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Kai Leitemo and Ulf Söderstrom

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Robust Monetary Policy in the New-Keynesian Framework

Kai Leitemo Ulf Söderström*

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Abstract

We study the effects of model uncertainty in a simple New-Keynesian model using robust control techniques. Due to the simple model structure, we are able to find closed-form solutions for the robust control problem, analyzing both instrument rules and targeting rules under different timing assumptions. In all cases but one, an increased preference for robustness makes monetary policy respond more aggressively to cost shocks but leaves the response to demand shocks unchanged. As a consequence, inflation is less volatile and output is more volatile than under the non-robust policy. Under one particular timing assumption, however, increasing the preference for robustness has no effect on the optimal targeting rule (nor on the economy).

Keywords: Knightian uncertainty, model uncertainty, robust control, min-max policies.

JEL Classification: E52, E58, F41.

Leitemo: Norwegian School of Management (BI), P.O. Box 580, 1302 Sandvika, Norway; kai.leitemo@bi.no; *Söderström*: Department of Economics and IGIER, Università Bocconi, Via Salasco 5, 20136 Milano, Italy; ulf.soderstrom@uni-bocconi.it. Financial support from the Financial Market Fund, a program under the Norwegian Research Council, is gratefully acknowledged. We are also grateful for comments from Paolo Giordani, Juha Kilponen, Tom Sargent, Paul Söderlind, Juha Tarkka and Jouko Vilmunen, as well as participants at a Bank of Finland workshop in May 2004 and the University of Copenhagen Conference on Dynamic Macroeconomic Theory in June 2004. In particular, we thank Carl Walsh for many insightful comments.

1 Introduction

The New-Keynesian approach to macroeconomic modelling has been dominating the monetary policy literature for almost a decade now. It has produced several important insights in the conduct of monetary policy, and is commonly used to provide policy prescriptions (see, e.g., Clarida et al., 1999). However, although based on the optimizing behaviour of private agents, as any model it rests on a set of assumptions that may or may not be good approximations of true economies. Since a complete description of reality cannot be given, a policymaker is likely to prefer basing policy on principles that are valid also if the assumptions on which the model is based differ from reality. That is, policy prescriptions should be robust to reasonable deviations from the benchmark model.

In this paper, we explore how monetary policy in the New-Keynesian model should be conducted in order to be robust to small changes in the specification of the model. We assume that the policymaker is endowed with a model that is believed to be the most likely description of reality, but fears that reality deviates from this model in ways that cannot be described by a known probability distribution. The policymaker wants to avoid particularly bad outcomes, and therefore wants policy to be robust against specification errors that could have particularly severe consequences.

This problem has recently been addressed by Hansen and Sargent (2004) using "robust control" techniques. Assuming that the policymaker is unable to formulate a probability distribution over plausible models, the robust policymaker designs policy for the worst possible outcome within a pre-specified set of models. We apply robust control techniques developed by Hansen and Sargent (2004) and Giordani and Söderlind (2004) to a simple New-Keynesian model of a closed economy. Due to its simple structure, we are able to solve the robust control problem analytically. This gives us policy prescriptions that are more general than in the previous literature, which has typically used numerical methods.

We analyze the effects of small degrees of robustness on both optimal instrument rules and targeting rules under two different timing assumptions. In the first timing assumption, the central bank chooses its optimal policy rule and a fictitious evil agent chooses the optimal degree of misspecification simultaneously, leading to a Nash equilibrium. Under the second timing assumption, the central bank chooses policy taking into account the choice of the evil agent, thus acting as a Stackelberg leader.

In all cases but one, an increased preference for robustness makes monetary policy respond more aggressively to cost shocks but leaves the response to demand shocks unchanged. This is because the central bank fears that cost shocks have larger impact on inflation, and it therefore counteracts these shocks more vigorously. As a consequence, in the most likely outcome of the model inflation is less volatile but output is more volatile than under the non-robust policy. There is one exception to this rule, however: under the Nash assumption, the optimal *targeting rule* is not affected by the central bank's preference for robustness, in contrast to the optimal *instrument rule*. In this case, therefore, if the central bank implements policy through the optimal targeting rule, its preference for robustness has no effects on the behavior of the economy.

We begin in Section 2 by presenting the model and the general features of the robust control problem. Section 3 derives the optimal robust policy under the two alternative timing assumptions, and discusses the results. Section 4 concludes and relates our work to the previous literature.

