

# On the Political Complementarity between Health Care and Social Security<sup>□</sup>

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## Abstract

The dramatic rise in the US social security and public health expenditure is only partially explained by the demographic trend, and may be due to the political complementarity between these two programs. We suggest that public health care increases the political constituency in favor of social security, and viceversa. Specifically, public health decreases the longevity differential between low and high-income individuals, therefore rising the retirement period, and the total pension benefits of the former relatively to the latter. This increases the political support for social security among the low-income young. We show that in a political equilibrium of a two-dimensional majoritarian election, a voting majority of low-income young and all retirees supports a large welfare state. Its composition between public health and social security is determined by intermediate (median) income types, who favor a combination of the two programs, since public health increases their longevity enough to make social security more attractive.

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## 1. Introduction

In the last few decades, the US have witnessed a dramatic increase in social security and public health care (Medicare) expenditure. Since both programs are mainly targeted to the elderly, the aging process represents a natural candidate to explain their rapid expansion. However, as argued by Mulligan and Sala-i-Martin (1999), demographics alone may not be sufficient to account for the full extent of this rise. To qualify this statement, Figure 1 displays the increase in the level of social security and public health care (Medicare) expenditure per elderly person, i.e., the ratio between social security and public health care (Medicare) expenditure as share of GNP and the number of elderly in the population, from 1960 to 1999. In a nutshell, the share of the elderly in the population has risen, and each of them has been receiving more resources, particularly as public health care. Moreover, this trend is expected to continue, thereby posing serious concerns on the sustainability of both systems.

The goal of this paper is to explain the contemporaneous expansion of social security and public health care. We suggest that the key element lies in the political complementarity between these two programs, which induces a multiplicative response to the aging process. With political complementarity, we characterize the fact that the existence of a social security program increases the political constituency in favor of public health care, and viceversa. The seed of this intuition was in Philipson and Becker (1998), who argued that social security induces the elderly to increase their private investment in health care, because the existence of an annuity – the old age pension – rises the value of longevity.

We identify a new link that goes from (public) health care to social security. Expenditure in public health increases longevity in a non-linear way, as its effect tends to be larger among low-income individuals than among well-off people (see Anand and Ravallion (1993), and Cutler and Richardson (1997 and 1998)). However, richer individuals tend to live longer, since income has a protective effect on health (see Deaton and Paxton (1998 and 1999) and Smith (1999) among others). Thus, for a given distribution of income, the expenditure in public health contributes to decrease the longevity differential between rich and poor individuals. As a result, the retirement period, and thus the total pension benefits, increases more for low-income individuals than for high income individuals, therefore rising the returns on social security for the low income workers, as opposed to high income ones.

The main contribution of the paper is to show that, for a sensible representation of the two programs, the political complementarity between social security and public health care exists, and pushes the size of the welfare system beyond what the demographic structure alone would have implied. Social security and public health care are sustained as a politico-economic equilibrium outcome of a dynamic majoritarian voting game. A voting majority of low-income young and all retirees supports a large welfare state, as in Tabellini (1990) and in Conde-Ruiz and Galasso (1999). Its composition between public health and social security is determined by intermediate (median) income types, who favor a combination of the two programs, since public health increases their longevity enough to make social security more attractive. On the other hand, in the case of exogenous longevity, that is, when public health has no effect on the longevity differential, there could be no political complementarity running from public health to social security. Every individual would either choose pure public health or pure social security, and the dimension of the welfare

state would be lower.

Three elements are crucial to our explanations. First, we emphasize the protective effect of income on health. As shown by Deaton and Paxton (1998 and 1999) and Smith (1999) among others, high-income individuals live longer. Second, we consider the redistributive effects of the health care expenditure in its double role of increasing the quality of life and of rising longevity. Although the largest share of the health care expenditure is targeted to the elderly (see Auerbach, Gokhale, and Kotlikoff (1992) and Cutler and Meara (1997)), there are evidence of within-cohort redistribution in favor of the low-income individuals (see van Doorslaer et al. (1999) and Lee, McClennan and Skinner (1999)). Analogously, the increase in longevity induced by the public health expenditure is stronger among low-income individuals than among high income ones (see Anand and Ravallion (1993), and Cutler and Richardson (1997 and 1998)). These aspects are critical to explain why health care is appealing to low-income individuals.

Third, we underline the intragenerational redistribution component of social security, due to the combination of social security contributions that are proportional to the labor income (up to a maximum) and regressive benefits. For the US, Boskin et al. (1987) and Galasso (2000) show that, within a given cohort, low income families obtain larger internal rates of return from social security than middle or high-income families. Like in Tabellini (1990) and Conde-Ruiz and Galasso (1999), because of this within cohorts redistribution element, social security may be appealing to low-income young.

We introduce a dynamically efficient overlapping generation economy with storage technology. Individuals differ in their income, and therefore in their longevity. Agents value their old age consumption and total health care, which is provided publicly and privately. Private health care is more efficient in increasing the quality of life, and therefore in providing direct utility. Public health care is less efficient in rising the quality of life, but it increases longevity. This effect on longevity is non linear, and is stronger for low-income agents.

The welfare state collects a proportional income tax on the young, which finances public health care expenditure to the old and social security transfers. Public health care is available in equal amount to every elderly person at the beginning of her old age, whereas the unfunded social security system pays out a lump sum pension during the entire retirement period, i.e., an annuity.

The size of the welfare state and its composition between the two systems are determined in a two-dimensional majority voting game by all agents alive at every election. As shown by Conde-Ruiz and Galasso (1999 and 2000), these types of voting games display two critical features. First, because of the multidimensionality of the issue space, the existence of a Condorcet winner of the majority voting game is not guaranteed. Second, if an equilibrium exists, in absence of a commitment device over future policies, young voters have no incentive to support any intergenerational transfer scheme. To overcome this problem, we follow Conde-Ruiz and Galasso (1999 and 2000) in adopting the notion of subgame perfect structure induced equilibrium, which combines the concept of structure induced equilibrium, introduced by Shepsle (1979), with the intergenerational implicit contract idea, originally presented by Hammond (1975).

The paper proceeds as follows: Section 2 describes the economic model and the welfare system. Section 3 discusses the voting game, and the equilibrium concept, while section 4 characterizes the politico-economic equilibria. Section 5 analyzes the case of exogenous

longevity, and the results are compared to the endogenous longevity case in section 6. Section 7 concludes. All proofs are in the appendix.

## 2. The Economic Model

We introduce an overlapping generation model with storage technology. Every period, there are two generations of non-altruistic agents, young and old. Population grows at a constant rate  $\lambda > 0$ . Individuals are endowed with a young age income, and retire in their old age.

Agents are assumed to be heterogeneous in their young age income,  $e$ , which is distributed on the support  $[\underline{e}; \bar{e}] \subset \mathbb{R}_+$ , according to the cumulative distribution function  $G(\cdot)$ . An individual born at time  $t$  is characterized by an income level and will therefore be denoted by  $e_t \in [\underline{e}; \bar{e}]$ . The distribution of abilities is assumed to have mean  $\bar{e}$ , and to be skewed

$$\int_{\underline{e}}^{\bar{e}} e dG(e) = \bar{e}; \quad G(\bar{e}) > \frac{1}{2}$$

Income has a protective effect on longevity: agents with higher income tend to live longer than agents with lower incomes. This protective effect of income has often been attributed to the easier access that high income agents have to private health care. Although recent studies have criticized this explanation, as well as the casual relation between income and health (see Smith 1999), the existence of a positive relationship between health and income, the “gradient,” is uncontroversial.

For analytical simplicity, we choose to disregard the demand for private (and public) young age health care, and to assume that longevity is directly related to income<sup>1</sup>. Every agent lives until the second period. Longevity in the second period, i.e., the fraction of the second period during which an agent is alive, depends on her income, and on the level of public health care expenditure.

Public health care is assumed to have a positive non linear effect on the longevity of the agents. For a given level of public health expenditure, low-income individuals enjoy larger gains in longevity than medium-to-high income agents. In other words, longevity displays decreasing return to health care<sup>2</sup>. Additionally, we choose to abstract from the rise in average longevity, and to concentrate on the change in the longevity differential between low and high income agents. Unlike Philipson and Becker (1998), we are mainly interested in the within-cohort redistributive impact of the public health expenditure, because of its interesting spill over effects on the political decision on social security, rather than in the average longevity gains, which, for a given income tax rate, would just reduce the lump sum pension to all agents.

<sup>1</sup> A recent literature has analyzed the effect of income inequality on mortality. Wilkinson (1996) provided evidence that more income inequality increases the average mortality in a country, whereas Deaton (1999), and Deaton and Paxson (1999) found no evidence of a direct impact of inequality on mortality.

<sup>2</sup> This greater effectiveness of public health care among the poor can be due to the higher initial longevity of the high-income individuals, and to the lower private health care consumption of the low-income agents. Evidence that, for a given level of income inequality, public health care has non-linear effects are in Anand and Ravallion (1993) – for cross countries data, and in Cutler and Richardson (1997 and 1998) – for data on individual agents.

The following longevity function,  $\pm(e; H_t)$ , captures all these characteristics. It identifies the fraction of the old age that a type- $e$  individual born at time  $t-1$  is alive for:

$$\pm_{e;t} = \pm(e; H_t) = \underline{e}_t \left( 1 + E_t \frac{\bar{H} - H_t}{\bar{H}} \right) \quad (2.1)$$

$$\text{with } E_t = \frac{e_{t-1} - \bar{e}}{e_{t-1}} \quad (2.2)$$

where  $H_t$  is the average expenditure in public health care at time  $t$ ,  $\bar{H}$  represents the upper bound on the public health expenditure,  $\underline{e}_t \in (0, 1)$  is the longevity of the average type  $\bar{e}_{t-1}$ , and  $E_t$  is a measure of the distance of a type  $e_{t-1}$  from the mean type  $\bar{e}_{t-1}$ .

Notice that  $\underline{e}_t (1 + E_t)$  represents the longevity of a type- $e$  agent in absence of public health care. Then, the income is the only determinant of longevity, and its protective effect is assumed to be linear. In this case, the longevity differential between the poorest,  $\underline{e} = e$ , and the richest,  $\bar{e} = e$ , individual is largest. As the public health care expenditure increases, this longevity differential decreases, although the average longevity should rise. Since we disregard the latter effect ( $\underline{e}$  is assumed to be constant), the public health expenditure rises the longevity of the individuals whose income is below the mean, but decreases the longevity of the others. This effect is shown in Figure 2 for the mean income type,  $\bar{e}$ , for a poor,  $e < \bar{e}$ , and a rich,  $e > \bar{e}$ , individual. In the limit, as the maximum amount of disposable resources is devoted to public health,  $H = \bar{H}$ , the longevity differential disappears,  $\pm_e = \underline{e}$ .

Agents value consumption and health care in old age only<sup>3</sup>, according to a Cobb-Douglas utility function:

$$U_{t+1}^i = c_{t+1}^i m_{t+1}^i \quad (2.3)$$

where  $c$  is consumption, and  $m$  is the health care. Subscripts indicate the calendar time and superscripts indicate the period when the agent was born.

As in Epple and Romano (1996), agents value public and private health care jointly, as a composite good,  $m$ . Public and private health care do, however, differ. Public health care plays a double role: it provides medical services that improve the quality of life of the individuals, therefore increasing their utility<sup>4</sup>; and it rises longevity, i.e., the quantity of life. On the other hand, in our setting, private health care may only improve the living standard of the individuals<sup>5</sup>. In particular, private health care is assumed to be more efficient than public health in providing the medical services that rises the quality of life (see the empirical evidence in Currie, Gruber and Fisher (1995) and Cutler and Gruber

<sup>3</sup>This assumption greatly simplifies the analysis, but entails some costs. First, we do not model the demand for private (and public) young age health care, and simply assume that longevity depends on the income. Second, we abstract from saving decisions, that are known to be relevant for the political sustainability of social security, see Boldrin and Rustichini (2000), Cooley and Soares (1998) and Galasso (1999).

