Monopolistic Competition when Income Matters

Paolo Bertoletti and Federico Etro[1]

University of Pavia and Ca' Foscari University of Venice

November 2013

Keywords: Monopolistic competition, Indirect additivity, Non-homotheticity, Pricing to market, Krugman model, Melitz model
JEL Codes: D11, D43, L11, F12

Abstract

We study monopolistic competition with preferences over differentiated goods characterized by a separable indirect utility rather than a separable direct utility as in the Dixit-Stiglitz model, with the CES case as the only common ground. Examples include linear and log-linear direct demands. In equilibrium with free entry, an increase of the number of consumers is neutral on prices, but increases proportionally the number of firms, just creating pure gains from variety. Contrary to the Dixit-Stiglitz model, an increase in consumer income increases prices and more than proportionally the number of varieties if and only if the price elasticity of demand is increasing. We also discuss extensions to an outside good, heterogeneous consumers, heterogeneous firms *à la* Melitz and endogenous quality. Finally, we provide an application to international trade generating pricing to market in a generalized Krugman model.

[1]We are grateful to Daron Acemoglu, Simon Anderson, Paolo Epifani, Gene Grossman, James Markusen, Volker Nocke, Mario Padula, Ina Simonovska, Yossi Spiegel, Jacques Thisse, Kresimir Zigic, three anonymous referees and especially Patrick Rey. Seminar participants at the 2013 Cresse Conference in Corfù, the HSE Center for Market Studies and Spatial Economics in St. Petersburg, the University of Pavia and Ca' Foscari University provided insightful comments. *Correspondence.* Paolo Bertoletti: Dept. of Economics and Management, University of Pavia, Via San Felice, 5, I-27100 Pavia, Italy. Tel: +390382986202, email: *paolo.bertoletti@unipv.it.* Federico Etro: Dept. of Economics, University of Venice Ca' Foscari, Sestiere Cannaregio, 30121, Fond.ta S.Giobbe 873, Venice, Italy. Tel: +390412349172, email: *federico.etro@unive.it.*

The theory of monopolistic competition introduced by Chamberlin (1933) analyzes markets in which a large number of monopolistic firms choose prices independently and entry is free. The formalization proposed by Dixit and Stiglitz (D-S, 1977: Section I), based on Constant Elasticity of Substitution (CES) preferences over differentiated goods, has become a workhorse model in modern economics. As well known, it implies constant markups and an endogenous number of firms that is proportional to both the number of consumers and their income (individual expenditure). Moreover, under firm heterogeneity, CES preferences imply that the market size has no selection effects on the efficiency of the active firms. These features have key consequences, for instance for the structure of and the gains from trade (Krugman, 1980 and Melitz, 2003) and for macroeconomics (see Blanchard and Kiyotaki, 1987 and Bilbiie *et al.*, 2012).

From an empirical point of view, however, the CES model has some drawbacks. Primarily, it cannot account for the variability of markups across countries and trade conditions, or over the business cycle. There is indeed a consistent evidence that markups are higher in richer countries (see Alexandria and Kaboski, 2011 and Fieler, 2012), and there is also some evidence that they are variable over the business cycle (for instance, Nekarda and Ramey, 2013, make a case for procyclical markups). Although the empirical analysis of the impact of market size on prices under monopolistic competition has rarely distinguished income and population effects, a recent work by Simonovska (2013) studies international pricing of traded goods (online sales of identical goods shipped abroad) controlling separately for country population and income effects: she estimates an elasticity of prices to *per capita* income between 0.05 and 0.11, but does not find a significant impact of population on prices.[2]

To account for these facts under monopolistic competition one has to depart from homothetic preferences. The general version of the additively separable *direct* utility function of D-S (1977, Section II) can be used as a source of variable markups (Krugman, 1979). However, it generates prices that can either decrease or increase in the number of consumers (Zhelobodko *et al.*, 2012), implying an ambiguous impact of trade integration on welfare and ambiguous selection effects under firm heterogeneity (see Dhingra and Morrow, 2012, and Bertoletti and Epifani, 2012). Moreover, in spite of non-homotheticity, the D-S model with free entry generates always a (rarely recognized in the literature) neutrality of the market structure with respect to income: markups and firm selection are unaffected by changes in consumers' expenditure. In this paper we propose an alternative model of monopolistic competition based on a different class of preferences, and argue that it can easily account for the stylized facts outlined above.

We assume that consumers' preferences can be represented by an additively separable *indirect* utility function. Such "indirect additivity" amounts to assume that the relative demand of two goods does not depend on the price of

[2] It is well known that strategic interactions in concentrated markets can rationalize a competition effect due to an increase in the number of consumers, and the empirical IO literature has shown this (see Campbell and Hopenhayn, 2005). However, these strategic effects ought to disappear in markets characterized by many firms and monopolistic competition.

other goods, while it depends on income unless preferences are homothetic. It is thus different from the "direct additivity" exploited by D-S, for which the marginal rate of substitution between any two goods does not depend on the consumption of other goods. In fact, duality theory (Hicks, 1969; Samuelson, 1969; Blackorby *et al.,* 1978) tell us that the case of CES preferences is the only common ground (which requires homotheticity) of direct and indirect additivity. An implication of indirect additivity is that the number of goods provided in the market does not affect their substitutability and thus the price elasticity of demand, while income can affect this elasticity with crucial consequences. By contrast, under direct additivity substitutability and thus demand elasticity are unaffected by the number of goods conditionally on the size of consumption.

Monopolistic competition with indirect additivity generalizes the neutrality of the number of consumers on the production structure which emerges in CES models, thereby yielding pure gains from varieties as in Krugman (1980), as we confirm in a two-country version of our model.[3] Moreover, we obtain markups that are variable in income/spending, with two appealing consequences. First, pricing to market emerges as a natural phenomenon: as long as demand is less elastic for richer consumers, we have higher markups in markets with higher individual income. Second, markups vary cyclically when demand shocks affect income/spending or supply shocks affect marginal costs (i.e., firms' productivity). Similar results hold in a two-sector extension with an outside good representing the rest of the economy, when consumers are heterogeneous in preferences and income, and even when firms differ in productivity *à la* Melitz (2003).

The comparative statics for business creation is also of interest. Consider the case where demand gets less elastic with income. Richer consumers induce firms to increase their markups, which triggers more than proportional entry of firms in the market. When firms are heterogeneous, this establishes a Darwinian mechanism that is absent in the Melitz model with CES preferences: less productive firms enter in booms (when income increases) and exit during downturns (a sort of "cleansing effect" of recessions). Finally, if firms can invest in the quality of their products, then, taking advantage of larger market shares, more productive firms tend to react to an increase in consumers' income by offering products of higher quality sold at higher prices.[4]

The work is organized as follows. In Section 1 we present our baseline model. In addition to characterizing the endogenous entry equilibrium, we discuss indirect versus direct additivity and present analytically workable examples of the former (leading to linear and loglinear demand) for which we recover the underlying direct utility functions. In Section 2 we extend the model in various directions and provide a welfare analysis. In Section 3 we apply our framework to an international trade model *à la* Krugman considering both costless trade between different countries and costly trade between identical countries. We conclude in Section 4. All the proofs are in the Appendix.

[3] On the importance of non-homotheticity in trade models see Markusen (2013).

[4] This is consistent with the so-called *Linder hypothesis*. For recent empirical support see Kugler and Verhoogen (2012).

1 The Model

Consider a market populated by (a number or mass of) L identical agents with income $E > 0$ to be spent in a mass of n differentiated goods under the following symmetric and separable *indirect* utility function:

$$V = \int_0^n v\left(\frac{p_j}{E}\right) dj, \tag{1}$$

where $p_j > 0$ is the price of variety j.[5] The expression on the RHS of (1) exploits the property of homogeneity of degree zero of the indirect utility, and crucially assumes additive separability, i.e. "indirect additivity". To satisfy sufficient conditions for (1) being an indirect utility function (while allowing for a possibly finite choke-off price $\overline{s}$ and obtaining well-behaved demand functions), we assume that the indirect sub-utility $v(s)$ is at least thrice differentiable, with $v(s) > 0$, $v'(s) < 0$ and $v''(s) > 0$ for any $s < \overline{s}$, $v(s) = 0$ for $s \geq \overline{s}$, and $\lim_{s\to\overline{s}} v(s), v'(s) = 0$. These assumptions imply that demand and extra utility are zero for a good that is not consumed.

The *Roy identity* provides the following direct individual demand function for good i:

$$x_i(p_i, E, \mu) = \frac{v'\left(\frac{p_i}{E}\right)}{\mu}, \tag{2}$$

where

$$\mu = \int_0^n v'\left(\frac{p_j}{E}\right)\frac{p_j}{E} dj. \tag{3}$$

This generates total market demand $q_i = x_i(p_i, E, \mu)L$. Notice that $\mu < 0$ is the negative of the marginal utility of income, *times* the income level E.

Examples of (1) include simple cases such as the isoelastic function $v(s) = s^{1-\theta}$ with $\theta > 1$, the exponential function $v(s) = e^{-\tau s}$ with $\tau > 0$, or the "addilog" function $v(s) = (a - s)^{1+\gamma}$ with $a, \gamma > 0$.[6] Note that preferences are homothetic only if $v(s)$ is isoelastic. Indeed, in such a case they are of the CES type, with indirect utility $V = E\left(\int_j p_j^{1-\theta} dj\right)^{1/(1-\theta)}$, where θ is the elasticity of substitution. By an important duality result (see Hicks, 1969; Samuelson, 1969; Blackorby *et al.,* 1978), CES preferences is the only class of preferences satisfying both direct and indirect addivitity. That is, it is the only case in which preferences can also be represented by an additively separable *direct* utility function as the one assumed by D-S, $U = \int_0^n u(x_j)\, dj$ for any well-behaved subutility $u(\cdot)$. Therefore, the indirect utility (1) encompasses a different class of (non-homothetic) preferences whose corresponding direct utility functions are non-additive (more on this in Section 1.2).

