Tacit Collusion, Cost Asymmetries and Mergers

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December 2002

Abstract

This paper contributes to the analysis of tacit collusion in quantity setting supergames involving cost-asymmetric firms. Asymmetry is dealt with by assuming that firms have a different share of a specific asset which affects marginal costs. The model extends optimal punishment schemes in the style of Abreu (1986, 1988) and provides conditions for industry-wide collusion to be enforced. From the analysis of the impact of asset transfers on the sustainability of tacit collusion, merger policy implications can be drawn. In particular, it is shown that if the merger induces an increase in the inequality of asset holdings, this will hinder collusion.

Keywords: Mergers; Joint dominance; Collusion; Optimal penal codes

JEL classification: D43; L41

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*I am grateful to Massimo Motta and Pierpaolo Battigalli for supervision and guidance. Moreover, I would like to thank Pedro Pita Barros, Luis Cabral, Robert Rothschild, Chiara Fumagalli, Patrick Rey, Larry Samuelson, John Sutton and seminar participants at the European University Institute, Universitat Pompeu Fabra, The University of Warwick, ISEG (Lisbon), IUI (Stockholm), 27th Conference of the EARIE (Lausanne), 16th Congress of the EEA (Lausanne) and at the ASSET 2001 Euroconference for their valuable comments on early versions of the paper. Special thanks are due to Karl Schlag for many helpful discussions and insightful suggestions. Of course, any errors remain my own. This article is based on chapter 2 of my dissertation. Financial support from Fundação para a Ciência e Tecnologia and Direcção Geral dos Assuntos Comunitários is gratefully acknowledged.

1 Introduction

It has long been recognized that "...the more cost functions differ from firm to firm, the more trouble the firms will have maintaining a common price policy, and the less likely joint maximization of profits will be"(Scherer (1980), p. 205). Unfortunately, however, most of the studies which have discussed the factors that facilitate or hinder tacit collusion have only examined the not very realistic case in which firms are perfectly symmetric in terms of their costs. The present paper investigates how asymmetry in cost functions across firms affects the scope for collusion and provides conditions under which a collusive outcome involving all firms in the market can be supported.

We employ a model in which cost asymmetric firms repeatedly set quantities and use *optimal penal codes* to enforce collusion. Asymmetry will be dealt with by assuming that firms have a different share of a specific asset (say, capital) which affects marginal costs. Therefore, a firm is considered "large" if it owns a large fraction of the capital stock, and "small" if it owns only a restricted proportion of the capital available in the industry.

We start by characterizing firms' incentives to deviate from the collusive phase as well as their incentives to deviate from the punishment scheme. We show that these incentives turn out to crucially depend on the asset holdings of the firms in the industry. Specifically, joint profit maximization implies that output is shifted away from small (inefficient) firms towards large (efficient) firms. This implies that the smallest firm in the industry is the one that has the highest potential to steal the business of its rivals and, hence, has the highest incentives to disrupt the collusive agreement. This paper thus provides a theoretical rational to the finding of Mason, Phillips and Nowell (1992) that, in experimental duopoly games, "low-cost agents are unable to induce high cost agents to collude" (pp. 665-666). In addition, it is also shown that the incentives to deviate are exactly reversed when the equilibrium calls for punishments. Following Abreu (1986, 1988), we assume that if a deviation occurs, all firms expand output for one period so as to drive price below cost and return to the most collusive sustainable output in the following periods, provided that every player went along with the first phase of the punishment. Since the largest firm is the one that proportionally loses most in the first period of the punishment, it will have the highest incentives to deviate from the punishment strategy.

We then identify a minimal threshold for the discount factor in order for collusion to be sustained and study the impact of changes in firms' asset holdings on this minimal threshold. In spite of the simplifying assumptions (namely, the particular demand and cost functions used), the results offer some interesting merger policy implications. Specifically, they allow us to discuss the issue of *joint or oligopolistic dominance* which has been gaining increasing importance in European merger control.[1] The analysis suggests that the evaluation of whether the structural change implied by a merger creates more favorable conditions for tacit collusion to arise between the remaining firms, depends on which firms the merger involves. In particular, it is shown that two different effects can be induced by a merger: (*i*) if firms were already colluding before the merger takes place, then the merger will only have effects on the scope for collusion if it affects the size of the largest firm in the industry. A merger increasing the size of the largest firm gives rise to a more asymmetric distribution of assets and this offsets the increased risk of anticompetitive behavior due to higher concentration; (*ii*) If, instead, firms were not colluding before the merger, then a merger might make collusion enforceable afterwards. This will occur when the merger involves very small (and, hence, inefficient) firms, which, as already mentioned, turn out to have very high incentives to disrupt the collusive agreement.

To the best of our knowledge, the only paper that discusses the impact of cost heterogeneity on the stability of tacit collusion is Rothschild (1999). In a repeated game setting, Rothschild shows that the stability of tacit collusion depends crucially, and in quite a complex way, on the relative efficiencies of the deviant and nondeviant firms. There exist, however, two major differences between Rothschild's framework and the setting used in this paper. First, while Rothschild assumes that firms adopt standard 'grim' trigger strategies, in this model firms' strategies incorporate optimal punishments with a *stick and carrot* structure in the style of Abreu (1986, 1988) to sustain a mutually desirable collusive outcome.[2] Specifically, Abreu's work is extended to consider a class of "proportional penal codes".[3] Second, in this

[1]The issue of joint dominance was first used by the European Commission in the *Nestlè/Perrier* case. However, only in more recent cases, such as the *Kali und Salz* case and the *Airtours/First Choice* case, it has become clear that this concept can be used to block mergers within the European merger control. For a detailed analysis on this see Motta (2000).

[2]As Vives (2000) observes, "in general, the threat of Nash reversion does not provide the most severe credible punishment to deviants to a collusive agreement. This fact is important because the more severe the punishment is, the more 'cooperative' outcomes can be sustained." (p. 311).

[3]In this particular class of penal codes, firms outputs along the punishment path are

paper costs are not exogenous but depend on assets, that is, on each firm's share in the industry capital. This fact allows for the discussion of the impact of transfer of asset holdings amongst firms on their incentives to collude.

In two recent papers, which are probably the closest to our paper, Compte, Jenny and Rey (1997) and, more recently, Kühn and Motta (1999), working respectively with a Bertrand supergame with asymmetric capacity constraints and with a differentiated goods framework where firms produce different numbers of products, discuss the joint dominance issue based on asset transfers. Both studies reach - despite different mechanisms at work - the same conclusion that a more symmetric industrial structure enhances collusion. However, while in Compte, Jenny and Rey (1997) firms are endowed with different capacities and it turns out that the largest firm (the one with the highest capacity) is the one that has the highest incentives to disrupt the collusive agreement, in Kühn and Motta (1999) firms' assets are product varieties, and the firm which tends to have the largest incentives to deviate is the one with the most limited range of products (the smallest one in the industry). For that reason, Kühn and Motta (1999) conclude that "the specific incentive structure for collusion for small and large firms may vary greatly depending on the type of asset one is concerned about" (p. 2). In the present paper, as already mentioned, firms own some share of an industry tangible asset (capital) which affects marginal costs. In line with the two previous works, the outcome which emerges is that a more asymmetric distribution of firms' asset holdings tends to hurt tacit collusion. Nevertheless, it also appears that the smallest firms constitute the main obstacle for the stability of the collusive agreements. Hence, although the asset under consideration implicitly captures the importance of firms' capacity, as in Compte, Jenny and Rey (1997), the results obtained regarding the mapping between firm's asset holdings and their incentives to collude are much closer to those obtained by Kühn and Motta (1999). In addition, new insights are drawn for practical application of competition policy.

The rest of the paper is organized as follows. The model is laid out in the next section. In section 3, the case of *perfect efficient collusion* is considered. In this section, firms are assumed to maximize joint profits during the collusive phases. Section 4, discusses the case of *perfect non-efficient collusion*, i.e., firms are assumed to coordinate on the jointly production of the monopoly (aggregate) quantity, but deviate away from the joint profit

proportional to their share in the industry capital.

maximization rule to try and enhance collusion sustainability. This section, therefore, discusses the trade-off that exists between efficiency (joint-profit maximization) and sustainability in an industry-wide cartel of the type presented in this paper. Finally, section 5 offers some concluding comments.

2 The model

Consider n firms which produce in the same market for infinitely many periods. Suppose they make output decisions simultaneously at the beginning of each period. Let $q_{i,t}$ be the quantity chosen by firm i, $i \in \{1, ..., n\}$, in period t, $t = 1, 2, ...$.

We assume that the industry inverse demand is piecewise linear:

$$p(Q) = \max\{0, a - bQ\}, \tag{1}$$

where $Q \in \left[0, n\frac{a}{b}\right]$ is the industry output, p is the price of the output and $a, b > 0$ are demand parameters.

Following Perry and Porter (1985), we assume that what distinguishes firms is the amount of capital they own. Total supply of capital is assumed to be fixed to the industry. For the sake of simplicity, the total quantity of capital is normalized to be one.[4] Let k_i be the fraction of the industry capital stock owned by firm i, $i \in \{1, ..., n\}$. Notice that the assumption of a fixed supply of the industry capital is a key feature of the model which will affect our discussion of the effects of changes on firms' size (as measured by k_i) and in the number of firms on the scope for collusion.

