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The Fiscal Theory of the Price Level: Identifying Restrictions and Empirical Evidence

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Abstract

This paper aims to test some implications of the fiscal theory of the price level (FTPL). We develop a model similar to Leeper (1991) and Woodford (1996), but extended so to generate real effects of fiscal policy also in the "Ricardian" regime, via an OLG demographic structure. We test on the data the predictions of the FTPL as incorporated in the model. We find that the US fiscal policy in the period 1960-1979 can be classified as "Non-Ricardian", while it is "Ricardian" since 1990. According to our analysis, the fiscal theory of the price level characterizes one phase of the post-war US history.

JEL codes: E42, E58, E61, E63

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1 Introduction

There has been recently a renewed interest in the study of the interactions between fiscal and monetary policy, exemplified by the works of Leeper (1991), Sims (1994), Woodford (1994, 1995, 1997 and 2001) and Cochrane (1998, 2000a,b). The main point emphasized by this line of research, that goes under the name of the "Fiscal Theory of the Price Level" (FTPL), is that the present value government budget constraint:

$$\frac{\text{nominal debt}}{\text{price}} = \text{discounted sum of expected primary surpluses} \qquad (1)$$

and fiscal policy play a crucial role in the determination of the price level.

This idea is in sharp contrast with conventional theories of price determination[1], according to which the stock of money (and thus the monetary authority) is the sole determinant of the price level and fiscal policy is (often implicitely) assumed to passively adjust primary surpluses to guarantee solvency of the government for any price level. Such a situation is what Woodford (1995) labels *Ricardian* price determination. The FTPL reverse the argument above: if the fiscal authority is free to choose primary surpluses independently of government debt, then it is the the price level that has to adjust to satisfy the present value government budget constraint. There is only one price level compatible with equilibrium and it is precisely pinned down by the present value government budget constraint. This alternative regime is called *Non Ricardian* in Woodford (1995).

The core distinction between the classical theory and the FTPL lies precisely in the interpretation of the present value budget constraint of the government. According to the conventional interpretation of the monetarist tradition, the government intertemporal equation is a constraint and holds for any price level. According to the FTPL, the government intertemporal equation is an equilibrium condition and as such selects the equilibrium price level.

While this may be seen as a minor difference, important policy issues hinge on the distinction between *Ricardian* and *Non Ricardian* regimes.

According to the standard interpretation that underlies much of the recent literature on monetary economics, a well-designed monetary policy is a necessary and sufficient condition to guarantee low inflation. An independent Central Bank, with a strong institutional commitment to guarantee price stability will automatically compel fiscal authorities to adopt the right fiscal policy.

[1] The classical reference is Sargent and Wallace (1981).

The policy implications of the FTPL are very different: a well designed monetary policy is not sufficient to guarantee stable prices, unless additional steps are taken to limit the freedom of the fiscal authority.

Many papers have dealt with theoretical issues concerning the FTPL, with its logical soundness and with its implication for the optimal policy mix[2]. The empirical contributions, on the other side, are very few, the reason being that it is very hard to find theoretical implications useful to identify regimes. The intertemporal government budget constraint (1) holds in equilibrium in both regimes. What differs between them is the causal link between prices and surpluses: in a Non Ricardian regime, equilibrium is restored with prices adjusting to expected surpluses; in a Ricardian world equilibrium is restored with expected surpluses responding to the price level. Given that we observe only equilibrium sequences in the data, it is impossible to infer something about causality from the budget constraint alone: the FTPL per se has no testable implications. Of course, this problem is not specific to the FTPL: every economic theory requires additional assumptions to deliver testable implications and the FTPL makes no exception. The most common example of this is the identification of supply and demand schedules from a set of equilibrium points: with no additional assumptions, the two curves cannot be identified.

This paper is one of the first attempts to break the observational equivalence, by imposing additional identifying assumptions derived from a model.

We develop a dynamic general equilibrium model in the New-neoclassical tradition, which will deliver either a fiscalist or a classical price level determination in function of the specification of monetary and fiscal policy rules. The model will produce regime-specific restrictions, in terms of impulse response of the real interest rate to tax shocks. We extend on the previous theoretical literature by adding to a neo-keynesian sticky-prices framework an overlapping generation demographic structure[3]. The motivation for doing this, that will be discussed in detail later, is to make our model compatible with the large body of empirical evidence that finds large positive effects of fiscal shocks on output[4].

We then turn to the empirical analysis, in which we study US data from 1960 on, by means of VAR techniques. We compute impulse responses to tax shocks and compare the theoretical impulse responses in the two regimes with those estimated from the VAR. Results show that the empirical impulse

[2]For references, see footnote 2 in Woodford (2001).

[3]Cushing (2000) develops a similar models with OLG structure and derive implications for the FTPL. In his model, however, output is exogenous.

[4]See Blanchard and Perotti (2000), Mountford and Uhlig (2002) or Fisher, Edelberg and Eichenbaum (2001).

responses to a fiscal shock for the period 1960-1979 are in line with the implications of the FTPL as predicted by our model. The period 1990-2003 is characterized by a Ricardian regime in which monetary policy is the nominal anchor and fiscal policy is taking care of debt. The intermediate period, 1982-1990, can be rationalized as a regime of uncoordinated policies in which monetary policy was actively fighting inflation, but fiscal policy was not responding to developments in real debt.

The paper is organized in six sections. In Section 2 we briefly review the fiscal theory of the price level. In Section 3 we discuss the available empirical evidence. In Section 4 we develop the model that will provide us with the restrictions to be tested. In Section 5 we go to the data and we show results of the VAR analysis. Section 6 concludes.

2 The FTPL and Dynamic General Equilibrium Models

Starting from the seminal paper of Leeper (1991), dynamic general equilibrium models embedding the two alternative ways in which the present value government budget constraint can be satisfied have been developed (see Woodford, 1996). Leeper considers a representative agent that derives utility from consumption and real money balances, living in an economy with exogenous output and flexible prices.

The flow budget constraint for real debt is standard:

$$b_t = \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} - s_t \tag{2}$$

The monetary authority is assumed to follow a nominal interest rate rule in response to inflation deviations from the steady state:

$$i_t = \phi_\pi \pi_t \tag{3}$$

The fiscal authority is assumed to fix lump-sum taxes τ_t in response to real debt deviations from steady state, b_{t-1}, according to:

$$\tau_t = \gamma b_{t-1} + \theta_t \tag{4}$$

where the term θ_t is a random shock to fiscal policy.

By iterating forward equation (2), taking expectations conditional to information up to time t and imposing the transversality condition derived from utility maximization, one obtains the intertemporal budget constraint of the government debt[5]:

$$b_t = E_t \sum_{j=t}^{\infty} \left(\prod_{i=t}^{j} \frac{1+\pi_{i+1}}{1+i_i} \right) s_j \tag{5}$$

We will come back in greater detail to this equation when discussing the empirical evidence, because the channels through which it is satisfied are critical for the issue Ricardian versus Non Ricardian.

If one considers "local" equilibria, the magnitude of the policy parameters ϕ_π and γ plays a critical role for the behavior of the economy.

If monetary policy is "active" in contrasting inflation (large ϕ_π), and fiscal policy responds "passively" to debt by increasing taxes (large γ), one obtains a Ricardian price determination: prices are under the control of the monetary authority and fiscal solvency is under the responsibility of fiscal policy. Tax shocks have no real effects in this case. Given that taxes respond strongly to debt, rational agents anticipate that expansionary tax shocks will be followed by a fiscal restriction of the same size in present discounted value to stabilize debt: equilibrium in equation (5) is restored through higher surpluses, s_t.

If fiscal policy is "active" and does not take into account debt financing (small γ), and monetary policy is "passive" in reacting to inflation (small ϕ_π), price determination is Non-Ricardian: the equilibrium price level will be the one that guarantee fiscal solvency in equation (5). In this regime an expansionary tax shock will stimulate aggregate demand through wealth effects. Agents know that taxes will not increase much in response to an increase in debt, they feel richer and try to consume more. If output is exogenous and prices are flexible (as in Leeper (1991)), the only effect of the tax shock is to drive prices up. From the perspective of fiscal solvency, this is precisely what is needed: inflation deflates the stock of nominal debt and there is no need for corrections in the surplus process. If one allows for the existence of a short-run Phillips curve, as in Woodford (1996), shocks to lump-sum taxes do stimulate output in the short run and Ricardian equivalence is violated: even if all the classical assumptions are satisfied, a reduction in lump-sum taxes financed by issuing bonds have real effects.

If both policies are passive (small ϕ_π and large γ), the equilibrium is indeterminate. None of the two policies is providing the nominal anchor to the system and there are many paths consistent with a rational expectations equilibirum.

[5]Cochrane (1998) label equation (5) as the *valuation equation* for government debt. The name comes from the close analogy between (5) and the equation defining the price of a stock: $\frac{\text{stock's nominal value}}{\text{stock price}}$ =discounted sum of expected real dividends

If both policies are active (large ϕ_π and small γ), there is no stationary solution for the linearized system. Equilibria of the non-linear version of the model do exist, but they involve explosive paths. For details on this, see Woodford (1996).