[5] Using the wage as *numeraire*, E can be interpreted as the labor endowment of each agent (in efficiency units).

[6] Here the choke-off price $\overline{s} = a$ can be made arbitrary large. Other examples are generalizations of the isoelastic function such as $v(s) = (s + b)^{1-\vartheta}$, with $\vartheta > 1$, or "mixtures" such as $v(s) = s^{1-\alpha} + s^{1-\vartheta}$ with $\vartheta \neq \alpha > 1$.

Suppose now that each variety is sold by a firm producing with constant marginal cost $c > 0$ and fixed cost $F > 0$. Accordingly, the profits of firm i can be written as:

$$\pi(p_i, E, \mu) = \frac{(p_i - c)v'\left(\frac{p_i}{E}\right)L}{\mu} - F. \tag{4}$$

Notice that μ is unaffected by the price choice of firm i. The most relevant implication of this functional form is that the elasticity of the *direct* demand corresponds to the (absolute value of the) elasticity of $v'(\cdot)$, which we define as

$$\theta(s) \equiv -\frac{v''(s)s}{v'(s)} > 0.$$

θ depends on the price as a fraction of income, p_i/E, but is independent from μ and L. Instead, in the D-S case, the elasticity of *inverse* demand is uniquely determined by the consumption level.[7] This difference will be crucial for the Chamberlinian analysis of monopolistic competition with free entry because market adjustments (needed to restore the zero-profit condition) take place through shifts of demand due to changes in the mass of firms, which affect the marginal utility of income.

1.1 Equilibrium under monopolistic competition

Any firm i maximizes (4) with respect to p_i. The FOC is:

$$v'\left(\frac{p_i}{E}\right) + \frac{(p_i - c)v''\left(\frac{p_i}{E}\right)}{E} = 0, \tag{5}$$

which requires that (locally) $v''(s)s + v'(s) > 0$, or equivalently $\theta(s) > 1$. Moreover the SOC requires $2\theta(s) > \zeta(s)$, where $\zeta(s) \equiv -v'''(s)s/v''(s)$ is a measure of demand curvature. Notice that $\theta'(s)s/\theta(s) = \theta(s) + 1 - \zeta(s)$, therefore $\theta' > 0$ if and only if $\theta > \zeta - 1$, in which case the demand becomes more elastic when the price goes up or income goes down.[8]

The FOC (5) can be rewritten as follows for the equilibrium price p^e:

$$\frac{p^e - c}{p^e} = \frac{1}{\theta\left(\frac{p^e}{E}\right)}, \tag{6}$$

where the familiar expression for the Lerner index equates the inverse of our expression for demand elasticity.[9] The pricing rule (6) shows that under indirect additivity the profit maximizing price is always independent from the mass of varieties supplied, because the latter does not affect the elasticity of demand.

[7] In the general D-S model the (individual) *inverse* demand of variety i is given by $p_i(x_i) = u'(x_i)/\lambda$, where λ is the marginal utility of income.

[8] If demand is (locally) concave ($v''' > 0$) the SOC is always satisfied and $\theta' > 0$. On the contrary, if demand is convex ($v''' < 0$) we may have $\theta' < 0$.

[9] To guarantee the existence of a solution to (6) we assume that $\overline{s}E > c$ (so that consumer willingness to pay is large enough) and that $\lim_{s \to \overline{s}} \theta(s) > \overline{s}E/(\overline{s}E - c)$. Notice that the SOC guarantees uniqueness of the equilibrium.

On the contrary, the optimal price grows with income if firms face a less elastic demand and *vice versa*, which provides a demand-side rationale for markups that are variable across markets (or over the business cycle). Consider the realistic case of $\theta' > 0$: then, the model is consistent with typical forms of pricing-to-market, i.e., the same good should be sold at a higher price in richer (or booming) markets.[10] Similarly, under the same assumption a change in the marginal cost is transmitted (pass-through) to prices in a less than proportional way (undershifting). Summing up, we have:

PROPOSITION 1. *Under indirect additivity and monopolistic competition the equilibrium prices are independent from the mass of active firms; they increase in the income of consumers, and less than proportionally in the marginal cost, if and only if the demand elasticity is increasing in the price.*

Since by symmetry the equilibrium profit is the same for all firms, and it is decreasing in their mass, we can characterize the endogenous market structure through the zero profit condition $(p-c)EL/np = F$. This and the pricing rule (6) jointly deliver the free-entry mass of firms and the production size of each firm:

$$n^e = \frac{EL}{F\theta\left(\frac{p^e}{E}\right)}, \qquad q^e = F\frac{\theta\left(\frac{p^e}{E}\right)-1}{c}. \tag{7}$$

The following proposition summarizes the comparative statics for n^e:

PROPOSITION 2. *Under indirect additivity, in a monopolistic competition equilibrium with endogenous entry the mass of firms increases proportionally with the number of consumers; it increases more than proportionally with the income of consumers and decreases with the marginal cost if and only if the demand elasticity is increasing in the price.*

As a corollary, the equilibrium production of each firm q^e in (7) is does not depend on the number of consumers, and it decreases with individual income and increases with the marginal cost if and only if the demand elasticity is increasing. To understand these comparative statics and their applications, it is convenient to think of changes in L as changes in the scale of the economy, of changes in E as demand shocks on the disposable income of consumers and of changes in c as supply shocks to firms' productivity. First of all, the impact of an increase in the number of consumers is always the same as under CES preferences: a larger scale of the market does not affect prices and production per firm, but simply attracts more firms without inducing any external effect on the market structure. This neutrality result and its key implications for the Krugman (1980) model extend from CES preferences to the entire class described by (1).[11]

An increase in the income/spending of consumers has more complex implications. Consider the realistic case where higher income makes demand more rigid

[10] However, it is immediate to verify that p^e/E is always decreasing in income.

[11] As an immediate consequence, increasing the population just induces gains from variety. This is a remarkable difference compared to the D-S model, where the existence of gains from variety can be guaranteed only when the equilibrium price is decreasing in the population (see Zhelobodko *et al.*, 2012, and Dhingra and Morrow, 2012).

$(\theta' > 0)$: then, a positive demand shock induces firms to increase their markups and to reduce sales accordingly, which in turn promotes business creation and increases the equilibrium number of active firms in a more than proportional way. Finally, consider an increase in firms' productivity associated with a reduction of the marginal cost (still assuming $\theta' > 0$): lower costs are translated less than proportionally to prices, which increases the markups and triggers additional entry. Accordingly, and contrary to what happens with CES preferences, our more general model allows demand and supply shocks to generate additional processes of business creation/destruction. This should alter the dynamics of macroeconomic models with endogenous entry (see Etro and Colciago, 2010, and Bilbiie *et al.*, 2012).

It is important to emphasize the differences of our setting with the general D-S model under non-homothetic preferences. Its free entry equilibrium can be summarized as follows:

$$\frac{p^e - c}{p^e} = r\left(\frac{q^e}{L}\right), \quad n^e = \frac{ELr\left(q^e/L\right)}{F} \quad \text{and} \quad q^e = \frac{F\left[1 - r\left(q^e/L\right)\right]}{cr\left(q^e/L\right)}, \tag{8}$$

where $r(x) = -u''(x)x/u'(x)$ is what Zhelobodko *et al.* (2012) call the "relative love for variety." Here both the price and the quantity do depend on the population L, which in turn affects non-linearly the number of firms: the exact impact depends on the sign of $r'(x)$. A more surprising result (hardly noticed in the literature) is that the equilibrium price and firm size are independent from income E (not only with CES).[12] In the general D-S model, free entry eliminates any impact of income in spite of non-homotheticity, and markups cannot be affected by changes in consumer spending over the business cycle.

In conclusion, we remark that our microfoundation of demand can be applied to the case of a (finite) small number of firms to analyze Bertrand or Cournot competition.[13] Then, a standard competition effect emerges: in particular, a larger or richer market attracts new firms, which intensifies competition and reduces the markups. As a consequence, the production of each firm increases and the equilibrium number of firms increases less than proportionally with the market size. This would match the evidence emphasized in the empirical literature on entry in concentrated markets (see Campbell and Hopenhayn, 2005, and the application to trade in Etro, 2013).

1.2 Examples and primal utility functions

Our results can be illustrated in simple examples with closed form solutions. For instance, consider the exponential function $v(s) = e^{-\tau s}$, which generates

[12] The reason of the different results is rooted in the market adjustment process. Since the profit expression with direct additivity is $\pi = \left(u'\left(x\right)/\lambda - c\right)Lx - F$, where $\lambda = \int_j u'\left(x_j\right)x_j dj/E$, there is a unique (symmetric) equilibrium (zero-profit) value of $\lambda = \left(nu'(x)x\right)/E$. On the contrary, under indirect additivity, there is a unique equilibrium value of $L/\mu = LE/\left[nv'\left(p/E\right)p\right]$.