2 Model

Our economic environment follows the standard New-Keynesian model with sticky prices that has been used extensively in the recent literature on monetary policy. The attractiveness of this model is due to the fact that it summarizes the behavior of rational optimizing agents in two equations: a New-Keynesian Phillips curve for inflation and a forward-looking IS equation for output.[1] Thus, the benchmark model is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \Sigma_\pi \varepsilon_t^\pi, \tag{1}$$

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1} \left[i_t - \mathrm{E}_t \pi_{t+1} \right] + \Sigma_x \varepsilon_t^x, \tag{2}$$

where π_t is the rate of inflation (the log change in the price level), x_t is the output gap (the log deviation of output from its natural level), and i_t is the one-period nominal interest rate, controlled by the central bank. The parameters β, κ, σ are all positive with $\beta < 1$, and depend on deep parameters describing preferences and technology. In particular, β is the discount factor of private agents, σ is the elasticity of intertemporal substitution, and κ depends negatively on the degree of price stickiness. Finally, ε_t^π and ε_t^x are, respectively, a cost shock (e.g., a shock to firms' markup) and a demand shock (e.g., a preference shock or a shock to the natural

[1] Early derivations of this model from microfoundations can be found in Rotemberg and Woodford (1997) or Goodfriend and King (1997); a thorough discussion is provided in Clarida et al. (1999) and Walsh (2003).

level of output). Although in the literature these are often assumed to be persistent, we will assume that they are white noise with mean zero and unit variance, as this allows for a closed-form solution of the robust control problem. The parameters Σ_π and Σ_x thus determine the variance of these shocks: for instance, the variance of the supply shock $\Sigma_\pi \varepsilon_t^\pi$ will be Σ_π^2.

While the central bank sees the model (1)–(2) as the most likely specification, it realizes that the true model may deviate from this benchmark model, although it is unable to specify a probability distribution for these deviations. To model such misspecification we follow Hansen and Sargent (2004), and introduce in each equation a second type of disturbance, denoted v_t^π and v_t^x. These disturbances are controlled by a fictitious "evil agent" who represents the policymaker's worst fears concerning specification errors. Thus, the model with misspecification is given by[2]

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \Sigma_\pi \left[v_t^\pi + \varepsilon_t^\pi \right], \tag{3}$$

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1} \left[i_t - \mathrm{E}_t \pi_{t+1} \right] + \Sigma_x \left[v_t^x + \varepsilon_t^x \right]. \tag{4}$$

The specification errors v_t^j will be allowed to feed back from the state variables, so although the errors enter the model as additive shocks, they may well disturb the model in the same way as multiplicative parameter uncertainty (see Hansen and Sargent, 2004).[3]

To be robust against specification errors, the central bank is assumed to follow a min-max strategy, and design policy for the worst possible outcome of the model, where the evil agent chooses the amount of misspecification v_t^j to maximize central bank loss, given some constraints (to be specified below). This model will be referred to as the *worst-case model*, and is the outcome against which the central bank wants policy to be robust. The most likely outcome of the model, on the other hand, is one where the central bank sets policy and agents form expectations to reflect misspecification in the worst-case model, but there is no such misspecification in practice (so all v_t^j are zero). We will refer to this model as the *approximating model.*

[2]The amount of misspecification, measured by v_t^j, is scaled by the parameter Σ_j, which determines the volatility of the shock in equation j. Intuitively, the specification error is disguised by the disturbance term ε_t^j, so if the disturbance has no variance, the specification error would be detected immediately. The larger is the variance of the disturbance, the larger can the specification error be without being detected.

[3]Onatski and Williams (2003) stress that the Hansen-Sargent approach to robustness does not capture all types of parameter uncertainty, and that the "robust" rules may be fragile to certain sources of uncertainty that are not captured by the robust control approach.

3 Robust monetary policy

3.1 Setting up the control problem

The central bank is assumed to minimize a standard objective function which is quadratic in deviations of inflation and the output gap from their zero target levels. To design the robust policy, the central bank takes into account a certain degree of model misspecification by minimizing its objective function in the worst possible model within a given set of plausible models. Depending on its preference for robustness, the central bank allocates a budget η to the evil agent, which is used to create misspecification. Thus the budget constraints for the evil agent are

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (v_t^{\pi})^2 \leq \eta, \tag{5}$$

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (v_t^{x})^2 \leq \eta, \tag{6}$$

and in a standard control problem we would have $\eta = 0$. Following Hansen and Sargent (2004) the robust monetary policy is obtained by solving the min-max problem

$$\min_{\{i_t\}} \max_{\{v_t^j\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right] \tag{7}$$

subject to the model with misspecification in equations (3)–(4) and the evil agent's budget constraints (5)–(6). The central bank thus sets the interest rate to minimize the value of its intertemporal loss function, while the evil agent sets its controls to maximize the central bank's loss, given the constraints on misspecification.

