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## WORKING PAPER SERIES

Heckscher-Ohlin Business Cycles

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**Working Paper n. 210**

This draft: March 2004

IGIER – Università Bocconi, Via Salasco 5, 20136 Milano –Italy  
<http://www.igier.uni-bocconi.it>

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# Heckscher-Ohlin Business Cycles<sup>1</sup>

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Final draft: 31 March 2004

<sup>1</sup>We are particularly indebted with T.J. Kehoe and an anonymous referee for useful comments and suggestions. We are also grateful to G. Benigno, M. Boldrin, L. Christiano, L. Dedola, C. Favero, F. Giavazzi, M. Ravn, S. Redding, G. Tabellini, K.M. Yi, and conference participants at Università Bocconi, the SED Meeting 2002, the Minnesota Macro Workshop, and Birkbeck College for valuable comments. All errors remain ours. The authors gratefully acknowledge financial support from Università Bocconi, the MURST, and CICYT (SEC2002-0026).

## **Abstract**

This paper introduces Heckscher-Ohlin trade features into a two-country dynamic stochastic general equilibrium model, and studies the international transmission of productivity shocks through trade in goods. This framework improves upon existing international real business cycle models in that it generates business cycle properties comparable with the empirical evidence regarding the terms of trade and the trade balance.

*Keywords:* International Trade, Heckscher-Ohlin, Business Cycles, Productivity Shocks.

*JEL codes:* E32, F11, F32, F41.

# 1 Introduction

This paper introduces comparative advantage elements into an International Real Business Cycles (IRBC) model, and studies the international transmission of country-specific productivity shocks through trade in goods. More specifically, we assume that comparative advantage is determined by cross-country differences in relative factor endowments. We focus on the Heckscher-Ohlin model’s factor price equalization (FPE) case, although we allow for cross-country differences in total factor productivity (TFP). This leads to a rather weak form of FPE, which has received empirical support from Trefler [29].<sup>1</sup> We introduce these Heckscher-Ohlin trade features into a two-country dynamic stochastic general equilibrium model, and pay special attention to the variables through which commodity trade is supposed to propagate business cycles across countries: the terms of trade, the trade balance, and the real exchange rate.

From a theoretical perspective, we view our model as a first attempt to bridge the gap between two literatures: international macroeconomics models that emphasize the cross-country transmission of business cycles via commodity trade should aim to reproduce the observed behavior of the prices and quantities through which countries interact. At the same time, they should perform this task on the basis of international trade models that explain why countries trade and their specialization patterns.

Regarding the IRBC literature, the paper is a natural follow-up on the literature triggered by Backus *et al.* [2], who study the international propagation of business cycles through trade under an exogenously given trade structure.<sup>2</sup> To assess the extent to which our model improves upon the latter approach from a quantitative point of view, we calibrate it on aggregate and sectoral OECD data, and compare its predictions with those of a complete specialization model in the spirit of Backus *et al.* [2], keeping the parameterization as equal as possible across models. Table 1 reports evidence on the behavior of the terms of trade and the trade balance for the US, and compares it with the predictions of our model and that of Backus *et al.* [2].<sup>3</sup>

A first important difference between our Heckscher-Ohlin model and the standard IRBC model concerns the correlation of the terms of trade (defined as the price of imports over the price of exports) with income. In the latter model, due to its complete

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<sup>1</sup>Kraay and Ventura [20], [21] also study the international transmission of business cycles in a Heckscher-Ohlin framework. Their work, however, is harder to relate to the standard IRBC models than ours due to their emphasis on how business cycles differ across rich and poor countries, and to the fact that they avoid capital accumulation in their model.

<sup>2</sup>In Backus *et al.* [2], countries trade due to the (Armington) assumption that production of different goods is country specific.

<sup>3</sup> $NX/GDP$  and  $ToT$  denote net trade (exports minus imports) over GDP and the terms of trade (import deflator over export deflator), respectively. For further details on data sources, see footnote 30, p. 20.

	Rel. Std. Dev.			Cor. with $GDP$			Cor. with $\frac{NX}{GDP}$		
	US	HO	CS	US	HO	CS	US	HO	CS
$\frac{NX}{GDP}$	0.63	0.28	0.05	-0.46	-0.80	-0.86	-	-	-
	-	<i>0.03</i>	<i>0.01</i>	-	<i>0.07</i>	<i>0.05</i>	-	-	-
$ToT$	1.71	0.10	0.68	-0.18	-0.62	0.35	0.10	0.18	-0.66
	-	<i>0.01</i>	<i>0.11</i>	-	<i>0.10</i>	<i>0.16</i>	-	<i>0.16</i>	<i>0.09</i>

Table 1: Summary of main results.

specialization assumption, the terms of trade are positively correlated with income for all countries. While the correlation between the terms of trade with income is positive for most OECD countries, that of the US is negative. In the Heckscher-Ohlin set-up, it is easy instead to generate correlations of different signs for different countries, since commodity prices are a function of world aggregate factor endowments. Therefore the dynamics of a country’s terms of trade is affected by both its comparative advantage and the dynamics of the world.

Assume, for example, that a large economy suffers a positive shock that raises the productivity of its production factors.<sup>4</sup> For standard business cycle reasons, a positive technology shock leads to a much larger increase in labor than in capital. This relative decrease in the amount of capital raises the world price of the capital intensive good relative to that of the labor intensive good. This reduces the terms of trade of the capital abundant country, and raises those of the labor abundant country. Based on empirical evidence from the international trade literature, in section 2 we argue that the US is capital abundant relative to the rest of the OECD once one measures factor endowments in efficiency units. In the light of our model, therefore, it is not so surprising that the correlation of the terms of trade with income is positive for most OECD countries, but negative for the US.

A second interesting result is related to the trade balance. In comparison with our Heckscher-Ohlin model, and under a similar parameterization, the model à la Backus *et al.* [2] can only reproduce long-run volumes of trade of magnitudes similar to those of the data if the Armington assumption’s home bias is extremely large. Ironically enough, this implies that the international propagation of business cycles through trade is quantitatively irrelevant in the Armington assumption model: whereas the trade-balance volatility generated by the Heckscher-Ohlin model is of the same order of magnitude as that in the data, the volatility generated by the model à la Backus *et al.* [2] is more than ten times smaller. The intuition is straightforward: in the Backus *et al.* [2] complete specialization assumption framework, the long-run volume of trade is determined by size of the “home

<sup>4</sup>Although productivity shocks are country specific, there are no Ricardian features in the model: we keep total factor productivity identical across sectors within a country.

bias” in consumption. Given that this is an exogenous feature of the model, it imposes a structural constraint on the volatility of the trade balance, too: the large “home bias” in consumption, together with the unitary elasticity of substitution among intermediate goods, implies that the import shares in income in both countries are equal to a tiny and roughly constant fraction of income, leaving a very limited role to trade as a transmission mechanism. In the Heckscher-Ohlin framework, instead, both the trade pattern and the volume of trade are endogenously determined by the dynamics of the relative factor endowments, under the assumption of identical technologies and preferences across countries, *i.e.* without assuming any “home bias”.

A common problem to the two models is the fact that, unlike in the data, both predict that the real exchange rate is necessarily less volatile than the terms of trade, provided that productivity shocks affect the freely traded intermediate goods. When we introduce productivity shocks to the nontraded final good rather than to the traded intermediate goods, the real exchange rate becomes more volatile than the terms of trade in both models. At the same time, the Heckscher-Ohlin model introduces additional constraints on the marginal utilities of leisure via factor price equalization, and predicts that the international correlation of output is higher than the international correlation of consumption. This result is, in general, hard to deliver in the IRBC literature. Finally, the Heckscher-Ohlin model has the property that temporary shocks generate permanent effects due to the fact that FPE prevents the two countries’ steady states from being uniquely determined. In a stochastic framework, this implies that all country-level simulated series are non-stationary and cross-country cointegrated.

The rest of the paper is structured as follows. In Section 2, we present a stochastic two-country model that combines Heckscher-Ohlin driven comparative advantage with the Ramsey model. In Section 3 we subject countries to stochastic productivity shocks, and study their impulse response functions and business cycle properties. We compare the predictions of the model with those of a model à la Backus *et al.* [2]. In Section 4 we introduce shocks to the nontraded sector and once again compare the performance of both models. Section 5 concludes.

## 2 The Model<sup>5</sup>

### 2.1 Households

The world consists of two countries, denoted by  $j = h, f$ . Each country is inhabited by a *continuum* of identical and infinitely lived households that can be aggregated into a representative household. The representative households' preferences over consumption and leisure flows are summarized by the following intertemporal utility function:<sup>6</sup>

$$U_{jt} = E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \frac{c_{js}^{1-\sigma} (1 - n_{js})^{\gamma(1-\sigma)}}{1 - \sigma} \right], \quad (1)$$

where  $\beta \in (0, 1)$  is the intertemporal discount factor;  $\sigma > 0$  the rate of intertemporal substitution;  $\gamma > 0$  the share of leisure in total utility;  $c_{jt}$  the per-capita consumption level in country  $j$  at date  $t$ ;  $n_{jt}$  the time share devoted to labor in country  $j$  at date  $t$ ; and  $E_t$  the expectation operator conditional on the information set available at date  $t$ . Households own both factors of production, capital  $k_{jt}$  and labor, and sell their services in competitive spot markets at prices  $r_{jt}$  and  $w_{jt}$ , respectively. Income is used to purchase a homogeneous final good in a competitive market at price  $p_{jt}$ . This final good can be consumed or invested. We assume that the final good is not traded internationally, and that neither capital nor labor is internationally mobile.<sup>7</sup>

We assume the existence of a complete set of Arrow-Debreu securities. We can then solve for the Pareto-optimal allocation, and hence for the Walrasian equilibrium, by maximizing the social welfare function  $U_t \equiv \sum_{j=h,f} \xi_j U_{jt}$ , where the  $\xi_j$ 's are strictly positive welfare weights such that  $\xi_h + \xi_f = 1$ ,<sup>8</sup> under the world-level budget constraint and the country-level accumulation equations:

$$\sum_{j=h,f} p_{jt} (c_{jt} + i_{jt}) = \sum_{j=h,f} (w_{jt} n_{jt} + r_{jt} k_{jt}), \quad (2)$$

$$k_{jt+1} = (1 - \delta) k_{jt} + \varphi \left( \frac{i_{jt}}{k_{jt}} \right) k_{jt}. \quad (3)$$

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<sup>5</sup>In the paper, we focus on the decentralized dynamic equilibrium for expositional reasons. The first order conditions of our perfectly competitive framework can be obtained as the solution to a benevolent social planner problem. A characterization of this (perhaps) more familiar framework is provided in the Appendix.

<sup>6</sup>For the sake of notational simplicity, we avoid to make variables state contingent explicitly.

<sup>7</sup>With this assumption we are only ruling out the possibility that capital and/or labor may flow across countries instantly.