[13] The strategic games generated under indirect additivity belong to the general class of aggregative games analyzed by Acemoglu and Jensen (2011) with fixed number of firms and Anderson *et al.* (2012) with endogenous entry.

the log-linear demand $q_i = const \cdot e^{-\tau p_i/E}$. The free-entry equilibrium reads as:

$$p^e = c + \frac{E}{\tau}, \quad n^e = \frac{E^2 L}{F(c\tau + E)}, \quad q^e = \frac{F\tau}{E}. \tag{9}$$

Another example is based on the addilog case $v(s) = (a-s)^{1+\gamma}$, that delivers the linear demand $q_i = const \cdot (a - p_i/E)$ when $\gamma = 1$. In general this leads to:

$$p^e = \frac{\gamma c + aE}{1+\gamma}, \quad n^e = \frac{(aE - c)\,EL}{F\,(aE + \gamma c)}, \quad q^e = \frac{F(1+\gamma)}{aE - c}. \tag{10}$$

In both examples $\theta' > 0$: therefore higher income makes demand more rigid, which leads firms to increase their prices and reduce their production, with a more than proportional increase in the number of firms. In addition, a marginal cost reduction is not fully translated on prices, which attracts more business creation and has a limited impact on firm size.

The Roy identity (2) can be used to recover the inverse demand $p_i(x_i, E, \mu) = Ev'^{-1}(\mu x_i)$ for each variety i. Employing the budget constraint $\int_j p_j x_j dj = E$, we obtain that μ is implicitly defined by $1 \equiv \int_j v'^{-1}(\mu x_j)\, x_j dj$. Simple expressions for μ arise if $v'^{-1}(\cdot)$ is homogenous or logarithmic (up to a linear transformation). This is the case of our two examples, where closed-form solutions are available. For the log-linear demand we obtain:

$$p_i(x_i, E, \mu) = \frac{E}{\tau}\left[\ln(-\tau/\mu) - \ln x_i\right] \quad \text{where } \mu = -\tau \exp\left(-\frac{\tau + \int_0^n x_j \ln x_j dj}{\int_0^n x_j dj}\right),$$

and in the addilog example we have:

$$p_i(x_i, E, \mu) = E\left[a - x_i^{1/\gamma}\left(\frac{-\mu}{1+\gamma}\right)^{1/\gamma}\right] \quad \text{where } \mu = -(1+\gamma)\left[\frac{a\int_0^n x_j dj - 1}{\int_0^n x_j^{\frac{1+\gamma}{\gamma}} dj}\right]^{\gamma}.$$

In both cases μ depends on two simple aggregators of the consumption levels.[14]

We can finally recover the primal of our indirect utility function by plugging the inverse demand in the indirect utility (1).[15] In general, we have:

$$U = \int_0^n v\left(v'^{-1}(\mu x_j)\right) dj \equiv \int_0^n u\left(|\mu x_j|\right) dj \quad \text{with} \quad 1 = \int_0^n v'^{-1}\left(\mu x_j\right) x_j dj \tag{11}$$

where the "subutility" u for each good is increasing in its consumption level. In spite of an "additive" functional form, this is not a small deviation from the D-S model because preferences are not directly separable: (11) shows that the marginal rate of substitution between two varieties is affected by the consumption

[14] Of course, our previous results could be re-derived by assuming that each firm i chooses its production level x_i to maximize $\pi_i = (p_i(x_i, E, \mu) - c)\, Lx_i - F$.

[15] Standard results ensure that, under our assumptions, preferences represented by (1) can be also represented by a well-behaved direct utility: see Blackorby *et al.*, (1978, Section 2.2.1).

of all the others through μ. In our two examples we reach:[16]

$$U = \int_0^n x_j dj \exp\left(-\frac{\tau + \int_0^n x_j \ln x_j dj}{\int_0^n x_j dj}\right) \quad \text{and} \quad U = \frac{\left(a\int_0^n x_j dj - 1\right)^{1+\gamma}}{\left(\int_0^n x_j^{\frac{1+\gamma}{\gamma}} dj\right)^{\gamma}}.$$

1.3 Alternative models of monopolistic competition

To clarify the role of the assumptions on preferences in monopolistic competition, it is important to understand that they affect the substitutability among varieties, and that it is the elasticity of substitution[17] which ultimately determines demand elasticity in a symmetric equilibrium (see also Bertoletti and Epifani, 2012).[18] In particular, indirect additivity amounts to assume that the optimal *consumption ratio* of any two goods i and j, x_i/x_j, does not depend on the price of any other good (notice that a similar assumption is implicit in empirical Logit models). This implies that in case of a common price $p_i = p_j$ the elasticity of substitution between varieties i and j does not depend either on the number of the other goods or on their prices, while it might depend on income. Instead, under direct additivity of preferences, it is the marginal rate of substitution between any two goods $u'(x_i)/u'(x_j)$ which is independent from the consumption of the other goods, leading to the property that their inverse *price ratio*, p_i/p_j, is independent from the quantities of the other goods consumed. As an implication, the elasticity of substitution between varieties i and j in the case of a common consumption level $x_i = x_j$ depends only on this consumption level.

More generally, notice that in models of monopolistic competition it is standard to assume that preferences, including non-separable preferences, are symmetric with respect to the goods.[19] In all cases, the optimal pricing rule is determined by the elasticity of demand, whose symmetric equilibrium value must be a function of the (common) price-income ratio and of the number of varieties, say $\theta^e(p/E, n)$. Obviously, alternative assumptions on preferences have different implications for θ^e. As well-known, CES preferences imply that θ^e is constant. Indirect additivity implies that $\theta^e(p/E)$ does not depend on the number of goods provided by the market: in other words, the introduction of a new variety does not affect the substitutability between any of the existing goods.[20] The D-S hypothesis of direct additivity implies that $\theta^e(np/E)$ depends on the product of the price-income ratio and the number of goods, as it is clear

[16] Another example generating closed form solutions arises if $v(s) = (s+b)^{1-\vartheta}$, with $\vartheta > 1$. Here the equilibrium price is $p^e = (\vartheta c + bE)/(\vartheta - 1)$ and the direct utility is $U = \left(\int_j x_j^{(\vartheta-1)/\vartheta} dj\right)^{\vartheta/(\vartheta-1)} \left(1 + b\int_j x_j dj\right)^{-1}$. Notice that $\theta' \gtrless 0$ if $b \gtrless 0$.

[17] The elasticity of substitution between goods i and j is a logarithmic derivative of x_i/x_j with respect to p_i/p_j: see Blackorby and Russell (1989) for a formal discussion of the concept.

[18] Mrázová and Neary (2013) investigate how the assumptions on demand determine the relevant comparative statics properties.

[19] A notable exception is D-S (1977: Section III).

[20] This result may hold beyond indirect additivity. Consider the following non-separable

from (8) after noticing that in a symmetric equilibrium $x = E/np$: in other words, θ^e is equally sensitive to changes in p/E or n. Notice that quasilinear preferences (linear with respect to an outside good) and homothetic preferences produce an equilibrium elasticity which is independent from income, $\theta^e(p, n)$. This is the case of the quadratic preferences adopted by Melitz and Ottaviano (2008), where θ^e increases with respect to both its arguments, and of the Logit demand system (see Anderson *et al.*, 2012), where θ^e is actually independent from n. At the other extreme are models in which demand elasticity does not depend on the price-income ratio but depends on the number of varieties, as under the homothetic Translog preferences employed by Feenstra (2003), where θ^e increases with n. Which one of these preferences is more plausible remains a key issue for future empirical research.

2 Extensions

2.1 Outside good and optimum product diversity

In this section we extend the model to an outside good representing the rest of the economy, as in many general equilibrium models with two sectors. Let us consider a sector producing a homogenous good under perfect competition and constant returns to scale, and a sector producing differentiated goods under monopolistic competition. We follow D-S (1977: Section II) and adopt an indirect utility that has an intersectoral Cobb-Douglas form:[21]

$$V = \left(\frac{E}{p^0}\right)^{\gamma} \left(\int_0^n v\left(\frac{p_j}{E}\right) dj\right)^{1-\gamma}, \tag{12}$$

where p^0 is the price of the outside good and $\gamma \in [0, 1]$: clearly (12) collapses to (1) for $\gamma = 0$. In the Appendix we show that the pricing rule for the differentiated goods remains the same as in (6), but the equilibrium mass of firms also depends on the elasticity of the indirect sub-utility v, defined as $\eta(s) \equiv -v'(s)s/v(s) > 0$, which reflects the relative importance of the differentiated goods.

symmetric indirect utility function:

$$V = \int_0^n (a - s_j)^2 dj - \frac{1}{n}\left[\int_0^n s_j dj\right]^2.$$

By Roy identity we obtain the demand function:

$$q_i = const \cdot [aE - (p_i - \bar{p})]$$

where $\bar{p} = \int_j p_j dj$ is the average price. This is essentially the functional form used in the textbook of Krugman *et al.* (2012, Ch. 8) to introduce monopolistic competition. However, here we have $\theta^e = p/aE$ and in equilibrium we obtain the price $p^e = c + aE$, which is increasing in income but independent from the number of consumers, with $n^e = aE^2L/F(c + aE)$.

[21] A general specification of intersectoral preferences would not change the pricing rule, but the equilibrium number of firms would not necessarily be linear in the number of consumers. Notice that the Cobb-Douglas model can be also reinterpreted as a two-period model where young agents have income E to be spent in the homogenous good or saved to consume the differentiated goods when old, with discount factor $(1 - \gamma)/\gamma$ and a zero interest rate.

