

# Intervention and Peace<sup>☆</sup>

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## Abstract

Intervention often does not lead to peace, but rather to prolonged conflict. Indeed, we document that it is an important source of prolonged conflicts. We introduce a theoretical model of the balance of power to explain why this should be the case and to analyze how peace can be achieved: either a cold peace between hostile neighbors or the peace of the strong dominating the weak. Non-intervention generally leads to peace after defeat of the weak. Cold peace can be achieved with sufficiently strong outside intervention. The latter is thus optimal if the goal of policy is to prevent the strong from dominating the weak.

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## 1. Introduction

One of the facts of warfare is that victory in battle weakens the opposition making further victories easier. There are exceptions in conflicts over large geographical areas - for example the French and German invasions of Russia failed despite initial successes due to the overstretch of supply - but in regional conflicts over good terrain, absent outside intervention typically one side eventually achieves enough success that it is ultimately able to win the war. Outright victory tends to lead to peace, albeit the peace of the strong ruling the weak. Examples of this are the Union victory in the U.S. Civil War, the defeat of Napoleon and the defeat of Germany and Japan in the Second World War.

When there is outside intervention the weak may be propped up for a long period of time or even indefinitely leading to prolonged and often very bloody conflicts that may last decades or even generations. This depends in part on the goals of the outside powers and their strength. In many cases they support a balance of power, either for selfish reasons, to assure weak opposition, as in the case of Britain supporting a balance of power in continental Europe over many centuries, or because different outside powers take different sides in a conflict. Two obvious examples of conflicts prolonged over decades by outside intervention maintaining a balance of power are the Vietnam War, and the conflict between Israel and the neighboring Arab countries. By contrast conflicts without outside intervention - such as World War II or the US Civil war - are typically short, lasting on the order of five years before one side wins. We do not address the issue of the relative desirability of short term peace versus long-term conflict, but instead try to develop a useful model of the length and nature of conflict and how it depends upon outside intervention.

The model we develop is a stochastic model of regional conflicts. Under modest assumptions, absent outside intervention one side will win - and relatively quickly - leading to a hegemony and a long peace - that of the conqueror over the vanquished. By contrast, outside intervention typically supports the weaker side and can lead to a balance of power rather than a hegemony. By doing so it typically prolongs conflict. It does so, however, protect the weak from the strong. Hence there is a trade-off: peace being desirable on the one-hand and the protection of the weak being desirable on the other. We find that when the latter is a priority the level of intervention is a relevant determinant of the nature and length of the conflict, with stronger intervention being generally preferable towards the goal of minimizing casualties.

We first develop a stochastic model of regional conflict, then we discuss the implications in a number of different conflicts historical and contemporary.

## 2. The Basic Model

We examine a conflict between two groups denoted by  $j = 1, 2$  that we refer to as *societies* - these might be nation-states, non-governmental organizations, or bands of rebels.<sup>3</sup> These societies

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<sup>3</sup>A related model with more societies can be found in Levine and Modica (2017).



a society gains land it gains strength because it commands greater resources, and as it loses land it loses strength because it commands fewer resources.

### *Conflict Resolution and the Markov Process*

Conflict is a risky business: weather, luck in warfare, heroism, being in the right place at the right time - all these things matter in determining the result of a conflict. To model this we introduce a stochastic model: we denote by  $1 \geq p_\epsilon(\varphi_{jt}, \varphi_{kt}) \geq 0$  the *conflict resolution function* - the probability that society  $j$  gains a unit of land from society  $k$ . Here  $\epsilon$  is a parameter that we explain below. Naturally  $p_\epsilon(\varphi_{1t}, \varphi_{2t}) + p_\epsilon(\varphi_{2t}, \varphi_{1t}) \leq 1$  since only one society can gain a unit of land. In fact, we assume that the inequality is strict: there is a chance that nothing happens and that neither society gains a unit of land from the other.

The critical - and obvious - assumption is that  $p_\epsilon(\varphi_{jt}, \varphi_{kt})$  is increasing in  $\varphi_{jt}$  and decreasing in  $\varphi_{kt}$ . The stronger is a society and the weaker its opponent the more likely it is to prevail and gain land. This is by no means a new idea: in military theory it is known as Lanchester's Law - although that refers to a deterministic differential equation rather than a stochastic process in which luck plays a role.

The conflict resolution function defines a Markov chain. Taking as the state variable the land holding  $L_{jt}$  of society  $j$  and letting  $L_{kt} = L - L_{jt}$  be the land holding of the opponent we see that with probability  $p_\epsilon(\varphi(\gamma_j, L_{jt}), \varphi(\gamma_k, L_{kt}))$  society  $j$  gains a unit of land to  $L_{j,t+1} = L_{jt} + 1$  and with probability  $p_\epsilon(\varphi(\gamma_k, L_{kt}), \varphi(\gamma_j, L_{jt}))$  it loses a unit of land to  $L_{j,t+1} = L_{jt} - 1$  and with the remaining probability keeps the original amount of land  $L_{j,t+1} = L_{jt}$ . Indeed, this is a special type of Markov chain called a birth-death process.

The crucial feature of the model is that as  $L_{jt}$  grows the probability it continues to grow goes up. That is, as society  $j$  gains land its aggregate strength increase, the aggregate strength of its opponent decreases and so the probability with which it wins land goes up. This says - in effect - that a balance of power between two societies is unlikely. Through random luck one society or the other will gain the upper hand, and when it does so it will not need so much luck to prevail over the other society resulting in a situation we refer to as *hegemony* - one of the two societies controls all the land. In a hegemony of  $j$  we have  $L_{jt} = L$ . We now introduce some formal tools for analyzing Markov processes which make this point more precisely.

### *Resistance*

As one society becomes predominant - controls most of the land - the chances that the weaker society is able to wrest a unit of land in the face of overwhelming opposition becomes very small. For example, on December 2, 1913 in Alsace-Lorraine a shoemaker Karl Blank laughed at German soldiers, and was beaten and paralyzed. Subsequently there were protests of up to 3,000 people: needless to say the probability that this "rebellion" would succeed in wresting control of the Alsace-Lorraine region from the German Empire was very small and indeed it did not succeed. By contrast on October 8 in Wuhan China a rebel Sun Wu was seriously injured by a bomb he was building resulting in his capture and imminent execution. To prevent this soldiers of the New Army in the

local barracks mutinied. The probability that this “rebellion” would succeed in wresting control of China away from the Emperor was also very small - and indeed numerous similar revolts over the past decade had collapsed resulting in the execution or exile of their leaders. This one, however, succeeded and less than a year later the Empire had fallen.

The idea that prevailing over overwhelming opposition is very unlikely but never-the-less can happen is captured by the parameter  $\epsilon > 0$  which is a “small” number. The idea that unlikely things can never-the-less happen is captured by assuming that for  $\epsilon > 0$  “everything is possible,” that is  $p_\epsilon(\varphi_{jt}, \varphi_{kt}) > 0$  for all  $(\varphi_{jt}, \varphi_{kt})$ . A basic mathematical tool for analyzing Markov process in which certain probabilities are small is the notion of resistance. Specifically, the *resistance* of society  $j$  to *losing* land to  $k$  is defined as

$$r(\varphi_{jt}, \varphi_{kt}) = \lim_{\epsilon \rightarrow 0} \frac{\log p_\epsilon(\varphi_{kt}, \varphi_{jt})}{\log \epsilon}.$$

Notice that  $r(\varphi_{jt}, \varphi_{kt}) > 0$  implies  $\lim_{\epsilon \rightarrow 0} p_\epsilon(\varphi_{kt}, \varphi_{jt}) = 0$ . To understand the formula, if  $p_\epsilon(\varphi_{kt}, \varphi_{jt}) = \epsilon^r$  then  $r(\varphi_{jt}, \varphi_{kt}) = r$ . Bigger  $r$ , that is greater resistance, means smaller  $p_\epsilon = \epsilon^r$ ; that is, small probability  $p_\epsilon(\varphi_{kt}, \varphi_{jt})$  of the opponent taking land away translates to greater resistance  $r(\varphi_{jt}, \varphi_{kt})$  of  $j$  to losing the land.

With the notion of resistance we can apply powerful tools from the theory of Markov chains to analyze the conflict model when  $\epsilon$  is small. We make three standard mathematical regularity assumptions: first that  $\lim_{\epsilon \rightarrow 0} p_\epsilon(\varphi_{kt}, \varphi_{jt}) = p_0(\varphi_{kt}, \varphi_{jt})$  exists, second that the resistance exists, and third that if the resistance is zero then  $p_0(\varphi_{kt}, \varphi_{jt}) > 0$ . In effect, we assume that positive resistance means zero probability of losing land in the limit and that zero resistance means positive probability of losing land in the limit.

### *Hegemonic Resistance and Weak Geographic Barriers*

Since the probability of winning land  $p_\epsilon(\varphi_{jt}, \varphi_{kt})$  increases in own aggregate power and decreases in opponent aggregate power, we make the same assumption concerning the resistance to losing land: if resistance of  $j$  to losing land  $r(\varphi_{jt}, \varphi_{kt})$  is positive it increases in own aggregate power and decreases in opponent aggregate power.

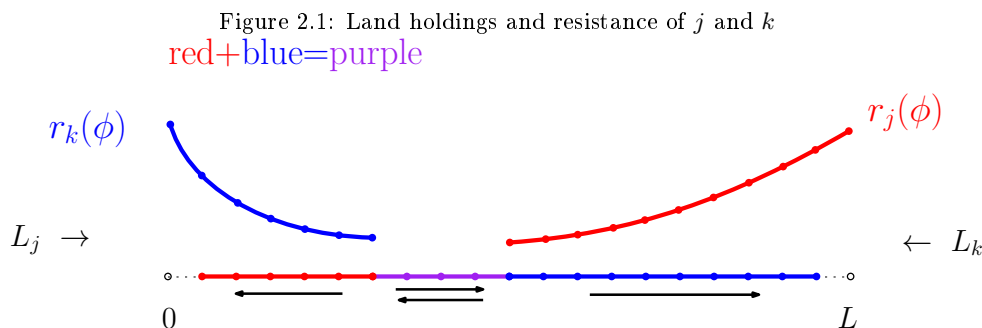
A hegemony once obtained will not persist forever. There can be internal dissension resulting in rebels seizing a unit of land. Since  $\varphi(\gamma_k, 0) = 0$ , if  $j$  has a hegemony the resistance to this is  $r_j^h = r(\varphi(\gamma_j, L), 0)$ , which we call the *hegemonic resistance*. We assume that the institutional strength of the two societies is great enough that they have positive resistance to losing land when they hold a hegemony: that is, we assume that  $r_j^h > 0$  for  $j = 1, 2$ . Observe that this implies that in the limiting  $p_0$ -process a hegemony, once entered, persists forever (it is absorbing); for small  $\epsilon$  it will last for a long time, and we will investigate “how long” this will be.

