Extracting Information from Asset Prices:
the Methodology of EMU Calculators

Carlo A. Favero*, Francesco Giavazzi*, Fabrizio Iacone+, Guido Tabellini*

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Abstract

This paper analyses how to extract market expectations from asset prices, with a particular example: using the term structure of interest rates to estimate the probability the market attaches to the event that a country, Italy, joins the European Monetary Union at a given date. The extraction of such a probability from the term structure is based on the presumption that the term structure contains valuable information regarding the markets' assessment of a country's chances to join the EMU. The case of Italy is interesting because in the survey regularly conducted by Reuters the probability of joining EMU in 1999 fluctuated between 0.07 and 0.15, while, during the same period, the measures of computed by financial houses -- also based on the term structure of interest rates -- have ranged between 0.5 and 0.8. The paper proposes a new method for computing these probabilities, and shows that the discrepancies between survey and market-based measures are not the result of market inefficiencies, but depend on an incorrect use of the term structure to compute probabilities. The technique proposed in the paper can also be used to distinguish between convergence of probabilities and convergence of fundamentals, that is to find out whether an observed reduction in interest rate spreads signals a higher probability of joining EMU at a given time, or simply reflects improved fundamentals.

* CEPR and Università Bocconi, + Università Bocconi
Corresponding author C.A. Favero, IGIER, Università Bocconi, Via Salasco 5, 20136 Milan, Italy,
E-Mail : Carlo.Favero@uni-bocconi.it

Non-Technical Abstract

Extracting market expectations from asset prices is a question which has recently attracted a great deal of interest both among market operators and central banks (for a recent review see Söderlind and Svensson, 1997.) This paper looks at one specific example: how can the term structure be used to estimate the probability the market attaches to the event that a country, Italy, joins the European Monetary Union at a given date. We have been drawn towards this example observing the striking difference that exists between the surveys that are regularly conducted among market participants, and the probabilities estimated extracting informatiom from the term structure. Table 1 reports five observations (over the interval January to May 1997) on the probability that Italy joins EMU on 1.1.1999. The surveys conducted by Reuters show a remarkably stable assessment of a Italy's chances – ranging from a minimum of 0.07 in February, to a maximum of 0.17 the previous month.[1] In the same table we report the probability computed accordingly to the *J.P. Morgan EMU Calculator*, regularly published in the Financial Times, and the *Credito Italiano EMU Calculator*, published in the Italian daily Corriere della Sera. The probabilities computed using these two techniques are very similar, but quite distant from the results of the survey. In particular, the survey reached a minimum in February, which does not coincide with the month in which the "calculators" show a minimum; both the survey and the "calculators" show a maximum in January, but the probability computed using the "calculator" is four times larger than that of the survey.

The aim of this paper is to investigate the sources of these discrepancies. They could be related to they way in which the "calculators" are constructed; alternatively they could be the result of market inefficiencies, or of risk premia terms.

We reach two main results :

- we show that probabilities computed by currently available EMU calculators are upward biased because they are based on average rather than instantaneous forward rates and on the consideration of a limited space of events (delayed entry is not explicitly considered when computed the probability of Itly's joining EMU in 1999)
- we model a PRE-EMU spread between Italy and Germany by explicitly estimating a Taylor's rule type reaction function for the Bank of Italy. Such model allows us to identify separately the component of the spread between Italian and German interest rates due to different fundamentals from the component of the spread due to the markets' assessment of the probability of Italy oining EMU in 1996. We show that tof the 214 basis points reduction in the difference between Italian and German forward rates with maturity in 1999 observed from March 96 to March 97, 150 basis points are to be attributed to the convergence of fundamentals while only 65 basis points are to be attributed to the modifications in markets' assessment of the probability of Italy joiningf EMU in 1999.

Our analysis of the difference between probabilities as computed in EMU calculators from probalities delivered by survey start by the consideration that probabilities of Italy's joining EMU in 1999 are computed in EMU calculators by mapping average forward rates into probabilities by assuming that

(i) currently observed forward interest rates with maturity in 1999 and settlement three to ten years later are a weighted average of the spot rate differential between a country and Germany in the case that country joins and of the spot rate differential in the case that country does not join, where the weights are precisely the probabilities of joining and its complement to one.

(ii) the spot rate differential in the case a country joins is zero

(iii) the spot rate differential attains a specific value in the case a country does not join

Probabilities are then derived as the ratio of the observed average forward rate differential to the spot rate differential assumed for the scenario in which a country is left out.

Leaving aside market inefficiencies, there are two possible sources of bias in computed probability assumption (ii) and (iii).

We claim that it is possible to identify a source of systematic bias in computed probability. Such bias is determined by the fact that average rather than instantaneous forward rates are used and by the omission of the inclusion in the space of the events a delayed entry by Italy.

To do so we first rule out that the source of the bias is generated by a "too high" value of the differentials in case of no entry in 1999 by substantially confirming the results obtained by financial houses with the explicit estimation of a reaction function for the Bank of Italy linking the interest rate spread between Italy and Germany to macroeconomic fundamentals.

We then construct an EMU calculator based on instantaneous forward rates with the consideration of three possible events Italy joins in 1999, Italy joins in 2001 and Italy does not join within 2001.

[1] Reuters polls each month 43 experts at banks, research houses, think tanks, universities and employers' associations across Europe. The list of panellist is available from the Reuters code <EMUPOLL37>. Poll details are on Reuters pages <EMUPOLL30> to <EMUPOLL37>.

We describe the perception by the market of Italy joining the EMU as a drop of Italian overnight rates from the PRE-EMU instantaneous forward rate curve to the German instantaneous forward rate curve. Using instantaneous forward rates allows to avoid the explicit consideration of the event Italy joins in 2001 when computing the probability of Italy joining in 1999. This is because the fact that markets discard the possibility of Italy joining in 1999 will put the overnight rate on the PRE-EMU curve both in the case Italy is expected to join before and after 2001. However this is not true when considering average forward rates. Think of thecase of average three-forward rates. We can interpret these rates as the average of the about nine hundred instantaneous forward rates expected to prevail between 1999 and 2002. It is clear that the curve of average rate when Italy is not expeced to join by 1999 but it is expected to join by 2001 will differ from the curve associated to the event Italy does not join in the three year following 1999. In fact in the first case the average forward rate will be an average of the PRE-EMU Italian rates and the German instantaneous forward rates, in the second case the average forward rate will be just an average of the PRE-EMU instantaneous forward rates. We show that in this case the omission from the space of the events of late entry in 2001 produces in upward bias of the probability of entry in 1999. This is because any mass attached to the late entry event is instead attributed to the entry in 1999 event. We consider the case of March 1997 to show that while the probability of Italy joining in 1999 computed on instantaneous forwad rates is 0.24, the same probability computed using average two-year forward rate is 0.41.
To understand the relevance of the argument consider the following “limit case” example. Markets are hundred per cent sure that Italy will join in 2001, therefore the probability of converging in 1999 is zero. In this case the Italian instantaneous forward rate for 1999 will be on the PRE-EMU curve and computation based on this rate will deliver the correct estimate of the probability of Italy joining in 1999. However the three-year average forward rate will be lower than the PRE-EMU forward rate being an average of two years of PRE-EMU Italian instantaneous forward rates and one year of German instantaneous forward rates. As a consequence the probability of Italy entering EMU in 1999 computed on the basis of forward rates (as one minus the ratio the average forward rate differential bewteen Italy and Germany to differential between the PRE-EMU Italian rate and the German rate) will be upward biased and different from zero.

Introduction

Extracting market expectations from asset prices is a question which has recently attracted a great deal of interest both among market operators and central banks (for a recent review see Söderlind and Svensson, 1997.) This paper looks at one specific example: how can the term structure be used to estimate the probability the market attaches to the event that a country, Italy, joins the European Monetary Union at a given date. We have been drawn towards this example observing the striking difference that exists between the surveys that are regularly conducted among market participants, and the probabilities estimated extracting informatiom from the term structure. Table 1 reports five observations (over the interval January to May 1997) on the probability that Italy joins EMU on 1.1.1999. The surveys conducted by Reuters show a remarkably stable assessment of a Italy's chances – ranging from a minimum of 0.07 in February, to a maximum of 0.17 the previous month.[2] In the same table we report the probability computed accordingly to the *J.P. Morgan EMU Calculator*, regularly published in the Financial Times, and the *Credito Italiano EMU Calculator*, published in the Italian daily Corriere della Sera. The probabilities computed using these two techniques are very similar, but quite distant from the results of the survey. In particular, the survey reached a minimum in February, which does not coincide with the month in which the "calculators" show a minimum; both the survey and the "calculators" show a maximum in January, but the probability computed using the "calculator" is four times larger than that of the survey.