3 The Empirical Evidence

There are few empirical works on the FTPL. The reason is that it is very hard to find testable implications.

The government's intertemporal budget constraint holds in equilibrium in both regimes and *per se* delivers no testable implications. By considering only equation (6), the two regimes are observationally equivalent. In the literature, there are different views on this point. Among the pessimistic, Kocherlakota and Phelan (1999) write: *"The distinction is on [...] how the government would have acted for price sequences other than [the equilibrium ones] [...]. The FTPL is not falsifiable"*

Among the optimistic, Christiano and Fitzgerald (2000) *"...view the FTPL as a starting point for a natural set of auxiliary assumptions which do restrict time series data, and then test those assumptions"* and Woodford (1998): *"Hypothesis about causality can never be confirmed or rejected [...] without the help of identifying assumptions"*.

We share these last comments about the need for additional identifying assumptions: this is exactly what this paper is about.

Let us review the available evidence. A first group of works derive implications for the FTPL from the stability analysis. As discussed above, if the valuation equation is embedded in a dynamic general equilibrium model, the coefficients associated with the policy rules play a key role. A Ricardian regime requires a large ϕ_π in the monetary rule *and* a large γ in the fiscal rule. A Non Ricardian regime requires a small ϕ_π in the monetary rule *and* a small γ in the fiscal rule. This observation can be used in the empirical analysis.

A work along these lines is Leeper (1989), in which various assumptions about the timing and the flow of information to agents are combined with different policy rules in an analysis of US data. Loyo (1998) shows evidence that a switch from a stable Non Ricardian regime to an unstable one has occurred in Brazil in mid 80's. Brazil was characterized by a "low ϕ_π" monetary policy, accompanied by a "low γ" fiscal policy. Around mid-80, Brazilian monetary policy switched to a "high ϕ_π" rule without any compensation in the fiscal side. The effect was a "fiscalist hyperinflation", as predicted by his model.

By using similar considerations, Woodford (1998) concludes that since before 1979 US monetary policy was passive, it follows that fiscal policy had to be active and thus that the regime was Non Ricardian. The opposite must hold after 1982.

A second group of authors, Cochrane (1998) and Canzoneri, Cumby and Diba (2001, CCD thereafter), works with the intertemporal budget constraint directly. For the sake of clarity, let us first rewrite the intertemporal budget constraint:

$$b_t = E_t \sum_{j=t}^{\infty} \left(\prod_{i=t}^{j} \frac{1+\pi_{i+1}}{1+i_i} \right) s_j = E_t \sum_{j=t}^{\infty} \left(\prod_{i=t}^{j} \alpha_i \right) s_j \tag{6}$$

in which we define the discount factor, $\alpha_i = \frac{1+\pi_{i+1}}{1+i_i}$.

Cochrane, pointing out the observational equivalence of the two regimes and stressing the lack of testable implications, provides an ingenious Non Ricardian interpretation of US data from 1960 on, by showing how an exogenous primary surplus can be made consistent with the intertemporal budget constraint and with the data.

CCD (2002) on the other side, propose to verify the empirical plausibility of the FTPL studying the response of debt to surplus shocks in a bivariate VAR in primary deficit and public debt. Their approach is based on the following idea. In a Ricardian regime, equation (6) holds for any price level chosen by the monetary authority. From the flow budget constraint (2), if surplus increases, real debt decreases ($s_t \uparrow \rightarrow b_t \downarrow$). In a Non Ricardian regime, the intertemporal budget constraint determines the price level. From equation (6), CCD identify three ways in which real debt can respond after a positive surplus shock, depending on the time-series properties of the surplus process and of discount factors:

$$s_t \uparrow \begin{cases} 1. & \text{if } Corr(s_t; s_{t+k}) = 0 & and & Corr(s_t; \alpha_{t+k}) = 0 & \rightarrow & b_t = \text{constant} \\ 2. & \text{if } Corr(s_t; s_{t+k}) > 0 & and & Corr(s_t; \alpha_{t+k}) > 0 & \rightarrow & b_t \uparrow \\ 3. & \text{if } Corr(s_t; s_{t+k}) < 0 & and & Corr(s_t; \alpha_{t+k}) < 0 & \rightarrow & b_t \downarrow \end{cases}$$

CCD argue that in cases 1 and 2 the response of b_t to a s_t shock allows to disentangle Ricardian from Non Ricardian regimes: if the response of b_t to s_t is negative, the regime is Ricardian; if the response of b_t to s_t is non-negative, the regime is Non Ricardian. The third case is the one that creates identification problems, as real debt moves in the same direction in both regimes. CCD's conclusion is that, since the response of debt to a surplus shock is always negative in the data and since they do not find evidence of the correlations characterizing case 3, Ricardian explanations are more plausible than Non Ricardian ones.

As noticed by Cochrane (1998), CCD's analysis takes equation (6) as the relevant equation in a Non Ricardian regime and the flow budget constraint (2) as the relevant one in a Ricardian regime. In fact, the two equations are two different ways to express the same thing: b_t, and as such, they both hold in both regimes. Let us equate the flow budget constraint (2) to equation (6):

$$b_t = \frac{1}{\alpha_t} b_{t-1} - s_t = E_t \sum_{j=t}^{\infty} \left(\prod_{i=t}^{j} \alpha_i \right) s_j \tag{7}$$

Let us consider a Non Ricardian regime, in which s_t follows an exogenous process and let us first assume that in response to a positive surplus shock the discount factor α_t does not move on impact. From the flow budget constraint on the left hand side, it is clear that b_t has to fall. Even in a Non Ricardian regime, an increase in s_t will reduce b_t. Using the response of real debt to a surplus shock as the identifying assumption delivers a test with no power: the hypothesis tested is always true, no matter how different side assumptions are specified.

What about the right hand side? It has to decrease as well, as it equals b_t. There is no other possibility. This automatically imposes some restrictions on the joint behavior of surpluses and discount factors on the right hand side of (7): the net effect has to be negative.

If α_t jumps on impact, the partial equilibrium, uniequational analyis presented above is not sufficient: in principle, b_t can move either up or down.

The general equilibrium model in the following Section will be based on the same intuition. In the model a surplus shock will reduce b_t, no matter the regime. We will show that the key to identify the regimes is not the response of b_t, but the endogenous response of discount factors α_{t+h}.

4 In Search for Identifying Assumptions: A Model

In this section we extend the analysis of Leeper (1991) and Woodford (1996). The model we have in mind, similar to Ghironi (2000), have the following ingredients: monopolistic competition between a continuum of firms producing differentiated goods, sticky prices (introduced through quadratic costs of price adjustment (Rotemberg, 1982)), endogenous labor supply and output and an OLG demographic structure[6].

The choice of adding the OLG demographic structure is motivated by the desire to make the predictions of our model in line with empirical evidence on the effects of fiscal policy. The available studies find, without exceptions, that there are real effects of fiscal policy[7].

In the optimizing representative agent models *á la* Leeper and Woodford, lump-sum tax shocks have no effects in a Ricardian regime, while they have real effects in a Non Ricardian world. If there is no alternative way to introduce real effects of fiscal shocks in the model, the only way to justify the empirical findings is indeed the FTPL. Conditional to the structure of the model, the mere fact that there are real effects of fiscal shocks would imply the validity of the FTPL. By introducing another friction in the model, we allow for an alternative explanation of the real effects of fiscal policy that does not require the FTPL to hold. By adding OLG, the model will deliver predictions for the behavior of output consistent with the data[8] under both regimes.

4.1 Consumers

At each instant in time the economy is populated by a continuum of agents of measure one. As in Blanchard (1985), in each period each agent faces a probability μ of dying. By assuming that the probability of dying is independent of age, μ represents also the fraction of agents which die in each period. Conversely, $1-\mu$ represents the probability of being still alive tomorrow. The presence of this term reduces the discount factor and makes agents more impatient, as they know that their time horizon is now shorter. As agents are uncertain about the time of their death, they stipulate insurance contracts with perfectly competitive insurance companies. Agents receive a certain amount of resources when alive in exchange of all their wealth in case of death. The no-profits condition in the insurance market generates an additional return on the assets held by the agents equal to $\frac{1}{1-\mu}$ (see Blanchard

[6]Cushing (2000) develops a similar models with OLG structure and derive implications for the FTPL. In his model, however, output is exogenous.

[7]See Blanchard and Perotti (2000), Mountford and Uhlig (2002) and Fisher, Edelberg and Eichenbaum (2001).

[8]The choice of introducing OLG to break Ricardian equivalence is of course one among other possible choices. Another reasonable choice would be the introduction of distortionary taxation, as in Schmitt-Grohé and Uribe (2004). It would be interesting to compare the outcomes of different modelling strategies for the issue at stake here. Our guess is nevertheless that distortionary taxation will generate effects very similar to those obtained by OLG.

(1985)). This additional term appears on the right hand side of their budget constraint below.

