# Tracking Greenspan: systematic and unsystematic monetary policy revisited Preliminary Version 

Domenico Giannone<br>ECARES, Université Libre de Bruxelles<br>Lucrezia Reichlin<br>ECARES, Université Libre de Bruxelles and CEPR<br>Luca Sala<br>ECARES, Université Libre de Bruxelles

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#### Abstract

This paper proposes a new framework to analyse systematic and unsystematic monetary policy within the same econometric model. As in Bernanke and Boivin, 2001, the model aims at capturing the following facts: monetary authorities use information from a large number of data series to extract a signal on current economic activity which is typically measured with error. Due to strong collinearity between macroeconomic time series, relevant information is obtained by regressing the observables on few aggregates. Collinearity implies that a large panel of time series, which constitutes the information available to policy makers, can be represented as a dynamic factor model a la Forni and Reichlin, 1998, Stock and Watson, 1999 and Forni et al., 2000. Here we show how, in this framework, shocks can be identified structurally and the parameters of monetary policy rules, conditional on these shocks, can be estimated. Our results for the US economy between 1982 and 2001 show that: (i) Two shocks capture $80 \%$ of the variance of key variables such as output and inflation at all horizons; (ii) The monetary shock mainly affects the term structure of interest rates, but has virtually no effect on output and inflation so that monetary policy affects the economy through its systematic behavior rather than by surprising agents; (iii) Since demand and technology have been the main forces for the dynamics of cyclical output and inflation during the Greenspan era, while supply shocks have been negligible, monetary authorities did not face any tradeoff between inflation and output. By stabilizing inflation conditionally on demand shocks, they also achieved output stabilization; (iv) Conditionally on demand, Greenspan followed the Taylor principle while, conditionally on technology, monetary policy did not respond.


Key words and phrases : Monetary policy, Taylor rules, monetary shocks, signal extraction, dynamic factor models.

## 1 Introduction

There are two ways in which the literature has modeled the transmission of monetary policy. A large set of studies has analysed the effect of unanticipated monetary policy shocks estimated and identified from small VAR models. Those studies implicitly assume that the agents' information set contains the present and past observations of the variables considered in the VAR and that policy affects the economy by surprising consumers and business. Monetary shocks are typically interpreted as arising from the difficulty of extracting the signal on current economic conditions from noisy data or as reflecting heterogeneity of views on how to implement policy (see, Christiano, Eichenbaum and Evans, 1999). Recently, the focus on policy shocks rather than to the systematic part of monetary policy has been questioned (eg Mc Callum, 1999). The criticism is based on the fact that the percentage of output variance due to monetary policy shocks in VARs is typically very low and that the bulk of variation in the federal fund rate is explained by observable macroeconomic variables rather than exogenous policy action. Another set of studies, on the other hand, has focus on the estimation of structural policy equations (this literature is based on Taylor, 1993) and analysed monetary policy as systematic response to variation in observable variables, typically output gap and inflation (Clarida, Gali and Gertler, 1998). In principle, policy rules reflecting systematic reaction of monetary policy to exogenous shocks could be identified in a structural VAR, however problems of measurement errors and uncertainty on the relevant information set of policy makers, make it difficult to interpret the estimated VAR coefficients (see again Christiano, Eichenbaum and Evans, 1999 for a discussion). This is why the two approaches, VAR and structural rules, have lived separate lives.

Our paper broadly takes the same view on shocks and systematic policy as that underlying the structural VAR approach, but proposes a new framework for empirical analysis which, we will argue, present several advantages with respect to VAR analysis. Our framework differs from the VAR model in three crucial aspects. First, we will explicitely model monetary shocks as those shocks arising from the Fed's missperception of the current value of target variables. Second, we will take into account that monetary authorities use all information available (many time series, possibly in the order of the hundreds) to extract information on current economic activity (on this point, see Bernanke and Boivin, 2001 and Evans, 2001). Third, we will identify the number (as well as the origin) of macroeconomic shocks with respect to which monetary policy react systematically. In our model, although monetary authorities use a large set of variables to assess current economic activity, they condition their behavior on a small number of key shocks driving the observable target variables. Within the same framework, we can identify the money shock and its effect throughout the economy as well as identify the other relevant shocks and the monetary policy reaction to them. We will then be able to evaluate policy, conditionally to the shocks driving the economy and this will allow us to track Greenspan from 1987 to today (and Volcker from 1983 to 1987) to reassess the consensus view on historical performance of the Federal Reserve.

To identify the number of key macroeconomic shocks as well as their economic origin, we follow Forni, Lippi and Reichlin (2002). In our model, each variable is driven by few macroeconomic shocks, common to the whole economy and by idiosyncratic
dynamics which is mostly interpreted as measurement error (see Forni, Hallin, Lippi and Reichlin, 2000). To extract information on the common shocks we will use an estimation method based on Forni, Hallin, Lippi and Reichlin (2002) and Stock and Watson (1999). The method provides a parsimonious way to take into account the large information set assumption when estimating shocks and propagation mechanisms. The goal will then be the identification of few shocks from many variables by imposing a minimal set of economically motivated restrictions. Since the shocks to be identified are less numerous than the variables we are conditioning on, with a set of minimal restrictions we easily obtain overidentification which we can then test (testing procedure in this context have been developed by Giannone, 2001). Having identified the shocks, we can estimate the monetary policy rule conditional on those shocks. We will argue that, unlike for unconditional rules, where parameters are functions of both policy action and shocks realization, the estimation of conditional rules allows us to identify policy parameters without ambigouity.

The paper is organised as follows. In Section 2 we briefly review the approach used in the literature to estimate systematic and unsystematic monetary policy. In Section 3 we report some stylized facts that motivate our approach. In Section 4 we outline our methodological strategy. Section 5 reports results on estimates of monetary and non-monetary shocks. Section 6 reports results on the systematic component of monetary policy, conditional on the shocks. Section 7 discusses the advantages of our methodology with respect to the structural VAR approach and standard estimation of policy rules. Section 9 briefly discusses the implications of our results for the evaluation of the Volcker-Greenspan era. Technical issues are discussed in the Appendices.

## 2 Systematic and unsystematic monetary policy: the SVAR view and the Taylor view

The literature on the empirics of the monetary transmission mechanism is huge and it is not our goal here to summarize it in details. To introduce our methodology, however, it is useful to remind the stylized basic structure of most of the empirical work on the subject.

Let us assume that the Federal Reserve uses the federal funds rate as an instrument of monetary policy. This assumption is typically motivated by institutional considerations (see, for example, Bernanke and Blinder, 1992). Systematic policy can be represented by the following linear rule:

$$
r_{t}=f\left(\Omega_{t}\right)+\eta_{t}
$$

which links the instrument to the variables in the information set of the policy makers $\Omega_{t}$. The function $f\left(\Omega_{t}\right)$ represents systematic policy while $\eta_{t}$ represents the unanticipated monetary shock. There are different views on how $\eta_{t}$ is generated. Typically, a justification for the monetary shock $\eta_{t}$ is found in changes in the preferences of monetary policy makers, conflicts of views on monetary policy generated by uncertainty on actual economic conditions, measurement errors (for a discussion, see Christiano, Eichenbaum and Evans, 1999).

A VAR specifies the DGP for the elements of $\Omega_{t}$. Typically, $\Omega_{t}$ will contain present and past values of target variables $\left(\tau_{t}\right)$, such as ouput and inflation, for example, and other variables used as leading indicators $\left(z_{t}\right)$, such as, for example the commodity price index. We can partition the vector of the variables of interest as follows:

$$
x_{t}=\left(z_{t}, \tau_{t}, r_{t}\right)
$$

and define

$$
\Omega_{t}=\overline{\operatorname{span}}\left\{x_{t-k}, \quad k \geq 0\right\}
$$

Following usual notation, a reduced form VAR is :

$$
A(L) x_{t}=v_{t}
$$

with $\mathrm{Evv}=V$.
Let us partition the structural shocks in the macroeconomy as non-monetary $\mu_{t}$ and monetary $\eta_{t}$ so that:

$$
\begin{equation*}
\epsilon_{t}=\left(\mu_{t}, \eta_{t}\right) . \tag{2.1}
\end{equation*}
$$

Assuming that the elements of $\epsilon_{t}$ are mutually orthogonal and imposing the normalizing condition $\mathrm{E} \epsilon \epsilon^{\prime}=I$, identification of the structural parameters of the model requires to impose $n(n-1) / 2$ restrictions. This is achieved by chosing an appropriate orthonormal rotation $v_{t}=Q \epsilon_{t}, Q Q^{\prime}=I$.

Typically, restrictions amount to informational assumptions about policy decisions. For example, in one of the benchmark model discussed in Christiano et al. (1999), it is assumed that policy makers at time $t$ know about current and lagged value of output and inflation, lagged values of the federal fund-rate, non-borrowed reserves, total reserves and the money stock and about current and past value of the commodity price index.

Under the assumption of recursivity (see again, Christiano, Eichenbaum and Evans, 1999), we can obtain just-identification and the money shock $\eta$ is identified as the shock associated to the row in the VAR corresponding to the federal fund rate. In this case, shocks and coefficients can be estimated by OLS.

