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## Gains from Coordination in a Multi-Sector Open Economy: Does It Pay to Be Different?\*

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#### Abstract

Do countries gain by coordinating their monetary policies if they have different economic structures? We address this issue in the context of a new open-economy macro model with a traded and a non-traded sector and more importantly, with a across-country asymmetry in the size of the traded sector. We study optimal monetary policy under independent and cooperating central banks, based on analytical expressions for welfare objectives derived from quadratic approximations to individual preferences. In the presence of asymmetric structures, a new source of gains from coordination emerges due to a terms-of-trade externality. This externality affects unfavorably the country that is more exposed to trade and its effects tend to be overlooked when national central banks act independently. The welfare gains from coordination are sizable and increase with the degree of asymmetry across countries and the degree of openness, and decrease with the within-country correlation of sectoral shocks.

JEL classification: E52, F41, F42

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## 1 Introduction

As countries become more interdependent through international trade, should they conduct monetary policies independently or should they coordinate their policies? In other words, are there gains from international monetary policy coordination? This question lies at the heart of the intellectual discussions about optimal monetary policy in open economies. The literature has produced a strong conclusion in favor of inward-looking policies and flexible exchange-rate regimes. This conclusion has been drawn not only in the traditional literature within the Mundell-Fleming framework that features ad hoc stabilizing policy goals, but also in the more recent New Open-Economy Macro (NOEM) literature that features optimizing individuals, monopolistic competition and nominal rigidities, with the representative household's utility function serving as a natural welfare metric for optimal policy. In the traditional literature, many have argued that the gains from coordination are likely to be small because a flexible exchange-rate system would effectively insulate impacts of foreign disturbances on domestic employment and output [e.g., Mundell (1961) and the survey by McKibbin (1997)]. In the NOEM literature pioneered by Obstfeld and Rogoff (1995), it has been shown that, although gains from coordination are theoretically possible, they are quantitatively small [e.g., Obstfeld and Rogoff (2000, 2002), Corsetti and Pesenti (2001)].

The remarkably strong conclusion about the lack of gains from coordination has stimulated a lively debate and a growing strand of literature in search of sources of coordination gains by enriching the simple framework built by Obstfeld and Rogoff (2000, 2002). Several potential sources have been identified. For instance, the gains from coordination can be related to the degree of exchange-rate pass-through [e.g., Devereux and Engel (2003), Duarte (2003), and Corsetti and Pesenti (2001)].<sup>1</sup> Even with perfect exchange-rate pass-through, inward-looking monetary policy can be suboptimal and be improved upon by coordination, depending on the values of the intertemporal elasticity and the elasticity of substitution between goods produced in different countries [e.g., Clarida, et al. (2002), Benigno and Benigno (2003), Pappa (2004), Sutherland (2002a), and Tsacharov (2004)]. Policy coordination may also produce welfare gains if the international financial markets are incomplete [e.g., Benigno (2001) and Sutherland (2002b)], policy makers have imperfect information [e.g., Dellas (2004)], or domestic shocks are imperfectly correlated across sectors [e.g., Canzoneri, et al. (2004)].

<sup>&</sup>lt;sup>1</sup>Corsetti and Dedola (2002) show that, if the distribution of traded goods requires local inputs, then international markets would be endogenously segmented, rendering exchange-rate pass-through incomplete. This feature also provides a scope for international monetary policy cooperation.

The present paper emphasizes the role of asymmetries in the production structure across countries in generating gains from policy coordination. To this end we build a two-country model in the spirit of the NOEM literature, with two production sectors within each county. One sector produces traded goods that enter the consumption basket in both countries, and the other sector produces non-traded goods that enter the domestic consumption basket only. To allow for real effects of monetary policy, we assume staggered price setting in both sectors.<sup>2</sup> A key point of departure from the NOEM literature is that we allow the share of traded goods in the consumption basket to be different across countries to capture an important cross-country difference in production and trading structures. As Figure 1 shows, a developed country has typically a much larger share of service value-added in GDP than does a developing country, and the traded component of services is small. In this sense, the asymmetric production and trading structure in our model can be interpreted broadly as characterizing countries at different stages of development. In the context of this model, we examine how the presence of multiple sectors and sectoral asymmetries across countries would affect macroeconomic stability and welfare under independent or cooperating central banks.

To help exposition, we assume log-utility in aggregate consumption, a unitary elasticity of substitution between domestically-produced traded goods and imported goods, and a unitary elasticity of substitution between traded and non-traded goods in the consumption baskets. Many authors have demonstrated that, in the absence of non-traded sectors, these assumptions would preclude the possibility for a national policy maker to manipulate the terms of trade to improve its own welfare, so that optimal policy has no international dimension and there are no gains from coordination.<sup>3</sup> Introducing a non-traded sector and sticky prices in both sectors renders exchange-rate pass-through incomplete even under producer-currency pricing; meanwhile, it creates a policy trade-off facing independent central banks when sectoral shocks are imperfectly correlated, which provides a potential scope for gains from policy coordination. Nonetheless, in the special case of our model with symmetric production structures

<sup>&</sup>lt;sup>2</sup>For analytical tractability, the literature on international welfare effects of monetary policy typically employs a simpler model with one-period predetermined prices [e.g., Obstfeld and Rogoff (1995, 2000a, 2002), Corsetti and Pesenti (2001), Canzoneri, et al. (2004)]. Assuming staggered price-setting as we do here instead of predetermined prices helps generate richer and arguably more realistic equilibrium dynamics [e.g., Clarida, et al. (2002), Kollmann (2002)] and is thus more appropriate for *quantitative* welfare analysis. In addition, as is well known in the closed-economy literature, staggered price-setting leads to inefficient price dispersion, giving rise to an additional source of inefficiency that optimal monetary policy needs to deal with.

<sup>&</sup>lt;sup>3</sup>See, for example, Clarida et al. (2002), Benigno and Benigno (2003), and Pappa (2004), among others.

across countries, the coordination gains are quantitatively negligible. Thus, without sectoral asymmetry, it is difficult to generate large welfare gains from coordination in our framework that could emerge, for example, from our specifications of the utility function, the sources of nominal rigidities, or other modifications suggested in the literature. The same set of assumptions also makes it possible to derive second-order approximations to the households' utility functions even in the presence of multiple sources of nominal rigidities and sectoral asymmetry, and helps us to obtain an analytical expression for the welfare criterion that can be used to compare outcomes of different policies. Despite their apparent restrictiveness, these assumptions do not prevent the model from generating significant coordination gains in the presence of cross-country asymmetries, nor do they prevent us from studying the sensitivity of the results to some key parameters in the model.

The literature has long emphasized the importance of the non-traded sector in understanding international business cycle fluctuations [e.g., Stockman and Tesar (1995), Baxter, et al (1998), Corsetti, et al. (2003), and Ghironi and Melitz (2003)] and real exchange rate movements [e.g., Rogoff (1996) and Burnstein et al (2003)]. Empirical studies suggest that, at least for the OECD countries, a substantial part of aggregate fluctuations originates from sectoral shocks rather than national disturbances [e.g., Stockman (1988), and Marimon and Zilibotti (1998)]; and within each country, the time series processes generating productivity shocks in traded and non-traded sectors are quite different [e.g., Canzoneri, et al. (1999)]. These studies cast doubts on the ability of models with a single traded sector in explaining the transmission of shocks across countries. Yet, it is remarkable that studies of optimal monetary policy in open economies typically abstract from the non-traded sector or other multi-sector features of the actual economy by assuming that each country is completely specialized in a single traded sector, with no distinctions between sector-specific and country-specific shocks.<sup>4</sup>

<sup>&</sup>lt;sup>4</sup>A few notable exceptions include Obstfeld and Rogoff (2002) and Hau (2000), whose models feature a traded and a non-traded sector, with perfectly correlated shocks; Canzoneri, et al. (2004), who examine a version of the model presented in Obstfeld and Rogoff (2002), but allow imperfect correlations between sectoral shocks; Tille (2002), who presents a two-country model that features incomplete specialization of the countries in two types of traded goods (but with no non-traded goods), so that a distinction between sectoral shocks and national shocks arises; and Huang and Liu (2004a), who study a model with multiple stages of production and trade in intermediate goods. Unlike our work here, all of these studies maintain symmetric production and trading structures across countries.

Our paper contributes to the literature in three aspects. First, we explicitly incorporate the non-traded sector into an open economy model, so that a monetary authority needs to confront a policy trade-off stemming from multiple sources of nominal rigidities and imperfectly correlated sector-specific shocks, whereas in the standard one-sector model with traded goods only, policy makers are not concerned about such trade-offs. Second, we make a methodological contribution to the literature by deriving an explicit expression for welfare under both independent central banks and a common planner. To our knowledge, we are the first to derive such a welfare criterion in an open economy with multiple sectors based on quadratic approximations of households' utility function.<sup>5</sup> Finally, the main value-added of the current paper in relation to the existing literature is that, by introducing non-traded goods and asymmetric production structures, without any further modifications, we are able to go beyond the special results obtained by Obstfeld and Rogoff (2002) concerning the welfare consequences of international monetary policy cooperation. Under asymmetric production structures, the terms-of-trade movements generate a negative externality for the country that is more open to trade, an externality that the planner tries to correct for yet independent central banks do not take into account, so that there are gains from policy coordination. The gains increase with the degree of asymmetry across countries, the degree of openness, and the difference in the degrees of price rigidity between the traded and the non-traded sector; but decrease with the within-country correlations of sectoral shocks. Under calibrated parameters, the gains from coordination are sizable (between 0.1 and 0.2 percent of steady-state consumption equivalence, depending on the degree of asymmetry in the size of the traded sector). To the extent that the production and trading structure in our model captures a difference between developing countries and developed ones, our results shed some light on the welfare consequences of international monetary policy coordination between countries at different stages of development.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines equilibrium dynamics. Section 4 discusses optimal monetary policy under independent central banks and under cooperation. Section 5 assesses the quantitative gains from policy coordination and studies their sensitivity to changes in a few key parameters in the model. Finally, Section 6 concludes. We focus on presenting the main results and intuitions in the text, and relegate detailed derivations to the Appendix.