<sup>4</sup>Since Grossman (1972) seminal contribution, health care has been assumed to provide utility, either directly or by increasing the utility from consumption, as in Epple and Romano (1996) and Philipson and Becker (1998).

<sup>5</sup>We acknowledge that private health care does, indeed, increase longevity. This effect is captured by the positive relation between income and longevity. Richer individuals are expected to spend more resources on private health and therefore to live longer.

(1996)). Thus, at time  $t$ , the health care services provided to an old agent,  $m_t$ , is equal to:

$$m_t = b_t + \theta H_t \quad (2.4)$$

where  $b_t$  and  $H_t$  are respectively the expenditure in private and public health care received by an old person, and  $\theta \in (0; 1)$  measures the efficiency gap between private and public health care.

A storage technology allows to transfer one unit of consumption today into  $(1 + R)$  units of consumption tomorrow. Additionally, we assume that  $R > \rho$ , and thus the economy is dynamically efficient. All private transfers of resources take place through this storage technology.

The budget constraint of a type- $e$  agent born at time  $t$  is

$$c_{t+1}^t + b_{t+1}^t \cdot (1 + R) = e_t(1 - \tau_t) + \pi_{e;t+1} P_{t+1} \quad (2.5)$$

where  $\tau_t$  is the tax rate on income at time  $t$ . Young agents are endowed with an initial income, on which they pay a tax. They take no economic decision, and save their net income for future consumption. When old, they are entitled to a (one-time) public health care and receive a lump sum pension for the remaining duration of their life. They use their pension income and their saving to finance their private consumption and their expenditure in private health care.

There exists a fundamental difference between pension transfers and public health care. A pension transfer is a lump sum annuity, which is paid to every agent for the entire duration of her old age. Although the pension is lump sum, and thus unrelated to income, since high income people enjoy higher longevity, they will receive a pension for a longer period, thus collecting a larger pension income. In every agent's budget constraint, the pension is thus multiplied by her longevity.

A public health care program entitles the elderly to a medical service. How can we measure the extent to which individuals that differ in their health status and longevity use this service? High income individuals have better health status, but they live longer, and may need more expensive medical services; whereas low income individuals have lower longevity, but may require a more intensive usage of the system while they are alive. For all individuals, however, the largest share of the cost of health care is concentrated in the last six months of their life (see Lee et al. (1999)). Thus, we choose to consider public health care as a lump sum expenditure, which occurs only once during the old age.

At time  $t + 1$ , an elderly person determines her demand for consumption and for private health care by maximizing her utility function, eq. 2.3, with respect to  $c_{t+1}^t$  and  $b_{t+1}^t$ , subject to the budget constraint at eq. 2.5. We call  $W_{e;t+1}^t$  the net wealth of a type- $e$  old agent at time  $t + 1$ :

$$W_{e;t+1}^t = e_t(1 - \tau_t)(1 + R) + \pi_{e;t+1} P_{t+1} + \theta H_{t+1} \quad (2.6)$$

The optimal demand for consumption and private health care of a type- $e$  old agent at time  $t + 1$  are respectively<sup>6</sup>:

$$\begin{aligned} c_{e;t+1}^{st} &= \beta W_{e;t+1}^t \\ b_{e;t+1}^{st} &= (1 - \beta) W_{e;t+1}^t - \theta H_{t+1} \end{aligned} \quad (2.7)$$

<sup>6</sup>Here, in order to ensure that  $b_{e;t+1}^{st} \geq 0$ , we assume that even the individual with the lowest income has a non-negative demand for private health, i.e., that  $\frac{e_t(1 - \tau_t)(1 + R) + \pi_{e;t+1} P_{t+1}}{H_{t+1}} \geq \frac{\theta}{1 - \beta}$ .

Unsurprisingly, richer individuals are willing to supplement public health with more private health care. Moreover, in line with the findings of Cutler and Gruber (1996), more public health care crowds out private health.

## 2.1. The Welfare State

Our welfare state consists of two instruments, which transfer resources across generations, from young (workers) to old agents (retirees): a public health program and a social security (or pension) system. At every time  $t$ , the young contribute a proportion,  $\zeta_t$ , of their income to the system, and every retiree is entitled to a lump sum one-time health care service,  $H_t$ , and receives a lump sum pension,  $P_t$ , for the remaining part of her old age,  $\pm e_t$ .

Notice that, although these systems are pay-as-you-go, i.e., current young finance the expenditure of current old, they both entail an element of intragenerational redistribution. In fact, both social security and public health care are financed through a proportional tax, and thus place a higher burden on the medium-to-high income young, whereas the benefits, i.e., pension and medical service, are unrelated to income.

In our setting, agents are endowed with a young age income, and thus the income tax creates no distortion. To introduce a distortionary effect of taxation, and thereby to avoid agents to have too extreme preferences over the welfare state, we assume that there exists a quadratic cost of taxation<sup>7</sup>.

The welfare state is assumed to be balanced every period, so that its total expenditure in both programs has to be equal to the amount of collected taxes,  $T_t$ . Let  $\zeta_t$  be the share of collected taxes,  $T_t$ , dedicated to social security, and  $(1 - \zeta_t)$  to public health care. Then, accounting for the quadratic cost of taxation, we have that the total amount of collected taxes is:

$$T_t = \zeta_t (1 - \zeta_t) \frac{Z}{e} \text{edG}(e_t) = \zeta_t (1 - \zeta_t) e_t \quad (2.8)$$

Notice that as the tax rate,  $\zeta_t$ , increases so does its distortionary effect. In particular, the maximum of the Laffer curve is reached for  $\zeta_t = 1/2$ . Finally, the total amount of resources is divided between pensions:

$$\zeta_t T_t = \frac{e_t P_t}{(1 + \gamma)}, \quad (2.9)$$

where  $P_t$  is the lump sum pension transfer paid to every retiree during her old age period, and public health care:

$$(1 - \zeta_t) T_t = \frac{H_t}{(1 + \gamma)}. \quad (2.10)$$

Finally, to simplify the algebra, we assume that the upper bound on the public health expenditure,  $\bar{H}$ , is equal to the maximum amount of collectable taxes, i.e.,  $\zeta = 1/2$ , entirely spent on health, i.e.,  $\zeta = 0$ , that is:  $\bar{H} = \frac{1}{4} (1 + \gamma) e_t$ .

<sup>7</sup>We choose this approach, rather than the more natural one – to endogenize the labor supply – because it allows us to obtain a close form solution of the voting game.

## 2.2. The Economic Equilibrium

We can now define the economic equilibrium as follows:

**Definition 2.1.** For a given sequence of the tax rate and of the pension share,  $\tau_t, \tau_{t+1}, g_{t=0}^1$ , and a given real interest rate,  $R$ , an economic equilibrium is a sequence of allocations,  $c_{t+1}^t(e_t); b_{t+1}^t(e_t)_{t=0, \dots, 1}$ ; such that:

- 2 in every period, agents maximize their utility function at eq. 2.3, with respect to  $c_{t+1}^t(e_t)$  and  $b_{t+1}^t(e_t)$ , subject to the budget constraint in eq.2.5;
- 2 the welfare budget constraints are balanced every period, and thus equations 2.8, 2.9 and 2.10 are satisfied; and
- 2 the goods market clears every period:

$$\frac{R_{\underline{e}}}{\underline{e}} b_{t+1}^t dG(e_{t+1}) + \frac{R_{\underline{e}}}{\underline{e}} c_{t+1}^t dG(e_{t+1}) = (1 - \tau_t) (1 + R) \frac{R_{\underline{e}}}{\underline{e}} e_{t+1} dG(e_{t+1}) + (1 + \tau_t) \frac{R_{\underline{e}}}{\underline{e}} e_t dG(e_t)$$

The utility level obtained in an economic equilibrium by the agents is represented by their indirect utility functions. For a type- $e_t$  young:

$$v_{e;t}^t(\tau_t; P_{t+1}; H_{t+1}; e_t) = \mu W_{e;t+1}^t$$

where  $\mu = (1 - \tau_t)^{(1 - \alpha)}$ . It is now useful to substitute the welfare state budget constraints in the above expression, and to express the individual types,  $e$ , in terms of differences from the mean type  $\bar{e}$ . Thus, we obtain an indirect utility function for a type- $E$  young that depends on current and future tax rates and on the future pension share:

$$V_{t;E}^t(\tau_t; \tau_{t+1}; \tau_{t+1}; E_t) = \mu \bar{e} (1 + R) f(1 + E_t) (1 - \tau_t) + [\tau_{t+1} (1 + E_t) + \tau_{t+1}^0 (1 - \tau_{t+1})] \alpha (1 + N) \tau_{t+1} (1 - \tau_{t+1}) - 4E_{t,t+1} (1 - \tau_{t+1}) \tau_{t+1}^2 (1 - \tau_{t+1})^2 \quad (2.11)$$

where  $(1 + N) = (1 + \tau_t) = (1 + R)$  represents the relative performance of the social security system with respect to the private savings. For a type- $E$  old individual at time  $t$  the indirect utility function is:

$$V_{t;E}^{t-1}(\tau_{t-1}; \tau_t; \tau_t; E_t) = V_{t-1;E}^{t-1}(\tau_{t-1}; \tau_t; \tau_t; E_t) \quad (2.12)$$

## 3. The Voting Game

The size and the composition of the welfare state are decided by the agents through a political system of majoritarian voting. Elections take place every period, and all persons alive, young and old, cast a ballot over  $\tau$ , the income tax, and  $\tau$ , the share of pension in the welfare state. Individual preferences over the two issues are represented by the indirect utility functions at equations 2.11 and 2.12, respectively for the young and the old. Notice that every agent has zero mass, and thus no individual vote could change the outcome of the election. To overcome this problem, individuals are assumed to vote sincerely.

This majoritarian voting game shares two important features with the games analyzed in Conde-Ruiz and Galasso (1999 and 2000). First, the issue space is bidimensional,



$(\zeta; \zeta)$ , and thus a Nash equilibrium may fail to exist, and second, the game is intrinsically dynamic, since it describes the interaction, or social contract, between successive generations of workers and retirees. We therefore use their concept of subgame perfect structure induced equilibrium<sup>8</sup>, which reduces the game to a dynamic issue-by-issue voting game.

Following their methodology, we first analyze the case of full commitment, in which voters determine the constant sequence of the parameters of the welfare state  $(\zeta; \zeta)$ . In absence of a state variable, this voting game is static, and the result in Shepsle (1979) [Theorem 3.1] can be applied to obtain the sufficient conditions for a (structure induced) equilibrium to exist. In particular, if preferences are single-peaked along every dimension of the issue space, a sufficient condition for  $(\zeta^*; \zeta^*)$  to be an equilibrium of the voting game with full commitment is that  $\zeta^*$  represents the outcome of a majority voting over the jurisdiction  $\zeta$ , when the other dimension is fixed at its level  $\zeta^*$ , and viceversa.

To use this theorem in our environment, we need to ensure that individuals' preferences are single peaked along the two dimensions,  $\zeta$  and  $\zeta$ . The following lemma describes a set of sufficient conditions.

**Lemma 3.1.** Individuals' preferences are single-peaked over  $\zeta$  for given  $\zeta$ . Individuals' preferences are single-peaked over  $\zeta$  for given  $\zeta$ , if  $E = \frac{1 + \theta(1 + \zeta)}{5 + 4\zeta}$  and  $E < 1$ .