The aim of this paper is to investigate the sources of these discrepancies. They could be related to they way in which the "calculators" are constructed; alternatively they could be the result of market inefficiencies, or of risk premia terms. In the second and third sections of the paper we discuss the construction of a "calculator", spelling out the assumptions that are needed in order to arrive at an estimate of probabilities. In the fourth section we use this framework to ask whether the convergence between Italian and German interest rates, observed since the third quarter of 1996, is the result of converging fundamentals or of a change in the assessment of Italy's chances to join EMU, related to market sentiment, but unrelated to fundamentals. Finally, in section 5, we assess the "EMU Calculators" currently in use.

Table 1: Probability that Italy joins EMU from the start, in 1999

	15th Jan 97	15th Feb 97	15th Mar 97	15th Apr 97	15th May 97
Reuters **EMU Survey**	0.17	0.07	0.12	0.15	0.14
Credito Ital. **EMU Calculator**	0.78	0.70	0.54	0.55	0.58
J.P. Morgan **EMU Calculator**	0.70	0.61	0.51	0.56	0.58

The construction of an EMU calculator

The first step involves the estimation of the term structure of spot rates for Italy and Germany under the assumption that the German curve is to be taken as the benchmark for the Euro curve after EMU has started. To the curve estimated on spot rates for each country we then associate the term structure of istantaneous forward rates: we interpret this curve as the sequence of overnight rates expected to prevail at any date in the future. Forward rates are interest rates on investments made at a future date, the *settlement date*, and expiring at a date further into the future, the *maturity date*. Instantaneous forward interest rates are the limit as the maturity date and the settlement date approach one another. The relationship between a "yield-to-maturity" and the instantaneous forward rate at that maturity is thus analogous to the relationship between marginal and average cost. The curve of instantaneous forward rates thus lies above the curve of spot rates, when this is positively sloped, and below the curve of spot rates, when this is negatively sloped.

[2] Reuters polls each month 43 experts at banks, research houses, think tanks, universities and employers' associations across Europe. The list of panellist is available from the Reuters code <EMUPOLL37>. Poll details are on Reuters pages <EMUPOLL30> to <EMUPOLL37>.

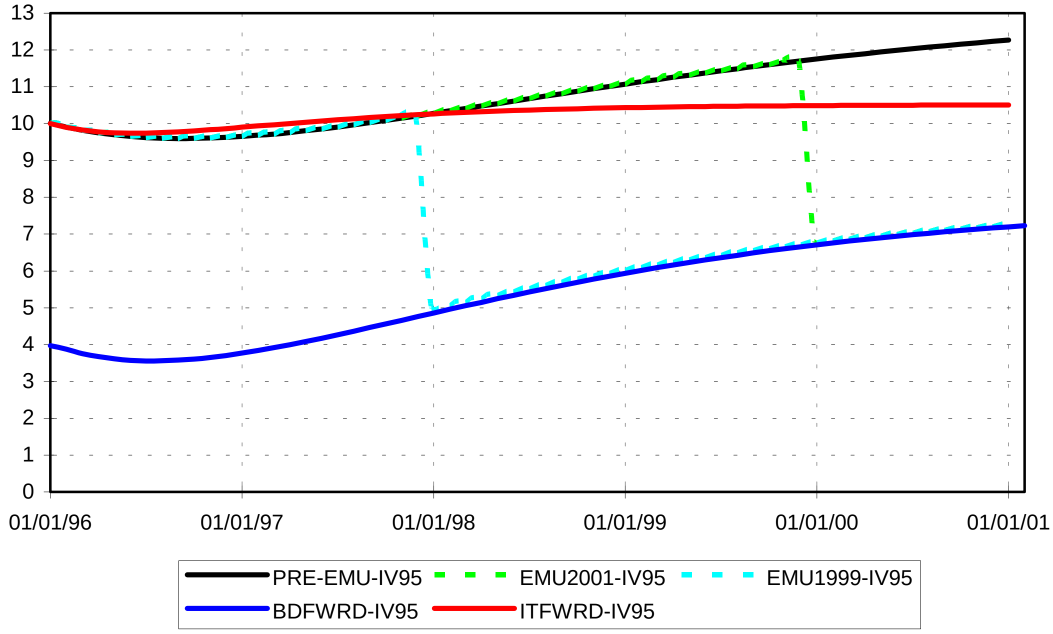
If the pure expectational model is valid, and there is no term premium, then instantaneous forward rates at future dates can be interpreted as the overnight spot rates expected to prevail at those future dates. If we think of the overnight rate as the rate controlled by the central bank, then the curve of instantaneous forward rates can be interpreted as an indicator of expected monetary policy, based on the pure expectational model.

Instantaneous forward rates are of particular interest in judging the likelihood of a country joining EMU, since in the monetary union the overnight rate will be the same for all the participating countries. We consider two scenarios: a first one in which Italy joins EMU on 1 January 1999; and a second one in which Italy joins EMU two years later, at the beginning of 2001. We describe the event of joining EMU by assuming that, from the entry date onward, the curve of Italian instantaneous forward rates coincides with that of Germany. We thus have two portions of hypothetical curves of instantaneous forward rates for Italy: one describing the path of overnight rates from 1999 onwards, if Italy joins in 1999; one describing the path of overnight rates from 2001 onwards, if Italy joins in 2001. Of each curve we also know the initial point, which is the current overnight rate, but there remain two gaps, one on each curve -- respectively from the initial observation to 1 January 1999, and from the initial observation to 1 January 2001.

Inbetween the current date, and the day EMU starts, there remains exchange rate risk, and overnight rates are determined by the interaction between market expectations and the central bank's monetary policy rule. We refer to the sequence of these overnight rates as the "PRE-EMU" curve. The level of overnight rates along such curve will be different for different countries reflecting differences in expectations and in monetary policy rules. In the next section we estimate such curve for Italy, relating the level of Italian overnight rates to German rates and to other fundamentals.

On the day the exchange rate risk disappears the overnight rates fall to the level of the corresponding German rates. Thus the "PRE-EMU" drops to the German instantaneous forward curve. If a country can join EMU at different dates – 1.1.1999, 1.1.2001, or later – there will be uncertainty as to when such drop will occur. In the case of Italy we consider three alternatives: joining in 1999, joining in 2001, and not joining by 2001. Figure 1 reports five curves -- the observed Italian and German instantaneous forward curves, the Italian curve in the case Italy joins in 1999, the Italian curve in the case Italy joins in 2001, and the PRE-EMU Italian curve.

Figure 1: Forward Rates and Convergence to EMU as of 1995:IV



In our approach the information on the date at which the forward overnight rate drops to the German overnight rate is unnecessary to identify the probability of entry in 1999 from the probability of entry in 2001. What we need, to be able to identify the probabilities, is simply that the instantaneous forward rate on 1.1.1999, in the case of no entry in 1999, lies on the PRE-EMU curve. (In Figure 1, for the sake of illustration, we have assumed that such jumps happen on 1.1.1998, in one case, and on 1.1.2000 in the other.) To map forward differentials into probabilities, we use the following formula:

$$\mathbf{ITFW}_t = \pi_{99,t}\,\mathbf{EMU99}_t + \pi_{01,t}\,\mathbf{EMU01}_t + (1-\boldsymbol{\pi}_{\mathbf{99,t}}-\boldsymbol{\pi}_{\mathbf{01,t}})\,\mathbf{PRE\text{-}EMU}_t \qquad (1)$$

where :

ITFW is the observed Italian instantaneous forward rates curve;
EMU99 is the theoretical Italian instantaneous forward rates curve associated to the event Italy joins in 1999;
EMU01 is the theoretical Italian instantaneous forward rates curve associated to the event Italy joins in 2001;
PRE-EMU is the theoretical Italian instantaneous forward rates curve associated to the event Italy does not join;
$\boldsymbol{\pi}_{\mathbf{99,t}}$ is the estimated probability at time t of Italy joining in 1999;
$\boldsymbol{\pi}_{\mathbf{01,t}}$ is the estimated probability at time t of Italy joining in 2001.

