\frac{1 + \delta_{-h}}{2} \frac{\theta_{i_h}^3}{(\theta_{i_h} + \theta_{\text{max}_{-h}})^2} + \frac{1 - \delta_{-h}}{2} \frac{\theta_{i_h}^3}{(\theta_{i_h} + \theta_{\text{max}_{-h}+1})^2}.
\]
By defining
\[
w_{i_h} = \beta \theta_{i_h} + \frac{1 + \delta_{-h}}{2} \frac{\theta_{i_h}^3}{(\theta_{i_h} + \theta_{\text{max}_{-h}})^2} + \frac{1 - \delta_{-h}}{2} \frac{\theta_{i_h}^3}{(\theta_{i_h} + \theta_{\text{max}_{-h}+1})^2},
\]
and noticing that \(w_{i_h}\) is strictly increasing in \(\theta_{i_h}\), it follows that the strategy profile described above is the unique best response for party \(h\) politicians. We can pin down the equilibrium value of \(\delta_h\) solving the system
\[
\delta_h = 1 - \frac{w_{\text{max}h+1}(\delta_{-h})}{w_{\text{max}h}(\delta_{-h})} \text{ for } h \in \{L, R\}.
\]
Since each equation of the system in (6) is a continuous function of \(\delta_{-h}\) that maps the unit interval into itself, a solution always exists. If \(\text{max}_L = \text{max}_R\), (6) has trivially a unique solution where \(\delta_h = \delta_{-h} = \delta^*\), and it is easy to show that \(\delta^*\) is decreasing in \(\beta\) and decreasing in \(\theta_{\text{max}_{h+1}}/\theta_{\text{max}_h}\). If instead \(\text{max}_L \neq \text{max}_R\), it must be the case that \(\delta_h \neq \delta_{-h}\), and tedious but straightforward algebra shows that the solution is still unique.
In order to show that this is the unique equilibrium of the operational phase, we apply the same argument as before and suppose that party R’s members play any strategy \( \sigma_j : \theta_j \rightarrow \Delta[0, b_j] \), \( j = \{max_R, \ldots, |K_R|\} \), where \( \Delta[0, b_j] \) denotes a probability distribution on the interval \([0, b_j]\) and \( b_j < B < \infty \). The profile \( \sigma = (\sigma_{max_R}, \ldots, \sigma_{|K_R|}) \) generates a probability of winning party R’s nomination \( q_j(\sigma) \in [0, 1] \) for \( j = \{1, \ldots, |K_R|\} \) such that \( \sum_j q_j(\sigma) = 1 \) and, if \( max_R > 1 \), \( q_j(\sigma) = 0 \) for \( j = \{\max_R\} \). The expected value of winning the nomination in party L is therefore

\[
\hat{w}_j = \beta \theta_{max_L+j-1} + \sum_{s=1}^{\{K_R\}} q_s(\sigma) \frac{\theta_{max_L+j-1}^3}{(\theta_{max_L+j-1} + \theta_s)^2},
\]

for \( j = \{1, \ldots, |K_L|\} \). Furthermore,

\[
\hat{w}_j - \hat{w}_{j+1} = \beta (\theta_{max_L+j-1} - \theta_{max_L+j}) + \sum_{s=1}^{\{K_R\}} q_s(\sigma) \left( \frac{\theta_{max_L+j-1}^3}{(\theta_{max_L+j-1} + \theta_s)^2} - \frac{\theta_{max_L+j}^3}{(\theta_{max_L+j} + \theta_s)^2} \right) > 0.
\]

Hence, for any strategy profile \( \sigma = (\sigma_{max_R}, \ldots, \sigma_{|K_R|}) \) of party R’s members, the operational phase of the game for party L’s members is an all-pay auction with complete information and strictly ordered expected valuations \( \hat{w}_j \) defined above, which has a unique equilibrium.

We now move to the recruitment phase of the game and show that there exists a \( \bar{\gamma}^{PR} \) such that a necessary and sufficient condition to have a mediocracy equilibrium is \( \gamma < \bar{\gamma}^{PR} \). In order to support a symmetric selection profile where aristocracy arises in equilibrium, i.e., \( \{\theta_1, \theta_2\}, h = \{R, L\} \), it is enough to check that a party does not want to deviate to a selection \( \{\theta_2, \theta_3\} \) (condition B).