<sup>&</sup>lt;sup>5</sup>For a comprehensive description of the general approach to deriving the welfare criterion for optimal policy based on quadratic approximations to households' utility functions, see Michael Woodford (2003).

## 2 The Model

Consider a world economy with two countries, home and foreign, each populated by a continuum of identical, infinitely-lived households. The representative household in each country is endowed with one unit of time, and derives utility from consuming a basket of final goods. The consumption basket consists of traded goods, either domestically produced or imported (e.g., manufacturing goods), and of non-traded goods (e.g., services). Final consumption goods are composites of differentiated intermediate goods produced in two sectors, a traded good sector, and a non-traded good sector. Production of intermediate goods requires domestic labor as the only input, which is supplied by domestic households. Labor is mobile across sectors, but not across countries. The production and preference structures in the two countries are symmetric except that the share of traded goods in the final consumption basket may differ.

Time is discrete. In each period of time t = 0, 1, ..., a productivity shock is realized in each intermediate-good sector. Firms and households make their optimizing decisions after observing the shocks. All agents have access to an international financial market, where they can trade a state-contingent nominal bond. The government in each country conducts monetary policy and uses lump-sum transfers to finance production subsidies.

#### 2.1 Representative Households

The preferences of households are symmetric across countries, so we focus on the representative household in the home country. The utility function is given by

$$\mathbf{E}\sum_{t=0}^{\infty}\beta^{t}[\ln C_{t}-\Psi L_{t}],\tag{2.1}$$

where  $0 < \beta < 1$  is a subjective discount factor,  $C_t > 0$  denotes consumption,  $L_t \in (0, 1)$  denotes hours worked, and E is an expectation operator.

The purchase of consumption goods is financed by labor income, profit income, and a lumpsum transfer from the government. In addition, the household has access to an international financial market, where state-contingent nominal bonds (denominated in home currency) can be traded. The period-budget constraint facing the household is given by

$$P_t C_t + E_t D_{t,t+1} B_{t+1} \le W_t L_t + B_t + \Pi_t + T_t, \quad t = 0, 1, \dots,$$
(2.2)

where  $P_t$  is the price level,  $B_{t+1}$  is the holdings of the state-contingent nominal bond that pays one unit of home currency in period t + 1 if a specified state is realized,  $D_{t,t+1}$  is the period-t price of such bonds,  $W_t$  is the nominal wage rate,  $\Pi_t$  is the profit income, and  $T_t$  is the lump-sum transfer from the government.

The household maximizes (2.1) subject to (2.2). The optimal labor supply decision implies

$$\Psi C_t = W_t / P_t, \tag{2.3}$$

which states that the marginal rate of substitution between leisure and consumption equals the real consumption wage. The optimal consumption-saving decision is described by

$$D_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}},$$
(2.4)

so that the intertemporal marginal rate of substitution equals the price of the state contingent bond. Define the nominal interest rate on a risk-free bond as  $R_t = [E_t D_{t,t+1}]^{-1}$ . Then (2.4) implies that

$$\frac{1}{C_t} = \beta \mathcal{E}_t \left[ \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}} R_t \right], \qquad (2.5)$$

which is the familiar intertemporal Euler equation.

The final consumption basket consists of traded goods (domestically produced and imported) and non-traded goods. Denote  $C_{Nt}$  the composite good that is non-traded, and  $C_{Tt}$ the composite of goods that are traded. Then we have

$$C_t = \bar{\alpha} C_{Tt}^{\alpha} C_{Nt}^{1-\alpha}, \quad \bar{\alpha} = \alpha^{-\alpha} (1-\alpha)^{\alpha-1}.$$
(2.6)

The traded component  $C_{Tt}$  is itself an aggregate of domestically produced good  $C_{Ht}$  and imported good  $C_{Ft}$ , that is,

$$C_{Tt} = \bar{\omega} C_{Ht}^{\omega} C_{Ft}^{1-\omega}, \quad \bar{\omega} = \omega^{-\omega} (1-\omega)^{\omega-1}.$$
(2.7)

Solving the household's expenditure-minimizing problem yields the following demand functions for non-traded and traded goods:

$$C_{Nt} = (1 - \alpha) P_t C_t / \bar{P}_{Nt}, \quad C_{Tt} = \alpha P_t C_t / \bar{P}_{Tt},$$
 (2.8)

where  $\bar{P}_{Nt}$  is the price of final non-traded goods, and  $\bar{P}_{Tt}$  is the price of final traded goods, which are related to the price level  $P_t$  by

$$P_t = \bar{P}_{Tt}^{\alpha} \bar{P}_{Nt}^{1-\alpha}.$$
(2.9)

The induced demand functions for domestically produced traded goods and for imported goods are respectively given by

$$C_{Ht} = \omega \bar{P}_{Tt} C_{Tt} / \bar{P}_{Ht}, \quad C_{Ft} = (1 - \omega) \bar{P}_{Tt} C_{Tt} / [\mathcal{E}_t \bar{P}_{Ft}^*],$$
 (2.10)

where  $\bar{P}_{Ht}$  is the price index of home-produced traded goods,  $\bar{P}_{Ft}^*$  is the price index of foreignproduced traded goods, and  $\mathcal{E}_t$  is the nominal exchange rate. These prices are related to  $\bar{P}_{Tt}$ by

$$\bar{P}_{Tt} = \bar{P}_{Ht}^{\omega} [\mathcal{E}_t \bar{P}_{Ft}^*]^{1-\omega} \tag{2.11}$$

Throughout our analysis, we assume that firms set prices in the sellers' local currency and the law-of-one-price holds, so that the cost of imported goods in the home consumption basket is simply the price of traded goods charged by foreign exporting firms, adjusted by the nominal exchange rate, as in (2.11).

## 2.2 Production Technologies and Optimal Pricing Rules

There are two sectors producing intermediate goods: a non-traded sector and a traded sector. In each sector, there is a continuum of firms producing differentiated products indexed in the interval [0, 1]. To produce intermediate goods in each sector requires labor input, with constant-returns-to-scale (CRS) technologies

$$Y_{Nt}(i) = A_{Nt}L_{Nt}(i), \quad i \in [0, 1],$$
(2.12)

and

$$Y_{Tt}(j) = Y_{Ht}(j) + Y_{Ht}^*(j) = A_{Tt}L_{Tt}(j), \quad j \in [0, 1],$$
(2.13)

where  $Y_{Nt}(i)$  is the output of type-*i* non-traded intermediate goods;  $Y_{Tt}(j)$  is the output of type-*j* traded intermediate goods, part of which is to be sold in the domestic market  $(Y_{Ht}(j))$ and the rest to be exported  $(Y_{Ht}^*(j))$ ;  $A_{Nt}$  and  $A_{Tt}$  are productivity shocks in the two sectors; and  $L_N$  and  $L_T$  are labor inputs in the non-traded and in the traded sector respectively. The logarithms of the productivity shocks in each sector follows a random-walk process, that is,

$$\ln(A_{k,t+1}) = \ln(A_{k,t}) + \varepsilon_{k,t+1}, \quad k \in \{N, T\},$$
(2.14)

where  $\varepsilon_{Nt}$  and  $\varepsilon_{Tt}$  are mean-zero, iid normal processes with finite variances given by  $\sigma_N^2$  and  $\sigma_T^2$ , respectively. We allow the innovations of the sectoral shocks to be correlated, with a correlation coefficient given by  $\rho_{NT} \in [-1, 1]$ .

There is a CES aggregation technology that transforms intermediate goods produced in each sector into final consumption goods according to

$$C_{Nt} = \left[\int_{0}^{1} Y_{Nt}(i)^{\frac{\theta_{N}-1}{\theta_{N}}} di\right]^{\frac{\theta_{N}}{\theta_{N}-1}}, \quad C_{Ht} = \left[\int_{0}^{1} Y_{Ht}(j)^{\frac{\theta_{T}-1}{\theta_{T}}} dj\right]^{\frac{\theta_{T}}{\theta_{T}-1}}, \quad (2.15)$$

where  $\theta_N$  and  $\theta_T$  denote elasticities of substitution between differentiated products in the two sectors. To ensure equilibrium existence, we assume that the  $\theta$ 's both exceed unity (see, for example, Blanchard and Kiyotaki (1987)).