We also want to capture the idea that we are modeling a region within which geographical and other barriers are weak. We do so by assuming that the weaker society - unprotected by barriers as it is - has no resistance to losing land. When geographical barriers are strong it may be that neither of two nearly equal powers has a realistic chance of taking land from the other. It is no mystery

why Switzerland is Switzerland or the Balkans are Balkanized - this is a matter of mountains and rugged terrain ill-suited for invasion. But our goal here is to study conflict that takes place in a region within which there are not important geographical barriers.<sup>4</sup> To capture this notice that monotonicity implies that if  $\varphi_{jt} \leq \varphi_{kt}$  then  $r(\varphi_{jt}, \varphi_{kt}) \leq r(\varphi_{kt}, \varphi_{jt})$ , that is the weaker has lower resistance. We assume it has zero resistance:

**Assumption 1.** *If  $0 < \varphi_{jt} \leq \varphi_{kt}$  then  $r(\varphi_{jt}, \varphi_{kt}) = 0$ .*

This implies that if the opposing society is at least as strong then the probability of losing land is positive in the  $p_0$ -process. Notice that this also crucially implies that at any given configuration if  $j$ 's resistance is strictly positive then  $k$ 's resistance has to be zero. The typical resistance configuration is illustrated in figure 2.1.



$j$ 's resistance to losing land is in red,  $k$ 's resistance in blue. The functions are defined at the discrete land values, the lines along the dots are drawn for ease of visualization. Resistance decreases with land holdings, and when it is zero the system moves with positive probability in the direction of no resistance - which in this and the following pictures is indicated by arrows. Notice that only hegemonies are absorbing in the limit process.

### 3. Stochastic Stability and Hegemony

We are now in a position to give a theoretical analysis of the Markov process. First, we have assumed that for  $\epsilon > 0$  we have  $p_\epsilon(\varphi_{jt}, \varphi_{kt}) + p_\epsilon(\varphi_{kt}, \varphi_{jt}) < 1$  and  $p_\epsilon(\varphi_{jt}, \varphi_{kt}) > 0$ : these two imply that for  $\epsilon > 0$  the Markov process is aperiodic and ergodic, which in turn implies that there is a unique ergodic probability distribution  $\mu_\epsilon$  over the state space describing how frequently the different states are visited. From Young (1993) and the regularity conditions on resistances, we also know that as  $\epsilon \rightarrow 0$  the ergodic distributions  $\mu_\epsilon$  have a unique limit  $\mu_0$  and that this limit places weight only on absorbing states for the limiting process  $p_0$ . For simplicity we will refer to these

<sup>4</sup>It is also the case that particular military technology may favor the defense so that even in the absence of geographical barriers invasion is impractical. However historical examples of this type are difficult to find. One example may be the stand-off between the Roman Empire and Persia: the two powers used incompatible military technologies, neither able to defeat the other.

states simply as *absorbing* with the understanding this is true only when  $\epsilon = 0$ . Those absorbing states that have positive probability in the limit distribution  $\mu_0$  are called *stochastically stable*: they represent the state or states which are observed “most of the time” when  $\epsilon$  is small.

It should be apparent that when  $\epsilon = 0$  the hegemonies of  $j = 1, 2$  are absorbing: since the hegemonic resistance is by assumption positive when  $\epsilon = 0$  the probability of leaving hegemony is also zero. By contrast, Assumption 1 implies that all the other states are transient when  $\epsilon = 0$ . The key idea is that the society that is weaker in aggregate power always has a positive probability of losing land to the strong: this further weakens them and strengthens the opposition - so that there is a positive probability from any state  $L_j$  of reaching a hegemony of the society which begins with greater aggregate power.

Hence our first result, following from Corollary 5 Levine and Modica (2016) section 6.3, formally captures the instability of a balance of power between two societies:

**Theorem 1.** *The stochastically stable state of the system is a hegemony of the society with the highest unit power  $\gamma_j$  or if the two are equal, both hegemonies  $j = 1, 2$  are stochastically stable.*

We can say a bit more than that: we can say relatively how likely the two hegemonies are. Let  $z_j$  denote the state in which  $j$  has a hegemony, that is  $L_j = L$ . If we compute the *relative ergodic resistance*

$$\rho(z_j, z_k) = \lim_{\epsilon \rightarrow 0} \frac{\log(\mu_\epsilon(z_j)/\mu_\epsilon(z_k))}{\log \epsilon}$$

then the order of magnitude of  $\mu_\epsilon(z_j)/\mu_\epsilon(z_k)$  is  $\epsilon^{\rho(z_j, z_k)}$ . To compute  $\rho(z_j, z_k)$  we introduce the concept of *radius* which in the current context is the total resistance of the path from one hegemony to another. Starting the hegemony of  $j$  the resistance to losing one unit of land is  $r_j^h$ . Once the first unit of land is lost, the resistance to losing the second unit of land is  $r(\varphi(\gamma_j, L-1), \varphi(\gamma_k, 1))$ . We can continue, computing the resistance to losing each additional unit of land until we reach a hegemony of  $-j$ . If we add together these number we can define the radius  $R_j = r_j^h + r(\varphi(\gamma_j, L-1), \varphi(\gamma_k, 1)) + \dots$  from a hegemony of  $j$  to a hegemony of  $k$ . From Levine and Modica (2016) we find that  $\rho(z_j, z_k) = R_k - R_j$ . That is, the relative frequency with which we see  $z_j$  to that with which we see  $z_k$  is to an order of magnitude  $\epsilon$  raised to a power equal to the difference between the least resistance of going from  $z_k$  to  $z_j$  and that of going the opposite direction. The harder it is to go from one hegemony to the other, the more frequently we see that hegemony. In particular, the stable hegemony is the one with higher radius.<sup>5</sup> Moreover, the expected length of time before leaving the hegemony  $z_j$  for the other  $z_k$  is of order  $\epsilon^{-R_j}$ .

### *Hegemonies are Common*

Our theory says we should generally see hegemony. The idea of history being dominated by hegemonic states may seem a strange one, but with some important exceptions it is borne out by

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<sup>5</sup>Except as otherwise noted the proofs of the theorems in this paper are applications of results in Levine and Modica (2016). A set of results for a related version of this model that allows for multiple societies can be found in Levine and Modica (2017).

historical facts.<sup>6</sup> Take, for example, the largely geographically isolated region of China: bounded by jungles in the South, deserts on the West, cold arid wasteland in the North and the Pacific Ocean in the East. We find that during the 2,234 years beginning from when we have decent historical records in 221 BCE the area was ruled by a hegemonic state roughly 72% of the time, with five interregna. Less reliable records exist for the area of Egypt, but in the 1,617 years from 2686 BCE to the end of the new Kingdom in 1069 BCE we see hegemonic rule 87% of the time with two interregna. In Persia during the 1,201 years from 550 BCE to 651 CE we see hegemony 84% of the time with two interregna. England has been largely hegemonic within the geographically confined area of the island of Britain for 947 years from 1066 CE to the present. The Roman Empire ruled the Mediterranean area as a hegemony for 422 years from the advent of Augustus in 27 BCE to the permanent division into Eastern and Western Empires in 395 CE and the Eastern Roman Empire lasted an additional 429 years until the advent of the Caliphate in 814 CE. The Caliphate itself lasted 444 years until the Mongol invasion in 1258. After a 259 year gap, the Ottoman Empire established a hegemony over the same general area for 304 years from the conquest of Egypt in 1517 CE to the Greek revolution in 1821 CE.

### *Hegemonies are not Ubiquitous*

While hegemonies are common in history, there are two glaring exceptions: except for brief periods neither the subcontinent of India nor, following the fall of the Western Roman Empire, the area of continental Europe were subject to a hegemonic state: indeed the situation, especially in Europe, can better be described as a balance of power between competing societies. Clearly the theory is deficient: it says that as one side gains an advantage it becomes more likely to gain additional advantage. Evidentially this was not true in Europe and India. To see why this might be the case the example of the Korean war is useful. In September 1950 North Korea was on the verge of dominating the South. On the 15th of September the United States launched an amphibious invasion reversing the situation. But rather than gaining resources and weakening the North Koreans the result of this success was the entry of China into the war on the side of North Korea - resulting in the United Nations forces being pushed back and ultimately a stalemate. The key point is: gaining land leads to greater weakness of the opponent only if it does not draw outside intervention.

The intervention of outsiders - protected typically by their own strong geographic barriers so not at risk in the conflict - is common in history. In Europe following the fall of Rome and up to around 1066 we have the continued interference of northerners - the Vikings and later Swedes were especially well protected by their own geography. Following 1066 we have the constant interference of England - also safe behind a water barrier: during this period we observe that England constantly intervened in continental conflicts but always to support the weaker side, and eventually this policy of balance of power became explicit.<sup>7</sup> India also was subject to repeated invasion from central Asia

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<sup>6</sup>See Levine and Modica (2012) for data and sources.

<sup>7</sup>It is not completely correct to view England and Scandinavia as "outsiders" as at various time they had continental



- protected not by water but by difficult desert and mountain terrain.<sup>8</sup> Of course China too was subject to outside influence - particularly that of the Mongols. However, the relative size of the Mongolia is quite small relative to China - less than half a percent of the population - while the population of Scandinavia was about 5% that of continental Europe, that in central Asia about 5% that of India, while England was about 8% of continental Europe.<sup>9</sup>

The role of England in maintaining a balance of power on the continent is well documented and notorious for its complete cynicism. From the rise of Spain following the discovery of America in 1492 through Brexit in 2016 British foreign policy has largely been aimed at preventing a hegemony over continental Europe. Many books (see for example Sheehan (1996)) have been written on the topic and few discussions of European history fail to remark upon the striking fact that Britain consistently changed sides in conflicts to support the losing side. Most dramatic perhaps is the shift to an alliance with France in 1904 in the face of the German threat. Note that until the advent of the European Union and the fall of the Iron Curtain this policy was quite successful. The latest effort to break up the continental hegemony of the EU - Brexit - may be less successful: contrary to the predictions of its advocates it seems to have strengthened pro-European sentiment on the continent.

To further advance the theory and understand the role of outside intervention in the balance of power we now extend the model to incorporate outside intervention

#### 4. Outside Intervention and the Balance of Power

We introduce *outside forces* with *intervention power*  $\varphi_0$ . Outsiders are assumed to be protected by geography, climate or sheer strength from action by the region in question. One example is the aforementioned case of Great Britain with respect to continental Europe. Currently the U.S. and Russia are outside forces with respect to the Middle East, being protected by distance, the ocean (in the case of the U.S.) and by military strength from Middle Eastern societies.<sup>10</sup>

These outside forces can *reinforce* society  $j$  and we focus on the case in which only one party to the conflict is the beneficiary of outside intervention at any moment of time. We assume that

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interests and conversely, but the key point is that they had a core area relatively safe from invasion. In a different direction Hoffman (2013) argues a role also for the Western Catholic church which in Europe acted as a balancing force much akin to the outsiders of our model.