The expected payoff of party h in an aristocracy equilibrium is

\[
\frac{\gamma}{2} + \left( 1 + \frac{w_{2h}(\delta^*)}{w_{1h}(\delta^*)} \right) \frac{w_{2h}(\delta^*)}{2},
\]

where

\[
w_{ih}(\delta) = \beta \theta_i + \frac{1 + \delta}{2} \frac{\theta_i^3}{(\theta_i + \theta_1)^2} + \frac{1 - \delta}{2} \frac{\theta_i^3}{(\theta_i + \theta_2)^2},
\]

and \( \delta^* \) is the unique solution to (6) when \( max_h = max_{-h} = 1 \). By deviating to \( \{\theta_2, \theta_3\} \) party h’s payoff is

\[
\gamma \hat{P}_h + \left( 1 + \frac{w_{3h}(\delta_{-h}^*)}{w_{2h}(\delta_{-h}^*)} \right) \frac{w_{3h}(\delta_{-h}^*)}{2},
\]

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where $\hat{P}_h < 1/2$, and $(\delta^*_h, \delta^*_{-h})$ solve (6) when $\max_{-h} = 1$ and $\max_h = 2$. Hence, a necessary and sufficient condition for party $h$ not to deviate is

$$\gamma > \gamma^{PR}_h \equiv \frac{1 + \frac{w_{3h}(\delta^*_{-h})}{w_{2h}(\delta^*_{-h})} - \frac{w_{3h}(\delta^*_{-h})}{w_{2h}(\delta^*)}}{1 - 2\hat{P}_h}. \quad (7)$$

By letting

$$\frac{\hat{P}_1}{P} = Pr(e_{1,2h} < e_{1,3h}) = \frac{1}{2} \frac{w_{3h}(\delta^*_{-h})}{w_{2h}(\delta^*_h)} = \frac{1}{2} (1 - \delta^*_h) < \rho_1,$$

and

$$\frac{\hat{P}_3}{P} = Pr(e_{1,1-h} < e_{1,2-h}) = \frac{1}{2} \frac{w_{2-h}(\delta^*_h)}{w_{1-h}(\delta^*_h)} = \frac{1}{2} (1 - \delta^*_h) < \rho_3,$$

it follows that

$$\hat{P}_h = \hat{P}_1 \left( \hat{P}_3 \frac{\theta_3}{\theta_2 + \theta_3} + (1 - \hat{P}_3) \frac{\theta_3}{\theta_1 + \theta_3} \right) + (1 - \hat{P}_1) \left( \frac{1}{2} \frac{\theta_2}{\theta_1 + \theta_2} \right) < \frac{1}{2}. \quad (8)$$

In a similar fashion it can be shown that a necessary and sufficient condition to support a symmetric selection profile where each party in the first stage selects only $\{\theta_{2h}, \theta_{3h}\}$, $h = \{R, L\}$ is $\gamma < \gamma^{PR}_h$. Since $\hat{P}_h < 1/2$, the denominator of (7) is always positive. Further, when $\theta_1 > \theta_2$ and $\theta_3$ approaches $\theta_2$, $w_{3h}(\delta^*_{-h})$ approaches $w_{2h}(\hat{\delta}^*_{-h})$, where $\hat{\delta}^*_{-h} \equiv \lim_{\theta_3 \to \theta_2} \delta^*_{-h}$, and the numerator of (7) simplifies to

$$2w_{2h}(\hat{\delta}^*_{-h}) - \left( 1 + \frac{w_{2h}(\delta^*)}{w_{1h}(\delta^*)} \right) w_{2h}(\delta^*) = 2w_{2h}(\hat{\delta}^*_{-h}) - (2 - \delta^*) w_{2h}(\delta^*).$$

The last expression is strictly positive since tedious algebra shows that it is increasing in $\beta$, $w_{3h}(\delta^*_{-h}) < w_{2h}(\delta^*)$ if and only if $\hat{\delta}^*_{-h} > \hat{\delta}^*$, and there exists a $\bar{\beta} > 0$ such that $\hat{\delta}^*_{-h} > \hat{\delta}^*$ if and only if $\beta < \bar{\beta}$. Note that contrary to the case of $\theta_2 > \theta_3$, when $\theta_1 > \theta_2$ and $\theta_2$ is exactly equal to $\theta_3$, the equilibrium of the operational phase of the game is not unique anymore (Baye, Kovenock, and de Vries (1993)). Here, we focus on the limit of the unique equilibrium described above, i.e., when $\theta_3 - \theta_2 < \epsilon$ for $\epsilon$ positive and small.