By solving the cost-minimizing problem of the aggregation sector, we obtain the demand functions for each type of intermediate goods:

$$Y_{Nt}^d(i) = \left[\frac{P_{Nt}(i)}{\bar{P}_{Nt}}\right]^{-\theta_N} C_{Nt}, \quad Y_{Ht}^d(j) = \left[\frac{P_{Ht}(j)}{\bar{P}_{Ht}}\right]^{-\theta_T} C_{Ht},$$
(2.16)

where  $P_{Nt}(i)$  is the price of type-*i* non-traded intermediate goods,  $P_{Ht}(j)$  is the price of type-*j* traded intermediate goods, and  $\bar{P}_{Nt} = \left[\int_0^1 P_{Nt}(i)^{1-\theta_N} dj\right]^{\frac{1}{1-\theta_N}}$  and  $\bar{P}_{Ht} = \left[\int_0^1 P_{Ht}(j)^{1-\theta_T} dj\right]^{\frac{1}{1-\theta_T}}$  are the corresponding price indices.

Firms are price takers in the input market and monopolistic competitors in the product markets. In each sector, firms stagger their pricing decisions in the spirit of Calvo (1983). Specifically, in each period of time, each firm receives an i.i.d. random signal that determines whether or not it can set a new price. The probability that a firm can adjust its price is  $1 - \gamma_k$ in sector  $k \in \{N, T\}$ . By the law of large numbers, a fraction  $1 - \gamma_k$  of all firms in sector kcan adjust prices, while the rest of the firms cannot.

If a firm who produces type-*i* non-traded goods can set a new price, it chooses  $P_{Nt}(i)$  to maximize its expected present value of profits

$$E_t \sum_{\tau=t}^{\infty} \gamma_N^{\tau-t} D_{t,\tau} [P_{Nt}(i)(1+\tau_N) - V_{N\tau}] Y_{N\tau}^d(i), \qquad (2.17)$$

where  $\tau_N$  is a production subsidy,  $V_{Nt}$  is the unit cost, which is identical across firms since all firms face the same input market, and  $Y_{Nt}^d(i)$  is the demand schedule for type *i* non-traded good described in (2.16). Regardless of whether a firm can adjust its price, it has to solve a cost-minimizing problem, the solution of which yields the unit cost function

$$V_{Nt} = W_t / A_{Nt}, \tag{2.18}$$

and a conditional factor demand function

$$L_{Nt} = \frac{1}{A_{Nt}} \int_0^1 Y_{Nt}^d(i) di.$$
 (2.19)

The solution to the profit-maximizing problem gives the optimal pricing rule

$$P_{Nt}(i) = \frac{\mu_N}{(1+\tau_N)} \frac{E_t \sum_{\tau=t}^{\infty} \gamma_N^{\tau-t} D_{t,\tau} V_{N\tau} Y_{N\tau}^d(i)}{E_t \sum_{\tau=t}^{\infty} \gamma_N^{\tau-t} D_{t,\tau} Y_{N\tau}^d(i)},$$
(2.20)

where  $\mu_N = \theta_N/(\theta_N - 1)$  measures the steady-state markup in sector N. Similarly, the optimal pricing rule for a firm that produces type-*j* traded good is given by

$$P_{Ht}(j) = \frac{\mu_T}{(1+\tau_T)} \frac{E_t \sum_{\tau=t}^{\infty} \gamma_T^{\tau-t} D_{t,\tau} V_{T\tau} [Y_{H\tau}^d(j) + Y_{H\tau}^{*d}(j)]}{E_t \sum_{\tau=t}^{\infty} \gamma_T^{\tau-t} D_{t,\tau} [Y_{H\tau}^d(j) + Y_{H\tau}^{*d}(j)]},$$
(2.21)

where  $\mu_T = \theta_T/(\theta_T - 1)$  measures the steady state markup in sector T. From solving the firm's cost-minimizing problem, we obtain the unit cost function

$$V_{Tt} = W_t / A_{Tt}, \tag{2.22}$$

and a conditional factor demand function

$$L_{Tt} = \frac{1}{A_{Tt}} \int_0^1 [Y_{Ht}^d(j) + Y_{Ht}^{*d}(j)] dj.$$
 (2.23)

The economic structure of the foreign country is similar, except that the share of traded good in the consumption basket may differ from that in the home country. In particular, the foreign consumption basket is given by

$$C_t^* = \bar{\alpha}^* C_{Tt}^{*\alpha^*} C_{Nt}^{*1-\alpha^*}, \quad \bar{\alpha}^* = \alpha^{*-\alpha} (1-\alpha^*)^{\alpha^*-1}, \tag{2.24}$$

where  $\alpha^*$  may not equal to  $\alpha$ . The foreign country's structure is otherwise symmetric to that of the home country's. In what follows, we denote all foreign variables with an asterisk and assume that all other parameters are identical to their counterparts in the home country.

### 2.3 Risk Sharing, Market Clearing, and Equilibrium

Since the state-contingent nominal bond is traded in the international financial market, the foreign household's optimal consumption-saving decision leads to

$$D_{t,t+1} = \beta \frac{C_t^*}{C_{t+1}^*} \frac{P_t^*}{P_{t+1}^*} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}.$$
(2.25)

By combining this equation with its home counterpart (2.4) and iterating with respect to t, we obtain a risk-sharing condition

$$Q_t = \phi_0 \frac{C_t}{C_t^*},$$
 (2.26)

where  $Q_t = \mathcal{E}_t P_t^* / P_t$  is the real exchange rate, and  $\phi_0 = Q_0 C_0^* / C_0$ . The risk-sharing condition links the real exchange rate to the marginal rate of substitution between consumption in the two countries, so that all households face identical relative price of consumption goods in the world market. In equilibrium, each country's labor market as well as the world bond market clear. Since labor is mobile within each country (but not across countries), labor market clearing implies that

$$L_{Nt} + L_{Tt} = L_t, \quad L_{Nt}^* + L_{Tt}^* = L_t^*.$$
 (2.27)

Also, in equilibrium, nominal bonds are in zero net supply in the world market, so that  $B_t + B_t^* = 0.$ 

Our goal is to analyze optimal monetary policy under two alternative monetary regimes. One in which each country tries to maximize its own households' welfare, taking the other country's policy actions as given; and the other in which a world planner tries to coordinate the two countries' monetary policy so as to maximize their collective welfare. For this purpose, we do not specify a particular monetary policy rule. Instead, we solve for the optimal policy that maximizes the welfare objective under each regime, subject to the private sector's optimizing conditions. For any given monetary policy, we can define an equilibrium for this world economy.

An equilibrium consists of allocations  $C_t$ ,  $C_{Nt}$ ,  $C_{Tt}$ ,  $L_t$ ,  $B_{t+1}$  for the home household and  $C_t^*$ ,  $C_{Nt}^*$ ,  $C_{Tt}^*$ ,  $L_t^*$ ,  $B_{t+1}^*$  for the foreign household; allocations  $Y_{Nt}(i)$ , and  $L_{Nt}(i)$ , and price  $P_{Nt}(i)$  for non-traded intermediate good producer  $i \in [0, 1]$  in the home country and  $Y_{Nt}^*(i)$ , and  $L_{Nt}^*(i)$ , and price  $P_{Nt}^*(i)$  for non-traded intermediate good producer  $i \in [0, 1]$  in the home country and  $Y_{Nt}^*(i)$ , and  $L_{Nt}^*(i)$ , and price  $P_{Nt}^*(i)$  for non-traded intermediate good producer  $i \in [0, 1]$  in the foreign country; allocations  $Y_{Ht}(j)$ ,  $Y_{Ht}^*(j)$ , and  $L_{Tt}(j)$ , and price  $P_{Ht}(j)$  for traded intermediate good producer  $j \in [0, 1]$  in the home country and  $Y_{Ft}^*(j)$ ,  $Y_{Ft}(j)$ , and  $L_{Tt}^*(j)$ , and price  $P_{Ft}^*(j)$  for traded intermediate good producer  $j \in [0, 1]$  in the home country and  $Y_{Ft}^*(j)$ ,  $Y_{Ft}(j)$ , and  $L_{Tt}^*(j)$ , and price  $P_{Ft}^*(j)$  for traded intermediate good producer  $j \in [0, 1]$  in the home country and  $Y_{Ft}^*(j)$ ,  $Y_{Ft}(j)$ , and  $L_{Tt}^*(j)$ , and price  $P_{Ft}^*(j)$  for traded intermediate good producer  $j \in [0, 1]$  in the foreign country; together with prices  $D_{t,t+1}$ ,  $\mathcal{E}_t$ ,  $Q_t$ ,  $P_t$ ,  $\bar{P}_{Nt}$ ,  $\bar{P}_{Tt}$ ,  $\bar{P}_{$ 

## 3 Equilibrium Dynamics

To facilitate analysis of optimal monetary policy, we first examine a useful benchmark in which price adjustments are flexible, and then describe the equilibrium dynamics under sticky prices. For ease of exposition, we assume that the production subsidies exactly offset the steady-state monopolistic distortions, so that the allocations in the flexible-price equilibrium are Pareto optimal.<sup>6</sup> We call these allocations the "natural rate" allocations, and deviations of the sticky-price equilibrium allocations from their natural rate levels the "gaps." In analyzing the equilibrium dynamics, we focus on log-deviations of equilibrium variables from their steady-state values (denoted by hatted variables).