<sup>8</sup>The exact nature of the asymmetry in the physical geographical barrier is uncertain, but it is a fact that India has been invaded numerous times successfully from Central Asia, but there have been no successful conquests of Central Asia from India. Phil Hoffman in a private communication suggests that part of the answer may lie in the fact that the area of Central Asia is well suited for raising horses and India is not, and that horses play a central military role in conflict between Central Asia and India.

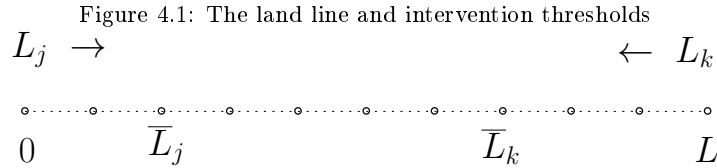
<sup>9</sup>Note that geographical factors matter in our argument only in so far as they give rise to outsiders who influence the evolution of the relationships between the other groups. An existing literature, including Diamond (1998), gives physical geography a direct role, arguing for example that the terrain of Western Europe is more defensible than that of China, hence less susceptible to hegemony. Besides this particular claim being challenged on physical grounds (Hoffman (2013)), such considerations have no bite in the Chinese case. Incidentally: while this discussion includes only the area of Europe, Asia and North Africa, it should be borne in mind that until modern times 90% of the world population lived in this area.

<sup>10</sup>Note that we are not considering here terrorism - which on the scale of conflict it is relatively minor.

these outside forces have a fixed aggregate power  $\varphi_0$ . If society  $j$  is reinforced its *combined power* is  $\phi_j = \varphi_j + \varphi_0$ , otherwise it is  $\phi_j = \varphi_j$ . Hence the conflict resolution is now given by  $p_\epsilon(\phi_{jt}, \phi_{kt})$  rather than  $p_\epsilon(\varphi_{jt}, \varphi_{kt})$ , and the resistance function is also changed accordingly. In the model with outside intervention the assumption of positive hegemonic resistance, which was  $r_j^h = r(\varphi(\gamma_j, L), 0)$ , will now be strengthened to  $r_j^h \equiv r(\varphi(\gamma_j, L), \varphi_0) > 0$ : resistance remain positive even when opponents are supported by outsiders.

The behavior of outside forces is determined by an intervention policy which we initially take to be exogenous. We study a simple but important intervention: intervention on behalf of the weak.<sup>11</sup> Specifically we assume there exist thresholds  $\bar{L}_j, \bar{L}_k$  with  $\bar{L}_j < L - \bar{L}_k$  and such that if  $L_{jt} \leq \bar{L}_j$  then outsiders reinforce society  $j$ . Notice that we are agnostic about whether it is different or the same outsiders who reinforce  $j$  as who reinforce  $k$ ; we discuss this further below when consider why outsiders might intervene. The assumption that the size of intervention  $\varphi_0$  is the same on both sides is a simplification enabling us to focus on  $\bar{L}_j$  as a measure of the strength of intervention.

It is useful at this point to denote combined power of  $j$  as  $\phi_j(L_{jt})$  since unit power  $\gamma_j$  is determined by  $j$  while intervention is determined by  $L_{jt}$  (because  $L_{kt} = L - L_{jt}$ ). The inequality  $\bar{L}_j + \bar{L}_k < L$  means that  $\bar{L}_j$  appears on the left of  $\bar{L}_k$ , as in figure 4.1 (where  $\bar{L}_j = 2$  and  $\bar{L}_k = 3$ ).



### *Stochastic Stability and the Balance of Power*

We shall see that there are two types of balance of power configurations that are absorbing when  $\epsilon = 0$  and hence candidates for being stochastically stable. One is a *cold peace*, where a single unit of land changes hands back and forth. It may be thought of as corresponding to border skirmishes - for example, the recent conflict between Israel and Lebanon which occasionally flares into the firing of rockets over the border or small border incursions. Such a continuing conflict is relatively low key and “peaceful.” The other is a *prolonged war*, where the two sides fight back and forth losing and gaining substantial amounts of land. The civil war in Syria in the last years is a case in point.

We now formally introduce the notion of balance of power segments.<sup>12</sup>

**Definition 1.** A *short balance of power* segment consist of two states, either with  $L_j = \bar{L}_j, \bar{L}_j + 1$  or  $L_k = \bar{L}_k, \bar{L}_k + 1$ . A *long* segment is all the states between  $L_j = \bar{L}_j$  and  $L_j = L - \bar{L}_k$  inclusive. Note that a segment can be both short and long if  $\bar{L}_j + \bar{L}_k = L - 1$  and that there can be a pair of short segments.

<sup>11</sup>Intervention on behalf of the strong may hasten hegemony but will not lead to a balance of power.

<sup>12</sup>We call them segments because they appear so in the graphical representations we use, see Figure 4.1.

We can extend the idea of an absorbing state to that of an *absorbing set* - meaning that when  $\epsilon = 0$  it is impossible to escape from the set, but that there is positive probability of moving about within the set. Hence segments are absorbing if at the left end  $j$  has positive resistance; at the right end  $k$  has positive resistance; and in the interior, if nonempty, both have zero resistance to losing land. Depending on the level of intervention some segments are absorbing and others are not. We distinguish four levels of intervention *on behalf of  $j$* :

**Definition 2** (Intervention strength). 1. *Very strong*. Intervention takes place when resistance is positive in the absence intervention:  $r(\varphi_j(\bar{L}_j), \varphi_k(L - \bar{L}_j)) > 0$

2. *Very weak*. Intervention is insufficient to give positive resistance:  $r(\varphi_j(\bar{L}_j) + \varphi_0, \varphi_k(L - \bar{L}_j)) = 0$

For the remaining cases we assume that #1 and #2 do not hold, that is  $r(\varphi_j(\bar{L}_j), \varphi_k(L - \bar{L}_j)) = 0$  and  $r(\phi_j(\bar{L}_j), \phi_k(L - \bar{L}_j)) > 0$ :

3. *Strong*. When  $j$  gains a unit of land above the threshold (thus losing support) the opponent has zero resistance to losing land:  $r(\phi_k(L - \bar{L}_j - 1), \phi_j(\bar{L}_j + 1)) = 0$

4. *Weak*. When  $j$  gains a unit of land above the threshold the opponent has positive resistance to losing land:  $r(\phi_k(L - \bar{L}_j - 1), \varphi_j(\bar{L}_j + 1)) > 0$

In Appendix 1 we characterize the relationship between intervention and the existence of absorbing segments of different types. The results are reported in the following

**Theorem 2.** *Existence, if any, of absorbing segments depending on the type of intervention on behalf of societies  $j$  and  $k$  can be summarized in the following table (where land is expressed in units of  $L_j$ ):*

|  | <i>strong <math>k</math></i>             | <i>weak <math>k</math></i>                   | <i>very weak/none/very strong <math>k</math></i> |
|--|--|--|--|
| <i>strong <math>j</math></i>                     | <i>long</i>                              | <i>short from <math>L - \bar{L}_k</math></i> | <i>none</i>                                      |
| <i>weak <math>j</math></i>                       | <i>short from <math>\bar{L}_j</math></i> | <i>pair of short</i>                         | <i>short from <math>\bar{L}_j</math></i>         |
| <i>very weak/none/very strong <math>j</math></i> | <i>none</i>                              | <i>short from <math>L - \bar{L}_k</math></i> | <i>none</i>                                      |

An important result concerns whether balance of power or hegemony is stochastically stable. As might be expected which is stochastically stable depends on the strength of the outside forces. In Appendix 3 we prove the following:

**Theorem 3.** *There exist  $\infty \geq \bar{\varphi}_0 \geq \underline{\varphi}_0 > 0$  such that if  $\varphi_0 > \bar{\varphi}_0$  and if intervention thresholds are positive on both sides on both sides there are stochastically stable balance of power segments but not stochastically stable hegemonies, while if  $\varphi_0 < \underline{\varphi}_0$  there are stochastically stable hegemonies but not stochastically stable - or even absorbing - balance of power segments.*

### War and Peace

To focus thinking it is useful to consider a simple case that highlights the main conclusions of the theoretical analysis. We suppose that  $j$  and  $k$  are equally strong so that  $\gamma_j = \gamma_k = \gamma$ , and that

the intervention policy is symmetric so that  $\bar{L}_j = \bar{L}_k$ . Hence intervention policy is indexed by a single scalar, the land threshold for intervention on behalf of both contenders. We assume that the number of units of land  $L$  is odd.<sup>13</sup> Finally we assume that the strength of the intervenor(s)  $\varphi_0$  is high enough that strong intervention is possible, but that it is ineffective for  $\bar{L}_j$  sufficiently small.

Start with  $\bar{L}_j$  small. In this case intervention is ineffective - there is no point in intervening when  $j$  has become so weak that they have lost even with outside help. In this case there is no balance of power segment, but rather a hegemony of one society: we refer to this as the *peace of the strong over the weak*. As  $\bar{L}_j$  increases, eventually the point is reached where intervention is weak. Now there is a pair of short balance of power segments, corresponding to a *cold peace*. As  $\bar{L}_j$  increases further intervention becomes strong and there is now a single long balance of power segment - a *prolonged war*. Hence conflict which was low increases substantially. As  $\bar{L}_j$  increases the length of the long segment shrinks reducing the scale of the conflict until eventually  $\bar{L}_j$  reaches the center and the long segment is also short and we are again at cold peace. We want to emphasize the non-monotonicity: a weak or sufficiently strong intervention leads to cold peace, but an intermediate intervention leads to prolonged war and it is the costliest in terms of lives and distress to the peoples and economies involved.

In addition to conflicts corresponding to absorbing states when  $\epsilon = 0$  we can have *transitional wars* corresponding to movement between different absorbing states - that is, from hegemony to balance of power and back, or between different balance of power segments. From Levine and Modica (2016) we know that these conflicts must be relatively short in the sense that the expected length of time before an absorbing configuration is reached is bounded independently of  $\epsilon$  while the length of time spent in an absorbing configuration grows without bound as  $\epsilon \rightarrow 0$ .