The cost function of a firm that owns a fraction k_i of the capital stock and produces q_i units of output is given by:

$$C_i(q_i, k_i) = cq_i + \frac{q_i^2}{2k_i}, \tag{2}$$

where $0 < c < a$,[5] $0 < k_i < 1$ and $\sum_{i=1}^{n} k_i = 1$. Without loss of generality, we assume that $k_1 \geq ... \geq k_n$,[6] $q_i \in \left[0, \frac{a}{b}\right]$ and fixed costs are taken to be zero.

[4]As pointed out by Perry and Porter (1985), "this supresses de novo entry into the industry" (p. 220).

[5]To exclude the trivial case in which production is not viable.

[6]Firms are ranked by decreasing efficiency.

The resulting marginal cost function is linearly increasing:

$$C_i'(q_i, k_i) = c + \frac{q_i}{k_i}. \tag{3}$$

Notice that the marginal cost function rotates about the intercept as the proportion of capital owned by firm i (k_i) increases or decreases. Hence, this way of characterizing efficiency differences amongst firms implicitly captures the importance of firms' capacity.[7]

Assume that in the absence of collusion, firms behave like Cournot competitors. A basic insight from the supergame literature is that nonstationary equilibria of quantity setting oligopoly repeated games are much larger sets than just the Cournot (Nash) equilibrium repeated in every round. Reductions in output below the Cournot levels can benefit all the players, but they also create incentives for some firms to undermine the tacitly collusive agreement by (secretly) expanding their individual output. Hence, in order for tacit collusion to be possible, firms have to use their ability to punish each other's deviations from any supposed equilibrium path, by using a *credible penal code*. A penal code is a rule which specifies what players should do in the event that a firm deviates either from the collusive path or from the behavior specified in the penal code. If a penal code is *credible*, then, in each period of the game, given that all the other firms have decided to follow the behavior prescribed by the penal code, each individual player maximizes the present value of its profits stream by also obeying the penal code. Subgame perfection is here used as the equilibrium concept. Unfortunately, as has been shown by different versions of the Folk Theorem,[8] there exists a large set of subgame perfect equilibrium strategies, if players are sufficiently patient. However, in order to carry out comparative statics exercises, which turn out to be particularly relevant to analyze the effects of mergers (and other capital transfers) on the scope for collusion, a plausible equilibrium will, in what follows, be selected and characterized within the set of subgame perfect equilibria.

A standard example of a *credible penal code* is the one proposed by Friedman (1971), which consists in a Cournot-Nash reversal forever. This penal code is easily seen to be credible since no player can gain by deviating in the

[7]If a firm is endowed with a small share of the industry capital, it will face a rapidly rising marginal cost curve.

[8]See, for example, Fudenberg and Maskin (1986).

punishment phase, because play there is just an infinite number of repetitions of a static (Nash) equilibrium. Notice, however, that such an infinite unforgiving punishment might seem rather extreme. This fact justifies the importance of two papers by Abreu (1986, 1988). In order to derive the highest level of profits which can be sustained by a fixed number of firms as a subgame perfect equilibrium, Abreu examined a class of more sophisticated punishments than reversion to the one-shot Nash equilibrium. He pointed out that, without loss of generality, attention can be restricted to what he defined as *simple penal code*. A simple penal code has a very simple structure. If firms conform with the strategies of the prescribed equilibrium, then they will earn the value of the best continuation equilibrium. If, instead, a single deviation occurs,[9] all firms (including the deviant one) revert to a punishment which gives the deviant its worst possible continuation equilibrium.[10] Assume that period t payoffs are received at the end of period t. For each simple penal code, there exists a vector $(v_1, ..., v_n)$, where v_i represents the present value of profits that firm $i \in \{1, ..., n\}$ receives after its deviation has occurred, discounted to the beginning of the first period after deviation.

Let q^c represent a collusive output vector. Denote the profit corresponding to firm i under collusion as $\pi_i^c(k_i)$ and the (one period) gain from deviation as $\pi_i^d(k_i)$. The collusive output is said to be sustainable if, for some simple penal code and for all i, the potential short-run gains from cheating are no greater than the present value of expected future losses which are due to the subsequent punishment. This trade-off is captured by the analysis of the incentive compatibility constraint:

$$\frac{\pi_i^c(k_i)}{1-\delta} \geq \pi_i^d(k_i) + v_i, \tag{4}$$

which can also be written as follows:

$$\left(\frac{\delta}{1-\delta}\right)\pi_i^c(k_i) - v_i \geq \pi_i^d(k_i) - \pi_i^c(k_i), \tag{5}$$

[9]Simultaneous deviations are ignored since in seeking a Nash or subgame-perfect equilibria, we ask only if a player can gain by deviating *assuming his opponents play as originally specified*.

[10]In other words, the most effective way to prevent a player from deviating is to threaten to respond to a deviation from a proposed strategy by playing the subgame perfect equilibrium of the infinitely repeated game which yields the lowest payoff of all such equilibria for the deviator.

where $\delta \in (0,1)$ is the common discount factor. If the condition in (4) (or, equivalently, (5)) holds for all i, then the collusive solution is self-enforcing for every single firm in the coalition.

In the analysis which follows, we consider the case of perfect efficient collusion. The analysis suggests that the extent to which an industry can sustain a stable collusive agreement depends crucially upon the asset distribution amongst coalition members. In particular, it is shown that small firms represent the main obstacle for industry-wide collusion.

3 The analysis of perfect efficient collusion

In this section, the case in which firms are assumed to adopt a joint profit maximization behavior on the collusive path is analyzed, concentrating particularly on understanding the effect of the structure of asset distribution amongst member firms on the scope for collusion. To do so, we start by computing the collusive and the optimal deviation profits for a generic firm i, owning a fraction k_i of the industry capital stock.

3.1 Collusive profits

In the specific case of full collusion, the coalition operates as a monopolist with n plants, so the marginal cost of production (mc) must be equalized amongst firms:[11]

$$mc = c + \frac{q_1}{k_1} = c + \frac{q_2}{k_2} = ... = c + \frac{q_n}{k_n}. \tag{6}$$

Using relation (6),[12] we can easily derive the 'aggregate' marginal cost of production ($mc(Q)$) by horizontally summing the individual marginal cost functions of the member firms:

$$mc(Q) = c + Q. \tag{7}$$

Hence, it is straightforward to show that the collusive aggregate quantity, individual output and market price are, respectively:

$$Q^c = \frac{a-c}{2b+1}, \tag{8}$$

[11] As opposed to models in which firms have different but constant marginal costs, the cartel problem at hand is not a trivial one.

[12] Notice that $\forall i \in \{1, ..., n\}, q_i = (mc - c) k_i$.

$$q_i^c = \frac{a-c}{2b+1} k_i, \tag{9}$$

and

$$p^c = \frac{a + b(a+c)}{2b+1}. \tag{10}$$

From the above expressions for the equilibrium price and individual quantity, it follows that the profit earned by firm i in a collusive period equals:

$$\pi_i^c(k_i) = \frac{1}{2}\frac{(a-c)^2}{2b+1} k_i > 0. \tag{11}$$

At this stage, it is worth mentioning that the allocation rule adopted by the cartel in order to share the joint profit maximizing output (as expressed by eq. (6)) reflects the firms' different sizes (as shown by eq. (9)). Since, as output increases, marginal cost rises more rapidly for a small firm than for a large firm, joint profit maximization implies that the smaller (and, hence, the more inefficient) a member firm is, the lower its share in the aggregate output is. Banning side payments, this implies a correspondingly smaller share in the joint profit (see eq. (11)).

3.2 Deviation profits

If firm i considers deviating from the collusive agreement, it assumes that all the opponents will keep their quantities constant at the collusive level in the current period. Hence, it takes as given the combined rival's (collusive) output and chooses its deviation quantity (q_i^d) by maximizing the following profit function:

$$\pi_i^d(q_1, ..., q_n; k_i) = \left(a - bq_i - b \sum_{\substack{j=1 \\ j \neq i}}^{n} q_j^c \right) q_i - cq_i - \frac{q_i^2}{2k_i}. \tag{12}$$

Making use of equation (9), one finds that $\sum_{\substack{j=1 \\ j \neq i}}^{n} q_j^c = \left(\frac{a-c}{2b+1}\right)(1-k_i)$. Therefore, (12) can be rewritten as follows:

$$\pi_i^d(q_i; k_i) = \left(a - bq_i - b\left(\frac{a-c}{2b+1}\right)(1-k_i) \right) q_i - cq_i - \frac{q_i^2}{2k_i}. \tag{13}$$

By maximizing (13) with respect to q_i, it turns out that firm $i's$ optimal deviation quantity equals,

$$q_i^d = \frac{(a-c)(1+b+bk_i)}{(2b+1)(2bk_i+1)}k_i. \tag{14}$$

Using eq. (14) to substitute for q_i in expression (13), one obtains:

$$\pi_i^d(k_i) = \frac{1}{2}\frac{(a-c)^2(1+b+bk_i)^2}{(2bk_i+1)(2b+1)^2}k_i > 0. \tag{15}$$

Using now expressions (11) and (15) in order to carry out a simple exercise of comparative statics, it can be shown that $\frac{\partial \pi_i^c(k_i)}{\partial k_i} = \frac{1}{2}\frac{(a-c)^2}{2b+1} > 0$ and also that $\frac{\partial \pi_i^d(k_i)}{\partial k_i} = \frac{1}{2}\frac{(a-c)^2(1+b+bk_i)(bk_i(4bk_i+3)+b+1)}{(2b+1)^2(2bk_i+1)^2} > 0$.[13] Hence, the more efficient a member firm is, the higher its share on the collusive profit, on the one hand, and the higher are its deviation profits, on the other.[14]

3.3 Distribution of assets and scope for collusion

Having defined the profits of a representative firm i both at the joint profit maximum and in a deviation scenario, respectively, we now turn to the study of the conditions which must be satisfied in order for a stable fully collusive outcome to exist. This section will show how the incentives to deviate from the fully collusive agreement depend on the asset holdings of the firm, focusing on pure strategy (subgame) perfect equilibria. In particular, we propose a specific class of perfect penal codes more severe than Cournot-Nash reversion, in the style of the ones which have been characterized in general by Abreu (1986, 1988).