Although we do not know the exact form of the curves EMU99 and EMU01, we can evaluate equation (1) using the implicit forward rates for January 2001:

$$\mathbf{ITFW}_t = \pi_{99,t}\,\mathbf{BFW}_t + \pi_{01,t}\,\mathbf{BFW}_t + (1-\boldsymbol{\pi}_{\mathbf{99,t}}-\boldsymbol{\pi}_{\mathbf{01,t}})\,\mathbf{PRE\text{-}EMU}_t \qquad (2)$$

where **BFW** is the instantaneous German forward rate. Similarly we can evaluate (1) using the implicit forward rates for January 1999:

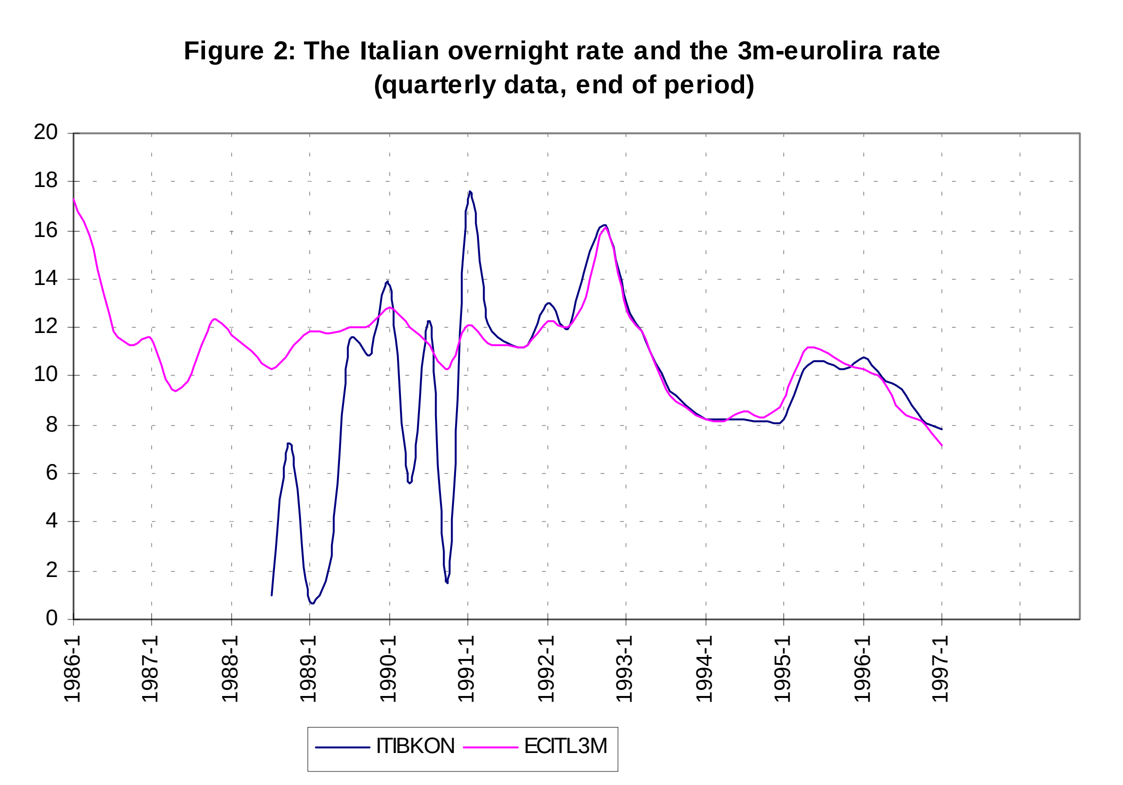
$$\mathbf{ITFW}_t = \pi_{99,t}\mathbf{BFW}_t + \pi_{01,t}\,\mathbf{PRE\text{-}EMU}_t + (1-\boldsymbol{\pi}_{\mathbf{99,t}}-\boldsymbol{\pi}_{\mathbf{01,t}})\mathbf{PRE\text{-}EMU}_t \qquad (3)$$

ITFW, BFW, and PRE-EMU are known. Equations (2) and (3) thus form a system in the two unknowns $\pi_{99,t}$ and $\pi_{01,r}$. The solutions to such system are our identified probabilities.

Forward Curves and probabilities of convergence

In the previous section we have shown that in order to identify the probabilities we must estimate three instantaneous forward curves: the Italian instantaneous forward curve, the German instantaneous forward curve, and the Italian forward curved associated to the PRE-EMU event. We estimate the first two curves using the Nelson-Siegel interpolant discussed in Svensson (1994) -- details of this estimation are provided in the Appendix. Our estimate of the PRE-EMU curve is based on a reaction function which explains the Italian short-term rate with macroeconomic fundamentals only. The obvious dependent variable should be the overnight rate, which is the observable equivalent of the instantaneous forward rate. However, as shown in Figure 2, the Italian overnight rate is extremely volatile[3] -- a result of the reserve requirement liquidity factors in the market for bank reserves prior to the reform of October 1990; in fact from then on the overnight rate moves very close to other short-term rates, such as the 3-months Euro rate.

We have thus specified our reaction function on 3-months Euro rates, taking 1987 as the initial date, and ending the estimation period in 1996:2— a date which allows us to simulate the model to obtain a PRE-EMU level of short-term interest rates for at least one-year. Our specification of the equation for short-term interest rates is a rule very much in the spirit of Taylor (1995), although adapted to an open economy. The estimated rule is reported on Table 2.



[3] A result of the Bank of Italy rules which, up to October 1990, forced Italian banks to meet the mandatory reserve requirement daily, rather than on average over the maintanance period.

TABLE 2: A rule for short-term Italian interest rates

Modelling IT3MCC by OLS The sample is: 1987 (1) to 1996 (2)

Variable	Coefficient	Std.Error	t-value	t-prob	PartR^2
Constant	0.0255049	0.0125307	2.035	0.0510	0.1250
IT3MCC_1	0.599078	0.126913	4.720	0.0001	0.4345
INFLGAP	0.314109	0.140926	2.229	0.0337	0.1463
GRDGAP	0.264385	0.107711	2.455	0.0203	0.1720
REUGR	-0.184274	0.0914312	-2.015	0.0532	0.1229
LUSDDM	-0.0793958	0.0231922	-3.423	0.0019	0.2878
LUSDM_1	0.0469158	0.0222621	2.107	0.0438	0.1328
BD3MCC	0.905901	0.281672	3.216	0.0032	0.2629
BD3MCC_1	-0.487597	0.329971	-1.478	0.1503	0.0700

R^2 = 0.77975 F(8, 29) = 12.834 [0.0000] σ = 0.007667534 DW = 1.74
RSS = 0.0017049411422 for 9 variables and 38 observations

Diagnostic Tests

AR 1- 3F(3, 26) = 0.318958 [0.8116] ARCH 3 F(3, 23) = 0.145167 [0.9317]
Normality Chi^2(2)= 10.613 [0.0050] ** Xiý F(16, 12) = 1.4256 [0.2703]
RESET F(1, 28) = 0.563669 [0.4590]

Solved Static Long Run equation

IT3MCC =	+0.063616	+1.043 BD3MCC	-0.45963 REUGR
(SE)	(0.029368)	(0.30382)	(0.26185)
	-0.081013 LUSDM	+0.78347 INFLGAP	+0.65944 GRDGAP
	(0.045129)	(0.30131)	(0.28175)

EQ(6) Modelling IT3MCC by OLS

Variable	Coefficient	Std.Error	t-value	t-prob	PartR^2
Constant	0.0309317	0.00944132	3.276	0.0028	0.2771
IT3MCC_1	0.576555	0.0950693	6.065	0.0000	0.5678
INFLGAP	0.228647	0.106887	2.139	0.0413	0.1405
GRDGAP	0.234366	0.0808244	2.900	0.0072	0.2309
REUGR	-0.190575	0.0684216	-2.785	0.0095	0.2170
LUSDM	-0.0659308	0.0175706	-3.752	0.0008	0.3346
LUSDM_1	0.0452027	0.0166604	2.713	0.0113	0.2082
BD3MCC	1.0064	0.211752	4.753	0.0001	0.4465
BD3MCC_1	-0.717288	0.251335	-2.854	0.0080	0.2253
i1992p4	0.0315236	0.00646128	4.879	0.0000	0.4595