### *Dynamics*

How long, relatively speaking, will the system spend in the absorbing sets we have encountered, for small  $\epsilon$ ? The analysis of the dynamics of the system hinges on two numbers describing an absorbing sets containing a point  $z$ : the *radius*  $R_z$  which represents roughly speaking the resistance to escaping “far” from that state; and the *modified radius*  $M_z \geq R_z$ , which is a broader measure but the same idea. The dynamics (for  $\epsilon > 0$ ) can be well described by these two numbers: we have an extension of the results concerning the relative ergodic resistance for hegemonies in the simple model without outside intervention. As before, the expected length of time before leaving an absorbing configuration to reach another is of order  $\epsilon^{-R_z}$ . Configurations with  $R_z = 0$  also have  $M_z = 0$  and are not absorbing. In particular the stable sets are those with maximum modified radius.

The radius of an absorbing set is the least resistance needed to reach a different absorbing set, and its computation in our setting is always simple. We already saw a special case of this in the model without outside intervention, where the radius of an hegemony is the resistance to reach the

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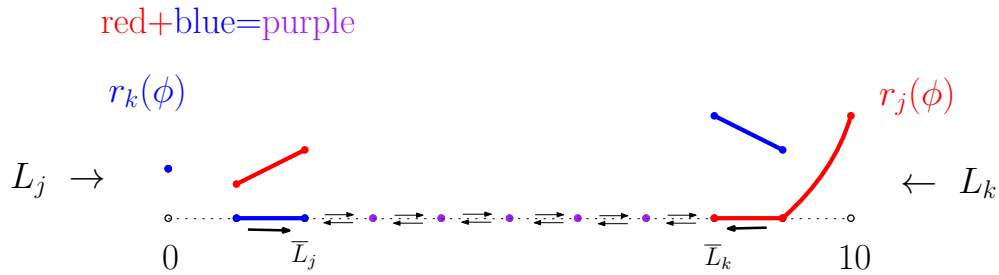
<sup>13</sup>So it is feasible for the two thresholds  $\bar{L}_j, \bar{L}_{-j}$  to be adjacent; this would be ruled out by symmetry if  $L$  is even.

only other absorbing set of the system, that is the other hegemony. The modified radius embodies the notion of the resistance needed to leave the set, but takes account of what path the system might take after leaving. It is more involved to calculate: the computation of the modified radii of the various absorbing sets we have seen is in Appendix 2.

As we indicated, the radius governs the length of time it takes to leave an absorbing set. Specifically if we let  $T_\epsilon(z)$  denote the expected hitting time before reaching a different absorbing set starting at  $z$  then  $T_\epsilon(z)$  is of order  $\epsilon^{-R_z}$ . By contrast the expected length of time it takes to actually make the leap from one absorbing set to another is much shorter: it is bounded independent of  $\epsilon$ . If we are interested in the relative amount of time spent in two absorbing sets containing the points  $z$  and  $z'$  respectively then the answer is given by the modified radius: the ratio of  $\mu_\epsilon(z)/\mu_\epsilon(z')$  is of order  $\epsilon^{M_{z'}-M_z}$ . This is proven in Levine and Modica (2016).

Consider for an illustration the case of an absorbing single long segment, as in figure 4.2. Here, as always,  $L_j$  is measured from left to right and  $L_k$  from right to left and the arrows in the figure denote the zero-resistance transitions. The radius is the least resistance to reach an hegemony. Hence we compute a *left radius*  $R_{jk}^\ell$  which is the total resistance to reach the left hegemony, a *right radius* similarly for the right hegemony, and conclude that for all  $z$  in the segment the radius is  $R_z = \min\{R_{jk}^\ell, R_{jk}^r\}$ . To wit, the left radius is computed as follows: start at the left endpoint  $\bar{L}_j$ ; reduce the land holding of  $j$  one unit at a time, that is from  $\bar{L}_j$  to  $\bar{L}_j - 1$  and so on. Each time compute the resistance to the land loss (we know from Theorem 2 that since this is absorbing the first step at least has positive resistance); add these numbers together and continue until  $j$  has lost all their land and become inactive. That is, we take  $r_j(\phi_j(\bar{L}_{jk}), \phi_k(L - \bar{L}_{jk}))$ , add to it  $r_j(\phi_j(\bar{L}_{jk} - 1), \phi_k(L - \bar{L}_{jk} + 1))$  and continue until all land is lost. This gives  $R_{jk}^\ell$ . Similarly we compute  $R_{jk}^r$  going rightward.

Figure 4.2: Single long balance of power segment



The long segment is between  $L_j = \bar{L}_j$  and  $L_j = L - \bar{L}_k$  (indicated in the figure by the label  $\bar{L}_k$  using the reverse axis).  $j$ 's resistance  $r_j$  jumps up at  $\bar{L}_j$  because of intervention. The double arrows indicate the zero resistance movements within the segment.

## 5. Balance of Power and Modern Conflict

We shortly turn to details of different configurations and illustrate them with examples. In addition to discussing specific cases, we gather the substantial post World War II conflicts in the

form of tables. Cases where one combatant did not occupy any land are excluded as the theory does not apply. For the rest we examined each postwar conflict in the Uppsala database. We excluded those marked as insignificant, those involving military coups, those involving invasions of minor powers by major powers (for example: 1956 invasion of Hungary) and guerrilla conflicts where the guerrillas did not control land and resources (for example: the Basque region). We examined each remaining conflict and believe that we have included the most significant. In some cases there were several intervention regimes: we discuss those separately. The data about individual conflicts is taken from Wikipedia. The tables show the region, the year in which the conflict began, and the number of years it lasted. Casualties (including civilian casualties) are reported in deaths per 100,000 per year which is the standard unit for reporting, for example, murder rates.<sup>14</sup> To put these numbers in context, note that the overall murder rate for Europe and Asia is about 3, for the entire world about 6, and for Africa about 12 and for the Americas about 16. So, for example, the death rate of 20 in the Sri Lankan civil war (a cold peace) is comparable to the murder rate in the Americas, while the death rate of 380 in the Syrian civil war (a prolonged war) is more than an order of magnitude higher. Following the casualty rates we list the parties and outside intervenors. In cases when war ended due to the withdrawal of intervention we report the “collapse” as the number of subsequent years until one side achieved victory. Entries in the table are arranged in chronological order.

Before examining the details, it is worth taking a broad overview of our findings. Intervention that either is designed to preserve the balance of power or which does so because of conflicting interests of the intervenors can lead either to a cold peace or a prolonged war. There is a large discontinuity in the amount of harm done in a cold peace and a prolonged war: in a cold peace death rates are on the order of relatively high murder rates, or in some cases lower, while in a transitional or prolonged war they are an order of magnitude larger than very high murder rates. Taking the Sri Lankan civil war as an example of cold peace we see that for 26 years the death rate was about 20, comparable to the highest murder rates in the world. Taking the breakup of India and Pakistan after the British withdrawal as an example of a transitional war it was vastly bloodier - the death rate was about 250. However, the Sri Lankan civil war lasted 26 years so the total is about 520, more than double that in India and Pakistan where the transitional war lasted only a year. Overall a cold peace does not seem to represent much of a savings in terms of casualties over non-intervention and a transitional war - but it does protect the weak. From a policy point if we were to take the point of view that, say, Lebanon posed a threat, then keeping it a bloody mess for three decades would surely neutralize that threat - but from a humanitarian point of view it represents a catastrophe. If we are to take a very cynical view of the conflict between Shia and Sunni, especially the current hot war in Syria, as a Western effort to preserve a balance of power that neutralizes the Arab world as a threat - the wave of refugees descending on Europe

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<sup>14</sup>Civilian casualties are the bulk of casualties and there are a wide range of estimates. We used the middle of the range of estimates.

with the consequent social and political problems shows that such an effort can have unintended consequences.

### 5.1. Anarchy

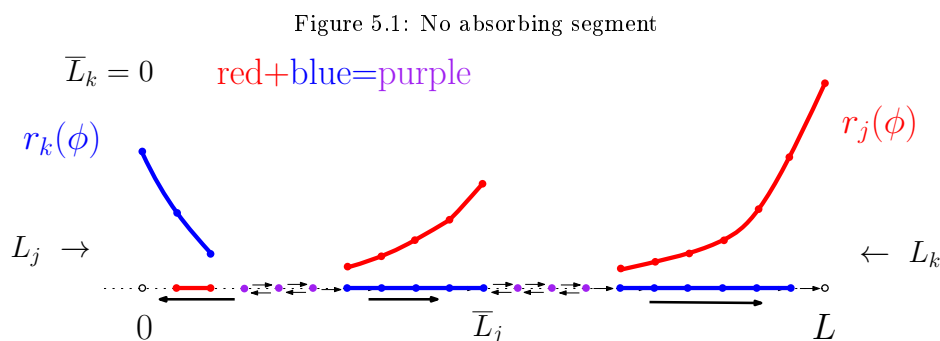
*Theory.* Our working assumption has been that hegemonic resistance is positive, but institutions may be so weak that hegemony is not absorbing. In the absence of outside intervention there are also no absorbing balance of power segments. In this case the theory predicts continual strife - at least until some group develops institutions adequate to persist.

*Discussion.* The situation in the Central African Republic, Somalia and Yemen appear to fall into this category. In the case of Somalia we quote from the US State Department website “There is no organized system of criminal justice, nor is there any recognized or established authority to administer a uniform application of due process. Enforcement of criminal laws is, therefore, haphazard to nonexistent.”

### 5.2. Transitional Wars

*Theory.* Transitional wars are the fast leap from one absorbing set to another: from hegemony to hegemony, from balance of power to hegemony, from hegemony to balance of power or from one balance of power to another. The most important case is when there is no outside intervention or outside intervention is too weak to matter. In this case we have only the transitional war of moving from one hegemony to another, or an unstable state where two societies hold land followed - relatively quickly - by the victory of one resulting in hegemony. However, transitional conflicts also occur with outside intervention in which the intervenor is attempting to help one side win and not to preserve a balance of power.

The case in which there is no absorbing segments and movement is directly from one hegemony to another is illustrated in figure 5.1.



*Discussion.* We observe a number of wars with little outside intervention: these generally result in hegemony and moreover, as the theory predicts, they are relatively short. See Table 1. The only one longer than 1 year is the Iran/Iraq war. Often these transitional wars occur after a change in

Table 1: Significant Modern Transitional Wars

| Region     | Start | Duration | Casualties | Parties                | Intervenors | Collapse |
|------------|-------|----------|------------|------------------------|-------------|----------|
| India      | 1947  | 1        | 250        | India<br>Pakistan      |             |          |
| Palestine  | 1948  | 0.75     | 700        | Israel<br>Arab         |             |          |
| Bangladesh | 1971  | 1        | 100        | Bangladesh<br>Pakistan | India[1]    | 1        |
| Iran/Iraq  | 1980  | 8        | 100        | Iran<br>Iraq           |             |          |
| Falklands  | 1982  | 0.2      | 0          | Argentina<br>UK        |             | 0.2      |
| Rwanda     | 1994  | 0.25     | 6800       | Hutu<br>Tutsu          |             | 0.25     |

*Table Notes*

1. India did not attempt to preserve a balance of power but helped Bangladesh to win the war.

intervention policy in the form of the withdrawal of outside forces leading to collapse of the balance of power. A number of well known conflicts have this character: after the British withdrawal from Palestine in 1948 war broke out between Israel and the Arab nations: this lasted less than a year. Similarly when Britain withdrew from India conflict broke out between India and Pakistan including conventional warfare over Kashmir. This lasted slightly longer than a year. About a year after Richard Nixon agreed to “peace with honor” in 1973 - meaning actually that he agreed to stop intervening - North Vietnam launched an assault on the South winning the war in about a year. In Eastern Europe after Gorbachev announced that the Soviet Union would cease intervention the fall of the Berlin wall came about in a matter of months.