In his paper, Abreu (1988) shows that repeated games with discounting may be completely analyzed in terms of simple strategy profiles. A simple strategy profile is "a rule specifying an initial path (i.e., an infinite stream of one-period action profiles), and *punishments* (also paths, and hence infinite streams) for any deviations from the initial path or from a previously

[13] Remember that the total quantity of capital was assumed to be exogenously given and normalized to be one. Therefore, when performing exercises of comparative statics with respect to k_i, we are simply comparing the effect of interchanging a more efficient firm with a less efficient one.

[14] As we will see below, this does not imply that the larger the firm, the higher its incentives to deviate from the collusive agreement.

prescribed punishment" (Abreu (1988), p. 383). More formally, as already mentioned, $q_{i,t}$ denotes the quantity chosen by firm i, $i \in \{1, ..., n\}$, in period t, $t = 1, 2, ...$. Let $q(t) \equiv (q_{1,t}, ..., q_{n,t})$. An action profile $\{q(t)\}_{t=1}^{\infty}$ is referred to as a path or punishment and is denoted by $P \in \Omega$, where Ω represents the set of paths.

Definition 1 *(cf. Abreu(1988)) Let $P^i \in \Omega$, $i = 0, 1, ..., n$. A simple strategy profile $SSP\,(P^0, P^1, ..., P^n)$ specifies: (i) play P^0 until some player deviates unilaterally from P^0; (ii) for any $j \in \{1, ..., n\}$, play P^j if the $j-$th player deviates unilaterally from P^i, $i = 0, 1, ..., n$, where P^i is an ongoing previously specified path; continue with P^i if no deviations occur or if two or more players deviate simultaneously.*

A simple strategy profile is therefore history-independent in the sense that it specifies the same punishment P^i for any deviation (from the initial path P^0 or from a previously prescribed punishment) by player i. When there is an unilateral deviation, the subsequent sequence of action profiles depends on the identity of the deviant and not on the history that preceded its deviation.[15] Now, a *simple penal code* is defined by an n-vector of punishments $(P^1, ..., P^n)$, where P^i is inflicted if player i deviates. Notice that the elements of simple strategy profiles that define a simple penal code differ only with respect to the initial path they prescribe.

3.3.1 Necessary and sufficient conditions for perfect efficient collusion

The following Lemma shows that under the assumptions of our model, a simple penal code exists which is an optimal penal code. A penal code is said to be optimal if it yields to the deviant player the lowest possible continuation payoff in any (subgame) perfect equilibrium of the model at hand.

Lemma 1 *An optimal simple penal code exists.*

Proof. To prove that a simple penal code exists, all we need to show is that the model fits Assumptions 2 to 4 in Abreu (1988).

[15] As was highlighted by Abreu (1988), "in no sense is there any need to 'make the punishment fit the crime'." (p. 385).

Notice that:

1. $(q_1, ..., q_n) \in \left[0, \frac{a}{b}\right]^n$, which is a compact topological space;

2. The one period profit function of a generic firm i, $\pi_i(q_1, ..., q_n; k_i)$, is continuous.

3. Since the cost functions $C_i(q_i, k_i)$ are strictly convex in the first argument $\left(\frac{\partial^2 C_i(q_i,k_i)}{\partial q_i^2} > 0\right)$ and the (inverse) demand function is piecewise linear, each firm's profit function is strictly quasi-concave in its own output. Moreover, the profit functions are continuous and their domain is compact. Hence, the one-shot (stage) game has a pure-strategy Nash equilibrium (a Cournot equilibrium exists) and,[16] therefore, the set of perfect equilibrium strategy profiles of the supergame with discounting is nonempty.

Hence, by Proposition 2 in Abreu (1988), an optimal simple penal code exists. ■

Lemma 1 establishes the existence of an $n-$vector of punishments. The $i-th$ vector is an infinite stream of action profiles specifying what each player should do in the event of a (single) deviation by firm i from the agreed upon initial path, or from a previously prescribed punishment. In the case player $i's$ specific punishment is imposed, this player will earn its lowest possible perfect equilibrium payoff. Notice, however, that this result does not provide us with the specific intertemporal structure of the optimal punishment paths. In what follows, we show that although asymmetry amongst firms' cost functions is assumed, the optimal punishments inflicted to deviant players may have very simple structures, as the following definition suggests.

Definition 2 $\sigma(q^1, q^2)$ *denotes a simple "proportional" two-phase penal code, where:*

- $q^j = (q_1^j, ..., q_n^j) \in \left[0, \frac{a}{b}\right]^n$, *for* $j = 1, 2$;
- $Q^j = \sum_{i=1}^{n} q_i^j$, *for* $j = 1, 2$;
- $q_i^j = k_i Q^j$,*for* $i \in \{1, ..., n\}, j = 1, 2$;
- $q_{i,t} = \begin{cases} q_i^1, & \text{if } t = 1 \\ q_i^2, & \text{if } t = 2, 3, ... \end{cases}$.

[16] See Appendix A.

Two remarks are in order at this point. First, notice that the proposed class of penal codes (proportional to capital shares) uses the *same* punishment path for *every* deviating firm. Second, this particular class of punishments has a two-phase structure, implying that punishments are stationary after the first period.

Definition 3 $v_i(q^1, q^2) \equiv \delta\left(\pi_i(q^1) + \left(\frac{\delta}{1-\delta}\right)\pi_i(q^2)\right)$*, where* $\pi_i(q^j)$*, for* $i \in \{1, ..., n\}, j = 1, 2$*, is the profit earned by firm* i *when* q^j *is the vector of quantities produced by all firms in the market.*

Abreu (1986) applies the systematic framework presented in Abreu (1988) to oligopolistic quantity-setting supergames. Our setting respects Assumptions (A2)-(A5) in Abreu (1986). However, instead of identical firms producing a homogeneous good at constant marginal cost, we consider cost asymmetric firms whose efficiency differences are characterized by the cost function (2). The next Lemma aims at showing that, under the assumptions of our model and if players are sufficiently patient, a "proportional" two-phase penal code exists, yielding every player a payoff of zero. Since zero is the minmax payoff for every firm in the component game, the proposed penal code turns out to be *globally optimal.*[17]

Lemma 2 *There exists a lower bound* $\underline{\delta} < 1$ *such that for every* $\delta \geq \underline{\delta}$ *there exists a pair* $(q^1, q^2) \in \left[0, \frac{a}{b}\right]^{2n}$ *such that* $\sigma(q^1, q^2)$ *is an optimal simple "proportional" two-phase penal code yielding* $v_i(q^1, q^2) = 0$ *for all* $i \in \{1, ..., n\}$ *if and only if* $\delta \geq \underline{\delta}$.

Proof. For any $(q^1, q^2) \in \left[0, \frac{a}{b}\right]^{2n}$, $v_i(q^1, q^2) = 0$ if and only if:

$$-\pi_i\left(q^1\right) = \delta\left(\pi_i\left(q^2\right) - \pi_i\left(q^1\right)\right). \tag{16}$$

On the other hand, $\sigma(q^1, q^2)$ is a perfect equilibrium if and only if no member firm has incentives to deviate from any phase of the punishment, that is, if and only if, for all i,

$$\pi_i^*\left(q_{-i}^1\right) - \pi_i\left(q^1\right) \leq \delta\left(\pi_i\left(q^2\right) - \pi_i\left(q^1\right)\right), \tag{17}$$

[17] Zero is the lowest possible payoff that firms are willing to accept in order to go along with the punishment, since firms can guarantee themselves a zero payoff by producing nothing forever.

$$\pi_i^*\left(q_{-i}^2\right) - \pi_i\left(q^2\right) \le \delta\left(\pi_i\left(q^2\right) - \pi_i\left(q^1\right)\right), \tag{18}$$

where $q_{-i}^j = \left(q_1^j, ..., q_{i-1}^j, q_{i+1}^j, ..., q_n^j\right)$, $j = 1, 2$, and $\pi_i^* : \left[0, \frac{a}{b}\right]^{n-1} \to \mathbf{R}$ denotes firm $i's$ best response profit, that is, $\pi_i^*\left(q_{-i}^j\right) = \max\left\{\pi_i(x, q_{-i}^j) \,\middle|\, x \in \left[0, \frac{a}{b}\right]\right\}$. Since $\sigma\left(q^1, q^2\right)$ satisfies (16), eqs. (17) and (18) can be rewritten as follows, respectively:

$$\pi_i^*\left(q_{-i}^1\right) = 0, \tag{19}$$

and,

$$\pi_i^*\left(q_{-i}^2\right) \le \pi_i\left(q^2\right) - \pi_i\left(q^1\right). \tag{20}$$