R^2 = 0.880953 F(9, 28) = 23.022 [0.0000] σ = 0.005736896 DW = 1.86
RSS = 0.00092153534533 for 10 variables and 38 observations

Diagnostic Tests

AR 1- 3F(3, 25) = 1.1308 [0.3556] ARCH 3 F(3, 22) = 0.248704 [0.8614]
Normality Chiý(2)= 0.634888 [0.7280] Xiý F(17, 10) = 0.29833 [0.9862]
RESET F(1, 27) = 0.175522 [0.6786]

Solved Static Long Run equation

IT3MCC =	+0.073048	+0.68266 BD3MCC	-0.45006 REUGR
(SE)	(0.021502)	(0.22988)	(0.18445)
	-0.048951 LUSDM	+0.53997 INFLGAP	+0.55347 GRDGAP
	(0.029612)	(0.21516)	(0.19165)
	+0.074445 i1992p4		
	(0.022078)		

The Italian short-term interest rate depends on its own lag, on the current and lagged level of the German short-term rate, and on three weakly exogenous variables: the inflation gap between Italy and Germany, defined as the difference between the headline annual CPI inflation (log of price in quarter t *minus* log of price in quarter t-4) between the two countries; the output gap between Italy and Germany, defined as the difference in annual GDP growth in the two countries; and the current and lagged level of the log of the US dollar-Deutschemark exchange rate. Because of the potential effect of German reunification on our estimated coefficients we have interacted both the inflation gap and the output gap with a reunification dummy, which takes a value of 1 in 1991, and zero anywhere else. The reunification effect is significant when interacted with the outuput gap, but not significant when interacted with the inflation gap: in the final specification we have thus kept only the product of the output gap and the reunification dummy (REUGER.)

The intuition behind our specification is that of a small open-economy Taylor-rule where the central bank has an objective function which includes, along with the usual macroeconomic variables such as inflation and growth, exchange rate stability. The objective of exchange rate stability is implemented by defining the target values of the macroeconomic variables as those assumed by these variables in the reference country -- Germany. The Lira-Deutschemark rate cannot be assumed to be weakly exogenous: it was thus replaced, in the estimation, with the dollar-Deutschemark rate, which we interpret as a weakly exogenous instrument correlated with the Italian Lira-Deutschemark rate (see Giavazzi and Giovannini, 1989.) Exogeneity of the macroeconomic variables is guaranteed if monetary policy takes some time -- at least one quarter -- to affect such variables – by now a standard assumption in the literature on monetary transmission mechanism which uses structural VAR models (Bernanke-Mihov, 1996, Christiano and Eichenbaum, 1992, Sims and Leeper and Zha, 1996.) [4]

Before commenting on the empirical results is worth mentioning that we have experimented with alternative specifications. In particular, we have included, on the right-hand-side of the equation, the Italian GDP gap -- deviation of actual GDP from a Hodrick-Prescott trend -- and a commodity price index. The first variable would be justified if we allow for the possibility that even a small open economy targets growth independently of the reference country; the second variable has an established tradition as a leading indicator for inflation, and a relevant argument in the reaction functions of central banks.[5] Neither the deviation of GDP form its trend, nor commodity prices turn out to be significant when added to our basic specification. On the basis of these results we concluded that there is no statistical evidence that the Italian central bank targets the deviation of GDP from its trend (as identified by the HP filter), and thus omitted this variable from the estimated equation. We have also omitted commodity prices: as we include the German policy rate, and considering that this rate reacts significantly to commodity prices (Bernanke and Mihov, Clarida and Gertler), we concluded that commodity prices would not play an independent role in our specification.

Table 2 reports our results. The coefficient of on the lagged dependent variable is 0,6 and that on short-term German interest rates is not significantly different from one. The coefficients on the inflation and output gaps are, respectively, 0,31 and 0,26 in the short-run, 0,78 and 0,66 in the long-run. These long-run coefficients are significantly different from zero, but not from 0,5. Both the short-run and the long-run coefficient on the dollar-Deutschemark exchange rate take a value of -0.08, implying that a one percentage point appreciation of the US dollar against the Deutschemark will be reflected in an eight basis points fall in Italian short-term rates. The estimated equation passes all the diagnostics with the exception of the normality of residuals. As shown in Figure 3, which displays the time series of actual and fitted Italian short-term rates, the absence of normality in the residuals may be due to the presence of an outlier in 1992:4. This is likely to be associated to a shock in the risk premium rather than a strucutral break of the model. This suspicion is confirmed by the results, also reported in Table 2, obtained including a point dummy for 1992:4. Including the dummy eliminates the non-normality problem, but does not alter the other coeffcients. To further investigate the effects of shocks to the risk premium we have simulated a shock of 315 basis points -- the value of the coefficient on the EMS crisis dummy. The inflation and output gaps, the US dollar-Deutschemark rate, and the German three-month rate are all set at their observed values in 1997:1. The model is initialised at the steady state. The simulation, reported in Figure 4, shows that a shock to the risk premium of the dimension observed on the occasion of the Italy's exit from the ERM is re-absorbed within two-years.

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[4] Note that our single-equation rule is consistent with the rules derived within VAR models, and, under the validity of our identifying assumption, the parameters estimated in a structural VAR would coincide with the ones delivered in our single-equation framework.

[5] The original Taylor rule does not include commodity prices as an explanatory variable in the determination of short-term interest rates. However, the structural VAR approach has shown that the omission of this variable may lead to misspecification of the reaction function, and to some puzzling impulse responses, *i.e.* prices declining in response to an expansionary monetary policy shock.

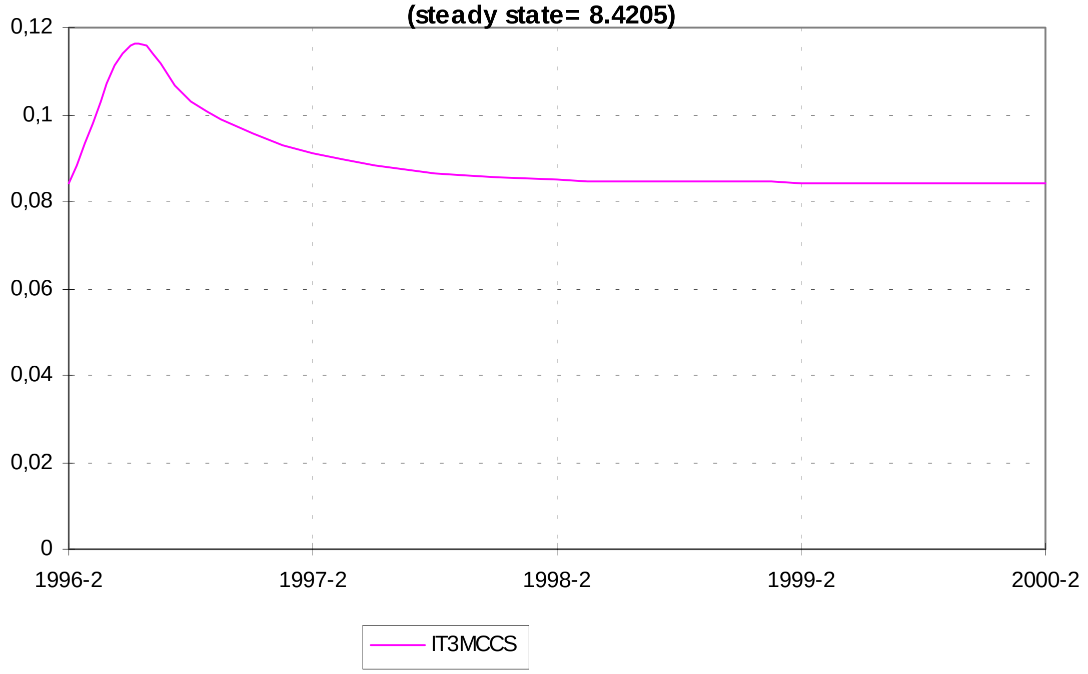


Figure 4: Simulating the effect of a 315 basis point shock to the risk premium on the Italian 3m-eurolira rate (steady state= 8.4205)

Having estimated the rule for Italian short-term rates we proceed as follows:

- we map this rule into the PRE-EMU curve, projecting it forward for two years. We use "Consensus" forecasts for Italian and German inflation and growth, and for the dollar-Deutschemark exchange rate; the path of short-term German rates is instead derived from the estimated German instantaneous forward curve;
- we fit a Nelson-Siegel forward function through ten points: the current overnight rate, the eight rates obtained projecting the rule, and an asymptot which corresponds to the long-run solution of the estimated rule. The ten points thus obtained are sufficient to identify the four parameters in the Nelson-Siegel forward function.