The Rwandan Civil War is an interesting case study in what happens without intervention. The conflict was largely ethnic between Hutu and Tutsi. On April 6, 1994 the plane of the Hutu President Habyarimana was shot down initiating the Hutu genocide perpetrated by the Tutsi. By July 3, 1994 - that is only about three months later - the Tutsi RPF overran the capital city of Kigale effectively ending the war. The absence of western intervention in this case is well known and usually discussed in the context of preventing the genocide. Given the time-line this was probably not feasible: the rapid forms of western intervention - air power, special forces - are ineffectual against large groups of people wielding machetes, and by the time massive numbers of ground troops could have been put in place it would have been far too late. Rather the lack of western intervention is a case study in how prolonged war can be avoided: the Tutsi won and the peace of the strong over the weak has prevailed since.

We want to emphasize just how short are transitional wars compared to the prolonged conflict brought about by insufficiently strong outside intervention. The transitional war that brought peace to Rwanda lasted months. Moreover, the length of wars appears largely unrelated to whether they are civil wars: although most prolonged wars are civil, absent outside intervention they tend to be relatively short, if not so short as in Rwanda. Reaching farther back in history, the U.S. Civil War,



bloody as it was, lasted only four years. World wars - in which outside intervention is not possible more or less by definition - also have been relatively short: four years in the case of World War I and six years in the case of World War II.

The overall point is that these transitional wars are short: less than a decade in length and often lasting only months. Hence although they are bloody, because they are short they are not necessarily more bloody than a cold peace that lasts many decades.

### 5.3. Cold Peace: Short Segments

*Theory.* Short segments involve a single unit of land that changes hands back and forth always at the threshold for intervention. The intervenor prevents “their side” from losing, but is unwilling to help them win. An absorbing single short segment starts from  $\bar{L}_j$  or from  $\bar{L}_k$ ; or it may be a long segment which is also short because  $\bar{L}_j$  and  $\bar{L}_k$  are one unit of land apart. Lastly, absorbing short segments may occur in pairs if both elements of the pair are absorbing. A short segment from  $\bar{L}_j$  is illustrated in figure 5.2; the case of a long segment which is also short is illustrated in figure 5.3; the case of a pair of segments is in figure 5.4. In the discussion that follows we confine attention to the case of a single segment as it is the relevant one in practice.

Figure 5.2: Short segment from  $\bar{L}_j$

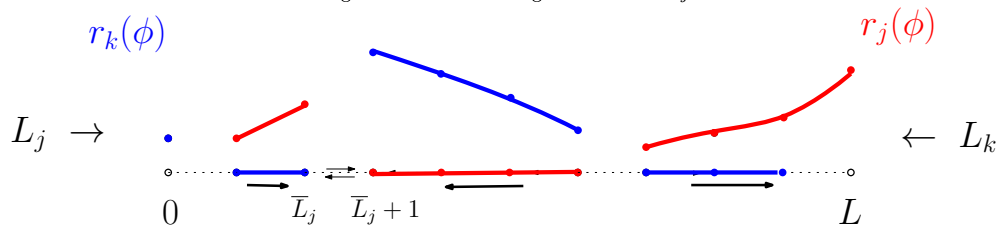


Figure 5.3: Long segment which is also short

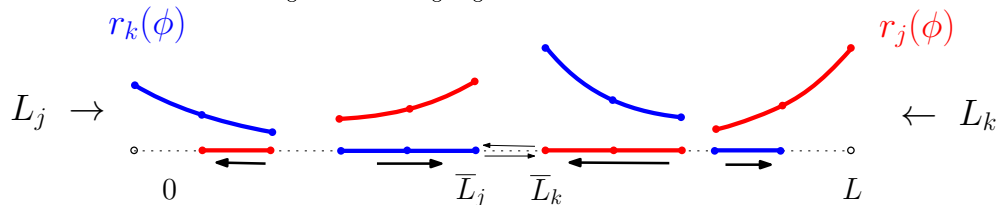
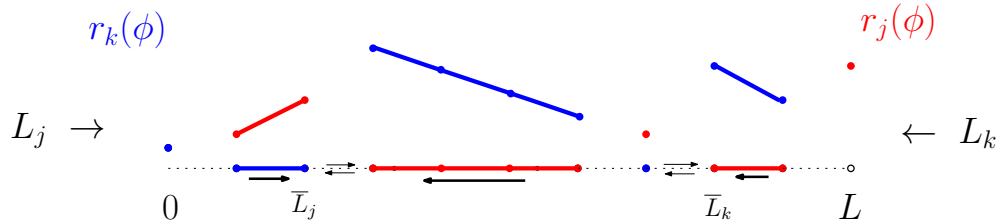


Figure 5.4: A pair of absorbing short segments



*Discussion.* There are two types of cold peace: a one-sided cold peace where weak intervention is matched against the strength of the opponent and a two sided cold peace where two outside

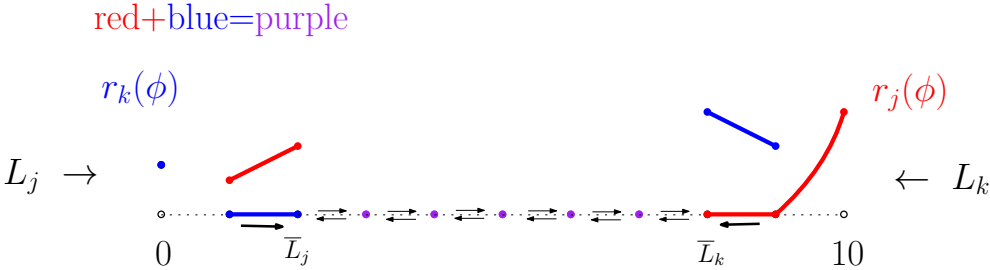
interveners stare eye-to-eye across a border. The former seems rare: one example is in Afghanistan, which has had varied intervention policies over the years since civil war began in 1978. Initially the Soviet Union intervened and a prolonged war resulted until the Soviet withdrawal in 1988. What followed seems to have been a cold peace where intervention was matched against the strength of the opponent: the Taliban overran most of the country, but a small enclave remained under the control of the Northern League propped up by foreign support. This persisted until the strong US intervention in 2001 resulting in the prolonged war that continues to this day.

The eye-to-eye stare across the border is a more common form of cold peace. Post World War II episodes are presented in Table 2. The classical example is the Iron Curtain, where military forces of the intervenors - the US on the West and Soviets on the East - sat for decades eye-to-eye in the literal sense. A more interesting case is in the Sinai where the intervention on both sides is by the US. Naturally we do not see US soldiers staring at each other eye to eye across the border, but the essential element of the Camp David accords was the promise of substantial military support (in the form of equipment and training) for both sides. We do note that in some places outside intervenors in the form of UN blue-helmets patrol the boundary - taking both sides in effect - but their job is merely to act as monitors - they have not even enough military strength to protect themselves should a shooting war break out.

5.4. Prolonged War: Long Segments

*Theory.* Prolonged war results when there is a single long segment that is...long. This is illustrated in figure 5.5 below. Notice that according to the definition there is positive resistance to leave at the two extremes and zero resistance to move in the interior. As we saw before in this case the radius is the minimum of left and right radii which are the resistances to reach left and right hegemony respectively.

Figure 5.5: Single long balance of power segment



*Discussion.* As we have indicated, this seems the least justifiable form of intervention. Data are collected in Table 3. The only rationale we can think of is that a region poses a particular danger and hence the importance of keeping it weak offsets the bloody harm of prolonged war. Yet, if we look at the record, Vietnam, Sudan, Angola, Lebanon, and Syria do not appear to have ever presented any great danger to the intervening powers. It is interesting that while the US intervention in Vietnam is widely criticized outside the US, it seems to be so for mostly the wrong reasons. Surely there was nothing wrong with supporting the South, for, despite all the

Table 2: Significant Modern Cold Peace

| Region                          | Start | Duration | Casualties | Parties                          | Interveners         | Collapse |
|---------------------------------|-------|----------|------------|----------------------------------|---------------------|----------|
| Gibraltar                       | 1713  | 304      | 0          | Spain<br>Gibraltar               | Britain             |          |
| Falklands                       | 1833  | 184      | 0          | Argentina<br>Falklands           | Britain             |          |
| Hong Kong                       | 1842  | 155      | 0          | China<br>Hong Kong               | Britain             | 0        |
| Iron Curtain                    | 1945  | 46       | 0          | Eastern Europe<br>Western Europe | Soviets<br>US       | 1        |
| Sinai                           | 1948  | 68+      | 1          | Israel<br>Egypt                  | West<br>Soviets [1] |          |
| Korea                           | 1950  | 66       | 8          | North<br>South                   | China<br>US         |          |
| Sri Lanka                       | 1983  | 26       | 20         | Government<br>Tigers             | India[2]            |          |
| Nagorno-Karabakh                | 1988  | 28       | 26 [3]     | Armenia<br>Azerbaijan            | Turkey<br>Russia    |          |
| Afghanistan - Northern Alliance | 1994  | 7        | 5 [4]      | Taliban<br>Northern Alliance     | United States       | 0.2      |

*Table Notes*

1. It should be noted that originally the Soviets supported Israel.
2. Indian troops left Sri Lanka in 1990 nineteen years before the Government victory. Prior to sending troops in 1987 the Indian government aided the Tigers through the intelligence agency RAW. It likely that this continued after withdrawal in 1990, but there is no information available about this, so we cannot say when or even if India stopped supporting the Tigers.
3. It is unclear in which population the casualties occurred. Virtually all deaths occurred during the six years of active war beginning in 1988. It is estimated that 28,000–38,000 died in that conflict. The population of Nagorno-Karabakh is only 147,000, but it is highly unlikely the bulk of casualties occurred among that population. We used the average of the population of Azerbaijan and Armenia as our base population.
4. Casualties in the war seem mostly to have involved Taliban massacres of civilians. The number are unclear but do not seem to exceed 10,000 which is the basis of our estimate of 5 per hundred thousand per year.