PROPOSITION 3. *In a Cobb-Douglas two-sector economy with indirect additivity and monopolistic competition with endogenous entry in the differentiated sector, an increase in the number of consumers is neutral on prices and increases linearly the mass of firms, but higher income increases prices if and only if the demand elasticity is increasing in the price.*

It is interesting to evaluate the welfare properties of this generalized equilibrium. As well known, firms do not fully internalize the welfare impact of their entry decision, which may lead to too many or too few firms.[22] The constrained optimal allocation (controlling prices and number of varieties under a zero profit constraint) is derived in the Appendix and provides a simple comparison with the decentralized equilibrium for any $\gamma \in [0,1]$:

PROPOSITION 4. *In a Cobb-Douglas two-sector economy with indirect additivity, monopolistic competition with endogenous entry generates excess (insufficient) entry with too little (much) production by each firm if the elasticity of the indirect sub-utility $\eta(\cdot)$ is everywhere increasing (decreasing) in the price.*

Paralleling D-S, an intuition for this result can be obtained by noticing that η approximates the ratio between the revenue of each firm and the additional utility generated by its variety. If $\eta' > (<)\ 0$ they diverge and at the margin each firm finds it more profitable to price higher (lower), i.e, to produce less (more), than what would be socially desirable. This, in turn, attracts too many (too few) firms.[23]

2.2 Heterogeneous consumers and income distribution

In this section we generalize our model to the case of consumers with different preferences and income. The model remains tractable and allows one to draw implications on the impact of income distribution on the market structure. We assume that there is a mass L of consumers of different "types". Types are distributed across the population according to the cumulative distribution function $C(h)$ with support $[0,1]$.[24] The consumer of type h has income E_h and indirect utility function given by:

$$V_h = \int_0^n v_h\left(\frac{p_j}{E_h}\right) dj. \tag{13}$$

In a symmetric equilibrium, each firm adopts a simple extension of the pricing rule (6) for homogenous consumers:

$$\frac{p^e - c}{p^e} = \frac{1}{\widetilde{\theta}\left(p^e, C\right)} \quad \text{with } \widetilde{\theta}\left(p, C\right) \equiv \int_0^1 \theta_h\left(\frac{p}{E_h}\right) \frac{E_h}{\bar{E}} dC\left(h\right), \tag{14}$$

[22] See the original D-S (1977) paper, Kuhn and Vives (1999) and Dhingra and Morrow (2012) for key references on this issue. Notice that the first-best allocation would require marginal cost pricing and subsidies to the firms. Details are available from the authors.

[23] One may find it more reasonable the case in which the elasticity of the sub-utility decreases when income gets higher, which requires $\eta' > 0$. This is the case for the exponential and addilog cases: accordingly, they both imply excess entry.

[24] We arrange consumer types in such a way that $h > k$ implies $E_h > E_k$, exclude any form of price discrimination (i.e., there is no market segmentation), and focus on the symmetric equilibrium.

where $\widetilde{\theta}$ is a weighted average of demand elasticities θ_h, and the weight is the consumer of type h's "fraction" of average income $\bar{E}$. Under free entry, the mass L of consumer is again neutral, but the distribution of types is not.[25]

PROPOSITION 5. *Under indirect additivity with heterogeneous consumers and monopolistic competition with endogenous entry, an increase in the mass of consumers is neutral on prices and increases linearly the number of firms. With identical preferences: 1) if the demand elasticity is increasing (decreasing) in the price, an improvement of the income distribution according to the likelihood-ratio dominance raises (decreases) prices and increases the mass of firms more (less) than proportionally to average income; 2) a mean preserving spread decreases prices and the mass of firms if and only if the demand elasticity is convex.*

The impact of an improvement of income distribution is in line with the baseline model, but the impact of inequality is in general ambiguous. Consider identical preferences with a demand elasticity increasing and convex with respect to the price, as in our addilog example: in such a case a mean preserving spread of the income distribution increases the average demand elasticity that is expected by the firms, which reduces prices and induces business destruction.

2.3 Heterogeneous firms and endogenous quality

Melitz (2003) has shown that under heterogeneous productivity of the firms and CES preferences there are no selection effects on the set of active firms when a market expands, for instance in a boom or when the country opens up to costless trade. However, under more general D-S preferences this neutrality holds for changes in income but not in the population, whose increase can give raise to ambiguous effects (depending on the shape of the relative love for variety). In particular, when prices are increasing with the size of consumption, an expansion of the market scale induces a selection effect, forcing the exit of the least productive firms, while less productive firms are able to survive during a contraction of the market (see Zhelobodko *et al.*, 2012 and Bertoletti and Epifani, 2012). In this section we show that under indirect additivity the number of consumers is always neutral, but it is income growth that has an impact, exerting an anti-selection effect as long as $\theta' > 0$: income growth allows less productive firms to survive, while low-productivity firms exit during downturns.

Following Melitz (2003), we assume that, upon paying a fixed entry cost F_e, each firm draws its marginal cost $c \in [\underline{c}, \infty)$ from a continuous cumulative distribution $G(c)$ with $\underline{c} > 0$. In the Appendix we show that the equilibrium price function $p(c)$ of an active c-firm is the same function of the marginal cost expressed in (6), that high-productivity firms produce more and are more profitable, and that they also charge lower markups if and only if $\theta' > 0$. Firms are active if their variable profits π_v cover the fixed cost F, that is if they have

[25] A special case arises if preferences are of the exponential type, i.e., $v_h = e^{-\tau_h p/E_h}$. In such a case $\theta_h(p/E_h) = \tau_h p/E_h$ and therefore $\widetilde{\theta} = p\overline{\tau}/\bar{E}$, where $\overline{\tau} = \int_h \tau_h dC(t)$: the market structure depends only on $\overline{\tau}$ and average income $\bar{E}$.

a marginal cost below the cut-off $\hat{c}$ satisfying:

$$\pi_v(\hat{c}) = \frac{[p(\hat{c}) - \hat{c}]\, v'(p(\hat{c})/E)L}{\mu} = F. \tag{15}$$

Moreover, the equilibrium must satisfy the endogenous entry condition:

$$\int_{\underline{c}}^{\hat{c}} [\pi_v(c) - F]\, dG(c) = F_e \tag{16}$$

i.e., firms must expect zero profit from entering in the market. The two equations determine $\hat{c}$ and μ in function of L, F, F_e and E, but in the Appendix we show that a change in L produces no selection effects: an increase of the population is completely neutral on all the prices and on the productivity cut-off beyond which firms are active, even when preferences are not CES. Instead, changes in income induce novel effects on the structure of production:

PROPOSITION 6. *Under indirect additivity, monopolistic competition with endogenous entry and cost heterogeneity between firms, an increase in population is neutral on prices and on the productivities of the active firms (it increases proportionally their mass), but higher income increases prices of all firms and makes less productive firms able to survive (an anti-selection effect) if and only if the demand elasticity is increasing with respect to the price.*

This result rationalizes a "cleansing effect" of recessions: these induce the exit of low-productivity firms leaving the high-productivity firms in the market, while expansionary shocks associated with higher spending make low-productivity firms able to survive. Notice that such a cyclical process cannot be reproduced in the baseline Melitz model or in its extension to directly additive preferences.

The heterogeneous costs model can be easily extended to take into account endogenous quality choices. This possibility has been recently explored to account for positive correlations of productivity with both quality and prices (see for instance Kugler and Verhoogen, 2012), but non-homothetic preferences are essential to explain a positive association of income with both quality and prices (the so-called *Linder hypothesis*). For simplicity, let us suppose that for a variety j with price p_j and quality $k_j \geq 0$ the sub-utility is given by $v_j = v\left(p_j/E\right)\varphi(k_j)$, where φ, $\varphi' > 0$ (higher quality increases both utility and demand without affecting demand elasticity), and $\lim_{k\to 0}\varphi(k) = 0$ (to avoid corner solutions). For simplicity, let us assume that a c-firm can produce goods of quality k at the marginal cost ck, obtaining variable profits $\pi_v = (p - ck)\, v'(p/E)\varphi(k)L/\mu$. Under some regularity conditions,[26] the equilibrium choices $p(c)$ and $k(c)$ satisfy $(p - ck)/p = 1/\theta(p/E)$ and $\theta(p/E) = 1 + \varepsilon(k)$, where $\varepsilon(k) \equiv \varphi'(k)k/\varphi(k)$ is the elasticity of demand with respect to quality. Price and quality are again independent from L, but their relation with productivity and consumers' income is more complex and can be derived through total differentiation as follows:

$$\begin{array}{lll} sign\left\{\frac{\partial p}{\partial c}\right\} = -sign\left\{\varepsilon'\right\} & \text{and} & sign\left\{\frac{\partial p}{\partial E}\right\} = sign\left\{\theta'\right\}, \\ sign\left\{\frac{\partial k}{\partial c}\right\} = -sign\left\{\theta'\right\} & \text{and} & sign\left\{\frac{\partial k}{\partial E}\right\} = sign\left\{\theta'\right\}. \end{array} \tag{17}$$

[26] The SOCs require $2\theta > \zeta$, $\xi \equiv k\varphi''/\varphi' < 2\left(\theta - 1\right)$, and $\xi\left[\zeta - 2\theta\right] > \left(\theta - 1\right)\left[2\zeta - 3\theta\right]$. Accordingly, it must be the case that $\theta' > 0$ if $\varepsilon' \geq 0$.