As a benchmark assumption, we will assume that the central bank and the evil agent play a Nash game, so their choice is optimal given the other player's choice. This is not the only possible assumption. In Section 3.3 below, we will analyze the case where the central bank is a Stackelberg leader, and so makes its policy choice taking into account how the evil agent will choose the specification errors. This will have some important consequences for the effect of robustness on policy and therefore on the economy.[4]

[4]Most applications (e.g., Giordani and Söderlind, 2004) have focused on the Nash assumption. Hansen and Sargent (2004, Ch. 6) show how different timing assumptions lead to the same equilibrium outcome in a model without forward-looking features. This is not the case here, where agents are forward-looking.

3.2 The Nash solution

In the Nash problem, the Lagrangian is given by

$$\begin{aligned} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \quad \Big\{ \quad & \pi_t^2 + \lambda x_t^2 - \theta \left(v_t^\pi\right)^2 - \theta \left(v_t^x\right)^2 \\ & -\mu_t^\pi \Big[\pi_t - \beta \mathrm{E}_t \pi_{t+1} - \kappa x_t - \Sigma_\pi v_t^\pi - \Sigma_\pi \varepsilon_t^\pi\Big] \\ & -\mu_t^x \Big[x_t - \mathrm{E}_t x_{t+1} + \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) - \Sigma_x v_t^x - \Sigma_x \varepsilon_t^x\Big] \Big\}, \end{aligned} \tag{8}$$

where μ_t^j are Lagrange multipliers on the constraints (3)–(4) and θ determines the set of models available to the evil agent against which the policymaker wants to be robust. This robustness parameter is inversely related to the evil agent's budget η: as the budget shrinks towards zero, so the degree of misspecification approaches zero, θ will approach infinity.

Throughout, we will focus on marginal amounts of model misspecification. For sufficiently large amounts of misspecification, the evil agent will be able to overturn any relationship in the model, so the approximating model (1)–(2) is not a good description of reality. We therefore want to consider reasonable degrees of model uncertainty that cannot be easily identified by the policymaker.[5] More specifically, we will analyze the effects of small increases in the preference for robustness starting from the non-robust policy, i.e., small decreases in θ starting from $\theta = \infty$.

We assume that neither the central bank nor the fictitious evil agent has access to any mechanism that allows them to commit to future policies. Consequently, we take expectations as given in the optimization and look for a discretionary equilibrium. From the first-order conditions we can derive the following optimality conditions relating inflation, output and the degree of misspecification to each other:

$$x_t = -\frac{\kappa}{\lambda}\pi_t \equiv -A_N \pi_t, \tag{9}$$

$$v_t^\pi = \frac{\Sigma_\pi}{\theta}\pi_t, \tag{10}$$

$$v_t^x = 0. \tag{11}$$

These optimality conditions immediately reveal some interesting features of our model.

First, the optimal inflation–output trade-off in equation (9) is not affected by the

[5] In numerical approaches to robust control, the amount of misspecification can be chosen such that the policymaker cannot distinguish between the approximating model and the worst-case model at reasonable statistical significance levels. See Hansen and Sargent (2004) and Giordani and Söderlind (2004).

preference for robustness. Irrespective of the central bank's fear of misspecification, the robust policy does not change the optimal relationship between the targeting variables inflation and output. The optimal specification errors in equation (10) increase inflation volatility, but do not change the channels through which the central bank can reduce such volatility, i.e., moving output in the opposite direction. In other words, the presence of model misspecification will not alter the central bank's optimal "targeting rule".[6] However, if instead the central bank implements policy using an "instrument rule," i.e., a rule for the interest rate, policy will be affected by the preference for robustness, since, as explained in more detail below, the central bank needs to move the interest rate more in order to induce the appropriate changes in the output gap.

Second, the central bank only worries about model misspecification in the Phillips curve, as the optimal misspecification in the IS equation is always zero. The policymaker is able to counteract any specification errors in the output equation by an appropriate adjustment of the interest rate, and these interest rate movements do not influence central bank loss independently. Therefore the central bank does not fear such specification errors.

Third, the amount of misspecification in the Phillips curve is increasing in inflation and in the variance of cost shocks, Σ_π. The central bank fears inflationary shocks as these force the central bank to reduce the output gap in order to achieve the desired trade-off between inflation and the output gap. The evil agent adds to these shocks through misspecification in the Phillips curve, and increases inflation further when inflation is already high. Furthermore, the larger is the variance of the cost shock ε_t^π, the more difficult it is for the central bank to identify misspecification. Therefore the central bank is more on its guard against specification errors.

