The Time Consistency of Optimal Monetary Policy with Heterogeneous Agents

Stefania Albanesi*
Bocconi University, IGIER and CEPR

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Abstract

This paper studies the structure and time consistency of optimal monetary policy from a public finance perspective in an economy where agents differ in transaction patterns and asset holdings.

I find that the presence of distributional effects breaks the link between time consistency and high inflation which characterizes representative agent models of optimal fiscal and monetary policy. For a large class of economies, optimal monetary policy is time consistent. I relate these findings to key historical episodes of inflation and deflation.

Keywords: Inflation, Heterogeneity, Distribution, Time Consistency

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1. Introduction

The purpose of this paper is to study the structure and the time consistency of optimal monetary policy from a public finance perspective in an economy where agents are heterogeneous in transaction patterns and asset holdings. The seminal work of Calvo (1978) and Lucas and Stokey (1983) illustrates that in a monetary economy a benevolent policymaker has the incentive to tax outstanding nominal assets via unanticipated inflation when lump-sum taxation is not available. On this basis, lack of commitment has been advocated as a potential explanation of persistent high inflation and high public deficits[1]. In the presence of nominal assets and distortionary taxation, rational agents anticipate the policymaker's incentive to revise policy in the direction of higher money growth. This leads to high inflation in equilibrium. Moreover, the equilibrium inflation rate is positively correlated with the level of outstanding nominal government debt.

I find that heterogeneity breaks the link between lack of commitment and high inflation which characterizes representative agent models of optimal fiscal and monetary policy. The incentive to generate unanticipated inflation depends crucially on the distribution of currency and other nominal assets, as well as on the distribution of political power. Optimal monetary policy is time consistent for a large class of economies.

Lucas and Stokey (1983) and Chamley (1985) argue that time consistency of optimal monetary policy can be achieved only if the monetary authority can commit to a path for nominal prices. Persson, Persson and Svensson (1987) identify a set of conditions under which it is possible to chose debt maturities and degree of indexation to make optimal monetary policy time consistent. The findings in this paper suggest that optimal monetary policy could be made time consistent by influencing the distribution of government debt. This argument is not new. Hamilton (1795) argued in favor of the Federal assumption of the states' war debt as a way to reduce the risk of monetization. Debt assumption would provide powerful government creditors with a strong incentive to support Federal tax legislation, making the use of inflation to raise revenues less likely.

I describe an economy in which households have identical preferences but differ in labor productivity. Households chose consumption and labor supply. They make purchases with currency or with a costly alternative payment technology. The fixed cost associated to avoiding the use of cash implies that households with lower labor productivity and lower income hold more currency as a fraction of

[1] Kydland and Prescott (1977), and Barro and Gordon (1983, a and b) emplore the consequences of the time inconsistency of monetary policy in an expectational Phillips curve environment.

total purchases. This feature of the economy is consistent with cross–sectional evidence on transaction patterns and asset holdings. Erosa and Ventura (2000) report that in the US low income households use cash for a greater fraction of their total purchases relative to high income households. Mulligan and Sala-i-Martin (2000) estimate the probability of adopting financial technologies that hedge against inflation and find that is positively related to the level of household wealth and inversely related to the level of education. Attanasio, Guiso and Jappelli (2001) find that the probability of using an interest bearing bank account increases with educational attainment, income and average consumption, based on cross-sectional household data for Italy.

First, I study optimal monetary and fiscal policy for a benevolent government with the ability to commit to future policy- the Ramsey equilibrium. I assume that the government issues money, nominal and real debt and collects labor income taxes to finance an exogenous stream of government spending and I trace out the Pareto frontier for this economy for the class of utility functions in which the Friedman rule is optimal in a corresponding representative agent economy. Monetary and fiscal policy have redistributional effects. Inflation weighs more heavily on low productivity households who use currency for a greater fraction of their purchases and unanticipated inflation hits holders of nominal assets. The share of labor income tax revenues collected from each type of household is proportional to the labor supplied.

I find that the optimality of the Friedman rule depends on the distributional preferences of the government as well as on the constraints on distribution enacted via labor income taxation. If the government has no constraints in setting labor income tax rates, the Friedman rule is optimal. If the government is subject to constraints on labor income tax rates which limit its ability to tax low productivity households, the Friedman rule is optimal if and only if the government wishes to distribute to low productivity households. Otherwise high rates of inflation and government deficits will result. This finding is related to the conditions for optimal uniform commodity taxation. As shown in Atkinson and Stiglitz (1976), when utility is weakly separable, uniform commodity taxation is optimal, even with distributional objectives, if the labor income tax schedule is sufficiently unconstrained.

I then explore the time consistency of optimal fiscal and monetary policy. Distributional goals have ambivalent effects on government incentives. On one hand, constraints on available distributional policy instruments may increase the incentive to revise pre-announced policy. For example, Pearce and Stacchetti (1997) study an economy where constraints on redistribution arise from incentive problems due to asymmetric information, and this gives rise to time inconsistency, despite the availability of lump sum taxation. On the other, the distributional costs associated with deviations that would be optimal with a representative agent may remove the incentive to deviate from the ex ante optimal policy. Rogers (1986) studies optimal wage and interest taxation in a two-period, multiple consumer economy, and finds that inconsistent interest tax increases may be moderated if they create an unacceptable utility distribution in the economy.

I characterize the sufficient conditions for time consistency of the optimal policy, under the assumption that in each period fiscal and monetary policy are chosen before households can adjust their holdings of currency and their pattern of transactions, following Svensson (1985). The time consistency of the Ramsey equilibrium depends on the balance between distribution and efficiency. The main finding is that, for a large class of economies, lack of commitment does not imply a higher equilibrium rate of inflation. First, the Ramsey equilibrium is time consistent if and only if the Friedman rule is optimal i.e. when the government wished to distribute to low productivity households. In this case, since unanticipated inflation is more costly for low productivity households, distributional and efficiency incentives are in conflict. It is then always possible to find a distribution and maturity structure for assets such that the government chooses to adhere with the Ramsey equilibrium policy, even if the outstanding nominal government debt is large. In addition, if high productivity households have a high Pareto weight and they hold a sufficiently large fraction of nominal assets the optimal inflation rate under no commitment is lower that in the Ramsey equilibrium. If these conditions do not hold, the time consistent policy involves higher inflation than under commitment and the discretionary inflation rate is higher than in an economy with no heterogeneity. This is because both efficiency and distributional incentives push the government towards higher than expected money growth.

To evaluate the relevance of these findings, I analyze a number of key historical episodes of large inflations and deflation. Descriptive accounts of these episodes provide clear evidence of the importance of distributional consequences of unanticipated changes in inflation in shaping government incentives, conditional on the political influence of different groups of agents on monetary and fiscal policy decisions.

The plan of the paper is as follows. Section 2 describes the model. Section 3 studies optimal fiscal and monetary policy under commitment. Section 4 characterizes the sufficient conditions for time consistency. Section 5 reviews a number of historical episodes of inflation and disinflation. Section 6 concludes.

2. A Cash-Credit Good Economy with Heterogenous Households

In this section, I describe a version of Lucas and Stokey's cash-credit good economy with two key modifications. First, there are two types of households with different labor productivity who in equilibrium purchase different quantities of cash and credit goods. Second, in each period trade in goods and labor precedes trade in assets. This timing, introduced by Svensson (1985), implies that households cannot adjust the amount of currency available for purchases in the current period to changes in the inflation rate. The economy is populated by households, firms and a government. Households consume cash and credit goods and supply labor. Firms have access to a linear production technology that requires labor for the production of consumption goods. They are perfectly competitive. The government finances an exogenous stream of spending by issuing nominal debt, printing money and taxing labor income at a uniform proportional rate. There is no uncertainty.

I now illustrate the problems faced by the agents in our economy in detail.

2.1. Firms

Firms live for one period. They hire labor to produce consumption goods with a linear technology, given by:

$$\sum_{j=1}^{2} y_{jt} \leq n_t.$$

Here y_{1t} is total production of cash goods and y_{2t} total production of credit goods at time t and n_t is aggregate labor. Perfect competition implies:

$$P_{1t} = P_{2t} = P_t = W_t, \tag{2.1}$$

where P_t is the price charged for consumption goods and W_t the nominal wage at time t.

Purchases of consumption goods without currency need to be arranged. The services required to arrange to purchase consumption goods on credit are provided by competitive financial firms. Their profit per good is given by:

$$\pi_t\left(j\right) - W_t\theta\left(j\right), \tag{2.2}$$

where $\theta\left(\cdot\right)$ is measured in efficiency units of labor and satisfies $\theta' > 0$. π_t is the dollar charge for arranging purchases of consumption good j without currency. Profit maximization implies: $\pi_t\left(j\right) = W_t\theta\left(j\right)$ for all t and all $j \in [0, 1]$.

2.2. Households

There is a continuum of unit measure of households, divided into two types, where $0 < \nu_i < 1$ is the fraction of type i agents, with $i = 1, 2$ and $\sum_i \nu_i = 1$. Households of the same type are identical. Households have preferences defined over consumption of cash goods c_{i1}, consumption of credit goods c_{i2} and over hours worked n_i. Preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t U\left(c_{it}, n_{it}\right),$$

$$c_i = \left[\left(1 - z_i\right) c_{i1}^{\rho} + z_i c_{i2}^{\rho}\right]^{\frac{1}{\rho}}, \tag{2.3}$$

where $\rho \in (0, 1)$ and z_i is the fraction of consumption goods purchased without the use of cash, and c_{i1t}, c_{i2t} is the level of consumption of goods purchased with and without currency, respectively. I assume:

$$U\left(c_i, n_i\right) = h\left(c_i\right) + v\left(n_i\right),$$

where h is strictly increasing and strictly concave, while v is strictly decreasing and concave.

Households choose z_{it}, purchase consumption goods, supply labor, accumulate currency and trade one-period nominal discount bonds in each period. They enter a period with M_{it} units of currency and are subject to a cash in advance constraint, given by:

$$P_t c_{i1t}\left(1 - z_{it}\right) - M_{it} \leq 0. \tag{2.4}$$

The asset market session follows trading in the goods and labor market. During the asset market session households receive labor income net of taxes, clear consumption liabilities and trade nominal and real bonds of different maturities issued by other households or by the government. Nominal (real) bonds purchased at time t entitle holders to one unit of currency (consumption) in the asset market section at $t+1$. I assume that the government and private agents are committed to debt repayments. This implies that agents are indifferent between holding privately or government issued bonds. The price in terms of currency of a nominal bond of maturity s at time t is $Q_{t,t+s}$. Analogously, the price in terms of currency of a real bond with maturity s at time t is $P_t q_{t,t+s}$. If the government does not issue debt, the bonds will be in zero net supply. Total holdings of nominal and real bonds by agent i at the end of time t are denoted with $B_{it,t+s}$ and $b_{it,t+s}$ for $i = 1, 2$ and $s > 0$.

Households face the following constraint on the asset market:

$$
\begin{aligned}
& M_{t+1}+\sum_{s>0}\left(Q_{t,t+s}B_{it,t+s}+q_{t,t+s}P_t b_{it,t+s}\right) && (2.5)\\
\leq\ & M_{it}+\sum_{\hat{t}=-1}^{t-1}\left(B_{i\hat{t},t}+P_t b_{i\hat{t},t}\right)-P_t c_{i1t}\left(1-z_{it}\right)-P_t c_{i2t} z_{it}-\int_{\underline{z}}^{z_{it}}\pi_t\left(j\right)dj+W_t\xi_i\left(1-\tau_t^i\right)n_{it},
\end{aligned}
$$

where W_t is the nominal wage, ξ_i denotes labor productivity, τ_t^i is the tax rate on labor income and $\int_{\underline{z}}^{z_{it}}\pi_t\left(j\right)dj$ the currency cost of arranging purchases of consumption goods with credit. In addition, the no-Ponzi game condition:

$$
\left(Q_{t,t+1}^{-1}M_{it+1}+B_{it+1}\right)\Phi_{t+1}+\sum_{s=1}^{\infty}\Phi_{t+s}W_{t+s}\left(1-\tau_{t+s}\right)\xi_i\geq 0, \tag{2.6}
$$

is also required, with $\Phi_t=\prod_{t'=0}^{t-1}Q_{t',t'+1}$, $\Phi_0=1$.

2.3. Government

The government finances an exogenous stream of consumption $\bar{g}$ and is subject to the following dynamic budget constraint:

$$
P_t\bar{g}_t+M_t+\sum_{\hat{t}=0}^{t-1}\left(B_{\hat{t},t}+P_t b_{\hat{t},t}\right)=\sum_{s>0}\left(Q_{t,t+s}B_{t,t+s}+q_{t,t+s}P_t b_{t,t+s}\right)+M_{t+1}+W_tT_t, \tag{2.7}
$$

where M_t, B_t , b_t are the supply of currency, nominal and real bonds, respectively, and:

$$
T_t=\sum_i \nu_i\tau_t^i\xi_i n_{it}.
$$

I will consider the possibility that government policy is constrained to satisfy certain restrictions, captured in the constraint:

$$
\kappa\left(\tau_t^1,\tau_t^2\right)\leq 0. \tag{2.8}
$$

I will refer to a particular specification of $\kappa\left(\cdot\right)$ as a *fiscal constitution*[2].

[2] The constraint (2.8) is introduced to capture the notion that broad features of the fiscal structure, which determine the fiscal instruments available, tend to remain in place for long time periods and acquire an aura of costitutionality, as argued in Buchanan (1967).

2.4. Private Sector Equilibrium

The timing of events in each period is as follows:

1. Households come into the period with holdings of currency and debt given by M_{it} and B_{it}, b_{it}. They choose z_{it}.