Table 3: Significant Modern Prolonged Wars

| Region      | Start | Duration | Casualties | Parties                       | Intervenors                   | Collapse |
|-------------|-------|----------|------------|-------------------------------|-------------------------------|----------|
| Vietnam     | 1955  | 20       | 170        | North<br>South                | Soviets<br>US                 | 1        |
| Sudan       | 1955  | 60       | 330        | North Sudan<br>Southern Sudan | Horn of Africa                |          |
| Angola      | 1975  | 27       | 86         | MPLA<br>UNITA                 | Soviets<br>South Africa       | 1 [1]    |
| Lebanon     | 1975  | 31       | 400        | Shia<br>Christian/Druze       | Syria<br>Israel               | 1        |
| El Salvador | 1979  | 12       | 138        | Government<br>FMLN            | United States<br>Soviet Union |          |
| Syria       | 2011  | 5+       | 380        | Government<br>Insurgents      | Russia, Iran<br>West          |          |

*Table Notes*

1. The date at which intervention on behalf of UNITA ceased is unclear. We dated it to May 2001 when DeBeers - the main source of funding and illicit weapons shipments to UNITA - ceased operation in Angola.

shortcomings of its government, there was no popular desire to be ruled by the equally bad or worse government in the North. Nor can there be much moral doubt about opposing the spread of communism: one need not look further than North Korea and Cuba - two of the most miserable places in the world - to see that. Nor is it clear why the direct involvement of the US is worse than indirect Russian involvement. From our point of view the US should be rather criticized for creating a prolonged and costly conflict by attempting to maintain a balance of power in the South.

*5.5. Radii and Modified Radii: Persistence and Frequency*

As we mentioned at the end of section 4, persistence and relative frequency of hegemonies and balance of power depend on radii and modified radii of the various absorbing sets. We confine attention to the case of a single segment, and it does not matter whether it is long or short (the case of two segments is discussed in Appendix 2). There are three absorbing sets: the two hegemonies and the balance of power segment. We may assume without loss of generality that the least resistance from the balance of power segment is to the hegemony of  $j$ : that is  $R_{jk}^r < R_{jk}^l$ . If the segment is short this is more “likely” to be the case if it is the one that is closer to the hegemony of  $j$ , that is the short segment from  $\bar{L}_k$  - this arises in the case of strong intervention for  $j$  and weak for  $k$ . From Proposition 1 in Appendix 2 we find for the hegemony of  $k$  the modified radius is equal to the radius; for the balance of power segment the modified radius is equal to the left radius  $R_{jk}^l$  which is to say it is larger than the radius; and for the hegemony of  $j$  it is equal to the radius of the hegemony plus the difference between the left and right radius of the balance of power segment, thus again greater than the radius. In a sense the balance of power segment and hegemony of  $j$  are more likely to be seen in the long run since there will be movement back and forth between the two of them more often than there is movement to the hegemony of  $k$ . The

segment is stochastically stable - has the highest modified radius - if and only if resistances from the segment to the hegemonies are higher than hegemonic resistances.<sup>15</sup> Notice also that increasing  $\varphi_0$  decreases the radius of the hegemonies but increases one of the radii of the segment so that stronger outside intervention makes the segment more likely to be stochastically stable.

This basic picture is consistent with our observation that hegemonies are relatively common in places such as China where outside forces are weak and uncommon in places like Europe and India where they are strong.

## 6. Reasons and Sources of Intervention

While we do not model the motivation of the intervenors nor yet consider a conflict that is more than two sided, our model does shed useful insights into both of these cases. We first examine the role and motivations of intervenors, then conflicts that involve more than two parties.

### *One Intervenor or Two?*

We have not explicitly specified if outside forces are meant to represent a single country or for example two external powers with possibly opposed interests. The conclusions of the model do not depend on which configuration applies, and in reality both cases have concrete examples. As we have already mentioned, England in continental Europe has played the role of a single intervenor in favor of the weaker sides quite independently of their nationality. In the case of the United States and Russia in the Middle East the situation is obviously different, with each external power supporting a different side. It may be worth looking at what the segment result implies in this case: the result asserts that against a strong intervenor in favor of  $k$ , if you play weak  $j$  you reach a short segment while if you play strong  $j$  you produce a long segment. Is the latter outcome worse? Not necessarily. Because if the long segment in the strong/strong case is also short it is central, while a weak/strong short segment leans on one side. And the central one is likely less bloody, because the parties have less hope to prevail hence are less willing to invest in warfare, while in the segment on one side the advantaged party has victory in sight and will keep pushing more fiercely. That is, the result does not imply that you should play weak against a strong opponent intervenor.

### *Objective function and costs*

We have seen how analysis of modified radii leads to the result that, even in the presence of outside intervention, the more likely stable configuration of power is one with a hegemony - and that is what we should and we do observe more often over the very long run. Expected time to leave an absorbing set depends instead on the unmodified radius (least resistance to another absorbing set). Considering the case of a single segment for simplicity, let us ask what segments have maximum radius - hence maximum expected duration. The answer is that it has to be a

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<sup>15</sup>This follows directly from the values of the modified radii, which are respectively: for  $k$ 's hegemony  $r_k^h$ ; for the segment  $R_{jk}^l$ ; for  $j$ 's hegemony  $r_j^h + R_{jk}^l - R_{jk}^r$ . See the appendix for details.

“central” short segment. To see this first observe that the segment should be short: the radius of any longer segment weakly increases if we shrink it to minimum length, from left or right. Next, consider a short segment: if we move right, up to some point the radius will be the increasing left radius, then the left one will become larger and the radius will be the decreasing right one; the maximum radius is found at the tilting point, reached by one or two adjacent segments.

In terms of intervention policy this says that the balance of power with the longest expected duration is a cold peace brought about by sufficiently strong intervention. This is what outside powers should do if avoiding hegemony is their goal. However, intervenors may have goals besides preserving the balance of power - in particular, as seems to be the case with Europe and Russia, they may be concerned about the cost of intervention and hence not intervene in sufficient strength to bring about cold peace.

On the other side of the coin there is the cost of war in terms of human lives and population distress. It appears that a short central segment is the least costly from this point of view. First because it is short. Secondly, the further away from hegemony the conflict develops the fainter the hopes of reaching hegemony on the part of contenders. Indeed owing to this, a central short segment might be argued to be the most promising step towards a peaceful conclusion of the hostilities.

What are the benefits to the intervenors of outside intervention? One is to prevent hegemony that may be threatening. Examples of this are Britain and the balance of power on the continent and quite likely the US and USSR keeping Europe and especially Germany divided after WWII. Another is to extract economic rents - this may have been the case with the British Indian Empire - although the economic history literature seems divided on whether they managed to do so or not. Another purely economic motivation may be the desire to sell arms to the conflicting nations. Somewhat in between keeping an enemy weak and economic exploitation is the desire to extend military influence and provide a buffer against more distant rivals - for example Russia seems to have intervened in Syria in order to preserve and expand their naval base in Syria. Similar motivation may be territory for the pure sake of territory - the “manifest destiny” of the United States seems to be of this sort. Finally, domestic popularity may play a role. The Falkland Island war was driven by the lack of popularity of the Argentine junta while European intervention in Libya and Syria was also influenced by the unpopularity of Cameron and Hollande.

### *Balkanization*

Our theory deals with only two contestants: but there are parts of the world that are Balkanized in the sense of the Balkans - or perhaps today the area of Syria and Iraq - where more than two contestants vie in a sort of free-for-all. While the details are beyond this paper and can be found in Levine and Modica (2017) we can give the general idea.

In some areas such as the Balkans there is difficult terrain so that it is hard for any side to prevail. If every society has positive resistance to losing land all states will be absorbing (at  $\epsilon = 0$  as usual). Since positive resistance events do occur over long periods of time, this means that there will be recurrent conflicts. This seems a relatively good description of the Balkans, which has been Balkanized and in a more or less perpetual state of conflict since about 1200 BCE with the

exception of periods of time when strong outside powers (Rome, the Ottomans) imposed peace. Each time, with the withdrawal of the outsiders low level periodic conflict seems to have more or less immediately resumed. Most recently large parts of the Balkans have been absorbed into the EU which may play much the same role as the Romans and Ottomans in bringing peace to the region - what will happen at some future time when that power should be withdrawn we suspect will be a resumption of the old divisions.

What about the case where terrain is good? Suppose we have a balkanization of more than two societies. We continue to maintain the assumption that the weakest society has no resistance to losing land. If outsiders cannot intervene simultaneously for both of the two weakest societies then one of them must have zero resistance to losing land, and we would expect there to be zero resistance to the strongest society winning the land. Such a change does not affect which are the two weakest societies: so collectively the two weakest societies always have zero resistance to losing a unit of land until one of them vanishes. Eventually we get down to two societies and the model we have developed above.

If there is a single outside intervenor this makes perfectly good sense - it is not very practical in a multi-way conflict to consistently intervene on behalf of two different clients. However, with more than one outside intervenor it may be that each intervenes consistently on behalf of a different client. Specifically: if outside intervention consistently takes place on behalf of both of the two weakest societies and is sufficiently strong as to give both positive resistance to losing land then it is easy to construct intervention policies that preserve the balkanization: as soon as one of the two weakest societies wins a unit of land intervention is withdrawn until it loses the land again. The key point is that we can have stability in a balkanization provided that there is more than one outside intervenor against the weak parties, but it is not so likely with just one.

The Middle East is the most obvious example of a region which is generally perceived as a balkanization. Never-the-less it can be usefully analyzed by breaking it down into two sub-regional conflicts: Israel versus Egypt in the South and Sunni versus Shiite in the North. In the South Israel and Egypt have intermittently fought from 1948 (the Arab-Israeli war listed in our table) to 1973 (the Yom Kippur war) after which peace negotiations began and ended with a further cold peace in 1979. This cold peace, enforced by strong US intervention on both sides takes the form of a treaty that has largely resulted in the cessation of bloodshed. This appears to be the most desirable form of cold peace.

The situation in the North is on the other hand a story of insufficiently strong intervention and bloody, prolonged wars. We have first the Lebanese civil war, then the Iran-Iraq war, then the conquest of Kuwait, followed by the liberation, the second Iraq war, and now by the Syrian civil war. As can be seen in our table some of these conflicts can be broken into separate regions which can be usefully analyzed by our methods.

The Kurds form a particularly interesting sub-case in the North. We have assumed that the weakest power is the most likely to lose land. But in a multi-lateral conflict the larger powers may be so focused on fighting each other that a smaller power is able to survive "in the shadows" so to

speak. Originally the Kurds were able to occupy land as a consequence of the civil disorder and no fly zone that followed the Iraqi defeat in Kuwait: Saddam Hussein's Sunni forces were tied up with defeating the Shia near Basra and especially the marsh Arabs. Following second Iraq war politically the Sunni's and Shia's were more concerned with each other than with the Kurds - who also received limited US support, very limited on account of the alliance with Turkey: Indeed the official US position has always been that Kurdistan should be part of a unified Iraq.