Agents born at time s maximize under perfect foresight the following utility function with respect to real consumption, c_t^s, leisure, L_t^s, nominal 1-period government bonds holdings, B_t^s and money holdings M_t^s:

$$\max \sum_{t=s}^{\infty} [(1-\mu)\beta]^t \left\{ [\delta \log c_t^s + (1-\delta) \log L_t^s] - \chi \left(\frac{M_t^s}{p_t} \right)^h \right\} \tag{8}$$

subject to the following sequence of budget constraints:

$$c_t^s + \frac{M_t^s}{p_t} + \frac{B_t^s}{p_t} = \frac{Y_t^s}{p_t} - \frac{T_t^s}{p_t} + \left[\frac{M_{t-1}^s}{p_t} + R_{t-1} \frac{B_{t-1}^s}{p_t} \right] \frac{1}{1-\mu} \tag{9}$$

where total income $Y_t^s = \Pi_t + w_t N_t^s$ is the sum of profits[9] and labor income. w_t is nominal wage (equal across firms, by assuming perfect competition in the labor market), and $N_t^s = 1 - L_t^s$ is labor supply.

All cohort-specific variables x are denoted as: x_j^k. The superscript k represents the date of birth, while the subscript j indicates the moment of time. Thus, c_t^s is real consumption at time t for an agent born at time s. The consumption index c_t^s is defined as:

$$c_t^s = \left[\int_0^1 (c_t^{s,z})^{\frac{\eta-1}{\eta}} \, dz \right]^{\frac{\eta}{\eta-1}}$$

where $c_t^{s,i}$ is consumption at time t of good i of the representative agent of the cohort born at time s and η is the elasticity of substitution between goods (assumed to be constant). The price index p_t is equivalently defined as:

$$p_t = \left[\int_0^1 (p_t^z)^{1-\eta} \, dz \right]^{\frac{1}{1-\eta}}$$

where p_t^i is the price of good i at time t.

At any time t an agent born at time s will decide how to allocate her resources coming from profits Π_t, labor income: $w_t N_t^s$, and capital income. The portfolio of assets available to agents is composed by a one-period nominal bond, B_{t-1}^s which pays a gross nominal interest rate $R_{t-1} = 1 + i_{t-1}$ issued by the government and fiat money M_{t-1}^s that pays no interest.
T_t^s are nominal (lump-sum) taxes, assumed equal for each cohort: $T_t^s = T_t$.

[9]By assuming distributed ownership, $\Pi(t)$ represents also aggregate profits.

4.2 Firms

The economy is populated by a continuum of firms, indexed by i, with $i \in [0,1]$, each producing a differentiated good with the following production function: $y_t^i = N_t^i$. Each firm chooses the price p_t^i and the demand of labor, L_t^i, to maximize profits from $t = 0$ to ∞, subject to three constraints: the demand for its good, an adjustment cost for changing its price: $\frac{\phi}{2}\left(\frac{p_t^i}{p_{t-1}^i} - (1+\pi_0)\right)^2$ (where π_0 is the steady state inflation rate) and the market clearing condition.

Real profits at time t are then:

$$\frac{\Pi_t^i}{p_t} = y_t^i \frac{p_t^i}{p_t} - \frac{w_t}{p_t} N_t^i - \frac{\phi}{2}\left(\frac{p_t^i}{p_{t-1}^i} - (1+\pi_0)\right)^2$$

In the maximization, each firm takes p_t, y_t and w_t as given.

4.3 Government

The government is composed by two authorities.

The monetary authority, the Central Bank, prints money, M_t.

The fiscal authority has the power to tax agents, decides about real public expenditures, g_t and issues a 1-period nominal bond, B_t to finance eventual primary deficits.

The budget constraint of the consolidated public sector, expressed in nominal terms, is:

$$B_t = R_{t-1}B_{t-1} + p_t g_t - T_t - M_t + M_{t-1}$$

We assume that seigniorage $M_t - M_{t-1}$ is perfectly rebated to agents via lump-sum transfers Υ_t. Let us define real taxes as: $\tau_t = \frac{T_t}{p_t}$

Consequently, the budget constraint can be rewritten in real terms as:

$$\frac{B_t}{p_t} = \frac{1+i_{t-1}}{1+\pi_t}\frac{B_{t-1}}{p_{t-1}} + g_t - \tau_t$$

The two authorities follow simple rules. The monetary authority follows a Taylor-type rule:

$$i_t = \phi_0 + \phi_\pi \pi_t \tag{10}$$

The fiscal authority follows a rule linking real taxes to the past value of real debt, as in Leeper (1991), plus an exogenous shock[10], θ_t, assumed to

[10]The presence of the random term θ_t, may seem at odds with the assumption of perfect foresight. Notice however that we will log-linearize the model around a deterministic

follow an AR(1) process, $\theta_t = \rho\theta_{t-1} + \xi_t$:

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \theta_t \tag{11}$$

4.4 Solving the Model

4.4.1 Consumers

Consumers solve their maximization problem in two steps.

First, they decide how to allocate the resources between money, bonds and consumption and how much labor effort to supply. The first order conditions for this problem are standard.

The Euler equation for consumption:

$$c^s_{t+1} = \frac{1+i_t}{1+\pi_{t+1}}\beta c^s_t \tag{12}$$

The labor supply equation:

$$N^s_t = 1 - \left(\frac{1-\delta}{\delta}\right)\frac{p_t}{w_t}c^s_t \tag{13}$$

and money demand:

$$\frac{M^s_t}{p_t} = \left[\frac{\chi}{\delta}c^s_t\frac{1+i_{t+1}}{i_{t+1}}\right]^h \tag{14}$$

After having decided the total amount of resources to be allocated to consumption, the agent decides the optimal allocation between the different goods supplied by the firms. The demand for good i is given by:

$$c^{s,i}_t = \left(\frac{p^i_t}{p_t}\right)^{-\eta} c^s_t \tag{15}$$

4.4.2 Government

We assume that the government demands all differentiated goods and that it has the same demand function as private consumers:

$$g^i_t = \left(\frac{p^i_t}{p_t}\right)^{-\eta} g_t$$

steady-state. Agents can be surprised by unexpected shocks that moves the economy away from the steady state. For a similar point, see Smets and Wouters (2002).

4.4.3 Firms

The firms maximize the infinite stream of discounted profits by choosing p_t^i and N_t^i.

The first order condition with respect to p_t^i gives:

$$p_t^i = p_t \lambda_t^i \Psi_t^i \tag{16}$$

where the mark-up Ψ_t^i is defined in Appendix 1. If prices are flexible ($\phi = 0$), the mark-up becomes a constant: $\Psi_t^i = \frac{\eta}{\eta - 1}$.

The first order condition with respect to N_t^i gives the real wage:

$$\frac{w_t}{p_t} = \lambda_t^i \tag{17}$$

Combining the two, one obtains the labor demand schedule:

$$\frac{w_t}{p_t} = \frac{p_t^i}{p_t} \frac{1}{\Psi_t^i} \tag{18}$$

4.5 Aggregation

Having derived the behavior of cohort-specific magnitudes, we now compute aggregate variables.

Details are reported in Appendix 1. Here we just report the aggregation equation for a generic variable k_t^s:

$$k_t = \sum_{s=-\infty}^{t} \mu(1-\mu)^{t-s} k_t^s \tag{19}$$

This equation simply takes into account the two aspect of cohort heterogeneity: the fact that the dimension of each cohort decreases as time passes and that each cohort may make different choices.

The aggregate Euler equation becomes:

$$c_t = \frac{1}{\beta(1+r_{t+1})} \left[\frac{1}{1-\mu} c_{t+1} - \frac{\mu}{1-\mu} c_{t+1}^{t+1} \right]$$

where newborn consumption, c_t^t, equals a fraction $\delta(1-\beta(1-\mu))$ of their after-tax lifetime resources inc_t:

$$c_t^t = \delta(1-\beta(1-\mu)) inc_t$$

with inc_t defined as:

$$inc_t = \frac{1-\mu}{1+r_{t+1}} inc_{t+1} + \left[\frac{w_t}{p_t} + \frac{\Pi_t}{p_t} - \tau_t\right]$$

4.6 A Special Case: $\mu = 0$

In this section we analyze the case in which agents are infinitely lived. In this simplified setup we provide the intuition behind the dynamics of the model and we show why the FTPL would be the only way to rationalize the empirical evidence.

The case $\mu = 0$ is similar to the model in Woodford (1996). We add to his framework the specification of a fiscal rule[11]. We log-linearize the model around a deterministic steady state[12] and we reduce the system to the following standard 3 equations:

$$\begin{cases} \hat{\pi}_{t+1} = \frac{1}{\beta}\hat{\pi}_t - k\hat{y}_t \\ \hat{b}_t = (\frac{1}{\beta} - \gamma)\hat{b}_{t-1} + \frac{1}{\beta}(\phi_\pi - 1)\hat{\pi}_{t-1} - \theta_t \\ \hat{y}_{t+1} = \frac{\bar{c}}{\bar{y}}(\phi_\pi - \frac{1}{\beta})\hat{\pi}_t + \left[1 + \frac{\bar{c}}{\bar{y}}k\right]\hat{y}_t \end{cases} \tag{20}$$

where $k = \dfrac{\bar{r}N(\eta-1)\bar{y}(\delta\bar{w} + (1-\delta))}{\bar{c}(1-\delta)\phi}$

The first equation is the neo-Keynesian forward-looking Phillips curve. The second equation is the government budget constraint. The third equation is the neo-Keynesian forward-looking IS curve.