2. The government sets policy subject to (2.7) and (2.8).

3. Households, firms and the government trade on the goods and labor markets. The households' purchases of cash goods are subject to (2.4). Equilibrium on the goods market requires:

$$\sum_{i=1,2} \nu_i \left(c_{i1t} \left(1 - z_{it}\right) + c_{i2t} z_{it} + \int_{\underline{z}}^{z_{it}} \theta\left(j\right) dj - \xi_i n_{it} \right) + \bar{g}_t = 0. \tag{2.9}$$

4. Asset markets open. Households purchase bonds and acquire currency to take into the following period subject to the constraint (2.5). Equilibrium in the asset market requires:

$$\begin{aligned} \sum_{i=1,2} \nu_i B_{it,t+s} &= B_{t,t+s}, \text{ for } s > 0, \\ \sum_{i=1,2} \nu_i b_{it,t+s} &= b_{t,t+s}, \text{ for } s > 0, \\ \sum_{i=1,2} \nu_i M_{it+1} &= M_{t+1}. \end{aligned} \tag{2.10}$$

Definition 2.1. *A private sector equilibrium is given by a government policy* $\{\bar{g}_t, \tau_t^i, M_{t+1}, B_{t,t+s}, b_{t,t+s}\}_{t\geq 0, s>0}$ *, a price system* $\{P_t, W_t, Q_{t,t+s}, q_{t,t+s}, \pi_t\left(j\right)\}_{t\geq 0, s>0, j\in[0,1]}$ *and an allocation* $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, B_{it,t+s}, b_{it,t+s}\}_{i=1,2,t\geq 0,s>0}$ *such that:*

1. *given the policy and the price system households and firm optimize;*

2. *government policy satisfies (2.7);*

3. *markets clear.*

The following proposition characterizes the competitive equilibrium.

Proposition 2.2. *An allocation* $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, B_{it,t+s}, b_{it,t+s}\}_{i=1,2,t\geq 0,s>0}$ *and a price system* $\{P_t, W_t, Q_{t,t+s}, q_{t,t+s}, \pi_t\left(j\right)\}_{t\geq 0,s>0,j\in[0,1]}$ *constitute a private sector* *equilibrium if and only if, for a given government policy* $\left\{\bar{g}_t, \tau_t^i, M_{t+1}, B_{t,t+s}, b_{t,t+s}\right\}_{t\geq 0, s>0}$, *(2.9), (2.7) and the following conditions are verified:*

$$0 < Q_{t,t+1} \leq 1,$$

$$W_t = P_t,$$

$$Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}} \frac{u_{i2,t+1}/z_{it+1}}{u_{i2,t}/z_{it}}, \tag{2.11}$$

$$q_{t,t+1} = \beta \frac{u_{i2,t+1}/z_{it+1}}{u_{i2,t}/z_{it}}, \tag{2.12}$$

$$\frac{-u_{i2t}/z_{it}}{u_{int}} = \frac{1}{\xi_i\left(1-\tau_t^i\right)} \text{ for } t \geq 0, \tag{2.13}$$

$$\frac{u_{i1t+1}/\left(1-z_{it}\right)}{u_{i2t+1}/z_{it}} = Q_{t,t+1}^{-1} \equiv R_{t+1}, \tag{2.14}$$

$$\begin{aligned} \left(R_t - 1\right)\left(P_{t+1}c_{i1t+1}\left(1-z_{it}\right) - M_{it+1}\right) &= 0, \\ P_{t+1}c_{i1t+1}\left(1-z_{it}\right) &\leq M_{it+1}, \end{aligned}$$

$$\left[\left(\frac{1}{\rho}-1\right)\left(1-R_s^{\frac{\rho}{\rho-1}}\right) - \frac{\theta\left(z_{is}\right)}{c_{i2s}}\right]\begin{cases} \leq 0 \text{ for } z_{is} = \underline{z}, \\ = 0 \text{ for } z_{is} \in \left(\underline{z}, \bar{z}\right), \\ \geq 0 \text{ for } z_{is} = \bar{z}. \end{cases} \tag{2.15}$$

for $t \geq 0$, *and:*

$$P_0 c_{i10}\left(1-z_{i0}\right) \leq M_{i0}, \tag{2.16}$$

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t \left[u_{i1t}c_{i1t} + u_{i2t}\hat{c}_{i2t} + u_{int}n_{it}\right] \qquad (2.17) \\ &= \hat{u}_{i10}\frac{M_{i0}}{P_0} + \hat{u}_{i20}\frac{B_{i(-1),0}}{P_0} + \hat{u}_{i20}\sum_{t=1}^{\infty}\frac{B_{i(-1),t}}{P_0}\prod_{j=1}^{t}R_j + \sum_{t=0}^{\infty}\beta^t\hat{u}_{i2t}b_{i(-1),t}. \end{aligned}$$

for $i = 1, 2$, *with* $C\left(z_{it}\right) = \int_{\underline{z}}^{z_{it}} \theta\left(j\right) dj$.

Here, $u_{ij} = \partial U\left(c_i, n_i\right)/\partial c_{ij}$, $u_{in} = U_2\left(c_i, n_i\right)$ and $\hat{c}_{i2} = c_{i2} + \frac{C(z_i)}{z_i}$, $\hat{u}_{i1} = u_{i1}/\left(1-z_i\right)$, $\hat{u}_{i2} = u_{i2}/z_i$ for $i, j = 1, 2$. Equation (2.17) is the households' intertemporal budget constraint and it incorporates the transversality condition. The proof of this proposition is in Appendix A.

3. Optimal Policy with Commitment

I define the Ramsey equilibrium as the private sector equilibrium which maximizes the government's objective function, given by the weighted sum of the households' lifetime utility. The Pareto weight on type i agents is η_i, with $\eta_1 + \eta_2 = 1$. I assume that Pareto weights are time-invariant. The case $\eta_i = \nu_i$ corresponds to a utilitarian government.

The Ramsey equilibrium outcome can be characterized by solving the "primal government problem", where the government chooses an allocation at time 0 subject to the constraint that it constitutes a private sector equilibrium. This problem's choice variables are $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}\}_{i=1,2,t\geq 0}$ and P_0. The level of P_0 determines the real value of nominal assets at time 0 and defines the boundary of the agents' intertemporal budget set. High values of P_0 amount to a tax on outstanding nominal wealth and on consumption of goods purchased with cash at time 0. The government is constrained to tax all nominal assets at the same rate. The extent to which each household is hit by this tax depends on the exogenous distribution of currency and bonds at time 0 and on liquidity preference.

Proposition 3.1. *An allocation* $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}\}_{i=1,2,t\geq 0}$ *and values of* $\{R_t\}_{t\geq 0}$ *and* P_0 *constitute a Ramsey equilibrium if and only if they solve the problem:*

$$\max_{P_0,\{c_{i1t},c_{i2t},n_{it},z_{it}\}_{i=1,2,t\geq 0}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1,2} \eta_i U\left(c_{it}, n_{it}\right)$$

subject to:

$$\frac{\hat{u}_{i1t}}{\hat{u}_{i2t}} = R_t, \text{ for } i = 1, 2, \tag{3.1}$$

$$R_t \geq 1, \tag{3.2}$$

$$\kappa\left(\frac{u_{12t}/z_{2t}}{u_{1nt}}\xi_1, \frac{u_{22t}/z_{2t}}{u_{2nt}}\xi_2\right) = 0, \tag{3.3}$$

(2.15) and (2.9) for all t*, as well as (2.17) and (2.16).*

A detailed proof of this characterization can be found in Chari and Kehoe (1998).

3.1. Properties of Optimal Policy for $t > 0$

In this section, I illustrate the key properties of Ramsey equilibrium policy for $t > 0$. For simplicity, assume that:

$$b_{i(-1),t} = B_{i(-1),t} = 0, \ i = 1, 2, \ t \geq 1. \tag{3.4}$$

The arguments hold more generally, however.

I first assume that the government is not subject to constraints in setting tax rates on labor. Then, the following proposition holds.

Proposition 3.2. *Assume:*

$$U(c,n) = h(c) + v(n), \tag{3.5}$$

and (3.4). The solution to the relaxed Ramsey problem where constraint (2.8) is not imposed, implies $R_t = 1$ for $t \geq 1$.

The proof is in Appendix B and is analogous to the proof of the optimality of the Friedman rule in the environment with a representative agent and distorting taxes analyzed by Christiano, Chari and Kehoe (1996). It relies on the homotheticity of the consumption aggregator, which implies a unitary income elasticity of money demand. Intuitively, if the government can set different labor tax rates for different agents, optimality requires equalizing the relative price of cash and credit goods.

I now assume that (2.8) belongs to general affine class:

$$\kappa_0 + \kappa_1 \left(1 - \tau_t^2\right) \leq 1 - \tau_t^1, \tag{3.6}$$

with $\kappa_0 \leq 1 - \kappa_1$ and $\kappa_1 > 0$. Constraints of this type impose that the tax rate on labor income imposed on type 2 households be sufficiently high, relative to the tax rate imposed on type 1 households.

Let $\bar{\eta}_1$ denote the Pareto weight such that constraint (3.6) is not binding for $t > 0$. For example, it can be verified that, for $u_{in} = -\gamma$ and $\kappa_0 = 0$, it is defined by the following equation:

$$\frac{\xi_2}{\xi_1}\kappa_1 = \frac{\bar{\eta}_1}{\nu_1}\left(\frac{\bar{\eta}_2}{\nu_2}\right)^{-1},$$

where $\bar{\eta}_2 = 1 - \bar{\eta}_1$. In addition, assume that, for all $c > 0$,

$$\frac{\partial C\left(z\left(R,c\right)\right)}{\partial z} > 0 \text{ for } R > 1, \tag{3.7}$$

where $z(R,c)$ is defined by (2.15). This assumption guarantees that $z_{2,t}$ is strictly interior for $R_t > 1$ and that $\theta(\cdot)$ is strictly positive for interior z's.

Proposition 3.3. *Assume (3.5), (3.4) and (3.7). In the Ramsey equilibrium under (3.6), $R_t = 1$ for $t \geq 1$ if and only if $\eta_1 \geq \bar{\eta}_1$.*

The proof is in Appendix B.

To understand this result, it is useful to define a type specific consumption price indexes, P_t^i, $\hat{P}_t^i$ for $i = 1, 2$ for $t > 0$:

$$P_t^i = \left[(1 - z_{it})\,(R_t)^{\frac{\rho}{\rho-1}} + z_{it}\right]^{\frac{\rho-1}{\rho}},$$

$$\hat{P}_t^i = P_t^i + \frac{C\,(z_{i,t})}{c_{it}}. \tag{3.8}$$

P_i^i measures the cost in efficiency units of labor of one unit of the consumption aggregator c_i for given z_{it}. $\hat{P}_i^i$ measures the cost in efficiency units of labor of one unit of c_i when z_{it} solves (2.15), including the cost of z_{it} [3]. Optimality implies $\tilde{P}_t^i \leq R_t$.

For a given level of R_t, (2.15) implies $z_{2t} > z_{1t}$ and $P_t^1 > P_t^2$ and $\hat{P}_t^1 \geq \hat{P}_t^2$. This implies that, for a given tax rate on labor income, the wedge between the marginal utility of leisure and the marginal utility of consumption is higher for low productivity households:

$$(1 - \tau_t)\,\frac{1}{\hat{P}_t^2} > (1 - \tau_t)\,\frac{1}{\hat{P}_t^1}. \tag{3.9}$$

Therefore, a departure from the Friedman rule is equivalent to a higher net real wage in efficiency units for high productivity households relative to low productivity households and amounts to redistribution in favor of high productivity households.

Assumptions (3.4) and (3.7) are not essential for the result in proposition 3.3, as will become clear in the section on numerical results. (3.4) is imposed since

[3] This price index is derived from the solution of the following static optimization problem:

$$\begin{aligned} & \max_{c_{i1}, c_{i2}, z_i} \left[(1 - z_i)\, c_{i1}^{\rho} + z_i c_{i2}^{\rho}\right]^{1/\rho} \text{ subject to} \\ w &= R c_{i1}\,(1 - z_i) + c_{i2} z_i + C\,(z_i)\,, \end{aligned}$$

where w is an exogeous endowment of real wealth. Let:

$$c_i = \left[(1 - z_i)\, c_{i1}^{\rho} + z_i c_{i2}^{\rho}\right]^{1/\rho},$$

and denote the expenditure function with $e(R;\theta)$ and the value function with $v\,(R; w, \theta)$. Then, the optimal value of c_i solves $c_i = v\,(R; w, \theta)$ and:

$$\hat{P}^i = \frac{e(R; w, \theta)}{c_i}.$$

the presence of real debt alters the conditions for optimality of the Friedman rule. Combining the first order conditions for c_{i1} and c_{i2}, assuming $R_t = 1$ and (3.6) non-binding yields:

$$(\eta_i + \lambda_i)\left(\hat{u}_{i1} - \hat{u}_{i2}\right) + \lambda_i \hat{u}_{i22,t} b_{i(-1),t} = 0.$$

Optimality of the Friedman rule requires

$$-\lambda_i \hat{u}_{i22t} b_{i(-1),t} \leq 0 \tag{3.10}$$

for all t and $i = 1, 2$. A similar condition would arise in a representative agent model[4].