Let $D = \left\{\delta \in (0,1) \middle| \left(\delta, q^1, q^2\right) \text{ satisfies Eqs. (16), (19) and (20) for some } \left(q^1, q^2\right) \in \left[0, \frac{a}{b}\right]^{2n}\right\}$. Continuity of $\pi_i(\cdot)$ and $\pi_i^*(\cdot)$ implies that D is closed.[18] Since demand is piecewise linear, $p(0) > C_i'(0)$ (that is, $a > c$) and $C_i''(q_i) > 0$ (assumptions (A2)-(A4) in Abreu (1986) hold in our framework), there exists $q^0 \in \left[0, \frac{a}{b}\right]^n$ and, therefore, a $Q^0 = \sum_{i=1}^n q_i^0$ such that $p\left(Q^0(1 - k_i)\right) \le C_i'(0) = c$.[19] Hence, by (A2) in Abreu (1986) and since $C_i''(q_i) > 0$, one has that $\pi_i\left(q^0\right) < 0$ and $\pi_i^*\left(q_{-i}^0\right) = 0$. Observe that there exists a proportional fully collusive output vector $q^c \in \left[0, \frac{a}{b}\right]^n$, where q_i^c is given by equation (9). Let $\delta' = \frac{-\pi_i\left(q^0\right)}{\pi_i(q^c) - \pi_i(q^0)} < 1$. Then, $\left(\delta', q^0, q^c\right)$ satisfies Eqs. (16), (19) and (20), and D is nonempty.[20] Let $\underline{\delta} = \min D$. Then, for $\delta < \underline{\delta}$ there exists no $\left(\delta, q^1, q^2\right)$ such that $\sigma\left(q^1, q^2\right)$ is a Perfect Equilibrium and $v_i\left(q^1, q^2\right) = 0$. So, the "only if" part of the proof is complete.

Let $\left(q^{1*}, q^{2*}\right) \in \left[0, \frac{a}{b}\right]^{2n}$ satisfy Eqs. (16), (19) and (20) for $\delta = \underline{\delta}$. Now consider $\widehat{\delta} \ge \underline{\delta}$ and let $\widehat{q^1} \in \left[0, \frac{a}{b}\right]^n$ satisfy Eq. (16) for $q^2 = q^{2*}$ and $\delta = \widehat{\delta}$. By (A2)-(A4) in Abreu (1986), $\widehat{q^1}$ exists, $-\pi_i\left(\widehat{q^1}\right) \ge -\pi_i\left(q^{1*}\right)$, and $\widehat{Q^1} \ge Q^{1*} \ge Q^0$, so that $\left(\widehat{\delta}, \widehat{q^1}, q^{2*}\right)$ satisfies Eqs. (16), (19) and (20). Thus, $\sigma\left(\widehat{q^1}, q^{2*}\right)$ is a perfect equilibrium and yields $v_i\left(\widehat{q^1}, q^{2*}\right) = 0$ for all $i \in \{1, ..., n\}$. ■

[18] Notice that the condition $\delta > 0$ in the definition of D is not binding/relevant. When $\delta = 0$, condition (16) is not satisfied. As a consequence, continuity of $\pi_i(\cdot)$ and $\pi_i^*(\cdot)$ implies that $\min D$ exists (and is positive).

[19] It is worth mentioning at this point that since in this setting $C_i'(0) = c$, Q^0 can be chosen to fit all possible deviating firm i.

[20] Notice that (18) is satisfied, since one can chose Q^0 to be high enough.

The intuition behind this result is as follows. A potential deviant has to trade off the short-run gains from deviation with the *future* discounted loss due to the restarting of a punishment phase.[21] Hence, the discount factor should be high enough for firms not to deviate from the punishment strategy. Besides, if δ is higher, firms can increase the severity of the punishment by appropriately choosing a higher aggregate output in the first phase of the punishment scheme (Q^1 in our notation). Therefore, there is some minimum discount factor, denoted $\underline{\delta}$, such that it is credible to impose a punishment which yields each firm a zero payoff.

Lemma 2 simplifies the characterization of the relationship between asset distribution and firms' incentives to collude, considered in the next Proposition.

Proposition 1 *Let $\underline{\delta}$ be as in Lemma 2. Perfect efficient collusion is sustainable in a subgame perfect equilibrium if and only if $\delta \geq \max\left\{\underline{\delta}, \widetilde{\delta}_n\right\}$, where $\widetilde{\delta}_n = \frac{b^2(1-k_n)^2}{(1+b+bk_n)^2}$.*

Proof. Notice that the incentive compatibility constraint (eq. (4)) can be rewritten in the following way:

$$\pi_i^c(k_i) - (1-\delta)\,\pi_i^d(k_i) \geq (1-\delta)\,v_i. \tag{21}$$

By Lemma 2, for every $\delta \geq \underline{\delta}$, an optimal "*proportional*" penal code exists yielding $v_i = 0, \forall i \in \{1, ..., n\}$. Hence, if $\delta \geq \underline{\delta}$, the r.h.s. of condition (21) equals zero.

Now, making use of eqs. (11) and (15), one concludes that $\pi_i^c(k_i) - (1-\delta)\,\pi_i^d(k_i) \geq 0$, if and only if, for all $i \in \{1, ..., n\}$:

$$\delta \geq \frac{b^2\,(1-k_i)^2}{(1+b+bk_i)^2} \equiv \widetilde{\delta}_i. \tag{22}$$

Notice also that, from (22), it can be easily shown that,

$$\frac{\partial \widetilde{\delta}_i}{\partial k_i} = -2b^2 \frac{(1-k_i)\,(2b+1)}{(1+b+bk_i)^3} < 0.$$

[21] Notice, in this direction, that, as pointed out by Abreu (1988), "the early stages of an optimal punishment must be more unpleasant than the remainder."(p. 385) Therefore, the punishment is made credible by the threat of being restarted should any player deviate from the punishment strategy.

Hence, the main problem is to prevent firms with rapidly rising marginal cost curves from deviating. Since in our setting k_n denotes the share in the capital corresponding to the smallest firm in the industry, the condition which should be taken into account in order to evaluate the stability of an industry-wide cartel whose members maximize joint profits at the collusive path is

$$\delta \geq \frac{b^2 (1 - k_n)^2}{(1 + b + bk_n)^2} \equiv \widetilde{\delta}_n. \tag{23}$$

Hence, if $\delta < \widetilde{\delta}_n$, then perfect efficient collusion cannot be sustainable, since (23) is a necessary condition under maximal punishments. Take now the case in which $\widetilde{\delta}_n < \underline{\delta}$. When this is the case, firms cannot enforce perfect *efficient* collusion when $\delta < \underline{\delta}$. This is so because firms are assumed to use most severe punishments strategies. These punishment strategies are optimal in the sense that given a certain fixed collusive allocation (namely, the one given by eq. (9)), firms can implement this same allocation with the lowest critical discount factor, $\underline{\delta}$. Thus, a perfect efficient collusion is sustainable if and only if $\delta \geq \max \left\{ \underline{\delta}, \widetilde{\delta}_n \right\}$. ■

Proposition 1 captures the fact that, as already shown, joint profit maximization implies that output is shifted away from small (inefficient) firms towards large (efficient) firms. As a result, the smallest firm is the one which is allotted a share in the collusive aggregate output that is too low with respect to its optimal deviation output. However, as was highlighted by Martin (1988), a small inefficient firm"may well judge that over the long run its bargaining power within the cartel will be tied to its market share. If this is the case, accepting a lower market share to maximize joint profit will amount to cutting its own throat within the cartel." (p. 137). Notice, in this direction, that, from eqs. (9) and (14), it is straightforward to show that:

$$\frac{q_i^d}{q_i^c} = \frac{1 + b + bk_i}{2bk_i + 1}. \tag{24}$$

For a collusive agreement to be stable, this ratio should not be too high for any member firm i. Working through some algebra, one can show that $\frac{\partial}{\partial k_i} \left(\frac{q_i^d}{q_i^c} \right) = -b \frac{2b+1}{(2bk_i+1)^2} < 0$. Therefore, the smaller the firm is, the higher its potential to profitably capture demand from its opponents by deviating.[22]

[22] At this point it is worth contrasting this result with the one obtained by Rothschild (1999). In his paper, Rothschild does not give sharp predictions as to the relationship

It should also be stressed at this point that the fact that small firms are the less keen to accept the collusive agreement relies on our assumption of absence of monetary transfers, which we think is the most realistic in most circumstances.

The previous proposition also highlights the fact that in order for firms to credibly participate in this efficient collusive scheme, they should be willing to comply with the collusive path, on the one hand, and with the punishment strategy, on the other. It was shown that on the collusive equilibrium path the incentive constraint which is binding is that of the smallest firm. In the next proposition it is shown that if the smallest firm is not too small (inefficient), then we can also identify the firm for which the incentive constraint is binding on the punishment path.

Proposition 2 *If the smallest firm in the industry is not too small, that is, if* $k_n \in [k^*, k_1]$*, where*

$$\begin{aligned} k^* &= \frac{b^3\left(k_1^2+6k_1+1\right)+2bk_1\left(1+4b\right)}{\left(1-k_1\right)^2 b^3}+ \\ &\quad \frac{\left(2b+1\right)\left(\left(2b+1\right)-\left(1+bk_1+b\right)\sqrt{\left(\left(2b+1\right)\left(1+2k_1 b\right)\right)}\right)}{\left(1-k_1\right)^2 b^3}, \end{aligned}$$

then $\underline{\delta} \geq \widetilde{\delta}_n$*, where* $\underline{\delta} = \frac{(1+2bk_1)(2b+1)}{(1+b+bk_1)^2}$.