<sup>8</sup>We implicitly assume that the population size is equal across countries. Following the Negishi-Mantel algorithm, the welfare weights are selected so as to make the initial steady state's cross-country net wealth transfer equal to zero.

Equation (3) describes the dynamics of capital.  $\delta \in (0, 1)$  is the exogenous depreciation rate. Following Baxter and Crucini [5], we introduce a cost of adjusting capital in equation (3), such that  $\varphi > 0$ ,  $\varphi' > 0$ , and  $\varphi'' < 0$ , and assume that it does not play any role in steady state, *i.e.*  $\varphi'(\frac{i}{k}) = 1$ . We define the elasticity of the adjustment cost near the steady state as  $\xi_\varphi \equiv -\varphi''(i/k)(i/k)$ ; the parameter  $\xi_\varphi$  is the only feature of the adjustment cost function that is relevant under our solution procedure.<sup>9</sup>

The budget constraint (2) can be rewritten as

$$p_{ht}(c_{ht} + i_{ht} + \pi_t) = y_{ht} \equiv w_{ht}n_{ht} + r_{ht}k_{ht}, \quad (4)$$

$$p_{ft}\left(c_{ft} + i_{ft} - \frac{p_{ht}}{p_{ft}}\pi_t\right) = y_{ft} \equiv w_{ft}n_{ft} + r_{ft}k_{ft}, \quad (5)$$

where  $\pi_t$  is the trade balance (net exports) in country  $h$ , expressed in units of the final good produced in the same country. The social welfare function is maximized subject to equations (3), (4), and (5), taking  $p_{jt}$ ,  $w_{jt}$ , and  $r_{jt}$  as given (these prices are determined in the static trade equilibrium). Under our assumptions, the first order conditions and the transversality conditions are necessary and sufficient for the dynamic optimization problem.

## 2.2 Firms

### 2.2.1 Final Goods

The final good is produced in each country by a continuum of competitive firms that use two intermediate goods with the following Cobb-Douglas production function:

$$Y_{jt} = x_{1jt}^\phi x_{2jt}^{1-\phi}, \quad (6)$$

where  $\phi \in (0, 1)$ ;  $Y_{jt}$  is the per-capita output level of the final good; and  $x_{ijt}$  denotes the amounts of intermediate good  $i$  used in the production of  $Y_{jt}$ .

### 2.2.2 Intermediate Goods

Intermediate goods are freely traded. We assume that the markets for intermediates are also competitive, and that firms in both countries have access to the same technologies

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<sup>9</sup>Notice that the adjustment cost applies only at the aggregate level: we assume that capital moves freely across sectors. Without adjustment costs, the actual international allocation of capital in the integrated equilibrium is undetermined: once the rate of returns are equalized across countries through trade in intermediate goods, the world capital stock becomes the only relevant state variable in the planner's dynamic problem. In the two-good model developed in Backus *et al.* [2], the adjustment cost is not necessary, since the country specificity of each intermediate good is enough to identify consumption and investment at the country level.



to produce them:

$$y_{ijt} = a_{jt} k_{ijt}^{\alpha_i} n_{ijt}^{1-\alpha_i}, \quad (7)$$

where  $\alpha_i \in (0, 1)$ .  $y_{ijt}$  denotes the amounts of intermediate good  $i$  produced in country  $j$  at date  $t$ , while  $k_{ijt}$  and  $n_{ijt}$  are respectively the amounts of capital and labor employed in the production of good  $i$ ;  $a_{jt}$  denotes the total factor productivity (TFP) level.<sup>10</sup> We assume that TFP follows an exogenous stationary stochastic Markov process. In particular, we assume that the logarithm of  $a_t \equiv [a_{ht}, a_{ft}]'$  is governed by a VAR(1):<sup>11</sup>

$$\ln a_{t+1} = B \ln a_t + \varepsilon_t, \quad (8)$$

where  $B$  is the persistence matrix, and  $\varepsilon_t \sim N(0, \Sigma)$  is an *iid* vector of innovations. Note that the current levels of TFP are known at date  $t$ .

### 2.3 Trade Equilibrium

We choose the final good produced in country  $f$  as the numeraire, *i.e.*  $p_{ft} = 1$ ; given free trade,  $p_{ht} = p_{ft} = 1$ . We assume that countries are similar enough in their relative factor endowments for the trade equilibrium to yield factor price equalization. The FPE theorem implies that international trade in intermediate goods acts as a substitute for trade in factors, equalizing wage and rental rates across countries.<sup>12</sup> Trefler [29] argues that FPE cannot be rejected by the data once one controls for cross-country factor-augmenting productivity differences. In other words, factor prices seem to be equal across countries when production factors are made comparable across nations. When FPE holds, the trade equilibrium yields the same resource allocation and prices as the world's integrated equilibrium, in which both goods and factors are perfectly mobile internationally. Since the integrated equilibrium behaves like a closed economy, factor prices only depend on world aggregates.<sup>13</sup> The wage rate  $w$  and the rate of return to capital  $r$  depend, respectively,

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<sup>10</sup>Our FPE setup does not allow for country-sector specific productivity shocks.

<sup>11</sup>The steady-state TFP level is normalized to one.

<sup>12</sup>In the Heckscher-Ohlin model, each country can produce all intermediate goods if the relative factor endowments of countries are similar enough. Provided there are at least as many goods as production factors, the equilibrium factor prices of countries depend only on goods prices through the competitive price conditions. Given free trade and identical technologies across countries, factor prices are equalized internationally. (See Dixit and Norman [12].)

<sup>13</sup>Given homotheticity in the production function of  $Y$ , world demand for intermediates does not depend on the way income is distributed across countries.

positively and negatively on the world's capital-labor ratio expressed in efficiency units:

$$w_t = \Gamma \left( \frac{s_N}{s_K} \right)^{s_K} \left( \frac{K_t}{N_t} \right)^{s_K}, \quad (9)$$

$$r_t = \Gamma \left( \frac{s_K}{s_N} \right)^{s_N} \left( \frac{K_t}{N_t} \right)^{s_N}, \quad (10)$$

where  $\frac{K_t}{N_t} \equiv \frac{a_{ht}k_{ht}+a_{ft}k_{ft}}{a_{ht}n_{ht}+a_{ft}n_{ft}}$ .  $\Gamma \equiv \phi^\phi (1-\phi)^{(1-\phi)} [\alpha_1^{\alpha_1} (1-\alpha_1)^{1-\alpha_1}]^\phi [\alpha_2^{\alpha_2} (1-\alpha_2)^{1-\alpha_2}]^{(1-\phi)}$ ,  $s_K \equiv \frac{rK}{wN+rK} = \phi\alpha_1 + (1-\phi)\alpha_2$ , and  $s_N \equiv 1 - s_K$  are positive constants. Factor prices per “raw” unit are simply  $w_{jt} = w_t a_{jt}$  and  $r_{jt} = r_t a_{jt}$ . Concerning the trade pattern, the capital-abundant (labor-abundant) country will export the capital-intensive (labor-intensive) good.

## 2.4 Dynamic Equilibrium

A dynamic recursive equilibrium under FPE can be summarized by equations (3), (4), (5), (9), (10), and:

$$\xi_h c_{ht}^{-\sigma} (1 - n_{ht})^{\gamma(1-\sigma)} = \xi_f c_{ft}^{-\sigma} (1 - n_{ft})^{\gamma(1-\sigma)}, \quad (11)$$

$$\xi_h c_{ht}^{1-\sigma} (1 - n_{ht})^{\gamma(1-\sigma)-1} \frac{a_{ft}}{a_{ht}} = \xi_f c_{ft}^{1-\sigma} (1 - n_{ft})^{\gamma(1-\sigma)-1}, \quad (12)$$

$$E_t \left\{ \lambda_{jt+1} \left[ \varphi' \left( \frac{i_{jt+1}}{k_{jt+1}} \right) r_{jt+1} + 1 - \delta + \Phi \left( \frac{i_{jt+1}}{k_{jt+1}} \right) \right] \right\} = \frac{\lambda_{jt}}{\beta}, \quad (13)$$

where  $\Phi \left( \frac{i_{jt}}{k_{jt}} \right) \equiv \varphi' \left( \frac{i_{jt}}{k_{jt}} \right) \left( \frac{i_{jt}}{k_{jt}} \right) - \varphi \left( \frac{i_{jt}}{k_{jt}} \right)$ , and the costate variable  $\lambda_{jt} = \xi_j c_{jt}^{-\sigma} (1 - n_{jt})^{\gamma(1-\sigma)} / \varphi'$  represents the shadow value of installed capital. The previous system of equations is valid if and only if the FPE condition is satisfied *ex-post* at all dates  $t \in [0, \infty)$ .<sup>14</sup>

Notice that under FPE and complete markets both the marginal utility of consumption and the marginal utility of leisure are equalized across countries. Combining equations (11) and (12) yields

$$\frac{c_{ht}}{c_{ft}} = \left( \frac{\xi_h}{\xi_f} \right)^{\frac{1}{\sigma+\gamma(\sigma-1)}} \left( \frac{a_{ht}}{a_{ft}} \right)^{\frac{\gamma(\sigma-1)}{\sigma+\gamma(\sigma-1)}}, \quad (14)$$

$$\frac{1 - n_{ht}}{1 - n_{ft}} = \left( \frac{\xi_h}{\xi_f} \right)^{\frac{1}{\sigma+\gamma(\sigma-1)}} \left( \frac{a_{ht}}{a_{ft}} \right)^{\frac{\sigma}{\gamma(1-\sigma)-\sigma}}. \quad (15)$$

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<sup>14</sup>More precisely, the first order conditions are valid only if the agents consider the possibility of leaving FPE as zero-probability event.

## 2.5 Steady State

The system is in steady state if and only if the following condition holds:

$$r = \frac{1}{\beta} - 1 + \delta. \quad (16)$$

Consider equation (10) evaluated at the steady state:

$$r = \Gamma \left( \frac{s_N}{s_K} \right)^{-s_N} \left( \frac{K}{N} \right)^{-s_N}. \quad (17)$$

It is easy to show that equations (16) and (17), together with the other first order conditions and resource constraints, characterize the integrated economy's steady state.