Propositions 3.2 and 3.3 point to a general principle, reminiscent of the results on uniform commodity taxation in Atkinson and Stiglitz (1976). They show that when utility is weakly separable, uniform commodity taxation is optimal, even with distributional objectives, if the labor income tax schedule is sufficiently unconstrained[5]. This result is based on the following logic. Since the wedge between leisure and consumption is only affected by the sub-utility derived from consumption, as long as resources are scarce and there are no constraints on redistribution, a benevolent government seeks to deliver this sub-utility in the cost minimizing way. Constraints on the labor income tax schedule may give rise to a conflict between efficiency and distribution, which induces the government to abandon uniform commodity taxation. For the model economy in this paper, unitary income elasticity of money demand implies that the cost minimizing way of delivering a given sub-utility from consumption is to follow the Friedman rule. However, this policy is sub-optimal when high productivity households have a high Pareto weight and the government faces constraints on redistribution towards them via labor income taxation.

3.2. Numerical Findings

To evaluate the impact of redistributional incentives I compute the Ramsey equilibrium as a function of the Pareto weight for a plausibly parametrized version of the economy. I focus on the utility specification:

$$U\left(c_i, n_i\right) = \frac{c_i^{1-\sigma} - 1}{1 - \sigma} + v\left(n_i\right), \text{ for } i = 1, 2, \ \sigma > 0, \tag{3.11}$$

[4] This issue does not arise in Chari, Christiano and Kehoe (1996) who do not consider real debt. Alvarez, Kehoe and Neumeyer (2001) also avoid this problem. Given the assumption of consumption taxes, only the marginal utility of labor appears in the intertemporal Euler equations in their formulation.

[5] In Atkinson and Stiglitz, imperfect information on the agents' labor productivity impose incentive compatibility constraints on labor income taxation. Here, I abstract from asymmetric imformation and focus on exogenous constraints on labor tax rates.

and I present results for:

$$v(n_i) = \psi \log(1-n_i), \ \psi > 0, \tag{3.12}$$
$$= -\gamma n_i, \ \gamma > 0. \tag{3.13}$$

The case corresponding to (3.13) is a useful benchmark. The absence of wealth effects on consumption implies that the distribution of cash holdings across agents at the end of any period only depends on government policy in the following period. When (3.13) is imposed, I restrict $0 < \sigma < 1$ to ensure that labor supply increases with the real wage. Parameter values are displayed below:

Table 1: Benchmark Parameter Values

$\underline{z}$	$\overline{z}$	β	γ	ν_1	θ	σ	ψ	ρ	ξ_1	ξ_2
0.10	0.654	0.97	3	0.56	0.021	0.8	3	0.5	1	1.8

I set $\nu_1 = 0.56$, which roughly matches the percentage of US households having no financial assets other than a checking account, according to the 1995 Survey of Consumer Finances. I assume $\rho = 0.5$ and parameterize the transactions technology as follows:

$$\begin{aligned} \theta(j) &= 0 \text{ for } j \leq \underline{z}, \\ &= \theta \text{ for } j \in (\underline{z}, \bar{z}) \\ &= \infty \text{ for } j \geq \bar{z}, \end{aligned}$$

where $0 \leq \underline{z} < \bar{z} \leq 1$. θ is set so that $z_2 = \bar{z}$ for $R \geq 1.10$, and $z_1 = \bar{z}$ for $R \geq 1.8$. I set $\bar{z} = 0.65$ and $\underline{z}= 0.10$. This implies that in equilibrium with $R = 1.06$, money demand velocity in the model is equal to 2.89[6] When (3.13) is imposed, the value of σ determines the interest elasticity of money demand. I set $\sigma = 0.8$. This implies that for both preference specifications the interest semi-elasticity of aggregate money demand is approximately equal to 4 at $R = 1.06$ [7]. The level of government consumption is set equal to 20% of total employment in equilibrium. Lastly, $\xi_1 = 1$ and $\xi_2 = 1.8$ imply that at a steady state with $R = 1.06$ and $\tau^i = 0.30$, labor income of high productivity households is 2.2 greater than labor

[6] Dotsey and Ireland (1996) report that **a**verage M1 velocity in the US for the post-war period is equal to 5.4.

[7] Computed as:

$$\frac{\partial \log(M/P)}{\partial \log(R)},$$

where M/P are aggregate real money balances. This number is slightly lower than estimates reported in the literature. For examples, Dotsey and Ireland (1996) report a value of this statistic of 5.9 for the US.

income of low productivity households. This percentage is approximately equal to the value of this statistic for the US (see Erosa and Ventura, 2000).

I analyze the Ramsey equilibrium under different constraints on direct distribution.

First, I consider affine restrictions on tax rates. Figure 1 displays the features of Ramsey equilibrium policy as a function of η_1 for $t > 0$, under (3.13) and (3.6) with equality at $\kappa_1 = 1$ and $\kappa_0 = 0$ i.e. under the constraint that the labor income tax rate is the same across types. Initial real and nominal debt holdings are set at 0 and the distribution of currency is symmetric. The top right panel exemplifies the result in proposition 3.3. The solid line represents the Ramsey equilibrium net nominal interest rate and the dashed line the value of the multiplier on the distribution constraint. There is a value of the Pareto weight, $\bar{\eta}_1$, for which the constraint on tax rates is not binding. For $\eta_1 \geq \bar{\eta}_1$, the Friedman rule is optimal. The tax rate on labor is increasing in η_1, even for $\eta_1 > \bar{\eta}_1$. This is due to the fact that for higher η_1 the multiplier on the implementability constraint on type 2 falls (and the one for type 1 increases). This reduces the shadow cost of raising distortionary taxes from type 2 and induces a rise in the optimal tax rate. The tax rate on labor varies from 0.10 to 0.28, while the net nominal interest rate from 34% to 0.

Figure 2 displays the properties of the Ramsey equilibrium under (3.13) and (3.6) with inequality at $\kappa_1 = 1$ and $\kappa_0 = 0$, i.e.

$$\tau^2 \geq \tau^1. \tag{3.14}$$

The net nominal interest rate is decreasing in η_1 and the Friedman rule obtains only if the multiplier on direct distribution is 0. The tax rate on type 2 households increases with η_1. As long as the constraint on distribution is binding, the tax rate on type 1 agents also increases with η_1. For $\eta_1 \geq \bar{\eta}_1$, it is decreasing with $\eta_{1,}$ since in this region of the Pareto space the government wishes to distribute to type 1 agents.

Figure 3 displays the features of Ramsey equilibrium policy as a function of η_1 for $t > 0$, under (3.13) and

$$T_{1t} \leq T_{2t} \text{ for all } t, \tag{3.15}$$

where $T_{it} = \tau_t^i \xi_i n_{it}$ for $i = 1, 2$.

Here, $\bar{\eta}_1$ corresponds to the value of η_1 for which the percentage of labor income tax revenues raised from type 1 (displayed in the bottom right panel) is equal to 50%. For this value of η_1, the multiplier on the distribution constraint is 0. The tax rate on type 2 agents increases with η_1, ranging from 0.05 to 0.52.

The tax rate on type 1 decreases for $\eta_1 > \bar{\eta}_1$ and ranges from 0.10 to 0.23. The net nominal interest rate peaks at 26% for $\eta_1 = 0.35$. The behavior of Ramsey policy under (3.15) is very similar to the behavior of Ramsey policy under (3.14).

Figure 4 displays the Ramsey equilibrium for the following restriction on tax rates:

$$\frac{T_{1t}}{W_t \xi_1 n_{1t}} \leq \frac{T_{2t}}{W_t \xi_2 n_{2t}} \text{ for all } t, \tag{3.16}$$

which corresponds to progressive taxation. As before, the Friedman rule is optimal only when the constraint of distribution does not bind. The net nominal interest rate peaks at 26% for the lowest value of η_1. The tax rate on type 2 systematically raises with η_1, from 0.12 to 0.41, while the tax rate on type 1 is increasing in η_1 as long as the constraint on distribution is binding and falls with η_1 otherwise. Under progressive taxation, the maximum share of labor income tax revenues raised from type 1 is 43% and falls below 30% when the constraint on distribution is not binding. Similar results hold under (3.12).

4. Sufficient Conditions for Time Consistency

In this section, I illustrate the potential sources of time inconsistency and derive the sufficient conditions for time consistency of the Ramsey equilibrium. In addition, I provide examples of economies in which the Ramsey equilibrium is not time consistent but the optimal deviation involves a fall in the realized inflation rate.

As in Lucas and Stokey (1983), the procedure to derive sufficient conditions for time consistency of the Ramsey equilibrium is the following. For any $t \geq 0$, define the Ramsey problem at period t analogously to the Ramsey problem for period 0. Then, the Ramsey problem at period t is time consistent for period $t+1$ if the continuation allocation of the solution to the Ramsey problem at period t solves the Ramsey problem at $t+1$. The Ramsey equilibrium is time consistent if the Ramsey problem at time t is time consistent for the Ramsey problem at $t+1$ for $t \geq 0$. In practice, it is sufficient to verify that initial conditions for the time 1 problem exist that would induce the government at time 1 to continue with the allocation that solves the Ramsey problem at time 0.

The source of time inconsistency in the Ramsey equilibrium is that decisions on asset holdings and on transaction patterns for $t = 1$ are sunk for a government optimizing at time 1. Therefore, the elasticity of the inflation tax base at time 1 is lower in the time 1 Ramsey problem relative to the time 0 Ramsey problem. The government's incentives for departing at time 1 from the continuation allocation implied by the solution to the time 0 Ramsey problem depend on the balance of efficiency and distributional incentives. To illustrate this, I compare features of the allocation for time 1 that solves the time 1 Ramsey problem with the continuation allocation of the time 0 Ramsey equilibrium under (3.13).

First, assume that real and nominal debt are in 0 net supply for all t -the government is constrained to run a balanced budget-, private holdings of real and nominal debt are 0[8], and (3.6) is imposed. Then, R_t just denotes the shadow price of cash goods relative to credit goods. If the Friedman rule is optimal, at the continuation of the time 0 Ramsey equilibrium, the price elasticity of cash good consumption at time 1 is the same at the time 0 and at the time 1 Ramsey equilibrium for small changes in P_1 since the cash in advance constraint is not binding. Larger increases in P_1 cause the cash in advance constraint to become binding, reducing the price elasticity of cash good consumption at time 1 in the time 1 Ramsey problem. For $\eta_1 > \bar{\eta}_1$, when the government wishes to distribute to type 1 agents, it is sub-optimal to increase P_1 in a way that generates a rise in the relative price of cash goods. Since the price of cash goods relative to credit goods is already at its lowest, there is no feasible deviation in the price level which increases utility. At $\eta_1 = \bar{\eta}_1$, distributional concerns are second order and the government faces the same incentives as in a representative agent economy. Given the lower price elasticity of cash good consumption when currency holdings and transaction patterns have already been set, it will be optimal to set a higher inflation than expected. However, the Svensson timing imposes an upper bound to the extent to which a change in P_1 is desirable, even from the standpoint of pure efficiency considerations[9]. For $\eta_1 < \bar{\eta}_1$, $R_t > 1$ and the cash in advance constraint is binding for $t > 0$ in the time 0 Ramsey equilibrium and the price elasticity of cash good consumption is 1, lower than in the time 0 Ramsey problem. Therefore, it is efficient to tax cash good consumption at a higher rate at time 1 by increasing P_1, relative to the value prescribed by the continuation of the time 0 Ramsey equilibrium. Since an increase in P_1 corresponds to a higher relative price of cash goods, the distributional effect reinforces the incentive arising from efficiency considerations and the deviation will be greater than with a

[8] Since under (3.13), asset positions are not pinned down for $t > 0$, such an equilibrium always exists.

[9] Nicolini (1998) also evaluates the conditions under which optimal monetary policy is time consistent when agents are heterogeneous in their ability to adjust currency holdings in response to unanticipated changes in inflation. He finds that optimal monetary policy is not time consistent in general but that for certain conditions the optimal deviation involves a fall in the rate of money growth. The conditions involve the distribution of the Pareto weights and the price elasticity of cash good consumption. Nicolini does not consider labor income taxation and stops short of analyzing the case with nominal government debt.

representative agent. This case corresponds to figure 5[10], where the solid line represents the continuation of the time 0 Ramsey equilibrium and the $*$ is the solution to the time 1 Ramsey problem at $t = 1$.

If outstanding nominal government debt is positive, the elasticity of the inflation tax base at time 1 is always lower in the time 1 Ramsey problem. If debt is evenly distributed across types of agents or $\eta_1 = \bar{\eta}_1$, i.e. distributional concerns are second order, the prevailing incentive is to reduce aggregate distortions. However, if the type of household having the highest Pareto weight also has large holdings of nominal government debt, distributional concerns would induce the government to set a lower P_1 in the time 1 Ramsey equilibrium, relative to the time 0 Ramsey equilibrium. Figure 6 displays two cases illustrating this. The top panel features a symmetric distribution of real and nominal debt holdings. Real debt holdings are 0 for both types of agents, and both types hold the same amount of currency and nominal debt. In the bottom panel, the aggregate nominal and real debt is 0 and real debt holdings are zero for both types. Type2 households are net creditors to type 1 households[11]. In figure 6 the Ramsey equilibrium is not time consistent. However, the features of the solution to the time 1 Ramsey equilibrium suggest that in the case $\eta_1 \geq \bar{\eta}_1$ it may be possible for the Ramsey equilibrium to be time consistent, even with positive outstanding nominal government debt.

The following proposition holds.

Proposition 4.1. *If the government is subject to (3.6), the Ramsey equilibrium is time consistent if and only if the Friedman rule is optimal.*

The proof is in appendix C. Here, I sketch it briefly. If the Friedman rule is optimal, by propositions 3.2 and 3.3, the constraint on distribution is not binding and the corresponding multiplier is 0. The multipliers on constraints (3.1) and (3.2) are also 0. Then, it is possible to show that $M_{i,1}$, $b_{i1,t}$ and $B_{i1,t}$ for $t \geq 1$ exist such that the continuation of the solution of the time 0 Ramsey problem satisfies the first order conditions for time 1 Ramsey problem.