Proof. From (16), one concludes that:

$$\delta = \frac{-\pi_i\left(q^1\right)}{\pi_i\left(q^2\right)-\pi_i\left(q^1\right)}. \tag{25}$$

Since, in this setting, $\pi_i\left(q^1\right) = (a-c-bQ^1)k_iQ^1 - \frac{(k_iQ^1)^2}{2k_i}$ and $\pi_i\left(q^2\right) = \left(a-c-bQ^2\right)k_iQ^2 - \frac{(k_iQ^2)^2}{2k_i}$, one can reevaluate (25), obtaining

$$\delta = Q^1 \frac{\left(2b+1\right)Q^1 - 2\left(a-c\right)}{\left(Q^1-Q^2\right)\left(\left(Q^1+Q^2\right)\left(2b+1\right)-2\left(a-c\right)\right)}, \tag{26}$$

between firms' cost conditions and their ability to sustain a collusive agreement to restrict output. His Proposition 4 shows that the most inefficient firms *might be* the ones that determine the stability a fully collusive agreement. However, in his result the propensity of these firms to deviate depends crucially upon the Cournot outputs of the nondeviant firms, and this in turn depends on the relative efficiencies of the deviant and nondeviant firms.

where we assume that $Q^1 > 2\left(\frac{a-c}{2b+1}\right)$ in order for δ to be positive. Now, from (26), one can easily show that:

$$\frac{\partial\delta}{\partial Q^2} = \frac{2Q^1\left(Q^1\left(2b+1\right)-2\left(a-c\right)\right)\left(\left(2b+1\right)Q^2-\left(a-c\right)\right)}{\left(Q^1-Q^2\right)^2\left(\left(Q^2+Q^1\right)\left(2b+1\right)-2\left(a-c\right)\right)^2}, \tag{27}$$

which is always *non-negative* for the following two reasons. First, we have just assumed that $Q^1 > 2\left(\frac{a-c}{2b+1}\right)$ in order for δ to be positive. Second, for all $i = \{1,...,n\}$, the individual profit function is strictly concave in its own output and the joint profit maximum is achieved when the aggregate output equals $\frac{a-c}{2b+1}$ (see eq. (8)). Hence, $Q^2 \geq \frac{a-c}{2b+1}$. Since we are looking for a minimum value for the discount factor δ, let us, therefore, set $Q^2 = \frac{a-c}{2b+1}$. Reevaluating (26) for this specific value of Q^2, one gets

$$\delta^* = \frac{Q^1\left(2b+1\right)\left(\left(2b+1\right)Q^1-2(a-c)\right)}{\left(\left(2b+1\right)Q^1-\left(a-c\right)\right)^2}. \tag{28}$$

Notice now that, from (28), it can be easily shown that

$$\frac{\partial\delta^*}{\partial Q^1} = \frac{2\left(2b+1\right)\left(a-c\right)^2}{\left(\left(2b+1\right)Q^1-\left(a-c\right)\right)^3} > 0. \tag{29}$$

Hence, in order to minimize δ^*, we want the lowest possible value of Q^1 for which conditions (19) and (20) hold. Notice that, as long as $Q^1 \leq \frac{a}{b}$, $\pi_i^*\left(q_{-i}^1\right) = \frac{1}{2}\left(a-c-bQ^1\left(1-k_i\right)\right)^2\frac{k_i}{1+2bk_i}$.[23] Hence, in order for condition (19) to hold, we must have that $Q^1 \geq \frac{a-c}{b(1-k_1)}$.[24] Let us, therefore, set $Q^1 = \frac{a-c}{b(1-k_1)}$. Now, using the analytical expressions of $\pi_i\left(q^1\right)$ and $\pi_i\left(q^2\right)$ specified above and given that $\pi_i^*\left(q_{-i}^2\right) = \frac{1}{2}\left(a-c-bQ^2\left(1-k_i\right)\right)^2\frac{k_i}{1+2bk_i}$, some algebra shows that when we set $Q^1 = \frac{a-c}{b(1-k_1)}$ and $Q^2 = \left(\frac{a-c}{2b+1}\right)$, condition (20) is satisfied if, for all i,

$$\begin{aligned} b^4\left(1-k_1\right)^2\left(1+k_i^2\right)-\left(2b+1\right)^2\left(1+2b\left(k_1+k_i\right)\right)- \\ 2b^4k_i\left(1+k_1^2+6k_1\right)-4b^2k_ik_1\left(1+4b\right) \leq 0. \end{aligned} \tag{30}$$

It can be easily shown that the derivative of the l.h.s. of the previous condition with respect to k_i is always negative. Therefore, the condition which

[23]In the case that $Q^1 > \frac{a}{b}$, $\pi_i^*\left(q_{-i}^1\right) = 0$ and, therefore, condition (19) is trivially satisfied.

[24]Notice that $\forall k_1 \in (0,1)\,, \frac{a-c}{b(1-k_1)} > 2\left(\frac{a-c}{2b+1}\right)$ and, therefore, δ^* is nonnegative.

is binding is that of the smallest firm in the industry (whose capital share is given by k_n). In addition, the l.h.s. of (30) is a polynomial of second degree in k_i and one of its roots is greater than one. Hence, after some manipulation, one concludes that in order for condition (20) to hold, one must have that $k_n \in [k^*, k_1]$, where

$$\begin{aligned} k^* &= \frac{b^3 \left(k_1^2 + 6k_1 + 1\right) + 2bk_1 \left(1 + 4b\right)}{\left(1 - k_1\right)^2 b^3} + \\ &\quad \frac{\left(2b + 1\right) \left(\left(2b + 1\right) - \left(1 + bk_1 + b\right) \sqrt{\left(\left(2b + 1\right) \left(1 + 2k_1 b\right)\right)}\right)}{\left(1 - k_1\right)^2 b^3}. \end{aligned} \tag{31}$$

If this is the case, i.e., if $k_n \in [k^*, k_1]$, making use of (28) and setting $Q^1 = \frac{a-c}{b(1-k_1)}$, we obtain:

$$\underline{\delta} = \frac{\left(1 + 2bk_1\right) \left(2b + 1\right)}{\left(b + 1 + bk_1\right)^2}. \tag{32}$$

In addition, making use of eqs. (23) and (32), one can easily show that if $k_n \in [k^*, k_1]$, then $\underline{\delta} \geq \widetilde{\delta}_n$.

This completes the proof of Proposition 2. ■

Notice that from (32), one concludes that $\underline{\delta}$ only depends on the capital share owned by the largest firm in the industry. The intuition here rests on the fact that in the first period of the punishment path the aggregate output produced has to be large enough such that a very sharp price cut occurs leading all firms to earn negative profits in this period. Besides, the largest firm is the one which is proportionally most affected by this price cut since it is the one with the highest market share in the agreement. As a result, a lower bound on the discount factor is clearly necessary. The discount factor has to be sufficiently high so that the largest firm (and, therefore, all the other firms) can recoup the one-period losses on the most attractive (second) phase of the punishment.

The insights of the two previous propositions can be summarized as follows. On the one hand, during the collusive phases the cartel maximizes its joint profit, which implies that the smaller the firm the lower its share in the collusive output. This implies that the incentive constraint that matters is that of the smallest firm. Hence, there exists a minimal discount factor $\widetilde{\delta}_n$, only depending on the share in the capital of the smallest firm, above which all firms in the industry find it optimal to keep the cartel agreement. On the other hand, if the smallest firm in the market is not too small, there exists a lower bound on the discount factor $\underline{\delta}$, which only depends on the size of the largest firm, above which an optimal "proportional" two-phase penal code exists. This penal code yields for every firm a continuation payoff of zero after a deviation has occurred. Studying the ranking between the identified thresholds for the discount factor, one concludes that the necessary and sufficient condition which must be met in order perfect efficient collusion to be sustainable is the following:

$$\delta \geq \frac{(1+2bk_1)(2b+1)}{(b+1+bk_1)^2} \equiv \underline{\delta}. \tag{33}$$

In the next subsection, we study the implications of changes in the distribution of asset holdings (due to mergers, transfers or split-offs) on the sustainability of tacit collusion. By doing so, we draw some merger policy implications.

3.3.2 The impact of mergers on collusion

In this setting, a specific asset (namely, capital) is introduced and assumed to affect firms' marginal costs (see eq. (3)). Hence, any merger gives rise to endogenous efficiency gains since it brings the individual capital of the merging firms under a single larger (and, hence, more efficient) resulting firm. A more delicate problem, however, is to understand the impact of merger-induced changes in firm's capital allocations on the sustainability of tacit collusion. This is the issue we address in the present section.

The common wisdom is that mergers tend to create structural conditions which facilitate collusion. The argument typically used is that the lower the number of market participants, the easier it will be for them to coordinate their actions (e.g. the easier it is to allocate market shares) or to monitor departures from agreed-upon output levels.[25] In what follows, however, it is shown that when cost-asymmetric firms co-exist in the market, two distinct effects can be induced by a merger: (*i*) If firms were already colluding before the merger, then the merger either has no effect on the scope for collusion or it hinders collusion; (*ii*) If, instead, before the merger collusion is not feasible, then a merger might make collusion enforceable afterwards.

[25] This is a well-established argument which extends at least as far back as Stigler (1964).

In our setting, the effect of a merger is not restricted to a decrease in the number of firms. A merger also gives rise to a different distribution of assets amongst the remaining firms (a different post-merger capital allocation). A natural question at this point is therefore which capital reallocations can be induced by a merger. Figure 1 shows that two cases should be considered when analyzing this question.