### 3.1 The Balanced-Trade Steady State and the Current Account

We begin by describing a balanced-trade steady state equilibrium, which is obtained by shutting off all the shocks (i.e., we set  $A_k = A_k^* = 1$  for  $k \in \{N, T\}$ ) and in which the net export is zero. The net export in the home country is given by

$$NX_{t} = \bar{P}_{Ht}C_{Ht}^{*} - \mathcal{E}_{t}\bar{P}_{Ft}^{*}C_{Ft}$$

$$= (1-\omega)\alpha^{*}\mathcal{E}_{t}P_{t}^{*}C_{t}^{*} - (1-\omega)\alpha P_{t}C_{t}$$

$$= (1-\omega)\alpha^{*}\mathcal{E}_{t}P_{t}^{*}C_{t}^{*} \left[1 - \frac{\alpha}{\alpha^{*}}Q_{t}^{-1}\frac{C_{t}}{C_{t}^{*}}\right]$$

$$= (1-\omega)\alpha^{*}\mathcal{E}_{t}P_{t}^{*}C_{t}^{*} \left[1 - \frac{\alpha}{\alpha^{*}}\phi_{0}^{-1}\right], \qquad (3.1)$$

where the second equality follows from the demand functions for final traded consumption goods as in (2.10) and its foreign counterpart, the third from the definition of the real exchange rate, and the last from the international risk-sharing condition (2.26). In the balanced-trade steady state, NX = 0 so that  $\phi_0$  is given by

$$\phi_0 = \frac{\alpha}{\alpha^*}.\tag{3.2}$$

Clearly, if the countries have symmetric structures, that is, if  $\alpha = \alpha^*$ , then we have  $\phi_0 = 1$  and, from the risk-sharing condition (2.26),  $C_t = Q_t C_t^*$ . Since the real exchange rate  $Q_t$  represents the relative price of foreign consumption basket in terms of home consumption, it follows that, under symmetric structures, international risk-sharing leads to equalized aggregation

<sup>&</sup>lt;sup>6</sup>Under certain conditions, monopolistic competition and the associated inflationary bias for monetary policy is not the only source of steady-state distortions; an independent central bank has also an incentive to manipulate the terms of trade to their own favor, in other words, there is also a deflationary bias [e.g., Clarida, et al. (2002) and Benigno and Benigno (2003)]. This result can be obtained in a world where the households in the two countries face an identical traded-consumption basket, that is, the expenditure share on traded goods produced by a given country coincides with the population size of that country. This is the assumption made, for example, by Clarida, et al. (2002). We do not make this assumption so that removing the monopolistic distortion renders the flexible-price equilibrium allocations Pareto optimal.

consumption (measured in identical units) across countries for each period t. Yet, in the presence of structural asymmetry, that is, in the more general case with  $\alpha \neq \alpha^*$ , we have  $\phi_0 \neq 1$  so that consumptions in the two countries are not necessarily equal (in conformable units) even with households trading the state-contingent assets in the international financial market. In this case, the  $\phi_0$  term represents a "risk-sharing wedge" that arises only in the presence of structural asymmetry in the global economy. It turns out, as we show below, the condition under which the risk-sharing wedge arises also leads to sizable welfare gains from international monetary policy coordination.<sup>7</sup>

Given that  $\phi_0 = \alpha/\alpha^*$ , equation (3.1) implies that the net export is zero not only in the steady state, but for all  $t \ge 0$ . With zero net export, along with the assumption that neither country has an initial outstanding debts, the equilibrium current account would be zero for all t. This result greatly simplifies our analytical derivations of the welfare criteria.

#### 3.2 The Flexible-Price Equilibrium and the Natural Rate

When price adjustments are flexible, firms' pricing decisions are synchronized, so that the optimal price set by a firm is a constant markup over its contemporaneous marginal cost and that the price index in each sector coincides with the pricing decision of a typical firm in that sector. We now describe the equilibrium dynamics when all prices are flexible.

Let  $S_t = \mathcal{E}_t \bar{P}_{Ft}^* / \bar{P}_{Ht}$  denote the home country's terms of trade. It is easy to show that the natural-rate level of the terms of trade, in log-deviation forms, is given by

$$\hat{s}_t^n = \hat{a}_{Tt} - \hat{a}_{Tt}^*, \tag{3.3}$$

Thus, an increase in the relative productivity in home's traded sector (relative to the foreign traded sector) tends to lower the relative price of traded goods produced in the home country, and thus leads to worsened terms of trade for that country.

Next, to a first-order approximation, the natural-rate level of home traded output and non-trade output are given by  $\hat{y}_{Tt}^n = \hat{a}_{Tt}$  and  $\hat{y}_{Nt}^n = \hat{a}_{Nt}$ , respectively. Thus, under flexible

<sup>&</sup>lt;sup>7</sup>Pesenti and Tille (2004) emphasize the importance of the risk-sharing wedge in analyzing gains from international monetary policy coordination in a one-sector open economy model with preset prices. The risk-sharing wedge in our model is somewhat different from theirs in that it is determined here by the balanced-trade steadystate conditions, so that it is independent of monetary policy; whereas in the Pesenti-Tille world, the wedge is given by the ratio of the expected marginal utility of consumption in the two countries, and is thus endogenous to policy. Such difference stems mainly from the different assumptions about the timing of portfolio choice decisions.

prices, each sector's output responds one-for-one with the sector-specific shocks, and there is no inter-sectoral or international spillover effects of shocks on production. It then follows from the production functions that the natural rates of sectoral employment are constant, that is,  $\hat{l}_{Tt}^n = \hat{l}_{Nt}^n = 0.$ 

Third, given the solutions for the sectoral outputs and the terms of trade above, we can solve for the natural-rate level of aggregate consumption, which is given by

$$\hat{c}_t^n = \alpha \hat{a}_{Tt} + (1 - \alpha) \hat{a}_{Nt} - \alpha (1 - \omega) \hat{s}_t^n.$$
(3.4)

Thus, aggregate consumption responds not only to domestic sectoral shocks, but also to movements in the terms of trade since part of the consumption basket consists of imported goods. An improved domestic productivity or terms of trade would raise the natural rate level of consumption.

Finally, the relative price of non-traded goods (in terms of traded goods) in the flexibleprice equilibrium can be obtained by using the pricing decision equations and the solution for the terms of trade:

$$\hat{q}_{Nt}^{n} \equiv \hat{\bar{p}}_{Nt} - \hat{\bar{p}}_{Tt} = \hat{a}_{Tt} - \hat{a}_{Nt} - (1 - \omega)\hat{s}_{t}^{n}.$$
(3.5)

Hence, in the flexible-price equilibrium, the relative price of non-traded goods decreases with the relative productivity of the non-traded sector. Further, an improvement in the terms of trade (i.e., a fall in  $\hat{s}_t^n$ ) would make imported goods relatively cheaper, so that the price of the traded basket would fall and the relative price of non-traded goods would rise.

### 3.3 The Sticky-Price Equilibrium

The sticky price equilibrium is characterized by the optimizing conditions derived in Section 2. Denote  $\tilde{x}_t = \hat{x}_t - \hat{x}_t^n$  the deviation of equilibrium variable  $\hat{x}_t$  under sticky prices from its own natural rate  $\hat{x}_t^n$ , that is, the gap. After log-linearizing, the private sector's optimizing

conditions under sticky prices can be summarized below:

$$\pi_{Nt} = \beta \mathbf{E}_t \pi_{N,t+1} + \kappa_N \left[ \tilde{c}_t - \alpha \tilde{q}_{Nt} \right], \qquad (3.6)$$

$$\pi_{Ht} = \beta E_t \pi_{H,t+1} + \kappa_T \left[ \tilde{c}_t + (1-\alpha) \tilde{q}_{Nt} + (1-\omega) \tilde{s}_t \right], \qquad (3.7)$$

$$\tilde{q}_{Nt} = \tilde{q}_{N,t-1} + \pi_{Nt} - \pi_{Ht} - (1-\omega)\Delta \tilde{s}_t + \Delta \hat{a}_{Nt} - \Delta \hat{a}_{Tt},$$
(3.8)

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - \{ \hat{r}_t - E_t \left[ \alpha (\pi_{H,t+1} + (1-\omega)\Delta \tilde{s}_{t+1}) + (1-\alpha)\pi_{N,t+1} \right] \}$$
(3.9)

$$\pi_{Nt}^{*} = \beta E_{t} \pi_{N,t+1}^{*} + \kappa_{N} \left[ \tilde{c}_{t}^{*} - \alpha^{*} \tilde{q}_{Nt}^{*} \right], \qquad (3.10)$$

$$\pi_{Ft}^* = \beta E_t \pi_{F,t+1}^* + \kappa_T \left[ \tilde{c}_t^* + (1 - \alpha^*) \tilde{q}_{Nt}^* - (1 - \omega) \tilde{s}_t \right], \qquad (3.11)$$

$$\tilde{q}_{Nt}^{*} = \tilde{q}_{N,t-1}^{*} + \pi_{Nt}^{*} - \pi_{Ft}^{*} + (1-\omega)\Delta\tilde{s}_{t} + \Delta\hat{a}_{Nt}^{*} - \Delta\hat{a}_{Tt}^{*}, \qquad (3.12)$$

$$\tilde{c}_t^* = \mathbf{E}_t \tilde{c}_{t+1}^* - \left\{ \hat{r}_t^* - \mathbf{E}_t \left[ \alpha^* (\pi_{F,t+1}^* - (1-\omega)\Delta \tilde{s}_{t+1}) + (1-\alpha^*)\pi_{N,t+1}^* \right] \right\}$$
(3.13)

$$\tilde{s}_{t} = \tilde{s}_{t-1} + \Delta \hat{e}_{t} + \pi^{*}_{Ft} - \pi_{Ht} + \Delta \hat{a}^{*}_{Tt} - \Delta \hat{a}_{Tt}, \qquad (3.14)$$

$$\tilde{c}_t - \tilde{c}_t^* = (2\omega - 1)\tilde{s}_t + (1 - \alpha^*)\tilde{q}_{Nt}^* - (1 - \alpha)\tilde{q}_{Nt}, \qquad (3.15)$$

where the  $\pi$ 's denote the sectoral inflation rates, and  $\kappa_i = \frac{(1-\beta\gamma_i)(1-\gamma_i)}{\gamma_i}$  is a constant that measures the responsiveness of the pricing decisions in sector  $i \in \{N, T\}$  to variations in the sectoral real marginal cost gaps.