At first sight, it would then seem that within the same framework one can identify both shocks and the parameters of the rule. However, there are many problems in interpreting the coefficients of the rules in terms of the behaviour of monetary authorities. First, policy makers observe data with errors (see our discussion below). Second, policy makers may have a larger information set than the econometricians. For example, the assumption that output is observed contemporaneously seems unrealistic since GDP is released quarterly and with a rather long delay. As Christiano, Eichenbaum and Evans (1999) put it:
"the policy rules parameters estimated by the econometrician are a convolution of the parameters of the rule implemented in real time by the policy maker and the parameters of the projection of the missing data onto the econometrician's data set. It is the convolution of
these two types of parameters which makes it difficult to assign behavioral interpretations to the econometrician's estimated policy rule parameters." (p. 55)

This is why the systematic component of monetary policy is typically evaluated outside the VAR framework, by structural estimation of an interest rate equation of the form

$$
r_{t}=\phi^{\prime} \tau_{t}+\eta_{t}
$$

where the instrument is related to observable variables, rather than exogenous shocks (see, for example, Clarida, Gali and Gertler, 1998).

## 3 Some stylized facts

## - Fact 1: Comovements

Let us look at some descriptive features of a panel of quarterly time series for the US economy. To be precise, we will analyze 479 time series. Let us call the $n \times 1$ vector of these variables suitably transformed to obtain stationarity, $x_{t}$ (see appendix 6 for details on data sources and data transformation).

Although our data set contains variables of different nature: real, nominal, financial and sectoral as well as aggregate data, one fact that emerges from the analysis of the covariance structure is that there is strong collinearity indicating that, indeed, macro time series comove, particularly at business cycle frequencies.

One way to estimate the degree of collinearity between elements of a panel of time series is to look at the eigenvalues of the covariance matrix and see how many we need to capture the bulk of the variance. Table 1 below shows results for our panel. In the first column we have the variance captured by the first $j=1, \ldots, 5$ eigenvalues (cumulated) in percentage of the total variance. In column 2 and 3 we have, respectively, the same ratio for frequencies higher than $\theta=\frac{\pi}{2}$, corresponding to cycles longer than 1 years, and $\theta=\frac{\pi}{4}$, corresponding to cycles longer than 2 years. We can see that, overall, the first three eigenvalues capture more than $60 \%$ of total variance on average and more than $70 \%$ at business cycle frequencies.

Table 1. Variance explained by the first 5 DPCs

| no. of DPC | all freq. | $>1$ year | $>2$ years |
| :--- | :--- | :--- | :--- |
| 1 | 0.34 | 0.41 | 0.45 |
| 2 | 0.51 | 0.56 | 0.61 |
| 3 | 0.63 | 0.67 | $\mathbf{0 . 7 1}$ |
| 4 | 0.72 | 0.76 | 0.80 |
| 5 | 0.79 | 0.82 | 0.85 |

Let us define the first 3 dynamic principal components as

$$
z_{h t}=p_{h}(L) x_{t}, \quad h=1, \ldots, 3
$$

where $L$ is the lag operator and $p_{h}(L)$ is a $(1 \times n)$ row vector of two-sided linear filters ${ }^{1}$.
In order to understand how strongly key variables of interest are correlated with the rest of the panel, let us project the latters on the span of the first three principal components. We have:

$$
x_{i t}=\gamma_{i t}+\zeta_{i t}
$$

where $\gamma_{i t}=c_{i}(L) z_{t}$ with $z_{t}=\left(z_{1 t} \cdots z_{3 t}\right)^{\prime}$ and $\zeta_{i t}$ is the residual vector.
Figure 1 below shows $\gamma_{i t}$ and $\zeta i t$ plotted over time and their spectral densities over the frequency domain for four variables of interest: the growth rate of GDP ( $\delta y$ ), consumption $(\delta c)$, inflation $(\delta \pi)$ and the federal funds rate $\left(\delta r_{t}\right)$.

Figure 1. Projection on the first 3 dynamic principal components: spectral densities (first column) and time series (second column)









Three features are worth noticing. First, the projection component (the $\gamma$ 's) captures the bulk of the variance in all cases. This suggests that three linear combinations of our large panel capture most of the variance of the variables of interest. Second, in all cases, the residuals have flat spectral shape, indicating that they can be modeled as white noise components, while all interesting dynamics is captured by the projection. This suggests that the residuals can be considered as measurement errors or a component with no temporal or cross-sectional persistence. Table 2 reports the variance

[^0]captured by the first $j=1, \ldots, 5$ eigenvalues (cumulated) in percentage of the total variance for selected series and details the results.

Table 2. Variance explained by the first 5 DPCs for selected series

|  | $\Delta y$ |  |  |  | $\Delta c$ |  |  |  | $\Delta \pi$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| no. of DPC | all | $>1 y$ | $>2 y$ | all | $>1 y$ | $>2 y$ | all | $>1 y$ | $>2 y$ | all | $>1 y$ | $>2 y$ |
| 1 | 0.60 | 0.74 | 0.78 | 0.41 | 0.43 | 0.49 | 0.39 | 0.47 | 0.50 | 0.59 | 0.66 | 0.68 |
| 2 | 0.72 | 0.82 | 0.88 | 0.60 | 0.63 | 0.65 | 0.75 | 0.79 | 0.83 | 0.73 | 0.79 | 0.81 |
| 3 | 0.80 | 0.86 | 0.90 | 0.72 | 0.74 | 0.77 | 0.82 | 0.84 | 0.86 | 0.80 | 0.85 | 0.87 |
| 4 | 0.86 | 0.91 | 0.93 | 0.80 | 0.86 | 0.89 | 0.86 | 0.88 | 0.90 | 0.84 | 0.88 | 0.90 |
| 5 | 0.90 | 0.93 | 0.95 | 0.85 | 0.89 | 0.92 | 0.88 | 0.90 | 0.92 | 0.87 | 0.91 | 0.92 |

Notice that for all variables, except consumption, the first two dynamic principal components capture around $80 \%$ of the variance of the selected variables if cycles longer than one year are considered, while the first three achieve $80 \%$ at all frequencies.

## -Fact 2: Current economic conditions are known with error

There are two major sources of uncertainty about the current state of real economic activity. First, GDP data and, to a less extent, industrial production and price data, are released with delay. Second, real variables are subject to statistical revisions. The length of publication delay and the size of revision errors vary by country (see Faust et al, 2001 for an interesting cross-country comparison), but they are generally recognized to be substantial. This fact is often misregarded in the literature where it is current practice to fit policy functions assuming that the values of the time series published at time $T+h$ are the same as those known at $T$. Recently, starting with the work of Orphanides (2001), standard results have been reconsidered using data prior to revisions, i.e. data available at the time of policy decisions. A different literature has dealt with the delay in data publications by suggesting techniques which allow to exploit information on data published with no delay to estimate recent data points in GDP (Altissimo et al., 2001). The two problems, delay and revision errors, are mostly treated separately, but they can be analyzed jointly, just considering that GDP or other variables of interest, are not population quantities, but statistical models. The goal of the econometrician and the policy maker is to extract a signal from noisy data (this way of looking at the problem is, for example, suggested by Evans, 2001). The signal, if successfully measured, will show higher correlation with revised data than to first releases.

Obviously, measurement error is larger in real quantities and price data than in financial data. In particular, the federal fund rate has basically zero measurement error. Moreover, certain variables such as asset prices, survey based indicators on business and consumer expectations are available with minimal delay and are often used formally or informally to assess current economic conditions. Although it is not clear what information policy makers exactly use, it is reasonable to conjecture that their judgement on the state of the economy is based on information coming from a large variety of sources and variable definitions. Below we will model this assumption explicitely.

## 4 Modeling strategy

Our modeling strategy is based on the stylized facts documented above and rests on three pillars.

## - Pillar 1: The large information set conjecture

Recent literature (Bernanke and Boivin, 2001, Evans, 2001, Favero and Marcellino, 2001) has suggested that monetary policy makers, when setting the level of the instrument, do not just look at few key variables, but instead exploit knowledge coming from rich data sets containing sectoral as well as aggregate variables, asset prices and key variables used to detect demand and supply conditions (following the notation of Section 2, this implies to assume that $\Omega_{t}$ is large).

## - Pillar 2: Signal extraction

We assume that most variables in the panel (2.1), especially quantities and prices of goods and services, are measured with error and that they can be decomposed into the sum of two orthogonal components, the signal $x_{i t}^{*}$ and the measurement error $e_{i t}$ :

$$
\begin{equation*}
x_{i t}=x_{i t}^{*}+e_{i t} . \tag{4.2}
\end{equation*}
$$

The problem for monetary policy makers is to extract the signal $\tau_{t}^{*}$ from $\tau_{t}$, and set $r_{t}$ according to some policy rule.

Given the large information set conjecture, we assume that the signal from $\tau_{t}$ is extracted using all information potentially available, i.e. by projecting $\tau_{t}$ onto the span of $x_{t}$ and its past. We have:

$$
\begin{equation*}
\tau_{t}^{*}=\operatorname{Proj}\left[\tau_{t} \mid \overline{\operatorname{span}}\left(x_{t-k}, k \geq 0\right)\right] . \tag{4.3}
\end{equation*}
$$

Notice that $\overline{\operatorname{span}}\left(x_{t-k}, k \geq 0\right)=\Omega_{t}$. Obviously, if the number of elements of $x_{t}$ is large (in our case $n=479$ ), we cannot use a VAR to estimate this projection since the latter has too many parameters. However, the collinearity of the panel can be exploited to extract relevant information from the large cross-section within a parsimonious framework.