The optimal interest rate rule for the central bank is designed for the *worst-case model*, i.e., the worst possible outcome of the model, given the restrictions on misspecification. This model is found by combining the optimality conditions (9)–(11) with the model with misspecification in (3)–(4). As there is no persistence in the model, and the central bank is able to completely offset the effects of demand shocks on the economy, the only relevant state variable is the cost shock ε_t^π. Thus, the solutions for all variables will depend on the cost shock only.[7] This also implies

[6]Walsh (2004) obtains a similar result in the New-Keynesian model, showing that the optimal implicit instrument rule using robust control methods is equivalent to the robustly optimal instrument rules of Giannoni and Woodford (2003), and thus does not depend on the central bank's preference for robustness.

[7]Note that we allow the evil agent only to respond to the same variables as the policymaker, i.e., the cost shock. This differs from the setup of Hansen and Sargent (2004) and Giordani and

that all expectations are zero, a feature that makes possible an analytical solution of the model.

Setting expectations to zero and using the optimal trade-off (9) and the optimal misspecification (10) in the misspecified Phillips curve (3) then gives

$$\begin{aligned}\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \Sigma_\pi \left[v_t^\pi + \varepsilon_t^\pi \right] \\ &= -\frac{\kappa^2}{\lambda}\pi_t + \frac{\Sigma_\pi^2}{\theta}\pi_t + \Sigma_\pi \varepsilon_t^\pi \\ &= a_N \Sigma_\pi \varepsilon_t^\pi, \end{aligned} \tag{12}$$

where

$$a_N \equiv \frac{\lambda}{\lambda(1-\Sigma_\pi^2/\theta)+\kappa^2}. \tag{13}$$

The optimal trade-off (9) then gives the worst-case solution for the output gap as

$$x_t = b_N \Sigma_\pi \varepsilon_t^\pi, \tag{14}$$

where

$$b_N \equiv -\frac{\kappa}{\lambda} a_N = -\frac{\kappa}{\lambda(1-\Sigma_\pi^2/\theta)+\kappa^2}. \tag{15}$$

For small amounts of misspecification, $a_N > 0$ and $b_N < 0$: a positive cost shock increases inflation, but reduces the output gap, as the central bank offsets the shock by tightening policy. More importantly, we see that a_N is decreasing in θ and b_N is increasing in θ. Thus, an increase in the preference for robustness (a decrease in θ) leads to an increase in the absolute value of both a_N and b_N. This implies that both inflation and output in the worst-case model become more volatile when the preference for robustness increases.[8]

Söderlind (2004), where the evil agent is allowed to respond also to lagged state variables, thus introducing persistence in the shocks. This is because they set up the model on its state-space form where the shocks are predetermined variables and are written as autoregressive processes without any persistence. The set of state variables then includes also lagged values of the shocks, and the evil agent is allowed to respond to all state variables. In our setup, the evil agent is not allowed to introduce serial correlation in the shocks, as there is no such persistence from the outset. This assumption is mainly for tractability, but is also consistent with the assumption in both approaches that the evil agent is not allowed to introduce additional state variables to increase the degree of serial correlation in the endogenous variables.

[8] The worst-case solution highlights the need to focus on small degrees of misspecification. If we endow the evil agent with a very large budget to create specification errors, so θ is very small, a_N and b_N will take the opposite signs, so the evil agent makes inflation fall and the output gap rise after a positive cost shock. This puts the policymaker in a very difficult (and unrealistic) situation. For this reason, we focus on small deviations from the non-robust solution, so θ is close to infinity. (See also Giordani and Söderlind, 2004, for a discussion.)

The worst-case solution for inflation implies that the amount of misspecification is given by

$$v_t^\pi = \frac{\lambda\Sigma_\pi^2}{\lambda(\theta - \Sigma_\pi^2) + \theta\kappa^2}\varepsilon_t^\pi. \tag{16}$$

Naturally, this is larger if Σ_π is large (so specification errors are more easily disguised) and θ is small (so the central bank's preference for robustness is large). In addition, the amount of misspecification is also larger if it is costly for the central bank to offset inflationary shocks, that is, if λ is large (so offsetting output movements are costly) or if κ is small (so large movements in output are needed to affect inflation). In the case when the central bank attaches no weight to stabilizing output ($\lambda = 0$), misspecification is not a problem, as the central bank in each period can adjust output costlessly to offset any inflationary shocks. Indeed, in this case inflation is zero in both the worst-case model above and the approximating model below.