---

[10] Here, parameters are as in Table 1. (3.6) is imposed with equality. The distribution of currency and $z_{i,1}$ at the beginning of time 1 are determined by the continuation of the solution of the time 0 Ramsey problem for each value of η_1.

[11] All parameters and constraints are as in figure 5. The distribution of nominal debt at $t = 1$ is:

$$\frac{B_{i0,1}}{M_{i1}} = 1, \text{ for } i = 1, 2, \ \frac{B_{0,1}}{M_1} = 1,$$

in the top panel, and:

$$\frac{B_{10,1}}{M_{11}} = -1, \ \frac{B_{20,1}}{M_{21}} = 4.79, \ \frac{B_{0,1}}{M_1} = 0.$$

Legend as in figure 5.

To show the converse, assume that the Ramsey equilibrium is time consistent and the Friedman rule does not hold. Then, the multipliers on the implementability constraints, on the distribution constraint and on the resource constraint are the same in the time 0 and time 1 Ramsey problem. In addition, the shadow value of outstanding nominal debt must be 0, otherwise the government would have an incentive to change P_1 to reduce the real value of debt and distortionary taxes in the future. Note that this condition does not imply the actual nominal debt is 0, as with a representative agent. By the fact that the Friedman rule does not hold, the cash in advance constraint must be binding for both types at time 1 in the solution of the time 0 and the time 1 Ramsey problem and the value of the multiplier on constraint (3.2) is positive. This entails a contraction.

The intuition behind this result is as follows. For $\eta_1 > \bar{\eta}_1$, the Friedman rule is optimal. This has two implications. First, the present discounted value of government's nominal liabilities is minimized. Second, as explained above, the price elasticity of cash good consumption at time 1 is the same in the time 0 and in the time 1 Ramsey equilibrium. Then, the government does not have an incentive to change P_1 to reduce distortionary taxation. In addition, the government wishes to distribute to type 1 households who hold more currency as a fraction of their total purchases. This distributional force makes the Ramsey equilibrium time consistent even when the value of nominal government debt is positive, thus strengthening the time consistency result. For $\eta_1 < \bar{\eta}_1$, the Friedman rule is not optimal. Then, unless the shadow value of nominal government debt is 0 at the continuation of the time 0 Ramsey equilibrium, there is an incentive to increase P_1 and lower R_t at all future dates to reduce the present discounted value of nominal liabilities outstanding at time 1. If the shadow value of nominal government debt is 0, the incentives to deviate from the continuation of the time 0 Ramsey equilibrium at time 1 arise from the distribution of currency only. Since the cash in advance constraint is binding at the continuation of the time 0 Ramsey equilibrium, the price elasticity of the inflation tax base at time 1 is lower in the time 1 Ramsey equilibrium than in the time 0 Ramsey equilibrium. In addition, $R_t > 1$ is optimal because it is a form of distribution to type 2 agents. But it is more efficient to carry out this distribution by raising the relative price of cash goods at time 1, i.e. increasing P_1, and reducing it at future dates. Therefore, both distribution and efficiency objectives generate an incentive to increase P_1 as long as R_t is strictly greater than 1.

The last part of the argument illustrates that the Friedman rule is a necessary and sufficient condition for time consistency of the Ramsey equilibrium holds even if the government is constrained to issue no nominal or real debt, so that currency is its only liability. In addition, the proposition also holds if the government is constrained to issue nominal debt only or nominal debt and real debt of one period maturity. The maturity structure of nominal debt is irrelevant for time consistency or for any other feature of the Ramsey equilibrium.

Figure 7 displays an example of currency and asset holdings for $t \geq 1$ that guarantee that the time 0 Ramsey equilibrium in time consistent for time 1 under $\eta_1 \geq \bar{\eta}_1$. All parameters are as in figure 5 and (3.6) is imposed as a weak inequality. In addition, $M_{1,0} = M_{2,0}$ and $b_{i(-1),t} = B_{i(-1),t} = 0$ for both i and all t is assumed. Here, I consider the class of currency and asset holdings for which the value of the multipliers of the implementability constraint is the same in the time 0 and the time 1 Ramsey equilibrium. This class is identified by the condition: $b_{i0,1} = -B_{i0,1}/P_1$ for $i = 1, 2$[12]. Asset positions are asymmetric. Type 1 households do not hold any nominal debt while type 2 households have negative nominal asset holdings. Type 2 households hold real debt at time 1. Households are always real debtors to the government in this example. Interestingly, the present discounted value of the real claims the government has on each type of household is equal to $|\frac{M_{i,1}-M_{i2}}{P_1}|$.

5. Empirical Relevance

The distribution of nominal wealth and the distribution of political power among classes of agents with different exposure to the effects of inflation played a crucial role in shaping monetary policy decisions in a number of historical episodes of large inflations and deflations.

Johnson (1970) provides a detailed description of the behavior of inflation in England in the aftermath of the Glorious Revolution:

> "When the Bank of England received its charter [1694] ... its directors cultivated all possible contact with parlamentarians, on whom they relied for periodic renewal of the charter."
>
> "Fierce dispute broke out as to what ... should be the remedy. To return to the good old standard would mean ... bankruptcy of many in trade and enrichment of old creditors. Devaluation would ... protect and stabilize domestic trade though initially hit the foreign trader. ... With landed property predominant in government the issue was never

[12] The sufficient conditions for time consistency do not uniquely determine $\{\lambda_i', b_{i0,1}, B_{i0,1}\}$ for $i = 1, 2$. Any triple that satisfied the conditions in the proof of proposition 4.1 and:

$$\lambda_i' (b_{i0,1} + B_{i0,1}) \leq 0,$$

will be valid.

> in doubt. The recoinage of 1897-1698 returned to Elisabeth's silver standard."
>
> The same political forces played a role in successive episodes of deflation in England, for example in 1815, as Johnson (1970) reports:
>
> "The unitary in monetary interest in the gold standard... included ...the owners of Consols sold to finance the war with Napoleon at a time of skyhigh prices and interest rates. ...In returning to gold, Lord Liverpool thus handed a large bonus to the landed gentry and to a new monied middle class".

Redistributional concerns can also account for the large monetization which occurred in France after the Revolution in 1789. White (1896) reports the following:

> "Mirabeu..showed that he was fully aware of the dangers of inflation, but he yielded to the pressure... partly because he thought it important to sell government lands rapidly to the people, and so develop speedily a large class of landholders, pledged to stand by the government who gave them their titles."
>
> "This outgrowth [in money] was the creation of a great debtor class in the nation, directly interested in the depreciation of the currency in which their debts were to be payed. The nucleus of this debtor class was formed by those who had purchased the Church lands from the Government"

Sargent and Velde (1995) document that the downpayments required to purchase church lands were in the range $12 - 30\%$. The rest of the payment was arranged through promissory notes repayed annually over a period ranging between 10 and 12 years at 5% interest.

Hamilton (1788) also highlights the importance of redistributional concerns for the credibility of government debt policy:

> "There are even dissimilar views ... as to the general principle of discharging the public debt. Some of them, either less impressed with the importance of national credit, or because they have little, if any, immediate interest in the question, feel an indifference, if not a repugnance, to the payment of the domestic debt at any rate. ... Others of them, a numerous body of whose citizens are creditors to the public beyond proportion ... in the total amount of the national debt, would be strenuous for some equitable and effective provision."

Based on this view, Hamilton (1795) argued in favor of the Federal assumption of the states' war debt. Debt assumption would provide powerful government creditors with a strong incentive to support the establishment of a Federal tax legislation, thus decreasing the risk of default or monetization.

Faust (1996) documents that the Federal Reserve Bank's structure is a response to public conflict over inflation's distributional consequences. Demands for debt relief through surprise inflation animated the US political debate from the Revolutionary War, through the *free silver* debate (1870-80's), up until the FED's founding in 1935. The intent behind the Fed's internal power structure is to balance voting power of the financial, agricultural, industrial and commercial interests in the US. The view underlying the distribution of voting rights was, in the words of J. Laurence Laughlin, a monetary economist writing in 1933, that:

> "Politicians find it easy to appeal to the underlying prejudice in favor of inflation in order to ... lift the burden of debt."

The 1935 debates over how to devide FOMC voting power between politically appointed governors and federal reserve president reflect this clearly. Steagall (Congretional Record, 1935, p13706) summarizes it as follows:

> "Under the bill ... the board will stand 5 to 7 giving the people of the country, as contradistinguished by private banking interests, control by a vote of 7 to 5 instead of by a vote of 3 to 2 [as proposed by the Senate]"

Caselli (1997) finds that, for a sample of highly indebted OECD countries in the time period 1970-1990, the interest cost of public debt, which is presumably inversely related to the perceived credibility of the government, depends positively on asymmetries in the distribution of taxes and negatively on the degree of identification of the government with a specific constituency.

6. Concluding Remarks

I describe a monetary economy in which households have different labor productivity, which implies that they are heterogeneous in transaction patterns and asset holdings. Heterogeneity implies that monetary policy has distributional effects and the time consistency of the Ramsey equilibrium depends on the balance between distribution and efficiency. Therefore, heterogeneity breaks the link between high inflation and time consistency. First, due to the distributional impact of expected inflation surprisingly high rates of inflation may be optimal even with commitment. Therefore, credibility of government policy does not imply low inflation. On the other hand, due to the distributional costs of unanticipated inflation, the optimal inflation rate under discretion is not necessarily higher than in the Ramsey equilibrium. Interestingly, if the Friedman rule is an outcome of the Ramsey equilibrium then optimal policy under commitment can always be made time consistent. In addition, economies in which nominal government debt is mostly held by the economic group having more political power are less subject to high inflation, independently of the level of nominal aggregate debt. If the Ramsey equilibrium is not time consistent, then the "bias" towards high inflation and large deficits due to lack of commitment is larger than in a representative agent economy, since in this case both efficiency and distributional objectives reinforce the government's incentive to deviate.

These findings contrast with the results for a representative agent economy, where -as shown in Lucas and Stokey (1983)- it is never possible to guarantee time consistency in a monetary economy where outstanding government debt is positive and denominated in nominal terms. Alvarez, Kehoe and Neumeyer (2001) describe a monetary economy with a representative agent in which the Ramsey equilibrium is time consistent if and only if the Friedman rule is optimal. Real and nominal debt of all maturities is required to establish this result. With heterogeneity, the proposition holds even if the government is restricted to issue nominal debt of one period maturity.

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7. Appendix

7.1. A: Characterization of Private Sector Equilibria

Assume that an allocation $\{c_{i1t}, c_{i2t}, n_{it}, z_{it},\ M_{it+1}, b_{it,t+s}, B_{it,t+s}\}_{i=1,2,t\geq 0,s>0}$, with $n_{it} > 0$ for $i = 1, 2$ and $t \geq 0$, and a price system $\{P_t,\ W_t,\ Q_{t,t+s},\ q_{t,t+s},\ \pi_t(j)\}_{t\geq 0, j\in[0,1]}$ constitute a private sector equilibrium for a given policy $\{\bar{g}_t, \tau_t, M_{t+1}, B_{t+1}\}_{t\geq 0}$. Then, conditions (2.1) and (2.2) derive from optimality of firm behavior, conditions (2.9) and (2.10) from clearing in the goods and assets markets. The other conditions follow from household optimization.

The Lagrangian for the household problem is given by:

$$
\begin{aligned}
L \;=\; & \sum_{t=0}^{\infty} \beta^t \Big\{ u^i(c_{it}, n_{it}) - \mu_{it}\left(P_t c_{i1t}(1-z_{it}) - M_{it}\right) \\
& -\lambda_{it} \Bigg[M_{it+1} + \sum_{s>0} \left(Q_{t,t+s} B_{it,t+s} + q_{t,t+s} P_t b_{it,t+s}\right) - M_{it} - \sum_{\hat{t}=0}^{t-1} \Big(B_{i\hat{t},t} + P_t b_{i\hat{t},t} \Big) \\
& -W_t(1-\tau_t)\xi_i n_{it} + P_t c_{i1t}(1-z_{it}) + P_t c_{i2t} z_{it} + \int_0^{z_{it}} \pi_t(j)\, dj \Bigg] \Big\},
\end{aligned}
$$

where c_{it} is defined in (2.3) and μ_{it}, λ_{it} are the multipliers on the cash in advance constraint and the wealth evolution equation, respectively. Denote with u_{ijt} and u_{int} the marginal utility of good j and of labor for households $i = 1, 2$.

The necessary conditions for household optimization are given by:

$$u_{i1t} = P_t(\mu_{it} + \lambda_{it})(1-z_{it}), \tag{7.1}$$

$$\mu_{it}(P_t c_{it}(1-z_{it}) - M_{it}) = 0,\ \mu_{it} \geq 0, \tag{7.2}$$

$$u_{i2t} = P_t \lambda_{it} z_{it}, \tag{7.3}$$

$$-u_{int} = W_t(1-\tau_t)\xi_i \lambda_{it}, \tag{7.4}$$

$$P_t c_{i1t}(\mu_{it} + \lambda_{it}) - P_t c_{i2t}\lambda_{it} - q_t(z_{it})\lambda_{it} \begin{cases} < 0 \text{ for } z_{it} = \underline{z}, \\ = 0 \text{ for } z_{it} \in (\underline{z}, \bar{z}), \\ > 0 \text{ for } z_{it} = \bar{z}, \end{cases} \tag{7.5}$$

$$\lambda_{it} = \beta\left(\lambda_{it+1} + \mu_{it+1}\right), \tag{7.6}$$

$$\lambda_{it} Q_{t,t+1} = \beta \lambda_{it+1}, \tag{7.7}$$

$$\lim_{T\to\infty} \beta^T \lambda_{iT} M_{iT} = 0,\ \lim_{T\to\infty} \beta^T \lambda_{iT} B_{it,T} = 0, \tag{7.8}$$

as well as (2.4) and (2.5). To see that (7.8) is a necessary condition for household optimization, suppose it does not hold and

$$\lim_{T\to\infty}\beta^T\lambda_{iT}M_{iT}>0,\ \lim_{T\to\infty}\beta^T\lambda_{iT}B_{it,T}>0.$$

(The strictly smaller case is rule out by (2.6).) Then, it is possible to construct a consumption sequence such that the budget constraint is satisfied in each period and utility for each type of household is greater, violating optimality.