It is worth mentioning that even in the case of $\theta_3 = \theta_2$ the equilibrium that we described above exists and it is the one that maximizes expected effort in the operational phase, see Baye, Kovenock, and de Vries (1993). In conclusion, mediocrity arises in PR if and only if $\gamma < \gamma^{PR}_h$, and, when $\theta_1 > \theta_2$ and $\theta_3$ approaches $\theta_2$, $\gamma^{PR}_h$ is strictly positive for all values of $\beta$. 

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Proof of Proposition 2

Proof. Using equations (4) and (7), let \( Q(\beta, \theta_2/\theta_1) \) denote the ratio \( \bar{\gamma}_{PR}/\bar{\gamma}_{FPP} \) when \( \theta_3 \) approaches \( \theta_2 \). Then, tedious algebra delivers that \( Q(\beta, \theta_2/\theta_1) \) is decreasing in \( \beta \) and therefore \( Q(\beta, \theta_2/\theta_1) > Q(1, \theta_2/\theta_1) \geq 1 \), where the last inequality follows from the fact that \( Q(1, \theta_2/\theta_1) \geq Q(1, 0) = 1 \).

Proof of Proposition 3

Proof. Let \( Z_s \) be the probability of electing a type \( \theta_1 \) in electoral system \( s \in \{FPP, PR\} \). Then in the case of FPP we have that

\[
Z_{FPP} = \frac{(1 + \alpha)^2}{4} + \frac{1 - \alpha^2}{2} \left(1 - \frac{h}{2}\right),
\]

where

\[
\alpha(\beta, h) = 1 - \frac{\sqrt{\beta^2 + 2\beta h (1 - h)} - \beta}{1 - h} \in (0, 1), \quad \text{and} \quad h = \frac{\theta_2}{\theta_1}.
\]

In the case of PR we have that

\[
Z_{PR} = \frac{(1 + \delta)^2}{4} + \frac{1 - \delta^2}{2} \frac{1}{1 + h},
\]

where \( \delta(\beta, h) \in (0, 1) \) is the unique solution to (6) when \( \max_L = \max_R = 1 \). Note that since \( Z_{FPP} \) is increasing in \( \alpha \) and \( 1 - h/2 > 1/(1 + h) \), then if \( \alpha \geq \delta \) it immediately follows that \( Z_{FPP} > Z_{PR} \). Since \( \alpha(\beta, 0) = \delta(\beta, 0) = 1 \) and \( \alpha(\beta, 1) = \delta(\beta, 1) = 0 \) and by definition

\[
\delta = 1 - \frac{h \left(2\beta + 1 + \frac{h^2}{(1+h)^2}\right) - \delta h \left(\frac{1}{4} - \frac{h^2}{(1+h)^2}\right)}{2\beta + 1 + \frac{1}{(1+h)^2} - \delta \left(\frac{1}{4} - \frac{h^2}{(1+h)^2}\right)}
\]

\[
\alpha = 1 - \frac{2\beta h}{2\beta + (1 - h)(1 - \alpha)},
\]

we have that when \( h \in (0, 1) \), \( \alpha = \delta = x \) if and only if

\[
\frac{2\beta + 1 + \frac{h^2}{(1+h)^2} - x h \left(\frac{1}{4} - \frac{h^2}{(1+h)^2}\right)}{2\beta + 1 + \frac{1}{(1+h)^2} - x \left(\frac{1}{4} - \frac{h^2}{(1+h)^2}\right)} = \frac{2\beta}{2\beta + (1 - h)(1 - \alpha)}.
\]
The last expression is quadratic in $x$, it admits two solutions, and it can be verified that only one solution is strictly smaller than 1. Therefore there exist a unique $\bar{h}(\beta) \in (0, 1)$ such that $\alpha (\beta, \bar{h}(\beta)) = \delta (\beta, \bar{h}(\beta))$. Further, since

$$\frac{\partial \alpha (\beta, h)}{\partial h} = -\frac{2\beta (2\beta + 1 - \alpha)}{(2\beta + (1 - h) (1 - \alpha))^2 + 2\beta h (1 - h)} < 0$$