Given the central bank's worst-case scenario, we can derive the optimal *interest rate rule* by using the worst-case output gap (14) and the optimal misspecification (11) in the misspecified IS equation (4). Setting expectations to zero and solving for the interest rate then gives

$$\begin{aligned} i_t &= -\sigma x_t + \sigma\Sigma_x\varepsilon_t^x \\ &= c_N\Sigma_\pi\varepsilon_t^\pi + \sigma\Sigma_x\varepsilon_t^x, \end{aligned} \tag{17}$$

where

$$c_N \equiv -\sigma b_N = \frac{\sigma\kappa}{\lambda(1 - \Sigma_\pi^2/\theta) + \kappa^2}. \tag{18}$$

For small degrees of misspecification, c_N is positive, so the central bank responds to a positive cost shock by tightening policy, and c_N is decreasing in θ. Thus, an increased preference for robustness leads the central bank to respond more aggressively to cost shocks, thereby increasing interest rate volatility. This is because the central bank fears that cost shocks have larger impact on inflation.[9]

The worst-case model derived so far represents the central bank's worst fears about model misspecification, given its preference for robustness, and the central bank designs its policy to guard against such bad outcomes. The consequences for the economy of this policy behavior are given by the most likely model outcome—the *approximating model*—where the policy rule and agents' expectations reflect the central bank's preference for robustness, but the actual misspecification is zero.

[9]As in Hansen and Sargent (2004), we note that the optimal robust policy is not certainty-equivalent, but the variance of cost shocks explicitly affects the optimal policy rule in equation (17). This is because the evil agent will allocate more misspecification to equations where the shock variance is large.

The solution for the approximating model are obtained by using the optimal robust interest rate rule from equation (17) in the original model (1)–(2), again setting expectations to zero. For output this gives

$$\begin{aligned} x_t &= -\sigma^{-1} i_t + \Sigma_x \varepsilon_t^x \\ &= \bar{b}_N \Sigma_\pi \varepsilon_t^\pi, \end{aligned} \tag{19}$$

where

$$\bar{b}_N \equiv -\frac{\kappa}{\lambda(1 - \Sigma_\pi^2/\theta) + \kappa^2}. \tag{20}$$

As there is no misspecification in the IS equation, the approximating model for output is the same as the worst-case model, so $\bar{b}_N = b_N$. Thus an increase in the preference for robustness will make output more volatile also in the most likely approximating model. For inflation, the approximating model is given by

$$\begin{aligned} \pi_t &= \kappa x_t + \Sigma_\pi \varepsilon_t^\pi \\ &= \bar{a}_N \Sigma_\pi \varepsilon_t^\pi, \end{aligned} \tag{21}$$

where

$$\bar{a}_N \equiv 1 + \kappa \bar{b}_N = 1 - \frac{\kappa^2}{\lambda(1 - \Sigma_\pi^2/\theta) + \kappa^2}, \tag{22}$$

which is positive and increasing in θ. Thus, an increased preference for robustness makes inflation less volatile in the most likely model.

To summarize, these results imply that a central bank that wants to be robust against model misspecification will fear that inflation is more influenced by cost shocks, and so is more volatile, as shown in equation (12). The optimal policy (given by the trade-off in equation (9)) then implies that the central bank induces more volatility also in output, see equation (14). The fear that inflation is more volatile leads the policymaker to contract the economy more in response to positive cost shocks, inducing more volatility in the interest rate, see equation (17). In the most likely model (the model without misspecification), this volatility in the interest rate leads to more output volatility but less volatility in inflation, see equations (19) and (21). As monetary policy responds more aggressively to cost shocks when the central bank's preference for robustness increases, the effects of these shocks on inflation are dampened, but at the cost of more output volatility.

3.3 The Stackelberg solution

Although the Nash timing assumption has been more commonly used in applied work, we now analyze an alternative timing assumption, where the central bank acts as a Stackelberg leader and designs policy taking into account the evil agent's optimal choice of the level of misspecification in equations (10) and (11):[10]

$$
\begin{aligned}
v_t^{\pi} &= \frac{\Sigma_\pi}{\theta}\pi_t, & (23)\\
v_t^{x} &= 0. & (24)
\end{aligned}
$$