Under suitable conditions on the variance-covariance of the $x$ 's which, roughly speaking, are satisfied when there is high collinearity (see Forni, Hallin, Lippi and Reichlin, 2000 for precise conditions), we can assume that $x_{t}$ follows a dynamic factor model:

$$
\begin{equation*}
x_{i t}=\chi_{i t}+\xi_{i t} \tag{4.4}
\end{equation*}
$$

where

$$
\chi_{i t}=b_{i}(L) u_{t}=\sum_{h=1}^{q} b_{i h}(L) u_{h t}
$$

is the common component, $u_{t}=\left(u_{1 t}, \ldots, u_{q t}\right)^{\prime}$ is the $q$-dimensional vector of the common shocks, which are unit variance white noises mutually orthogonal at all leads and lags, $b_{i}(L)=b_{i 1}(L), \ldots, b_{i q}(L)$ is a row vector of polynomials in the lag operator of possibly infinite order and the idiosyncratic component $\xi_{i t}$ is orthogonal to $u_{t-k}$ for any $k$ and $i$.

The common component, which is identified under the conditions given in Forni et al. (2000), captures that part of the series which is correlated with the rest of the panel while the idiosyncratic component contains measurement error and locally cross-correlated components.

Abstracting for the moment from estimation issues, we assume that by extracting the common components from target variables, we clean them from measurement error. We have:

$$
\tau_{t}^{*}=\chi_{\tau, t}
$$

Under these assumptions, $\tau_{t}^{*}$ can be recovered by projecting on the span of the common components of $x_{t}$ (or, which is the same, on the span of the common factors $\left.u_{t h}\right)$ rather than $x_{t}$ itself. Since the latter is of reduced rank $q$, this projection is feasible. Under our assumptions, $\tau^{*}$ is equal to its common component $\chi_{\tau t}$. We have:

$$
\begin{equation*}
\tau_{t}^{*}=\chi_{\tau t}=\operatorname{Proj}\left[\tau_{t} \mid \overline{\operatorname{span}}\left(u_{t-k}, k \geq 0\right)\right] \tag{4.5}
\end{equation*}
$$

## - Pillar 3: Policy function

In Section 2, we expressed the policy rule as a function relating the instrument $r_{t}$ and the information set $\Omega_{t}$ :

$$
r_{t}=f\left(\Omega_{t}\right)+\eta_{t}
$$

Given, the argument above, we can be more specific and say:

$$
f\left(\Omega_{t}\right)=\phi(L) \tau_{t}^{*}=g\left(u_{t}\right)
$$

where $\tau_{t}^{*}$ is not known and has to be estimated on the basis of information in $\Omega_{t}$, i.e. the span of the present and past $x$ 's, where $x$ is potentially very large. The estimate is obtained from (4.5) as $\chi_{\tau t}$. The unfeasible problem of estimating a large VAR on $x$ has then be transformed in a reduced rank regression problem of regressing the $x$ 's on a few number of common shocks.

The policy function can then be written in three alternative forms:

$$
\begin{equation*}
r_{t}=\phi(L) \tau_{t}^{*}+\eta_{t}=\phi(L) \chi_{\tau t}+\eta_{t}=\phi(L) b_{\tau}(L) u_{t}+\eta_{t} \tag{4.6}
\end{equation*}
$$

In the first two forms the federal fund rate is expressed as a function of the signal extracted from observable target variables while, in the third, we obtain it as a function of unobservable exogenous macroeconomic shocks.

Notice that the number of shocks is not necessarily the same as the number of target variables and, clearly, if the targets are more numerous than the shocks, coefficients cannot be identified. We will comment on this point later on. Let us here say that the number of common shocks can be inferred from the behavior of the eigenvalues of the covariances of the $x$ 's (see, Forni et al., 2000 on this point).

Once the number of shocks is selected, we must identify the shocks structurally and label them as technology, demand, etc.. Had the money shock affected the whole economy, one element of $u_{t}$ should certainly be identified as "money" and distingished from other shocks. The policy function can be expressed in its conditional form as a function of the money shock $u_{t}^{m}$ and the vector of the non-money shocks $u_{t}^{n m}$. We have:

$$
\begin{equation*}
r_{t}=\phi^{n m} b_{\tau}^{n m}(L) u_{t}^{n m}+\phi^{m} b_{\tau}^{m}(L) u_{t}^{m}+\eta_{t} . \tag{4.7}
\end{equation*}
$$

Notice that, since the monetary shock $\eta_{t}$, beside having a pervasive impact, may also have a local impact (affecting, for example, only nominal variables), we can split it in its "common" and "idiosyncratic" component:

$$
\eta_{t}=u_{t}^{m}+\xi_{r t} .
$$

It is easily seen that $\eta_{t}$ in (4.6) and (4.7) is not orthogonal to the regressors. While this would be a problem in a standard estimation of the rule (even in the absence of forward looking behaviour), it is not a problem in our framework since the coefficients of the rules and the exogenous shocks can be identified and estimated from:

$$
\begin{equation*}
r_{t}=b_{r}^{n m}(L) u_{t}^{n m}+b_{r}(L) u_{t}^{m}+\xi_{r, t} \tag{4.8}
\end{equation*}
$$

where $b_{r}^{n m}(L)$ provide information on the systematic reaction of monetary policy to unanticipated shocks and $b_{r}(L) u_{t}^{m}$ represents the feedbacks. Of course, the money shock may turn out not to propagate throughout the whole economy and to have only a local effect, as for example, to nominal variables. In that case the money shock will be idiosyncratic, i.e. poorly cross-sectionally correlated in the sense defined by Forni et al. (2000). We will investigate this possibility later on.

Let us here stress two points and make two remarks.
Point 1. In order to identify policy behavior, it is important to define rules conditionally on shocks rather than estimating them in terms of the variables themselves. This point can be best illustrated by the following example.
Example: Inflation targeting.
Assume that there are two shocks in the economy, demand and technology, so that inflation and the federal fund rate can be expressed as the sum of a component conditional on demand and a component conditional on technology. With obvious notation we have;

$$
\begin{aligned}
& \pi_{t}=\pi_{t}^{d}+\pi_{t}^{t e k} \\
& r_{t}=r_{t}^{d}+r_{t}^{t e k}
\end{aligned}
$$

where $r_{t}^{d}=\phi_{\pi}^{d} \pi_{t}^{d}$ and $r_{t}^{t e k}=\phi_{\pi}^{t e k} \pi_{t}^{t e k}$.
The coefficients of the conditional rule are:

$$
\begin{aligned}
\phi_{\pi}^{d} & =\frac{\operatorname{cov}\left(\pi_{t}^{d}, r_{t}^{d}\right)}{\operatorname{var}\left(\pi_{t}^{d}\right)} \\
\phi_{\pi}^{t e k} & =\frac{\operatorname{cov}\left(\pi_{t}^{t e k}, r_{t}^{t e k}\right)}{\operatorname{var}\left(\pi_{t}^{t e k}\right)}
\end{aligned}
$$

The corresponding unconditional rule is:

$$
r_{t}=\tilde{\phi}_{\pi}\left(\pi_{t}^{d}+\pi_{t}^{t e k}\right)
$$

and the estimated coefficient is derived as:

$$
\tilde{\phi}_{\pi}=\frac{\operatorname{cov}\left(\pi_{t}^{t e k}+\pi_{t}^{d}, r_{t}^{t e k}+r_{t}^{d}\right)}{\operatorname{var}\left(\pi_{t}^{t e k}+\pi_{t}^{d}\right)}
$$

Under the assumption that $\operatorname{cov}\left(\pi_{t}^{t e k}, r_{t}^{t e k}\right)=0$, the expression above simplifies to:

$$
\tilde{\phi}_{\pi}=\phi_{\pi}^{d} \frac{\operatorname{var}\left(\pi_{t}^{d}\right)}{\operatorname{var}\left(\pi_{t}^{t e k}+\pi_{t}^{d}\right)}
$$

Clearly the coefficient of the unconditional rule depends on the relative weight of the variance of inflation explained by demand and that explained by supply. Assume that the Central Bank does not react to the technology shocks so that $\phi_{\pi}^{t e k}=0$. If the Central Bank follows the Taylor principle conditionally on demand, so that $\phi_{\pi}^{d}$ is larger than one, we may observe an unconditional parameter larger or smaller than one depending on the ratio between the variance of inflation explained by demand and the variance of inflation explained by technology. If this ratio decreases, causing, other things being equal, a drop of $\tilde{\phi}$ to a value less than one, we may wrongly conclude that the Central Bank has not followed a stabilizing rule.