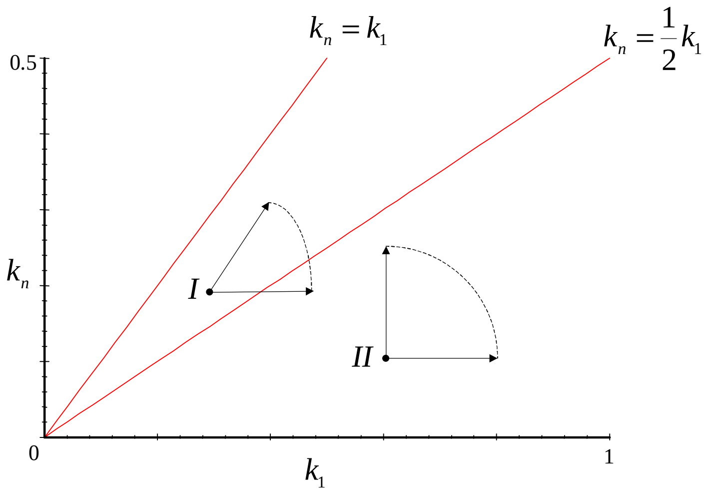

Figure 1: Merger Effects on Capital Allocation.

First, consider the situations in which before the merger $k_n > \frac{1}{2}k_1$ (e.g. allocation I in Figure 1). When this is the case, any merger will lead to an increase in the size of the largest firm. Notice, in this direction, that even if the merging firms are the two smallest firms in the industry, then the size of the resulting merged firm $(k_n + k_{n-1})$ will certainly be greater than k_1. Take now the case in which the largest firm merges with any other firm but the smallest one. This will lead to an increase in the size of the largest competitor, but the size of the smallest firm will remain unaffected. We, thus, move along the horizontal arrow starting from point I in the Figure. Hence, the capital reallocation induced by any merger can be represented by an arrow starting from point I which lies between the two arrows that form the acute angle presented in the diagram.

Next, take any pre-merger capital allocation such that $k_n < \frac{1}{2}k_1$ (e.g. allocation II in Figure 1). We can still have mergers affecting the size of the largest firm but not that of the smallest firm (we move along the horizontal arrow starting from point II in the picture). However, now we can also have a merger in which the size of the smallest firm increases but the size of the largest one remains unaffected. Take, for instance, the situation in which before the merger there are two equal-sized smallest firms. If they decide to merge, then the size of the new smallest firm will be $k_n' = \min\{2k_n, k_{n-1}\}$, but the size of the largest firm in the market remains unchanged since $2k_n \leq k_1$. As a result, if allocation II in the diagram is the pre-merger capital allocation, the capital reallocation induced by any merger can be represented by an arrow which starts from II and lies within the right angle formed by the arrows shown in Figure 1.

The previous discussion identified the possible capital reallocations which a merger can induce. Since in this setting both the initial number of firms is given and there is a fixed supply of the industry capital, we can now also identify natural bounds within which capital shares k_1 and k_n can vary. Figure 2 shows a symmetry line along which $k_n = k_1 = \frac{1}{n}$. Since, by definition $k_n \leq k_1$, any feasible capital allocation must lie within the region below this symmetry line.

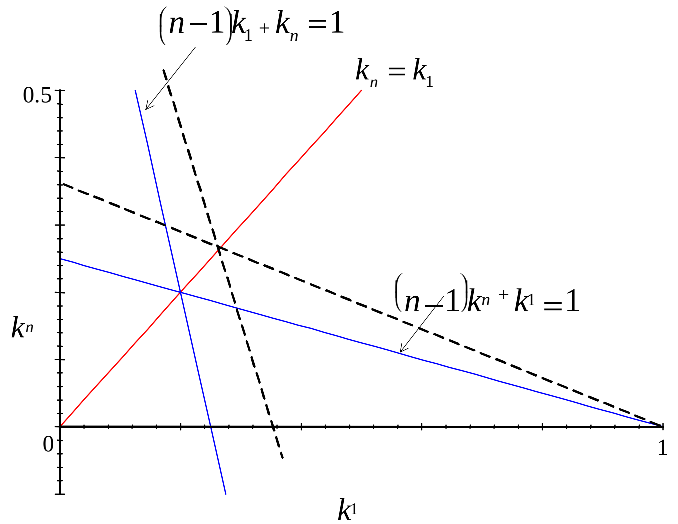

Figure 2: Merger Effects on the Feasibility Region.

The two other solid straight lines represent two extreme cases regarding the industry configuration. In the first case, there exists a large firm owning a share k_1 of the industry capital and the residual capital share $(1 - k_1)$ is equally shared by the remaining (small) firms. In the second case, the industry is composed of a single small firm and $n - 1$ symmetric large firms. In order for the capital constraint to hold before the merger, the pre-merger capital allocation (k_n, k_1) must lie within the triangle formed by these two solid straight lines. If two firms decide to merge, then the number of independent firms in the market is reduced by one. Therefore, the two extreme cases for the industry configuration are now represented by the dashed lines along which $k_1 + (n - 2)k_n = 1$ and $(n - 2)k_1 + k_n = 1$, respectively. As a result, the feasibility region in which the capital constraint holds after the merger is now represented by the triangle formed by the dashed lines.

We know that in this framework small firms have the highest incentives to deviate from the collusive path. This explains why in Proposition 2 a minimal level of efficiency k^* for the smallest firm is required in order for perfect efficient collusion to be enforceable, that is, in order for the smallest firm not to have incentives to disrupt the collusive agreement. Some algebra shows that $\forall b > 0 \; \forall k_1 \in [k_n, 1)\,, \frac{\partial k^*}{\partial k_1} < 0$. This is illustrated in Figure 3 by a solid curve along which $k_n = k^*$.[26] Notice that collusion can be enforced for every pair (k_n, k_1) above this solid locus. In addition, $\lim\limits_{k_1 \to 1} k^* = -\frac{1}{2b}$. Hence, as shown in Figure 3, k^* assumes negative values for high enough values of the capital share owned by the largest firm in the market. This is so because, according to the punishment scheme firms adopt in this setting, all the firms in the market must earn a negative profit in the first phase of the punishment. This implies that a lower bound exists for the aggregate output produced in this first phase of the punishment. More precisely, condition (19) implies that $Q^1 \geq \frac{a-c}{b(1-k_1)}$. Since this lower bound obviously increases with the size of the largest firm, an increase in k_1 leads to an increase in the first period losses of every firm in the market, that is, induces an increase of the severity of the punishment. As a result, if the largest firm is sufficiently large, then the severity of the punishment becomes so high that any small firm, no matter how small it is, will have no incentives to disrupt the collusive agreement.

[26]As is shown in Proposition 2, the minimal level of efficiency k^* does not depend on the number of firms in the industry, n (see eq. (31)). Therefore, the solid curve $k_n = k^*$ is unaffected by a merger.

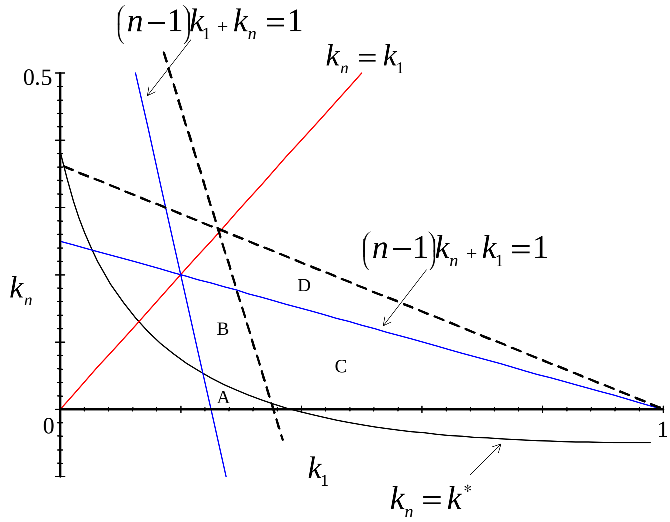

Figure 3: Merger Effects on the Scope for Collusion (Panel a)

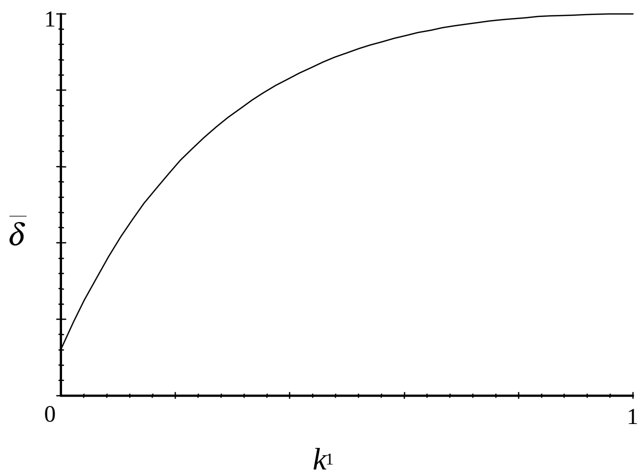

Figure 4: Merger Effects on the Scope for Collusion (Panel b)

In order to be able to discuss the impact of mergers on the scope for collusion, we also need to know how the minimal discount factor reacts to changes in firms' capital allocations. From (32), one concludes that the sustainability of tacit collusion only depends on the share in the capital of the largest firm. In addition, some simple algebra shows that:

$$\frac{\partial \underline{\delta}}{\partial k_1} = \frac{2b^2 (2b+1)(1-k_1)}{(1+b+bk_1)^3} > 0, \tag{34}$$

and

$$\frac{\partial^2 \underline{\delta}}{\partial k_1^2} = 2b^2 (2b+1) \frac{2b(k_1-2)-1}{(b+1+bk_1)^4} < 0. \tag{35}$$

Hence, an increase in the size of the largest firm will induce an increase in the minimal threshold on the discount factor above which perfect efficient collusion is sustainable. This is illustrated in Figure 4. The intuition which underlies this result is the following. The larger the largest firm is, the higher will be its share on the one period losses due to the first phase of the punishment strategy. Therefore, the more weight has to be attached to the future stream of payoffs in order for this firm to comply with the punishment strategy.