In matrix notation, the system can be rewritten as:

$$\begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{b}_t \\ \hat{y}_{t+1} \\ \hat{z}_{t+1}(=\hat{\pi}_t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & 0 & -\frac{k}{\beta} & 0 \\ 0 & \frac{1}{\beta} - \gamma & 0 & \frac{1}{\beta}(\phi_\pi - 1) \\ \varphi(\phi_\pi - \frac{1}{\beta}) & 0 & 1 + \varphi(\frac{k}{\beta} + \phi_y) & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{b}_{t-1} \\ \hat{y}_t \\ \hat{z}_t(=\hat{\pi}_{t-1}) \end{bmatrix} - \begin{bmatrix} 0 \\ \theta_t \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

where: $\vartheta = \frac{\bar{c}}{\bar{y}}$.

As there are two forward-looking variables in the system, the existence of a unique stationary solution requires the existence of two eigenvalues of the transition matrix outside the unit circle (Blanchard and Kahn, 1980).

[11] The exogenous deficit specification assumed in Woodford (1996) will just be the special case $\gamma = 0$.

[12] Appendix 1 reports the full log-linearized system and the steady-state values of all the variables.

The two stability regions in function of the policy parameters: $[\phi_\pi, \gamma]$, can be characterized as follows:

$$\text{Non Ricardian region:} \quad \begin{cases} \left|\frac{1-\gamma\beta}{\beta}\right| < 1 \\ \\ \phi_\pi < 1 \end{cases}$$

$$\text{Ricardian region:} \quad \begin{cases} \left|\frac{1-\gamma\beta}{\beta}\right| > 1 \\ \\ \phi_\pi > 1 \end{cases}$$

As discussed in Section 2, large ϕ_π and small γ delivers Ricardian price determination, while small ϕ_π and large γ characterize Non Ricardian regimes.

To better understand the different dynamics of the model in the two regimes, in the following sub-section we compute impulse responses.

4.6.1 Impulse response to tax shocks

Let us concentrate on the dynamics of the system in response to shocks to lump-sum taxes. Consider the effects of an unexpected and temporary reduction in $\hat{\tau}_t$. Impulse response functions of the relevant variables to a temporary tax decrease in both regimes are reported in Figure 1[13].

[Figure 1]

Consider first the case of a Non Ricardian regime, with ϕ_π and γ small. The immediate effect of a tax reduction is to increase the stock of real debt.

From the government budget constraint:

$$\hat{b}_t = (\frac{1}{\beta} - \gamma)\hat{b}_{t-1} + \frac{1}{\beta}(\phi_\pi - 1)\hat{\pi}_{t-1} - \theta_t$$

it is easy to see that in the case of a small γ, if inflation remains at its steady state value, real debt explodes, as $\frac{1}{\beta} - \gamma$ is larger than one. As γ is small, agents understand that future taxes will not be increased much in response to the increase in debt. This in turn stimulates aggregate demand: public debt is net wealth, in opposition to the Ricardian equivalence result in Barro (1974). The increase in inflation is caused by the excess demand for goods and is needed to reduce the real interest rate and the interest burden on real debt (thanks to the small ϕ_π).

[13] The Non-Ricardian regime is simulated with: $\gamma = 0$ and $\phi_\pi = .5$. The Ricardian regime is simulated with: $\gamma = .1$ and $\phi_\pi = 1.5$. The process for the shock has been set as an AR(1) with a coefficient of 0.5.

As output is endogenous (as in Woodford (1996)), the increase in demand is accompanied by an increase in output. Given that in this model seigniorage financing is ruled out by assumption, the channels through which fiscal policy affects inflation are different from the traditional analysis of monetary-fiscal policy interactions *à la* Sargent and Wallace (1981), as noted by Woodford (1996).

In the case of a Ricardian fiscal policy, when ϕ_π and γ are sufficiently big, the adjustment dynamics have a different flavour. In response to a reduction in $\hat{\tau}_t$ real debt increases. This time agents know that taxes will be increased in the future sufficiently (γ is sufficiently "big") to guarantee stability of real debt. No wealth effect will affect demand and nothing else will have to adjust. The only effect of a tax decrease will be a temporary increase in real debt that will be absorbed at a constant real interest rate by higher taxes. We are back to Ricardian Equivalence: the way in which a given stream of government expenditures is financed is irrelevant for the dynamics of prices, nominal, real interest rates and output.

Under the assumption $\mu = 0$, fiscal shocks have real effects only in Non Ricardian regimes, while they are neutral in Ricardian regimes.

Empirical evidence points to significant real effects of fiscal shocks. It is immediate to conclude that the only way to rationalize this finding in the above model is the FTPL.

In what follows, we move to the general case in which $\mu > 0$. We show that in that setup fiscal shocks generate real effects independently of the regime.

4.7 The General Case: $\mu > 0$

In this Section we allow for μ, the probability of death, greater than zero. In this case, one more variable has to be added to the system (21), as we have to keep track of the lifetime after-tax resources at time t, inc_t. The dynamic system is composed by 4 equations, one for each element of the vector $a_{t+1} = [\pi_{t+1}, y_{t+1}, inc_{t+1}, b_t]$. We compute numerically the solution of the model[14]. Figure 2 shows that the stability regions are similar to those obtained in the case without OLG.

[Figure 2]

Small values of ϕ_π must be matched with small values for γ and viceversa.

[14]Steady state values are reported in the Appendix.

Impulse responses from this model in response to a reduction in taxes are reported in Figure 3[15]. Differently from the responses in the previous Section, the model now generates real effects also in the Ricardian regime: output reacts positively to a tax cut.

[Figure 3]

Agents alive at the time of the shock know that they will pay only a fraction of the additional taxes that will be levied in the future to pay back the additional debt and increase the demand for goods[16].

4.7.1 Implications for the Empirical Analysis

There are some features to be highlighted here, which will be important for the empirical analysis.

The first is the response of real debt to a fiscal expansion. In both regimes, a fiscal expansion increases real debt. This is in contrast with the key identifying assumption in CCD (2001), as discussed above.

The second is the response of discount factors (or equivalently, of real interest rates). The response of r_t to a positive fiscal shock is positive in a R regime but negative in a Non Ricardian regime. This will be the crucial response to focus on in the empirical analysis.

A third aspect worth to be discussed here is the issue of dynamic (in)efficiency and Ponzi-games. The real interest rate has been sometimes negative in the period from 1960 to 1979. This raises the issue of the possibility of Ponzi-games by the government. In other words, a negative real interest rate may allow the government to roll over his debt, without ever increasing taxes and making the government budget constraint always satisfied. It this were the case, the government budget constraint might not pin down the price level. However, as it has been shown by Blanchard and Weil (2002), a negative real interest rate is neither a necessary nor a sufficient condition to allow Ponzi-games and it is not sufficient to conclude that the price level was not pinned down by the government budget constraint.

[15] The Non-Ricardian regime is simulated with: $\gamma = 0$ and $\phi_\pi = .5$. The Ricardian regime is simulated with: $\gamma = .1$ and $\phi_\pi = 1.5$. The process for the shock has been set as an AR(1) with a coefficient of 0.5.

[16] A word of caution about the magnitude of the parameter μ is warranted. Reasonable values for μ generates only tiny real effects. Our aim here is not to generate empirically sensible quantitative results, but simply to produce conditional correlations useful to understand the empirical evidence.

5 What Happened in the US?

So far we have built a theoretical model that implies different responses of the economy to shocks depending on the regimes followed by the fiscal and the monetary authority.

Evidence available for the US economy (see Clarida, Gali and Gertler, 1998 and 2000) suggests that one of these parameters, namely ϕ_π, has changed at the beginning of the eighties. Clarida, Gali and Gertler (2000) find evidence that before '79 the coefficient ϕ_π was smaller than one, while it has become significantly larger than one after '82. The implication for our model is thus that a regime shift occurred around 1980 in the monetary policy rule. If we believe that the economy was on a stationary path both before '79 and after '82, this should automatically imply a simultaneous increase in the parameter γ in the fiscal rule (see Woodford, 1998 on this point)[17,18].

We use the evidence on the shift in the parameter ϕ_π and split the sample in two parts. The first one go from 1960 to 1979 and as $\phi_\pi < 1$ we would expect it to be characterized as a "Non Ricardian" regime. The second period go from 1983 to 2003 and as $\phi_\pi > 1$, it should be identified as a "Ricardian" regime. The idea is now to go to the data and study, given the robust features of the model in both regimes, the adjustment path of the "critical" variables.

Before doing this, we can see many interesting facts by simply inspecting graphically the characteristics of the series in Figure 4[19].

[Figure 4]

In Table 1 we report correlations between relevant variables in the subsample 1960-1979[20]:

Table 1: Correlations (1960 - 1979)

	d	r	Δb_{tot}	Gap
d	1			
r	-.85	1		
Δb_{tot}	.63	-.62	1	
Gap	-.56	.55	-.37	1

[17] Clarida, Gali and Gertler (2000) do not consider the possibility of a Non-Ricardian fiscal policy. As a consequence, they propose an interpretation of pre-79 US data based on an indeterminate equilibrium and on the role played by sunspot shocks in explaining the data.