First, notice that under CES preferences ($\theta' = 0$) quality is (endogenously) independent from productivity and consumers' income, while when demand is isoelastic in quality ($\varepsilon' = 0$) the price is the same for all firms and more productive firms invest more in quality. Under the standard assumption $\theta' > 0$, more productive firms produce goods of higher quality. Moreover, they can even invest so much to sell them at higher prices compared to low productivity firms: this happens when the demand becomes more sensible to quality for products of higher quality (that is if $\varepsilon' > 0$).[27] Finally, in line with the *Linder hypothesis*, higher income induces specialization in high quality, high price goods. Given the price and quality choices, variable profits are still increasing in productivity and income, and the free-entry mechanism operates as before.

3 Pricing to Market in a Two-country Model

One of the main limits of the trade models based on monopolistic competition with CES preferences (Krugman, 1980 and Melitz, 2003) is their inability in providing simple reasons why firms should adopt different markups in different countries. It is well known that pricing to market is a pervasive phenomenon: identical products tend to be sold at different markups in different countries and in particular prices appear to be positively correlated with per capita income (Alessandria and Kaboski, 2011) but not with country population (Simonovska, 2013). In this section we generalize the Krugman (1980) model to indirectly additive preferences and emphasize its implications for the structure of trade.[28]

We consider trade between two countries sharing the same preferences (1) and technology, as embedded into the costs c and F, which are given in labor units, but possibly with different numbers of consumers L and L^* and different *productivity* (i.e., labor endowment in efficiency units). In particular, we assume that the labor endowments of consumers in the Home and Foreign countries are respectively e and e^*, so that income levels are $E = we$ and $E^* = w^*e^*$. Accordingly, the marginal and fixed costs in the domestic and foreign countries are respectively wc and wF and w^*c and w^*F.[29] Let us assume that to export each firm bears an "iceberg" cost $d \geq 1$, and, as standard, let us rule out the possibility of parallel imports aimed at arbitraging away price differentials (i.e.: international markets are segmented). Consider the profit of a firm i, based in the Home country, which can choose two different prices for domestic sales p_i and exports p_{zi}:

[27] For empirical evidence in this direction see, for instance, Kugler and Verhoogen (2012).

[28] Several recent papers have studied trade in multi-country models with non-homothetic preferences. Bertoletti and Epifani (2012) consider the general D-S model but focus on identical countries. Behrens and Murata (2012) and Simonovska (2013) use specific types of D-S preferences: the former paper assumes that market are not segmented while the latter deals with the case of international price discrimination. Finally, Fajgelbaum *et al.* (2011) consider products of different qualities within a Logit demand system.

[29] Identical results would emerge assuming different productivities (affecting proportionally both marginal and fixed cost), and equal labor endowments. This would be reflected on the equilibrium wages and through this on incomes.

$$\pi_i = \frac{(p_i - wc)v'\left(\frac{p_i}{E}\right)L}{\mu} + \frac{(p_{zi} - wcd)v'\left(\frac{p_{zi}}{E^*}\right)L^*}{\mu^*} - wF, \tag{18}$$

where μ and μ^* are the Home and Foreign values of (3). A symmetric expression holds for a Foreign firm j, choosing prices p_j^* and p_{zj}^*.

The optimal price rules for the Home firms are:

$$\frac{p - wc}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \quad \frac{p_z - wcd}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E^*}\right)}, \tag{19}$$

and the optimal price rules for the Foreign firms are similarly obtained. Therefore, four different prices emerge in the symmetric equilibrium, with $\mu = nv'\left(\frac{p}{E}\right)\frac{p}{E} + n^*v'\left(\frac{p_z^*}{E}\right)\frac{p_z^*}{E}$ and $\mu^* = nv'\left(\frac{p_z}{E^*}\right)\frac{p_z}{E^*} + n^*v'\left(\frac{p^*}{E^*}\right)\frac{p^*}{E^*}$. The endogenous entry condition for the firms of the Home country reads as:

$$\frac{(p - wc)v'\left(\frac{p}{E}\right)L}{\mu} + \frac{(p_z - wcd)v'\left(\frac{p_z}{E^*}\right)L^*}{\mu^*} = wF, \tag{20}$$

and a corresponding one holds for the firms of the Foreign country. We can normalize the home wage to unity, $w = 1$, and close the model with the domestic resource constraint (or, equivalently, the labor market clearing condition):

$$eL = n\left[cxL + cdx_zL^* + F\right], \tag{21}$$

where $x = v'(p/E)/\mu$ and $x_z = v'\left(p_z/E^*\right)/\mu^*$. Trade balance holds residually. This provides a system of seven equation in seven unknowns (p, p_z, p^*, p_z^*, w^*, n and n^*). With non-homothetic preferences, population and productivity of a country have a distinct impact on the relative wages, with complex implications for price differentials and the structure of trade. However, we can study the main insights of the model focusing on the two cases traditionally analyzed in the literature: costless trade between different countries, and costly trade between identical countries. The former case is obtained setting $d = 1$, and is characterized as follows:

PROPOSITION 7. *Under indirect additivity, monopolistic competition with endogenous entry and costless trade, firms adopt a higher price in the country with higher per-capita income; opening up to trade reduces (increases) the number of firms in the country with higher (lower) per-capita income if and only if the demand elasticity is increasing, and generates pure gains from variety.*

Since costless trade induces factor price equalization (otherwise the zero-profit condition would not be satisfied in both countries), the price rules show immediately the emergence of pricing to market: under the standard assumption $\theta' > 0$, prices of identical goods are higher in the country with the higher per-capita income because demand is more rigid compared to the other country, and these prices are independent from the population sizes. Consumers enjoy new varieties produced abroad and bought at the same price of the domestic goods. Nevertheless, opening up to trade induces a redistribution of firms

and production across countries which is absent in the Krugman (1980) model. Firms exporting to the country with poorer consumers sell there at a lower mark up and face entry of foreign firms in the domestic market: accordingly, they obtain lower variable profits, which leads to business destruction at home. The country with richer consumers is then characterized by a process of concentration in fewer and larger firms. On the contrary, business creation takes place in the country with poorer consumers, where firms start selling abroad at higher mark ups and reduce their size. Finally, prices and the total number of firms across countries remain the same as in autarky, therefore the gains from trade are always pure gains from variety as in Krugman (1980).[30]

The second case we consider, the one of costly trade, is obtained by setting $d > 1$ with $L = L^*$ and $e = e^*$, and is characterized as follows:

PROPOSITION 8. *Under indirect additivity, monopolistic competition with endogenous entry and costly trade between identical countries, opening up to trade reduces the markup on the exported goods and the mass of firms in each country relative to autarky if and only if the demand elasticity is increasing.*

Because also transport costs are symmetric, wages and prices are equalized in both countries. However, the markup applied to goods sold at home and abroad is not the same when preferences are not homothetic. In particular, the markup (on the marginal cost cd) is lower for the exported goods if $\theta' > 0$, because firms undershift transport costs on prices. This shows a different form of pricing to market, which has the additional consequence of affecting the entry process compared to the neutrality of the Krugman (1980) model: as long as the average markup diminishes because of undershifting of the transport costs on export prices, opening up generates a process of business destruction in both countries. Welfare gains from trade, therefore, do not derive from pure gains from variety (as in the Krugman model with transport costs), but potentially also from a downward pressure on the markup of the imported goods.

Notice that our setting breaks the neutrality of changes in trade costs and income on the structure of trade which holds in the Krugman (1980) model. First, a reduction in transport costs reduces the price of exports, but simultaneously increases their markups, which affects the number of firms as well. Second, under some additional conditions, richer countries trade relatively more between themselves than poorer countries, which is in line with the evidence (for instance see Fieler, 2011).[31] We believe that the assumption of indirect additiv-

[30] Augmenting the model with strategic interactions would generate a competition effect of trade on markups leading also to gains from competition (see Etro, 2013, for a related discussion).

[31] The export share on GDP can be derived as:

$$\frac{Exports}{EL} = \frac{v'(p_z/E)p_z}{v'(p_z/E)p_z + v'(p/E)\,p}$$

where prices satisfy (19). Under CES preferences this ratio is $1/(1+d^{\theta-1})$, therefore the export share is independent from income. Under non-homotheticity, weak conditions satisfied with linear and log-linear demand ($\zeta' > 0$ is sufficient) guarantee that the export share increases with income because the *relative* demand for imported goods becomes more elastic (in line with the *Linder hypothesis*).

ity of preferences will prove useful also to disentangle the impacts of income and population on other aspects of trade (such as the emergence of multinational and multiproduct firms, or the role of trade policy).

Finally, it is important to stress that, if preferences are not homothetic, a sort of pricing to market arises in a multi-country setting also under direct additivity (see Markusen, 2013 and Simonovska, 2013). The mechanism is simple: a larger income implies a larger individual consumption for each variety, which in turn affects markups. However, exactly for this reason, direct additivity also preserves the (ambiguous) impact of the number of consumers on markups. For example, the model of Simonovska (2013) predicts a negative relation between country population and prices for which she does not find clear support in the data, making her empirical findings more in line with the model presented here.