This procedure allows us to map fundamentals into a PRE-EMU curve, and thus to associate the level of the PRE-EMU Italian interest rate to the evolution of fundamentals. We have now all the necessary information for the computation of probabilities: we show in Figure 5 the estimated curves for the actual Italian forward rate, for the German forward rate, and for the Italian PRE-EMU forward rates. On the basis of these curves probabilities of convergence have been computed, and are reported in Table 3.

TABLE 3: The Probability of Italy Joining EMU in 1999 and in 2001.

date	$\pi_{99,t}$	$\pi_{01,t}$	$1-\pi_{99,t}-\pi_{01,t}$	ITFW99	ITFW01	BFW99	BFW01	NOEU99	NOEU01
11/12/95	0,12692	0,22262	0,65046	10,433	10,507	6,017	7,229	11,075	12,268
11/03/96	0,30337	0,10690	0,58973	9,772	10,378	6,496	7,597	11,199	12,312
10/06/96	0,23875	0,23944	0,52181	8,806	9,367	6,052	7,464	9,669	11,111
09/09/96	0,31362	0,20886	0,47748	8,230	9,006	5,394	7,028	9,526	11,171
09/12/96	0,52379	0,17154	0,30467	6,406	7,426	4,584	6,249	8,410	10,113
10/03/97	0,24311	0,24926	0,50763	6,707	7,416	4,440	5,947	7,436	8,842

ITFW99 is the observed Italian instantaneous forward rate for January 1999, at date t

ITFW01 is the observed Italian instantaneous forward rate for January 2001, at date t

BFW99 is the observed German instantaneous forward rate for January 1999, at date t

BFW01 is the observed German instantaneous forward rate for January 2001, at date t

NOEU99 is the theoretical Italian instantaneous forward rate associated to the event Italy does not join EMU for January 1999, at date t

NOEU01 is the theoretical Italian instantaneous forward rate associated to the event Italy does not join EMU for January 2001, at date t

$\pi_{99,t}$ is the estimated probability at date t of Italy joining the EMU in 1999

$\pi_{01,t}$ is the estimated probability at date t of Italy joining the EMU in 2001

$\pi_{99,t}$ and $\pi_{01,t}$ are derived solving recursively the following two equations system :

$ITFW01_t = \pi_{99,t}BFW01_t + \pi_{01,t}\, BFW01_t + (1-\pi_{99,t}-\pi_{01,t})PRE\text{-}EMU01_t$

$ITFW99_t = \pi_{99,t}BFW99_t + \pi_{01,t}\, PRE\text{-}EMU99_t + (1-\pi_{99,t}-\pi_{01,t})PRE\text{-}EMU99_t$

Two features of our results are worth commenting. First, the substantial decrease in Italian forward rates from the beginning to the end of our sample are associated more with a decrease in the PRE-EMU forward rates than with an increase in the probability of Italy joining EMU. Second, the probabilities of Italy joining in 1999, computed according to our procedure, are uniformly lower than those derived using the EMU calculators presented in Table 1, although our probabilities of convergence in 2001 are much closer to the probabilities of convergence in 1999 computed by the EMU calculators. We shall devote the next two sections to a closer investigation of these results.

Figure 5.1:The estimated Italian forward rates

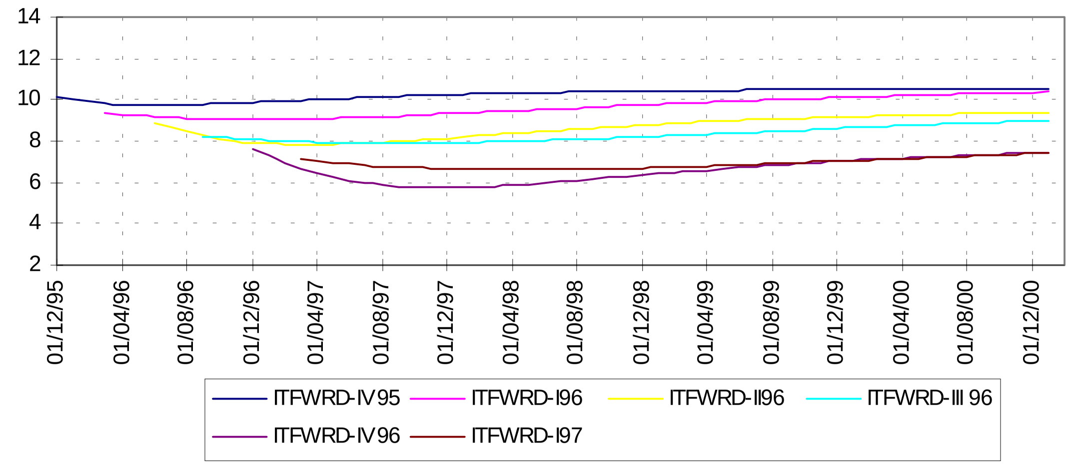


Figure 5.2:The estimated German forward rates

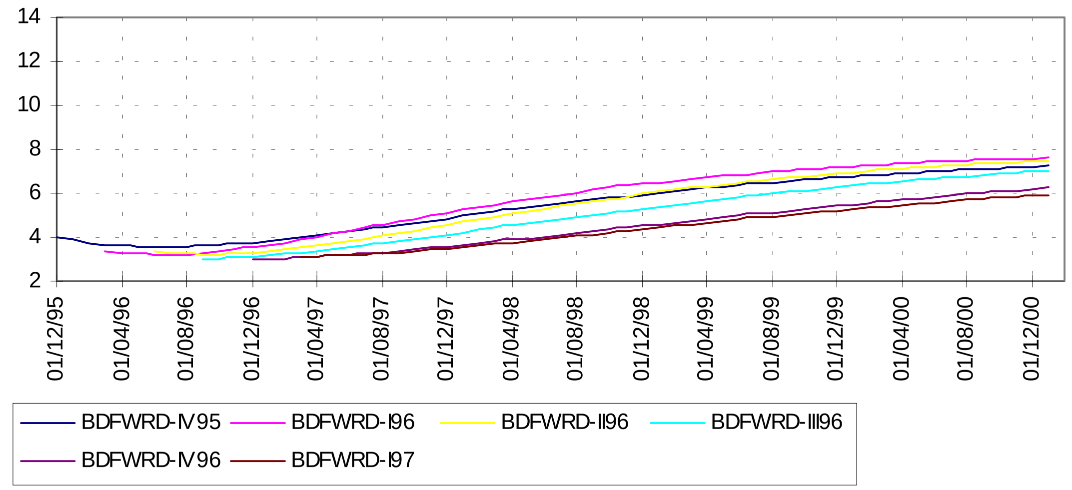
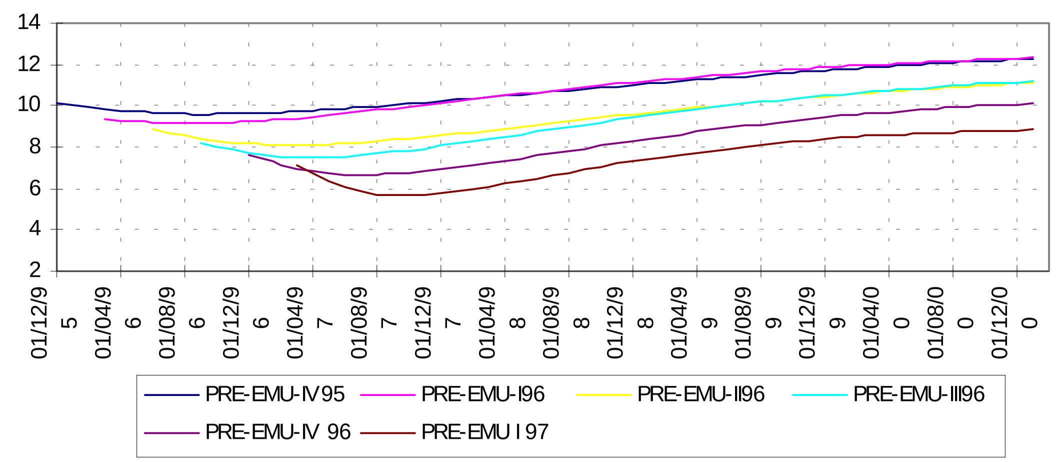


Figure 5.3:The estimated PRE-EMU Italian forward rates



Convergence of probabilities or convergence of fundamentals ?