[18] Favero and Monacelli (2003) estimate regime switching monetary and fiscal rules on US post-1960 data. They do find evidence of regime switching in both rules.

[19] Data are for the US, at quarterly frequency. Detailed data descriptions are in Appendix 2. Shaded areas correspond to the period 1979:4 - 1982:3.

[20] d is primary deficit; r is the real (ex-post) real interest rate; b_{tot} is total real debt and Gap is a measure of detrended output constructed by the St. Louis Fed.

We can notice a strong positive link between deficit and variations in debt (.63) and a negative correlation between real interest rate and variations in real debt (-.62). An interesting figure is the very strong and negative correlation between primary deficit and the real interest rate (-.85). The output gap is negatively correlated with deficit (-.56) and variations in debt (-.37) and it is positively correlated with the real interest rate (.55).

We now turn to the most recent period. In Table 2 we report correlations for the period 1983-2003.

Table 2: Correlations (1983 - 2003)

	d	r	Δb_{tot}	Gap
d	1			
r	-.16	1		
Δb_{tot}	.79	-.10	1	
Gap	-.86	-.12	-.66	1

We can see sizeable differences between Tables 1 and 2. The correlation between the deficit and the real interest rate increases from -.85 to -.16, the one between the output gap and the real interest rate falls from .55 to -.12. The increased correlation between the defict and the output gap (from .-.56 to -.86) seems to point out a more important role for countercyclical fiscal policy in the second period than before 1979.

To summarize, we see significant differences in these figures on the two sub-samples. We take this as an interesting observation pointing to changes in the underlying data generating process. However, by simply considering unconditional correlations, we cannot disentangle the automatic response of the relevant variables to business cycle developments from the response to fiscal shocks. This is why we move to a more refined analysis below, trying to move from simple comovements to cause-effect relationships.

5.1 Understanding the Causality: 1960 - 1979

In order to disentangle the automatic response of fiscal variables to business cycle from the response to fiscal shocks we estimate a VAR on the following quarterly variables for the US[21]: real GDP (Y), real receipts (T), real expenditures (G), real interest rate (federal funds rate - inflation) (R) and real (total) debt (B).

We use a triangular identification scheme, with ordering as above[22].

The two shocks we want to concentrate on are the one to real GDP, that should capture the automatic response of fiscal policy to shocks to the real

[21] Data definitions are in Appendix 2.

[22] We estimated the VAR with a constant and 2 lags. 2 lags are sufficient to obtain white noise residuals.

economy and the one to taxes that should capture the effect of a "pure" fiscal shock (as the one in the model economy). Results are reported in Figure 5[23].

[Figure 5]

The responses to real GDP are as expected. In response to a positive GDP shock, taxes increase. Debt significantly decreases and the real interest rate increases. This is what we would expect from a countercyclical policy mix responding endogenously to developments in the real economy.

The response to a pure tax shock delivers many interesting insights.

First, in response to an expansionary tax shock, real debt decreases significantly (and stays below trend for at least 3 years), as predicted by the model and second, real output increases significantly for approximately four years.

Most important for what concern our test is the positive and statistically significant response of the real interest rate (lasting for one year). This is what should happen in a Non Ricardian regime, according to the model.

We conclude this section recalling the main result. We identified two shocks to the US economy in the period from 1960 to 1979. One is a real GDP shock to which taxes respond endogenously. The other is an exogenous tax cut that stimulates the economy. Once these two effects are distinguished, it is possible to see that the response of the system to the "pure" fiscal shock is compatible with a Non Ricardian regime and with the Fiscal Theory of the Price Level as embedded in our model.

5.2 Understanding the Causality: The After-1983 Period

Let us now estimate the same VAR as before on the sample 1983:1-2003:2.

We employ the same specification and the same identification. Impulse responses to a GDP shock and to a tax shock are displayed in Figure 6.

[Figure 6]

The response of the system to a GDP shock is very similar in the two subperiods and points once more to the automatic stabilizing role of fiscal policy. In response to a positive GDP shock, taxes increase significantly. Expenditures respond positively after about 4 years and real debt decreases significantly.

[23] 95% confidence intervals were obtained using the bias- corrected bootstrap procedure proposed by Inoue and Kilian (2002).

The effect of a tax shock on the real economy is zero (in contrast to the previous period). The response of the real interest rate is now not different from zero. The point estimate of the response of real debt is now much bigger than in the previous period and after 4 years it is still significantly negative. We interpret this as due to an uncoordinated change in regime. The effect of the shift in monetary policy to a "high ϕ_π" after 82 matched with a "low γ" fiscal policy (as it was before 82) was to move the economy from a stable region to an unstable one.

Recall that, as predicted by our model, a "high ϕ_π" economy in which monetary policy increase the real interest rate in response to inflation produces a stable outcome only if matched with a "high γ" fiscal policy, in which taxes respond to real debt. The uncoordinated shift in 1982 triggered a debt explosion, through both channels identified before: an increased real interest rate and a deficit unresponsive to the increasing debt.

5.2.1 The After-1990 Period

Narrative evidence identifies a shift in the conduct of fiscal policy at the end of the eighties[24,25]. It was at about that time that commentators and policymakers started to raise concerns about the sustainability of debt. By re-estimating our VAR from 1990:1 on, we obtain the impulse responses in Figure 7. Once again, tax shocks (which now display very short lived dynamics) do not have significant effects on real GDP. The point estimate of the response of real debt is much smaller than before and it becomes insignificantly different from zero after few quarters. The most interesting result is the negative (though not strongly statistically significant) response of the real interest rate, as predicted by the model in response to a tax shock in a Ricardian regime.[26]

[Figure 7]

We conclude this section by summarizing our findings.

[24] I thank Philippe Weil for pointing out this to me.

[25] Favero and Monacelli (2003) by using Markov switching techniques estimate the shift in the fiscal policy rule at the end of the eighties as well.

[26] We have also modified in some directions our analysis to check for robustness of our results. We have experimented by estimating all our VARs with three and four lags. Results are unchanged. We have used the following specification for our VARs: Real GDP, Real Primary Deficit, Real Interest Rate and Real Debt with both 2 and 4 lags. Results once again are robust and in line with our model: in response to a positive deficit shock, real interest rate decrease significantly in the 60-79 sample and do not decrease significantly after 83. Finally, we have tried with other orderings in the Choleski decomposition: results are robust.

1. US policy mix was stable before 79 and was characterized by a Non Ricardian fiscal policy and a "passive" monetary policy (low γ and low ϕ_π).

2. After the sudden shift to an "active" monetary policy, US economy was on an transitional (possibly non-stationary) path (low γ and high ϕ_π), in which the costs of adjustment were felt through an exploding debt.

3. Only at the beginning of the nineties, when fiscal policy became truly "Ricardian", US economy was back to a stable path, characterized by a Ricardian policy mix (high γ and high ϕ_π).

4. Though this is not the focus of the paper we think it is important to stress that our results also highlight the fact that estimates on different time periods generate different responses. This finding sheds some shadows on results derived by estimating fiscal VARs on the whole post-WWII sample.

6 Conclusions

In this paper we have tested some implications of the Fiscal Theory of the Price Level.

The restrictions were derived from a general equilibrium model with sticky prices, monopolistic competition and an OLG demographic structure so to generate real effects of fiscal policy also under a "Ricardian" fiscal policy.

Using US data and VAR techniques, we found that the adjustment paths for the period 1960-1979 are in line with what predicted by our model under a Non Ricardian policy mix.

The period 1990-2003 is characterized by a Ricardian fiscal policy.

The intermediate period, 1982-1990, can be rationalized as a regime in which policies were uncoordinated: an active monetary policy was matched with a Non Ricardian fiscal policy. The outcome was an exploding debt.

As far as we know this paper is the first one that finds some evidence of the existence of some time periods and some countries in which the FTPL applies.

Of course, we do not think this work clarifies all the issues concerning the FTPL. Quite the opposite. What we have shown is that, once endowed with ancillary assumptions about the economic environment, the FTPL do have testable implications. The results of any empirical analysis will of course be conditional to those assumptions. It would be interesting to compare our predictions with those generated by alternative models

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Appendix 1: Solving the Model

In the appendix, we provide some more details about the solution of the model.