We therefore restrict the support of ability type of young and old individuals, in order to have that  $E = (e - e) / (e - e) \in [E; 1]$ , that is  $e \in [e(1 + E); 2e]$ .

The second step to find a subgame perfect structure induced equilibrium is to show that the (structure induced) equilibrium outcomes of the game with commitment are also subgame perfect equilibrium outcomes of the voting game without commitment<sup>9</sup>. In the game with no commitment, voters may only pin down the current values of  $\zeta$  and  $\zeta$ , although they may expect their current voting behavior to affect future voters' decisions. We will return to this point at the end of the next section.

## 4. Politico-Economic Equilibria

In this section, the individual votes over the each dimension of the issue space,  $(\zeta; \zeta)$ , are examined issue-by-issue. Initially, we assume that current voters can determine future policies, i.e., there exists commitment. Thus, voters cast a ballot over a constant sequence of  $\zeta$ , for a given constant sequence of  $\zeta$ , and viceversa. For each dimension,  $\zeta$  and  $\zeta$ , votes are then ordered to identify the median votes, which, by Shepsle (1979) theorem 3.1, represent the structure induced equilibrium outcome of the game with commitment. The results are then generalized to the game without commitment.

### 4.1. Voting over the Size of the Welfare State

Regardless of the composition of the welfare state, the elderly are net recipients from the system. Therefore, they will choose the tax rate that maximizes its size.

**Lemma 4.1.** For any share of pension in the welfare state,  $\zeta$ , the most preferred tax rate by any type-E old individual,  $\zeta_E^0$ , is equal to  $1/2$ .

<sup>8</sup>See the appendix for a formal definition, and Conde-Ruiz and Galasso (2000) for a detailed discussion.

<sup>9</sup>A full specification of the voting game without commitment is in the Appendix.

Today's young individuals may be willing to vote in favor of the welfare state, and thus to bear the cost of a current transfer, if their vote will also determine its future size, and thus their future benefits. In the game with commitment, a type-E young individual chooses her vote,  $\tau_E^Y$ , by maximizing her indirect utility function at eq.2.11 with respect to a constant sequence of tax rates,  $\tau_t = \tau_{t+1} = \tau_E^Y$ . The next lemma characterizes the vote of the young.

**Lemma 4.2.** For a given share of pension,  $\alpha$ , the most preferred tax rate by any type-E young individual is positive,  $\tau_E^Y > 0$ , if  $E < \bar{E}(\alpha)$ , and it is equal to zero,  $\tau_E^Y = 0$ , if  $E \geq \bar{E}(\alpha)$ , where  $\bar{E}(\alpha) = \frac{\alpha(1+N)(1+i)}{1+i+N}$ . Moreover,  $\tau_E^Y$  is weakly decreasing in  $E$ :  $\frac{\partial \tau_E^Y}{\partial E} \leq 0$ .

Lemma 4.2 suggests that the political support to the welfare state relies heavily on its within-cohort redistribution component. While relatively high young types,  $E \geq \bar{E}(\alpha)$ , oppose the system, among the low-income young the preferred size of the welfare state is decreasing with the voter's types. Rich young individuals,  $E > 0$ , pay more taxes than the average, but receive the same public health expenditure and old age unitary pension as everybody else. Although they live longer, and thus enjoy a larger total pension transfer, this extra longevity is not sufficient to compensate for the higher contribution they make in youth. This result is strengthened if the public health share of the welfare expenditure is increased, since public health reduces the longevity differential among types, and thus the total pension of the wealthy. Also intermediate young types,  $\bar{E}(\alpha) > E > 0$ , choose not to sustain the welfare state, despite receiving in old age more resources than they contribute in youth. In fact, the welfare state constitutes an inefficient technology to transfer resources into the future, and their young age contributions exceed the present value of their benefits. Only low-income young types,  $E < \bar{E}(\alpha)$ , are net recipients<sup>10</sup> and therefore vote for a positive welfare system, although they experience shorter longevity and thus enjoy smaller total pension transfers.

The next lemma constitutes an important step towards our main result. It characterizes the relation between the size of the system chosen by a type  $E < \bar{E}(\alpha)$  young individual and the pension share,  $\alpha$ , and discusses the complementarity between the two welfare systems.

**Lemma 4.3.** The most preferred tax rate by any type-E young individual, with  $E < \bar{E}(\alpha)$ , is weakly increasing for  $\alpha < \alpha^0$ , and decreasing for  $\alpha > \alpha^0$ , where  $\alpha^0 = \frac{1}{2} \frac{1+E}{1+E+i}$ .

For  $\alpha = 1$ , the welfare state is a pure social security system. But the longevity of the low-income individuals, and thus their total pension benefits, is too low to induce them to support it. As part of the expenditure is devoted to health care,  $\alpha < 1$ , low-income agents experience an improvement in the quality of their life. Additionally, the longevity

<sup>10</sup>Notice that the mass of young voters in favor of the system, i.e.,  $E < \bar{E}(\alpha)$ , depends on the relative share of the two welfare programs,  $\alpha$ . While a pure social security system,  $\alpha = 1$ , would receive no support, as the share of health care increases so does the mass of voters. The reason is that the low-income young greatly enjoy public health, which redistributes resources in their favor, and decreases the longevity differential.

differential decreases, their total pension benefits begin to rise, and they are now willing to support the welfare state. Indeed, as the share of health care increases, their most preferred size of the welfare state rises because the existence of public health improves their total pension benefits. However, as the share of public health becomes too large, this complementarity between the two programs is reduced: the longevity differential keeps decreasing, but not enough to compensate the reduction in the unitary pension, and hence the agents choose to downsize the welfare state.

It is now straightforward to order all individuals' votes on the size of the welfare state, for a given pension share, and to identify the median voter's type. Agents can be ranked according to their age and type, as shown at Figure 3, with elderly and then low-income young choosing larger sizes. The median voter is a type- $i$   $E_{m_\zeta}$  young agent who divides the electorate in halves:  $G(E_{m_\zeta}) = \frac{1}{2}(1 + \lambda)$ . For a given pension share,  $\tau_\zeta$ , we identify her most preferred tax rate as  $\zeta_{E_{m_\zeta}}(\tau_\zeta)$ .

#### 4.2. Voting over the Composition of the Welfare State

When the issue at stake is the pension share,  $\tau_\zeta$ , for a given size of the system,  $\zeta$ , votes only differ according to the voters' type, and the voters' age plays no role. This is not surprising. In the game with commitment, today's decision will be in place tomorrow as well. And the composition of the welfare state is only relevant in old age, when the benefits from the two programs are received. Thus, a type-E young and a type-E old share the same voting decision: they choose their vote,  $\tau_{\zeta E}$ , by maximizing their indirect utility function at eq.2.11 and eq.2.12

Lemma 4.4. For a given tax rate,  $\zeta$ , the most preferred social security share,  $\tau_{\zeta E}$ , by a type-E (young and old) individual is the following:

- (i)  $\tau_{\zeta E} = 1$ , if  $E > 0$ ;
- (ii)  $\tau_{\zeta E} = \min \left\{ 1; \frac{1}{2} \left[ i + \frac{1 - i + \lambda E}{8E\zeta(1 - \zeta)} \right] \right\}$ , if  $E \in [i(1 - \lambda); 0]$ ;
- (iii)  $\tau_{\zeta E} = \max \left\{ 0; \frac{1}{2} \left[ i + \frac{1 - i + \lambda E}{8E\zeta(1 - \zeta)} \right] \right\}$ , if  $E < i(1 - \lambda)$ .

Moreover,  $\tau_{\zeta E}$  is weakly increasing in  $E$ , i.e.,  $\frac{\partial \tau_{\zeta E}}{\partial E} \geq 0$ . And  $\tau_{\zeta E}$  is weakly increasing in  $\zeta$  if  $E \in [i(1 - \lambda); 0]$ , and weakly decreasing in  $\zeta$  if  $i(1 - \lambda) > E > 0$ .

Lemma 4.4 characterizes how the preferred composition of the welfare state depends on the individual type. Rich agents, whose type is above the mean,  $E > 0$ , vote for a pure social security system, since public health reduces the longevity gap and increases the redistributive element of the system. Intermediate types (cases ii and iii) exploit the complementarity between the two programs, and hence favor a combination of the two, in order to increase their relative longevity and to receive an old age pension<sup>11</sup>.

The relation between the composition of the system chosen by a type-E individual and its size,  $\zeta$ , depends on the voter's type. The votes of the high income types (case i) are unaffected by changes in the size. Among the agents with intermediate types, the poorer

<sup>11</sup>Notice that for very small dimension,  $\zeta \rightarrow 0$ , preferences over the composition of the welfare state are extremely polarized:  $\tau_{\zeta E} = 0$  if  $E \in [i(1 - \lambda); 0]$ , and  $\tau_{\zeta E} = 1$  if  $E > i(1 - \lambda)$ .

(case iii) will respond to a rise in the size of the system with an increase of the pension share. In fact, a larger unitary pension compensates a lower longevity. Relatively richer agents (case ii), on the other hand, will trade off a lower pensions for more public health.

Following the previous lemma, we can order the votes on the composition of the system according to the voters' types, as shown in Figure 4. The median voter is the low-income type,  $E_{m_s}$ , who divides the electorate in halves:  $G(E_{m_s}) = 1/2$ . For a given size of the system,  $\zeta$ , we identify her most preferred composition as  $s_{E_{m_s}}(\zeta)$ .

Figures 3 and 4 show a different ordering along the two dimensions of the policy space. In fact, in deciding the size of the system, the age of the voters plays an important role, since the elderly favor the largest system, whereas only individual types matter in the composition. As a result, the median voter over the direction  $s$  has a higher type than the median voter over the dimension  $\zeta$ :  $E_{m_s} > E_{m_\zeta}$ .

### 4.3. Characterization of Equilibria

The previous sections have separately analyzed the voting behavior of all individuals along the two dimensions of the issue space, i.e., size and composition of the welfare state, under the assumption of commitment. Since preferences are single peaked, we can now apply Shepsle's (1979) result and characterize the structure induced equilibria of the game with commitment.

**Proposition 4.5.** There exists a structure induced equilibrium,  $(\zeta^*, s^*)$ , of the voting game with commitment, such that:

- (A)  $(\zeta^* = 0; s^* = 1)$  if  $E_{m_s} > i(1 + \theta)$  and  $E_{m_\zeta} > 8 E_{m_s}$ ;
- (B)  $(\zeta^* = 0; s^* = 0)$  if  $E_{m_s} < i(1 + \theta)$  and  $E_{m_\zeta} > \frac{1}{2}(s)$ ;
- (C)  $(\zeta^* > 0; s^* = 0)$  if  $E_{m_s} < i(1 + \theta)$ , and  $-(E_{m_s}) \cdot E_{m_\zeta} < \frac{1}{2}(s)$ ;
- (D)  $\zeta^* > 0; 0 < s^* < \frac{1}{2}$  if  $E_{m_s} < i(1 + \theta)$ ,  $E_{m_\zeta} < \frac{1}{2}(s)$  and  $E_{m_\zeta} < -(E_{m_s})$

$$\text{where } -(E_{m_s}) = i(1 + \theta) \frac{P}{i(1 + \theta) - E_{m_s}}$$

If the median voter over  $s$  is sufficiently rich (case A), she will prefer a large share of pension, but then no young individual will be willing to support the system. If, on the other hand, the median voter over  $s$  prefers more health care, the size of the system will depend on the type of the median voter over  $\zeta$ ,  $E_{m_\zeta}$ . The poorer the median voter, the larger the system will be. Moreover, a sufficiently poor median voter (case D), will exploit the complementarity between health care and social security, and hence will choose a larger system.