Let us now turn to the discussion of merger effects on the scope for collusion. We know that before the merger the initial capital allocation belongs to the triangle formed by areas A, B and C in Figure 3, whereas after the merger, the final capital allocation (k_n, k_1) has to lie somewhere on the triangle formed by areas C and D in the same diagram. Hence, two different scenarios should be taken into account. First, we consider the case in which firms were already colluding before the merger takes place. Then, we analyze the situation in which industry-wide collusion was not feasible before the merger.

If before the merger firms were already colluding, this means that the initial capital allocation (k_n, k_1) lies somewhere in regions B or C of Figure 3. When this is the case, any merger leading to an increase in the size of the largest firm will hurt collusion. All other mergers will have no impact on the scope for collusion. In particular, if before the merger $k_n > \frac{1}{2}k_1$, then any merger will affect the size of the largest competitor and, hence, hurt collusion. This result stresses the fact that a merger induces a more asymmetric post-merger industry configuration when it increases the size of the largest firm.[27] Thus, this analysis stresses the fact that if collusion

[27] In our framework, merger parties are taken to be exogenous. However, if the analysis was extended to endogenize mergers (split-offs) decisions, one would probably conclude

already exists in the industry, there may be no room for an improvement in firms' ability to collude after a merger has taken place.

Notice, however, that if before the merger the number of firms is sufficiently high,[28] then the pre-merger capital allocation might lie in region A of Figure 3. If this is the case, then it may be rational for more efficient firms to merge with very small and inefficient firms if by doing so collusion turns out to be enforceable after the merger takes place (that is, if after the merger the smallest firm size $k'_n \geq k^*$). Therefore, according to the present model, only in this case does a merger have anti-competitive effects as the common wisdom would suggest.

As a final remark, notice that (34) also shows that a split-off reducing the size of the largest firm in the industry tends to improve the scope for collusion and, hence, should give rise to antitrust concerns since it contributes to a more symmetric distribution of capital shares amongst the existing firms.

Our results therefore reveal that the conventional wisdom that mergers tend to enhance collusion, whereas split-offs have pro-competitive effects, may actually give misleading predictions about the facility of collusion after an asset transfer takes place, if we disregard the fact that asymmetries in cost functions tend to make coordination amongst oligopolists less likely. More importantly, this analysis clearly suggests that, as was emphasized by Compte, Jenny and Rey (1997), a systematic analysis of market shares and concentration indexes, such as the Herfindahl-Hirshman Index (HHI), does not always provide a reliable guide to assess potential effects on the level of competition in the market induced by a horizontal merger.[29] Antitrust authorities, when assessing whether a merger between two firms is likely to enhance oligopolistic coordination in the market, should give special atten-

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that in the case where collusion was already taking place before the merger, any merger affecting the size of the largest firm would fail to occur in equilibrium. This would be justified by the fact that not only the merger would hurt collusion possibilities, but it would also have no effect on merging firm's profits. Indeed, the sum of the profits of the merging parties equals the (aggregate) profit of the resulting firm (see eq. (11)). Notice, however, that in such a situation a split-off of the largest competitor would decrease asymmetries in the distribution of asset holdings and, therefore, would possibly be part of the equilibrium since it would enhance collusion possibilities.

[28] More precisely, if $n > \frac{2b^3+(2b+1)^2-\left(\sqrt{(2b+1)}\right)^3(b+1)}{(b+1)\left(b^2+3b+1-\left(\sqrt{(2b+1)}\right)^3\right)}$.

[29] As was highlighted by Fisher (1987), a "serious analysis of market power and oligopoly cannot be subsumed in a few spuriously precise measurements." (p. 39).

tion to firms' cost conditions and to the degree of post-merger symmetry among the firms in the industry.[30] Even though any merger gives rise to an increase in the size of the merged parties and also reduces the number of competitors operating in the (relevant) market, this fact is not enough to conclude that the scope for collusion increases with the merger. It might well happen that asymmetries outweigh the collusion-enhancing effects of a proposed merger.

4 Perfect non-efficient collusion

The previous analysis suggests that in an industry consistent with the assumptions of our model, the smallest and largest firms in the agreement play a crucial role on the stability of perfect efficient collusion. Being allotted a very small share in the collusive output (and profits), small firms may have no incentive to credibly participate in the collusive agreement. On the other hand, when the equilibrium calls for punishments, the largest firm is the most penalized in the punishment first phase. It is, therefore, natural to wonder whether stability of the collusive agreement could be enhanced if large firms accepted to transfer part of their output share to the smallest ones, therefore allowing small firms to produce a disproportionate (higher) share of the collusive aggregate output. This section deals with precisely this issue by studying a situation in which firms accept a distortion of the efficient allocation rule (6) in order to try and enlarge the set of discount factors for which perfect industry-wide collusion is sustainable. Proposition 3 shows that there is a trade-off between efficiency and the stability of perfect collusion.

Proposition 3 *Let $\underline{\delta}$ be as in Proposition 2. There exists a $\delta^{ne} < \underline{\delta}$ such that there exists a subgame perfect equilibrium in which perfect (non-efficient) collusion is enforced for every $\delta \geq \delta^{ne}$.*

Proof. Consider an initial situation in which $k_n = k^*$ and $\delta = \underline{\delta}$, where k^* and $\underline{\delta}$ are defined by equations (31) and (32), respectively. Hence, we depart from a situation in which the two incentive constrains that matter along an optimal maximal punishment - that of the smallest firm to stick to the collusive output and that of the largest firm to stick to the first phase of the punishment - are just binding. Assume also, without loss of generality,

[30] It is important to note that the HHI index tends to *penalize* asymmetry.

that the number of largest and smallest firms in the market is the same. Let $\widetilde{k_i}$ denote the output share of firm i, whereas k_i continues to denote the share in the industry capital owned by firm i, $i \in \{1,...,n\}$. Let us now consider an ε-transfer of *output share* from the largest to the smallest firm(s). When this is the case, $\widetilde{k_1} = k_1 - \varepsilon$, $\widetilde{k_n} = k_n + \varepsilon$, where $\varepsilon < k_{n-1} - k_n$, whereas $\widetilde{k_j} = k_j$, for all $j \notin \{1, n\}$. Firms are assumed to keep on using a simple proportional two-phase penal code $\sigma\left(\widetilde{q^1}, \widetilde{q^2}\right)$, where $\widetilde{Q^1} = \frac{a-c}{b(1-k_1)}$, $\widetilde{Q^2} = \frac{a-c}{2b+1}$ and $\widetilde{q_i^j} = \widetilde{k_i}\widetilde{Q^j}$, for $i \in \{1,...,n\}$, $j = 1,2$. In order for $\sigma\left(\widetilde{q^1}, \widetilde{q^2}\right)$ to be a perfect equilibrium, one must have that, for all i,

$$\pi_i^*\left(\widetilde{q_{-i}^1}\right) - \pi_i\left(\widetilde{q^1}\right) - \underline{\delta}\left(\pi_i\left(\widetilde{q^2}\right) - \pi_i\left(\widetilde{q^1}\right)\right) \leq 0, \tag{36}$$

and

$$\pi_i^*\left(\widetilde{q_{-i}^2}\right) - \pi_i\left(\widetilde{q^2}\right) - \underline{\delta}\left(\pi_i\left(\widetilde{q^2}\right) - \pi_i\left(\widetilde{q^1}\right)\right) \leq 0, \tag{37}$$

where $\widetilde{q_{-i}^j} = \left(\widetilde{q_1^j}, ..., \widetilde{q_{i-1}^j}, \widetilde{q_{i+1}^j}, ..., \widetilde{q_n^j}\right)$, $j = 1,2$, and $\pi_i^* : \left[0, \frac{a}{b}\right]^{n-1} \to \mathbf{R}$ denotes firm $i's$ best response profit. Since we are assuming an initial situation in which $k_n = k^*$ and $\delta = \underline{\delta}$, then the previous analysis shows that when $\varepsilon = 0$ (initially), condition (36) is binding for the largest firm(s) in the market (firm 1), whereas condition (37) is binding for the smallest firm in the agreement (firm n). To show that there exists a subgame perfect equilibrium in which firms enforce perfect collusion within a larger set of discount factors by accepting a distortion of the efficient allocation rule (6), it suffices to show that the binding incentive constraints (36) and (37) are both relaxed when there is a transfer of output share from the largest to the smallest firms in the market.