Notice that, although the importance of conditional estimates is aknowledged in the theoretical literature (eg. Woodford, 2002 and Clarida, Gali and Gertler, 1999), the empirical literature generally takes an unconditional approach where $r_{t}$ is expressed as a function of observable variables rather than in terms of exogenous shocks. ${ }^{2}$
Point 2. The second point is that the fact that our model has many variables driven by the same few shocks, has important consequences on the identification of policy parameters inthe Taylor rule. If the number of target variables $m$ is smaller or equal than the number of shocks $q$, the parameters of the rule are identified by the relation

[^1]between (4.8) and (4.6). It is easily seen that equation (4.8) implies the following restrictions:
$$
\phi^{m}(L)=\left[b_{r}^{m}(L)-1\right] b_{\tau}^{m}(L)^{-1}
$$
and
$$
\phi^{n m}(L)=b_{r}^{n m}(L) b_{\tau}^{n m}(L)^{-1} .
$$

The identification of the parameters of the rule conditional on the shock, on the other hand, will not be identified in this case. For $q$ shocks and $m$ target variables, we need to identify $q \times m$ parameters from $q$ equations. We will return to this point in Session 6.
Remark 1. $\chi_{\tau t}$ has to be estimated. A consistent estimate can be obtained by projecting $x_{i t}$ on the appropriate number of static principal components as in Stock and Watson, 1999 or as the projection on the appropriate number of dynamic principal components as in Forni, Hallin, Lippi and Reichlin, 2000. If $q=3$ the estimated common component (the signal) of the target variables GDP and inflation is exactly what we have illustrated in Figure 1 in Section 2. Recall that, for both output and inflation, the measurement error captured by the residual idiosyncratic component exhibits a flat spectrum while the business cycle dynamics is entirely captured by the estimated common component. This should persuade the reader that by extracting the common component we are indeed extracting the signal.
Remark 2. The task of the structural identification of the $u$ 's requires to impose identification restrictions as it is usually done in the VAR literature. We turn to this point in the next Section.

## 5 Shocks

Before identifying the common shocks, we have to determine their number $q$ on the basis of some statistical criterion. Although a number of criteria have been suggested by the literature (Forni et al, 2000 and Liska, 2002), no formal test is available. The suggested criteria are based on the values of the dynamic eigenvalues of the observations. Table 1 in Section 2 reported the percentage of the total variance explained by the first five dynamic principal components. We can see that three eigenvalues capture over $60 \%$ of the variance when all frequencies are considered and over $70 \%$ at business cycle frequencies. Just the first two eigenvalues capture more than $50 \%$ of the total variance. From Figure 1, we have also seen that these numbers are even larger when one considers explained variance of key variables such as output, consumption and inflation. We will then try two alternative choices, $q=2$ and $q=3$.

We will start analysing the case $q=3$ and procede to identify three shocks. Identification requires two steps. First, we need to obtain a representation of the $u$ 's on the past of the $\chi$ 's. This allows us to recover the common shocks from a reduced rank regression on the common components. Second, we need to impose restrictions to reach
identification. Notice that in the factor model framework, we need to identify $q$ shocks from $n$ observable time series unlike in VARs where the number of shocks is the same as the number of series. The consequence, as it has been pointed out by Forni, Lippi and Reichlin (2001), is that we can easily obtained over-identifying restrictions which can then be tested using the framework developed by Giannone (2001).

Let us discuss the first point first. We will assume a restricted version of (4.4) which is in line with Stock and Watson (1999) and Forni et al. (2001):

$$
\begin{equation*}
x_{i t}=c_{i}(L) f_{t}+\xi_{i t} \tag{5.9}
\end{equation*}
$$

and

$$
\begin{equation*}
f_{t}-a_{1} f_{t-1}-\ldots-a_{p} f_{t-p}=a(L) f_{t}=B u_{t} \tag{5.10}
\end{equation*}
$$

We assume that $c_{i}(L)$ is of finite order $s$ and that the $q \times 1$ vector of the factors has an autoregressive structure of order $p$. We will then have:

$$
b_{i}(L)=c_{i}(L) a(L)^{-1} B
$$

Under the assumptions above, the model can be written in its stacked form and estimated by static principal components. Details are spelled out in Appendices 3 and 4. Let us here say that the identification problem consists in recovering a vector of structural shocks of dimension $q$. If the loadings have finite lag structure of order $s$, the model can be written is stacked form where each observable is a linear function of the first $r=q(s+1)$ static principal components of the $x$ 's. In this case, recovering the span of the $q$ dimensional shocks $u_{t}$ is the same as recovering the span of the $r$ dimensional vector of the $u$ 's and their $s$ lags. Identification is then achieved in two steps. In the first, we recover a linear combination of $r$ shocks and we compute their first $q$ static principal components: this gives us a linear combination of the $q$ shocks of interest. In the second step, we fix a $q$-dimensional rotation of the principal components.

Let us define the stacked $r$-dimensional common shocks as $v_{t}=D u_{t}$ with variancecovariance matrix $\Sigma=D D^{\prime}=P M P^{\prime}$ where the last term of the equality expresses it in terms of the matrix $P$ of its eigenvectors and the matrix $M$ of its eigenvalues. The $q$-vector of orthogonal common shocks can be estimated as the $q$ normalized non-zero static principal components of the $v$ residuals, i.e. as:

$$
\omega_{t}=M^{-1 / 2} P^{\prime} v_{t}
$$

The latter obviously spans the same space spanned by the $u_{t}$. We have $u_{t}=R^{\prime} \omega_{t}$ where $R$ is an orthonormal matrix which rotates $\omega_{t}$. In the case $q=3, R$ can be parameterized as:

$$
R=\left(\begin{array}{ccc}
\cos (a) & \sin (a) & 0  \tag{5.11}\\
-\sin (a) & \cos (a) & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos (b) & 0 & \sin (b) \\
0 & 1 & 0 \\
-\sin (b) & 0 & \cos (b)
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos (c) & \sin (c) \\
0 & -\sin (c) & \cos (c)
\end{array}\right)
$$

with $(a, b, c) \in[0,2 \pi]$.
Since the rotation matrix is of dimension $q \times q$ and the $u$ 's are orthonormal, justidentification is obtained with $q$ restrictions.

The impulse response functions of $x_{i t}$, associated with the common shocks $\omega$ 's are given by $b_{i}(L) R$.

Structural identification consists in the selection of a particular matrix $R$. Given the orthonormality of the $u$ 's this implies that we have to choose three parameters in order to reach just-identification. In our model, where $n \gg q$, by adopting a minimal set of "consensus" restrictions, we easily reach over-identification.

We have in mind four types of potentially relevant shocks: technology, demand, supply and money.

A broad class of theoretical models suggests that the identified shocks should satisfy the following restrictions on long-run multipliers (LR) (see Francis and Ramey, 2001 for a review) and conditional correlations (CC)

LR.1) demand, supply and monetary shocks do not have permanent effects on productivity and real wages;

LR.2) technology shocks do not permanently affect labor supply;
LR.3) monetary shocks do not have permanent effects on real variables such as consumption, investment, output, labor supply;
CC.1) supply shocks have opposite effects on prices and output;
CC.2) demand and money shocks affect prices and output in the same direction;

Notice that long-run restrictions do not allow to discriminate between demand and supply shocks.

Restrictions LR1-3 and CC1-3 are summarized in the table below where columns LR show the zero restrictions on the long-run multipliers and columns CC show the sign of the conditional response:

|  | tek |  | demand |  | supply |  | money |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LR | CC | LR | CC | LR | CC | LR | CC |
| $y / l$ | $\times$ | $\times$ | 0 | $\times$ | 0 | $\times$ | 0 | $\times$ |
| $w$ | $\times$ | $\times$ | 0 | $\times$ | 0 | $\times$ | 0 | $\times$ |
| $l$ | 0 | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | 0 | $\times$ |
| $y$ | $\times$ | $\times$ | $\times$ | + | $\times$ | + | 0 | + |
| $c$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | 0 | $\times$ |
| $i$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | 0 | $\times$ |
| $\pi$ | $\times$ | $\times$ | $\times$ | + | $\times$ | - | $\times$ | + |

Long-run overidentifying restrictions can be tested as follows. Denote by $\Delta G_{t}$ a vector process consisting of the first differences of labor productivity, real wage, hours per capita, GDP, consumption and investment. Let us define $B(1)=\left[b_{1}(1)^{\prime} \cdots b_{n}(1)^{\prime}\right]^{\prime}$ and call $B_{\Delta G}(1)$ the $6 \times q$ matrix of the long run structural multipliers of the three common
shocks to the variables in vector $G$. Over-identifying constraints on the long run multipliers can be expressed as $\Omega \operatorname{vec}\left(\mathrm{B}_{\Delta \mathrm{G}}(1)\right)=0$ where $\Omega$ has been chosen appropriately. The null hypothesis has the following form:

$$
\begin{equation*}
H_{0}: \Omega \operatorname{vec}\left(\mathrm{B}_{\Delta \mathrm{G}}(1)\right)=0 \tag{5.12}
\end{equation*}
$$

The test for the overidentifying restrictions is based on the minimal distance of the estimated $\Omega \operatorname{vec}\left(\mathrm{B}_{\Delta \mathrm{G}}(1)\right)$ from the null (see appendix 4 for details). The test statistics is similar to the Hansen J-statistics and is distributed as a chi-squared with degrees of freedom equal to the number of overidentifying restrictions.