4 Conclusion

We have studied monopolistic competition with non-homothetic preferences satisfying indirect additivity, an alternative (and not less plausible) setting compared to Dixit and Stiglitz (1977). Under reasonable conditions (namely more rigid demand for higher income), it generates two main predictions that are in contrast with the standard approach and await for additional empirical tests: a higher individual income should increase markups and more than proportionally the number of firms, while the number of consumers should be neutral. Our framework is highly tractable and encompasses a number of analytically solvable cases as those with linear or log-linear direct demands. Therefore it could be applied to analyze complex issues usually considered exclusive territory for CES modeling: in particular, indirect additivity could be useful in building closed- and open-economy models of imperfect competition with heterogeneous firms and consumers, possibly in a dynamic general equilibrium framework.

References

Acemoglu, D. and M. K. Jensen, 2011, Aggregate Comparative Statics, mimeo, M.I.T

Alessandria, G. and J. P. Kaboski, 2011, Pricing-to-Market and the Failure of Absolute PPP, *American Economic Journal: Macroeconomics*, 3, 1, 91-127

Anderson, S., N. Erkal and D. Piccinin, 2012, Aggregative Games and Entry, mimeo, University of Virginia

Behrens, K. and Y. Murata, 2012, Trade, Competition, and Efficiency, *Journal of International Economics*, 87, 1, 1-17

Bertoletti, P. and P. Epifani, 2012, Monopolistic Competition: CES Redux?, mimeo, University of Pavia

Bilbiie, F., F. Ghironi and M. Melitz, 2012, Endogenous Entry, Product Variety, and Business Cycles, *Journal of Political Economy*, 120, 2, 304-45

Blackorby, C. and R. Russell, 1989, Will the Real Elasticicity of Substitution Plase Stand Up? (A Comparison of the Allen/Uzawa and Morishima Elasticities), *The American Economic Review*, 79, 882-8

Blackorby, C., D. Primont and R. Russell, 1978, *Duality, Separability, and Functional Structure: theory and economic applications*, North Holland

Blanchard, O. and N. Kiyotaki, 1987, Monopolistic Competition and the Effects of Aggregate Demand, *The American Economic Review*, 77, 4, 647-66

Campbell, J. and H. Hopenhayn, 2005, Market Size Matters, *Journal of Industrial Economics*, 53, 1, 1-25

Chamberlin, E. H., 1933, *The Theory of Monopolistic Competition: A Reorientation of the Theory of Value*, Harvard University Press

Dhingra, S. and J. Morrow, 2012, The Impact of Integration on Productivity and Welfare Distortions Under Monopolistic Competition, CEP dp 1130

Dixit, A. and J. Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, *The American Economic Review*, 67, 297-308

Etro, F., 2013, Endogenous Market Structures and International Trade, *Scandinavian Journal of Economics*, in press

Etro, F. and A. Colciago, 2010, Endogenous Market Structures and the Business Cycle, *The Economic Journal*, 120, 1201-34

Fajgelbaum, P., G. Grossman and E. Helpman, 2011, Income Distribution, Product Quality, and International Trade, *Journal of Political Economy*, 119, 4, 721-65

Feenstra, R., 2003, A Homothetic Utility Function for Monopolistic Competition Models, without Constant Price Elasticity, *Economics Letters*, 78, 1, 79-86

Fieler, A. C., 2011, Non-homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation, *Econometrica*, 79, 4, 1069-01

Hicks, J., 1969, Direct and Indirect Additivity, *Econometrica*, 37, 2, 353-4

Krugman, P., 1979, Increasing Returns, Monopolistic Competition and International Trade, *Journal of International Economics*, 9, 469-79

Krugman, P., 1980, Scale Economies, Product Differentiation, and the Pattern of Trade, *The American Economic Review*, 70, 950-9

Krugman, P., M. Obstfeld and M. Melitz, 2012, *International Economics: Theory and Policy*, Ninth Edition, Global Edition, Pearson Ed. Ltd

Kugler, M. and E. Verhoogen, 2012, Prices, Plant Size, and Product Quality, *Review of Economic Studies*, 79, 1, 307-39

Kuhn, K.-U. and X. Vives, 1999, Excess Entry, Vertical Integration, and Welfare, *RAND Journal of Economics*, 30, 4, 575-603

Markusen, J., 2013, Putting per-capita Income back into Trade Theory, *Journal of International Economics*, 90, 2, 255-65

Melitz, M., 2003, The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica*, 71, 6, 1695-725

Melitz, M. and G. Ottaviano, 2008, Market Size, Trade, and Productivity, *Review of Economic Studies* 75, 1, 295-316

Mrázová, M. and P. Neary, 2013, Not so Demanding: Preference Structure, Firm Behavior, and Welfare, mimeo, University of Oxford

Nekarda, C. and V. Ramey, 2013, The Cyclical Behavior of the Price-Cost Markup, NBER WP 19099

Samuelson, P., 1969, Corrected Formulation of Direct and Indirect Additivity, *Econometrica*, 37, 2, 355-59

Simonovska, I., 2013, Income Differences and Prices of Tradables, mimeo, University of California, Davis

Shaked, M. and J. G. Shanthikumar, 1994, *Stochastic Orders and Their Applications*, San Diego Academic Press

Zhelobodko, E., S. Kokovin, M. Parenti and J.-F. Thisse, 2012, Monopolistic Competition in General Equilibrium: Beyond the CES, *Econometrica*, 80, 6, 2765-84

Appendix

PROOF OF PROPOSITION 1. By using $\theta' p/(\theta E) = \theta + 1 - \zeta \gtrless 0$ if and only if $\theta + 1 \gtrless \zeta$, the result follows from the total differentiation of (6):

$$\frac{\partial \ln p^e}{\partial \ln n} = 0, \quad \frac{\partial \ln p^e}{\partial \ln E} = \frac{\theta + 1 - \zeta}{2\theta - \zeta} \quad \text{and} \quad \frac{\partial \ln p^e}{\partial \ln c} = 1 - \frac{\theta + 1 - \zeta}{2\theta - \zeta}. \tag{22}$$

after noticing that $2\theta - \zeta > 0$ from the SOC. □

PROOF OF PROPOSITION 2. Using the comparative statics in (22) and differentiating (7) we obtain:

$$\frac{\partial \ln n^e}{\partial \ln L} = 1, \ \frac{\partial \ln n^e}{\partial \ln E} = 1 + \frac{(\theta + 1 - \zeta)(\theta - 1)}{2\theta - \zeta} \ \text{ and } \ \frac{\partial \ln n^e}{\partial \ln c} = -\frac{(\theta + 1 - \zeta)(\theta - 1)}{2\theta - \zeta}$$ □

PROOF OF PROPOSITION 3. By the Roy identity, the demand of each differentiated good is given by:

$$x_i = \frac{v'(p_i/E)}{\int_0^n \left[v'(p_j/E) \frac{p_j}{E} - \frac{\gamma}{1-\gamma} v(p_j/E) \right] dj},$$

and the profits of each firm i are given by:

$$\pi_i = \frac{v'(p_i/E)(p_i - c)L}{\int_0^n \left[v'(p_j/E) p_j/E - \frac{\gamma}{1-\gamma} v(p_j/E) \right] dj} - F, \tag{23}$$

where the denominator is unaffected by p_i. It is immediate to verify that, independently from the value of γ, each firm adopts the same pricing rule as in (6) and the comparative static properties of the profit-maximizing price p^e are then the same as in Proposition 1. The number of goods produced in the free-entry equilibrium can be derived as:

$$n^e = \frac{EL(1-\gamma)\eta(p^e/E)}{F\theta(p^e/E)\left[(1-\gamma)\eta(p^e/E) + \gamma\right]}, \tag{24}$$

which now depends on γ and both the elasticities θ and η: changes in income affect prices and the allocation of expenditure between the differentiated goods and the outside one. Nevertheless, the number of firms is always proportional to L. □

PROOF OF PROPOSITION 4. We compare the market performance with a constrained optimal allocation which maximizes utility under a zero-profit condition for the firms. The problem boils down to:

$$\max_{n,p} nv\left(p/E\right) \quad s.t. \;\; p = c + \frac{F}{x\left(p,n,E\right)L},$$

where

$$x\left(p,n,E\right) = \frac{v'\left(p/E\right)}{n\left[v'\left(p/E\right)\frac{p}{E} - \frac{\gamma}{1-\gamma}v\left(p/E\right)\right]}$$

is the symmetric demand of a variety. Notice that the zero-profit constraint implictly defines n as a function of p that is continuous on $[c, \overline{s}E]$, with $n(c) = 0$ and $\lim_{p\to\overline{s}E} n(p)$ which is a finite number. Accordingly, the objective function is null for $p = c$ and $p \to \overline{s}E$, and positive for at least some intermediate price due to our assumptions. Therefore, there must exist an internal optimum satisfying the FOCs:

$$\begin{aligned} v\left(p/E\right) &= -\rho\frac{F}{Lx^2}\frac{\partial x}{\partial n}, \\ nv'\left(p/E\right)/E &= -\rho\left[1 + \frac{F}{Lx^2}\frac{\partial x}{\partial p}\right]. \end{aligned}$$

where ρ is the relevant Lagrange multiplier. They imply:

$$\eta(p/E) = -\frac{\frac{xpL}{F} + \frac{\partial \ln x(p,n,E)}{\partial \ln p}}{\frac{\partial \ln x(p,n,E)}{\partial \ln n}} = \frac{p}{p-c} + \frac{\partial \ln x\left(p,n,E\right)}{\partial \ln p},$$