Equations (3.6) and (3.7) describe the sectoral Phillips-curve relations in the home country. These relations are forward-looking in that a sector's period-t inflation rate depends solely upon current and expected future marginal cost gaps in that sector. The marginal cost gap in each sector depends positively on the aggregate output gap but negatively on the sector's relative price gap. Additionally, the marginal cost in the home country's traded sector depends positively on its terms-of-trade gap, so that a terms-of-trade improvement (i.e., a fall in  $\tilde{s}_t$ ) leads to a fall in the real marginal cost in the home traded sector, but has no direct effect on the marginal cost in the non-traded sector.

Equation (3.8) describes the law of motion of the relative-price gap for home non-traded goods. Equation (3.9) is a log-linearized version of the intertemporal Euler equation (2.5) for the home household. Equations (3.10)-(3.13) are the foreign counterparts of (3.6)-(3.9), and can be interpreted similarly. Equation (3.14) describes the law of motion of the terms-of-trade gap. Finally, equation (3.15) is derived from the international risk-sharing condition (2.26) along with the price-index relations. It implies that, despite the assumption of producercurrency pricing and the existence of state-contingent assets, aggregate consumption does not equalize across countries if the share of non-traded sectors are positive (i.e., if the  $\alpha$ 's are less than 1) or there is home-bias in the consumption of traded goods (i.e.,  $\omega > 1/2$ ). In general, complete consumption insurance across countries can not be achieved by trading the statecontingent assets because of fluctuations in the relative prices of non-traded goods or in the terms of trade.

Before we proceed to characterize optimal monetary policy, it is necessary to find out whether or not, in a two-sector model like this, the national monetary authority faces a policy trade-off in stabilizing the gaps and sectoral inflation rates. If not, then optimal independent monetary policy would be able to replicate the efficient flexible-price allocations and there would be no need for cooperation. Woodford (2003, Chapter 3) shows that, in a closed economy with two sectors, if the degree of price stickiness is identical across sectors, then the sectoral Phillips curve relations can be reduced to an aggregate Phillips-curve that is identical to that in a one-sector model, so that the trade-off between price stability and stabilizing output gap fluctuations disappears, regardless of whether or not the sectoral shocks are correlated. Is this still the case in our two-sector open economy environment? To answer this question, consider the special case with  $\kappa_N = \kappa_T = \kappa$  so that the two sectors have identical durations of price contracts. Define a domestic inflation index as  $\hat{\pi}_{Dt} = \alpha \hat{\pi}_{Ht} + (1 - \alpha) \hat{\pi}_{Nt}$ . Then, by taking a weighted average of the sectoral Phillips curves in (3.6) and (3.7), we obtain

$$\pi_{Dt} = \beta \mathbf{E}_t \pi_{D,t+1} + \kappa \tilde{c}_t + \kappa \alpha (1-\omega) \tilde{s}_t.$$

In the special case of a closed-economy (with  $\omega = 1$ ), this relation reduces to an aggregate Phillips curve that implies no trade-off between output stability and price stability: the national central bank is able to close the output gap by simply setting the domestic inflation index  $\pi_{Dt} =$ 0. In an open economy as the one presented here, however, fluctuations in the terms-of-trade gap act as an endogenous "cost-push shock" that introduces a trade-off between stabilizing the output gap and the domestic inflation index, unless the traded sector is entirely shut off (i.e., with  $\alpha = 0$ ). It turns out that it is in general not possible to implement the flexible price allocations in this open economy.

**Proposition 1.** In the presence of nominal rigidities in both sectors and sector-specific shocks, it is not possible to implement the flexible-price allocations unless the domestic sectoral shocks are perfectly correlated.

**Proof:** By contradiction. Suppose that the flexible-price allocations could be replicated. Then the output gap, the relative price gap, and the terms-of-trade gap would all be closed, that is,  $\tilde{c}_t = \tilde{q}_t = \tilde{s}_t = 0$  for all t. It follows from (3.6) and (3.7) that  $\pi_{Nt} = \pi_{Ht} = 0$  for all t. But given  $\tilde{q}_t = 0$  and  $\tilde{s}_t = 0$  for all t, (3.8) implies that  $\pi_{Nt} - \pi_{Ht} = \Delta \hat{a}_{Tt} - \Delta \hat{a}_{Nt}$ , contradicting  $\pi_{Nt} = \pi_{Ht} = 0$  unless  $\Delta \hat{a}_{Tt} = \Delta \hat{a}_{Nt}$  for all t. Q.E.D.

Although the flexible-price equilibrium allocations are Pareto optimal, the existence of the trade-off between stabilizing the gaps and inflation rates stated in Proposition 1 renders optimal monetary policy second best. In the next section, we define the optimal monetary policy problems and characterize allocations under cooperative and non-cooperative policies.

## 4 Optimal Monetary Policy

Optimal monetary policy entails maximizing a social objective function subject to the private sector's optimizing conditions. A natural welfare criterion in our model is the representative households' expected life-time utility. Following the approach described in Benigno and Woodford (2004), we derive an analytical, quadratic expression for the welfare criterion based on second-order approximations to the representative households' utility functions and to the private sectors' optimizing conditions (except for those exact log-linear relations). We substitute all relevant second-order relations into the objective function to obtain a quadratic expression for the welfare objective. Finally, upon obtaining this objective, we solve for the allocations under optimal monetary policy by maximizing the quadratic objective subject to the set of log-linearized equilibrium conditions (3.6)-(3.15).<sup>8</sup> In this final step, we are essentially solving a linear-quadratic (LQ) problem with rational expectations. The LQ approach has become a popular tool in studying optimal monetary policy in closed economy models with a single sector (e.g., Rotemberg and Woodford (1997)) or multiple sectors (e.g., Erceg, et al. (2000), Huang and Liu (2004b)), and in open economy models with a single traded sector (e.g., Clarida, et al. (2002), Benigno and Benigno (2003), Gali and Monacelli (2002), and Pappa (2004)). We are the first to derive an analytical expression for the welfare objective in an open economy model with multiple sectors and multiple sources of nominal rigidity, for both a regime with independent central banks (i.e., the Nash regime) and one with cooperating central banks (i.e., the cooperating regime).<sup>9</sup>

 $<sup>^8{\</sup>rm The}$  details of this procedure is described in the Appendix.

<sup>&</sup>lt;sup>9</sup>Our approach differs slightly from that adopted in the open-economy papers mentioned here [e.g., Clarida, et al. (2002), Benigno and Benigno (2003), Gali and Monacelli (2002), and Pappa (2004)] in that we do not limit ourselves from the outset to taking first-order approximations to the private sectors' optimizing conditions. An alternative solution method is to take second-order approximations throughout the model and then to compute

#### 4.1 Independent Central Banks

A regime with independent central banks is one in which the national monetary authority in each country seeks to maximize the welfare of its own households, taking as given equilibrium variables and monetary policy in the other country. We refer to this regime as the "Nash regime" and a national central bank under this regime a "Nash central bank."

In the appendix, we show that, based on second-order approximations to the representative household's utility function, the welfare objective function for the Nash central bank in the home country is given by

$$W_t^{Nash} = E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t^{Nash} + \text{t.i.p.} + O\left(\|\xi\|^3\right),$$
(4.1)

with the period loss function given by

$$L_t^{Nash} = \tilde{c}_t^2 + \alpha (1 - \alpha) \tilde{q}_{Nt}^2 + (1 - \alpha) \theta_N \kappa_N^{-1} \pi_{Nt}^2 + \alpha \theta_T \kappa_T^{-1} \pi_{Ht}^2, \qquad (4.2)$$

where "t.i.p." represents the terms independent of policy, and  $O\left(\|\xi\|^3\right)$  denotes terms that are of third or higher order in an appropriate bound on the amplitude of the shocks. The approximated welfare objective for the foreign central bank is analogous. Under the Nash regime, the home central bank solves its optimal monetary policy problem by maximizing the quadratic welfare objective function (4.1) subject to the private sector's optimizing conditions (3.6)-(3.15). Similar for the foreign central bank.