Combining (7.1)-(7.3) yields (2.14), while (7.3) and (7.4) determine (7.11). The expression in (2.11) follows from (7.4), (7.7) and (2.1), while (2.16) follows from (7.1)-(7.3) at $t=0$.

To derive (2.17), multiply (2.5) by λ_{it} and apply (7.2) and (7.6). Use (7.1), (7.3)-(7.5), multiply by β^t and sum over t from 0 to T. Let T go to infinity and apply (7.8). This yields:

$$\sum_{t=0}^{\infty}\beta^t\left(u_{i1t}c_{i1t}+u_{i2t}\left(c_{i2t}+\frac{C\left(z_{it}\right)}{z_{i,t}}-\frac{B_{i(-1),t}}{P_tz_{it}}-\frac{b_{i(-1),t}}{z_{it}}\right)+u_{int}n_{it}\right)=\frac{u_{i10}}{1-z_{i0}}\frac{M_{i0}}{P_0}. \tag{7.9}$$

From (7.6)-(7.7):

$$P_t=\beta^t\frac{\hat{u}_{i2t}}{\hat{u}_{i20}}P_0\prod_{j=1}^{t}R_j \text{ for } t>1,$$

with $\prod_{j=1}^{1}R_j\equiv R_1$, $\prod_{j=1}^{0}R_j\equiv 1$. Substitute into (7.9), to obtain (2.17).

Now assume that an allocation $\{c_{i1t},c_{i2t},n_{it},z_{it},M_{it+1},B_{it+1}\}_{i=1,2,t\geq 0}$, with $n_{it}>0$ for $i=1,2$ and $t\geq 0$, and a price system $\{P_t,W_t,Q_t,q_t\left(j\right)\}_{t\geq 0,j\in[0,1]}$ satisfy (2.1)-(2.17) and (2.9) for a given policy $\{\bar{g}_t,\tau_t,M_{t+1},B_{t+1}\}_{t\geq 0}$ for which (2.7) holds. Then, by (2.1) and (2.12) industrial and credit services firms optimize.

To see that household optimization conditions are satisfied consider an alternative candidate plan $\{c'_{i1t},c'_{i2t},n'_{it},z'_{it}\}_{i=1,2,t\geq 0}$ which satisfies the intertemporal budget constraint for the price system $\{P_t,W_t,Q_t,q_t\left(j\right)\}_{t\geq 0,j\in[0,1]}$. This implies that:

$$\Delta\equiv\lim_{T\to\infty}\beta^t\left\{u_{i1t}\left(c_{i1t}-c'_{i1t}\right)+u_{i2t}\left(c_{i2t}+\frac{C\left(z_{it}\right)}{z_{it}}-c'_{i2t}-\frac{C\left(z'_{it}\right)}{z'_{it}}\right)-\gamma\left(n_{it}-n'_{it}\right)\right\}\geq 0,$$

using (2.11) and the fact that $\{c_{i1t},c_{i2t},n_{it},z_{it}\}_{i=1,2,t\geq 0}$ satisfies (2.14)-(2.17) and that the intertemporal budget constraint holds as a weak inequality using (2.6) and (2.5) for the price system $\{P_t,W_t,Q_t,q_t\left(j\right)\}_{t\geq 0,j\in[0,1]}$. By concavity of u^i :

$$D\equiv\lim_{T\to\infty}\sum_{t=0}^{T}\beta^t\left(u^i\left(c_{it},n_{it}\right)-u^i\left(c'_{it},n'_{it}\right)\right)\geq\Delta,$$

where c'_{it} is defined by (2.3). This establishes the result since (2.10) and (2.9) guarantee market clearing.

7.2. B: Solving the Ramsey problem

The Lagrangian for the Ramsey problem can be written as:

$$\begin{aligned}
\Lambda &= \sum_{t=0}^{\infty}\beta^t\sum_i\left\{\eta_i U\left(c_{it},n_{it}\right)+\lambda_i\left(u_{i1t}c_{i1t}+u_{i2t}\hat{c}_{i2t}+u_{int}n_{it}\right)\right\}\\
&-\sum_{t=1}^{\infty}\beta^t\left[\mu_t\left(1-R_t\right)+\sum_i\mu_{it}\left(\frac{\hat{u}^i_{1t}}{\hat{u}^i_{2t}}-R_t\right)\right]-\sum_{t=0}^{\infty}\beta^t\zeta_t\left(\frac{u^1_{nt}}{\hat{u}^1_{2t}}-\frac{u^2_{nt}}{\hat{u}^2_{2t}}\right)\\
&-\sum_i\lambda_i\left(\hat{u}_{i1,0}\frac{M_{i0}}{P_0}+\sum_{t=0}^{\infty}\beta^t\hat{u}_{i2t}b_{i(-1),t}+\hat{u}_{i20}\sum_{t=0}^{\infty}\frac{B_{i(-1),t}}{P_0}\prod_{j=1}^{t}R_j\right)\\
&-\sum_i\mu_{i0}\left(c^i_{10}\left(1-z_{i,0}\right)-\frac{M^i_0}{P_0}\right),
\end{aligned}$$

with choice variables $c_{i1,t},\ c_{i2,t},\ z_{i,t},\ n_{it},\ R_t,\ P_0$.

The first order conditions for the Ramsey problem at time 0 are as follows.

For $t>0$ and $i=1,2:$

$$0=\eta_i u_{i1t}+\lambda_i\left(u_{i1t}+u_{i11t}c_{i1t}+u_{i12,t}c_{i2,t}\right)-\frac{\mu_{it}}{\hat{u}_{i2t}}\hat{u}_{i11t}-\nu_i\left(1-z_{i,t}\right)\omega_t, \quad (7.10)$$

$$\begin{aligned}
0 &= \eta_i u_{i2t}+\lambda_i\left(\mathrm{u}_{i12,t}\,c_{i1,t}+u_{i2,t}+u_{i22t}\hat{c}_{i2t}\right)-\nu_i z_{i,t}\omega_t \qquad (7.11)\\
&+\frac{\mu_{it}}{\hat{u}_{i2t}}\frac{\hat{u}_{i1t}}{\hat{u}_{i2t}}\hat{u}_{i22t}+\frac{\zeta_{it}}{\hat{u}_{i2t}}\frac{u_{int}}{\hat{u}_{i2t}}\hat{u}_{i22t}\\
&-\lambda_i\hat{u}_{i22t}b_{i(-1),t},
\end{aligned}$$

where $\zeta_{it}=(-1)^{i+1}\zeta_t$, for $i=1,2$,

$$\mu_t\left(1-R_t\right)=0,\ \mu_t\geq 0,\ r_t\geq 1, \quad (7.12)$$

$$\mu_{it}\left(\frac{\hat{u}_{i1t}}{\hat{u}_{i2t}}-R_t\right)=0,$$

$$\mu_t+\sum_i\left(\mu_{it}-\frac{\lambda_i\hat{u}_{i2,0}}{R_t}\sum_{s=t+1}^{\infty}\frac{B_{i(-1),s}}{P_0}\prod_{j=1}^{s}R_j\right)=0. \quad (7.13)$$

For $t \geq 0$:

$$0 = \eta_i u_{int} + \lambda_i \left(u_{int} + u_{innt} n_{it} \right) - \frac{\zeta_{it}}{\hat{u}_{i2t}} u_{innt} + \nu_i \omega_t, \tag{7.14}$$

$$\zeta_t \left(\frac{u_{1nt}}{\hat{u}_{12t}} - \frac{u_{2nt}}{\hat{u}_{22t}} \right) = 0,\ \frac{u_{1nt}}{\hat{u}_{12,t}} \leq \frac{u_{2nt}}{\hat{u}_{22,t}},\ \zeta_t \geq 0. \tag{7.15}$$

For $t = 0$:

$$0 = \eta_i u_{i1,0} + \lambda_i \left(u_{i1,0} + u_{i11,0} c_{i1,0} + u_{i12,0} c_{i2,0} \right) - \mu_{i0} \left(1 - z_{i,0} \right) - \lambda_i \hat{u}_{i11,0} M_{i,0} - \nu_i \left(1 - z_{i,0} \right) \omega_0, \tag{7.16}$$

$$\begin{aligned} 0 \;=\; & \eta_i u_{i20} + \lambda_i \left(u_{i12,0} c_{i1,0} + u_{i20} + u_{i220} c_{i20} \right) - \nu_i z_{i,0} \omega_0 \\ & + \frac{\zeta_{i0}}{\hat{u}_{i20}} \frac{u_{in0}}{\hat{u}_{i20}} \hat{u}_{i220} \\ & - \lambda_i \hat{u}_{i22,0} \left(b_{i(-1),0} + \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{7.17}$$

$$\sum_{i=1,2} \left(-\lambda_i \hat{u}_{i1,0} M_{i0} - \lambda_i \hat{u}_{i20} \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j + \mu_{i0} M_{i0} \right) \left(\frac{-1}{P_0^2} \right) = 0, \tag{7.18}$$

$$\mu_{i0} \left(c_{i10} \left(1 - z_{i,0} \right) - \frac{M_{i0}}{P_0} \right) = 0,\ \mu_{i0} \geq 0,\ c_{i10} \left(1 - z_{i,0} \right) \leq \frac{M_{i0}}{P_0}.$$

Proof of Proposition 3.2

Combining (7.10) and (7.11) yields:

$$R = \max \left\{ 1, \frac{\eta_i + \lambda_i + \lambda_i \frac{(\hat{u}_{i21} c_{i1} + \hat{u}_{i22} \hat{c}_{i2})}{\hat{u}_{i2}} + \zeta_i \frac{u_{in}}{(\hat{u}_{i2})^2} \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} + \mu_i R \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} - \lambda_i \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} b_i}{\eta_i + \lambda_i + \lambda_i \frac{(\hat{u}_{i12} \hat{c}_{i2} + \hat{u}_{i11} c_{i1})}{\hat{u}_{i1}} - \mu_i R \frac{\hat{u}_{i11}}{\hat{u}_{i1}(1 - z_i)}} \right\} \tag{7.19}$$

If (3.6) is not imposed, the first order conditions for the corresponding relaxed Ramsey problem are the same as for the original with $\zeta_t \equiv 0$ for $t \geq 0$. Consider the doubly relaxed Ramsey problem where constraints (3.1) and (3.2) are not imposed. By homotheticity of h^i and the fact that $C\left(\underline{z}\right) = 0$ at $\hat{u}_{i1} = \hat{u}_{i2}$ by (2.15):

$$\frac{u_{i11} c_{i1} + u_{i12} c_{i2}}{u_{i1}} = \frac{u_{i12} c_{i1} + u_{i22} c_{i2}}{u_{i2}},$$

or equivalently:

$$\frac{\hat{u}_{i11}c_{i1}+\hat{u}_{i12}\hat{c}_{i2}}{\hat{u}_{i1}}=\frac{\hat{u}_{i12}c_{i1}+\hat{u}_{i22}\hat{c}_{i2}}{\hat{u}_{i2}}.$$

Hence, $b_i = 0$ for both i and (7.19) imply $R = 1$. The solution to the doubly relaxed Ramsey problem satisfies the Ramsey problem without constraint (3.6). Therefore, $R = 1$ is a necessary condition for the solution to that problem. QED

Proof of Proposition 3.3

To prove necessity, suppose to the contrary that $R_t > 1$ and $\eta_1 \geq \bar{\eta}_1$. Then, by (7.10) and (7.11) and (7.19):

$$+\zeta_i\frac{u_{in}}{(\hat{u}_{i2})^2}\frac{\hat{u}_{i22}}{\hat{u}_{i2}z_i}+\mu_iR\frac{\hat{u}_{i22}}{\hat{u}_{i2}z_i}+\frac{\hat{u}_{i22}}{\hat{u}_{i2}z_i}C\left(z_i\right)\geq-\mu_iR\frac{\hat{u}_{i11}}{\hat{u}_{i1}\left(1-z_i\right)}+\frac{\hat{u}_{i12}}{\hat{u}_{i1}z_i}C\left(z_i\right),$$

for $i = 2$. By definition of $\bar{\eta}_i$ and (7.11), $\zeta_t > 0$ for $\eta_2 > \bar{\eta}_2$ and $\zeta_t = 0$ for $\eta_1 \geq \bar{\eta}_1$, so that:

$$\mu_iR\left(\frac{\hat{u}_{i22}}{\hat{u}_{i2}z_i}+\frac{\hat{u}_{i11}}{\hat{u}_{i1}\left(1-z_i\right)}\right)\geq\frac{C\left(z_i\right)}{z_i}\left(\frac{\hat{u}_{i12}}{\hat{u}_{i1}}-\frac{\hat{u}_{i22}}{\hat{u}_{i2}}\right).$$

This implies $\mu_i \leq 0$ for both i and $\mu_2 < 0$ by (3.7). Then, (7.13) implies $\mu > 0$, which contradicts $R_t > 1$ by (7.12).