Firms

Each firm i maximizes the sum of discounted profits:

$$\max_{\{p_t^i,\, N_t^i\}} \sum_{t=0}^{\infty} \frac{1}{\prod_{i=0}^{t}(1+r_{t+i})} \left\{ \begin{array}{c} \underbrace{\left[\frac{p_t^i}{p_t}\right]^{1-\eta} y_t}_{\text{revenues}} - \underbrace{\left[\frac{w_t^i}{p_t} N_t^i - \frac{\phi}{2}\left(\frac{p_t^i}{p_{t-1}^i} - 1 - \pi_0\right)^2\right]}_{\text{costs}} \\ -\lambda_t^i \underbrace{\left[\left(\frac{p_t^i}{p_t}\right)^{-\eta} y_t - N_t^i\right]}_{\text{dem } = \text{ supply}} \end{array} \right\}$$

First order condition with respect to p_t^i gives:

$$p_t^i = p_t \lambda_t^i \Psi_t^i$$

Where the mark-up Ψ_t^i:

$$\Psi_t^i = \frac{\eta y_t \left(\frac{p_t^i}{p_t}\right)^{-\eta}}{\frac{p_t}{p_t^i}\phi\left[\left(\frac{p_t^i}{p_{t-1}^i} - \Pi_0\right)\frac{p_t^i}{p_{t-1}^i} - \left(\frac{p_{t+1}^i}{p_t^i} - \Pi_0\right)\frac{p_{t+1}^i}{p_t^i}\right] + (\eta - 1)\left(\frac{p_t^i}{p_t}\right)^{-\eta} y_t}$$

where: $\Pi_0 = 1 + \pi_0$.

The first order condition with respect to N_t^i gives:

$$\frac{w_t}{p_t} = \lambda_t^i$$

Combining the two, one gets the labor demand schedule:

$$\frac{w_t}{p_t} = \frac{p_t^i}{p_t}\frac{1}{\Psi_t^i}$$

Aggregation

The aggregation formula is the following:

$$x_t = \sum_{s=-\infty}^{t} \mu(1-\mu)^{t-s} x_t^s$$

The equation simply takes into account the two aspect of cohort heterogeneity: the fact that the dimension of each cohort decreases as time passes and that each cohort makes different choices.

This equation takes into account the fact that population is constant at each period at 1:

$$\sum_{s=-\infty}^{t} \mu(1-\mu)^{t-s} = \mu \sum_{s=-\infty}^{t} (1-\mu)^{t-s} = \mu \sum_{s=-\infty}^{0} (1-\mu)^{t-s-t} = \mu \sum_{s=0}^{+\infty} (1-\mu)^{s} = \frac{\mu}{1-(1-\mu)} = 1$$

The only equation that require special care in the aggregation is the Euler equation:

$$\begin{aligned} c_t^s &= \frac{1+i_t}{(1+\pi_{t+1})\beta} c_{t+1}^s \Rightarrow \\ \sum_{s=-\infty}^{t} \mu(1-\mu)^{t-s} c_t^s &= \frac{1+i_t}{(1+\pi_{t+1})\beta} \sum_{s=-\infty}^{t} \mu(1-\mu)^{t-s} c_{t+1}^s \end{aligned} \tag{22}$$

We know that:

$$\begin{aligned} c_{t+1} &= \sum_{s=-\infty}^{t+1} \mu(1-\mu)^{t+1-s} c_{t+1}^s \\ &= \sum_{s=-\infty}^{t} \mu(1-\mu)^{t+1-s} c_{t+1}^s + \mu c_{t+1}^{t+1} \end{aligned}$$

Dividing both sides by $1-\mu$, we see that the summation on the right hand side of equation (22) can be expressed as:

$$\sum_{s=-\infty}^{t} \mu(1-\mu)^{t-s} c_{t+1}^s = \frac{c_{t+1}}{1-\mu} - \frac{\mu}{1-\mu} c_{t+1}^{t+1}$$

Substituting this expression in equation (22) we finally obtain the aggregate Euler equation:

$$c_t = \frac{1}{\beta(1+r_{t+1})} \left[\frac{1}{1-\mu} c_{t+1} - \frac{\mu}{1-\mu} c_{t+1}^{t+1} \right]$$

This equation simply states that the fact that newborn do not have any financial wealth in the first period of their life reduces aggregate consumption by a fraction proportional to their number: $\frac{\mu}{1-\mu}$.

The next step is to compute c_{t+1}^{t+1}. Newborn consumption can be expressed as a fraction of their life-time resources.

From the agents' budget constraint:

$$c_t^s + \frac{B_t^s}{p_t} = \frac{Y_t^s}{p_t} - \frac{T_t^s}{p_t} + R_{t-1}\frac{B_{t-1}^s}{p_t}\frac{1}{(1-\mu)}$$

Solve it forward and substitute recursively for b_t^s. Impose the appropriate no-Ponzi game condition.

$$\sum_{t=j}^{\infty} x(j,t)c_t^s = \sum_{t=j}^{\infty} x(j,t)\left[\frac{\Pi_t}{p_t} - t_t^s\right] + \sum_{t=j}^{\infty} x(j,t)\left[\frac{w_t}{p_t}L_t^s\right]$$

where the discount factor x_t^j is such that: $x_j^j = 1$ and $x_t^j = (1-\mu)^{j-t-1}\prod_{i=t}^{j-1}\frac{1}{1+r_i}$, for $j > t$.

Substitute L_t^s with its expression: $L_t^s = 1 - \left(\frac{1-\delta}{\delta}\right)\frac{p_t}{w_t}c_t^s$.

After some tedious computations, one obtains:

$$\sum_{t=j}^{\infty} x_t^j \frac{c_t^s}{\delta} = \sum_{t=j}^{\infty} x_t^j\left[\frac{\Pi_t}{p_t} - \tau_t + \frac{w_t}{p_t}\right]$$

Substituting the Euler equation for consumption:

$$c_j^j \sum_{t=j}^{\infty} x_t^j \frac{1}{\delta}\frac{1}{x_t^j}\beta^t = \sum_{t=j}^{\infty} x_t^j\left[\frac{\Pi_t}{p_t} - \tau_t + \frac{w_t}{p_t}\right]$$

Finally, newborn's consumption can be expressed as:

$$c_j^j = \delta(1-\beta(1-\mu))inc_j$$

where:

$$inc_j = \sum_{t=j}^{\infty} x_t^j\left[\frac{\Pi_t}{p_t} - \tau_t + \frac{w_t}{p_t}\right] \tag{23}$$

represents total life-time resources for all the agents alive at time j.

A forward-looking difference equation for inc_t can be easily deduced from equation (23):

$$inc_t = \frac{1-\mu}{1+r_{t+1}}inc_{t+1} + \left[\frac{w_t}{p_t} + \frac{\Pi_t}{p_t} - \tau_t\right]$$

For the sake of clarity, we report all the aggregate equations.
Aggregate money demand:

$$\frac{M_t}{p_t} = \left[\frac{\chi}{\delta} c_t \frac{1+i_{t+1}}{i_{t+1}}\right]^h$$

Aggregate labor demand:

$$N_t = 1 - \left(\frac{1-\delta}{\delta}\right) \frac{p_t}{w_t} c_t$$

Pricing function:

$$1 = \lambda_t \Psi_t$$

Mark-up:

$$\Psi_t = \left[\frac{\eta y_t}{\phi\left[\left(\pi_t - \pi_0\right)\left(1+\pi_t\right) - \left(\pi_{t+1} - \pi_0\right)\frac{(1+\pi_{t+1})}{(1+r_{t+1})}\right] + (\eta - 1) y_t}\right]$$

Real wage:

$$\frac{w_t}{p_t} = \lambda_t$$

Aggregate profits:

$$\frac{\Pi_t}{p_t} = y_t - \left[N_t \frac{w_t}{p_t} + \frac{\phi}{2}\left(\pi_t - \pi_0\right)^2\right]$$

Government's budget constraint:

$$b_t = b_{t-1} \frac{1+i_{t-1}}{1+\pi_t} + \left[g_t - \tau_t\right]$$

Resources constraint:

$$y_t = c_t + g_t$$

Fisher equation

$$1 + r_{t-1} = \frac{1+i_{t-1}}{1+\pi_t}$$

Taylor rule:

$$i_{t+1} = \phi(\pi_t - \pi_0, y_t - \bar{y})$$

Fiscal rule:

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \eta_t$$

The Complete Log-Linearized System

$$\hat{N}_t = \frac{1-\delta}{\delta}\frac{\bar{c}}{\bar{w}\bar{N}}\hat{w}_t - \frac{1-\delta}{\delta}\frac{\bar{c}}{\bar{w}\bar{N}}\hat{c}_t$$

$$\hat{m}_t = h\hat{c}_t - \frac{h}{(\bar{\imath}-1)}\hat{\imath}_{t+1}$$

$$0 = \bar{\Psi}\bar{\lambda}(\hat{\Psi}_t + \hat{\lambda}_t)$$

$$\hat{y}_t = \frac{\bar{w}\bar{N}}{\bar{\lambda}\bar{y}}(\hat{w}_t + \hat{N}_t) - \hat{\lambda}_t$$

$$\hat{\Psi}_t = \frac{\eta\phi\bar{\pi}}{\bar{\Psi}(\eta-1)^2\bar{y}}(\frac{\hat{\pi}_{t+1}}{\bar{r}} - \hat{\pi}_t)$$