The welfare criterion described in (4.1) and (4.2) reveals that a Nash central bank seeks to bring its domestic equilibrium allocation close to that under flexible prices through minimizing variations in the final output gap (i.e., consumption gap) and the relative-price gap, and through stabilizing the domestic sectoral inflation rates. It is not surprising that foreign variables do not appear in the home central bank's objective function since, by the definition of the Nash regime, they are treated as terms independent of policy. A somewhat more surprising result is that the terms of trade do not enter the objective function either. This result emerges since, as we show in the Appendix (Section A.1), under our specifications of preferences and

approximate optimal policy rules through non-linear simulations of the second-order system [e.g., Pesenti and Tille (2004), Sutherland (2002b), Tille (2002), Tscharov (2004)]. A main advantage of our approach, and the standard LQ approach described by Woodford (2003) as well, is that it allows us to obtain an analytical and explicit description of the objective function for optimal policy.

aggregation technologies, the home terms of trade can be expressed in terms of foreign variables alone (see (A.1.7)), and vice versa for the foreign terms of trade (see (A.1.6)), so that each country's central bank perceives its terms of trade as independent of policy.<sup>10</sup>

As is evident from the loss function (4.2), a national central bank faces a trade-off between stabilizing domestic gaps and inflation rates, so that it cannot implement the flexible-price allocations that are Pareto optimal (Proposition 1), unless the size of one sector approaches zero (i.e.,  $\alpha = 1$  or  $\alpha = 0$ ), or there is only one source of nominal rigidity (i.e.,  $\gamma_T = 0$  or  $\gamma_N = 0$ , or the shocks are perfectly correlated (i.e.,  $\Delta \hat{a}_{Tt} = \Delta \hat{a}_{Nt}$ ). In general, allocations under optimal policy are second best, and the social welfare under optimal policy depends on the relative weights in front of each of the four variable that the central bank cares about. The relative price gap receives a weight that is concave in the parameter  $\alpha$  that measures the relative size of the traded sector, and the weight reaches its maximum when  $\alpha = 0.5$ . When the size distribution of sectors is skewed, however, the sector with a greater share receives a larger weight in front of its sectoral inflation rate, and fluctuations in the relative-price gap become less of a concern for optimal policy. Holding the size of each sector constant, the weight of a sector's inflation rate increases with the elasticity of substitution between differentiated goods produced in that sector (i.e., increases with  $\theta_i$ ) and with the sector's price-rigidity (i.e., decreases with  $\kappa_i$ ). Yet, a sector with more rigid prices does not necessarily receive a larger weight for its inflation in the loss function, since the weight here is scaled by the relative size of the sector.

An important issue of concern, in the spirit of Obstfeld and Rogoff (2002), is then: From a global perspective, would the lack of international monetary policy coordination incur substantive welfare losses? Obstfeld and Rogoff find that the answer is "no" in their model with a single source of nominal rigidity. We revisit this issue below in a context with multiple sources of nominal rigidities and with potential structural differences across countries in the form of differing sizes of traded sectors. For this purpose, we first derive a welfare objective function

<sup>&</sup>lt;sup>10</sup>Under more general specifications of preferences and aggregation technologies, for instance, with non-unitary values of intertemporal (or intratemporal) elasticity of substitution, a Nash central bank may be tempted to manipulate its terms of trade to engage in beggar-thy-neighbor policies, instead of trying to implement the flexible-price equilibrium. See, for example, Benigno and Benigno (2003), Clarida, et al. (2002), and Pappa (2004). Since our focus is to evaluate optimal monetary policy under asymmetric structures, we simplify the model by assuming log-utility and Cobb-Douglas aggregation technologies so that it is possible to derive analytical expressions for approximated welfare objective functions.

for the policymakers under the cooperating regime in the next subsection, and then examine the quantitative welfare gains from coordination in Section 5.

#### 4.2 Cooperating Central Banks

A regime with cooperating central banks is one in which monetary policy decisions are delegated to a supranational monetary institution (i.e., a social planner), who seeks to maximize a weighted average of national welfare in the two countries. Unlike a Nash central bank, the planner here does not take any country's variables as given. To maintain symmetry of the model (other than the potentially different size of the traded sectors), we assume that the planner assigns equal weights (half) to each member country's national welfare.

As in the Nash regime, the social planner here cannot replicate the flexible-price equilibrium allocations and thus seeks to minimize deviations of allocations from the natural-rate levels. Yet, a critical difference between the cooperating regime and the Nash regime is that the social planner does care about the welfare implications of changes in the terms of trade. It turns out that, when the size of the traded sector differs across countries, the terms of trade plays a particularly important role in determining the social welfare.

In the appendix, we show that the planner's welfare objective can also be derived by second-order approximations to a weighted average of the households' utility functions in the two countries. It is given by

$$W_{t}^{Planner} = \frac{1}{2} E_{0} \sum_{t=0}^{\infty} \beta^{t} [U_{t} + U_{t}^{*}] = -\frac{1}{4} E_{0} \sum_{t=0}^{\infty} \beta^{t} L_{t}^{Planner} + \text{t.i.p.} + O\left(\|\xi\|^{3}\right), \quad (4.3)$$

$$L_{t}^{Planner} = \alpha \left[\tilde{m}c_{Ht}^{2} + \theta_{T}\kappa_{T}^{-1}\pi_{Ht}^{2}\right] + (1-\alpha) \left[\tilde{m}c_{Nt}^{2} + \theta_{N}\kappa_{N}^{-1}\pi_{Nt}^{2}\right]$$

$$+ \alpha^{*} \left[\tilde{m}c_{Ft}^{*2} + \theta_{T}^{*}\kappa_{T}^{*-1}\pi_{Ft}^{*2}\right] + (1-\alpha^{*}) \left[\tilde{m}c_{Nt}^{*2} + \theta_{N}^{*}\kappa_{N}^{*-1}\pi_{Nt}^{*2}\right]$$

$$+ 2(\alpha - \alpha^{*})(1-\omega)\tilde{s}_{t}, \quad (4.4)$$

where the marginal cost terms are given by

$$\tilde{m}c_{Ht} = \tilde{c}_t + (1 - \alpha)\tilde{q}_{Nt} + (1 - \omega)\tilde{s}_t, \quad \tilde{m}c_{Nt} = \tilde{c}_t - \alpha\tilde{q}_{Nt}, \tilde{m}c_{Ft}^* = \tilde{c}_t^* + (1 - \alpha^*)\tilde{q}_{Nt}^* - (1 - \omega)\tilde{s}_t, \quad \tilde{m}c_{Nt}^* = \tilde{c}_t^* - \alpha^*\tilde{q}_{Nt}^*.$$

A striking feature of the planner's welfare loss function here compared to that under the Nash regime is the presence of a linear term involving the terms-of-trade gap.<sup>11</sup> The loss

<sup>&</sup>lt;sup>11</sup>The planner's objective function is not derived as a simple average of the two Nash central banks objective functions. A Nash central bank takes foreign equilibrium variables and monetary policy as given, so that, as

function (4.4) for the planner reveals that the planner cares not only about the variations in the gaps and the sectoral inflation rates in the member countries, it also cares about the *level* of the terms-of-trade gap. In particular, the planner tries to move the terms-of-trade gap in a direction that favors the country with a larger traded sector. For example, if the home country has a larger traded sector (i.e., if  $\alpha - \alpha^* > 0$ ), then it would be in the planner's interest to improve the home country's terms of trade (i.e., to lower  $\tilde{s}_t$ ).

Why? Suppose the planner tries to do the opposite, that is, to allow home country's terms of trade to deteriorate. Then an expenditure switching effect would lead to increased world demand for home traded goods, forcing home workers in the traded sector to work harder. As the size of the traded sector in the home country is larger than that in the foreign country, it follows that a larger fraction of the world population would have to work harder, which is apparently against the social planner's interest.

Of course, the planner would also like to minimize variations in the sectoral marginal-cost gaps, and in particular, variations in the terms-of-trade gap as well. It thus tries to balance the desire to set the terms-of-trade gap in favor of the country with a relatively large traded sector against the need to stabilize the terms-of-trade gap. In the symmetric case when countries have an equal size of the traded sector, the level of the terms of trade drops out of the planner's objective function, and stabilizing the gaps (including the terms-of-trade gap) and the inflation rates becomes the sole objective for the planner.

To make the dependence of the planner's objective on the gaps and inflation rates more explicit, we expand the quadratic terms in the loss function (4.4) and collect terms to obtain

$$L_t^{Planner} = \tilde{c}_t^2 + \alpha (1-\alpha) \tilde{q}_{Nt}^2 + (1-\alpha) \theta_N \kappa_N^{-1} \pi_{Nt}^2 + \alpha \theta_T \kappa_T^{-1} \pi_{Ht}^2 + \tilde{c}_t^{*2} + \alpha^* (1-\alpha^*) \tilde{q}_{Nt}^{*2} + (1-\alpha^*) \theta_N \kappa_N^{-1} \pi_{Nt}^{*2} + \alpha^* \theta_T \kappa_T^{-1} \pi_{Ft}^{*2} + (1-\omega) \left[ (\alpha+\alpha^*)(1-\omega) + 2\alpha^* (2\omega-1) \right] \tilde{s}_t^2 + 2(1-\omega)(\alpha-\alpha^*) \tilde{s}_t \left[ \tilde{c}_t + (1-\alpha) \tilde{q}_{Nt} + 1 \right].$$
(4.5)

The terms in the first two lines of this expression of the loss function are identical to those in the national welfare objective functions under the Nash regime (i.e., the loss function (4.2) and its foreign counterpart). The rest of the terms in the loss function here reflect the planner's concerns about variations in and the level of the terms-of-trade gap and its covariance with the terms of trade can be expressed as a function of foreign variables alone (see equations (A.1.6) and (A.1.7) in the appendix), it is treated as a term independent of policy (t.i.p.) by a Nash central bank. Yet, under the cooperating regime, the foreign variables that we have included in the t.i.p. term in deriving the objective function (4.2) for the Nash central bank will now matter for the social planner.