The Lagrangian for the central bank's optimal choice of policy is then given by

$$
\begin{aligned}
\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \quad \Big\{ \quad & \pi_t^2 + \lambda x_t^2 - \mu_t^\pi \left[\pi_t - \beta \mathrm{E}_t \pi_{t+1} - \kappa x_t - \frac{\Sigma_\pi^2}{\theta}\pi_t - \Sigma_\pi \varepsilon_t^\pi \right] \qquad (25)\\
& -\mu_t^x \left[x_t - \mathrm{E}_t x_{t+1} + \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) - \Sigma_x \varepsilon_t^x \right] \Big\},
\end{aligned}
$$

and the first-order conditions for the central bank give the optimal trade-off

$$
x_t = -\frac{\kappa}{\lambda\left(1-\Sigma_\pi^2/\theta\right)}\pi_t \equiv -A_S \pi_t. \qquad (26)
$$

While in the Nash model the optimal trade-off is independent of the preference for robustness, this is no longer true in the Stackelberg model, when the central bank foresees the evil agent's choice and adapts its policy accordingly. Now the trade-off is steeper than in the Nash model (A_S is larger than A_N), so the central bank implicitly puts more weight on stabilizing inflation than in the Nash model. Moreover, a larger preference for robustness makes the trade-off even steeper (A_S increases), further increasing the implicit weight on stabilizing inflation.

Again we find the *worst-case model* for inflation by setting expectations to zero and using the optimal trade-off (26) and the optimal misspecification (10) in the misspecified Phillips curve (3). This yields

$$
\begin{aligned}
\pi_t &= \kappa x_t + \Sigma_\pi v_t^\pi + \Sigma_\pi \varepsilon_t^\pi \\
&= a_S \Sigma_\pi \varepsilon_t^\pi, & (27)
\end{aligned}
$$

where

$$
a_S \equiv \frac{\lambda(1-\Sigma_\pi^2/\theta)}{\lambda(1-\Sigma_\pi^2/\theta)^2+\kappa^2}, \qquad (28)
$$

[10] We are grateful to Carl Walsh for suggesting this alternative timing assumption.

which is positive when the preference for robustness is small. Similarly, the worst-case model for the output gap is

$$x_t = b_S \Sigma_\pi \varepsilon_t^\pi, \tag{29}$$

where

$$b_S \equiv -A_S a_S = -\frac{\kappa}{\lambda(1-\Sigma_\pi^2/\theta)^2+\kappa^2}, \tag{30}$$

which is negative and increasing in θ. Because the trade-off is steeper in the Stackelberg model than in the Nash model, inflation is less volatile ($a_S < a_N$), but output is more volatile ($|b_S| > |b_N|$). An increased preference for robustness makes output in the worst-case model more volatile (because the trade-off becomes steeper), while the effect on inflation depends on parameters. If κ is large relative to λ, inflation in the worst-case model becomes less volatile when the preference for robustness increases, so a_S falls, but if κ is small relative to λ we get the opposite effect.[11]

The amount of misspecification is then given by

$$v_t^\pi = \frac{\lambda(1-\Sigma_\pi^2/\theta)\Sigma_\pi^2}{\lambda\theta(1-\Sigma_\pi^2/\theta)^2+\theta\kappa^2}\varepsilon_t^\pi, \tag{31}$$

which is smaller than in the Nash solution, but is still increasing in Σ_π and λ and decreasing in θ and κ.

The optimal policy rule is given by

$$\begin{aligned} i_t &= -\sigma x_t + \sigma\Sigma_x\varepsilon_t^x \\ &= c_S\Sigma_\pi\varepsilon_t^\pi + \sigma\Sigma_x\varepsilon_t^x, \end{aligned} \tag{32}$$

where

$$c_S \equiv -\sigma b_S = \frac{\sigma\kappa}{\lambda(1-\Sigma_\pi^2/\theta)^2+\kappa^2}, \tag{33}$$

[11] To see that $a_S < a_N$, note that

$$a_S = \left[1+\kappa A_S - \frac{\Sigma_\pi^2}{\theta}\right]^{-1} < \left[1+\kappa A_N - \frac{\Sigma_\pi^2}{\theta}\right]^{-1} = a_N,$$

as $A_S > A_N$. The effects of increased robustness on inflation in the worst-case model are given by

$$\begin{aligned} \frac{\partial a_S}{\partial\theta} &= \frac{\lambda\left[\lambda(1-\Sigma_\pi^2/\theta)^2+\kappa^2\right]-2\lambda^2(1-\Sigma_\pi^2/\theta)^2}{\left[\lambda(1-\Sigma_\pi^2/\theta)^2+\kappa^2\right]^2}\frac{\Sigma_\pi^2}{\theta^2} \\ &= \frac{\lambda\left[\kappa^2-\lambda(1-\Sigma_\pi^2/\theta)^2\right]}{\left[\lambda(1-\Sigma_\pi^2/\theta)^2+\kappa^2\right]^2}\frac{\Sigma_\pi^2}{\theta^2}, \end{aligned}$$

whose sign depends on the relative size of κ and λ.

which is positive and decreasing in θ. As in the Nash model, the central bank counters inflationary shocks by increasing the interest rate ($c_S > 0$) for small degrees of misspecification, but the central bank now foresees the evil agent's choice and responds more aggressively, so $c_S > c_N$. An increase in the preference for robustness will increase c_S further and make the interest rate more volatile.