To prove sufficiency, assume $\eta_2 > \bar{\eta}_2$ and $R_t = 1$. Then, by (7.19) and C$(z_i) =$ 0 for $R_t = 1$:

$$+\zeta_i\frac{u_{in}}{(\hat{u}_{i2})}\frac{\hat{u}_{i22}}{\hat{u}_{i2}z_i}=-\mu_iR\left(\frac{\hat{u}_{i22}}{\hat{u}_{i2}z_i}+\frac{\hat{u}_{i11}}{\hat{u}_{i1}\left(1-z_i\right)}\right),$$

for $i = 1, 2$. This simplifies to:

$$+\zeta_i\frac{u_{in}}{\hat{u}_{i2}}=-\mu_i\left(1+\frac{u_{i11}}{u_{i22}}\right).$$

Since at $R = 1$, $\frac{u_{i11}}{u_{i22}}$ is the same for both i, dividing through by $\left(1+\frac{u_{i11}}{u_{i22}}\right)$, summing the above equality across i and using (7.15), implies:

$$\sum_i\mu_i=0.$$

By (7.13), this implies $\mu = 0$, which contradicts $R = 1$.QED

7.3. C: Sufficient Conditions for Time Consistency

Part 1: If the FR is optimal, the Ramsey equilibrium is time consistent.

If the Friedman rule is optimal, the distribution constraint is not binding so that $\zeta_t = 0$ for $t \geq 0$. In addition, when the Friedman rule is optimal, the Ramsey equilibrium allocation solves the relaxed problem where the constraints corresponding to multipliers μ_t, μ_{it} for $t > 0$ are dropped. Therefore, the first order conditions for the time 0 Ramsey problem at $t > 0$ can be written as:

$$0 = \eta_i u_{i1t} + \lambda_i \left(u_{i1t} + u_{i11t} c_{i1t} + u_{i21t} \hat{c}_{i2t} \right) - \nu_i \omega_t \left(1 - z_{i,t} \right),$$

$$\begin{aligned} 0 &= \eta_i u_{i2t} + \lambda_i \left(u_{i12t} c_{i1t} + \mathrm{u}_{i2t} + u_{i22t} \hat{c}_{i2t} \right) - \nu_i \omega_t z_{i,t} \\ &\quad - \lambda_i \hat{u}_{i22t} b_{i(-1),t}, \end{aligned}$$

$$0 = \eta_i u_{int} + \lambda_i \left(u_{int} + u_{innt} n_{it} \right) + \nu_i \xi_i \omega_t,$$

plus (2.15) and (3.10) for all t and $i = 1, 2$.

If the Friedman rule is optimal and $g_t = g$, the Ramsey problem at time 0 has a constant continuation allocation at $t = 1$, as long as (3.10) is satisfied, even if $b_{i(-1),t}$ varies with t. For simplicity assume $b_{i(-1),t} = b_i$ for all t.

The continuation allocation of the time 0 Ramsey equilibrium will also satisfy the first order conditions for $t > 1$ of the time 1 Ramsey equilibrium at the Friedman rule, since optimality of the Friedman rule does not depend on the value of the multipliers in the implementability constraints, as shown in the proof of proposition 3.3. Denote with a prime the multipliers associated with the time 1 Ramsey equilibrium. Then, $\mu'_{it} = 0$ for $t > 1$, $\zeta'_t = 0$ and $z_{i,t} = \underline{z}$ for $t > 1$.

The first order conditions for $t = 1$ in the time 1 Ramsey problem are given by:

$$0 = \eta_i u_{i1,1} + \lambda'_i \left(u_{i1,1} + u_{i11,1} c_{i1,1} + u_{i2,1} \hat{c}_{i2,1} \right) - \mu'_{i1} \left(1 - z_{i,1} \right) - \lambda'_i \hat{u}_{i11,1} \frac{M_{i,1}}{P_1} - \nu_i \omega'_1 \left(1 - z_{i,1} \right),$$

$$\begin{aligned} 0 &= \eta_i u_{i2,1} + \lambda'_i \left(u_{i1,1} c_{i1,1} + \mathrm{u}_{i21} + u_{i22,1} \hat{c}_{i2,1} \right) - \nu_i \omega'_1 z_{i,1} \\ &\quad + \frac{\zeta'_{i1}}{\hat{u}_{i2,1}} \frac{u_{in1}}{\hat{u}_{i2,1}} \hat{u}_{i22,1} \\ &\quad - \lambda'_i \hat{u}_{i22,1} \left(b_{i0,1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{7.20}$$

$$\sum_{i=1,2}\left(-\lambda_i'\hat{u}_{i1,1}\frac{M_{i,1}}{P_1}-\lambda_i'\hat{u}_{i2,1}\left(\sum_{t=1}^{\infty}\frac{B_{i0,t}}{P_1}\prod_{j=1}^{t}R_j\right)+\mu_{i1}'\frac{M_{i,1}}{P_1}\right)\left(\frac{-1}{P_1}\right)=0, \quad (7.21)$$

$$\mu_{i1}'\left(c_{i1,1}\left(1-z_{i,1}\right)-\frac{M_{i,1}}{P_1}\right)=0,\ \mu_{i1}'\geq 0,\ c_{i1,1}\left(1-z_{i,1}\right)\leq\frac{M_{i1}}{P_1}, \quad (7.22)$$

$$P_1=P_0\beta\frac{\hat{u}_{i1,1}}{\hat{u}_{i2,0}},$$

plus (7.14) rewritten for $t\geq 1$, and (7.10), (7.11), (2.15) with primed multipliers appropriately replacing the ones for the time 0 Ramsey problem. $z_{i,1}$ is chosen ahead to government re-optimization and it is taken as given.

Since the solution to the time 0 Ramsey problem is stationary and this must also be the case for the solution to the time 1 Ramsey problem, time subscripts are dropped for convenience where possible from this point on.

I need to show that it is possible to find λ_i', $B_{i1,t}$ with $t\geq 1$, $b_{i0,t}=b_i'$ and M_{i1} so that the continuation allocation for the time 0 Ramsey problem solves the time 1 Ramsey problem.

To obtain b_i' combine the first order conditions for n_i and c_{i2} for $t>1$ in the time 1 Ramsey problem:

$$\left(\eta_i+\lambda_i'\right)\left(\hat{u}_{i2}+u_{in}\right)+\lambda_i'\left(\hat{u}_{i12}c_{i1}+\hat{u}_{i22}\hat{c}_{i2}+u_{inn}n_i\right)=\lambda_i'\hat{u}_{i22}b_i',$$

where $\hat{u}_{i11}=u_{i11}/\left(1-z_i\right)$, $\hat{u}_{i12}=u_{i12}/\left(1-z_i\right)$, $\hat{u}_{i22}=u_{i22}/z_i$ and $\hat{u}_{i21}=u_{i21}/z_i$. This implies:

$$b_i'=\left(\frac{\eta_i}{\lambda_i'}+1\right)\left(\frac{\hat{u}_{i2}+u_{in}}{\hat{u}_{i22}}\right)+\left(\frac{\hat{u}_{i12}}{\hat{u}_{i22}}c_{i1}+\hat{c}_{i2}+\frac{u_{inn}}{\hat{u}_{i22}}n_i\right), \quad (7.23)$$

where b_i' automatically satisfies (3.10), since by optimality of the Friedman rule, b_i satisfies (3.10)[13].

For the continuation of the time 0 equilibrium allocation to solve the first order condition for c_{i1t} at $t=1$ in the time 1 Ramsey problem, the multipliers on the cash in advance constraint for $t=1$ must solve:

$$\begin{aligned}\mu_{i1}' &= \left(\eta_i+\lambda_i'\right)\left(\hat{u}_{i1}+u_{in}\right)+\lambda_i'\left(\hat{u}_{i11}c_{i1}+\hat{u}_{i12}\hat{c}_{i2}+u_{inn}n_i\right)-\lambda_i'\frac{\hat{u}_{i11,1}}{1-z_{i,1}}\frac{M_{i,1}}{P_1}\\ &= \left(\eta_i+\lambda_i'\right)\left(\hat{u}_{i1}+u_{in}\right)+\lambda_i'\left(\hat{u}_{i12}\hat{c}_{i2}+u_{inn}n_i\right)+\lambda_i'\frac{\hat{u}_{i11,1}}{1-z_{i,1}}\left(c_{i1,1}\left(1-z_{i,1}\right)-\frac{M_{i,1}}{P_1}\right).\end{aligned}$$

[13] If real debt is exogenously assumed to be 0, this step of the proof still holds, since the conditions for optimality of the Friedman rule do not depend on the value of λ_i' and are automatically verified if real debt is 0 in the time 0 and time 1 Ramsey equilibrium.

Then, by the complementary slackness condition on the time 1 cash in advance constraint:

$$\mu_{i1}' = \max\{0, (\eta_i + \lambda_i')(\hat{u}_{i1} + u_{in}) + \lambda_i'(\hat{u}_{i12}\hat{c}_{i2} + u_{inn}n_i)\}. \tag{7.24}$$

Homotheticity of h^i for a given z_i implies:

$$\frac{u_{i11}c_{i1} + u_{i12}c_{i2}}{u_{i1}} = \frac{u_{i12}c_{i1} + u_{i22}c_{i2}}{u_{i2}}.$$

Then, by $C(\underline{z}) = 0$ and $\hat{u}_{i1} = \hat{u}_{i2}$ at the Friedman rule:

$$\frac{\hat{u}_{i11}c_{i1} + \hat{u}_{i12}\hat{c}_{i2}}{\hat{u}_{i1}} = \frac{\hat{u}_{i12}c_{i1} + \hat{u}_{i22}\hat{c}_{i2}}{\hat{u}_{i2}}.$$

Using (7.21) and (7.24):

$$\sum_{i=1,2}\left(-\lambda_i'\hat{u}_{i2,1}\sum_{t=1}^{\infty}\frac{B_{i0,t}}{P_1} + \max\{-\lambda_i'\hat{u}_{i1,1}, \eta_i\hat{u}_{i1,1} + (\eta_i + \lambda_i')u_{in} + \lambda_i'(\hat{u}_{i12}\hat{c}_{i2} + u_{inn}n_i)\}\frac{M_{i,1}}{P_1}\right) = 0, \tag{7.25}$$

it is possible to pin down the distribution of $\sum_{t=1}^{\infty} B_{i0,t}$ as a function of λ_1', λ_2' and the distribution of currency. The distribution of currency is pinned down by money demand at the end of period 0. These values of b_i', $\sum_{t=1}^{\infty} B_{i1,t}$ and M_{i1} can be substituted into the time 1 implementability constraint to solve for λ_i'. QED

Part 2: If the Ramsey equilibrium is time consistent, the Friedman rule is optimal.

Assume by contradiction that the Ramsey equilibrium is time consistent and the Friedman rule is not optimal.

By evaluating (7.14) at time 1 in the time 0 and time 1 Ramsey equilibrium, with $n_{i,1}$ set at the value implied by the continuation of the time 0 Ramsey equilibrium, we obtain four equations in four unknowns λ_i' for $i = 1, 2$, ζ', ω'. This implies that the value of the multipliers on the implementability constraints, on the distribution constraint and on the resource constraint must be the same in the time 1 and time 0 Ramsey equilibrium:

$$\lambda_i' = \lambda_i \text{ for } i = 1, 2, \tag{7.26}$$

$$\zeta = \zeta',\ \omega = \omega'. \tag{7.27}$$

By the fact that the Ramsey equilibrium is time consistent and that the Friedman rule is not optimal:

$$\sum_i \lambda_i \hat{u}_{i2} \sum_{t=2}^{\infty} B_{i0,t} = 0. \tag{7.28}$$

If this condition did not hold, it would be possible to increase the value of the planner's objective in the time 1 Ramsey equilibrium by decreasing R and increasing P_1. This would reduce the present discounted value of nominal liabilities of the government, thus relaxing both implementability constraints, while satisfying the resource constraint and other optimality conditions. In particular, the first order conditions for R, given by the analogue of (7.13) for the time 1 Ramsey equilibrium $t > 1$:

$$\mu_t' + \sum_i \left(\mu_{i,t}' - \lambda_i \frac{\hat{u}_{i2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{i0,s}}{P_1} \prod_{j=1}^{s} R_j \right) = 0, \tag{7.29}$$

would still be satisfied, since the value of λ_i is fixed.