Notice that this proposition does not provide a complete characterization of the structure induced equilibria of the game. In fact, interior equilibria could arise in cases A and C, if the reaction function  $\zeta^*(s) = \zeta_{E_{m_\zeta}}(s)$ , which represents the decision of the median voter over  $\zeta$ , becomes sufficiently steep, and crosses the reaction function  $s^*(\zeta) = s_{E_{m_s}}(\zeta)$ , of the median voter over  $s$ , as shown in Figures 5 and 6. Were these equilibria to exist, we would have a case of multiple equilibria. Even in this case, however, as we will discuss in section 6, the main message of the paper would not be affected.

What happens if we relax the assumption of commitment and consider a game in which voters may only determine the current size and composition of the welfare system? The result in proposition 4.5 generalize to a game without commitment:

**Proposition 4.6.** Every pair  $(\hat{\zeta}^y; \hat{\zeta}^o)$ , which constitutes a structure induced equilibrium of the voting game with commitment, is a (subgame perfect structure induced) equilibrium of the game without commitment.

The intuition is straightforward. Old agents' voting behavior does not depend on tomorrow's policy and thus on the existence of commitment. Low-income young individual, who were in favor of the welfare state in the case of commitment, will also be willing to enter an "implicit contract" among successive generations of voters to sustain the welfare state. This "implicit contract," or social norm, specifies that if current young support the existing welfare system, they will be rewarded with a corresponding transfer of resources (pension and health care) in their old age, or they will be punished.

## 5. Exogenous Longevity

In order to analyze the impact of the political complementarity, which runs from health care to social security through the reduction in the longevity differential, on the size of the welfare system, we now examine the case of exogenous longevity. In this section, longevity is assumed to depend exclusively on the agent's type, and not to be affected by public health expenditure. At time  $t$ , a type-E individual enjoys the following longevity:

$$\pm_{e;t}^x = \bar{\zeta}_t (1 + E_t) \quad (5.1)$$

where the subscript  $x$  indicates the variables in the exogenous longevity environment.

Agents solve the same economic problem as in section 2, and obtain the optimal demand of consumption and private health care at eq.2.7, where the wealth  $W_{e;t+1}^t$ , previously defined at eq.2.6, will now depend on the exogenous longevity,  $\pm_{e;t}^x$ , rather than on  $\pm_{e;t}$ . Notice that the only difference between the two cases lies in the total pension transfer, which, in the exogenous longevity case, depends entirely on the agent's type, whereas it is also affected by health care expenditure if longevity is endogenous.

The agents' preferences over the policy space  $(\zeta; \zeta)$  are again represented by the indirect utility function, which, for a type-E young at time  $t$  is equal to:

$$V_{t;E}^{t;x}(\zeta_t; \zeta_{t+1}; \zeta_{t+1}; E_t) = \mu e(1 + R) \quad (5.2)$$

$$f(1 + E_t)(1 - \zeta_t) + [\zeta_{t+1}(1 + E_t) + \bar{\zeta}(1 - \zeta_{t+1})](1 + N)\zeta_{t+1}(1 - \zeta_{t+1})g$$

and for a type-E old individual is:

$$V_{t;E}^{t-1;x}(\zeta_{t-1}; \zeta_t; \zeta_t; E_t) = V_{t-1;E}^{t-1;x}(\zeta_{t-1}; \zeta_t; \zeta_t; E_t) \quad (5.3)$$

We can now turn to the agents' voting behavior. Following the methodology explained in section 3, we first analyze the voting game with commitment, and then generalize the results to the game without commitment.

### 5.1. Voting over $\zeta$ and $\zeta_s$

For any composition of the welfare state, a type-E old agent will vote  $\zeta_E^{O;x} = 1=2$ , in order to maximize the size of the system, from which, as in the previous case, she is a net recipient.. A type-E young individual may benefit from the welfare state depending on her type. For a given pension share  $\zeta_s$ , she will choose the tax rate,  $\zeta_E^{Y;x}$ , which maximizes her indirect utility at eq.5.2.

Lemma 5.1. For a given share of pension,  $\zeta_s$ , the most preferred tax rate by any type-E young individual,  $\zeta_E^{Y;x}$ , is the following:

$$(i) \zeta_E^{Y;x} = \frac{1}{2} \text{ if } \frac{(1+E)}{2(1+N)[\zeta_s(1+E_i^{\otimes})+\otimes]} > 0, \text{ if } E < \zeta_s^{\otimes}(\zeta_s), \text{ and}$$

$$(ii) \zeta_E^{Y;x} = 0, \text{ if } E \geq \zeta_s^{\otimes}(\zeta_s).$$

Moreover,  $\zeta_E^{Y;x}$  is weakly decreasing in  $E$  and in  $\zeta_s$ ,  $\frac{\partial \zeta_E^{Y;x}}{\partial E} \leq 0$  and  $\frac{\partial \zeta_E^{Y;x}}{\partial \zeta_s} \leq 0$ .

As in the previous case, relatively rich young agents,  $E \geq \zeta_s^{\otimes}(\zeta_s)$ , will oppose the welfare state,  $\zeta_E^{Y;x} = 0$ . Low-income young choose a positive tax rate, which is larger the lower the voter's type. These agents enjoy a lower than average longevity, and thus prefer health care to social security, since health care provides a one-time old age benefit, whereas social security pays an annuity. Additionally, health care does not improve their longevity, and thus their most preferred tax rate decreases as the share of pension increases. In the extreme case of a pure social security system, the size would be zero.

The ordering of the votes over  $\zeta$ , and thus the median voter,  $E_{m\zeta}^x$ , is the same as in the endogenous longevity case:  $E_{m\zeta}^x = E_{m\zeta}$  (see figure 3).

When voting over the composition of the system,  $\zeta_s$ , the only relevant characteristic of the agent is their type. For a given size,  $\zeta$ , a type-E young and old individual choose the same pension share  $\zeta_s^x$ .

Lemma 5.2. For any tax rate,  $\zeta$ , a type-E (young and old) individual prefers a pure health care system,  $\zeta_s^x = 0$ , if  $E \leq \zeta_s^{\otimes}(1 - \zeta_s^{\otimes})$ , and a pure social security system,  $\zeta_s^x = 1$ , if  $E > \zeta_s^{\otimes}(1 - \zeta_s^{\otimes})$ .

Since health care does not affect the longevity differential, and thus the relative return on social security, the voting behavior becomes more polarized. Relatively rich individuals live longer, and favor a pure social security system, since the benefits from the total pension transfer,  $1 + E$ , exceeds the benefits from public health care,  $\otimes$ . The opposite is true for poorer individuals,  $E \leq \zeta_s^{\otimes}(1 - \zeta_s^{\otimes})$ . Votes over the dimension  $\zeta_s$  can easily be ordered in two groups, according to the voters' type. Finally, notice that the median voter's type,  $E_{m\zeta_s}^x$ , coincides with the median voter in the case of endogenous longevity,  $E_{m\zeta_s}$ .

### 5.2. Characterization of Equilibria

Applying Shesple's (1979) result to the voting game with commitment, we can characterize the structure induced equilibria as follows.

**Proposition 5.3.** There exists a structure induced equilibrium,  $(\tau_X^a; \tau_X^y)$ , of the voting game with commitment, such that:

- (A)  $(\tau_X^a = 0; \tau_X^y = 1)$  if  $E_{m_y} > i(1 + \beta)$  and  $E_{m_z} > E_{m_z}^0$ ;
- (B)  $(\tau_X^a = 0; \tau_X^y = 0)$  if  $E_{m_y} < i(1 + \beta)$  and  $E_{m_z} > E_{m_z}^0$ ;
- (C)  $(\tau_X^a > 0; \tau_X^y = 0)$  if  $E_{m_y} < i(1 + \beta)$  and  $E_{m_z} < E_{m_z}^0$ .

As in the previous section, the existence of a welfare state,  $\tau > 0$ , requires both median voters to be relatively poor. However, in this case, the welfare state may only consist of a pure health care system. In fact, in absence of the effect of public health on the longevity differential, a pension system is never sustained because of the lack of support by the low-income young. As in the case of endogenous longevity, these structure induced equilibrium outcomes can easily be generalized, in a game without commitment, to subgame perfect structure induced equilibrium outcomes.

## 6. Comparing Equilibria

We can now compare the equilibria obtained in the previous sections under the hypothesis of endogenous and exogenous longevity. For a given distribution of income, and therefore of initial health status, we aim at isolating the impact of the political complementarity on the size of the welfare system. In particular, we want to characterize the specific effect on the dimension of the welfare of the reduction in the longevity differential induced by the public health expenditure. The results are summarized in the following proposition.

**Proposition 6.1.** If  $E_{m_y} \geq i$ , the welfare state, i.e., the equilibrium tax rate, is weakly larger in the case of endogenous longevity. In particular:

- (A)  $\tau^a > \tau_X^a = 0$  and  $\tau^y = \tau_X^y = 1$ , if  $E_{m_y} > i(1 + \beta)$  and  $E_{m_z} > E_{m_z}^0$ ;
- (B)  $\tau^a = \tau_X^a = 0$  and  $\tau^y = \tau_X^y = 0$ , if  $E_{m_y} < i(1 + \beta)$  and  $E_{m_z} > E_{m_z}^0$ ;
- (C)  $\tau^a > \tau_X^a = \frac{1}{2} i \frac{1 + E_{m_z}}{2\beta(1 + N)}$  and  $\tau^y = \tau_X^y = 0$ , if  $E_{m_y} < i(1 + \beta)$ ,  $E_{m_z} < E_{m_z}^0$  and  $E_{m_z} > E_{m_z}^0 - (E_{m_y})$ ;
- (D)  $\tau^a > \tau_X^a$  and  $\tau^y > \tau_X^y = 0$ , if  $E_{m_y} < i(1 + \beta)$ ,  $E_{m_y} < \tilde{A}(E_{m_z})$ ,  $E_{m_z} < E_{m_z}^0$ , and  $E_{m_z} < E_{m_z}^0 - (E_{m_y})$ ,

where  $\tilde{A}(E_{m_z}) = (1 + \beta) E_{m_z} = 1 + \beta i E_{m_z} = 1 + \beta i \frac{(1 + E_{m_z})^2}{2\beta(1 + N)^2}$ , and

$\alpha = (E_{m_z}; E_{m_y}) \mid E_{m_z} < E_{m_z}^0, E_{m_z} < E_{m_z}^0 - (E_{m_y})$  and  $\tilde{A}(E_{m_z}) \cdot E_{m_y} < i(1 + \beta) \mu$   
 $[\underline{E}; 1] \in [\underline{E}; 1]$

In the first three cases, the equilibrium size of the welfare state is not affected by the assumption on the longevity, except if multiple equilibria arise. In case (A), a relatively rich median voter over  $y$  chooses a pure social security system, which no young individual is willing to sustain. Here, the inequality may arise in the rather unrealistic

situation, described in Figure 5, in which the median voter over  $\ell$  is extremely poor, and the complementarity between the two programs is strong enough even for large values of  $\tau$ . In case (B), the median voter over  $\ell$  is a relatively high-income type, who opposes the welfare state,  $\ell = 0$ . In case C, a low-income median voter over  $\tau$  prefers a pure health care system. The median voter over  $\ell$  is relatively rich. She does not benefit from the complementarity between the two programs, and hence votes the same tax rate as in the exogenous longevity case. Here, the inequality may again arise if there are multiple equilibria as shown in Figure 6.