We first test for the existence of two shocks long-run neutral neutral on laborproductivity and output. The zero restrictions imposed on the long-run multipliers are:

|  | tek | 2nd shock | 3rd shock |
| :--- | :---: | :---: | :---: |
| $y / l$ | $\times$ | 0 | 0 |
| $y$ | $\times$ | 0 | 0 |

We have one overidentifying restriction (four total restrictions minus three restrictions needed for just-identification), and the test statistics is 4.88 with a p-value of 0.03 . The null is hence rejected at $5 \%$ level. This result rules out the possibility that there are two non-technolgy shocks long-run neutral on output.

We then test for the existence of a shock that is long-run neutral on the selected real variables. The zero restrictions imposed on the long-run multipliers are:

|  | tek | demand | money |
| :--- | :---: | :---: | :---: |
| $y / l$ | $\times$ | 0 | 0 |
| $w$ | $\times$ | 0 | 0 |
| $l$ | 0 | $\times$ | 0 |
| $y$ | $\times$ | $\times$ | 0 |
| $c$ | $\times$ | $\times$ | 0 |
| $i$ | $\times$ | $\times$ | 0 |

We have 6 overidentifying restrictions and the test statistics is equal to 4.72 , with a p-value of 0.58 . These results are thus compatible with the existence of two shocks long-run neutral on productivity and only one shock long-run neutral on real variables. Long-run restrictions, however, are not sufficient to disentangle the following cases:

| a) | Tek | Dem | Money |
| :---: | :---: | :---: | :---: |
| b) | Tek | Dem | Supply |
| c) | Tek | Money | Supply |

To select one of these cases, we perform two types of exercises. In exercise 1, we impose the identification restriction for the technology shock that we have tested above. This implies to fix $a$ and $b$ in (5.11). We identify $c$ by imposing the other condition tested, namely that only one shock is long-run neutral on output. Figure 2 illustrates results on the impulse response functions of GDP, labor productivity, inflation and the
federal fund rate to the identified shocks. Notice that the results correspond to those just-identified restrictions which are a sub-set of the overidentified restrictions which we have tested. These are: technology shock is the only one with permanent effects on labor productivity and there is only one shock which is long-run neutral on output. ${ }^{3}$

Figure 2. Impulse response functions of key variables to the common shocks


The first column displays the effects of a technology shock. The second shock has been labeled demand, because it has significantly positive conditional correlations between output and prices. In addition, it is non-neutral in the long-run on output and thus cannot be confused with a monetary shock. Concerning the last shock, we are not able to discriminate between money and supply. The third shock displays long-run neutrality on output as required for a money shock, but conditional correlation between output and prices is negative (though not significant), as required for a supply shock. In conclusion, we can safely rule out case c, but we still remain uncertain between cases a and b since, the conditional correlations are not statistically significnt.

In order to check for the robustness of our conclusions, we performed a second exercise. In exercise 2, we fix $a$ and $b$ as above and examine rotations corresponding to those values of $c \in(0, \pi)$ such that the third shock has opposite effects on prices

[^2]and output in the first year after a shock. These are the rotations compatible with the existence of a supply shock.

Figure 3 below illustrates the impulse response functions corresponding to those values of $c$ (with a grid 0.01). The similarity of the responses with the one obtained in the previous just-identified exercise is striking. In particular, the demand shock has the same impulses and the long-run effect of the supply shock on output appears to be negligible. In addition, qualitative results are quite robust as the responses are not much affected by the choice of a particular rotation.

Figure 3. Impulse response functions of key variables to the common shocks


Notice that the long-run neutral shock on output, which can possibly be identified as money, does not satify the traditional sign restrictions suggested by theoretical models because it has an opposite effect on prices and output.

We conclude that the there are two major macroeconomic shocks in the economy, technology and demand. The third shock is small and can be labeled supply. We will keep this interpretation fron now on.

One feature is immediately evident. The supply shock, in the Greenspan's regime, has been very small, suggesting that it might have been irrelevant for policy.

As for the other shocks, we can see that, in the short-run, the main effect on output is caused by the demand shock, while, in the long-run, technology prevails.

Figure 4 reports the variance decompositions for the same variables as in Figure 2 and also complements the analysis by illustrating results for two other variables of interest: consumption and investment. Notice that the technology shock explains almost all the variance of consumption at all horizons, suggesting that only permanent
shocks affect consumption while demand explains the bulk of investment variance with technology catching up in the long-run. The Figure on investment helps us intepreting the nature of our non-monetary demand shock and suggest that this is the type of investment shock suggested by Rotemberg and Woodford, 1992. In a separate paper (Giannone, Reichlin and Sala, 2002) we exploit the (rich) cross-sectional information to interpret it more precisely. We skip the discussion here since this is not the focus of the paper.

Figure 4. Variance decompositions


Y


Y/L





Given the small or insignificant effect of the supply shock, a reasonable characterization of the macroeconomy corresponds to the choice $q=2$ with only two pervasive and sizeable shock, and supply, at least in this sample, not being one of them. To check for the robustness of our results on the impulses of demand technology, we have derived results for the choice $q=2$. From Figures 5 and 6 it is easily seen that results are basically unchanged from those based on $q=3$. From now on we will conduct the analysis under alternative specifications, $q=2$ and $q=3$.

Figure 5. Impulse response functions of key variables to the common shocks ( $\mathrm{q}=2$ )









Figure 6. Variance decompositions (q=2)


Y




C



Since the money shock is found not to be common to all blocks of variables in the system, while the idiosyncratic shock account for $27 \%(q=2)$ or $20 \%(q=3)$ of the federal fund rate, it is natural to ask whether the correlation structure of the idiosyncratic variance matrix reveals any interesting feature of the money shock. Table 3 below reports correlation coefficients between the idiosyncratic components of the federal fund rate and those of selected series.

Table 3. Correlation between idio components of selected series and $r_{t}$

|  | $\Delta y$ | $\Delta \pi$ | $\Delta r^{1 \text { yrs }}$ | $\Delta r^{3 y r s}$ | $\Delta r^{5 y r s}$ | $\Delta r^{10 y r s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 shocks | 0.22 | -0.21 | 0.50 | 0.40 | 0.31 | 0.26 |
| 3 shocks | 0.15 | -0.01 | 0.43 | 0.35 | 0.26 | 0.05 |

The correlation is estimated to be rather strong with respect to other interest rates and declining with maturity ( 1 year, 3 years, 5 years and 10 years). This simply reflects local correlations due to the term structure. Notice also that the correlation with output and inflation have the correct sign.

Our finding of a negligible effect of the monetary shock on output confirms the result of most of the literature (see Faust, 1998 for an analysis of the robustness of this conclusion). As showed in Faust the larger the dimension of the model, the smaller the percentage of the variance of GDP explained by the monetary shock and it is not surprising, therefore, that, by conditioning on a large information set, we obtain a small effect of monetary shocks on output.

If monetary shocks has no pervasive effect, the interesting part of the story is in the analysis of the systematic component of policy. This is where we now turn.

## 6 Conditional Monetary rules

An optimal monetary policy, designed to fully stabilize prices, should call for a differentiated response of the policy instrument conditional to the origin of the shock (Woodford, 2002, Clarida, Gali and Gertler, 1999). Our framework allows to estimate conditional rules. Therefore, our estimates will provide richer results than in the classical Taylor rule case since the systematic response to, say, inflation, will be split between the response to the component of inflation generated by a technological shock, the component of inflation generated by a demand shock, and so on.

The three shocks we have identified are demand, technology and supply so that, with obvious notation, we have:

$$
u_{t}=\left(u_{t}^{d}, u_{t}^{t e k}, u_{t}^{s}\right)^{\prime}
$$

- Unrestricted conditional policy rule

Given results of the previous Section, the conditional unrestricted rules estimated from the factor model can be written as:

$$
\begin{equation*}
\tilde{r}_{t}=b_{r}^{d}(L) u_{t}^{d}+b_{r}^{s}(L) u_{t}^{s}+b_{r}^{t e k}(L) u_{t}^{t e k} . \tag{6.13}
\end{equation*}
$$

Let us here recall some features of the impulse response functions that we can exploit in the analysis of monetary poicy rule.

1. The federal fund rate responds mainly to the demand shock while the response to the technology and supply shocks is very small.
2. The effect of the demand shock is larger on the interest rate than on inflation indicating that it affects the real interest rate positively (Taylor principle).
3. The technology shock has a negative effect on inflation in the short-run and a neutral effect at one year horizon or longer.
4. In response to a positive demand shock, both output and inflation increase; in response to a technology shock, inflation at one year horizon or longer does not move while the output response is large and significant only at horizons longer than one year and a half; in response to a supply shock, we have both a decrease in ouput and an increase in inflation.

Without imposing any restriction on the particular policy rule, we can already draw some conclusions on policy by analysing the impulse response functions to different variables in the system. In particular, the fourth feature higlighted above tells us that the supply shock is the only one amongst the three that calls for a response of monetary policy that implies a tradeoff between inflation and output. Only in that case, an increase of the federal fund rate targeted at inflation would have a cost in terms of cyclical output. Since, in the Greenspan era, the importance of supply shocks seems to have been negligible and technology shocks have been favorable, we may infer that the usual Phillips curve tradeoff has not burdened monetary policy in this particular sample period.