Syria has high resistance to losing land, especially with Russian support. The other major party to the conflict in Iraq and Syria, the ISIS, appear to be zealots, unstable, having no resistance to losing land and who consequently must either win quickly or vanish forever. As they have not won quickly we expect they will shortly vanish forever except perhaps as a rump group of stateless terrorists like their predecessor Al Qaeda. ISIS land, our model predicts, can go to either Syria/Iraq or to the Kurds. Then there are two possible scenarios:

(1) The US withdraws support for Kurds. In this case at some point Kurds will lose land to Syria and Iraq and Kurdistan will vanish;

(2) The US holds its support for Kurds. In this case our model predicts a balance of power between Kurds and Syria - but reality is a little different: owing to US concern with Turkey, sustained US support for Kurds is possible only if the Kurds compromise on their requests for an independent state and commit in advance to be part of a united Syria (like the Iraqi Kurds in Iraq) when ISIS is defeated. If this is the case - and we think it indeed is the more likely scenario - the outcome may be a balance of power within the context of particular states.

## 7. Conclusion

The model we have studied sheds some light on the issues related to intervention and peace, in particular on the trade-off between having the contenders reaching peace as quickly as possible - which usually happens with the strong dominating the weak - and protecting the weak - which may prolong the conflict. We have further seen that if the goal of protecting the weak is predominant, then to minimize the costs of war intervention should be strong enough to avoid going back and forth between states where one part in turn is considerably stronger than the other, and reduce the war to what we have called a "cold peace" - which can be thought of as "border skirmishes", and hopefully end in reaching an unarmed negotiation stage.

However, much is left to understand. The great success story of peace is the de facto US occupation of Western Europe after World War II, its role in NATO and promoting the European Union and encouragement of European politicians, especially in France and Britain, that led to durable peace and democratic institutions. This was enormously costly, and this kind of peace - real peace - has lacked success elsewhere. Indeed US efforts at nation-building outside Western Europe and Japan has been an abysmal failure. The greatest success, ending the Israel-Arab conflict, has succeeded only in creating a cold peace propped up by continued and costly US intervention.

An important question is to understand why the U.S. was so successful in Europe and Japan and so unsuccessful elsewhere. Was it simply the willingness to commit resources on a massive scale - a



huge war effort, military occupation on a giant scale, money poured into reconstruction? One may say it is the cold war - the willingness to actively support Europe to counteract the Soviets - but the cold war effort to intervene in Vietnam was as colossal a failure as the European intervention was a success. Certainly understanding the success in Europe and Japan and failure elsewhere is something that needs to be understood. If it is simply a matter of resources and willingness to spend them, then perhaps the US success in Europe holds no useful lesson for peace.

In a similar vein we may wish to study earlier successful and unsuccessful attempts at nation building: for example, the British legacy in India is a stable and relatively peaceful democracy. The French legacy in their colonies is poor - and the Belgian horrific. To understand success and failure here is to understand whether or not hegemony is a good idea. From the analysis here, however, it is reasonable to conclude that intervention to prevent hegemony needs to be strong enough (or weak enough) to bring about a cold peace - intervention that brings about prolonged war cannot be good from the point of view of peace.

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## Appendix 1: Balance of Power Segments

**Lemma 1.** *In a  $j, k$  balance of power segment there are no states below the intervention thresholds, that is if  $L_j(z) < \bar{L}_j$ , or  $L_k(z) < \bar{L}_k$ , then  $z$  is not in a  $j, k$  balance of power segment.*

*Proof.* Consider  $j$ . Suppose first that  $r_j^0(\phi_j(L_j(z)), \phi_k(L - L_j(z))) > 0$ . Then there is zero resistance to  $j$  increasing land, but positive resistance to  $j$  decreasing land. Hence if  $L_j(z)$  is in a segment it could only be a left endpoint. If this is the case then at  $z'$  where  $j$  has one more unit of land there would have to be zero resistance to  $j$  losing a unit of land. This would imply that at  $L_j(z)$  there is outside help, that is  $L_j(z) = \bar{L}_j$  - hence that in the segment  $L_j \geq \bar{L}_j$ .

Next suppose that  $r_j^0(\phi_j(L_j(z)), \phi_k(L - L_j(z))) = 0$ . Then if  $z$  was part of a balance of power segment it should be  $L_j(z) > \bar{L}_j$ , where  $j$  has positive resistance to losing land. But then it cannot be  $L_j(z) < \bar{L}_j$  because below  $\bar{L}_j$  resistance of  $j$  decreases with its land.  $\square$

**Lemma 2.** *Very strong and very weak intervention on behalf of  $j$  against  $k$  are ineffective in the sense that the balance of power segments remain unchanged if we take  $\bar{L}_j = 0$ .*

*Proof.* In the case of very weak intervention the resistances are the same with or without intervention (recall that at  $L_j > \bar{L}_j$  there is no intervention) so the balance of power segments are the same.

The effect of very strong intervention is only to slow down the fall of resistance as  $j$  loses land, and it must fall to zero in any case because  $k$ 's hegemonic resistance is positive even with intervention in favor of  $j$ .  $\square$

**Lemma 3.** *There is a paired balance of power segment if and only if intervention is weak for both  $j, k$  in which case the balance of power segments are the two short segments.*

*Proof.* Suppose intervention is weak for both  $j$  and  $k$ . The two segments in the assertion, namely from  $\bar{L}_j$  to  $\bar{L}_j + 1$  going to the right and from  $\bar{L}_k$  to  $\bar{L}_k + 1$  going to the left form indeed a paired segment, directly from the definitions. From Lemma 1 we know that no state to the left of  $\bar{L}_j$  or to the right of  $\bar{L}_k$ , so what we have to show is that states between  $\bar{L}_j + 1$  and  $\bar{L}_k + 1$  do not belong to segments ("between" refers as usual to our preferred visualization). By hypothesis  $r_k = 0$  at  $L_k = \bar{L}_k + 1$ , and it will be zero up to some  $\bar{L}_k + \ell_k$ ; analogously,  $r_j = 0$  from  $\bar{L}_j + 1$  up to some  $\bar{L}_j + \ell_j$  (going left in our pictures). If  $\bar{L}_k + \ell_k$  and  $\bar{L}_j + \ell_j$  do not overlap they must be one unit of land apart (formally, if  $\bar{L}_k + \ell_k + \bar{L}_j + \ell_j < L$  their sum must be exactly  $L - 1$ ), because at the state where one resistance becomes positive the other must be zero; thus from  $\bar{L}_j + \ell_j$  down to  $\bar{L}_j + 1$  there is zero resistance, and from  $\bar{L}_j + \ell_j$  up-left to  $\bar{L}_k + 1$  too; and in the "hole" between two adjacent states there is positive resistance both ways. Therefore in this case there is no segment between the two segments. Suppose now that if  $\bar{L}_k + \ell_k$  and  $\bar{L}_j + \ell_j$  do overlap. The only difference from the previous case is that instead of the hole there is an interval between the two segments where points are linked by two-way zero-resistance; from the left end point of the interval there is zero resistance down to  $\bar{L}_j + 1$ , and from the right end point analogously up to  $\bar{L}_k + 1$ . None of these states can belong to a segment, as before.

Conversely, suppose there is a paired balance of power segment. Let us call “walls” the extremes of the segments. At the left wall of the rightmost segment  $j$ 's resistance is positive, then going down, at the right wall of the leftmost segment  $j$ 's resistance is zero (since  $k$  has positive resistance); and at the left wall of the left segment  $j$ 's resistance is positive again. Therefore here is where support for  $j$  occurs, that is the left wall of the left segment is  $\bar{L}_j$ . Similarly the right wall of the right segment is  $\bar{L}_k$ . Starting from there going down to the left wall of the same segment there is no support for  $j$  but still its resistance is positive there. If the right segment were not short, in its interior  $j$ 's resistance would have to be zero, but it is again positive at the left wall, which is impossible given there is no external support in that range. Thus the right segment is indeed the short one in the assertion, which implies intervention in support of  $k$  is weak as asserted. Notice in passing that  $j$ 's resistance would be positive at the right wall if it were not for intervention in favor of  $k$ . The argument for the left segment is the same.  $\square$

**Lemma 4.** *If intervention for  $j$  is weak and that for  $k$  is strong or there is no intervention for  $k$  there is a single short balance of power segment starting at  $\bar{L}_j$  and the intervention on behalf of  $k$  is ineffective.*

*Proof.* Existence of a short segment starting at  $\bar{L}_j$  follows as in Lemma 3 from weakness of intervention for  $j$ : at the threshold  $j$  has positive resistance (since the intervention is not very weak) and so  $k$  has zero, we can move only to the right; but when we move one unit to the right, by definition of weak intervention  $k$  has positive resistance, hence  $j$  has zero and we can move only to the left; hence as we can move back and forth only between  $\bar{L}_j$  and  $\bar{L}_j + 1$  we have a short segment. Since we know (Lemma 1) that states on the left of  $\bar{L}_j$  cannot be part of a segment, to establish uniqueness we only need to look to the right  $\bar{L}_j + 1$ .

Suppose there is no intervention for  $k$ . At  $\bar{L}_j + 1$  there is positive resistance by  $k$ , and increasing  $j$ 's land, as long as this is the case there is zero resistance to going left one step; at some point  $k$ 's resistance may become zero (so there is a hole there) and from then on to  $j$ 's hegemony  $k$ 's resistance must remain null. Thus we have no other segment.

Suppose now that intervention for  $k$  is strong. Then it must be the case that  $\bar{L}_k$  is to the right of  $\bar{L}_j + 1$ , for otherwise  $j$ 's resistance at  $\bar{L}_j$  would be zero. From  $\bar{L}_j + 1$  up to where  $L_k = \bar{L}_k$  we must have null resistance by  $j$  - since it is zero where  $L_k = \bar{L}_k$ , going left  $j$  has less land and no support in the range. And to the right of  $\bar{L}_k$  the situation is essentially as in the previous case: as we increase  $j$ 's land, the system can move one step to the left without resistance until a possible “hole” after which  $j$ 's resistance becomes positive, then right to  $j$ 's hegemony without resistance. Again there is no segment in the range.