Does the observed convergence of Italian and German interest rates, observed from the second half of 1996 onwards, depend on the convergence of fundamentals (inflation and out gaps), or is it the result of an increase in the probability that Italy joins EMU ? The Banca d'Italia has recently proposed (see Bollettino Economico, February 1997) a methodology that allows to identify the relative contribution of these factors. It shows that the reduction in the spread between Italian and German interest rates is more related to the convergence of fundamentals than to a change in the market assessment of the probability of Italy joining EMU. We will adapt and extend the proposed methodology to investigate the same issue within our framework.

As shown in the previuos section, our probabilities of Italian convergence, $\pi_{99,t}$ and $\pi_{01,t}$, are derived by solving recursively the following two-equations system:

$$ITFW01_t = \pi_{99,t}BFW01_t + \pi_{01,t}\,BFW01_t + (1-\pi_{99,t}-\pi_{01,t})PRE\text{-}EMU01_t \quad (4)$$

$$ITFW99_t = \pi_{99,t}BFW99_t + \pi_{01,t}\,PRE\text{-}EMU99_t + (1-\pi_{99,t}-\pi_{01,t})PRE\text{-}EMU99_t \quad (5)$$

by subtracting, respectively, BFW01 and BFW99 both from the left and the right-hand side of the two equations, we can write the following relationships, involving spreads:

$$SP01_t=(1-\pi_{99,t}-\pi_{01,t})\,SPNE01_t \quad (6)$$

$$SP99_t=(1-\pi_{99,t})\,SPNE99_t \quad (7)$$

where SP99=ITFW99-BFW99, SP01=ITFW01-BFW01, SPNE99=NOEU99-BFW99, and SPNE01=NOEU01-BFW01.

Differentiating (6) and (7), we can decompose the time variation of the forward spread between Italy and Germany in its components:

$$\Delta SP99_t=(1-\pi_{99,t})\,\Delta SPNE99_t - SPNE99_{t-1}\,\Delta\pi_{99,t} \quad (8)$$

$$\Delta SP01_t=(1-\pi_{99,t}-\pi_{01,t})\,\Delta SPNE01_t - SPNE01_{t-1}\,\Delta\pi_{99,t} - SPNE01_{t-1}\,\Delta\pi_{01,t} \quad (9)$$

Consider the forward spread for January 1999. The term $(1-\pi_{99,t})\,\Delta SPNE99_t$ captures that part of the change in the spread which is not related to the change in probability but rather to the change in the PRE-EMU spread, which only reflects fundamentals. The term $(-SPNE99_{t-1}\,\Delta\pi_{99,t})$ captures instead the change in the spread due to the change in markets' perception of probabilities, and is thus not directly related to fundamentals. Similarly for the forward spread in January 2001, the term $(1-\pi_{99,t}-\pi_{01,t})\,\Delta SPNE01_t$ reflects the effect of fundamentals, while the term $(-SPNE01_{t-1}\,\Delta\pi_{99,t} - SPNE01_{t-1}\,\Delta\pi_{01,t})$ reflects the probability effect.

Within this framework we can decompose the time variation of the forward spread into a component related to fundamentals, and one related to market sentiment (probabilities) but unrelated to fundamentals. Let's consider the overall spreads first: the top panel of Table 4 shows that the forward spread between Italy and Germany for January 1999 has dropped from 441 basis points at the end of 1995, to 226 basis points at the end of the first quarter of 1997. Similarly, the forward spread for January 2001 has decreased form 328 basis points to 147. Note, however, that the PRE-EMU spreads also fell -- from 505 to 299 basis points and from 504 to 290 basis points, respectively. The lower panel of Table 4 shows the decomposition period by period, and over the entire interval. Out of a total reduction of 214 basis points in the forward spread for January 1999, 149 basis points can be attributed to the direct effect of fundamentals, only 65 to the probability effect. Similarly, out of a total reduction of 181 basis points in the forward spread for January 2001, 109 are accounted for by fundamentals, and only 76 by the probability effect -- 64 accounted for by a change in the probability of Italy converging in 1999, and 8 by a change in the probability of Italy converging in 2001.

We conclude that the observed convergence between Italian and German interest rates is more a convergence of fundamentals than a convergence of probabilities – thus confirming the observation made by the Banca d'Italia.

TABLE 4 : ASSESSING CONVERGENCE

date	SP99	SP01	SPNE99	SPNE01	$\pi_{99,t}$	$\pi_{01,t}$
11/12/95	4.416	3.278	5.058	5.039	0.126917	0.222619
11/03/96	3.277	2.780	4.704	4.715	0.303365	0.106896
10/06/96	2.753	1.903	3.617	3.647	0.238746	0.239438
09/09/96	2.836	1.978	4.132	4.143	0.313622	0.208896
09/12/96	1.822	1.177	3.826	3.863	0.523787	0.171540
10/03/97	2.267	1.470	2.996	2.895	0.243106	0.249261

SP99=ITFW99-BFW99
SP01=ITFW01-BFW01
SPNE99=NOEU99-BFW99
SPNE01=NOEU01-BFW01
ITFW99 is the observed Italian instantaneous forward rate for January 1999, at date t
ITFW01 is the observed Italian instantaneous forward rate for January 2001, at date t
BFW99 is the observed German instantaneous forward rate for January 1999, at date t
BFW01 is the observed German instantaneous forward rate for January 2001, at date t
NOEU99 is the theoretical Italian instantaneous forward rate associated to the event Italy does not join EMU for January 1999, at date t
NOEU01 is the theoretical Italian instantaneous forward rate associated to the event Italy does not join EMU for January 2001, at date t
$\pi_{99,t}$ is the estimated probability at date t of Italy joining the EMU in 1999
$\pi_{01,t}$ is the estimated probability at date t of Italy joining the EMU in 2001

$\pi_{99,t}$ and $\pi_{01,t}$ are derived solving recursively the following two equations system :

$ITFW01_t = \pi_{99,t}BFW01_t + \pi_{01,t}\, BFW01_t + (1-\pi_{99,t}-\pi_{01,t})PRE\text{-}EMU01_t$

$ITFW99_t = \pi_{99,t}BFW99_t + \pi_{01,t}\, PRE\text{-}EMU99_t + (1-\pi_{99,t}-\pi_{01,t})PRE\text{-}EMU99_t$

Decomposition of the time variation in the spread
between Italian and German forward rates

date	ΔSP99	FUND99	PR99	ΔSP01	FUND01	PR0199	PR01
11/03/96	-1.139	-0.24664	-0.89244	-0.497	-0.19146	-0.88915	0.583149
10/06/96	-0.524	-0.82755	0.30395	-0.877	-0.55724	0.304645	-0.62487
09/09/96	0.083	0.353473	-0.2708	0.075	0.23699	-0.27304	0.111376
09/12/96	-1.014	-0.14555	-0.86833	-0.801	-0.08526	-0.8707	0.154762
10/03/97	0.445	-0.62846	1.073887	0.293	-0.49144	1.084305	-0.30024
TOTAL	-2.148	-1.495	-0.654	-1.808	-1.088	-0.644	-0.076
percent	100	69,6	30,4	100	60,2	35,6	4,2

$\Delta SP99_t = (1-\pi_{99,t})\, \Delta SPNE99_t - SPNE99_{t-1}\, \Delta\pi_{99,t} = FUND99+PR99$

$\Delta SP01_t = (1-\pi_{99,t}-\pi_{01,t})\, \Delta SPNE01_t - SPNE01_{t-1}\, \Delta\pi_{99,t} - SPNE01_{t-1}\Delta\pi_{99,t-1} = FUND99+PR0199+PR01$

Evaluating EMU Calculators

The probability that Italy joins EMU in 1999, computing according to the technique we have described, is significantly lower than that computed by J.P. Morgan and Credito Italiano, and reported in Table 1. Why is there a difference, and which technique should be used ?