Let us start with condition (36). This condition is binding for firm 1 when $\varepsilon = 0$. In addition, in this setting, $\pi_1^*\left(\widetilde{q_{-1}^1}\right) = 0$. Hence, one can rewrite condition (36) for the largest firm, obtaining

$$(1-\underline{\delta})\left(-\pi_1\left(\widetilde{q^1}\right)\right) - \underline{\delta}\pi_1\left(\widetilde{q^2}\right) \leq 0, \tag{38}$$

where $\pi_1\left(\widetilde{q^j}\right) = \left(a - c - b\widetilde{Q^j}\right)(k_1 - \varepsilon)\widetilde{Q^j} - \frac{((k_1-\varepsilon)\widetilde{Q^j})^2}{2k_1}$, for $j = 1,2$. Let $L_1 = (1-\underline{\delta})\left(-\pi_1\left(\widetilde{q^1}\right)\right) - \underline{\delta}\pi_1\left(\widetilde{q^2}\right)$. Then, after some algebra and making use of the fact that $\widetilde{Q^1} = \frac{a-c}{b(1-k_1)}$, $\widetilde{Q^2} = \frac{a-c}{2b+1}$ and $\underline{\delta} = \frac{(1+2bk_1)(2b+1)}{(b+1+bk_1)^2}$, one concludes that

$$\frac{\partial L_1}{\partial \varepsilon} = -\left(a-c\right)^2 \frac{k_1 - 2\varepsilon}{k_1\left(2b+1\right)\left(b+1+bk_1\right)} < 0.$$

Hence, the incentive constraint of the largest firm to stick to the first phase of the punishment is relaxed when there is a (marginal) transfer of output share from the largest to the smallest firm(s).

Let us now analyze the effect of the above-mentioned transfer of output share on the incentive constraint of the smallest firm to stick to the collusive output. Condition (37) for the smallest firm n is given by

$$\pi_n^*\left(\widetilde{q_{-n}^2}\right) - \pi_n\left(\widetilde{q^2}\right) - \underline{\delta}\left(\pi_n\left(\widetilde{q^2}\right) - \pi_n\left(\widetilde{q^1}\right)\right) \leq 0, \tag{39}$$

where we have that $\pi_n^*\left(\widetilde{q_{-n}^2}\right) = \frac{1}{2}\left(a-c-b\left(1-\left(k_n+\varepsilon\right)\right)\widetilde{Q^2}\right)^2 \frac{k_n}{1+2bk_n}$ and $\pi_n\left(\widetilde{q^j}\right) = \left(a-c-b\widetilde{Q^j}\right)\left(k_n+\varepsilon\right)\widetilde{Q^j} - \frac{\left(\left(k_n+\varepsilon\right)\widetilde{Q^j}\right)^2}{2k_n}$, for $j = 1, 2$. Now, let $L_n = \pi_n^*\left(\widetilde{q_{-n}^2}\right) - \pi_n\left(\widetilde{q^2}\right) - \underline{\delta}\left(\pi_n\left(\widetilde{q^2}\right) - \pi_n\left(\widetilde{q^1}\right)\right)$. Some algebra shows that

$$\begin{aligned}\frac{\partial L_n}{\partial \varepsilon} &= (a-c)^2\left(1+b\left(1+k_n+\varepsilon\right)\right)k_n\frac{b}{\left(2b+1\right)^2\left(1+2bk_n\right)} \\ &\quad -\left(1+\underline{\delta}\right)\left(\left(a-c\right)^2\frac{bk_n-\varepsilon}{\left(2b+1\right)^2 k_n}\right) - \underline{\delta}\left(a-c\right)^2\frac{k_n\left(1+bk_1\right)+\varepsilon}{b^2\left(1-k_1\right)^2 k_n}.\end{aligned}$$

Now, notice that

$$\begin{aligned}&(a-c)^2\left(1+b\left(1+k_n+\varepsilon\right)\right)k_n\frac{b}{\left(2b+1\right)^2\left(1+2bk_n\right)} - \left(a-c\right)^2\frac{bk_n-\varepsilon}{\left(2b+1\right)^2 k_n} = \\ &\qquad = \left(a-c\right)^2\left(1+bk_n\right)\frac{\varepsilon\left(1+bk_n\right)-bk_n\left(1-k_n\right)}{\left(2b+1\right)^2\left(1+2bk_n\right)k_n}.\end{aligned}$$

Hence, $\varepsilon < bk_n\left(1-k_n\right)/\left(1+bk_n\right) < 1$ is a sufficient condition for $\frac{\partial L_n}{\partial \varepsilon} < 0$, i.e., for sufficiently low values of ε, the incentive constraint of the smallest firm to stick to the collusive agreement is also relaxed by the above-mentioned (marginal) transfer of output share. This completes the proof of Proposition 3. ■

Hence, by accepting a distortion in the optimal allocation rule, firms can enlarge the set of discount factors for which perfect collusion can be credibly enforced.

Two remarks are in order before closing this section. First, notice that by transferring output share from the larger to the smallest firms in the market, this exercise is lessening asymmetries between market participants, thus contributing to increase collusion possibilities. In this way, the previous Proposition confirms and extends the general intuition developed in the previous section. Second, it should be noted that the (extended) proportional two-phase penal code used in the proof of Proposition 3 is not globally optimal. It does not drive *all* firms to their minmax payoff if a deviation occurs, but should drive the largest and smallest firms in the agreement close to the (zero) minmax payoff. Studying alternative output allocation rules as well as proposing other types of optimal punishments within a more general framework will be dealt with in future research.

5 Conclusion

This paper has explored the relationship between the distribution of a tangible industry asset which affects firms' marginal costs and the scope for collusion. In particular, we have found the conditions under which a (perfect) collusive outcome can be enforced when an infinitely repeated game is played between cost asymmetric firms which produce a homogeneous good and adopt optimal punishments in the style of Abreu (1986, 1988) that guarantee a prospective deviant zero profits (the lowest profits consistent with individual rationality) in the event a deviation occurs. The results obtained embody some important insights for practical application of competition policy.

First, we show that the sustainability of perfect efficient collusion crucially depends on the asset holdings of the firms involved in the agreement. In particular, it has been found that the smallest (and, hence, most inefficient) firm in the agreement, being the one which is allotted the lowest share in the collusive aggregate output, represents the main obstacle for collusion to be enforced because this share may be too low with respect to its optimal deviation output. On the other hand, if the punishment is started, then the largest firm is the one which is proportionally most penalized in the first (severe) phase of the punishment. Therefore, this firm faces the greatest incentives to deviate from the first period of the punishment strategy.

Second, it is shown that if firms accept to diverge from the joint-profit maximization behavior, then they can enhance collusion possibilities. Perfect inefficient collusion can be enforced within a larger set of discount factors. Therefore, in our setting, there exists a trade-off between efficiency and sustainability of perfect collusive agreements.

A distinct issue is also addressed, which is the impact of changes in the distribution of firms' asset holdings on the likelihood of collusion. Some important policy implications can be derived from the results. In particular, they shed some light on the complex problem of assessing the potential *joint dominance* (pro-collusive) effects induced by a merger. It turns out that when asymmetric firms co-exist in the industry the impact of a merger depends on which firms it involves. More specifically, it is shown that a merger can induce two distinct effects. First, if firms in the market were already colluding before the merger, then a merger either has no effect on the possibility of collusion or it harms that possibility. The latter case will happen when the merger affects the size of the largest firm in the market. This result stresses the fact that although the number of competitors is reduced with the merger, which tends to facilitate collusion, this effect is more than compensated for by a more asymmetric post-merger industry configuration. Second, if before the merger collusion is not feasible, then a merger might make collusion possible afterwards. This will happen when the merger involves very small and inefficient firms that are not able to credibly participate in a collusive scheme before the merger takes place.

This analysis thus suggests that a systematic analysis of market shares and concentration indexes, such as the Herfindahl-Hirshman Index (HHI), does not always provide a reliable guide to assess potential effects on the level of competition in the market induced by a horizontal merger. Once post-merger concentration appears to be high, then, among other things, firms' cost conditions and asset holdings' distribution must be an important part of the analysis, since asymmetries may offset any increased risk of post-merger anticompetitive behavior.

It remains to be seen whether our results are robust to changes in the model assumptions: different, or more general, functional forms for cost and demand, and firm heterogeneity. It would also be interesting to analyze under which conditions a subset of large firms could reach a collusive agreement without involving small firms (partial collusion). All these questions seem to deserve further research.

A The Cournot equilibrium

Consider the general case in which inverse demand is given by (1) and total cost function of firm i is represented by eq. (2). Firm i chooses q_i to maximize profits:

$$\pi_i (q_1, ..., q_n) = (a - bQ)q_i - \left(cq_i + \frac{q_i^2}{2k_i} \right). \tag{40}$$

$\pi_i (\cdot)$ is strictly concave in q_i. Hence, the choice of q_i results from the following first order condition:

$$p - c = \left(b + \frac{1}{k_i} \right) q_i. \tag{41}$$

Define $\beta_i = \frac{bk_i}{1+bk_i}$ and $B = \sum_{i=1}^{n} \beta_i$. Now, from (41), it can be shown that $\frac{q_j}{q_i} = \frac{\beta_j}{\beta_i}$, for $i, j = \{1, .., n\}$, $i \neq j$. Hence, the following results can be easily derived:

$$q_i^n = \left(\frac{a-c}{b} \right) \frac{\beta_i}{1+B}, \tag{42}$$

$$Q^n = \left(\frac{a-c}{b} \right) \left(\frac{B}{1+B} \right), \tag{43}$$

$$p = \frac{a+cB}{1+B}, \tag{44}$$

$$s_i^n \equiv \frac{q_i}{Q} = \frac{\beta_i}{B}, \tag{45}$$

$$\pi_i^n(k_i) = \frac{1}{2} (a-c)^2 \beta_i \frac{2k_i b - \beta_i}{b^2 (1+B)^2 k_i}. \tag{46}$$

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