Equations (14) and (15) suggest that, under complete markets, the fact that the world is a stationary system is enough to pin down the steady-state levels of consumption and worked hours at the country level. However, this is not enough to fully pin down the country-level steady state: any combination of  $k_h$  and  $k_f$  such that FPE holds and

$$\frac{k_h + k_f}{n_h + n_f} = \frac{K}{N} \quad (18)$$

is compatible with the steady state. The unique world-level steady state is therefore compatible with a multiplicity of steady states at the country level. These steady states are fully characterized by the cross-country distribution of capital stocks.<sup>15</sup>

The multiplicity of steady states does not imply the indeterminacy of the model's solution: once the initial conditions  $k_{j0}$  are exogenously given, the transitional dynamics leads the system to a unique and non-degenerate steady state. This can be understood as follows: (i) the world as a whole is a standard stationary Ramsey economy with a well specified steady state, characterized by a unique value of  $K/N$ ; (ii) given the initial conditions and our assumptions on the functional forms, the adjustment paths for all country-level variables are uniquely determined; (iii) equation (18) and the FPE condition imply that  $(k/n)_{\min} \leq k_j/n_j \leq (k/n)_{\max}$  for some  $(k/n)_{\max} > (k/n)_{\min} > 0$ . In other words, the world reaches a steady state in which equations (16) and (17) hold, and both  $k_h$  and  $k_f$  are strictly positive. Such a steady state may be characterized by different

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<sup>15</sup>Giavazzi and Wyplosz [13] show that a similar result holds in a two-country model under rational expectations and perfect capital mobility. In our set-up, international trade in goods is a substitute for perfect capital mobility. Becker [7] obtains this result in a heterogeneous-agents closed economy, due basically to the same economic mechanism: if all agents face the same rate of return, and the latter depends on the aggregate capital stock only, the steady-state wealth and income distributions are indeterminate. Lucas and Stokey [23] discuss the issue in detail, and suggest that the multiplicity of stationary equilibria can be avoided by introducing the hypothesis of increasing marginal impatience. See also Stiglitz [27] and Baxter [4].

values of income, investment and capital across countries.

## 2.6 Calibration

To solve and simulate the model numerically, we need to calibrate a large set of parameters. In particular, there are three sets of parameters that distinguish our framework from much of the work in the international real business cycles literature: the sectoral factor intensities  $\alpha_i$ , the share of sector 1 in value added  $\phi$ , and the initial capital-labor ratios for each country. The sectoral factor intensities and the cross-country distribution of capital-labor ratios will determine the trade pattern.

To match our model with the data, we initially consider a multisectoral version of our set-up in which the final consumption good is produced using  $m > 2$  intermediate goods, *i.e.*  $Y_{jt} = \prod_{i=1}^m x_{ijt}^{\phi_i}$  where  $\sum_{i=1}^m \phi_i = 1$ . In the integrated equilibrium,  $\phi_i$  equals the share of sector  $i$  in the world's total value added. The OECD publishes estimates of sectoral value added for 24 countries and 28 sectors in its *Annual National Accounts Detailed Tables*. Internationally comparable estimates for all sectors and all countries are available only for 1995 and 1996. Given that figures do not change significantly over this period, we focus on 1995. After converting all observations in US\$ with the OECD PPP exchange rates, we aggregate sectors across countries and calculate the share of each sector in total OECD value added. We calibrate the  $\phi_i$ 's to match these observed shares.

Each intermediate good is produced with the Cobb-Douglas production function defined in (7). In equilibrium,  $\alpha_i$  equals sector  $i$ 's capital share in value added. The OECD *Annual National Accounts* provide data on compensations of employees (wages and salaries), total employment, and the share of employees in total employment at the sectoral level for a smaller set of countries. Under the assumption of identical technologies, we focus on the US and, for comparability, on year 1995. As a first approximation, the sectoral labor share can be obtained as the share of compensation of employees in value added. With this approach, the share of income perceived by the self-employed as a remuneration of their own work is recorded as capital income rather than labor income.<sup>16</sup> We follow Gollin [15], and adjust the sectoral labor share using data on the share of employees over total employment: we assume that self-employed workers earn a wage comparable to the competitive market wage earned by employees, and correct the sectoral labor shares by dividing them by the sectoral share of employees in total employment. This correction changes the labor share significantly in a few cases only. We compute the sectoral capital share as one minus the corresponding labor share.<sup>17</sup> Table 2 summarizes

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<sup>16</sup>The OECD does not report proprietors income separately. Therefore the procedure outlined in Cooley and Prescott [10] to correct for this problem cannot be applied here.

<sup>17</sup>We consider land as a component of the capital stock.

Sector	$(1 - \alpha_i)$ (Not Adj.)	$(1 - \alpha_i)$ (Adj.)	$\phi_i$
Electricity, gas and water supply	0.26	0.26	0.026
Man. of coke, refined petroleum prod. and nuclear fuel	0.34	0.34	0.006
Mining and quarrying	0.34	0.35	0.010
Real estate, renting and business activities	0.31	0.37	0.175
Man. of chemicals, chem. prod. and man-made fibres	0.44	0.44	0.021
Man. of food products, beverages and tobacco	0.47	0.47	0.024
Man. of leather and leather products	0.53	0.55	0.002
Financial intermediation	0.55	0.57	0.061
Transport, storage and communication	0.55	0.59	0.069
Hotels and restaurants	0.59	0.62	0.016
Wholesale and retail trade; repair services	0.57	0.62	0.143
Man. of electrical and optical equipment	0.63	0.63	0.027
Agriculture, hunting, forestry, and fishing	0.34	0.65	0.023
Manufacturing n.e.c.	0.60	0.66	0.008
Man. of basic metals and fabricated metal products	0.67	0.68	0.024
Man. of wood and wood products	0.63	0.69	0.006
Man. of pulp, paper and paper prod.; pub. and printing	0.66	0.70	0.019
Man. of other non-metallic mineral products	0.68	0.70	0.008
Man. of machinery and equipment n.e.c.	0.75	0.76	0.020
Man. of rubber and plastic products	0.77	0.77	0.007
Man. of textiles and textile products	0.76	0.78	0.008
Man. of transport equipment	0.78	0.78	0.021
Other community, social and pers. services	0.70	0.83	0.061
Construction	0.67	0.85	0.055
Health and social work	0.81	0.88	0.050
Public admin. and defence; comp. social security	0.89	0.89	0.083
Education	0.94	0.95	0.026
Private households with employed persons	1.00	1.00	0.002

Table 2: Sectoral labor shares and shares in value added at the world level.

these results: we report the unadjusted and adjusted sectoral labor shares, as well as the sectoral shares in world value added. These estimates imply a world capital share equal to  $s_K = 0.37$ .

We aggregate our 28 sectors in two larger sectors: a labor-intensive sector and a capital-intensive one. Any such aggregation seems in principle arbitrary. However, in the integrated equilibrium, the dynamics of the system is influenced exclusively by the world capital share  $s_K$ . Therefore any aggregation that leaves  $s_K$  unaffected will not bias the

results in any way (apart from pinning down the trade pattern).<sup>18</sup> Note that

$$s_K = \sum_{i=1}^m \phi_i \alpha_i = \sum_{i=1}^z \phi_i \frac{\sum_{i=1}^z \phi_i \alpha_i}{\sum_{i=1}^z \phi_i} + \sum_{i=z+1}^m \phi_i \frac{\sum_{i=z+1}^m \phi_i \alpha_i}{\sum_{i=z+1}^m \phi_i} = \phi \alpha_1 + (1 - \phi) \alpha_2 \quad (19)$$

for any  $z$  and adequately defined  $\phi$ ,  $\alpha_1$ , and  $\alpha_2$ . For the sake of symmetry, we order the sectors according to their capital intensities and choose  $z = 14$ ; the resulting parameterization is  $\phi = 0.61$ ,  $\alpha_1 = 0.49$ , and  $\alpha_2 = 0.17$ .

Once we set the elasticity of intertemporal substitution to  $\sigma = 2$  and the intertemporal discount factor to  $\beta = 0.99$ , we are left with the preference parameter  $\gamma$  and the depreciation rate  $\delta$ . The usual procedure consists in calibrating these parameters in order to match the observed long-run time share devoted to labor and the long-run capital-income ratio. In our framework, however, the country-level capital-income ratio depends directly on the country-level capital-labor ratios. Since the steady state of our model is *ex-ante* indeterminate, we need to specify a starting point (the initial steady state) to pin down the dynamic equilibrium.

We assume that a share  $1/2 < \omega < 1$  of the world's capital stock is in country  $h$ . In this initial asymmetric steady state, capital-abundant country  $h$  will be a net exporter of the capital-intensive good 1, while labor-abundant country  $f$  will be a net exporter of the labor-intensive good 2. The value of  $\omega$  influences the trade volume directly: the farther the capital-labor ratios, the larger the volume of trade, defined as the ratio between imports plus exports over income. We calibrate  $\gamma$ ,  $\delta$ , and  $\omega$  jointly in order to make country  $h$  reproduce the US values for the time share devoted to labor (0.31), the yearly capital/income ratio (3.32), and the average US trade volume's GDP share (0.15).<sup>19</sup> The implied values are  $\gamma = 1.86$ ,  $\delta = 0.019$ , and  $\omega = 0.547$ . This parameterization implies a capital share equal to 0.39 in country  $h$ . Finally, the elasticity of the adjustment cost  $\xi_\varphi$  is calibrated so as to have country  $h$  display the observed US relative volatility of investment. The corresponding value is  $\xi_\varphi = 1/35$ .

Hence, our calibration is based on the assumption that the US is capital abundant with respect to the rest of the world. Is there some empirical support for this assumption? As Treffer [29] noted, what matters here are not the factor endowments in raw terms, but the factor endowments in *efficiency units*.<sup>20</sup> How to measure these endowments is still an unresolved issue from an empirical point of view. However, the results in Treffer [29]

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<sup>18</sup>The only key variable that depends on the particular aggregation we use is the terms of trade. (See equation (21) below.) We have performed robustness checks that show the results reported in the paper for the terms of trade do not change significantly for other aggregations.

<sup>19</sup>The values for the US time share and capital/income ratio have been taken from Cooley and Prescott [10]. The US volume of trade's GDP share is computed as the average of the value of imports plus exports over GDP (all variables in nominal terms) over the 1960-2001 period.

<sup>20</sup>Our model can be easily extended to allow for different levels of efficiency across factors.

	Raw units	Eff. units
	$\frac{K_j}{N_j} / \frac{K_{US}}{N_{US}}$	$\frac{a_{j,K}K_j}{a_{j,N}N_j} / \frac{a_{US,K}K_{US}}{a_{US,N}N_{US}}$
Austria	1.03	0.90
Belgium	1.12	0.91
Canada	1.14	1.02
Denmark	0.91	0.84
Finland	1.06	0.91
France	1.18	0.89
Germany	1.17	0.93
Greece	0.66	0.84
Ireland	0.81	0.86
Italy	1.11	0.89
Japan	0.93	0.94
Netherlands	1.10	0.97
New Zealand	0.88	0.88
Norway	1.35	1.02
Portugal	0.49	0.76
Spain	0.77	0.87
Sweden	0.79	0.93
Switzerland	1.50	0.89
UK	0.76	0.97
USA	1.00	1.00
<b>RoW</b>	<b>0.99</b>	<b>0.93</b>

Table 3: Capital-labor ratios in efficiency units.

provide some evidence in favour of our assumption:

Trefler calibrates the relative productivity levels  $a_K$  and  $a_N$  for a set of countries in order to make the Heckscher-Ohlin-Vanek model match the data on the net factor content of trade. Using his raw factor endowment and productivity level data, we can compute the capital-labor ratios in efficiency units implied by his calibration. The US is capital abundant *vis-à-vis* all OECD countries in the sample, but for Canada and Norway. More interestingly, the Rest of the World aggregate (similar to EU15, Canada and Japan) is labor abundant *vis-à-vis* the US, and the ratio of the capital-labor ratios, 0.93, is not far from the 0.83 calibrated in our paper. If the world works as a HOV model, then the US must be capital abundant (in efficiency units) to make the model fit the data.