## - Conditional restricted policy rule

Let us assume that policy targets inflation (or inflationary expectations). The policy coefficient (or better the filter) is implicit in our estimates of the impulse response functions. We have:

$$
\begin{equation*}
r_{r}=\phi_{\pi}(L) \pi_{t}=\phi_{\pi}^{d}(L) b_{\pi}^{d}(L) u_{t}^{d}+\phi_{\pi}^{s}(L) b_{\pi}^{s}(L) u_{t}^{d}+\phi_{\pi}^{t e k}(L) b_{\pi}^{d}(L) u_{\text {tek }}^{d} \tag{6.14}
\end{equation*}
$$

and the filters are identified from (6.13) and (6.14) as the following ratios:

$$
\phi_{\pi}^{j}(L)=b_{r}^{j}(L)\left[b_{\pi}^{j}(L)\right]^{-1}
$$

with $j=d, t e k, s$.

Table 4 reports two summary statistics describing the filter: $\phi_{\pi}^{j}(1)=\sum_{h} \phi_{\pi, h}^{j}$, which provides information on the long-run relation between the two variables and the mean $\operatorname{lag} \mathrm{ML}^{j}=-\sum_{h} \phi_{\pi, h}^{j} h / \sum_{j} \phi_{\pi, h}^{j}{ }^{4}$.

Table 4. Summary of the filter
equation: $\Delta r_{t}^{j}=\phi_{\pi}^{j}(L) \Delta \pi_{t}^{j}$

|  | Tek | Dem | Supply |
| :---: | :---: | :---: | :---: |
| $\phi_{\pi}^{j}(1)$ | 0.58 | $1.59^{* *}$ | -1.55 |
| $\mathrm{ML}^{j}$ | -1.22 | 0.50 | -2.40 |

**: different from zero at $5 \%$ level

Monetary policy responds to inflation with a coefficient significantly larger than one, conditionally on demand shocks, confirming what appeared to be the case from visual inspections of the impulse response functions. The response to the technology shock, on the other hand, is not significantly different from zero and neither is the response to the supply shock, although a value less than one is within the boostrapped confidence intervals. It should be pointed out that the bootstrap confidence intervals are very small for the coefficients of the filter conditional on demand while rather large for those conditional on technology and supply. The result on the response of the federal fund rate to inflation, conditional on demand does not only gives us a point estimate larger than one but tells us that a coefficient smaller than one is rejected at the $5 \%$ significance level. This is indeed a very strong evidence on the Taylor principle, conditionally on demand.

The estimates on the mean lag are not very reliable, not significant and not robust. If we believe the point estimates, we would conclude that the Federal Funds Rate leads inflation by half a quarter indicating a slightly forward-looking behaviour.

Notice that since the "clean" variables are collinear, it makes no sense to include an extra variables amongst the Central Bank target: the output gap is collinear with inflation, no matter what is the definition we want to give to it. However, to shed some light on the relative weight of output and inflation in the policy function, we can express inflation in terms of output and find the corresponding filter. We have:

$$
\Delta \pi_{t}^{j}=\gamma^{j}(L) \Delta x_{t}^{j}
$$

and $\Delta r_{t}=\phi_{\pi}^{j}(L) \gamma^{j}(L) b_{x}^{j}(L) u_{t}^{j}$
Table 5 reports results on $\gamma^{j}(1)$ and the mean lag.

Table 5. Summary of the filter

$$
\Delta \pi_{t}=\gamma^{j}(L) b_{x}^{j}(L) u_{t}^{j}
$$

[^3]|  | Tek | Dem | Supply |
| :---: | :---: | :---: | :---: |
| $\gamma^{j}(1)$ | -4.85 | $1.93^{* *}$ | -0.25 |
| $\mathrm{ML}^{j}$ | -9.88 | 0.87 | 35.43 |

**: different from zero at $5 \%$ level

Table 5 shows two results. First, the long run coefficient linking inflation and output conditionally on demand is larger than that linking the federal fund rate and inflation, suggesting that the long-run relation between the federal fund rate and output, conditionally on demand, is about 3 . Since ouput conditional on demand seems to lead inflation, this may simply mean that the federal fund rate is set so as to target future inflation, but, of course, we cannot really identify the two coefficients separately. Notice, that the sign of the long-run relations tell us that the long-run Phillips curve is positively sloped if conditional on demand, but negativey sloped if conditional on supply and technology. Finally, let us remark that since there is no consensus on what is the correct measure of the output gap, this result should be interpreted with caution and it can be related to results on the role of the output gap in policy functions found in the literature only if we are ready to accept the notion that the output gap is that part of output generated by the demand shock. A measure such as the Hodrick-Prescott filter on output, however, would give us an estimate just slighy lower than one, therefore not changing our qualitative result.

## 7 Structural VAR, rules and structural factor models

Let us go back to Section 2 and to the problems outlined there about the difficulties in interpreting the parameters of the rule within a structural VAR framework. What are the advantages of structural factor models such as the one used here over the structural VAR framework for understanding monetary rules and monetary shocks?

It is clear that structural factor models belong to the same family of structural VARs as far as view of modelization of policy. Factor models, however, present few advantages. Let us suggest the following list:

1. Information set $\Omega_{t}$. Since in our approach to the estimation of the factors we are not constrained by the dimension of the panel, $\Omega_{t}$ can be "as large as we want". In other words, we can have an encompassing informational assumption where apriori both the econometrician and the policy makers use all information available and then they estimate the approximate rank of the covariance matrix at hand to use a statistically based reduction criterion.
2. Measurement error. The problem of interpretation caused by measurement error present in the SVAR modelization, is addressed explicitely in the factor methodology where we model the signal extraction problem solved by monetary authorities.
3. Identification In a factor model we can use information on $n$ variables to identify $q$ shocks and $n \gg q$. This implies that even a small set of consensus restrictions
generate testable over-identified restrictions. In general, the indeterminacy problem of structural VAR is the same in structural factor models, but while in VAR the indeterminacy is generated by a $n$-dimensional rotation, in structural factor models the rotation is only of dimension $q$.
4. Systematic policy. As in SVARs, information on systematic policy is obtained from the row corresponding to the federal funds rate, but the coeffcients estimates are free from those interpretation problems due to measurement error and heterogeneity of information outlined in Section 2.

As for the estimation of rules, the advantage of our framework is that we obtain an expression of the federal fund rate in terms of exogenous shocks and policy can therefore be evaluated conditionally on the particular shock hitting the economy. We have seen that, when the rule is expressed in terms of observable variables rather than shocks the policy parameters have an ambiguous interpretation. It is also important to remark, that our results indicate that the relevant stochastic rank of the economy is between two and three which implies that macrovariables are collinear and that it is more transparent to write the rule in terms of one variable only, conditional on different shocks rather than in terms of different unconditional variables.

## 8 What about Greenspan?

As it has been widely commented on, the Greenspan era has been an era of remarkable output and inflation stability. Is this a result of luck or wisdom? We have shown that policy has been wise in two dimensions. First, the Central Bank has distinguished between sources of change, technology versus demand, and has reacted exploiting information on the different dynamic effects of these sources. Conditionally on technology, the federal fund rate was not adjusted while, conditionally on demand, it has followed the Taylor principle, which is stabilizing. But could have Greenspan done worse? The shock that poses a dilemma for policy, since it affects output downward and inflation upward, is the supply shock and we have shown that, during the Greenspan era, supply shocks have been negligible. Therefore Greenspan did not face any tradeoff and, as a consequence, the stabilization performance of his regime is largely due to luck. In the previous regime, where supply shocks have been larger, was the Central Bank policy different? This question goes beyond the scope of this work and it is one we are analysing in a separate paper.

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## Appendix 1: Dynamic eigenvalues

We start by estimating the spectral-density matrix of $X_{t}=\left(\begin{array}{lll}x_{1 t} & \cdots & x_{n t}\end{array}\right)^{\prime}$. Let us denote the theoretical matrix by $\boldsymbol{\Sigma}(\theta)$ and its estimate by $\hat{\boldsymbol{\Sigma}}(\theta)$. The estimation is accomplished by using a Bartlett lag-window of size $M=12$, i.e. by computing the sample auto-covariance matrices $\hat{\boldsymbol{\Gamma}}_{k}$, multiplying them by the weights $w_{k}=1-\frac{|k|}{m+1}$ and applying the discrete Fourier transform:

$$
\hat{\boldsymbol{\Sigma}}_{x}(\theta)=\frac{1}{2 \pi} \sum_{k=-m}^{m} w_{k} \cdot \hat{\boldsymbol{\Gamma}}_{k} \cdot e^{-i \theta k}
$$

The spectra were evaluated at 101 equally spaced frequencies in the interval $[-\pi, \pi]$, i.e. at the frequencies $\theta_{h}=\frac{2 \pi h}{100}, h=-50, \ldots, 50$.

Then we performed the dynamic principal component decomposition (see Brillinger, 1981). For each frequency of the grid, we computed the eigenvalues and eigenvectors of $\hat{\Sigma}(\theta)$. By ordering the eigenvalues in descending order for each frequency and collecting values corresponding to different frequencies, the eigenvalue and eigenvector functions $\hat{\lambda}_{j}(\theta)$ and $\hat{U}_{j}(\theta), j=1, \ldots, n$, are obtained. The function $\hat{\lambda}_{j}(\theta)$ can be interpreted as the (sample) spectral density of the $j$-th principal component series and, in analogy with the standard static principal component analysis, the ratio

$$
p_{j}=\int_{-\pi}^{\pi} \lambda_{j}(\theta) d \theta / \sum_{h=1}^{n} \int_{-\pi}^{\pi} \lambda_{h}(\theta) d \theta
$$

represents the contribution of the $j$-th principal component series to the total variance in the system.