and

$$\left. \frac{\partial \alpha (\beta, h)}{\partial h} \right|_{h=0} = -1 \quad \text{and} \quad \left. \frac{\partial \alpha (\beta, h)}{\partial h} \right|_{h=1} = -1 - \frac{1}{2\beta},$$

we have that $\alpha > \delta$ if and only if $h > \bar{h}(\beta)$. Hence, we can conclude that when $h > \bar{h}(\beta)$ the probability of electing a type $\theta_1$ is higher in FPP than in PR. Finally, since

$$\lim_{h \to 0} \frac{Z_{FPP}}{Z_{PR}} |_{\beta=1} \geq 1 \geq \lim_{h \to 0} \frac{Z_{FPP}}{Z_{PR}} |_{\beta=0},$$

there exist an $h^* (\beta)$ such that $Z_{FPP} \geq Z_{PR}$ if $h \geq h^* (\beta)$, and there exist a $\beta \in (0, 1)$ such that $h^* (\beta) > 0$ if $\beta < \beta^*$. 

\begin{proof}

Consider first the case of $\gamma \leq \min \{\bar{\gamma}^{PR}, \bar{\gamma}^{FPP}\}$ or $\gamma \geq \max \{\bar{\gamma}^{PR}, \bar{\gamma}^{FPP}\}$ and let $h \equiv \theta_{\max+1}/\theta_{\max}$ and let $Pr (\theta_x, \theta_y)$ denote the equilibrium probability that the election is contested between politicians of quality $\theta_x$ and $\theta_y$. Then, the expected total campaign effort of electoral candidates in FPP is equal to

$$Pr (\theta_{\max}, \theta_{\max}) \theta_{\max} + Pr (\theta_{\max+1}, \theta_{\max+1}) \theta_{\max+1} + 2 Pr (\theta_{\max}, \theta_{\max+1}) \frac{\theta_{\max+1}}{2} \left(1 + \frac{\theta_{\max+1}}{\theta_{\max}}\right) = \theta_{\max} \left(\frac{(1 + \alpha)^2}{4} + \frac{(1 - \alpha)^2}{4} + h + (1 - \alpha^2) \frac{h (1 + h)}{4}\right) > \frac{\theta_{\max}}{2},$$

\end{proof}
where the last inequality follows from the fact that the term in parentheses is increasing in
\( \alpha \), and 
\[
\alpha = \left( 1 - h - \sqrt{\beta^2 + 2h\beta (1 - h) + \beta} \right) / (1 - h)
\]
is decreasing in \( \beta \). Hence,
\[
\frac{(1 + \alpha)^2}{4} + \frac{(1 - \alpha)^2}{4} h + \left( 1 - \alpha^2 \right) \frac{h (1 + h)}{4} > \frac{(1 + \alpha|_{\beta=1})^2}{4} + \frac{(1 - \alpha|_{\beta=1})^2}{4} h + \left( 1 - \alpha^2|_{\beta=1} \right) \frac{h (1 + h)}{4},
\]
and the last expression is only a function of \( h \) and it is always bigger than \( 1/2 \). On the other hand, the expected total campaign effort of electoral candidates in PR is equal to
\[
\Pr(\theta_{\max}, \theta_{\max}) \frac{\theta_{\max}}{2} + \Pr(\theta_{\max+1}, \theta_{\max+1}) \frac{\theta_{\max+1}}{2} + 2 \Pr(\theta_{\max}, \theta_{\max+1}) \frac{\theta_{\max}\theta_{\max+1}}{\theta_{\max} + \theta_{\max+1}} = \frac{\theta_{\max}}{2} \left( \frac{(1 + \delta)^2}{4} + \frac{(1 - \delta)^2}{4} h + \left( 1 - \delta^2 \right) \frac{h}{1 + h} \right) < \frac{\theta_{\max}}{2},
\]
since
\[
\frac{(1 + \delta)^2}{4} + \frac{(1 - \delta)^2}{4} h + \left( 1 - \delta^2 \right) \frac{h}{1 + h} < \left( \left( 1 + \delta^2 \right) + \left( 1 - \delta^2 \right) \right) \frac{1}{2} = 1.
\]
Finally, since when \( \theta_3 \) is relatively close to \( \theta_2 \), the only case left is \( \gamma \in (\gamma^{FPP}, \gamma^{PR}) \), and in this case it is immediate to check that the expected total campaign effort of electoral candidates is higher in FPP than in PR, we are done.
References