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{G}}{\bar{y}}\hat{G}_t$$

$$\hat{\Pi}_t = \frac{\bar{y}}{\bar{\Pi}}\hat{y}_t - \frac{\bar{N}\bar{w}}{\bar{\Pi}}\hat{N}_t - \frac{\bar{N}\bar{w}}{\bar{\Pi}}\hat{w}_t$$

$$\hat{b}_t = \frac{\bar{\imath}}{\bar{\pi}}\hat{b}_{t-1} + \frac{\bar{\imath}}{\bar{\pi}}\hat{\imath}_{t-1} - \frac{\bar{\imath}}{\bar{\pi}}\hat{\pi}_{t-1} - \frac{\bar{G}}{\bar{b}}\hat{G}_t - \frac{\bar{t}}{\bar{b}}\hat{t}_t$$

$$\hat{c}_t = \frac{1}{\beta\bar{r}(1-\mu)}\hat{c}_{t+1} - \frac{\mu}{1-\mu}\frac{\bar{C}}{\beta\bar{r}\bar{c}}\hat{C}_{t+1} - \frac{1}{\beta\bar{c}\bar{r}}\left(\frac{\bar{c}}{1-\mu} - \frac{\mu}{1-\mu}\bar{C}\right)\hat{r}_t$$

$$\hat{C}_{t+1} = \delta(1-\beta)\frac{i\bar{n}c}{\bar{C}}i\hat{n}c_{t+1}$$

$$i\hat{n}c_t = \frac{1}{\bar{r}}i\hat{n}c_{t+1} - \frac{1}{\bar{r}}\hat{r}_t + \frac{\bar{w}}{i\bar{n}c}\hat{w}_t + \frac{\bar{\Pi}}{i\bar{n}c}\hat{\Pi}_t - \frac{\bar{t}}{i\bar{n}c}\hat{t}_t$$

$$\hat{y}_t = \frac{\bar{N}}{\bar{y}}\hat{N}_t$$

$$\hat{r}_t = \frac{\bar{\imath}}{\bar{r}\bar{\pi}}\hat{\imath}_t - \frac{\bar{\imath}}{\bar{r}}\hat{\pi}_{t+1}$$

$$\hat{\imath}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t$$

$$\hat{t}_t = \gamma\frac{\bar{b}}{\bar{t}}\hat{b}_t + \eta_t$$

Steady state values

Case with OLG
$\beta = .96$
$\delta = 0.8$
$\eta = 4$
$g = 0.4$
$\phi = 200$
$\mu = 0.05$
$b = 0.37$
$\tau = 0.42$
$r = 0.045$
$\pi_0 = 0$

Case without OLG
$\beta = .96$
$\delta = 0.8$
$\eta = 4$
$g = 0.4$
$\phi = 200$
$\mu = 0$
$b = 0.48$
$\tau = 0.42$
$r = \frac{1}{\beta} - 1 = 0.0417$
$\pi_0 = 0$

Appendix 2: Data

- **Interest Rate:** Federal Funds Rate, Federal Reserve Board: H15. Average of monthly figures.
- **Real Potential GDP:** constructed from the Fred dataset, FRB St. Louis
- **GDP:** NIPA - Real Gross Domestic Product, Chained Dollars, (Bil. $, SAAR) Table 1.1.6, Line 1
- **Deflator:** NIPA - Implicit Price Deflators for Gross Domestic Product, Table 1.1.9 - Line 1
- **Current expenditures, government:** NIPA (Bil. $, SAAR) - Table 3.1, Line 15
- **Current receipts, government:** NIPA (Bil. $, SAAR) - Table 3.1, Line 1
- **Interest payments, government** - NIPA (Bil. $, SAAR) - Table 3.1, Line 22
- **Federal Debt** - FRB Flow of Funds - Table D.3 - LA314102005
- **State and Local Debt** - FRB Flow of Funds - Table D.3 - LA214102005

Figure 1. Impulse responses - $\mu = 0$

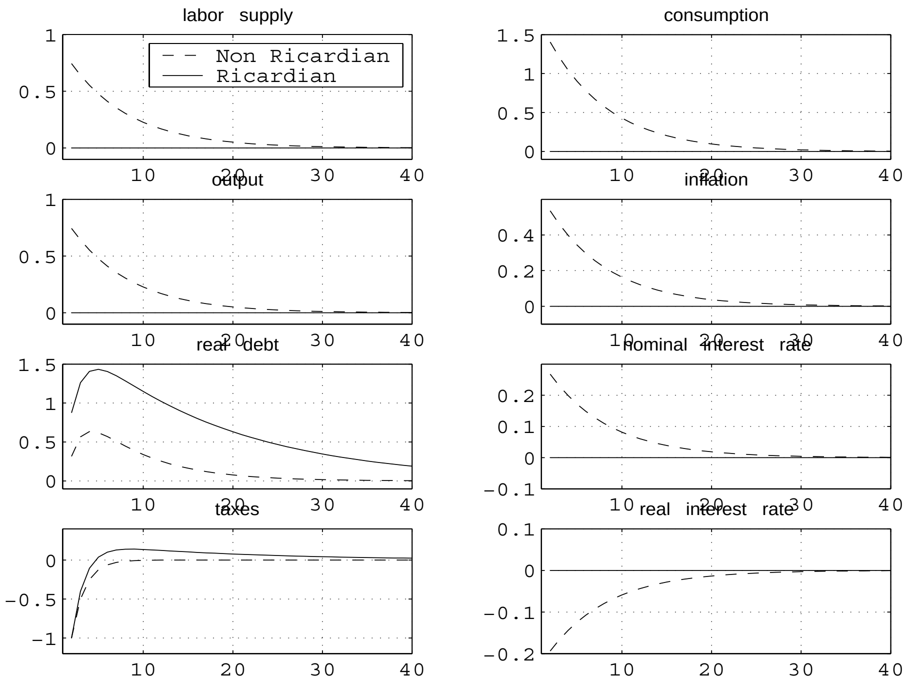


Figure 2. Stability regions - $\mu > 0$

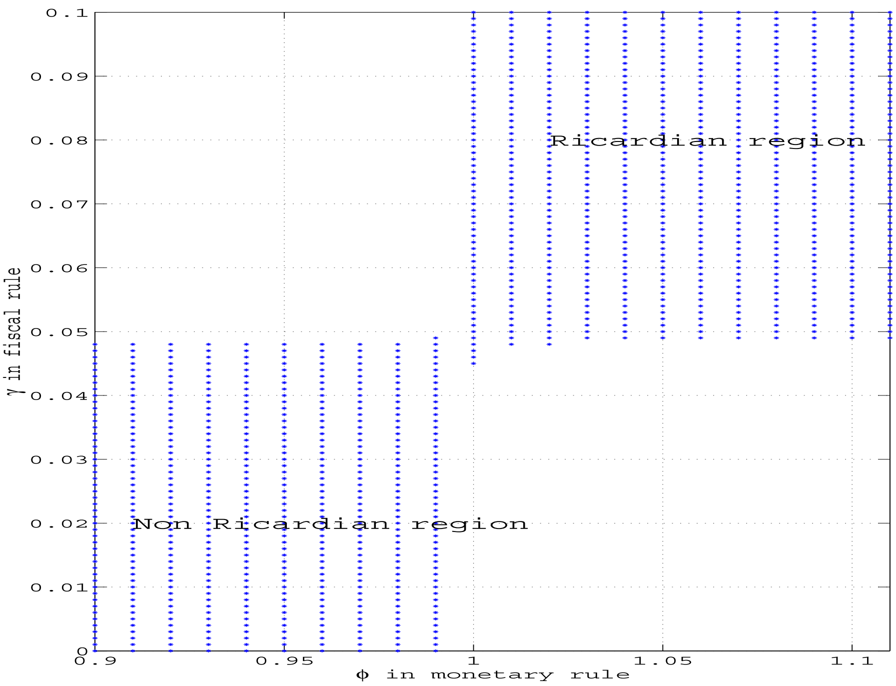
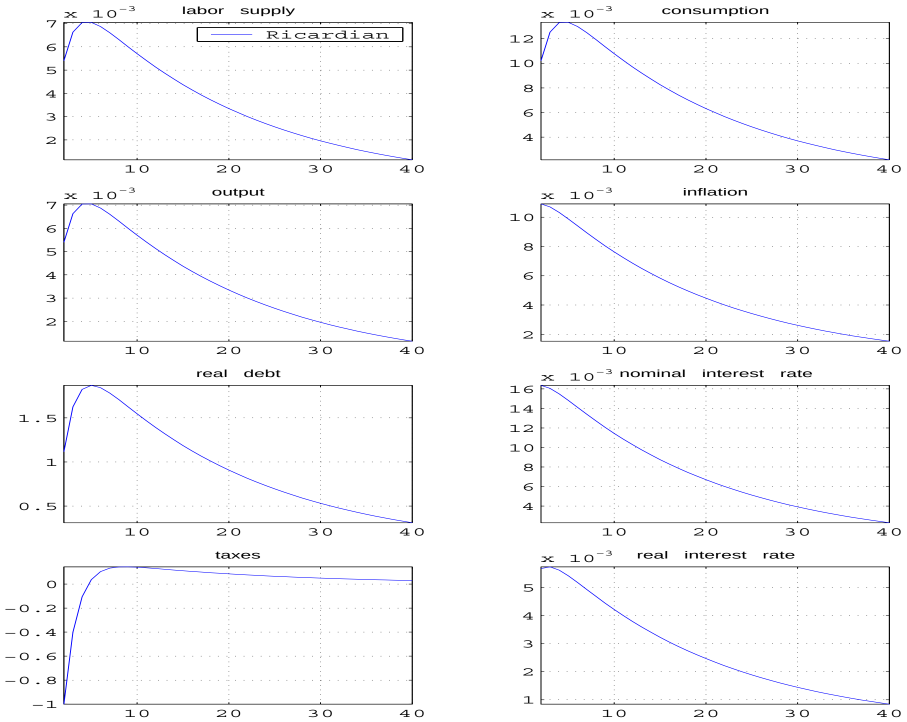