The J.P. Morgan and the Credito Italiano "EMU Calculators" both use the following assumption to compute the probability that Italy joins in 1999:

$$SP99_{m,t} = (1 - \pi_{99,t})\, SPNE99_{m,t} \qquad (10)$$

where $SP99_{m,t}$ is the observed spread between the Italian and German m-year forward rates with settlement in 1999, and $SPNE99_{m,t}$ is the theoretical spread between the Italian and German m-year forward rates with settlement in 1999 in the event "Italy does not enter EMU in 1999". m is set to three-years by Credito Italiano, and to five years by J.P. Morgan. The two institutions also use different assumptions to pin down the value of the differential in the event Italy does not join EMU – our PRE-EMU spread. Credito Italiano sets this spread equal to its average in 1993 -- on the presumption that in 1993 the start of EMU was not incorporated in market prices. J.P. Morgan, instead, identifies this spread by claiming that there is an international price for the risks of policy, inflation, and volatility, whose aggregation constitutes the total spread. Therefore "if EMU were not around, the spread between Italian and German bonds would be highly correlated with international measures of risks such as US-Australia, US-Canada, Brady-Treasury, the US long bond-2yr bond spread, as well as the order of performance of non-European currencies and market volatilities." [J.P. Morgan, 1997] J.P. Morgan thus estimates a regression of the observed spread on an average non-European spreads, the level of American market rates, the steepness of the American yield curve, and non-European measures of volatility, using daily data over the sample January 1989-December 1991. On the basis of the estimated coefficient of the regression in a no-covergence period they are able to map current observations on the regressors into a PRE-EMU level of the spread. The PRE-EMU spread thus computed can be updated daily, thus at a much higher frequency than our measure of the PRE-EMU spread, which is based on macroeconomic fundamentals updated quarterly.

The choice of a different PRE-EMU spread is an obvious candidate to explain the divergence between the probabilities we compute, and those computed by the two institutions. We shall rule out this explanation: the probabilities computed by J.P. Morgan and Credito Italiano move very closely to one another, and the level of the PRE-EMU spread based on the measure of the price of risk computed by J.P. Morgan is not very different from the level of PRE-EMU spread based on macroeconomic fundamentals used in this paper -- the difference never exceeds 40 basis within our sample. This difference, as we shall now show, depends instead on the use, by the two financial houses, of forward rates with a maturity date that is different from the settlement date. While in this paper we use instanteneous forward rates, that is, in practice, interest rates with settlement some time in the future, and maturity a day after, both financial hoses use forward rates with settlement some time in the future, and maturity some years after -- three years in the case of Credit, and five years in the case of J.P. Morgan. The use of non instanteneous forward rates produces an upward bias in the estimated probability of entry in 1999 whenever the option of entry in 2001 is not considered.

To prove this point we shall compare two methods to compute probabilities: one based on instantaneous forward rates, the other based on two-year forward rates, using, while maintaining the same model for the determination of the PRE-EMU spread. We consider two possible entry dates, 1 January 1999, and 1 January 2001. The probabilities of entry in 1999 are then determined by the following equation:

$$ITFW99_{t,m} = \pi_{99,t} BFW99_{t,m} + \pi_{01,t}\, EMU01_{t,m} + (1 - \pi_{99,t} - \pi_{01,t}) PRE\text{-}EMU99_{t,m} \qquad (11)$$

where t refers to the date in which forward rates for January 1999 are computed, and m refers to the settlement date. Equation (11) can be re-written in term of spreads by subtracting BFW99 on both sides:

$$SP99_{t,m} = (1 - \pi_{99,t})\, SPNE99_{t,m} \quad + \pi_{01,t}\, (EMU01_{t,m} - PRE\text{-}EMU99_{t,m}) \qquad (12)$$

where SP99 = ITFW99-BFW99, SPNE99 = NOEU99-BFW99.

"EMU Calculators" compute the probability of entry in 1999 without considering late entry in 2001 as an alternative. Therefore, they compute the probability as follows:

$$\hat{\pi}_{99,t} = 1 - \frac{SP99_{t,m}}{SPNE99_{t,m}}$$

This is an upward biased estimate of the correctly computed probability, which, in fact, is:

$$\pi_{99,t} = 1 - \frac{SP99_{t,m}}{SPNE99_{t,m}} + \pi_{01,t} \frac{\left(EMU01_{t,m} - PREEMU99_{t,m}\right)}{SPNE99_{t,m}}$$

The bias is upward because the probability of entry in 2001 is non-negative, and the level of forward Italian rates in case of late entry (*EMU01*) is not higher than the level of forward rates in the PRE-EMU case.

There are only two cases in which the probability computed by “EMU Calculators” is not biased. The first obviously occurs when $\pi_{01,t}$ is zero; the second occurs when $\left(EMU01_{t,m} - PREEMU99_{t,m}\right)$ is zero. There is just one case where this second condition is satisfied: when the distance between the maturity and the settlement of the forward rate collapses to zero, that is when instantaneous forward rates are considered . This is the case of our own “EMU Calculator”, but not of those used by the two financial houses, which consider three-year or longer forward rates.

The intuition behind this result is easier to understand if we think of a forward rate, say with a distance of three years between maturity and settlement, as the average of the instantaneous forward rates over the same period. Entry into EMU in 2001 can be thought of as a succession of instantaneous forward rates beginning at no-entry level, and, at some point between 1999 and 2001, dropping to the German instantaneous forward curve. Given that German instantaneous forward rates lie consistently below the instantaneous forward curve associated with the event “Italy does not join”, the average forward rate associated to the event “Italy enters in 2001” must be lower than the average forward rate associated to the event “for the time being Italy remains out”. Therefore, the “EMU Calculators” computed by J.P. Morgan and Credito Italiano produce probabilities of entry in 1999 that are upward biased.

Consider, as an example, March 10, 1997, when our methodology delivers an estimate of 0,24 for the probability of entry in 1999, and 0,24 for the probability of entry in 2001. If we compute the same probabilities using 2-year average forward rates, instead of instantaneous forward rates, without any change in the PRE-EMU scenario, the resulting probability of entry in 1999 is 0,41. The upward bias of 0,17 (==,41-0,24) is explained by the 0,24 probability of entry in 2001, if entry in 2001 is described as a drop from of the instantaneous forward rate from the PRE-EMU curve to the German curve occurring in August 1999. A full comparison of the results provided by alternative “EMU Calculators” for March 10, 1997 is shown in Table 5.

Table 5: Probabilities (as of March 10, 1997) of Italy joining EMU in 1999

	Reuters **EMU Survey**	Credito Ital. **EMU Calculator**	J.P. Morgan **EMU Calculator**	Our methodology using instantaneous forward rates	Our Methodology using average 2-year forward rates
	0.12	0.46	0.57	0.24	0.41

Conclusions

Our work was aimed at extracting information on the probability of Italy’s joining the EMU at given dates from the term structure of interest rates. We started from the observation of the remarkable differences between probabilities of Italy joining EMU in 1999 estimated by financial houses in the first five months of 1997 and probabilities of Italy

joining EMU in 1999 reported in the Reuter survey over the same period. In fact, the estimated probabilities by financial houses range from 0.51 to 0.78, while the probabilities derived by the survey range from 0.14 to 0.17.
We reach two main results :

- we show that probabilities computed by currently available EMU calculators are upward biased because they are based on average rather than instantaneous forward rates and on the consideration of a limited space of events (delayed entry is not explicitly considered when computed the probability of Itly's joining EMU in 1999)
- we model a PRE-EMU spread between Italy and Germany by explicitly estimating a Taylor's rule type reaction function for the Bank of Italy. Such model allows us to identify separately the component of the spread between Italian and German interest rates due to different fundamentals from the component of the spread due to the markets' assessment of the probability of Italy oining EMU in 1996. We show that tof the 214 basis points reduction in the difference between Italian and German forward rates with maturity in 1999 observed from March 96 to March 97, 150 basis points are to be attributed to the convergence of fundamentals while only 65 basis points are to be attributed to the modifications in markets' assessment of the probability of Italy joiningf EMU in 1999.