As we have just seen the two cases yield the same stable configuration, so that intervention for  $k$  is ineffective.  $\square$

**Lemma 5.** *If intervention for  $j$  is strong and there is no intervention for  $k$  there is no recurrent balance of power segment.*

*Proof.* By Lemma 1 no state below  $\bar{L}_j$  can belong to a segment; at  $L_j = \bar{L}_j + 1$  resistance by  $k$  is zero, and absent intervention in its favor  $k$ 's resistance remains null until  $j$  reaches hegemony, by monotonicity. Conclusion follows.  $\square$

**Lemma 6.** *If intervention for both is strong then there is a single long recurrent balance of power segment from  $\bar{L}_j$  to  $L - \bar{L}_k$ .*

*Proof.* Again by Lemma 1 states with  $L_j < \bar{L}_j$  or  $L_k < \bar{L}_k$  cannot be part of a segment. By hypothesis at  $L_j = \bar{L}_j + 1$  resistance by  $k$  is zero, hence it remains zero up to where  $L_k = \bar{L}_k + 1$ , then becomes positive at  $\bar{L}_k$  (where necessarily  $j$ 's resistance is null). Similarly, going left from  $\bar{L}_k$  by hypothesis  $j$ 's resistance is zero where  $L_k = \bar{L}_k + 1$  and by monotonicity it remains zero until where  $L_j = \bar{L}_j + 1$ , then it is positive at  $L_j = \bar{L}_j$  (where  $k$ 's resistance is null). This is what we had to show.  $\square$

## Appendix 2: Modified Radii

In this appendix we compute the modified radii of hegemonies and balance of power segments. The attribute of an absorbing set that determines the relative time the process spends on it is the *modified radius*, which we are going to compute for the absorbing sets we are interested in here. The general definition is given in Levine and Modica (2016), section 6.3, we sketch here the concept to apply it to the present setting. We say that a family of absorbing sets forms a *circuit* if any two of them are connected by a least resistance path not touching absorbing sets outside the family. The family of all absorbing sets of the process can be partitioned into circuits, and then one defines circuits of circuits - 2nd-order circuits we may say - by taking as modified resistance the incremental resistance needed to move from one to the other over that needed to move within the circuit. Continuing this way one gets to 3rd-order circuits and so on (modified resistance becomes accordingly increments of increments etc.) until all circuits of order  $K - 1$  form a single circuit of order  $K$ . The modified radius of an absorbing set is computed as the least total modified resistance needed to move out of the  $k$ -order circuits in which the absorbing set is contained, all the way up to the order  $K - 1$ . We now apply this procedure starting with a particularly simple case.

### *The case of rightward cascades*

We say that a path between two absorbing sets is *direct* if it does not touch other absorbing sets along the way and has finite resistance. A *rightward cascade*  $x_1 \rightarrow x_2 \dots \rightarrow x_n$  is a sequence of absorbing sets such that the only direct paths from  $x_i$  to other absorbing sets lead to the adjacent  $x_j$ 's. Denote the resistance of going back to  $x_{i-1}$  by  $R_k^l$  and of going forward by  $R_i^r$  (the left and right radii). To be a rightward cascade we require that  $R_i^r < R_i^l$  for  $i \neq 1, n$ , and we define the difference  $\Delta_i = r_{il} - r_{ir}$  which is by assumption strictly positive. In this case the hierarchy of circuits and the modified radii are as follows. The only non singleton circuit is given by  $\{x_{n-1}, x_n\}$ ; then one order up we find the 2-nd order circuit  $\{x_{n-2}, \{x_{n-1}, x_n\}\}$ , all the others being singletons. Proceeding this way we arrive at the two-element partition  $\{x_1, \{x_2, \{x_3 \dots \{x_{n-1}, x_n\}\} \dots\}$  after

which we have a single circuit. So from  $x_1$  we only need to move right to  $x_2$ , while from all the others we need to move up to  $x_1$ . For example for  $n = 4$  we have  $\{x_1, \{x_2, \{x_3, x_4\}\}\}$ ; to get out from  $x_1$  we only need  $R_1^r$ ; from  $x_2$  we need  $R_2^l$  to move out to  $x_1$  (notice that this is incremental as well since there is zero resistance to move within the singleton circuit formed by  $x_2$  alone); from  $x_3$  we need  $R_3^l$  to step out of the circuit to  $x_2$  and then the incremental resistance of moving from  $x_2$  to  $x_1$  which is  $\Delta_2$  (for the resistance of moving from  $x_2$  within this 2nd-order circuit circuit is  $R_2^r$ ), so the modified radius is  $R_3^l + \Delta_2$ ; from  $x_4$  we need to first reach  $x_3$  to get out of the circuit plus the increments up to  $x_1$ , so the modified radius is  $R_4^l + \Delta_3 + \Delta_2$ . The general rule is then as stated in the following

**Proposition 1.** *The modified radius of  $x_i$ , denoted by  $M_i$ , is given by  $M_i = R_i^l + \sum_{1 < k < i} \Delta_k$  for  $i > 1$ , while  $M_1 = R_1^r$ .*

### *Cascade with One Balance of Power Segment*

It is convenient to denote the hegemony of  $k$  by  $H$  that of  $j$  by  $h$  and the balance of power segment by  $B$  in the following. Notice that in this case we must have a cascade and we may assume without loss of generality the cascade is to the right  $H \rightarrow B \rightarrow h$ . Apply Proposition 1 to find,  $M_H = R_H^r$ ,  $M_B = R_B^l$ ,  $M_h = R_h^l + (R_B^l - R_B^r)$ .

### *Two Balance of Power Segments*

#### *case 1: Cascade*

Let  $B$  denote the segment on the left and  $b$  the one on the right. Assume without loss of generality the cascade is to the right:  $H \rightarrow B \rightarrow b \rightarrow h$ . Apply Proposition 1 to find  $M_h = R_h^l + (R_b^l - R_b^r) + (R_B^l - R_B^r)$ ,  $M_b = R_b^l + (R_B^l - R_B^r)$ ,  $M_B = R_B^l$ ,  $M_H = R_H^r$ .

*Comments.* We have  $M_b = M_B + R_b^l - R_B^r$ ; if  $R_b^l > R_B^r$  as we may suppose, then  $M_b > M_B$ . Also  $M_h = R_h^l - R_b^r + M_B + R_b^l - R_B^r = R_h^l - R_b^r + M_b$ ; hence, assuming  $R_b^l > R_B^r$  that is  $M_b > M_B$ , then  $M_h \geq M_b \iff R_h^l \geq R_b^r$ . So if  $R_h^l > R_b^r$  the stable set is a hegemony. Otherwise  $b$  is stable if  $R_H^r$  is low enough.

#### *case 2: Two Circuits*

Then both balance of power segments point at the adjoining hegemony and the circuits are  $\{H, B\}, \{b, h\}$ . At the higher order both form a single circuit. The incremental resistances to reach the other circuit are  $R_B^r - R_B^l$  and  $R_b^l - R_b^r$  so the modified radii are  $M_H = R_H^r + R_B^r - R_B^l$ ,  $M_B = R_B^r$ ,  $M_b = R_b^l$ ,  $M_h = R_h^l + R_b^l - R_b^r$ .

#### *case 3: Balance of Power Segments Point at Each Other - Three Circuits $H, \{B, b\}, h$*

In the circuit the radii are  $R_B^r, R_b^l$  and the incremental resistance of going to  $H$  is  $R_B^l - R_B^r$ , while that of going to  $h$  is  $R_b^r - R_b^l$ . Suppose without loss of generality  $R_b^r - R_b^l < R_B^l - R_B^r$  so that the 2nd-order circuit is  $\{\{B, b\}, h\}$ . The modified radius of  $H$  is just  $M_H = R_H^r$ . For  $B$  there is the resistance of going to  $h$  which is  $R_B^r + (R_b^r - R_b^l)$  - path to  $b$  then incremental resistance to reach  $h$ ; then the modified resistance of going from this to  $H$ , which is the difference between incremental resistance

of going to  $H$  and to  $h$  (incremental with respect to remaining in  $\{B, b\}$ , that is  $R_B^l - R_B^r - (R_b^r - R_b^l)$ ). So the modified radius of  $B$  is  $M_B = R_B^r + (R_b^r - R_b^l) + (R_B^l - R_B^r - (R_b^r - R_b^l)) = R_B^l$ . Since the difference of the modified radius of  $b$  with the modified radius of  $B$  must be  $R_b^l - R_B^r$  both being in the same circuit we have  $M_b = R_b^l + (R_B^l - R_B^r)$ . For  $h$ : we must first get to  $\{B, b\}$  - which costs  $R_h^l$ ; then there is the step to  $H$ , whose modified resistance we have already calculated; thus  $M_h = R_h^l + (R_B^l - R_B^r) - (R_b^r - R_b^l)$ . An alternative, sleeker derivation of  $M_h$ : the modified radius is the cost of going to  $H$  from  $b$ , that is  $R_b^l + (R_B^l - R_B^r)$ , plus the extra cost of starting from  $h$ , which is the difference  $R_h^l - R_b^r$  between the radii of  $h$  and  $b$  in the 2nd-order circuit  $\{\{B, b\}, h\}$ ; the result is  $M_h = R_b^l + (R_B^l - R_B^r) + R_h^l - R_b^r$  which is the same value as before.

*Comments* (assuming supercircuit  $H, \{\{B, b\}, h\}$ ). Let us suppose  $R_b^l \approx R_B^r$  so that  $M_B \approx M_b$ . Next,  $M_H \geq M_B \iff R_H^r \geq R_B^l$ ; and  $M_h = M_b + R_h^l - R_b^r$  so  $M_h \geq M_b \iff R_h^l \geq R_b^r$ . Stability is like in the single segment case: if hegemonic resistances are higher than resistance from segments to hegemony then a hegemony is stable; otherwise the stable set is a segment (or both).

### Appendix 3: Stochastic Stability of Hegemony and Balance of Power

**Theorem.** *[Theorem 3 in the text] There exist  $\infty \geq \bar{\varphi}_0 \geq \underline{\varphi}_0 > 0$  such that if  $\varphi_0 > \bar{\varphi}_0$  and if intervention thresholds are positive on both sides there are stochastically stable balance of power segments but not stochastically stable hegemonies, while if  $\varphi_0 < \underline{\varphi}_0$  there are stochastically stable hegemonies but not stochastically stable - or even absorbing - balance of power segments.*

*Proof.* Hegemonic resistance is decreasing in  $\varphi_0$  and by assumption positive for  $\varphi_0 = 0$ . At  $\varphi_0 = 0$  there are no balance of power segments with positive radius and by continuity this is the case for all sufficiently small  $\varphi_0$ . Hence there is  $\underline{\varphi}_0$  below which there are stochastically stable hegemonies but not balance of power segments.

Hegemonic resistance by assumption falls to zero for all sufficiently large  $\varphi_0$  while for sufficiently large  $\varphi_0$  the resistance to passing an intervention threshold is positive. Hence there is a  $\bar{\varphi}_0$  above which there are stochastically stable balance of power segments but not hegemonies.  $\square$