Case (D) is the most interesting one. The political complementarity between social security and health care arises, and hence the welfare state is larger with endogenous than with exogenous longevity. Let's see why. The median voter over  $\ell$  is a low-income type. Due to her low longevity, she tends to favor health care. However, as shown in Lemma 4.3, when public health expenditure is very large, she benefits from an increase in the pension share, since the loss in longevity is compensated by an increase in pension benefits. In this case, she will respond to a rise in the pension share with a corresponding increase in the size of the system, until a certain threshold,  $\tau^0$ , is reached. After this pension share, the size of the system would be reduced. The median voter over  $\tau$  is a low type too, and thus prefers a composition of the welfare state more oriented towards public health care. Moreover, since  $E_{m_\tau} < \tau(1 - \tau)$ , by Lemma 4.4, the median voter responds to an increase in the dimension of the system with a rise in the pension share. The sufficient condition,  $E_{m_\tau} < \bar{A}(E_{m_\ell})$ , guarantees that the median voter over  $\tau$  has a sufficiently low income, and hence prefers more health care. As a result, she will not push the composition of the welfare state so far towards social security as to induce the median voter over  $\ell$  to downsize the system. Therefore, the complementarity between the programs is preserved, and the welfare state is larger under endogenous than under exogenous longevity.

Figure 7 summarizes the restrictions imposed on the ability types of the individuals, and characterizes the equilibria under endogenous and exogenous longevity for different combinations of the median voters' types. Area I represents case (C) in proposition 6.1: the dimension of the welfare state is typically unaffected by the longevity type. Area II corresponds to case (D): the median voter over  $\tau$  has sufficiently low type to keep the composition of the welfare state towards more public health, and thus to guarantee the existence of political complementarity. In area III, on the other hand, the comparison between endogenous and exogenous longevity is ambiguous, since the median voter over  $\tau$  is relatively rich, and may push the welfare state towards too much social security, thereby inducing its size to be reduced, by the median voter over  $\ell$ , even below the level of exogenous longevity.

## 7. Conclusion

Public opinion and policymakers have become increasingly concerned with the rise in public health and social security expenditure. Since both programs generate a flow of resources from the workers to the retirees, the major suspect in explaining this increasing trend is the aging process. However, demographic dynamics may only be held partially responsible for this rise in health care and social security expenditure. The number of recipients from these programs, the elderly, has certainly increased, but so has done the per capita resources that they have received, particularly in health care.



We suggest that some political features of these two programs may be responsible for a multiplicative effect, which enlarges the impact of the aging process. Health care and social security are political complements in that the existence of health care increases the political support in favor of social security, and viceversa. Philipson and Becker (1998) emphasize the link from social security to public health. The existence of an annuity, the old age pension, increases the value of longevity and, hence, increases the demand for public health.

We focus on the opposite direction, from public health to social security. We argue that public health reduces the longevity differential between low and high-income agents, and hence allows low-income individuals to enjoy larger retirement periods, relatively to high-income agents. This effect fosters the within cohort redistributive component of social security, and increases the political support to this program among the low-income individuals. In a two-dimensional voting model, in which voters determine the size and the composition of the welfare state, we show that this political complementarity leads to the adoption of a large welfare system, in which the public health component is large, relatively to social security.

## A. Appendix

### Lemma 3.1

To prove that the preferences of every agent are single peaked over each dimension,  $\zeta$  and  $\zeta_s$ , it is sufficient to show that her indirect utility function is quasi concave over each dimension.

Consider first the dimension  $\zeta_s$  for a given  $\zeta$ . For a type-E agent, the second derivative w.r.t.  $\zeta_s$  of her indirect utility function, eq. 2.11, is  $8eE\zeta^2(1 - \zeta)^2$ . Since  $e > 0$ , preferences are clearly single peaked if  $E < 0$ . For  $E > 0$ , the indirect utility function is convex. However, since the first derivative w.r.t.  $\zeta_s$  evaluated in  $\zeta_s = 0$  is positive, these agents simply prefer higher  $\zeta_s$  to lower  $\zeta_s$ , and preferences are still single peaked, with a maximum in  $\zeta_s = 1$ .

Consider now the dimension  $\zeta$  for a given  $\zeta_s$ . For a type-E old agent, the second derivative w.r.t.  $\zeta$  of her indirect utility function, eq. 2.12, is

$$\text{SOC}_E^O(\zeta) = 2 \frac{n}{i_s} (1 + E) i_s^{\otimes} (1 - i_s) i_s^3 - 4E_s (1 - i_s)^3 - 1 - 6\zeta + 6\zeta^2 \quad \text{---}$$

Notice that  $1 - 6\zeta + 6\zeta^2 \geq 0$  for  $\zeta \in [\frac{1}{2} - \frac{1}{6}\sqrt{3}, \frac{1}{2} + \frac{1}{6}\sqrt{3}]$ . Thus, it is easy to see that for  $E \leq 0$  and  $\zeta \in [\frac{1}{2} - \frac{1}{6}\sqrt{3}, \frac{1}{2} + \frac{1}{6}\sqrt{3}]$  or  $\zeta \in [\frac{1}{2} + \frac{1}{6}\sqrt{3}, 1]$ , and for  $E < 0$  and  $\zeta \in [\frac{1}{2} - \frac{1}{6}\sqrt{3}, \frac{1}{2} + \frac{1}{6}\sqrt{3}]$ , then  $\text{SOC}_E^O(\zeta) \leq 0$ . For  $E > 0$  and  $\zeta \in [\frac{1}{2} - \frac{1}{6}\sqrt{3}, \frac{1}{2} + \frac{1}{6}\sqrt{3}]$ , the last term in the  $\text{SOC}_E^O(\zeta)$  is positive. This term is maximum for  $\zeta = 1/2$ , thus if  $\text{SOC}_E^O(\zeta = 1/2) < 0 \Rightarrow \text{SOC}_E^O(\zeta) < 0 \forall \zeta$ . It is straightforward to see that  $\text{SOC}_E^O(\zeta = 1/2) < 0$  if  $E < \frac{+ \otimes (1 - i_s)}{i_s (1 - 2i_s)} > 1$ . Finally, the last term in the  $\text{SOC}_E^O(\zeta)$  is also positive for  $E < 0$  and  $\zeta \in [\frac{1}{2} - \frac{1}{6}\sqrt{3}, \frac{1}{2} + \frac{1}{6}\sqrt{3}]$  or  $\zeta \in [\frac{1}{2} + \frac{1}{6}\sqrt{3}, 1]$ , and is largest for  $\zeta = 0$  or  $\zeta = 1$ . Thus if  $\text{SOC}_E^O(\zeta = 0) < 0 \Rightarrow \text{SOC}_E^O(\zeta) < 0 \forall \zeta$ . It is easy to see that  $\text{SOC}_E^O(\zeta = 0) < 0$  if  $E > \underline{E} = i_s \frac{+ \otimes (1 - i_s)}{(5i_s - 4)}$ .

For a type-E young agent, the second derivative w.r.t.  $\zeta_s$  of her indirect utility function is the same as for a type-E old agent, except for a multiplicative constant,  $1 + N$ , and thus the same restrictions apply, which proves the lemma. ■

### Voting Game without Commitment

We consider that voters can only determine current size and composition of the welfare state, although they may expect their vote to condition future voters' decisions. We define the voting game with no commitment as follows.

The sequence of tax rates and pension shares until  $t - 1$  constitutes the public history of the game at time  $t$ ,  $h_t = (i_0^y; i_0^o; \dots; (i_{t-1}^y; i_{t-1}^o)) \in X_t$ , where  $X_t$  is the set of all possible history at time  $t$ .

An action for a type E young individual at time  $t$  is a pair of tax rates and pension shares,  $a_{t,E}^y = (i_t^y; i_t^o) \in \mathcal{A}$ , where  $\mathcal{A} = \{ (i_t^y; i_t^o) : i_t^y \in [0; 1]; i_t^o \in [0; 1] \}$ . Analogously, an action for a type E old individual at time  $t$  is  $a_{t,E}^o = (i_t^y; i_t^o) \in \mathcal{A}$ . We call  $a_t$  the action profile of all individuals (young and old) at time  $t$ :  $a_t = (a_t^y [ a_t^o)$  where  $a_t^y = \prod_{E \in [E; 1]} a_{t,E}^y$  and  $a_t^o = \prod_{E \in [E; 1]} a_{t,E}^o$ .

For a type E young individual a strategy at time  $t$  is a mapping from the history of the game into the action space:  $s_{t,E}^y : h_t \rightarrow \mathcal{A}$ , and analogously for a type E old individual at time  $t$ :  $s_{t,E}^o : h_t \rightarrow \mathcal{A}$ . The strategy profile played by all individuals at time  $t$  is denoted by  $s_t = (s_t^y [ s_t^o)$  where  $s_t^y = \prod_{E \in [E; 1]} s_{t,E}^y$  and  $s_t^o = \prod_{E \in [E; 1]} s_{t,E}^o$ .

At time  $t$ , for a given action profile,  $a_t$ , the pair  $(\hat{\zeta}_t^m; \hat{s}_t^m)$  represents the medians of the distributions of tax rates. We take  $(\hat{\zeta}_t^m; \hat{s}_t^m)$  to be the outcome function of the voting game at time  $t$ . This outcome function corresponds to the structure induced equilibrium outcome of the voting game with commitment, according to Shepsle' (1979) results. The history of the game is updated according to the outcome function; at time  $t + 1$ :  $h_{t+1} = f(\hat{\zeta}_0; \hat{s}_0; \dots; (\hat{\zeta}_{t-1}; \hat{s}_{t-1}); (\hat{\zeta}_t^m; \hat{s}_t^m)) \in X_{t+1}$ .

For every agent, the payoff function corresponds to her indirect utility. Formally, for a given sequence of action profiles,  $(a_0; \dots; a_t; a_{t+1}; \dots)$ , and of corresponding realizations,  $((\hat{\zeta}_0; \hat{s}_0); \dots; (\hat{\zeta}_t; \hat{s}_t); (\hat{\zeta}_{t+1}; \hat{s}_{t+1}); \dots)$ , the payoff function for a type E young individual at time  $t$  is  $V_{t;E}^t(\hat{\zeta}_t; \hat{\zeta}_{t+1}; \hat{s}_{t+1}; E_t)$ , as defined in eq. 2.11, and for a type E old agent is  $V_{t;E}^{t-1}(\hat{\zeta}_{t-1}; \hat{\zeta}_t; \hat{s}_t; E_t)$ , according to eq. 2.12.

Let  $s_{t;\mathbf{b}}^y = s_t^y = s_{t;\mathbf{b}}^y$  be the strategy profile at time  $t$  for all young individuals except for type  $\mathbf{b}$ , and let  $s_{t;\mathbf{b}}^o = s_t^o = s_{t;\mathbf{b}}^o$  be the strategy profile at time  $t$  for all old individuals except for the type  $\mathbf{b}$ . Then, at time  $t$ , a type  $\mathbf{b}$  young individual maximizes

$$V_{t;\mathbf{b}}^t(s_0; \dots; s_{t;\mathbf{b}}^y; s_{t;\mathbf{b}}^y; s_t^o; s_{t+1}; \dots) = V_{t;E}^t(\hat{\zeta}_t^m; \hat{\zeta}_{t+1}^m; \hat{s}_{t+1}^m; \mathbf{b}_t)$$

and a type  $\mathbf{b}$  old individual maximizes

$$V_{t;\mathbf{b}}^{t-1}(s_0; \dots; s_{t;\mathbf{b}}^o; s_{t;\mathbf{b}}^o; s_t^y; s_{t+1}; \dots) = V_{t;E}^{t-1}(\hat{\zeta}_t^m; \hat{\zeta}_t^m; \mathbf{b}_t)$$

where, according to our previous definition of the outcome function,  $(\hat{\zeta}_t^m; \hat{s}_t^m)$  and  $(\hat{\zeta}_{t+1}^m; \hat{s}_{t+1}^m)$  are, respectively, the medians among the actions over the size and composition of the welfare state played at time  $t$  and  $t + 1$ .