Our analysis of the difference between probabilities as computed in EMU calculators from probalities delivered by survey start by the consideration that probabilities of Italy's joining EMU in 1999 are computed in EMU calculators by mapping average forward rates into probabilities by assuming that
(i) currently observed forward interest rates with maturity in 1999 and settlement three to ten years later are a weighted average of the spot rate differential between a country and Germany in the case that country joins and of the spot rate differential in the case that country does not join, where the weights are precisely the probabilities of joining and its complement to one.
(ii) the spot rate differential in the case a country joins is zero
(iii) the spot rate differential attains a specific value in the case a country does not join
Probabilities are then derived as the ratio of the observed average forward rate differential to the spot rate differential assumed for the scenario in which a country is left out.
Leaving aside market inefficiencies, there are two possible sources of bias in computed probability assumption (ii) and (iii).
We claim that it is possible to identify a source of systematic bias in computed probability. Such bias is determined by the fact that average rather than instantaneous forward rates are used and by the omission of the inclusion in the space of the events a delayed entry by Italy.
To do so we first rule out that the source of the bias is generated by a "too high" value of the differentials in case of no entry in 1999 by substantially confirming the results obtained by financial houses with the explicit estimation of a reaction function for the Bank of Italy linking the interest rate spread between Italy and Germany to macroeconomic fundamentals.
We then construct an EMU calculator based on instantaneous forward rates with the consideration of three possible events Italy joins in 1999, Italy joins in 2001 and Italy does not join within 2001.
We describe the perception by the market of Italy joining the EMU as a drop of Italian overnight rates from the PRE-EMU instantaneous forward rate curve to the German instantaneous forward rate curve. Using instantaneous forward rates allows to avoid the explicit consideration of the event Italy joins in 2001 when computing the probability of Italy joining in 1999. This is because the fact that markets discard the possibility of Italy joining in 1999 will put the overnight rate on the PRE-EMU curve both in the case Italy is expected to join before and after 2001. However this is not true when considering average forward rates. Think of thecase of average three-forward rates. We can interpret these rates as the average of the about nine hundred instantaneous forward rates expected to prevail between 1999 and 2002. It is clear that the curve of average rate when Italy is not expeced to join by 1999 but it is expected to join by 2001 will differ from the curve associated to the event Italy does not join in the three year following 1999. In fact in the first case the average forward rate will be an average of the PRE-EMU Italian rates and the German instantaneous forward rates, in the second case the average forward rate will be just an average of the PRE-EMU instantaneous forward rates. We show that in this case the omission from the space of the events of late entry in 2001 produces in upward bias of the probability of entry in 1999. This is because any mass attached to the late entry event is instead attributed to the entry in 1999 event. We consider the case of March 1997 to show that while the probability of Italy joining in 1999 computed on instantaneous forwad rates is 0.24, the same probability computed using average two-year forward rate is 0.41.
To understand the relevance of the argument consider the following "limit case" example. Markets are hundred per cent sure that Italy will join in 2001, therefore the probability of converging in 1999 is zero. In this case the Italian instantaneous forward rate for 1999 will be on the PRE-EMU curve and computation based on this rate will deliver the correct estimate of the probability of Italy joining in 1999. However the three-year average forward rate will be lower than the PRE-EMU forward rate being an average of two years of PRE-EMU Italian instantaneous forward rates and one year of German instantaneous forward rates. As a consequence the probability of Italy entering EMU in 1999 computed on the basis of forward rates (as one minus the ratio the average forward rate differential bewteen Italy and Germany to differential between the PRE-EMU Italian rate and the German rate) will be upward biased and different from zero.

Appendix: Spot Rates, the Expectational Model and Forward Rates

To illustrate our derivation of spot and forward rate functions, let us start by considering a zero coupon bond issued at time t, with a face value of 1, maturity of m years and price PZC_{mt}. The simple yield Y_{mt} is related to the price as follows:

$$(1)\quad PZC_{mt} = \frac{1}{(1+Y_{mt})^{m}}$$

Define the spot rate r_{mt} as $\log(1+Y_{mt})$, which is the continuously compounded yield, and define the discount function D_{mt} as the price at time t of a zero coupon that pays one unit at time t+m. We then have :

$$(2)\qquad PZC_{mt} = \exp(-mr_{mt}) = D_{mt}$$

Consider now a coupon bond that pays a coupon rate of c percent annually, and pays a face value of 1 at maturity. The price of the bond at trade date is given by the following formula:

$$(3)\qquad P_{mt} = \sum_{k=1}^{m} cD_{kt} + D_{mt}$$

Given the prices of coupon bonds, spot rates on zero-coupon equivalent can be derived by fitting a discount function based on the following specification for the spot rates:

$$(4)\ r_{kt} = \beta_0 + \beta_1 \frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} + \beta_2\left(\frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} - \exp\left(-\frac{k}{\tau_1}\right)\right) + \beta_3\left(\frac{1-\exp\left(-\frac{k}{\tau_2}\right)}{\frac{k}{\tau_2}} - \exp\left(-\frac{k}{\tau_2}\right)\right)$$

The above specification has been originally introduced by Svensson (1994) as an extension of the parametrization proposed by Nelson and Siegel(1987). Note that our estimated spot rate differs from the yield to maturity often quoted for coupon bonds. In fact the quoted yield to maturity, y_{mt}, is defined by the following relation:

$$(5)\qquad P_{mt} = \sum_{k=1}^{m} c\exp(-ky_{mt}) + \exp(-my_{mt})$$

Yield to maturities are averages of spot rates up to the date of maturity. While in general spot rates defined by (3) vary with the maturity, the yield to maturity defined by (5) is constant. Henceforth, the term structure of interest rates estimated on yields to maturity is only valid when the term structure of spot rates is flat. Moreover, the yield to maturity for a bond with a given maturity depends on the coupon rate, the so-called “coupon effect”. Spot rates instead are free from such an effect.

Implied forward rates can be computed from spot rates. A forward rate at time t with trade date t+t’, and settlement date t+T can be computed as the return on an investment strategy based on buying zero-coupon bonds at time t maturing at time t+T, and selling at time t zero-coupon bonds maturing at time t+t’. The forward rate is related to the spot rate according to the following formula:

$$(6)\qquad f_{T+t,t'+t,t} = \frac{Tr_{T,t} - t'\, r_{t',t}}{T-t'}$$

The forward rate for a 1-year investment with settlement in 2 years, and maturity in 3 years is thus equal to three times the 3-year spot rate, minus twice the two year spot rate.

The instantaneous forward rate is the rate on a forward contract with an infinitesimal investment after the settlement date:

(7) $$f_{mt} = \lim_{T \to m} f_{T+t,m+t,t}$$

In practice we identify the instantaneous forward rate with an overnight forward rate, *i.e.* a forward rate with maturity one day after the settlement. The relationship between the instantaneous forward rate and the spot rate is then:

(8) $$r_{mt} = \frac{\int_{\tau=t}^{t+m} f_{\tau t} \, d\tau}{m}$$ or, equivalently

(9) $$f_{mt} = r_{mt} + m \frac{\partial r_{m,t}}{\partial m}$$

Given specification (4) for the spot rate, the resulting forward function is:

(10) $$f_{kt} = \beta_0 + \beta_1 \exp\left(-\frac{k}{\tau_1}\right) + \beta_2 \frac{k}{\tau_1} \exp\left(-\frac{k}{\tau_1}\right) + \beta_3 \frac{k}{\tau_2} \exp\left(-\frac{k}{\tau_2}\right)$$

Therefore, as k goes to zero, the spot and the forward rate coincide at $\beta_0+\beta_1$, and as k goes to infinity the spot and the forward rate coincide at β_0. The forward rate function features a constant, an exponential term decreasing when β_1 is positive, and two “hump shape” terms.

We estimate a term structure of spot rates based on the observation of the overnight rate, the Euro 1-month, 3-month, 6-month and 12-month rates. We then consider the 2-year, 3-year, 5-year, 7-year and 10-year fixed interest rates on swaps. We use Euro-rates as spot rates, because they are zero-coupon bonds. We then consider swap rates as the long-term rates to be associated to Euro rates. Fixed interest rates swaps facilate international comparisons because they are not affected by different taxation regimes, and by default risk. By fitting the discout function to the data, and minimizing error in the yield space, we then estimate spot and forward rates.

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