It is easily computed that:

$$\eta' = \frac{\eta\left[1 - \theta + \eta\right]}{p/E}. \tag{25}$$

Since

$$\frac{\partial \ln x\left(p,n,E\right)}{\partial \ln p} = \frac{\gamma\frac{d\ln\eta(p/E)}{d\ln p/E}}{\gamma + (1-\gamma)\eta} - 1 = \frac{(2\gamma-1)\eta - \gamma\theta}{\gamma + (1-\gamma)\eta},$$

we obtain:

$$\frac{p^* - c}{p^*} = \frac{\gamma + (1-\gamma)\eta}{(1-\gamma)\eta\left(1+\eta\right) + \gamma\theta}. \tag{26}$$

Finally, the optimal mass of firms is:

$$n^* = \frac{(1-\gamma)\eta LE}{F\left[(1-\gamma)\eta\left(1+\eta\right) + \gamma\theta\right]}. \tag{27}$$

Compare (26) with (6): the RHS of (26) is larger (smaller) than $1/\theta$ if (everywhere) $\theta \gtrless 1 + \eta$. Since it follows from (25) that $\eta' \lesseqgtr 0$ is equivalent to $1/\left(1+\eta\right) \gtrless 1/\theta$, then (everywhere) $\eta' \lesseqgtr 0$ is equivalent to $p^e \lesseqgtr p^*$, which in turn implies $x^* \lesseqgtr x^e$ by the zero-profit constraint. Using the fact that the RHS of (27) is larger (smaller) than

the RHS of (24) if η' is smaller (larger) than zero we obtain $n^e \lesseqgtr n^*$, which completes the proof. □

PROOF OF PROPOSITION 5. The demand of a consumer h for good i can be written as $x_{hi}(p_i, E_h, \mu_h) = v_h'\left(p_i/E_h\right)/\mu_h$, where $\mu_h = \int_j v_h'\left(p_j/E_h\right)\left(p_j/E_h\right)dj$. The profits of firm i are given by:

$$\pi(p_i) = (p_i - c)L\int_0^1 x_{hi}(p_i, E_h, \mu_h)dC\left(h\right) - F,$$

which implies that the profit-maximizing price p_i satisfies the FOC:

$$(p_i - c)\int_0^1 \frac{v_h''\left(\frac{p_i}{E_h}\right)}{\mu_h E_h}dC\left(h\right) + \int_0^1 \frac{v_h'\left(\frac{p_i}{E_h}\right)}{\mu_h}dC\left(h\right) = 0.$$

Symmetric pricing implies $\mu_h = nv_h'\left(p^e/E_h\right)\left(p^e/E_h\right)$ and thus:

$$\frac{p^e - c}{p^e} = \frac{-\int_0^1 \frac{E_h}{np^e}dC\left(h\right)}{\int_0^1 \frac{v_h''\left(\frac{p^e}{E_h}\right)}{nv_h'\left(p^e/E_h\right)}dC\left(h\right)} = \frac{1}{\int_0^1 \theta_h\left(\frac{p^e}{E_h}\right)\frac{E_h}{\bar{E}}dC\left(h\right)} = \frac{1}{\widetilde{\theta}\left(p^e, C\right)},$$

where $\theta_h \equiv -v_h''\left(p/E_h\right)p/\left(v_h'\left(p/E_h\right)E_h\right)$ and $\bar{E} = \int_h E_h dC\left(h\right)$. The price rule is thus independent from L. Endogenous entry implies the following mass of firms:

$$n^e = \frac{\bar{E}L}{F\widetilde{\theta}\left(p^e, C\right)},$$

which proves the first part of the proposition since L affects linearly n^e.

To prove the second part, suppose that all consumers share the same preferences. It is then convenient to rewrite $\widetilde{\theta}$ directly as $\widetilde{\theta}\left(p, I\right) = \int_{E_0}^{E_1}\theta\left(p/E\right)\frac{E}{\bar{E}}dI(E)$, where $I\left(\cdot\right)$ is the income distribution function implied by $C\left(\cdot\right)$. Consider a change in I according to likelihood ratio dominance: i.e., a change from I^0 to I^1 such that $i^1(E)/i^1(T) \geq i^0(E)/i^0(T)$ for all $E > T$, $E, T \in [E_0, E_1]$, where $i\left(\cdot\right) = I'\left(\cdot\right)$ is the relevant density function. This implies that I^1 also (first-order) stochastically dominates I^0 (see for instance Shaked and Shanthikumar, 1994): thus, by a well-known result, this raises the average income (i.e., $\bar{E}^1 > \bar{E}^0$). We can write:

$$\widetilde{\theta}\left(p^e, I\right) = \int_{E_0}^{E_1}\theta\left(\frac{p^e}{E}\right)d\Phi(E; I) \quad \text{with } \Phi(E; I) = \int_{E_0}^{E}\frac{T}{\bar{E}}dI(T),$$

where the cumulative distribution function Φ has density $\phi = \Phi'$. Notice that:

$$\frac{\phi(E; I^1)}{\phi(T; I^1)} = \frac{Ei^1(E)}{Ti^1(T)} \geq \frac{Ei^0(E)}{Ti^0(T)} = \frac{\phi(E; I^0)}{\phi(T; I^0)} \quad \text{for all } E > T,\ E, T \in [E_0, E_1].$$

Accordingly, $\Phi(E; I^1)$ dominates in terms of the likelihood ratio $\Phi(E; I^0)$ and it must then be the case that $\Phi(E; I^1) \leq \Phi(E; I^0)$ for all $E \in [E_0, E_1]$ (i.e., the

former distribution first-order stochastically dominates the latter). It follows that when $\theta(p/E)$ is a decreasing (increasing) function of E an improvement of income distribution according to likelihood-ratio dominance implies $\widetilde{\theta}\left(p, I^1\right) \leq (\geq)\widetilde{\theta}\left(p, I^0\right)$ for all p, which in turn decreases (increases) the equilibrium value of $\widetilde{\theta}$, and thus raises (decreases) the equilibrium price level and the mass of active firms more (less) than proportionally to the rise of average income.

Suppose now that I^1 is a mean-preserving spread of I^0. Then $\bar{E}^1 = \bar{E}^0 = \overline{E}$. The function $\theta(p/E)\, E/\overline{E}$ in the definition of $\widetilde{\theta}(p, I)$ is a concave (convex) function with respect to E if and only if $\theta'' < (>)0$. By a standard result it must then be the case that $\widetilde{\theta}\left(p, I^0\right) > (<)\widetilde{\theta}\left(p, I^1\right)$ when $\theta'' < (>)0$. It follows that a mean-preserving spread decreases (increase) the equilibrium value of $\widetilde{\theta}$, and then raises (decreases) prices and the mass of firms when θ is a concave (convex) function of the price. □

Proof of Proposition 6. Let us start analyzing the price choices for the active firms. The variable profits of a c-firm are given by $\pi_v = (p - c)\, v'(p/E)L/\mu$, where $\mu = n\int_{\underline{c}}^{\widehat{c}} v'(p(c)/E)(p(c)/E)\frac{dG(c)}{G(\widehat{c})}$ is independent of its price choice. Therefore, the pricing rule (6) applies to all firms. We denote with $p = p(c)$ the profit-maximizing price of a c-firm, with $x(c) = v'(p(c)/E)/\mu$ the individual consumption of its product, and with:

$$\pi_v(c) = [p(c) - c]\, v'(p(c)/E)L/\mu$$

its variable profit for given μ. Note that the optimal price of firm c does not depend upon L and μ and follows the same comparative statics as in Proposition 1, with $\partial \ln p(c)/\partial \ln c \gtrless 1$ and $\partial \ln p(c)/\partial \ln E \gtrless 0$ if and only if $\theta' \gtrless 0$. Moreover, the FOCs and SOCs for profit maximization imply the following elasticities with respect to the marginal cost (for given μ):

$$\frac{\partial \ln x(c)}{\partial \ln c} = -\theta\left(p\left(c\right)/E\right)\frac{\partial \ln p(c)}{\partial \ln c} < 0, \tag{28}$$

$$\frac{\partial \ln p(c)x(c)}{\partial \ln c} = \frac{-\left(\theta\left(p\left(c\right)/E\right) - 1\right)^2}{2\theta\left(p\left(c\right)/E\right) - \zeta\left(p\left(c\right)/E\right)} < 0, \tag{29}$$

$$\frac{\partial \ln \pi_v(c)}{\partial \ln c} = 1 - \theta\left(p\left(c\right)/E\right) < 0. \tag{30}$$

Accordingly, high-productivity (low-c) firms are larger, make more revenues, and are more profitable, as in Melitz (2003), but they charge lower (higher) markups if θ is increasing (decreasing). In addition, again for given μ, we have the following elasticities with respect to income:

$$\frac{\partial \ln x(c)}{\partial \ln E} = \frac{\theta\left(p\left(c\right)/E\right)^2 - \theta\left(p\left(c\right)/E\right)}{2\theta\left(p\left(c\right)/E\right) - \zeta\left(p\left(c\right)/E\right)} > 0, \tag{31}$$

$$\frac{\partial \ln p(c)x(c)}{\partial \ln E} = \frac{\theta\left(p\left(c\right)/E\right)^2 + 1 - \zeta\left(p\left(c\right)/E\right)}{2\theta\left(p\left(c\right)/E\right) - \zeta\left(p\left(c\right)/E\right)} > 1, \tag{32}$$

$$\frac{\partial \ln \pi_v(c)}{\partial \ln E} = \theta\left(p\left(c\right)/E\right) > 1. \tag{33}$$