As previously argued, to deal with the two-dimensionality of the issue space, and to allow for intergenerational implicit contracts to arise, our equilibrium concept combines subgame perfection with the notion of structure induced equilibrium. We can now define a subgame perfect structure induced equilibrium of the voting game as follows:

**Definition A.1 (SPSIE).** A voting strategy profile  $s = f(s_t^y [s_t^o])_{t=0}^1$  is a Subgame Perfect Structure Induced Equilibrium (SPSIE) if the following conditions are satisfied:

- 1.  $s$  is a subgame perfect equilibrium.
- 2. At every time  $t$ , the equilibrium outcome associated to  $s$  is a Structure Induced Equilibrium of the static game with commitment. ■

**Lemma 4.1**

Trivial. For  $E \in [E; 1]$ , the indirect utility function at eq.2.12 is concave w.r.t.  $\hat{\zeta}$ , and is maximized at  $\hat{\zeta} = 1=2$ . ■

**Lemma 4.2**

Notice that the first order condition over  $\hat{\zeta}$  in the optimization problem of a type-E young voter is equal to the first order condition of a type-E old voter decreased by  $1 + E$ , i.e.,  $FOC_E^Y(\hat{\zeta}) = FOC_E^O(\hat{\zeta}) - (1 + E)$ . By Lemma 3.1, since  $E \in [E; 1]$ , the indirect

utility function is concave over  $\zeta$ , and thus a sufficient condition for a type-E young voter to maximize her indirect utility function in an interior, i.e., for  $\zeta > 0$ , is that the first order condition, evaluated at  $\zeta = 0$ , is strictly positive,  $FOC_E^Y(\zeta = 0) > 0$ . It is easy to see that if  $E < \bar{E}(\zeta) = \frac{\theta(1+N)(1+\zeta)}{1+\zeta(1+N)}$ ; 1, then  $FOC_E^Y(\zeta = 0) > 0$ .

Finally, to prove that  $\frac{\partial \zeta_E^Y}{\partial E} > 0$ , notice that for  $E > \bar{E}(\zeta)$  then  $\zeta_E^Y = 0$ , and  $\frac{\partial \zeta_E^Y}{\partial E} = 0$ . To examine the other case,  $E < \bar{E}(\zeta)$ , we differentiate the  $FOC_E^Y(\zeta)$  w.r.t.  $\zeta_E^Y$  and  $E$ , and evaluate it at  $\zeta = \zeta_E^Y$ . We obtain that  $\frac{\partial \zeta_E^Y}{\partial E} = \frac{\frac{\partial FOC_E^Y}{\partial E}}{SOC_E^Y(\zeta_E^Y)} < 0$ , since  $SOC_E^Y(\zeta_E^Y) < 0$ ,

and it is easy to see that  $\frac{\partial FOC_E^Y}{\partial E} < 0$ . ■

#### Lemma 4.3

For  $E < \bar{E}(\zeta)$ , by total differentiating the  $FOC_E^Y(\zeta)$  at  $\zeta = \zeta_E^Y$ , we have that  $\frac{\partial \zeta_E^Y}{\partial \zeta} = \frac{\frac{\partial FOC_E^Y}{\partial \zeta}}{SOC_E^Y(\zeta_E^Y)} < 0$ . Since  $SOC_E^Y(\zeta_E^Y) < 0$ , then  $\text{sign} \frac{\partial \zeta_E^Y}{\partial \zeta} = \text{sign} \frac{\partial FOC_E^Y}{\partial \zeta}$ , where  $\frac{\partial FOC_E^Y}{\partial \zeta} = (1 + E \theta) \zeta_E^Y (1 + \zeta_E^Y) = 0$  for  $\zeta = \frac{1}{2}$ ;  $\frac{(1+E\theta)}{16E\zeta^Y(1+\zeta^Y)}$ . Thus,  $\frac{\partial \zeta_E^Y}{\partial \zeta} > 0$  if  $\zeta < \frac{1}{2}$ , and  $\frac{\partial \zeta_E^Y}{\partial \zeta} < 0$  if  $\zeta > \frac{1}{2}$ . Notice that  $E < \bar{E}(\zeta)$  implies that  $E < \frac{1}{1+\theta}$ , and thus  $\frac{1}{2} < 1=2$ . ■

#### Lemma 4.4

Case (i) follows from lemma 3.1: recall that for  $E > 0$ ,  $FOC_E(\zeta = 0) > 0$  and  $SOC_E(\zeta) > 0$ . Case (iv): for  $E < \frac{1+\theta}{1+4\zeta(1+\zeta)}$ ,  $FOC_E(\zeta = 0) < 0$  and  $SOC_E(\zeta) < 0$ , thus  $\zeta_E = 0$ . Case (ii) and (iii): for  $E \geq \frac{1+\theta}{1+4\zeta(1+\zeta)}$ ; 0 then  $FOC_E(\zeta = 0) > 0$  and  $SOC_E(\zeta) < 0$ . The intermediate cases may arise, since  $FOC_E(\zeta_E) = 0$  for  $\zeta_E = \frac{1}{2}$ ;  $\frac{1+\theta+E}{8E\zeta(1+\zeta)}$ . However, for  $E \geq \frac{1+\theta}{1+4\zeta(1+\zeta)}$ ; 0, case (ii),  $\zeta_E$  could be greater than one, and thus we need to impose that  $\zeta_E = \min \left\{ \frac{1}{2}; \frac{1+\theta+E}{8E\zeta(1+\zeta)}; 1 \right\}$ . Analogously, for  $E \geq \frac{1+\theta}{1+4\zeta(1+\zeta)}$ ;  $\frac{1}{1+\theta}$ , case (iii),  $\zeta_E$  could be lower than zero, and thus we need to impose that  $\zeta_E = \max \left\{ 0; \frac{1+\theta+E}{8E\zeta(1+\zeta)} \right\}$ .

Moreover, it is straightforward to see that  $\frac{\partial \zeta_E}{\partial E} > 0$  for  $\zeta_E \in (0; 1)$

and  $\frac{\partial \zeta_E}{\partial E} = 0$  for  $\zeta_E = 0; 1$ . Finally,  $\frac{\partial \zeta_E}{\partial \zeta} = \frac{(1+\theta+E)(1+2\zeta)}{8E\zeta^2(1+\zeta)^2}$ , which is non-negative for  $E < \frac{1}{1+\theta}$ , and negative for  $0 > E > \frac{1}{1+\theta}$ . ■

#### Proposition 4.5

By Shepsle's (1979) Theorem 3.1, a structured induced equilibrium is a pair  $(\zeta^m; \zeta^m)$ , in which  $\zeta^m$  is the median vote over the dimension  $\zeta$ , when the other dimension is fixed at the level  $\zeta^m$ , and  $\zeta^m$  is the median vote over the dimension  $\zeta$ , when the other dimension is fixed at the level  $\zeta^m$ . We have previously identified the median voters, respectively over the dimension  $\zeta$  and  $\zeta$ , with a type- $E_{m_\zeta}$  young and a type- $E_{m_\zeta}$  (young or old) individual. Since the ordering of the votes over one dimension, e.g.,  $\zeta$ , is not affected by the value of the other dimension, e.g.,  $\zeta$ , the median vote over  $\zeta$  and  $\zeta$  always coincides with the votes of a type- $E_{m_\zeta}$  young and a type- $E_{m_\zeta}$  (young or old) individual. We can now analyze the different cases.

Case (A) is trivial. If  $E_{m_s} < i(1 + \theta)$ , then  $\zeta^s = 1$ , and for  $\zeta^s = 1$ ,  $\zeta^s = 0$  if  $E_{m_s} < i(1 + \theta)$ .  
 Case (B) is trivial too. If  $E_{m_s} < i(1 + \theta)$ , then  $\zeta^s = 0$ , and for  $\zeta^s = 0$ , if  $E_{m_s} < i(1 + \theta)$ , then  $\zeta^s = 0$ .

Case (C): If  $E_{m_s} < i(1 + \theta)$ , the median voter will vote according to Lemma 4.4, case iii and thus  $\zeta_{E_{m_s}}(\zeta) = \max\{0, \frac{1}{2} i \frac{1 + E_{m_s}}{8 E_{m_s} \zeta (1 + \theta)}\}$ , or case iv and thus  $\zeta_{E_{m_s}}(\zeta) = 0$ . If  $E_{m_s} < i(1 + \theta)$ , then  $\zeta_{E_{m_s}}(\zeta) > 0$ . Notice that for  $\zeta = 0$ ,  $\zeta_{E_{m_s}}(0) = \frac{1}{2} i \frac{1 + E_{m_s}}{2 \theta (1 + N)}$ . In order to have a structure induced equilibrium at  $(\zeta^s = \zeta_{E_{m_s}}(0) > 0; \zeta^s = 0)$ , we thus need to have that  $\zeta_{E_{m_s}} = 0$  for  $\zeta_{E_{m_s}} = \frac{1}{2} i \frac{1 + E_{m_s}}{2 \theta (1 + N)}$ . By substituting this value of  $\zeta_{E_{m_s}}$  in  $\zeta_{E_{m_s}}(\zeta)$  at Lemma 4.4, case iii, it is easy to see that  $\zeta_{E_{m_s}} = 0$ , if  $E_{m_s} < - (E_{m_s}) = i(1 + \theta)(1 + N) - i(1 + \theta) = E_{m_s}$ .

Case (D): If  $E_{m_s} < i(1 + \theta)$  and  $E_{m_s} < i(1 + \theta)$ , as in case C, but  $E_{m_s} < - (E_{m_s})$ , there is no equilibrium at  $\zeta^s = 0$ , since  $\zeta_{E_{m_s}}(\zeta = 0)$  is greater than the maximum  $\zeta$  such that  $\zeta_{E_{m_s}}(\zeta) = 0$ . In other words, at  $\zeta = 0$ , the reaction function  $\zeta_{E_{m_s}}(\zeta)$ , which represents the decision of the median voter  $E_{m_s}$  over  $\zeta$ , is above the reaction function  $\zeta_{E_{m_s}}(\zeta)$ , which represents the decision of the median voter  $E_{m_s}$  over  $\zeta$ . Notice that  $\zeta_{E_{m_s}}(\zeta)$  is continuous and bounded above by  $1/2$ , whereas by lemma 4.4,  $\zeta_{E_{m_s}}(\zeta)$  is continuous and weakly increasing in  $\zeta$  for  $E_{m_s} < i(1 + \theta)$ . Therefore, the two reaction functions will cross in a point  $(\zeta^s > 0; 0 < \zeta^s < 1/2)$ , which constitutes a structure induced equilibrium since  $\zeta_{E_{m_s}}(\zeta^s) = \zeta^s$  and  $\zeta_{E_{m_s}}(\zeta^s) = \zeta^s$ . ■

**Proposition 4.6**

Suppose  $(\zeta^s; \zeta^s)$  is a structure induced equilibrium outcome of the voting game with commitment. Let us define the following realization of the public history of the game:

$$X_t^0 = fh_t \geq X_{t-1} \zeta_k = 0; k = 0; \dots; t \geq 1$$

and

$$X_t^1 = fh_t \geq X_{t-1} \zeta_0 \geq f_0; 1; \dots; t \geq 1; \zeta_t = 0 \text{ if } t < t_0 \text{ and } \zeta_t = \zeta_0 \text{ if } t \geq t_0$$

notice that  $X_t^0 \setminus X_t^1 = \dots$

Consider the following strategy  $s = (s_{t,E}^y; s_{t,E}^o)$ , for a type E young:

i) if  $E < E_{m_s}$

$$s_{t,E}^y = \begin{cases} (\zeta^s; \zeta_{t,E}(\zeta^s)) & \text{if } fh_t \geq X_{t-1}^0 [X_t^1] \\ (0; \zeta_{t,E}(0)) & \text{if } fh_t \geq X_{t-1}^0 [X_t^0] \end{cases}$$

ii) if  $E > E_{m_s}$

$$s_{t,E}^y = \begin{cases} (\zeta_{t,E}^y(\zeta^s); \zeta_{t,E}(\zeta^s)) & \text{if } fh_t \geq X_{t-1}^0 [X_t^1] \\ (0; \zeta_{t,E}(0)) & \text{if } fh_t \geq X_{t-1}^0 [X_t^0] \end{cases}$$

and for a type-E old individual

$$s_{t,E}^o = (1/2; \zeta_{t,E}(\zeta^s)) \text{ if } fh_t \geq X_t$$

where  $\zeta_{t,E}^y(\zeta^s)$  is defined in Lemma 4.2, and  $\zeta_{t,E}(\zeta^s)$  in Lemma 4.4.