In the IRBC literature it is customary to approximate TFP with the standard Solow residual, defined as  $\ln s_{jt} \equiv \ln y_{jt} - s_N \ln n_{jt} - s_K \ln k_{jt}$ .<sup>21</sup> The joint stochastic properties of TFP are usually estimated by running a VAR(1) on the country-level proxies for the Solow residuals, as in Backus *et al.* [1] and others. We aggregate the EU15 European countries,

<sup>21</sup>In our framework, the Solow residuals remains a good empirical proxy for TFP:  $\ln s_{jt} = \ln a_{jt}$  up to a constant.

Canada, and Japan into a “Rest of the World” country. We take quarterly constant-price series for GDP and investment from the OECD *Quarterly National Accounts* for all countries over the 1961:I-2001:IV period. All series are then transformed in constant US\$ using the appropriate OECD PPP exchange rates. We build series for the physical capital stock using the permanent inventory method. As a proxy for the labor input, we take Total Civilian Employment, again from the *Quarterly National Accounts*, for the 1973:I-2001:IV period.<sup>22</sup> Finally, the “Rest of the World” is obtained by summing up the country-level variables. The joint stochastic properties of TFP are estimated by running a symmetric VAR(1) on the Solow residuals over the 1973:I-2001:IV period using the Full Information Maximum Likelihood approach. The estimation results are the following ( $p$ -values for standard  $t$ -tests in parenthesis):

$$B = \begin{bmatrix} 0.94 & 0.014 \\ (0.00) & (0.61) \\ 0.014 & 0.94 \\ (0.61) & (0.00) \end{bmatrix}, \quad \Sigma = 10^{-6} \begin{bmatrix} 9.28 & 2.05 \\ 2.05 & 4.43 \end{bmatrix}. \quad (20)$$

The implied shocks’ standard deviations are 0.003 in country  $h$  and 0.002 in country  $f$ , while the shocks’ international correlation is 0.32. Given that technological spillovers are clearly not significantly different from zero, we set the out-of-the-diagonal elements in the persistence matrix to zero.

To summarize, our benchmark parameterization is the following:

$$\begin{aligned} \sigma &= 2, & \beta &= 0.99, & \gamma &= 1.86, \\ \delta &= 0.019, & \omega &= 0.547, & \xi_\varphi &= 1/35, \\ \alpha_1 &= 0.49, & \alpha_2 &= 0.17, & \phi &= 0.61, \\ B_{jj} &= 0.94, & \sigma_{h\varepsilon} &= 0.003, & \sigma_{f\varepsilon} &= 0.002, & Cor &= 0.32. \end{aligned}$$

### 3 Results

The model is log-linearized around the initial asymmetric steady state, and solved with the standard King *et al.* [19] procedure. We study the impulse response functions and the stochastic properties of the approximated model.<sup>23</sup>

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<sup>22</sup>The OECD does not report Total Civilian Employment at the quarterly frequency for a few minor European countries: we use all available data in our computations.

<sup>23</sup>Giavazzi and Wyplosz [14] show that linear dynamic systems characterized by the indeterminacy of the stationary equilibrium (*i.e.* singularity of the transition matrix) converge to a unique stationary equilibrium for any set of initial conditions, if the stability conditions are satisfied.



To clarify the contribution of our Heckscher-Ohlin (HO) trade structure, we compare our impulse-response functions with their counterparts for a standard Complete Specialization (CS) model developed along the Backus *et al.* [2] tradition, and formally described in the Appendix. For comparability reasons, we keep the same parameterization but for  $\phi$ , the Cobb-Douglas exponent in the final good production function. (The need for this modification is discussed in the Appendix). This exponent is calibrated to make country  $h$  match the long-run US trade volume's GDP share under complete specialization.

### 3.1 Impulse-Response Functions

In this section we discuss the dynamic response of our HO model to an unexpected positive shock to  $a_h$ . We assume that  $a_h$  suddenly increases by 1% at date  $h$ , and solve for the corresponding impulse-response functions over a 60-quarter time horizon. The impulse responses for the country-level main variables are plotted in Figures 1 and 2. The simulated series are expressed in percentage deviations from the initial steady state (the trade balance is the only variable expressed in levels.)

The world reacts to the productivity shock as in a closed-economy RBC model: income, consumption, investment, and the time share devoted to labor increase on impact and then converge slowly to their initial steady state values. Figures 1-2 tell quite a different story as far as the country-level variables are concerned. To understand the properties of our model, we need to study how prices and quantities react to the productivity shock on impact and during the transition to the final steady state.

**Impact** We focus on each of the two transmission channels that are at work in our framework separately: international trade in goods and international risk sharing. We analyze them in turn, starting with the reaction of factor prices under our trade regime and their effect on the main aggregate quantities.

For the sake of the discussion, let us assume for the moment that  $n_j$  does not vary on impact, and consider the effect of the shock on prices. For constant  $n_j$ 's, we can show that if  $k_h/n_h > k_f/n_f$ ,  $\partial w/\partial a_h > 0$  and  $\partial r/\partial a_h < 0$ . An increase in  $a_h$  raises country  $h$ 's endowment of both capital and labor in efficiency units in the same proportions. Given that country  $h$  is capital-abundant, this implies an increase in the world's capital-labor ratio in efficiency units. Changes in  $a_h$  also have a direct effect on country  $h$  factor prices in raw units:  $\partial w_h/\partial a_h > 0$  and  $\partial r_h/\partial a_h > 0$ . In country  $f$ , however, factor prices in raw units equal factor prices in efficiency units, since  $a_f$  is not affected by  $a_h$ .

The increase in the wage and rental rates has the following effects in country  $h$ : (i) the rise in the wage raises labor supply through the intertemporal labor/leisure substitution effect; (ii) the increase in the rental rate raises the slope of the consumption path, and

- *ceteris paribus* - leads consumption to fall and investment to rise; (iii) the rise in the time share allocated to labor reduces the amount of leisure enjoyed by the representative household, and this tends - *ceteris paribus* - to reduce consumption; (iv) the increase in factor prices raises income, and therefore stimulates both consumption and investment. As a result, income, consumption, investment and the labor input in country  $h$  react positively on impact; consumption and investment react proportionally less and more than income, respectively. The change in country  $f$ 's factor prices has the following effects on impact: (i) the rise in the wage rate increases labor supply; (ii) the fall in the rental rate tends - *ceteris paribus* - to increase consumption and decrease investment; (iii) the fall in leisure tends - *ceteris paribus* - to decrease consumption; (iv) the rise in labor income increases total income, and therefore stimulates both consumption and investment.

The joint increase in  $n_h$  and  $n_f$  raises world labor supply. For a given world capital stock, this reduces the wage rate and raises the rental rate. In country  $h$ , the increase in the wage rate generated directly by the productivity shock is partially dampened, while the rental rate increases even further. In country  $f$ , the wage rate falls, while the rental rate rises. It turns out that this “labor supply” effect dominates and yields a reduction of the wage rate and an increase of the rental rate on impact.<sup>24</sup>

The overall change in country  $f$ 's factor prices has the following effects on impact: (i) the fall in the wage rate reduces labor supply; (ii) the rise in the rental rate tends - *ceteris paribus* - to decrease consumption and increase investment; (iii) the rise in leisure tends - *ceteris paribus* - to increase consumption; (iv) the fall in labor income decreases total income, and therefore depresses both consumption and investment. As a result, income, investment and the labor input in country  $f$  react negatively on impact, while consumption increases slightly.

Under complete markets, international consumption risk sharing is an additional channel of propagation. Given that the final good is non-tradable, this transmission mechanism works through trade in intermediate goods: a positive, persistent, and transitory shock in one of the two countries generates (i) a positive wealth effect in both countries, increasing - *ceteris paribus* - consumption and leisure; (ii) an incentive to transfer resources to the temporarily more productive country, that has strong effects on the dynamics of investment. The second mechanism dominates on impact: investment falls dramatically in country 2 and rises sharply in country  $h$ .

**Transition** During the transition towards the final steady state, three main forces are at work: (i) the stochastic properties of TFP drive  $a_h$  slowly back to its long-run value;

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<sup>24</sup>The intuition is clear:  $a_{ht}$  affects both the numerator and the denominator of the world capital-labor ratio in efficiency units, while the  $n_j$ 's affect the denominator only.

(ii) the process of capital accumulation raises the capital stock in country  $h$  and reduces the capital stock in country  $f$ ; (iii) international consumption risk sharing drives the consumption levels and the number of hours worked back to their common steady-state values rapidly, as predicted by equations (14) and (15).

Initially, the positive growth of capital in country  $h$  is higher in modulus than the negative growth rate of capital in country  $f$ , and therefore the world capital stock increases. This reverses the impact variations of factor prices gradually, and leads to a decrease in the labor supply in country  $h$ , while the opposite happens in country  $f$ . The negative growth rate of labor in country  $h$  dominates, and the world labor input converges to its initial steady state value. After a while, the world capital stock reverses its dynamics, and starts to converge to its initial steady state value. As a consequence, factor prices tend to converge to their original steady-state values. Factor prices in raw units are different across countries only as long as the productivity levels  $a_h$  and  $a_f$  differ. The convergence of  $a_h$  equalizes factor prices in raw units across countries, while the process of capital accumulation drives the world to its initial steady state.

**Long-run effects** As soon as both countries share the same rental rate, their consumption paths become similar enough to prevent country-level capital stocks from converging to their initial steady-state levels. The capital stock  $k_h$  remains permanently higher than before the shock, while  $k_f$  remains permanently lower. Since the world capital stock must reach its initial steady-state level, the increase in  $k_h$  exactly offsets the decrease in  $k_f$ . This permanent difference in the capital stocks implies permanent symmetric differences in income and investment. Since GDP in country  $f$  ends up being permanently lower than in the initial steady state, a permanent inflow of resources from country  $h$  is needed to finance the optimal consumption level. In other words, country  $h$  runs a trade surplus in the new steady state: households in country  $f$  will only transfer resources to country  $h$  on impact if these resources will be paid back in the long run with a permanent transfer.

The labor share in country  $h$  converges to a permanently lower value, while the opposite happens in country  $f$ . The investment share, the capital-income ratio, and the capital-labor ratio converge to higher values in country  $h$  and to lower ones in country  $f$ . The joint dynamics of capital and labor also have a permanent effect on the average productivity of labor, defined as the ratio between total income and the labor input. The transitory shock to productivity raises the labor productivity in country  $h$  and lowers it in country  $f$  permanently, since countries  $h$  and  $f$  become capital-abundant and capital-scarce, respectively.