Figure 3. Impulse responses - Ricardian regime - $\mu > 0$



Impulse responses - Non Ricardian regime - $\mu > 0$

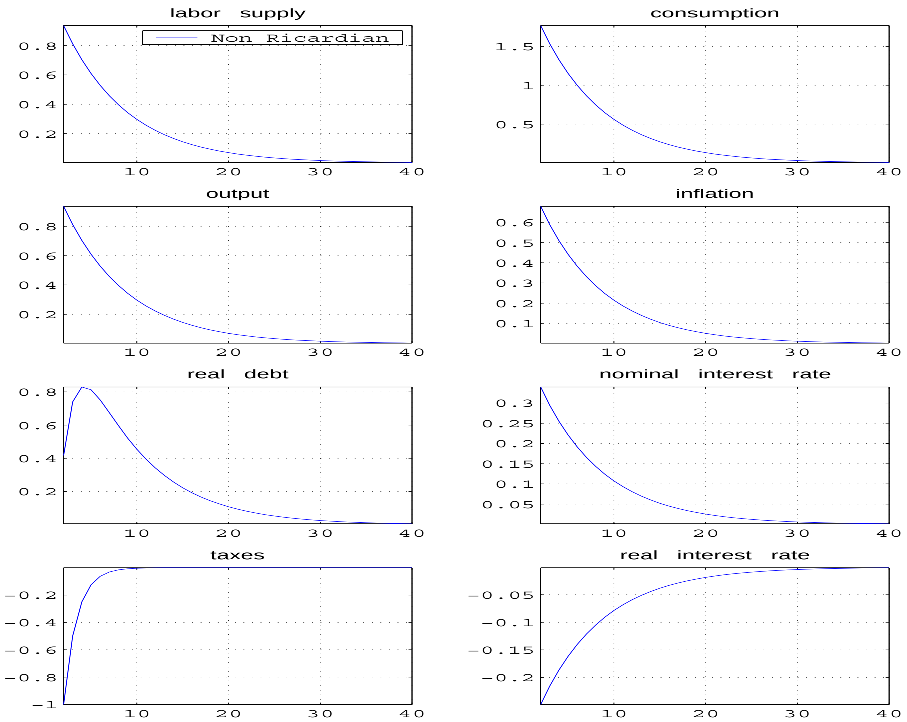


Figure 4. US Data - 1960:2003

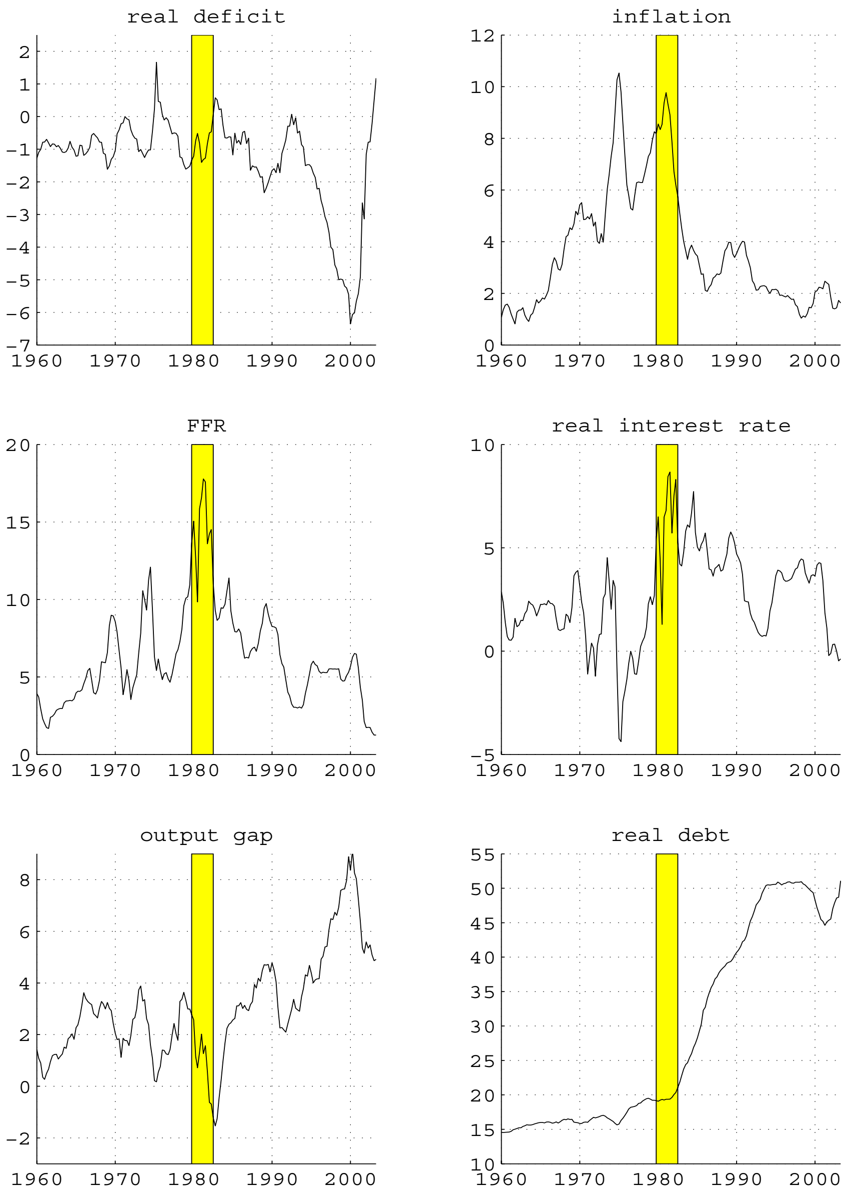


Figure 5. Impulse responses - 1960-1979

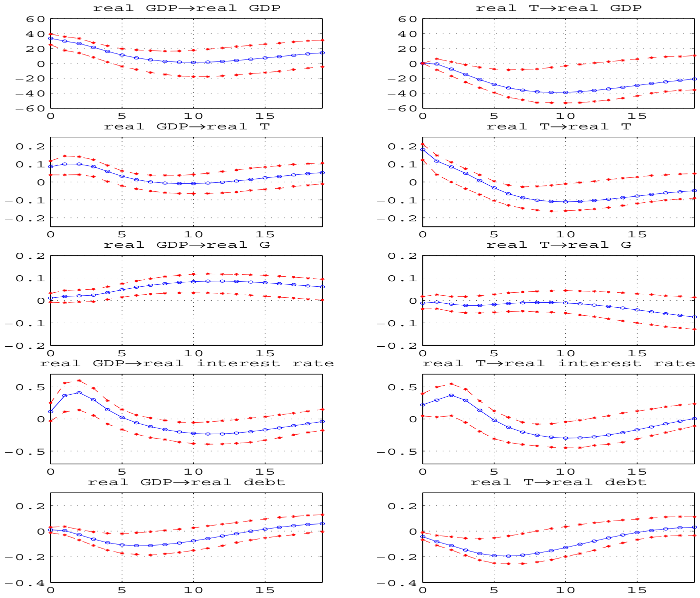


Figure 6. Impulse responses - 1983-2003

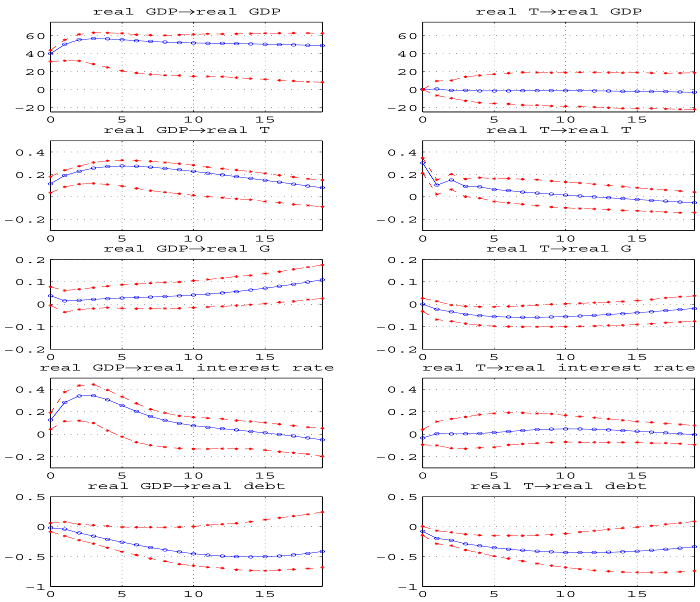


Figure 7. Impulse responses - 1990-2003