other gap variables. In the special case with no trade (i.e., with  $\omega = 1$ ), the terms of trade drops out from the loss function. In general, the planner seeks to minimize deviations of the equilibrium allocations from the flexible-price allocations, facing trade-offs between stabilizing the gaps and the inflation rates, as well as the trade-off between the stabilization goals and the desire to move the terms-of-trade gap in the direction that favors the country that is more open to trade. The weights on the variability of the terms-of-trade gap increases with the total exposure of the two countries to international trade measured by  $(\alpha + \alpha^*)(1 - \omega)$ , and also with the degree of home-bias measured by  $2\omega - 1$ .<sup>12</sup> If  $\omega = 0.5$ , the home bias effect drops out. The covariance terms and the linear term enter the loss function only if the two countries have asymmetric production structures, that is, only if  $\alpha \neq \alpha^*$ .<sup>13</sup>

The planner seeks to maximize the social welfare (4.3) subject to the private sector's optimizing conditions (3.6) - (3.15). Our goal is to solve this problem using the linear-quadratic (LQ) approach. Yet, the presence of the linear term in the welfare objective function requires further treatment. To express the objective function in a quadratic form, we follow Benigno and Woodford (2004) by taking second-order approximations to the pricing equations in the traded sectors in the two countries. Specifically, we show in the Appendix that the infinite discounted sum of the level of the terms-of-trade gap can be written as

$$\mathbf{E}_{0}\sum_{t=0}^{\infty}\beta^{t}\tilde{s}_{t} = V_{0} + \frac{1}{2}\mathbf{E}_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\left[\tilde{m}c_{Ft}^{*2} + \theta_{T}^{*}\kappa_{T}^{*-1}\pi_{Ft}^{*2}\right] - \left[\tilde{m}c_{Ht}^{2} + \theta_{T}\kappa_{T}^{-1}\pi_{Ht}^{2}\right]\right\} + O\left(\|\xi\|^{3}\right), \quad (4.6)$$

where  $V_0$  is a transitory term defined in the Appendix. Replacing the linear term in the welfare objective function (4.3), we obtain

$$W^{Planner} = -\frac{1}{2}(1-\omega)(\alpha-\alpha^{*})V_{0} - \frac{1}{4}E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\bar{\alpha}\left[\tilde{m}c_{Ht}^{2} + \theta_{T}\kappa_{T}^{-1}\pi_{Ht}^{2}\right] + (1-\alpha)\left[\tilde{m}c_{Nt}^{2} + \theta_{N}\kappa_{N}^{-1}\pi_{Nt}^{2}\right] + \bar{\alpha}^{*}\left[\tilde{m}c_{Ft}^{*2} + \theta_{T}^{*}\kappa_{T}^{*-1}\pi_{Ft}^{*2}\right] + (1-\alpha^{*})\left[\tilde{m}c_{Nt}^{*2} + \theta_{N}^{*}\kappa_{N}^{*-1}\pi_{Nt}^{*2}\right]\right\} + \text{t.i.p.} + O\left(\|\xi\|^{3}\right), \quad (4.7)$$

<sup>13</sup>The covariance term can be rewritten as  $\tilde{s}_t \tilde{c}_{Tt}$ . The loss function then suggests that, if home has a larger traded sector so that it is more open to international trade, then, a worsening of the home's terms of trade (i.e., an increase in  $\tilde{s}_t$ ) would incur welfare losses for the planner both directly (through increasing the labor efforts for home traded-sector workers) and indirectly if the home consumption demand for traded goods also increases, as the latter effect would further increase world demand for home traded goods and thus forcing home workers in the traded sector to work even harder.

<sup>&</sup>lt;sup>12</sup>As is evident from (2.8) and (2.10), the share of expenditures on imported goods in total consumption expenditure is given by  $\alpha(1-\omega)$  for the home country (and  $\alpha^*(1-\omega)$ ) for the foreign country). We use this share as a measure of a country's degree of openness.

where  $\bar{\alpha} = \alpha \omega + \alpha^*(1 - \omega)$  and  $\bar{\alpha}^* = \alpha^* \omega + \alpha(1 - \omega)$  are the new weights assigned to the variations in the traded-sector variables, while the weight assigned to the each country's non-traded sector remains equal to the domestic relative size of that sector.

Thus, asymmetric structure affects welfare through both the weights in front of tradedsector variables and the period-zero terms summarized in  $V_0$  in the objective function. We are interested in characterizing optimal monetary policy from a timeless perspective, that is, optimal policy from period 0 onward subject to the set of private sector's optimizing conditions as well as some given value of  $V_0$ . Apparently, given the value of  $V_0$ , higher values of the welfare objective in (4.7) correspond to higher values of its quadratic components. Thus, we can rank policies in terms of their implied values of the quadratic components of the welfare objective. For the purpose of characterizing optimal monetary policy from a timeless perspective, it then suffices to maximize the infinite discounted sum of the quadratic components in (4.7), subject to log-linear approximations to the private sector's optimizing conditions (3.6) - (3.15) as well as log-linear approximations to the term  $V_0$  that describes the initial commitments, the latter of which is given by  $\hat{V}_0 = \kappa_T^{-1} \pi_{H0} - \kappa_T^{*-1} \pi_{F0}^*$ .

## 5 Gains from Coordination

The analysis above reveals that there are potential gains from coordination, since independent central banks do not take into account the asymmetric effects of terms-of-trade movements on consumption and labor efforts across countries, while the world planner tries to internalize this terms-of-trade externality when conducting optimal monetary policy. In this section we quantify the welfare gains from coordination and relate the gains to the degree of asymmetry in production and trading structures across countries. We also study the sensitivity of the results to variations in some key parameters in the model.

#### 5.1 Parameter Calibration

Since it is difficult to obtain closed-form solutions for equilibrium allocations under optimal monetary policy, we resort to numerical simulations to calculate the welfare outcomes of different policy regimes under calibrated parameter values. In our baseline experiments here, we assign values to all parameters based on the standard international business cycle literature, except that we allow the shares of the traded sectors  $\alpha$  and  $\alpha^*$  to vary in the interval [0, 1]. The calibrated parameter values are summarized in Table 1.

 Table 1: Parameter Calibration

β	discount factor	0.99
$1-\omega$	steady-state share of imports in traded consumption basket	0.5
$\theta_T$	elasticity of substitution among traded varieties	10
$\theta_N$	elasticity of substitution among non-traded varieties	10
$\gamma_T$	degree of price stickiness in the traded sector	0.75
$\gamma_N$	degree of price stickiness in the non-traded sector	0.75
$\sigma_j$	standard deviation of shocks in sector $j \in \{T, N\}$	0.01

We set  $\beta = 0.99$ , so that the steady-state annualized real interest rate is 4 percent. We set  $\omega = 0.5$ , so that there is no steady-state home bias in the traded consumption baskets. Since the steady-state share of imports in the home country's GDP is given by  $\alpha(1-\omega)$ , if we consider a traded-sector share of  $\alpha = \alpha^* = 0.3$  as a benchmark value, then  $\omega = 0.5$  implies that the steady-state share of imports in GDP is 0.15, corresponding the calibrated value of trade openness for the US economy (e.g., Backus, et al. (1995)). We set  $\theta_T = \theta_N = 10$ , so that the steady state markup is 11 percent; and  $\gamma_T = \gamma_N = 0.75$ , so that the Calvo pricing contracts in each sector last for one year on average. We set the standard deviation of the innovations to sectoral productivity shocks to 0.01. In our baseline experiment, we assume that the shocks are uncorrelated across sectors and across countries.

### 5.2 Symmetric structures

We first consider the special case with symmetric structures across countries, that is, with  $\alpha = \alpha^*$ . In this case, the level of the terms of trade drops out of the social planner's objective function (4.4). As such, unlike the case with asymmetric structures, the planner does not face the trade-off between stabilizing the terms-of-trade gap and engineering the terms-of-trade policy in favor of the country with a larger traded sector. In this case, stabilizing the gaps and the inflation rates becomes the sole objective for the planner. Since the world planner cares about the variability of the terms-of-trade gap while independent national central banks do not, it is theoretically possible for the countries to gain by coordinating their policies. The question is then: How large is the welfare gain from coordination when the countries have symmetric structure?

The answer, as revealed by Figure 2, is that the gains from coordination between symmetric countries are negligible under calibrated parameters, despite that shocks in the two sectors are uncorrelated. Indeed, the figure suggests that the welfare losses under optimal independent monetary policy (solid line) and those under cooperating policy (dashed line) are almost identical when  $\alpha = \alpha^*$ , where we measure the welfare loss under each regime as a percentage of steady-state consumption. Thus, under symmetric structures, the planner's optimal policy can be approximately implemented by an inward-looking policy that targets domestic inflation and marginal costs, such as the one under the Nash regime. This result is similar to that obtained in the literature [e.g., Corsetti and Pesenti (2001), Obstfeld and Rogoff (2000, 2002), and Tille (2002)].<sup>14</sup>

Figure 2 also bears out the main result established in Proposition 1: optimal polices can replicate the flexible-price equilibrium (so that the welfare loss is zero) only if one sector is shut off (i.e.,  $\alpha = 0$  or  $\alpha = 1$ ). In general, with two sources of nominal rigidities within each country and imperfectly correlated domestic shocks, optimal monetary policy faces a non-trivial tradeoff. Neither the Nash regime nor the cooperating regime can bring the equilibrium allocations to the efficiency frontier. Further, the welfare losses display a hump shape with respect to  $\alpha$ : as  $\alpha$  rises from 0 to 1, the welfare loss initially rises, reaching a peak at  $\alpha = 0.5$ , and declines thereafter.