Finally, the *approximating model* for output is given by

$$\begin{aligned} x_t &= -\sigma^{-1} i_t + \Sigma_x \varepsilon_t^x \\ &= \bar{b}_S \Sigma_\pi \varepsilon_t^\pi, \end{aligned} \tag{34}$$

where

$$\bar{b}_S \equiv -\frac{\kappa}{\lambda(1-\Sigma_\pi^2/\theta)^2 + \kappa^2}, \tag{35}$$

so again $\bar{b}_S = b_S$ as there is no misspecification in the IS equation. For inflation we obtain

$$\begin{aligned} \pi_t &= \kappa x_t + \Sigma_\pi \varepsilon_t^\pi \\ &= \bar{a}_S \Sigma_\pi \varepsilon_t^\pi, \end{aligned} \tag{36}$$

where

$$\bar{a}_S \equiv 1 + \kappa \bar{b}_S = 1 - \frac{\kappa^2}{\lambda(1-\Sigma_\pi^2/\theta)^2 + \kappa^2}, \tag{37}$$

which is positive and increasing in θ.

We note that $\bar{a}_S$ is positive and smaller than $\bar{a}_N$ for small degrees of misspecification. Conversely, $\bar{b}_S$ is negative and larger (in absolute terms) than $\bar{b}_N$. Increasing the preference for robustness will increase $\bar{b}_S$ in absolute terms, just as b_S, but decrease $\bar{a}_S$, so inflation becomes less volatile and output becomes more volatile in the approximating model.

In the Stackelberg model the central bank knows that the worst-case misspecification raises inflation volatility, and it guards against these fears by focusing more on reducing inflation volatility, at the cost of increased volatility in the output gap. Thus, the optimal trade-off between inflation and output in equation (26) is steeper and monetary policy responds more aggressively to inflationary shocks in the Stackelberg model. This implies that inflation in the approximating model is less volatile than in the Nash solution, while output is more volatile.

A larger preference for robustness makes the central bank focus even more on reducing inflation volatility, giving an even steeper trade-off in equation (26), and the central bank puts more weight on reducing inflation volatility, again at the cost of more volatility in output. Thus, the interest rate and output become more volatile, while inflation in the approximating model becomes less volatile. The only ambiguity concerns the effects on inflation in the worst-case model. On the one hand, an increase in the preference for robustness makes the central bank reduce inflation volatility more aggressively, but on the other hand, the evil agent induces more volatility in inflation. In the Nash solution, where the trade-off did not depend on the preference for robustness, the net effect was a reduction in inflation volatility, but in the Stackelberg model, there is also an effect through the optimal trade-off, and the net effect depends on parameter values.

Comparing the two solutions, we see that the weight on output stabilization in the Nash model, λ, is everywhere multiplied by the term $(1 - \Sigma_\pi^2/\theta)$ in the Stackelberg model. For sufficiently large θ, this term is positive but smaller than one, so the central bank in the Stackelberg model behaves as a Nash central bank with a larger weight on stabilizing inflation (a smaller λ). As the preference for robustness increases, this weight increases further, and the difference between the Nash central bank and the Stackelberg central bank becomes greater. However, although the choice of timing assumption matters for the quantitative results—the size of response coefficients and the effects of increased robustness—the qualitative results under the two assumptions are the same when the central bank implements policy using the optimal instrument rule.

When policy is implemented using the optimal *targeting rule*, this is no longer true. In the Nash solution, the optimal targeting rule given by equation (9) is independent of the preference for robustness, so if policy is implemented using the targeting rule, the central bank's preference for robustness will not affect the behavior of the economy. In the Stackelberg solution this is no longer the case: if policy is implemented using the optimal targeting rule (26) rather than the interest rate rule (32), the approximating model is given by

$$\begin{aligned} \pi_t &= \kappa x_t + \Sigma_\pi \varepsilon_t^\pi \\ &= \hat{a}_S \Sigma_\pi \varepsilon_t^\pi, && (38) \\ x_t &= \hat{b}_S \Sigma_\pi \varepsilon_t^\pi, && (39) \end{aligned}$$

where

$$\begin{aligned} \hat{a}_S &\equiv [1 + \kappa A_S]^{-1} = \frac{\lambda(1 - \Sigma_\pi^2/\theta)}{\lambda(1 - \Sigma_\pi^2/\theta) + \kappa^2}, && (40) \\ \hat{b}_S &\equiv -\frac{\kappa}{\lambda(1 - \Sigma_\pi^2/\theta) + \kappa^2}. && (41) \end{aligned}$$

Thus, under the Stackelberg targeting rule policy, increasing the preference for robustness makes inflation in the approximating model less volatile and output more volatile, as the central bank implicitly puts more weight on stabilizing inflation rather than output. These results mimic those under the optimal instrument rule under either the Nash or the Stackelberg assumption.