The size of each firm increases with E (for given μ), and revenues and profits increase more than proportionally. However, each price increases with respect to income only when $\theta' > 0$, and decreases otherwise. The set of active firms is the set of firms productive enough to obtain positive profits. Denote by $\hat{c}$ the marginal cost cutoff, namely the value of c satisfying the zero cutoff profit condition $\pi_v(\hat{c}) = F$, or:

$$[p(\hat{c}) - \hat{c}]\, v'(p(\hat{c})/E)L = \mu F. \tag{34}$$

The relation (34) implicitly defines $\hat{c} = \hat{c}(E, \mu F/L)$. Differentiating it yields:

$$\frac{\partial \ln \hat{c}}{\partial \ln E} = \frac{\theta\left(p\left(\hat{c}\right)/E\right)}{\theta\left(p\left(\hat{c}\right)/E\right) - 1} > 0, \tag{35}$$

$$\frac{\partial \hat{c}}{\partial \mu}\frac{\mu}{\hat{c}} = \frac{\partial \ln \hat{c}}{\partial \ln F} = -\frac{\partial \ln \hat{c}}{\partial \ln L} = \frac{1}{1 - \theta\left(p\left(\hat{c}\right)/E\right)} < 0. \tag{36}$$

Endogenous entry of firms in the market implies that expected profits

$$E\left\{\pi\right\} = \int_{\underline{c}}^{\hat{c}} \left[\pi_v(c) - F\right] dG(c) \tag{37}$$

must be equal to the sunk entry cost F_e. The profits decrease when the absolute value of μ increases, that is $\partial E\left\{\pi\right\}/\partial\mu > 0$. Accordingly, the condition $E\left\{\pi\right\} = F_e$ pins down uniquely the equilibrium value of μ as a function $\mu\left(E, L, F, F_e\right)$. In particular, using (34) the free entry condition can be written as:

$$\int_{\underline{c}}^{\hat{c}} \left\{ \frac{[p(c) - c]\, v'(p(c)/E)}{[p(\hat{c}) - \hat{c}]\, v'(p(\hat{c})/E)} - 1 \right\} dG(c) = \frac{F_e}{F}. \tag{38}$$

The system $\left\{(34), (38)\right\}$ can actually be seen as determining $\hat{c}$ and μ in function of L, F, F_e and E. The second equation fixes $\hat{c}$ and is independent from L, and the first one determines μ as linear with respect to L. The cut-off $\hat{c}$ is therefore independent of market scale, because μ proportionally adjusts in such a way to keep constant the ratio L/μ and thus the variable profit of the cut-off firm. As a consequence, as in Melitz (2003), a change in L produces no selection effect, even when preferences are not CES. Also notice that a raise of F requires an increase of μ less than proportional (otherwise the value of the expected variable profit would increase more than proportionally), and this in turn decreases $\hat{c}$ (a selection effect), while an increase of F_e by increasing the equilibrium value of μ raises $\hat{c}$ (an anti-selection effect). The impact of income E is more complex. Since by (33) and (35) an increase of E raises $E\left\{\pi\right\}$, it must decrease the equilibrium value of μ. In particular:

$$\frac{\partial \mu}{\partial E}\frac{E}{\mu} = -\frac{\frac{\partial \mathbf{E}[\pi]}{\partial E}}{\frac{\partial \mathbf{E}[\pi]}{\partial \mu}}\frac{E}{\mu} = \overline{\theta}\left(\hat{c}\right) > 0, \tag{39}$$

where

$$\overline{\theta}\left(\hat{c}\right) = \left[\int_{\underline{c}}^{\hat{c}} \frac{1}{\theta\left(p\left(c\right)/E\right)} \frac{p(c)x(c)}{\int_{\underline{c}}^{\hat{c}} p(c)x(c)dG(c)} dG(c) \right]^{-1} \tag{40}$$

is the harmonic mean of the θ values according to $G(\cdot)$ and $\hat{c}$. Computing the total derivative of $\hat{c}$ with respect to E we obtain:

$$\frac{d\ln\hat{c}}{d\ln E}=\left[\frac{\partial\hat{c}}{\partial E}+\frac{\partial\hat{c}}{\partial\mu}\frac{\partial\mu}{\partial E}\right]\frac{E}{\overline{c}}=\frac{\theta\left(p\left(\hat{c}\right)/E\right)-\overline{\theta}\left(\hat{c}\right)}{\theta\left(p\left(\hat{c}\right)/E\right)-1}, \tag{41}$$

which is positive if and only if (everywhere) $\theta'>0$: that is, in addition of increasing (decreasing) the mark up of the infra-marginal firms, a rise of E creates an anti-selection (selection) effect if θ increases (decreases) with respect to the price. To close the model, the expected mass of active firms n is determined by the budget constraint, requiring average expenditure to equal E/n, and thus:

$$n=\frac{E}{\int_{\underline{c}}^{\hat{c}}p(c)x(c)\frac{dG(c)}{G(\hat{c})}}. \tag{42}$$

Since an increase of the mass of consumers L affects proportionally μ, and thus proportionally reduces individual consumption $x(c)$, it follows from (42) that it also proportionally increases the mass of varieties. □

Proof of Proposition 7. Let us assume $d=1$. In such a case each firm faces the same demand functions, independently from the country in which it is based. However, the firms based in the Home country have a cost advantage (disadvantage) with respect to firms from the Foreign country if $w<(>)\ w^*$. Since a necessary condition for a monopolistic equilibrium with endogenous entry in both countries is $\pi=\pi^*=0$, it follows that it must be $w/w^*=1$. Accordingly, we can normalize the common wage to $w=w^*=1$ (which restores the notation of the baseline model with $E=e$ and $E^*=e^*$), and conclude that in a symmetric equilibrium $p=p_z^*$ and $p^*=p_z$. This means that all firms adopt the same price in the same country, with:

$$\frac{p-c}{p}=\frac{1}{\theta\left(\frac{p}{E}\right)},\qquad\frac{p^*-c}{p^*}=\frac{1}{\theta\left(\frac{p^*}{E^*}\right)}, \tag{43}$$

where $p>p^*$ when $E>E^*$ if and only if $\theta'>0$ (everywhere). The opening of costless trade has no impact on prices and mark ups relative to autarky, extending this property of the Krugman (1980) model to our entire class of indirectly additive preferences. From symmetry we infer that all firms have the same profit and using the price rules in (19) the zero-profit constraint provides the total mass of firms as:

$$n+n^*=\frac{EL}{F\theta\left(\frac{p}{E}\right)}+\frac{E^*L^*}{F\theta\left(\frac{p^*}{E^*}\right)}, \tag{44}$$

This is the sum of the masses of firms emerging under the autarky equilibrium in each separate country, say $n^a=EL/F\theta\left(p/E\right)$ and $n^{a*}=E^*L^*/F\theta\left(p^*/E^*\right)$, therefore the total mass of firms remains the same. This implies that after opening up to trade welfare unambiguously increases because of the increase in the number of consumed varieties. However, the mass of firms active in each country is not the same as in autarky. In fact, by using the resource constraints one can obtain:

$$\frac{n}{n^*}=\frac{EL}{E^*L^*}\lessgtr\frac{n^a}{n^{a*}}=\frac{EL\theta\left(p^*/E^*\right)}{E^*L^*\theta\left(p/E\right)}\ \text{ if } E>E^*\ \text{ and }\ \theta'\gtreqless 0, \tag{45}$$

where we used the fact that $p/E \lesseqgtr p^*/E^*$ if $E > E^*$ and $\theta' \gtreqless 0$. Since the total number of firms is constant, the number of firms in the rich (poor) country must decrease (increase) if $\theta' > (<)0$. Finally, the resource constraint of each contry implies that whenever the domestic mass of firms increases (decreases) their size must decrease (increase). □

PROOF OF PROPOSITION 8. Let us assume $d > 1$ but $L = L^*$ and $e = e^*$. In such a case, all the equilibrium variables must be the same across countries by symmetry. Therefore we can again normalize $w = w^* = 1$, which implies that $E = E^*$. The internal prices and the prices of exports must be the same in both countries, i.e., $p = p^*$ and $p_z = p_z^*$. These prices satisfy:

$$\frac{p-c}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \qquad \frac{p_z - dc}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E}\right)}.$$

By Proposition 1 we know that $p_z > p$ and $(p-c)/p \gtreqless (p_z - dc)/p_z$ if and only if $\theta' \gtreqless 0$. By symmetry the number of firms in each country is the same, say n, but this does not need to be the same as in autarky, $n^a = EL/F\theta(p/E)$. To find the number of firms in each country after opening up to trade, let us combine the free entry condition (20) with the price rules (19) to obtain:

$$\frac{pxL}{\theta\left(\frac{p}{E}\right)} + \frac{p_z x_z L}{\theta\left(\frac{p_z}{E^*}\right)} = F$$

By using $E = n\left(px + p_z x_z\right)$ the number of firms can be derived as follows:

$$n = \frac{EL}{F}\left[\theta\left(\frac{p}{E}\right)^{-1}\frac{px}{px + p_z x_z} + \theta\left(\frac{p_z}{E}\right)^{-1}\frac{p_z x_z}{px + p_z x_z}\right]. \tag{46}$$

Notice that the parenthesis in (46) is a weighted average of $1/\theta\left(p/E\right)$ and $1/\theta\left(p_z/E\right)$. Under CES preferences this is a constant: the number of firms is the same as in autarky (independent from the transport costs). Otherwise, since $p_z > p$, we have $n \lesseqgtr n^a$ if and only if $\theta' \gtreqless 0$. □