Although the world as a whole is a stationary system, some country-level variables are non-stationary from a stochastic point of view. Aggregate income, investment, and capital

in each country are unit-root processes, and are cointegrated with the corresponding variables in the other country: they are individually non-stationary, but their sum is actually stationary. This implies that the country-level steady states to which the system tends after a shock are different from the initial ones, but endogenously determined by the adjustment process itself.

In our model, investing in physical capital is the only way to accumulate wealth over time. Hence, productivity shocks have permanent effects on the cross-country wealth distribution, and indirectly on the income distribution. Baxter and Crucini [6] show that in a standard IRBC model with restricted asset markets, in which only riskless bonds are internationally traded, the steady-state level of asset holdings - a sufficient statistic for the cross-country wealth distribution - is not invariant to productivity shocks. In a different framework, Obstfeld and Rogoff [25] show that demand shocks can have permanent effects on the cross-country consumption differential and wealth distribution when again asset markets are restricted to riskless bonds. In both these contributions, the restricted financial markets introduce a more or less direct link between consumption growth rates, via the common interest rate. Similarly, in our model the consumption/leisure paths are eventually driven in both countries by the same interest rate, but this international link is generated by trade in goods rather than trade in bonds.<sup>25</sup>

**Terms of trade** On impact, the productivity shock raises the labor supply in country  $h$  and reduces it in country  $f$ , leaving the capital stocks unaltered. The model's Heckscher-Ohlin trade structure implies that in country  $h$  resources flow from the capital-intensive sector to the labor-intensive sector, while the opposite happens in country  $f$ . The productivity shock in country  $h$  simply exacerbates this trade pattern in the new steady state. As a result, during the transition country  $h$  gradually reallocates both factors to the capital-intensive sector, while the opposite happens in country  $f$ .

In Figure 4 we plot country  $h$ 's terms of trade, defined as the price of imports over the price of exports. The pattern of trade is affected permanently by transitory shocks to productivity. For small deviations from the initial asymmetric steady state, country  $h$  remains a net exporter of the capital intensive good and a net importer of the labor intensive one. The  $p_2/p_1$  price ratio corresponds therefore to country  $h$ 's terms of trade.

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<sup>25</sup>Hall [16] shows that in a partial equilibrium model of the life-cycle permanent income hypothesis the marginal utility of consumption follows a martingale - and the consumption level is therefore a unit root process - if the real interest rate is exogenous. Similar results emerge in a small open economy framework, as pointed out by Sen and Turnovsky [26] and Correia *et al.* [11]. In our set-up, the equilibrium real interest rate is endogenous at the world level, but partly exogenous at the country level. Hence, Hall's result applies to a limited extent at the country level.

In the Technical Appendix we show that

$$\frac{p_{2t}}{p_{1t}} = \Xi \left( \frac{K_t}{N_t} \right)^{\alpha_1 - \alpha_2}, \quad (21)$$

where  $\Xi$  is a positive constant. Given  $\alpha_1 > \alpha_2$ , the initial increase in  $n_{jt}$  has on impact a negative effect on the terms of trade in country  $h$ ; the process of capital accumulation reverses this effect soon, raising the terms of trade. The latter deviate positively from the steady state during the remaining part of the transition. Since the steady-state capital-labor ratio is uniquely pinned down at the world level, so should be the steady-state value of the terms of trade. Figure 4 confirms that the terms of trade worsen on impact, but then improve quite remarkably during the transition to the steady state. In the long run, the terms of trade are not affected by the productivity shock permanently.

**Heckscher-Ohlin vs. Complete Specialization** Figures 3-4 report the impulse-response functions for the CS model, which tell quite a different story. From a qualitative point of view, the reaction of country  $h$  is very similar in both models, except for the fact that the CS model is completely stationary. That is, no long-run effect of productivity shocks can emerge in that framework. However, there are some striking quantitative differences. First, the reaction of investment on impact is much less vigorous. Second, and more importantly, the trade balance in country  $h$  decreases on impact as in the HO model, but the size of its decrease is almost six times smaller.

As far as country  $h$  is concerned, the only relevant qualitative difference between the two frameworks regards the behavior of the terms of trade: in the HO model, the terms of trade react negatively on impact, while exactly the opposite happens in the CS model. In the latter model, a positive shock to productivity in country  $j$  raises output, reducing the price of country  $j$ 's intermediate good: this implies that the terms of trade react positively on impact. In contrast, in the HO model productivity shocks have no direct effect on the terms of trade, since intermediate good prices depend only on the capital-labor ratio (in efficiency units) at the world level.

If we move to country  $f$ , instead, many more striking differences emerge: aggregate variables in that country seem hardly affected by the productivity shock in country  $h$ . Income, consumption, hours worked, and factor prices hardly react on impact: when the CS model is calibrated to replicate the observed US long-run trade volume's GDP share and technological spillovers are ruled out, international trade becomes irrelevant as a transmission mechanism. This clearly contrasts with the behavior of the HO model, in which international trade plays a much more important role.

This striking result can be understood as follows: under a complete specialization

assumption, and assuming a unitary elasticity of substitution among intermediate goods, the import-export shares in income will depend directly on the “home bias” parameter  $\phi$ . In order to match the observed long-run US trade volume share in income, the parameter  $\phi$  has to be set to a very high value, equal to 0.925 (see the Appendix for more details). Since the import shares in total expenditure are constant and equal to  $1 - \phi$  in both countries, the import shares in income in both countries will be a tiny and roughly constant fraction of income, leaving a very limited role to trade as a transmission mechanism. Given that the final good is not traded, trade in intermediate goods remains the only way to smooth consumption internationally, and the need to assume a high “home bias” limits this channel enormously.<sup>26</sup> Note that equation (57) can be rewritten as:

$$\pi_t = \frac{1 - \phi}{1 - 2\phi} \left( 1 - \hat{e}_t \frac{\hat{y}_{ft}}{\hat{y}_{ht}} \right), \quad (22)$$

where the “hat” identifies percentage deviations from the steady state. The dynamics of the net trade/ GDP ratio is driven exclusively by the real GDP ratio, adjusted for the real exchange rate: this is a direct consequence of the “Cobb-Douglas” and “home bias” assumptions. When  $\phi = 0.925$ , the value of  $(1 - \phi) / (1 - 2\phi)$  is  $-0.064$ . From Figures 3-4 we conclude that, on impact,  $\hat{y}_{h1} \simeq 1.2\%$ ,  $\hat{y}_{f1} \simeq 0.08\%$ , and  $\hat{e}_1 \simeq 0.5\%$ . Hence,  $\pi_1 = -0.06$ , as implied by Figure 4. The dynamics of  $\pi_t$  can be enhanced by assuming implausibly large import shares in income.

In the HO framework, instead, the trade pattern and the trade volume are both endogenously determined by comparative advantage considerations, and no “ad hoc” exogenous assumption is needed to reproduce the long-run trade volume apart from fixing the (*ex-ante* undetermined) initial conditions. In particular, both countries share the same final good production function, with no distinction between “home” and “foreign” intermediate goods: *i.e.* the “home bias,” so essential in the complete specialization framework to limit the extent of trade, plays absolutely no role in the HO framework, and therefore does not introduce an exogenous constraint on the dynamics of the trade balance.

## 3.2 Stochastic Properties

To study the stochastic properties of our model, we simulate it for 10.000 times over a 116 quarter horizon, drawing the shocks from a multivariate normal distribution. At each round we check whether the FPE condition holds and country 1 remains the capital-

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<sup>26</sup>In the Appendix we remind the reader that in the CS model countries do not need to be identical in steady state, and will not be if the factor shares are different across countries. Let us stress here that the very low volatility of the trade balance does not depend on this difference in country size: similar results occur in the symmetric case too.

abundant country over the whole simulation horizon.<sup>27</sup> Rounds that do not satisfy these requirements are dropped from the experiment. We then apply the Hodrick and Prescott [18] filter<sup>28</sup> to the simulated series, and compute the standard business cycles statistics for the cyclical components in country  $h$ : the relative volatility of all main aggregate variables, measured as the ratio between the standard deviation of each variable and the standard deviation of income, their correlation with income and net trade, and their international correlations. Table 4 reports the averages and standard deviations of these statistics over the whole sample. We also report the observed counterparts for the US based on quarterly OECD data over the 1973:I-2001:IV period.<sup>29</sup> The average and standard deviation of the observed international correlations are taken from Maffezzoli ([24], Tab. A.7).<sup>30</sup> Finally, Table 4 also reports the simulated statistics for the Complete Specialization counterpart, obtained under the same conditions. Note that  $y$  is GDP,  $c$  consumption,  $i$  investment,  $n$  labor,  $\pi/y$  net trade over GDP,  $p$  the terms of trade.

As far as the national business cycle properties are concerned, the HO model generates a relative volatility of consumption lower than the observed one, as is usual with this class of models. The relative volatilities of both the trade balance and the terms of trade are also below the observed ones. Similar results obtain in the CS framework: in particular, the relative volatility of the trade balance is particularly small (less than one tenth of the observed one).<sup>31</sup>

In the HO framework, the correlations of the trade balance and the terms of trade with income have the right signs, although they are larger in absolute terms than the

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<sup>27</sup>We performed several simulations of 10,000 rounds each and the model *never* left the FPE region.

<sup>28</sup>We Hodrick-Prescott filter the simulated series for two main reasons: (i) for comparison purposes with the existing literature; (ii) to extract the unit roots and obtain stationary cyclical components. Canova [8] and others, however, show that applying the Hodrick-Prescott filter to integrated series is likely to induce spurious results. Furthermore, since the Hodrick-Prescott is a univariate procedure, it ignores by construction the cointegrating relationships that link different variables in the same country and the same variables across countries. Note, however, that the same filtering procedure is used to extract the cyclical component from the data: in this sense, the model is compared to the data under the same conditions.

<sup>29</sup>The trade balance has been computed as the ratio between net exports (exports minus imports) over GDP, both at current prices; we Hodrick-Prescott filtered the series without logging it. The terms of trade are obtained as the ratio between the import and export deflators.

<sup>30</sup>The data set regards ten OECD countries (Australia, Austria, Canada, France, Germany, Italy, Japan, Switzerland, UK and USA). The sample period is 1970:1-1997:4 (for Australia, Germany, and Switzerland the sample period is shorter for some variables); sources are the OECD's *Quarterly National Accounts* integrated by OECD's *Quarterly Labour Statistics*. Variables are GDP, private consumption, private fixed investment (all at constant prices), and civilian employment. All variables are expressed in logarithms, deseasonalized (for series not deseasonalized at the origin, the X-11 program was used), and Hodrick-Prescott filtered. The reported statistics are averages and standard deviations across all country pairs.