4 Conclusions

Our analysis shows that a central bank that wants to be robust against particularly bad outcomes will typically respond more aggressively to cost shocks, as the central bank fears that inflationary shocks have larger effects on the economy. As a consequence, inflation is more stable and output more volatile than under the standard non-robust policy. Moreover, the policy response to demand shocks is not affected by the preference for robustness, as the central bank is always able to neutralize the impact of such shocks on output and inflation by appropriate (and costless) changes in the interest rate. We have shown these results to hold for the optimal instrument rule under both the Nash and Stackelberg timing assumptions as well as for the optimal targeting rule under the Stackelberg assumption. Under the Nash assumption, however, the optimal targeting rule is not affected by the central bank's preference for robustness, as in Walsh (2004).

The result that a preference for robustness leads to more volatility in the interest rate is reminiscent of that in Söderström (2002). Using a Bayesian approach in a backward-looking model, he shows that increased uncertainty about the persistence of inflation makes monetary policy more aggressive, as the central bank wants to reduce the probability that inflation moves away from the target in the future. Here there is no persistence in inflation, so current inflation has no information about its future path, but as the robust central bank fears that inflation is more responsive to shocks, the central bank responds more aggressively to these shocks.

Results similar to ours have also been reached within the applied robust control literature, using numerical methods. For instance, Giordani and Söderlind (2004) analyze optimal robust policy (under both commitment and discretion) in the standard New-Keynesian model with persistent shocks. They show that robustness makes the central bank fear that shocks are more persistent, and therefore the optimal robust policy is more aggressive than the non-robust policy. That result is valid for typical parameterizations of the model. Although our model does not include persistence, we are able to show that robustness always leads to more aggressive policy, in any parameterization of the model.

Giannoni (2002) also studies the New-Keynesian with persistent demand shocks, where the central bank is uncertain about the slope of the IS equation and the Phillips curve, but the private sector faces no such uncertainty. Calculating optimized Taylor rule parameters, he finds that monetary policy is always more aggressive when uncertainty increases, as the central bank puts a larger weight on stabilizing inflation *and* output relative to smoothing interest rates.

We expect our conclusions to be fairly robust as long as two assumptions are fulfilled: the central bank influences inflation through aggregate demand only, with no other influence of the interest rate on inflation; and interest rate fluctuations in themselves do not affect social loss. The first of these assumptions is common in the closed-economy literature. In a companion paper (Leitemo and Söderström, 2004), we extend the analysis to consider a New-Keynesian model of a small open economy, in which policy moves influence the economy also through the exchange rate. In that paper, we find that there is a role for specification errors also in the IS equation, as the central bank cannot costlessly offset demand shocks without affecting inflation through the exchange rate. Furthermore, we show that robust policy can be either more or less aggressive than the non-robust policy, depending on the type of shock and the source of misspecification. Another extension that may change our conclusions is when social loss depends directly on fluctuations in the interest rate, as in Giannoni and Woodford (2004). Also in that case, policy moves would be costly, introducing a role for specification errors in the IS equation.

We believe that policymaking in practice to a large extent is about avoiding particularly bad outcomes, either due to policy or to private sector behaviour. Given our limited understanding of the economy, robust control techniques provide a methodology to identify and avoid policy moves that risk moving the economy toward bad states. Of course, there are alternative techniques to analyze optimal policy under uncertainty, in particular, the Bayesian approach pioneered by Brainard (1967). This approach tends to suggest that policy should be more attenuated in the face of uncertainty, in order to reduce the impact of uncertainty on the outcome.[12] These recommendations are intuitively appealing. Intuition also tells us, however, that policy actions under uncertainty need to be resolute in order to avoid particularly bad outcomes, as in the robust control approach. Thus, both approaches seem to capture important aspects of the scenario facing policymakers, and therefore provide insights into the optimal behavior of policy in an uncertain world.

[12]Exceptions to this rule are demonstrated by Craine (1979) and Söderström (2002).

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