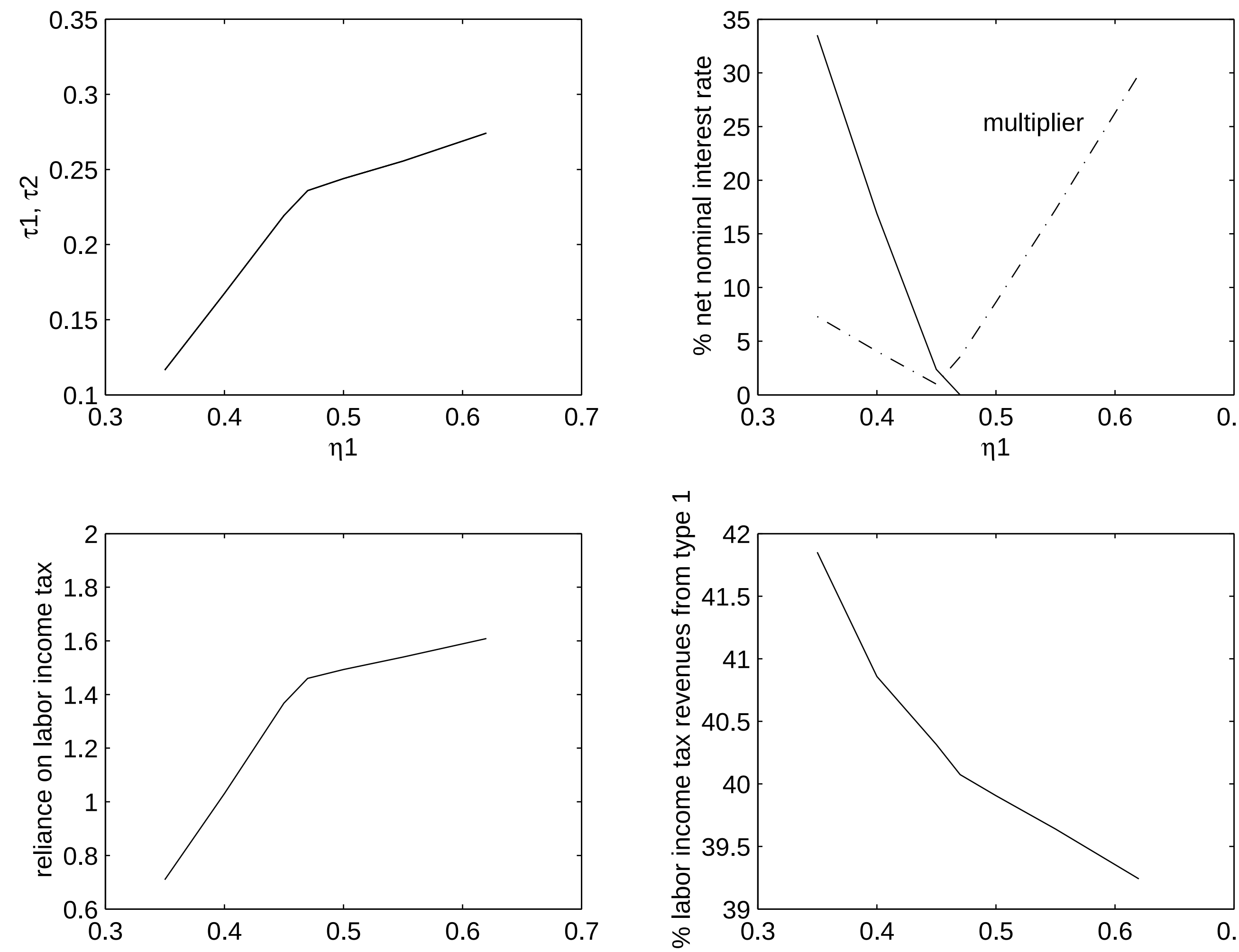
Using the first order condition for $c_{i1,1}$ in the time 0 and time 1 Ramsey equilibrium:

$$\mu_{i,1}' - \lambda_i \hat{u}_{i,11} \frac{M_{i,1}}{1 - z_{i,1}} = -\mu_{i,1} R \frac{\hat{u}_{i,11}}{(1 - z_{i,1})\, \hat{u}_{i,1}} \tag{7.30}$$

since $\mu_{i,1}' \geq 0$, $\lambda_i > 0$ and $u_{i11} < 0$, $\mu_{i,1} > 0$ for $i = 1, 2$.

But by (7.26)-(7.27) and (7.10) for $t > 1$ in the time 0 and time 1 Ramsey equilibrium, $\mu_{i,t} = \mu_{i,t}'$ for $t > 1$. Then, by (7.29) and (7.28): $\mu_t' = -\sum_i \mu_{i,t}' < 0$, which contradicts $\mu_t' \geq 0$. QED

Figure 1


τ1, τ2
0.35
0.3
0.25
0.2
0.15
0.1
0.3
0.4
0.5
0.6
0.7
η1
% net nominal interest rate
35
30
25
20
15
10
5
0
multiplier
η1
reliance on labor income tax
2
1.8
1.6
1.4
1.2
1
0.8
0.6
η1
% labor income tax revenues from type 1
42
41.5
41
40.5
40
39.5
39
η1

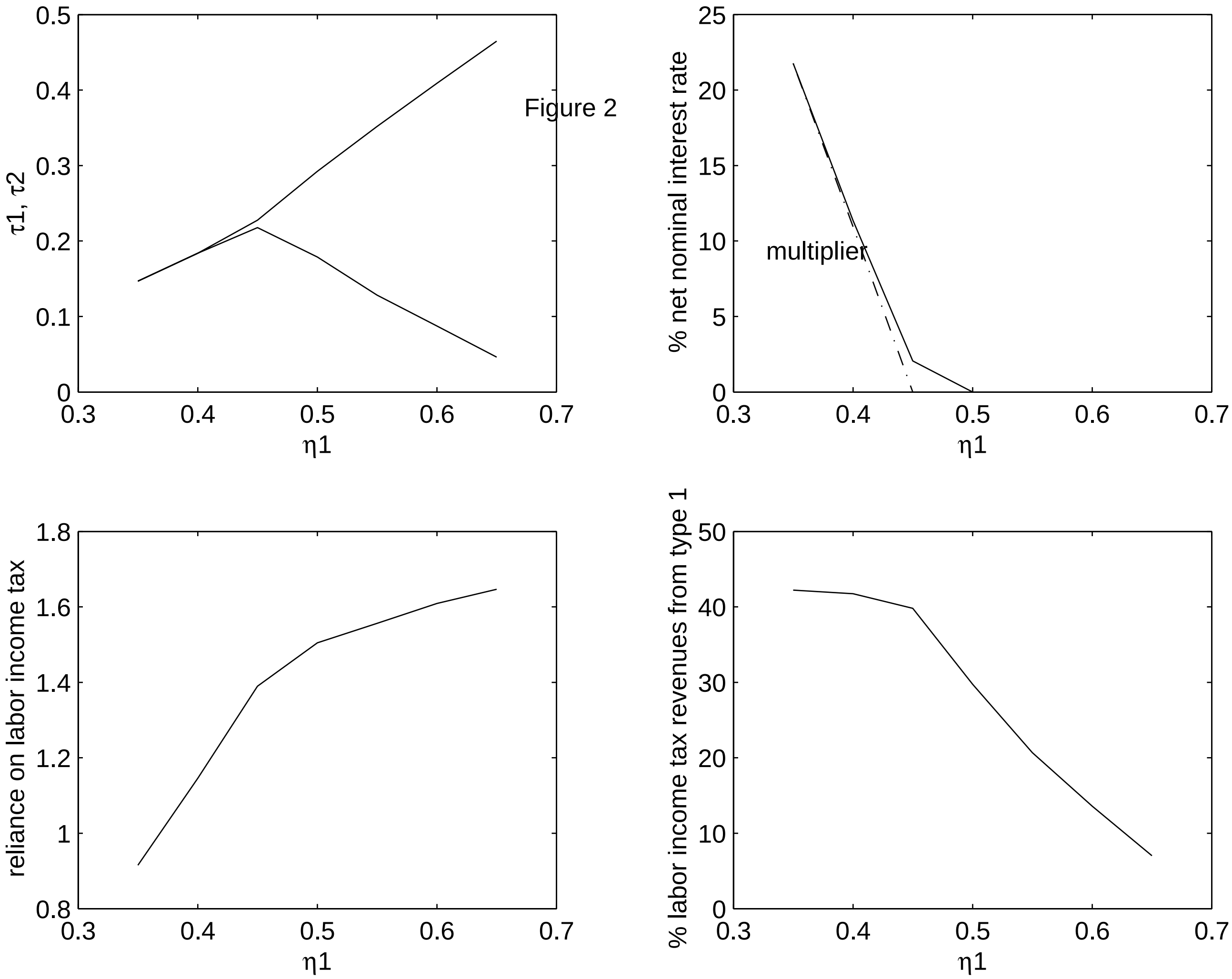
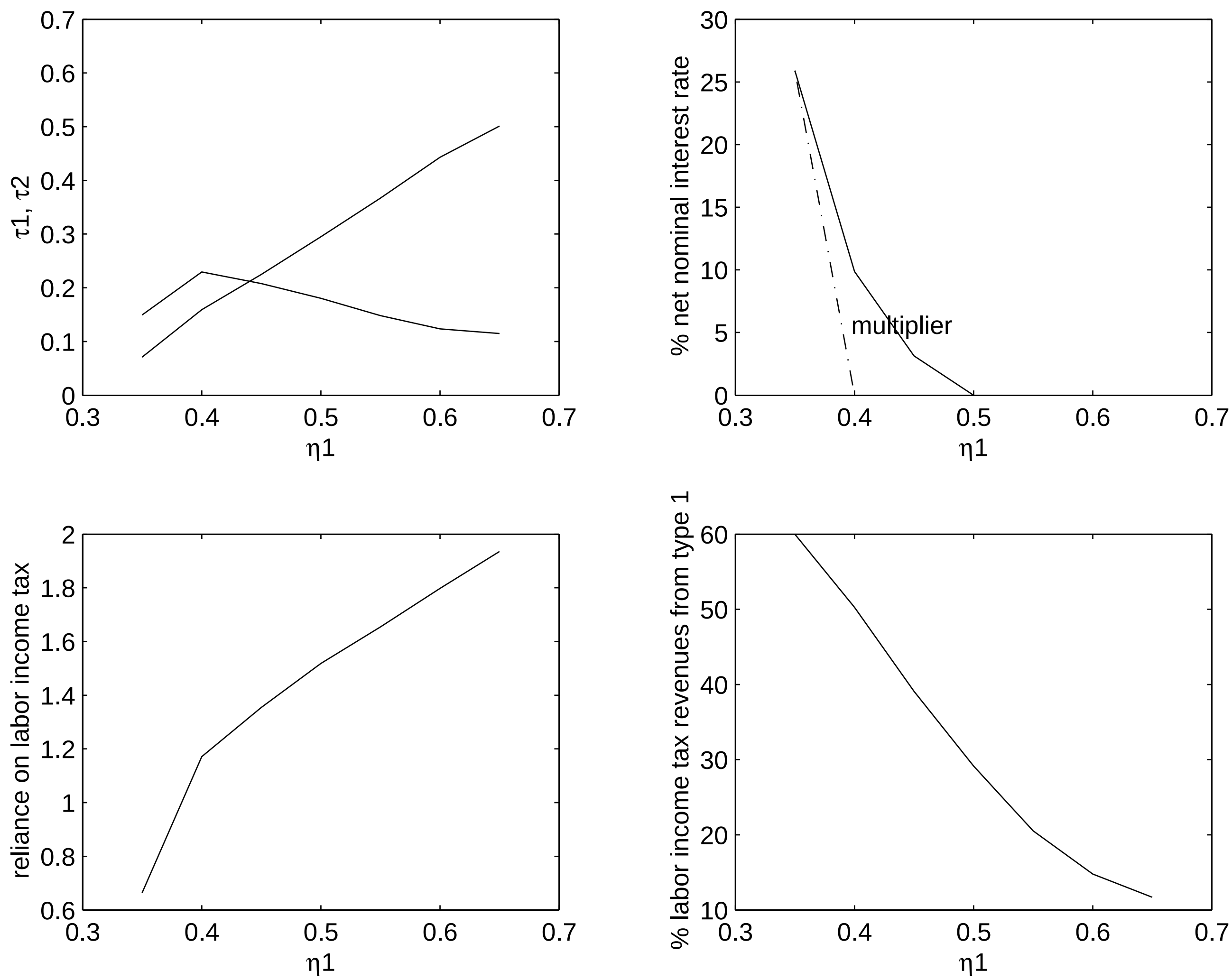

τ1, τ2
η1
% net nominal interest rate
multiplier
η1
reliance on labor income tax
η1
% labor income tax revenues from type 1
η1

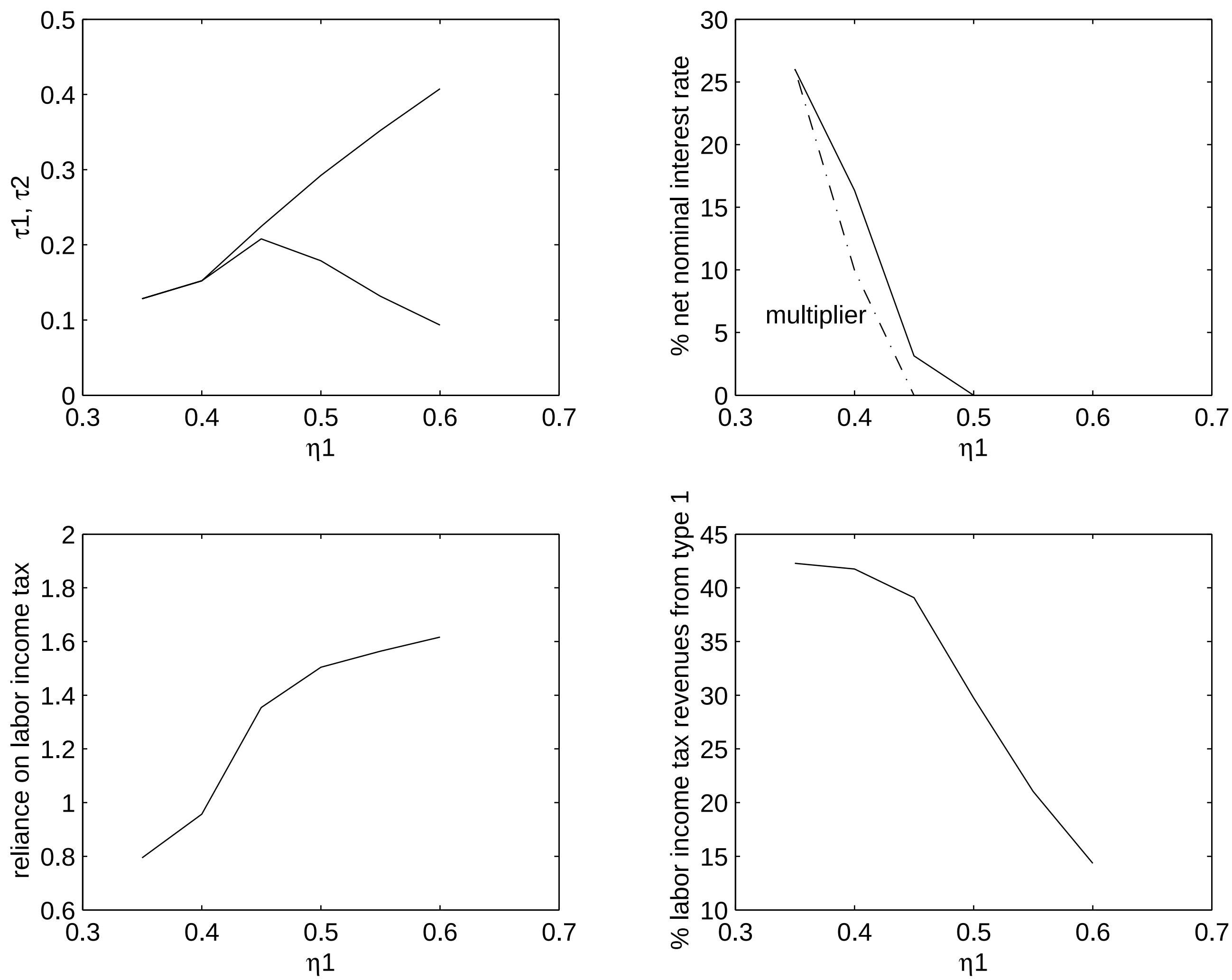
Figure 2

Figure 3



τ1, τ2
η1
% net nominal interest rate
multiplier
η1
reliance on labor income tax
η1
% labor income tax revenues from type 1
η1