<sup>31</sup>We acknowledge that the volatility of the terms of trade in our HO model is less than a tenth of the observed one, whereas the CS does not fare as badly in this dimension. The reader should keep in mind, however, that the CS model does not predict the right sign for the correlation of the terms of trade with the trade balance.

	Rel. Std. Dev.			Cor. with $y$			Cor. with $\pi/y$			Inter. Cor.		
	US	HO	CS	US	HO	CS	US	HO	CS	Data	HO	CS
$y$	-	-	-	-	-	-	-	-	-	0.42	0.07	0.41
	-	-	-	-	-	-	-	-	-	<i>0.20</i>	<i>0.16</i>	<i>0.15</i>
$c$	0.80	0.37	0.39	0.87	0.98	0.99	-	-	-	0.23	0.48	0.44
	-	<i>0.01</i>	<i>0.00</i>	-	<i>0.01</i>	<i>0.00</i>	-	-	-	<i>0.22</i>	<i>0.14</i>	<i>0.15</i>
$i$	3.83	3.83	2.43	0.94	0.98	1.00	-	-	-	0.31	-0.49	0.28
	-	<i>0.14</i>	<i>0.02</i>	-	<i>0.01</i>	<i>0.00</i>	-	-	-	<i>0.21</i>	<i>0.13</i>	<i>0.17</i>
$n$	0.64	0.55	0.44	0.88	0.97	1.00	-	-	-	0.35	-0.43	0.39
	-	<i>0.02</i>	<i>0.00</i>	-	<i>0.01</i>	<i>0.00</i>	-	-	-	<i>0.21</i>	<i>0.14</i>	<i>0.15</i>
$\frac{\pi}{y}$	0.63	0.28	0.05	-0.46	-0.80	-0.86	-	-	-	-	-	-
	-	<i>0.03</i>	<i>0.01</i>	-	<i>0.07</i>	<i>0.05</i>	-	-	-	-	-	-
$p$	1.71	0.10	0.68	-0.18	-0.62	0.35	0.10	0.18	-0.66	-	-	-
	-	<i>0.01</i>	<i>0.11</i>	-	<i>0.10</i>	<i>0.16</i>	-	<i>0.16</i>	<i>0.09</i>	-	-	-

Table 4: Business cycle properties (shocks to the traded sectors).

observed values. The CS model delivers instead a *positive* correlation of the terms of trade with income. Heathcote and Perri [17] show that the Backus *et al.* [2] model generates a significantly positive correlation between the terms of trade and income due to the complete specialization assumption. In that model, a positive shock to productivity in country  $j$  raises output, reducing the price of country  $j$ 's intermediate good. This implies that the terms of trade react positively on impact, being therefore highly correlated with income. Exactly the same mechanism is at work in our CS framework. In contrast, in the HO model productivity shocks have no direct effect on the terms of trade, since intermediate good prices depend only on the capital-labor ratio at the world level.

Backus *et al.* [2] and our CS model yield a significantly negative correlation between the terms of trade and the trade balance: the terms of trade react positively to the shock, while the trade balance response is negative. In the HO model, both the terms of trade and the trade balance react negatively on impact to a positive productivity shock in the capital-abundant country, and are therefore positively correlated; the simulated statistic is furthermore not very different from the observed one. The intuition is clear: a positive shock to  $a_h$  leads country  $h$  to borrow from abroad, and it causes  $n_h$  and subsequently  $N$  to rise on impact. This triggers a fall in both  $K/N$  and country  $h$ 's terms of trade  $p_2/p_1$ . The correlation between the trade balance and the terms of trade is negative instead for the labor-abundant country: a positive shock to  $a_f$  leads country  $f$  to borrow from abroad and to a fall in  $K/N$ , but to a rise in country  $f$ 's terms of trade  $p_1/p_2$ . This implies a positive correlation between income and the terms of trade, and a negative correlation between the trade balance and the terms of trade. In this respect, it is interesting to note that Backus *et al.* [2] report a positive correlation between net exports and the terms of



trade for the US, and negative correlations for most of the other countries in their sample. In fact, in their sample, most of the countries exhibiting a negative correlation between the terms of trade and the trade balance also exhibit a positive correlation between the terms of trade and income.

A look at the international business cycle properties suggests that the HO model is unable to solve the quantity puzzle: even if the international correlation of income is slightly positive, the international correlations of investment and hours worked are significantly negative. Furthermore, the international correlation of consumption is significantly higher than the correlation of income. The CS model seems to perform better, since it reproduces positive international correlations for all variables. However, this first impression is misleading: the CS model generates positive international correlations by simply reducing the importance of trade as a transmission mechanism. These positive correlations simply reproduce the international correlation of productivity shocks, equal to 0.32.

## 4 Shocks to the Non-traded Sector

In our set-up, preferences and technologies - apart from productivity levels - are identical across countries. Under free trade, our model is therefore unable to produce any dynamics of the real exchange rate, since the price levels in both countries always remain the same. Backus *et al.* [2] and our CS model are able to bypass the problem by assuming a cross-country asymmetry - a “home bias” - in the final good’s production function.

To study the predictions of our model as far as the real exchange rate is concerned, we assume that productivity shocks affect the non-traded final good sector instead of the traded intermediate good sectors. In other words, the production function for the final good becomes

$$Y_{jt} = a_{jt}x_{1jt}^{\phi}x_{2jt}^{1-\phi}. \quad (23)$$

We interpret the stochastic components  $a_{jt}$  literally as TFP levels, and in fact they represent the total productivity of factors used in the final good sector. However, productivity shocks in the consumption good sector can be interpreted also as demand shocks for the intermediate goods.

The relative price of the final good in country  $h$  in terms of the numeraire becomes  $p_{ht} = a_{ft}/a_{ht}$ . We define the real exchange rate as the ratio between the price of consumption in country  $f$  over the price of consumption in country  $h$ , *i.e.*  $e_t \equiv p_{ft}/p_{ht} = a_{ht}/a_{ft}$ . Hence, productivity shocks in our framework can be interpreted also as shocks to the real exchange rate.

Table 5 reports the simulated statistics for our models, as well as their estimated em-

	Rel. Std. Dev.			Cor. with $y$			Cor. with $\pi/y$			Inter. Cor.		
	US	HO	CS	US	HO	CS	US	HO	CS	Data	HO	CS
$y$	-	-	-	-	-	-	-	-	-	0.42	0.25	0.31
	-	-	-	-	-	-	-	-	-	<i>0.20</i>	<i>0.16</i>	<i>0.15</i>
$c$	0.80	0.53	0.47	0.87	1.00	1.00	-	-	-	0.23	0.20	0.33
	-	<i>0.01</i>	<i>0.00</i>	-	<i>0.00</i>	<i>0.00</i>	-	-	-	<i>0.22</i>	<i>0.17</i>	<i>0.15</i>
$i$	3.83	3.83	2.23	0.94	0.96	1.00	-	-	-	0.31	-0.51	0.22
	-	<i>0.20</i>	<i>0.01</i>	-	<i>0.02</i>	<i>0.00</i>	-	-	-	<i>0.21</i>	<i>0.13</i>	<i>0.16</i>
$n$	0.64	0.38	0.38	0.88	0.99	1.00	-	-	-	0.35	0.02	0.30
	-	<i>0.01</i>	<i>0.00</i>	-	<i>0.00</i>	<i>0.00</i>	-	-	-	<i>0.21</i>	<i>0.17</i>	<i>0.15</i>
$\frac{\pi}{y}$	0.63	0.44	0.03	-0.46	-0.72	-0.97	-	-	-	-	-	-
	-	<i>0.05</i>	<i>0.00</i>	-	<i>0.08</i>	<i>0.01</i>	-	-	-	-	-	-
$e$	5.79	0.83	0.84	0.09	0.70	0.48	0.16	-0.99	-0.49	-	-	-
	-	<i>0.10</i>	<i>0.14</i>	-	<i>0.09</i>	<i>0.13</i>	-	<i>0.01</i>	<i>0.12</i>	-	-	-
$p$	1.71	0.10	0.29	-0.18	-0.73	-0.73	0.10	0.26	0.79	-	-	-
	-	<i>0.01</i>	<i>0.03</i>	-	<i>0.07</i>	<i>0.08</i>	-	<i>0.16</i>	<i>0.05</i>	-	-	-

Table 5: Business cycle properties (shocks to the non-traded sector).

pirical counterparts for the US.  $e$  denotes the real exchange rate.<sup>32</sup> The relative volatilities of both the terms of trade and the real exchange rate are much lower than their estimated values, but we replicate their relative rank: the terms of trade are less volatile than the real exchange rate. Notice that in our model the volatility of the real exchange rate has a one-to-one relationship with the volatility of  $a_h/a_f$ , whereas the volatility of  $p_2/p_1$  has a one-to-one relationship with the volatility of  $K/N$ . That is, the volatility of productivity shocks affects the volatility of a country's real exchange rate directly, while its effect on the volatility of a country's terms of trade is indirect and dampened by the fact that intermediate goods prices are formed at the world level.

The relative volatilities of worked hours and the trade balance are slightly smaller than in the data. The correlations with income of the trade balance, the real exchange rate, and the terms of trade have the right signs, although they are larger in absolute terms than the observed value; in particular, the correlation of the real exchange rate with income largely exceed its observed value. Our model generates a significantly negative correlation of the trade balance with the real exchange rate, which is at odds with the positive correlation in the US data. The terms of trade, instead, remain positively correlated with the trade balance, as in the data.

The qualitative performance of the HO and CS models is quite similar as far as the

<sup>32</sup>We define the real exchange rate as the inverse of the price-adjusted and trade-weighted Broad Index of the foreign exchange value of the U.S. dollar. The quarterly data for the 1973:1-2001:4 period come from the Board of Governors and have been logged and Hodrick-Prescott filtered. Note that the elasticity of the adjustment cost had to be recalibrated to make the model match the observed volatility of investment: the new value is  $\xi_\varphi = 1/9.4$ .

national stochastic properties are concerned: all statistics share the same sign. In particular, when the shocks hit the non-traded sector, the CS model generates a negative correlation of the terms of trade with income, and therefore a positive correlation of the terms of trade with the trade balance. However, there are some important quantitative differences. The most striking one regards the dynamics of the trade balance: when the CS model is calibrated to reproduce the actual long-run trade volume's GDP share, the trade balance's relative volatility is dramatically small (in our particular case, it is 21 times lower than the observed one). This result confirms that in the CS model international trade in intermediate goods seems not to be a quantitatively relevant transmission mechanism.

Concerning the international business cycle properties, the CS model still generates an international correlation of consumption that is higher than the correlation of income, but these positive international correlations mainly reflect the exogenous positive international correlation of productivity shocks, given the negligible dynamics of the trade balance. The HO model's international business cycle properties exhibit striking changes: the international correlation of GDP is now significantly positive, and *greater* than the correlation of consumption. The international correlation of employment is positive, although not significantly. However, the international correlation of investment is significantly negative.