Since by definition of SIE,  $\zeta^m = \zeta_{t;E_{m_\zeta}}(\zeta^m)$  and  $\zeta^m = \zeta_{t;E_{m_\zeta}}(\zeta^m)$ , it is easy to see that:

$$\begin{aligned} \zeta_{t;E_{m_\zeta}}(\zeta^m) &= \zeta^m \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m} \\ \zeta_{t;E_{m_\zeta}}(\zeta^m) &= \zeta^m \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m} \end{aligned}$$

Recall that the outcome function of the voting game at time  $t$  is the median in every dimension of the distribution of actions,  $(\zeta_t^m; \zeta_t^m)$ , then it is straightforward to see that the previous strategy profile  $S_{t;E}^y; S_{t;E}^0$  constitute a subgame perfect equilibrium of the voting game with no commitment, with equilibrium outcome  $(\zeta^m; \zeta^m)$ .

#### Lemma 5.1

First notice that young agents' preferences over  $\zeta$  are single peaked, since the indirect utility function at eq. 5.2 is concave in  $\zeta$ . This function has a maximum at  $\zeta = \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , which is positive if  $E < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ .  $\zeta_E^{Y;x}$  is weakly decreasing in  $E$  since  $\frac{\partial \zeta_E^{Y;x}}{\partial E} = 0$  for  $E = \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$  and  $\frac{\partial \zeta_E^{Y;x}}{\partial E} = \frac{1}{2} \frac{1 + E_{m_\zeta}}{2(1 + N)[1 + E_{m_\zeta} + \zeta^m]^2} > 0$  for  $E < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ .  $\zeta_E^{Y;x}$  is weakly decreasing in  $\zeta$  since  $\frac{\partial \zeta_E^{Y;x}}{\partial \zeta} = 0$  for  $E = \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$  and  $\frac{\partial \zeta_E^{Y;x}}{\partial \zeta} = \frac{(1 + E_{m_\zeta})(1 + E_{m_\zeta} + \zeta^m)}{2(1 + N)[1 + E_{m_\zeta} + \zeta^m]^2} < 0$  for  $E < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , because  $E < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$  implies  $E < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ . ■

#### Lemma 5.2

Trivial. From the indirect utility functions at eq. 5.2 and 5.3, we have that  $FOC_E(\zeta) = \mu E(1 + R)(1 + E_{m_\zeta} + E)$ , and  $SOC_E(\zeta) = 0$ . Thus,  $\zeta_E^x = 0$  if  $E < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , and  $\zeta_E^x = 1$  if  $E > \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ . Notice that if  $E = \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , the agent is indifferent between  $\zeta_E^x = 0$  and  $\zeta_E^x = 1$ . We break the indifference in favor of  $\zeta_E^x = 0$ . ■

#### Proposition 5.3

As in Proposition 4.5, we apply Shepsle's (1979) Theorem 3.1. All cases are trivial. Case (A): if  $E_{m_\zeta} > \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , then  $\zeta_X^m = 1$ , and for  $\zeta = 1$ ,  $\zeta_X^m = 0 \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ . Case (B) and (C): if  $E_{m_\zeta} < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , then  $\zeta_X^m = 0$ . For  $\zeta_X^m = 0$ , if  $E_{m_\zeta} > \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , then  $\zeta_X^m = 0$  (case (B)), whereas if  $E_{m_\zeta} < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ , then  $\zeta_X^m > 0$  (case (C)). ■

#### Proposition 6.1

Recall that in the two cases of endogenous and exogenous longevity the two median voters coincides, i.e.,  $E_{m_\zeta} = E_{m_\zeta}^x$ , and  $E_{m_\zeta} = E_{m_\zeta}^x$ . Therefore, by Proposition 4.5 and 5.3, we immediately obtain case B, and cases A and C, with the equality signs holding, i.e.e, in case A ( $\zeta^m = \zeta_X^m = 0$ ,  $\zeta^m = \zeta_X^m = 1$ ); and in case C ( $\zeta^m = \zeta_X^m > 0$ ,  $\zeta^m = \zeta_X^m = 0$ ). However, in the discussion of Proposition 4.5, we acknowledged that interior equilibria may arise in cases A and C. Were these equilibria to exist, they would be on the downward sloping part of the reaction function  $\zeta_{E_{m_\zeta}}(\zeta)$ , in case A, and on the upward sloping part of  $\zeta_{E_{m_\zeta}}(\zeta)$ , in case C. Thus, in for these interior equilibria, we would have ( $\zeta^m > \zeta_X^m = 0$ ,  $\zeta^m < \zeta_X^m = 1$ ) in case A, and ( $\zeta^m > \zeta_X^m > 0$ ,  $\zeta^m > \zeta_X^m = 0$ ) in case C.

Case (D) is more interesting. By Proposition 4.5 and 5.3, we know that if  $E_{m_\zeta} < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$ ,  $E_{m_\zeta} < \frac{1}{2} \frac{1 + E_{m_\zeta}}{1 + E_{m_\zeta} + \zeta^m}$  and  $E_{m_\zeta} < - (E_{m_\zeta})$ , then ( $\zeta^m > 0$ ;  $\zeta^m > 0$ ) and ( $\zeta_X^m > 0$ ;  $\zeta_X^m = 0$ ). Notice that  $\zeta_{E_{m_\zeta}}(\zeta = 0) = \zeta_X^m$ . We will establish a sufficient condition for the two reaction functions  $\zeta_{E_{m_\zeta}}(\zeta)$  and  $\zeta_{E_{m_\zeta}}(\zeta)$  to cross at a point ( $\zeta^m > 0$ ;  $\zeta_X^m > 0$ ), which lies above the horizontal line  $\zeta = \zeta_X^m = \zeta_{E_{m_\zeta}}(\zeta = 0)$ . Let  $e_\zeta$  be the value of the reaction function of the median voter  $E_{m_\zeta}$  at  $\zeta_X^m$ :

$$e_\zeta = \zeta_{E_{m_\zeta}}(\zeta_X^m) = \frac{1}{2} \frac{1 + E_{m_\zeta} + \zeta_X^m}{2E_{m_\zeta} + [4\zeta_X^m(1 + \zeta_X^m)]}$$

Let  $b_s$  be the positive value of  $s$  such that the reaction function of the median voter  $E_{m_i}$  is equal to  $i_x^*$ :

$$b_s = s > 0 \text{ j } i_{E_{m_i}}(b_s) = i_x^* = 1 - i \frac{1 + E_{m_i} i^*}{2E_{m_i} [4i_x^* (1 - i_x^*)]}$$

Notice that by Lemma 4.3,  $i_{E_{m_i}}(s)$  is increasing first and then decreasing, whereas, by Lemma 4.4,  $i_{E_{m_i}}(i)$  is increasing for  $s > 0$ . Then, a sufficient condition for  $i^* > i_x^*$  is that the reaction function  $i_{E_{m_i}}(i)$  crosses the horizontal line  $i = i_x^*$  to the left of the reaction function  $i_{E_{m_i}}(s)$ , that is  $b_s > e_s$ , which, after simple algebra, can be stated as

$$E_{m_s} < \tilde{A}(E_{m_i}) = \frac{(1 - i^*)E_{m_i}}{1 - i^* E_{m_i} - 1 - \frac{(1 + E_{m_i})^2}{2(1 + N)^2}}. \blacksquare$$

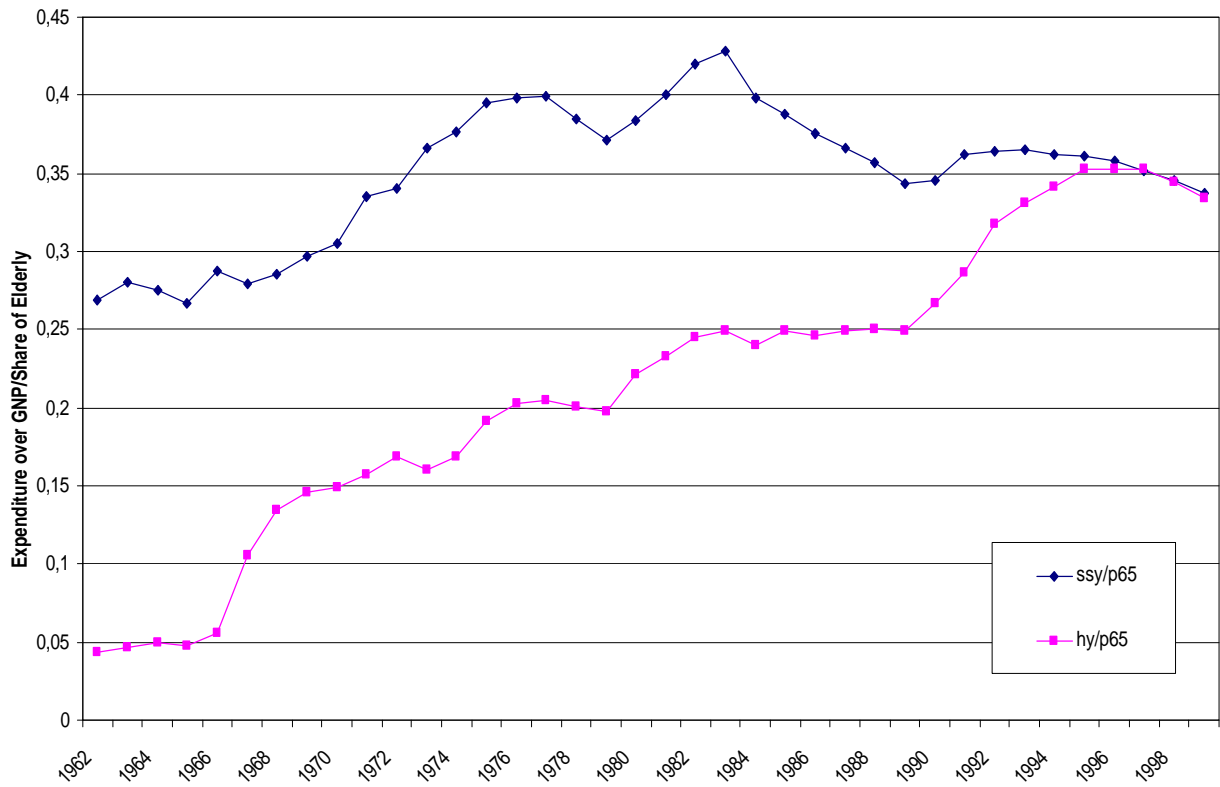
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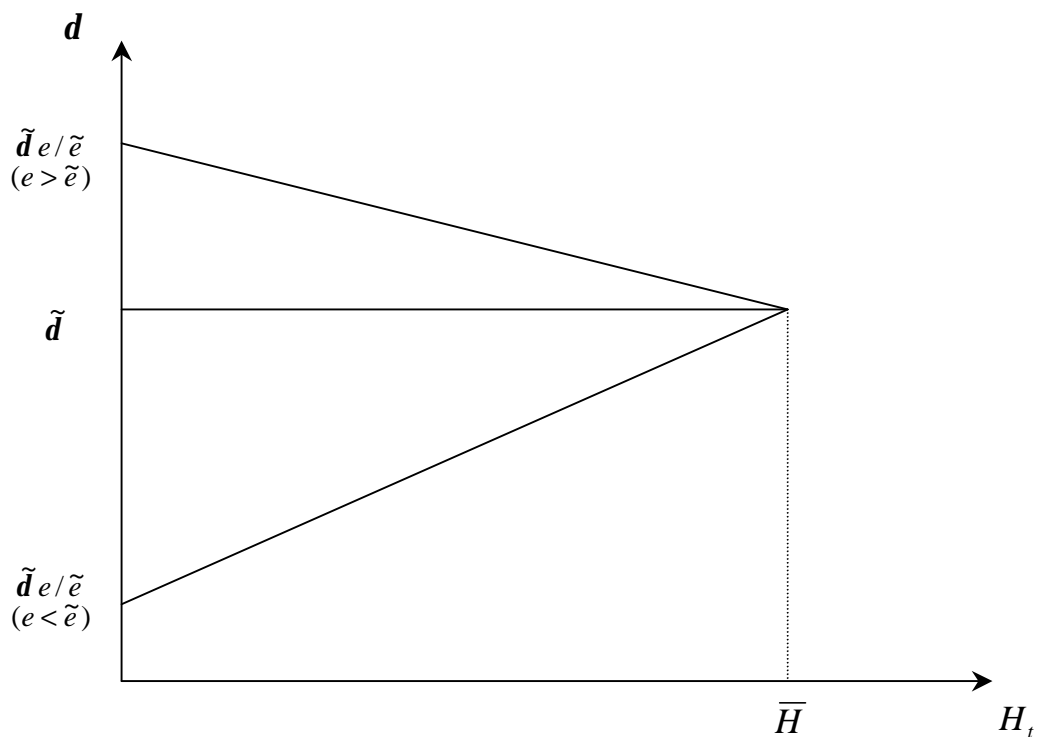