Figure 4


τ1, τ2
η1
% net nominal interest rate
multiplier
η1
reliance on labor income tax
η1
% labor income tax revenues from type 1
η1

No real or nominal debt

Figure 5

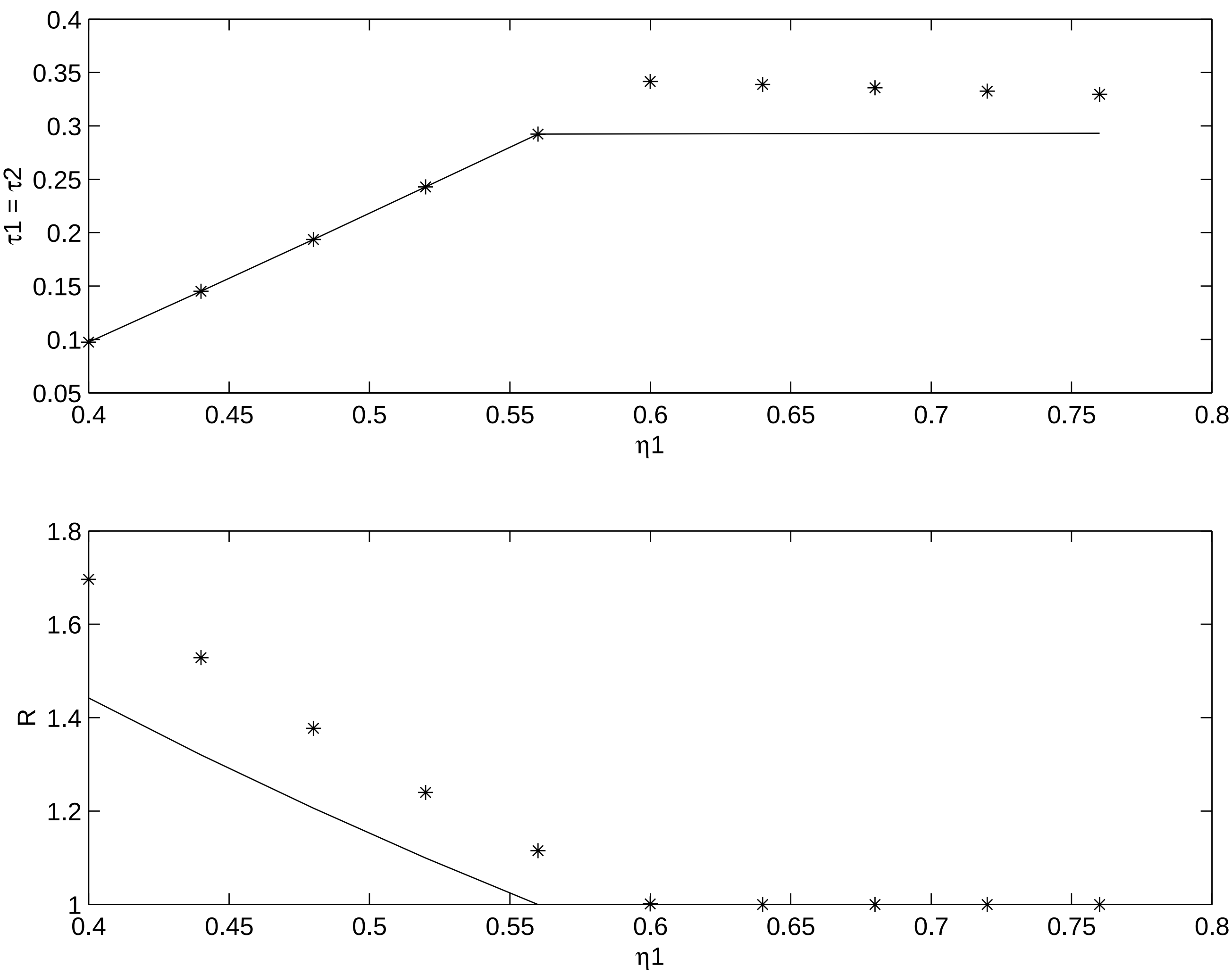


Figure 6

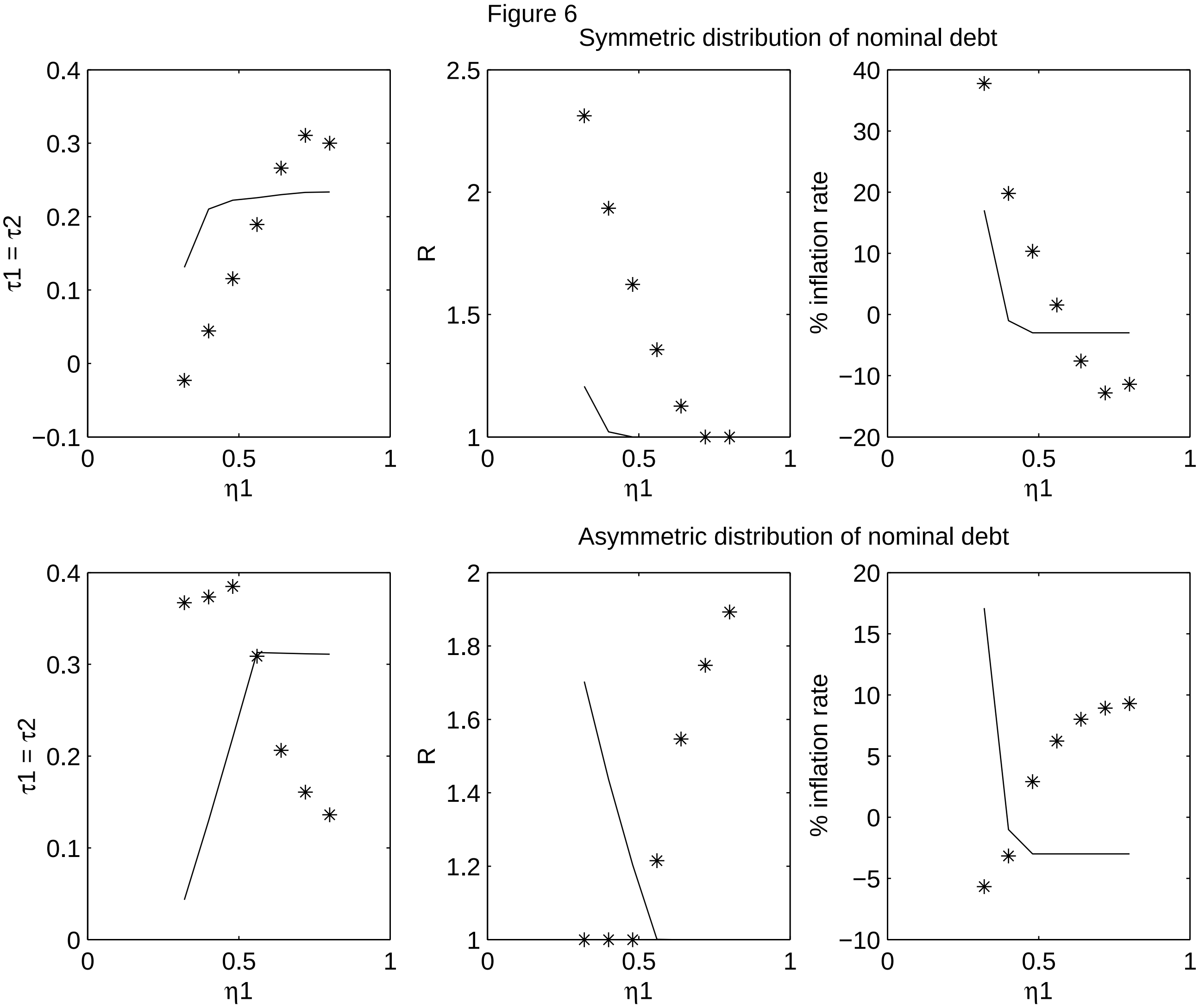
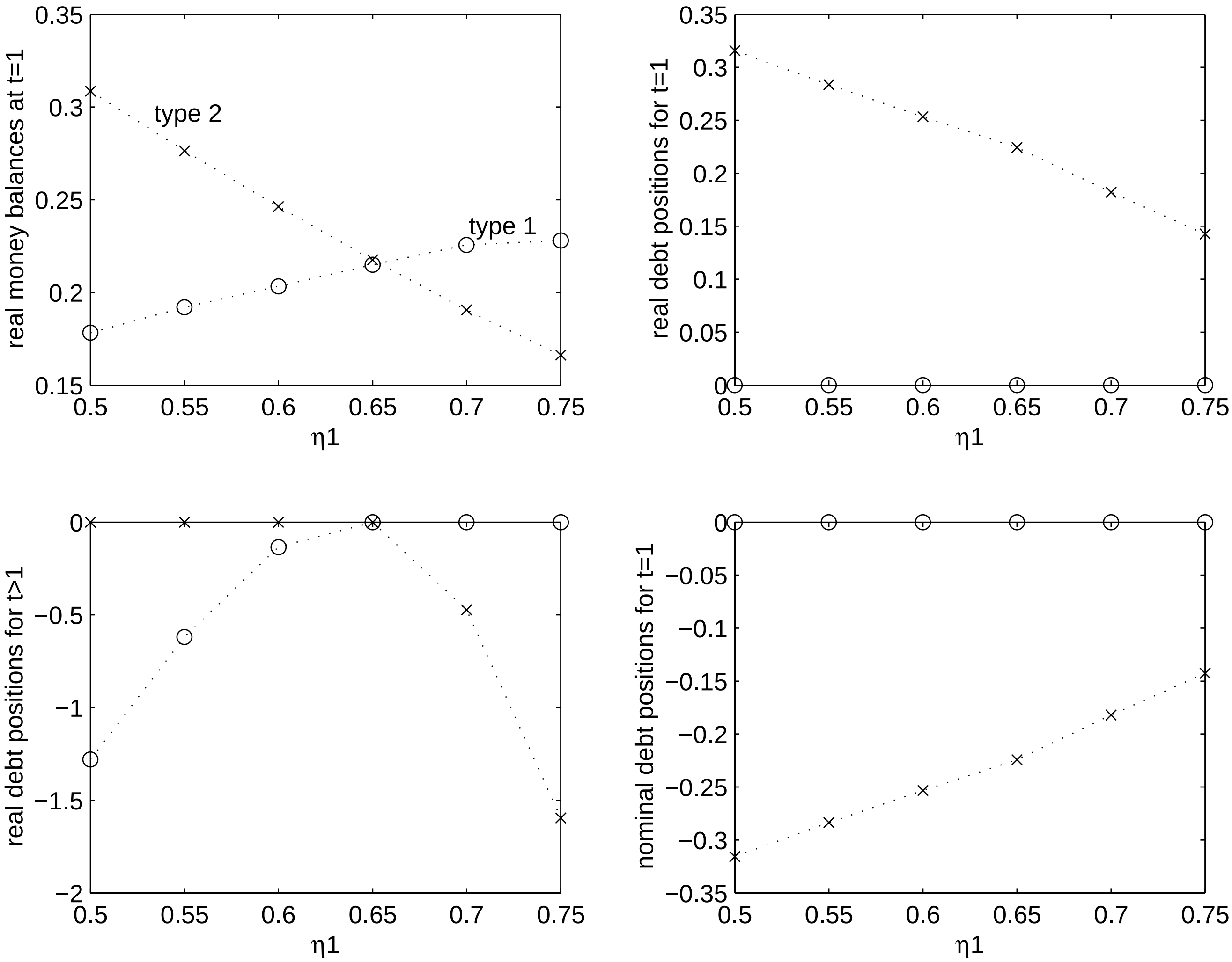

Symmetric distribution of nominal debt
Asymmetric distribution of nominal debt
τ1 = τ2
R
% inflation rate
η1
0.4
0.3
0.2
0.1
0
−0.1
2.5
2
1.5
1
40
30
20
10
−10
−20
1.8
1.6
1.4
1.2
15
5
−5
0.5

Figure 7


real money balances at t=1
type 2
type 1
η1
real debt positions for t=1
η1
real debt positions for t>1
η1
nominal debt positions for t=1
η1