Canova and Ravn [9] show that some testable restrictions implied by international consumption risk sharing in the standard IRBC framework are strongly rejected by the data. Lewis [22] suggests that financial market restrictions can help explain the apparent lack of international consumption risk sharing in the data. In our model, the further restrictions introduced by FPE - in particular the cross-country equality of the marginal utility of leisure - can help explain this empirical puzzle, once we focus on shocks to the non-traded sector.<sup>33</sup> The key relationship to understand this result is equation (15): we can rewrite it in terms of deviations from the steady state as  $\hat{c}_h - \hat{c}_f = \kappa(\hat{a}_h - \hat{a}_f)$ , where  $\kappa \equiv \gamma(\sigma - 1) / [\sigma + \gamma(\sigma - 1)]$ .<sup>34</sup> Note that, since  $\gamma > 0$  and  $\sigma > 0$  by assumption,  $\kappa < 0$  only if  $\gamma / (1 + \gamma) < \sigma < 1$ . Given that  $\text{std}(\hat{a}_h) = \text{std}(\hat{a}_f)$  by assumption, and  $\text{std}(\hat{c}_h) \approx \text{std}(\hat{c}_f)$  due to the symmetry of our set-up, we can show that

$$\text{cor}(\hat{c}_j, \hat{c}_{-j}) = 1 - \kappa [\text{cor}(\hat{c}_j, \hat{a}_j) - \text{cor}(\hat{c}_j, \hat{a}_{-j})] \frac{\text{std}(\hat{a}_j)}{\text{std}(\hat{c}_j)}. \quad (24)$$

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<sup>33</sup>Our findings are related to those in Stockman and Tesar [28]. In a different framework, they show that taste shocks help explaining the observed properties of the data, while productivity shocks hitting the traded and non-traded sectors do not.

<sup>34</sup>Equation (15) implies also that  $\ln(c_j/c_{-j}) = \kappa \ln e_j$ , where  $e_j$  is the real exchange rate. In other words, the model predicts a close relationship between consumption ratios and bilateral real exchange rates as long as  $\kappa \gg 0$ . Backus and Smith [3] find little empirical evidence for this relationship in OECD time series. Under our benchmark parameterization,  $\kappa$  is strictly positive; hence we are left with a puzzle that cannot be addressed in our framework.

The international correlation of consumption is smaller than one as long as: (i) if  $\kappa > 0$ ,  $\text{cor}(\hat{c}_j, \hat{a}_j) > \text{cor}(\hat{c}_j, \hat{a}_{-j})$ ; (ii) if  $\kappa < 0$ ,  $\text{cor}(\hat{c}_j, \hat{a}_j) < \text{cor}(\hat{c}_j, \hat{a}_{-j})$ . It decreases - *ceteris paribus* - if: (i)  $\kappa$  increases; (ii) the correlation of consumption with TFP in the same country increases; (iii) the correlation of consumption with TFP in the other country decreases; (iv) the volatility of consumption decreases with respect to the volatility of TFP. Hence, since consumption levels are usually highly correlated with TFP in the same country and less volatile than TFP, the international correlation of consumption is likely to be significantly lower than one. Under our benchmark parameterization, it turns out to be lower than the correlation of GDP.

## 5 Concluding Remarks

Understanding the international transmission of business cycles through commodity trade requires modelling how the production structures of countries, their factor prices and trade patterns evolve over the cycle. This paper shows that combining standard international trade and macro models may help us in this direction, improving upon a whole generation of IRBC models that have neglected the endogenous reaction of trade patterns to the cycle.

Although FPE captures the idea that countries' factor prices and production structures depend on world fundamentals, it might be too strong an assumption from an empirical perspective. Current research in international trade endeavors to uncover more realistic trade models that might bring new insights into international macroeconomics.

## 6 Appendix

### 6.1 The Planner's Problem

The first order conditions that characterize a dynamic equilibrium can be obtained by solving a more familiar social planner problem. In an integrated equilibrium, the planner maximizes the intertemporal utility function (see equation 1):

$$U_t = E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \sum_{j=h,f} \xi_j \frac{c_{js}^{1-\sigma} (1 - n_{js})^{\gamma(1-\sigma)}}{1 - \sigma} \right] \quad (25)$$

under the following resource constraints:

$$c_{ht} + i_{ht} = (v_t x_{1t})^\phi (v_t x_{2t})^{1-\phi}, \quad (26)$$

$$c_{ft} + i_{ft} = [(1 - v_t) x_{1t}]^\phi [(1 - v_t) x_{2t}]^{1-\phi}, \quad (27)$$

$$x_{1t} = (\varsigma_t K_t)^{\alpha_1} (\theta_t N_t)^{1-\alpha_1}, \quad (28)$$

$$x_{2t} = [(1 - \varsigma_t) K_t]^{\alpha_2} [(1 - \theta_t) N_t]^{1-\alpha_2}, \quad (29)$$

$$k_{jt+1} = (1 - \delta) k_{jt} + \varphi \left( \frac{i_{jt}}{k_{jt}} \right) k_{jt}, \quad (30)$$

where  $K_t \equiv \sum_{j=h,f} a_{jt} k_{jt}$  and  $N_t \equiv \sum_{j=h,f} a_{jt} n_{jt}$ . The first order conditions with respect to  $c_{jt}$ ,  $i_{jt}$ ,  $n_{jt}$ ,  $v_t$ ,  $\varsigma_t$ ,  $\theta_t$ , and  $k_{jt+1}$  can be simplified and arranged to obtain the same conditions discussed in the main text.

## 6.2 The FPE Condition

This Appendix discusses the implementation of the FPE condition, which is based on the integrated equilibrium's resource allocation. See Dixit and Norman [12] for a theoretical discussion of the FPE condition.

### 6.2.1 Integrated Equilibrium's Resource Allocation

Employment and capital in *efficiency units* are allocated as follows:

$$N_{2t} = \frac{(1 - \alpha_2)(1 - \phi)Y_t}{w_t}, \quad (31)$$

$$N_{1t} = \frac{(1 - \alpha_1)\phi Y_t}{w_t}, \quad (32)$$

$$K_{2t} = \frac{\alpha_2(1 - \phi)Y_t}{r_t}, \quad (33)$$

$$K_{1t} = \frac{\alpha_1\phi Y_t}{r_t}, \quad (34)$$

where  $N_{2t}$  ( $K_{2t}$ ) and  $N_{1t}$  ( $K_{1t}$ ) denote the amount of labor (capital) in efficiency units allocated at the world level to sector 2 and 1, respectively, and  $Y_t \equiv \sum_{j=h,f} p_{jt} y_{jt}$  denotes the integrated equilibrium's world income. Capital-labour intensities are therefore given by:

$$\frac{K_{2t}}{N_{2t}} = \frac{\alpha_2}{1 - \alpha_2} \frac{w_t}{r_t}, \quad (35)$$

$$\frac{K_{1t}}{N_{1t}} = \frac{\alpha_1}{1 - \alpha_1} \frac{w_t}{r_t}. \quad (36)$$

### 6.2.2 Trade Equilibrium's Resource Allocation

Under FPE, countries allocate resources according to the trade equilibrium's capital-labor intensities and their own factor endowments:

$$N_{2,ht} = \frac{\left(\frac{K_{1t}}{N_{1t}} - \frac{K_{ht}}{N_{ht}}\right)}{\left(\frac{K_{1t}}{N_{1t}} - \frac{K_{2t}}{N_{2t}}\right)} N_{ht} \quad (37)$$

$$N_{2,ht} = N_{ht} - N_{2,ht} \quad (38)$$

$$K_{2,ht} = \frac{K_{2t}}{N_{2t}} N_{2,ht} \quad (39)$$

$$K_{1,ht} = K_{ht} - K_{2,ht} \quad (40)$$

$$N_{2,ft} = N_{2t} - N_{2,ht} \quad (41)$$

$$N_{1,ft} = N_{1t} - N_{1,ht} \quad (42)$$

$$K_{2,ft} = K_{2t} - K_{2,ht} \quad (43)$$

$$K_{1,ft} = K_{1t} - K_{1,ht} \quad (44)$$

### 6.2.3 The Factor Price Equalization Condition

In the implementation of the FPE condition we need to distinguish several cases.

**Case 1:**  $N_1 < N_2$

1.  $N_h \in (0, N_1]$  :

$$\frac{K_2}{N_2} \leq \frac{K_h}{N_h} \leq \frac{K_1}{N_1} \quad (45)$$

2.  $N_h \in (N_1, N_2]$  :

$$\frac{K_2}{N_2} \leq \frac{K_h}{N_h} \leq \frac{\left[K_1 + \frac{K_2}{N_2}(N_h - N_1)\right]}{N_h} \quad (46)$$

3.  $N_h \in (N_2, N)$  :

$$\frac{\left[N_2 + \frac{K_1}{N_1}(N_h - N_2)\right]}{N_h} \leq \frac{K_h}{N_h} \leq \frac{\left[K_1 + \frac{K_2}{N_2}(N_h - N_1)\right]}{N_h} \quad (47)$$

**Case 2:**  $N_2 < N_1$

1.  $N_h \in (0, N_2]$  :

$$\frac{K_2}{N_2} \leq \frac{K_h}{N_h} \leq \frac{K_1}{N_1} \quad (48)$$

2.  $N_h \in (N_2, N_1]$  :

$$\frac{\left[ K_2 + \frac{K_1}{N_1}(N_h - N_2) \right]}{N_h} \leq \frac{K_h}{N_h} \leq \frac{K_1}{N_1} \quad (49)$$

3.  $N_h \in (N_1, N)$  :

$$\frac{\left[ K_2 + \frac{K_1}{N_1}(N_h - N_2) \right]}{N_h} \leq \frac{K_h}{N_h} \leq \frac{\left[ K_1 + \frac{K_2}{N_2}(N_h - N_1) \right]}{N_h} \quad (50)$$

**Case 3:**  $N_2 = N_1$

1.  $N_h \in (0, N_2]$  :

$$\frac{K_2}{N_2} \leq \frac{K_1}{N_1} \leq \frac{K_1}{N_1} \quad (51)$$

2.  $N_h \in (N_2, N)$  :

$$\frac{\left[ K_2 + \frac{K_1}{N_1}(N_h - N_2) \right]}{N_h} \leq \frac{K_h}{N_h} \leq \frac{\left[ K_1 + \frac{K_2}{N_2}(N_h - N_1) \right]}{N_h} \quad (52)$$

### 6.3 The Complete Specialization Model

Following Backus *et al.* [2], we assume that, for technological reasons,<sup>35</sup> countries specialize in the production of a single good, labeled 1 for country  $h$  and 2 for country  $f$ . Hence, households in both countries face the following resource constraints:

$$x_{1ht} + x_{1ft} = a_{ht} k_{ht}^{\alpha_1} n_{ht}^{1-\alpha_1}, \quad (53)$$

$$x_{2ht} + x_{2ft} = a_{ft} k_{ft}^{\alpha_2} n_{ft}^{1-\alpha_2}, \quad (54)$$

where  $x_{1jt}$  and  $x_{2jt}$  are the quantities of the two goods consumed in country  $j$  at date  $t$ . A final good, used for consumption and investment, is produced in each country using foreign and domestic goods via Cobb-Douglas production functions. We introduce a “home bias” by assuming that production functions are not symmetric across countries: domestic goods are relatively more important than foreign goods. Formally, households face the following additional resource constraints:

$$c_{ht} + i_{ht} = x_{1ht}^{\phi} x_{2ht}^{1-\phi}, \quad (55)$$

$$c_{ft} + i_{ft} = x_{1ft}^{1-\phi} x_{2ft}^{\phi}, \quad (56)$$

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<sup>35</sup>More precisely, we have to assume that the TFP level in the labor-intensive (capital-intensive) sector drops to zero in country  $h$  (country  $f$ ). This is the only way to make sure that each country produces only one of the two goods. See Baxter [4].

where  $\phi$  is the share of domestic goods in the final good production.<sup>36</sup> The capital stocks evolve according to equation (3).