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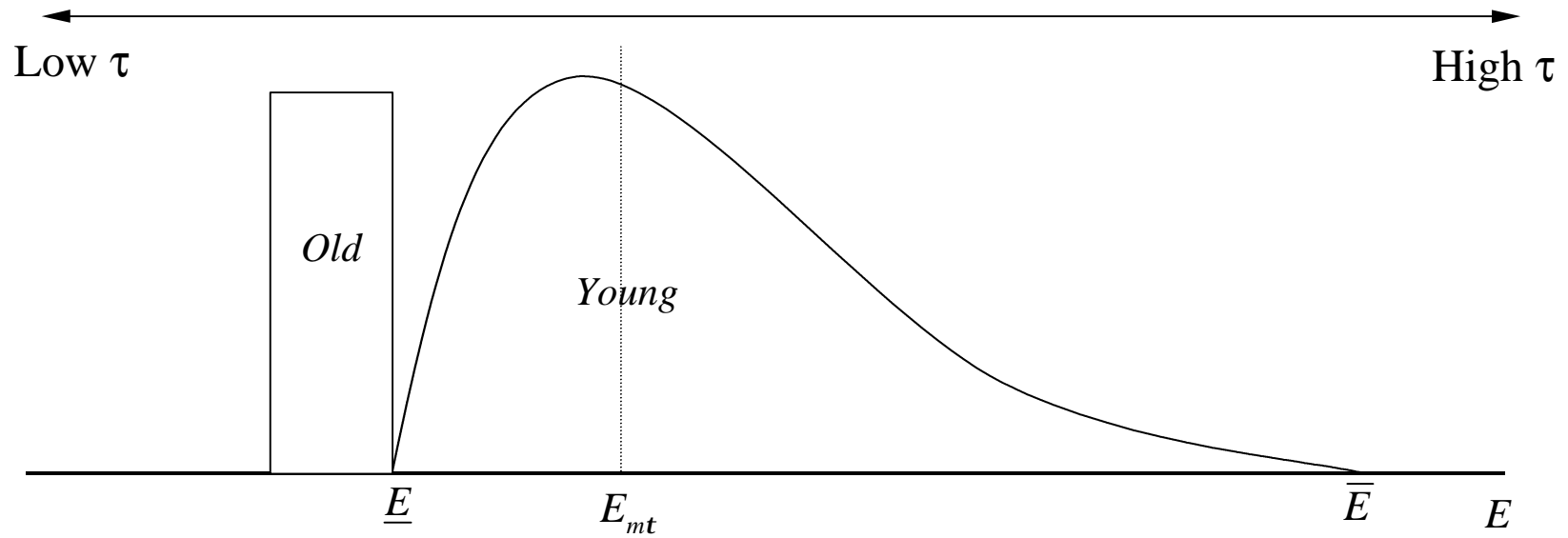
**Figure 1: US Social Security and Health Care**



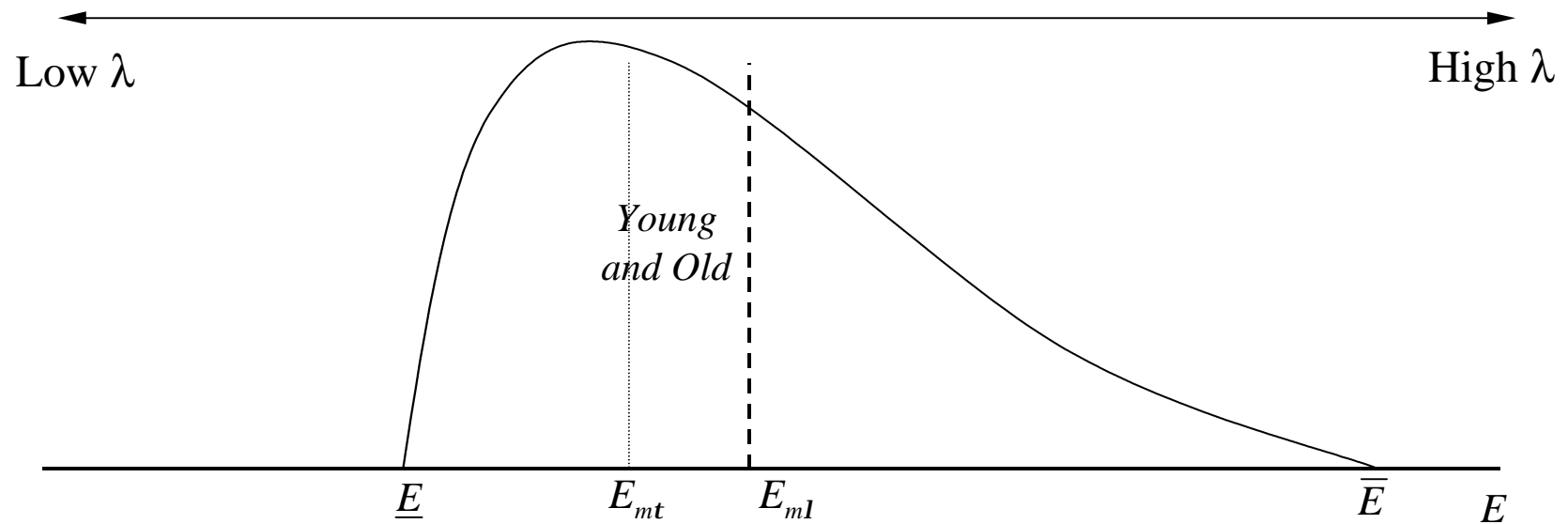
**Figure 2: Longevity Function  $d(e, H_t)$**



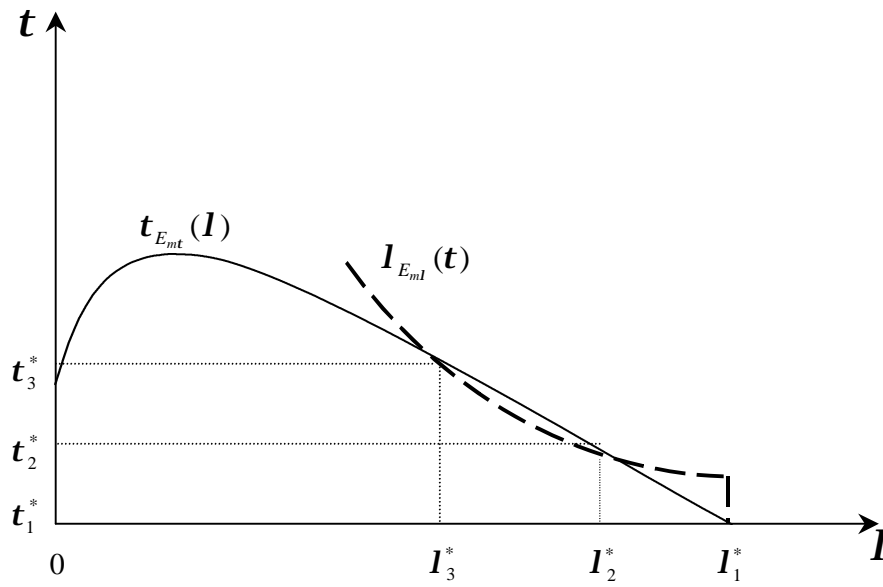
**Figure 3: Voting over  $\tau$**



**Figure 4: Voting over  $\lambda$**



**Figure 5: Case (A)**



**Figure 6: Case (D)**

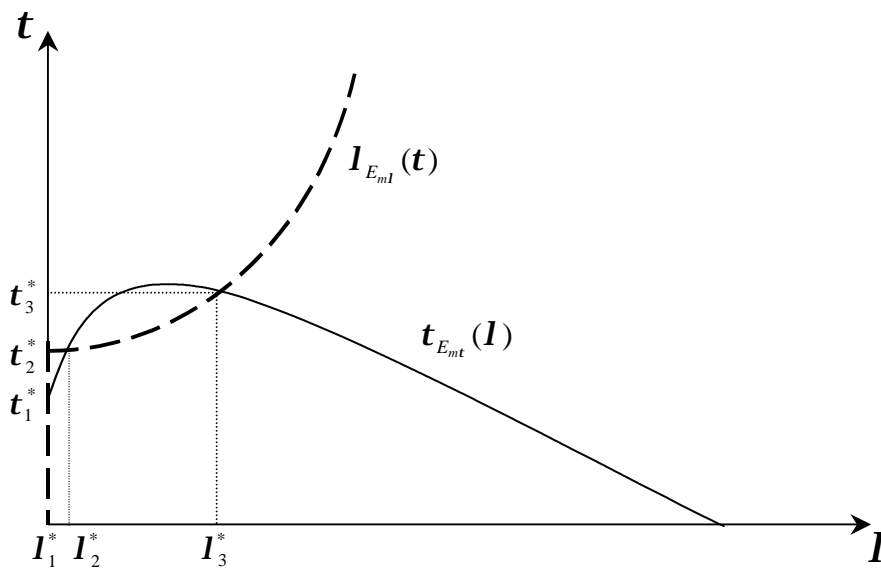


Figure 7