## Appendix 2: Link between the static factor representation and the dynamic representation

Given that the filters $c_{i}(L)$ are assumed to be of finite order, (4.4) can be rewritten as:

$$
x_{i t}=C_{i}^{\prime} F_{t}+\xi_{i t}
$$

where $x_{i t}$ is of dimension $n \times 1, C_{i}$ of dimension $r \times n$, where $r=q(s+1)$ and $F_{t}$ :

$$
F_{t}=\left[\begin{array}{llll}
f_{t}^{\prime} & f_{t-1}^{\prime} & \cdots & f_{t-s}^{\prime}
\end{array}\right]^{\prime}
$$

of dimension $r \times 1$.
Written in this form, $F_{t}$ can be estimated consistently using the first $r$ static principal components of the $x_{i t}$ 's (Stock and Watson, 1999).

Having obtained the $F_{t}$ 's, the $q$ common shocks can be derived from a VAR on the $F_{t}$ 's of the form $A(L) F_{t}=D u_{t}$. Identification can be obtained by fixing a static rotation of these shocks.

Let us explore this point further. If the length of the filter $a(L)$ is $p \leq s+1$, we can write $F_{t}$ in $\operatorname{VAR}(1)$ form.

We have:

$$
F_{t}=A F_{t-1}+D u_{t}
$$

where:

$$
A=\left(\begin{array}{cccccccc}
a_{1} & a_{2} & \cdots & a_{p} & 0 & \cdots & & 0 \\
I & 0 & \cdots & 0 & 0 & \cdots & & 0 \\
& & \ddots & & & & & \\
& & & & & \ddots & & \\
0 & 0 & \cdots & 0 & 0 & \cdots & I & 0
\end{array}\right)
$$

and:

$$
D=\left(\begin{array}{c}
B \\
0 \\
\vdots \\
0
\end{array}\right)
$$

If the length of the filter $a(L)$ is $p>s+1$, we can follow the same procedure as above, but we need to impose a larger number of lags of $F_{t}$ in order to span the space of the $u_{t}$ 's. In this case, $F_{t}$ has a $\operatorname{VAR}(p-s)$ representation:

$$
F_{t}=A_{1} F_{t-1}+\ldots+A_{p-s} F_{t-p+s}+D u_{t}
$$

where the matrices $A_{1} \ldots A_{p-s}$ have dimension $q(s+1) \times q(s+1)$. As an extreme, consider the case $s=0$. In this case $F_{t}=f_{t}$ and the $\operatorname{VAR}(p)$ will simply be equation (5.10) :

$$
f_{t}=a_{1} f_{t-1}+\ldots+a_{p} f_{t-p}+B u_{t}
$$

Once the VAR form for $F_{t}$ is obtained, we can obtain the residual $v_{t}=D u_{t}$. Notice that the dimension of $F_{t}, r$ and the dimension of $u_{t}, q$, are unknown. Once $r$ is specified, however, $q$ can be identified from the rank of the variance-covariance matrix of $v_{t}, \Sigma=D D^{\prime}$. In the empirical application we specified $r$ following the information criterion proposed by Bai and Ng (2001) and found $r=4$.

Write $N$ for a version of $\Sigma^{1 / 2}$, i.e. $N=D Q$ with $Q$ a $q \times q$ orthonormal matrix, and $N^{+}$for its generalized inverse ${ }^{5}$.

The $q$-vector of orthogonal common shocks can be expressed as:

$$
\omega_{t}=N^{+} v_{t}
$$

The latter obviously spans the same space spanned by the $u_{t}$.

[^4]We also have $\chi_{t}=C F_{t}$ which implies

$$
\chi_{t}=K \chi_{t-1}+C D u_{t}
$$

where $K=C A C^{\prime}$.
We are now ready for the second step. Just-identification of the $u$ 's consists in identifying an orthonormal matrix $R$ which rotates $\omega_{t}$. Since the rotation matrix is of dimension $q \times q$ and the $u$ 's are orthonormal, just-identification is obtained with $q(q-1) / 2$ restrictions.

The impulse response function of $x_{i t}$ associated with the common shocks $u$ 's are given by:

$$
C_{i} \Theta(L) u_{t}
$$

where $u_{t}=R^{\prime} \omega_{t}, \Theta(L)=A(L)^{-1} N R$.

## Appendix 3: Estimation

We estimate the static factors $f_{t}$ as linear combinations of (the present of) the observable variables $x_{i t}, i=1, \ldots, n$. We need only a set of $r=q(s+1)$ variables forming a basis for the linear space spanned by the $u_{h t}$ 's and their lags. We can then obtain $\hat{\chi}_{j t}$ by projecting $\chi_{j t}$ on such factors. Two strategies have been used in the literature, Stock and Watson, 1999 and Forni et al., 2001. Forni et al., 2000 propose a method based on frequency domain which is what we used in Section 2.

Otherwise we have followed Stock and Watson and estimated $F_{t}$ as the first $r$ static principal components of the $x$ 's:

$$
\hat{F}_{t}=\Lambda^{-1 / 2} V^{\prime} X_{t}
$$

where $\Lambda$ is the $r \times r$ diagonal matrix of the first $r$ eigenvalues of the sample covariance matrix of the $X$ 's, $V$ is the ( $n \times r$ ) matrix of the corresponding eigenvectors and

$$
X_{t}=\left[\begin{array}{lll}
x_{1 t}^{\prime} & x_{2 t}^{\prime} & \cdots x_{n t}^{\prime}
\end{array}\right]^{\prime} .
$$

Stock and Watson (1998) show that the static projection $\hat{\chi}_{i t}=\hat{C}_{i}^{\prime} \hat{F}_{t}$ converges to $\chi_{i t}$ as both the number of series $(n)$ and the sample size $(T)$ tend to infinity, where

$$
\hat{C}_{i}=\left(\frac{1}{T} \sum_{t=1}^{T} \hat{F}_{t} \hat{F}_{t}^{\prime}\right)^{-1}\left(\frac{1}{T} \sum_{t=1}^{T} \hat{F}_{t} x_{i t}\right)=V_{i} \Lambda^{1 / 2}
$$

and $V_{i}$ denote the $i$-th row of $V$.
Consider now the OLS estimator of a VAR on the estimated factors ${ }^{6}$ :

$$
\hat{A}(L) \hat{F}_{t}=\hat{v}_{t}, \quad \hat{\Sigma}=\frac{1}{T-p+1} \sum_{t=t+1}^{T} \hat{v}_{t} \hat{v}_{t}^{\prime}
$$

[^5]and write $\hat{N}=\hat{P} \hat{M}^{1 / 2}$, where $\hat{M}$ is the $q \times q$ diagonal matrix of the first $q$ eigenvalues of the $\hat{\Sigma}$ and $\hat{M}$ is the $(r \times q)$ matrix of the corresponding eigenvectors.

We will show that $\hat{C}_{i} \hat{A}(L)^{-1} \hat{N}$ is a consistent estimator of the population impulse response functions $C_{i} A(L)^{-1} N$. This result is an immediate consequence of the following lemma.

Lemma. Under assumptions A-E given in Bai (2001), there exists an invertible matrix $H$ (whose dependence on $T$ and $n$ is dropped for notational simplicity) such that:
(i) $\frac{1}{T} \sum_{t=1}^{T}\left\|\hat{F}_{t}-H^{\prime} F_{t}\right\|^{2}=O_{p}\left(\min \left\{T^{-1}, n^{-1}\right\}\right)$
(ii) $\left(\hat{C}_{i}-H^{-1} C_{i}\right)=O_{p}\left(\min \left\{T^{-1 / 2}, n^{-1}\right\}\right)$
(iii) $\left\|\hat{\Gamma}_{h}-H^{\prime} \Gamma_{\mathrm{h}} \mathrm{H}\right\|=\left\|\frac{1}{\mathrm{~T}-\mathrm{h}} \sum_{\mathrm{t}=\mathrm{h}+1}^{\mathrm{T}} \hat{\mathrm{F}}_{\mathrm{t}} \hat{\mathrm{F}}_{\mathrm{t}-\mathrm{h}}^{\prime}-\mathrm{H}^{\prime} \mathrm{E}\left[\mathrm{F}_{\mathrm{t}} \mathrm{F}_{\mathrm{t}-\mathrm{h}}^{\prime}\right] \mathrm{H}\right\|=\mathrm{o}_{\mathrm{p}}(1)$
as $\min \{T, n\} \rightarrow \infty$.
Proof. For (i) and (ii) see Bai(2001) Lemma A. 1 and Theorem 2, respectively.
To prove part (iii), notice that from Holder's inequality and from the triangular inequality, it follows:

$$
\begin{aligned}
& \left\|\frac{1}{T-h} \sum_{t=h+1}^{T} \hat{F}_{t} \hat{F}_{t-h}^{\prime}-H \mathrm{E}\left[\mathrm{~F}_{\mathrm{t}} \mathrm{~F}_{\mathrm{t}-\mathrm{h}}^{\prime}\right] \mathrm{H}^{\prime}\right\| \\
= & \| \frac{1}{T-h} \sum_{t=h+1}^{T} \hat{F}_{t}\left(\hat{F}_{t-h}-H^{\prime} F_{t-h}\right)^{\prime}+\frac{1}{T-h} \sum_{t=h+1}^{T}\left(\hat{F}_{t}-H^{\prime} F_{t}\right) F_{t-h}^{\prime} H \\
& +\frac{1}{T-h} \sum_{t=h+1}^{T} H F_{t} F_{t-h}^{\prime} H^{\prime}-H \mathrm{E}\left[\mathrm{~F}_{\mathrm{t}} \mathrm{~F}_{\mathrm{t}-\mathrm{h}}^{\prime}\right] \mathrm{H}^{\prime} \| \\
\leq & \sqrt{\frac{1}{T-h} \sum_{t=h+1}^{T}\left\|\hat{F}_{t}\right\|^{2}} \sqrt{\frac{1}{T-h} \sum_{t=h+1}^{T}\left\|\hat{F}_{t-h}-H^{\prime} F_{t-h}\right\|^{2}} \\
& +\sqrt{\frac{1}{T-h} \sum_{t=h+1}^{T} \| \hat{F}_{t}-\left.H^{\prime} F_{t}\right|^{2}} \sqrt{\sum_{t=h+1}^{T}\left\|H^{\prime} F_{t-h}\right\|^{2}} \\
& +\left\|\frac{1}{T-h} \sum_{t=h+1}^{T} H F_{t} F_{t-h}^{\prime} H^{\prime}-H \mathrm{E}\left[\mathrm{~F}_{\mathrm{t}} \mathrm{~F}_{\mathrm{t}-\mathrm{h}}^{\prime}\right] \mathrm{H}^{\prime}\right\|
\end{aligned}
$$

The last term is $o_{p}(1)$ as $T \rightarrow \infty$. The first and second terms are $o_{p}(1)$ from (i) and because both $\frac{1}{T-h} \sum_{t=h+1}^{T}\left\|F_{t}\right\|^{2}$ and $\frac{1}{T-h} \sum_{t=h+1}^{T}\left\|\hat{F}_{t}\right\|^{2}$ are $O_{p}(1)$ as $\min \{T, n\} \rightarrow \infty$ (cfr. Bai 2001).

Proposition. For each $i$,

$$
\hat{C}_{i} \hat{A}(L)^{-1} \hat{N} \xrightarrow{p} C_{i} A(L)^{-1} N \quad \text { as } \min \{T, n\} \rightarrow \infty
$$

Proof. $\hat{A}(L), \hat{M}$ and $\hat{P}$ are continuous functions of the sample covariance matrices $\hat{\Gamma}_{h}$, so from result (iii) of the previous lemma and the continuous mapping theorem:
(iv) $\left\|\hat{A}(L)-H A(L) H^{\prime-1}\right\|=o_{p}(1)$
(v) $\left\|\hat{P} \hat{M}^{1 / 2}-H^{\prime} N\right\|=o_{p}(1)$
as $\min \{T, n\} \rightarrow \infty$, where $N$ is a version of $\Sigma^{1 / 2}$
Putting togheter (ii), (iv) and (v), the result follows.

## Appendix 4: Testing

Consider a vector process $G_{t}$ consisting a set of difference stationary series, subset of $x_{t}$ and write $B_{G}(1)$ for the relative long run multiplier. We consider the following hypothesis testing problem:

$$
H_{0}: \Omega \operatorname{vec}\left(\mathrm{B}_{\mathrm{G}}(1)\right)=0
$$

Denote by $\hat{B}(1)=\hat{U}_{q}\left(\hat{\Lambda}_{q}\right)^{1 / 2}$, where $\hat{\Lambda}_{q}=\operatorname{diag}\left(\hat{\lambda}_{1}(1), \ldots \hat{\lambda}_{q}(1)\right)$ and $\hat{V}_{q}=\left(\hat{U}_{1}, \ldots, \hat{U}_{1}\right)$. It can be shown that there exist a unitary $(q \times q)$ matrix, $Q$, such that

$$
\sqrt{\frac{3 T}{2 m}}[\operatorname{vec}(\hat{\mathrm{~B}}(1) \mathrm{Q}-\mathrm{B}(1))] \xrightarrow{d} \mathrm{~N}(0, \boldsymbol{\Theta}(\mathrm{Q}))
$$

as both the cross-sectional and the time dimensions tend to infinity at appropriate rate.

Write $\hat{B}_{G}(1)$ the matrix formed by taking the rows of $\hat{B}(1)$ relative to the variable in $G$. Then it can be shown that if $H_{0}$ holds then:

$$
\hat{J}=\frac{3 T}{2 m}\left(\min _{Q \mid Q Q^{\prime}=I_{q}}\left\{\Omega \operatorname{vec}\left(\hat{\mathrm{~B}}_{\mathrm{G}}(1) \mathrm{Q}\right)\right)^{\prime}(\boldsymbol{\Theta}(\mathrm{Q}))^{-1}\left(\Omega \operatorname{vec}\left(\hat{\mathrm{~B}}_{\mathrm{G}}(1) \mathrm{Q}\right)\right\}\right) \xrightarrow{d} \chi_{(v)}^{2}
$$

where $v$ is equal to the number of overidentifying restriction. Define

$$
\left.Q^{*}=: \operatorname{argmin}_{\mathrm{Q} \mid \mathrm{QQ}}{ }^{\prime}=\mathrm{I}_{\mathrm{q}} \Omega \operatorname{vec}\left(\hat{\mathrm{~B}}_{\mathrm{G}}(1) \mathrm{Q}\right)\right)^{\prime}(\boldsymbol{\Theta}(\mathrm{Q}))^{-1}\left(\Omega \mathrm{vec}\left(\hat{\mathrm{~B}}_{\mathrm{G}}(1) \mathrm{Q}\right)\right.
$$

$\hat{B}_{G}(1) Q^{*}$ is a natural estimator for the structural long-run multiplier $B_{G}(1)$ under $H_{0}$.

## Appendix 5: Asymptotic distribution and Monte Carlo bands for the impulse response functions

Under suitable assumptions on the asymptotic behavior of $n$ and $T(T \ll n)$, the error in the estimation of the factors can be ignored (notes available from the authors). This
implies that, in the construction of the confidence bands, we can treat the factors as known.

In addition, uncertainty regarding the matrix of eigenvectors $V$ becomes asymptotically negligible. The only source of uncertainty to be taken into account in computing confidence intervals is the one regarding the estimation of the VAR for $F_{t}$.

After having estimated the model and computed the impulse response functions as explained in the text, we compute 500 new series for the factors $F_{t}$ and we re-estimate and re-identify the whole model.

Each new sample $j$ is generated as follows. We first extract with replacement from the series of structural shocks: $\left\{\hat{u}_{i}\right\}_{i=p+1}^{T}$ and generate a new series of shocks $\left\{\hat{u}_{i}^{(j)}\right\}_{i=p+1}^{T}$.

Second, we use the new series of shocks, the estimated $\hat{A}, \hat{P}$ and $\hat{M}$ matrices to compute the new $F_{t}^{(j)}$.

After having obtained a new $F_{t}^{(j)}$ vector, we perform the whole estimation and identification procedure to get new impulse response functions. We sort the 500 responses to obtain the empirical distribution and finally we select the $\frac{\alpha}{2}$-th and the ( $1-\frac{\alpha}{2}$ )-th percentile to obtain the $\alpha$ probability level confidence intervals. In the Figures, we report $95 \%$ confidence intervals.

## Appendix 6: Data and data treatment

0: no transformation
1: first difference
2. logarithm

3: first difference of logarithm
4: fourth difference of first difference of logarithm


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[^0]:    ${ }^{1} z_{h t}$ and $z_{k t}$ are mutually orthogonal at any lead and lag for $h \neq k$ and the filters are normalized in such as to guarantee orthogonality $h \neq k$ and unitary variance. For more details, see Brillinger (1981) and Appendix 1.

[^1]:    ${ }^{2}$ Exception is Gali, López-Salido, Vallés (2000) who, however, only conditions with respect to the technology shock.

[^2]:    ${ }^{3}$ An alternative natural choice would have been to chose that rotation that minimizes the distance with the restrictions which we have tested. Notice, however, that, if the restrictions hold, it is sufficient to impose a subset of just-identifying restrictions to have a consistent estimator of the structural model. The strategy we have chosen is motivated by simplicity. The difficulty of estimating the "minimum distance" matrix is that Giannone's test is developed in the frequency domain and the translation in the time domain is not straightforward.

[^3]:    ${ }^{4}$ Since variables are non-stationary, they are all expressed in first differences.

[^4]:    ${ }^{5}$ A particular choice is $N=P M^{1 / 2}$ and $N^{+}=M^{-1 / 2} P^{\prime}$ where $M$ is the $q \times q$ diagonal matrix of the non zero eigenvalues of $\Sigma$ and $P$ is the $r \times q$ matrix of the corresponding eigenvectors

[^5]:    ${ }^{6}$ In the case in which the order of the VAR is larger than one, we cannot use OLS estimation because of collinearity and we perform a reduced rank regression, i.e. we regress $F_{t}$ on the first $r+(p-1)$ principal components of the variance matrix of the stacked vector of the lagged $F$ 's.

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