We can easily show that:

$$\pi_t = \frac{p_{1ht}x_{1ft} - p_{2ht}x_{2ht}}{y_{ht}} = \frac{1 - \phi}{1 - 2\phi} \left( 1 - e_t \frac{y_{ft}}{y_{ht}} \right), \quad (57)$$

where  $e_t = p_{1,ht}/p_{1,ft}$ . Note that in steady state  $y_h \neq y_f$  and  $e \neq 1$  as long as  $\alpha_1 \neq \alpha_2$ : the corresponding values can be found by numerically solving the system of first order conditions evaluated at the steady state.<sup>37</sup>

The different role of  $\phi$  in the final good production function is the only structural difference between our Heckscher-Ohlin setup and the Complete Specialization framework. This difference is unavoidable: with Cobb-Douglas functional forms and under complete specialization, cross-country symmetry implies that the GDP of each country is simply a constant share of world GDP. While the Heckscher-Ohlin and the Complete Specialization frameworks share all the remaining parameters,  $\phi$  has to be calibrated separately. Following again Backus *et al.* [2], we calibrate  $\phi$  to make country  $h$ 's steady-state trade volume share in GDP match the US long-run trade volume share (0.15). The corresponding value for  $\phi$  is 0.925.

This approach guarantees that, in both models, the key feature that drives the trade volume in steady state, (the difference in the relative factor endowments for the HO model and the “home bias” for the CS model) is calibrated to make country  $h$ 's steady-state trade volume match its observed counterpart for the US.

## 7 References

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<sup>36</sup>Backus *et al.* [2] assume a CES production function for the final consumption good, and favour a calibration characterized by a high elasticity of substitution between intermediate goods. For comparability reasons, we stick to the Cobb-Douglas case. Heathcote and Perri [17] estimate the elasticity of substitution in the Backus *et al.* [2] framework, and obtain a value of 0.9. Hence, our Cobb-Douglas seems not to be completely at odds with the empirical evidence.

<sup>37</sup>A detailed Technical Appendix is available from the authors upon request.



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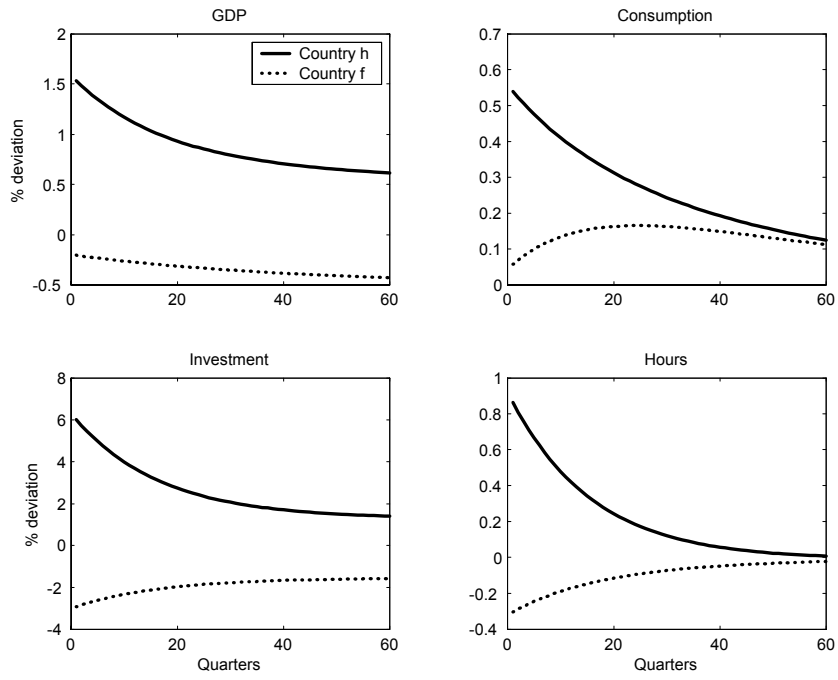


Figure 1: Income and its components (HO - shock to traded sectors).

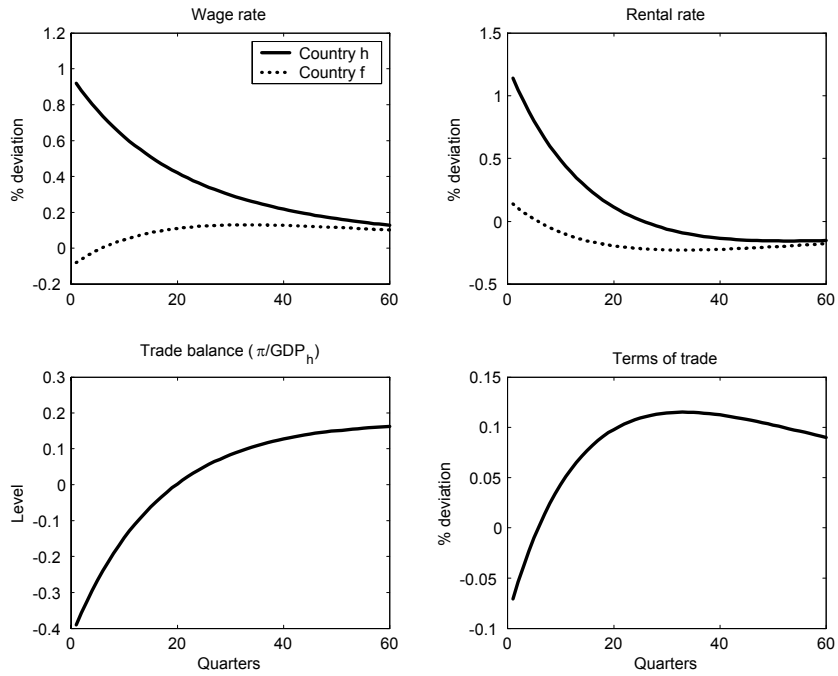


Figure 2: Wages and rental rates (HO - shock to traded sectors).

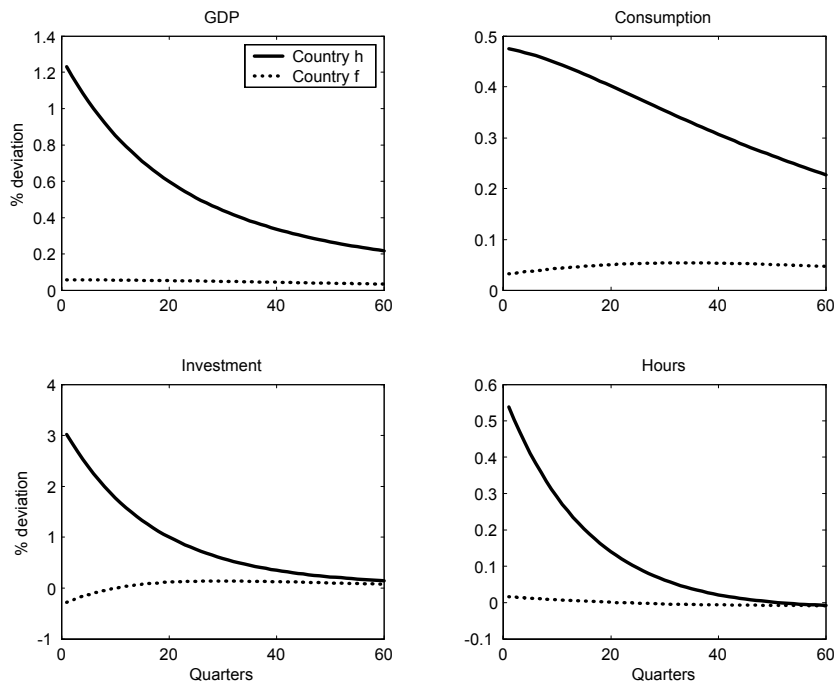


Figure 3: Income and its components (CS - shock to traded sector).

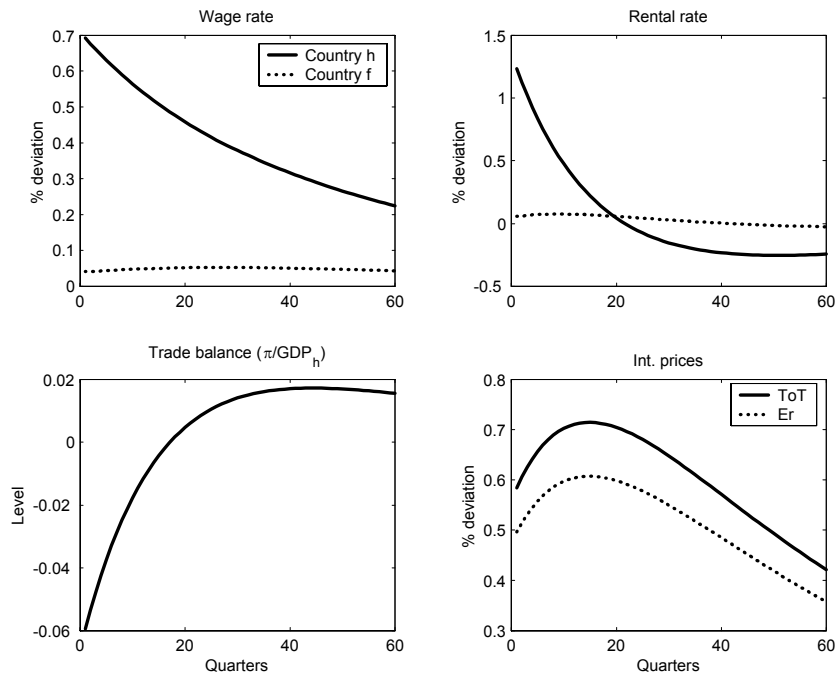


Figure 4: Wages, rental rates, international variables (CS - shock to traded sector).