Why the hump shape? With a flexible exchange rate, exchange-rate adjustments can be used to insulate the country from foreign shocks. Thus, the hump-shaped relation between the welfare losses and  $\alpha$  primarily reflects the effectiveness of domestic relative-price adjustments in stabilizing the consumption gaps and sectoral inflation rates in face of domestic sector-specific shocks. To make this connection more transparent, let's inspect, for instance, the period loss function (4.2) for an independent central bank. In the loss function, sectoral inflation rates

<sup>&</sup>lt;sup>14</sup>The symmetric version of our model is similar to the one studied by Canzoneri, et al. (2004), although their welfare results are somewhat different. They argue that, when domestic sectoral shocks are imperfectly correlated, it is possible to obtain significant welfare gains by coordinating monetary policy. Such differences may arise for two reasons. First, we assume Calvo pricing contracts so that there is equilibrium price-dispersion within each sector and thus sectoral inflation rates enter the planner's objective function along with the gaps, whereas they assume one-period predetermined prices so that there is no inflation goal in their policy objective. Second, the measures of welfare gains from coordination are different: they use an "R-ratio" in the spirit of Obstfeld and Rogoff (2000), while we rely on comparing steady-state consumption equivalence under alternative policy regimes based on welfare measures derived from second-order approximations to the representative households' utility functions. As noted by Canzoneri, et al. (p. 20), the welfare gains from coordination in their model are also negligible if measured in consumption equivalence.

receive weights proportional to the relative sizes of the sectors, whereas the relative-price gap (i.e., the  $\tilde{q}_{Nt}$  term) receives a weight that is concave in  $\alpha$ , reaching its maximum when  $\alpha = 0.5$ . As  $\alpha$  moves away from 0.5, the weight in front of the relative-price gap becomes smaller, so that the monetary authority cares less about fluctuations in the relative price, and it can rely more effectively on relative-price adjustments to insulate the impacts of sector-specific shocks on the consumption gap and the sectoral inflation rates. The further is  $\alpha$  from 0.5, the smaller the weight in front of the relative-price gap becomes, the more effective the monetary authority can use relative-price adjustments to insulate domestic sector-specific shocks, and thus the lower the welfare losses become under optimal policy. Conversely, the closer the value of  $\alpha$  is to 0.5, the greater the weight the relative-price gap receives, the less the policymakers are willing to adjust the relative-price gap, leading to higher welfare losses. A similar logic applies to explaining the hump shape in the welfare losses under cooperation in the case with symmetric structures.

#### 5.3 Asymmetric structures

When the countries' have asymmetric structures (i.e.,  $\alpha \neq \alpha^*$ ), the *level* of the home country's terms-of-trade gap becomes an independent source of concern for the social planner, as revealed by the planner's period loss function (4.4). The weight assigned to this new term is proportional to the degree of asymmetry measured by  $\alpha - \alpha^*$ . As we have discussed above, independent central banks do not care about the terms of trade movements, while the planner does. Yet, in the symmetric case with  $\alpha = \alpha^*$ , the planner's ability to manipulate the terms-of-trade movements does not yield any significant welfare gains relative to the Nash regime. Now, with asymmetric structure, as the *level* of the terms-of-trade also enters the planner's objective, the planner needs to balance the need to stabilize the gaps, including the terms-of-trade gap, against the desire to manipulate the terms-of-trade movements in favor of the country with a larger traded sector. A natural question is then: Does the presence of asymmetric structures represent a new source of welfare gains from coordination? If so, how big are these gains?

Figure 3 provides an answer. There, we plot the relative welfare losses under Nash central banks relative to those under cooperating central banks in the  $(\alpha, \alpha^*)$  space. Naturally, the relative losses here measure the gains from coordination. The gains are close to zero along the diagonal of the space, confirming the results under symmetric structures (i.e., with  $\alpha = \alpha^*$ ) discussed in the previous subsection. As we move away from the diagonal so that the difference

between  $\alpha$  and  $\alpha^*$  enlarges, the gains from coordination also increase, until reaching a maximum of 1.5 percent of steady-state consumption at the edge of the grid. If we take  $\alpha = 0.3$  as a benchmark value for the size of the home traded sector, then the welfare gains of coordination rises from 0.008% to 0.085% and then to 0.213% of steady-state consumption equivalence as the value of  $\alpha^*$  increases from 0.3 (symmetric structures) to 0.5 and then to 0.7.

The externality stemming from terms-of-trade movements identified here under asymmetric structures should not be confused with the usual "terms-of-trade externality" studied in the NOEM literature (e.g., Corsetti and Pesenti (2001), Benigno and Benigno (2003), and Pappa (2004), among others). In the special case of our model with symmetric structures, as it is typically assumed in the NOEM literature, there is no important interdependence between countries from a stabilization point of view. As such, there are no significant gains from coordination. Yet, with structural asymmetry, small movements in the terms of trade following country-specific productivity shocks tend to have large destabilizing effects, especially in the country that is more open to trade. Since independent central banks do not internalize this externality while the world planner does, there are welfare losses due to policy competition.

To help further illustrate the intuition, we plot in Figure 4 the gains from coordination as a function of  $\alpha$ , while we allow  $\alpha^*$  to take three different values: 0.3, 0.5, and 0.7. An interesting pattern emerges: for a given degree of asymmetry, the gains from policy cooperation are smaller when the countries are more exposed to trade. That is, the gains from coordination depend negatively on  $\alpha + \alpha^*$ . For example, when  $\alpha^* = 0.3$ , gains from coordination are bigger for  $\alpha = 0.1$  than for  $\alpha = 0.5$ . The same is true if we compare gains for  $\alpha^* = 0.5$  when  $\alpha = 0.1$  and when  $\alpha = 0.9$ . Some intuition behind this result can be drawn by inspecting the welfare objective (4.5) for the social planner. For a given degree of asymmetry, the weight in front of the linear term of the terms of trade remains unaffected, while the weight in front of the variability of the terms of trade gap increases with  $\alpha + \alpha^*$ . As such, the trade-off that the planner faces between moving the terms of trade in favor of the country which is more open to trade and stabilizing the terms-of-trade gap is tilted towards the latter target, so that cooperating policies would deliver more similar macroeconomic outcomes to those under the Nash regime.

#### 5.4 Sensitivity

#### 5.4.1 Home bias

We have so far considered the welfare gains from coordination with no home bias in the traded consumption basket (i.e., with  $\omega = 0.5$ ). As we have demonstrated in Section 4.2, the planner's concern about the terms-of-trade externality depends on both the degree of cross-country asymmetry (i.e.,  $\alpha - \alpha^*$ ) and the share of imported goods in the traded baskets (i.e.,  $1-\omega$ ). To the extent that home bias (i.e.,  $\omega \neq 0.5$ ) would affect the planner's optimal trade-off between its stabilization goals and the desire to manipulate the terms-of-trade gap in favor of the country that is more open to trade, the presence of home bias has also implications on the gains from coordination.

To understand how the presence of home bias would affect welfare under the two alternative policy regimes, we revisit the planner's objective function (4.4) under cooperation. Suppose, for example, that  $\alpha - \alpha^* > 0$ . Then, it is clear that the importance of the terms-of-trade externality as reflected by the linear term in the loss function (4.4) increases with the share of imports in the traded baskets (measured by  $1 - \omega$ ). For this reason, introducing home bias by increasing the size of  $\omega$  would make the terms-of-trade externality less important under optimal cooperating monetary policy, and thus the gains from coordination should become smaller.<sup>15</sup>

Figure 5 bears this intuition out. The figure plots the welfare losses under the two alternative regimes (the upper panel) and the welfare gains from coordination (the lower panel) for  $\omega \in [0, 0.9]$ , where we have fixed  $\alpha = 0.3$  and  $\alpha^* = 0.7$  to capture the structural asymmetry between the two countries.<sup>16</sup> It shows that, as the degree of home bias measured by  $\omega$  rises (i.e., as the countries rely less on consuming imported goods and are thus less exposed to international trade), not only do the welfare losses under both regimes become smaller, but the welfare gains from coordination also decline.

#### 5.4.2 Correlation of Shocks

In our baseline experiments, we have assumed that the sectoral shocks are uncorrelated both within a country and between countries. In contrast, the NOEM literature frequently assumes

<sup>&</sup>lt;sup>15</sup>Obviously, the same logic goes through if  $\alpha^* - \alpha > 0$ .

<sup>&</sup>lt;sup>16</sup>In the rest of the sensitivity analysis, we keep the  $\alpha$ 's at these values.

that shocks are perfectly correlated within each country but uncorrelated across countries. We now examine the sensitivity of our results to correlations between the shocks.

Figure 6 displays the welfare gains from coordination as the correlations between sectoral shocks vary in the interval [-1, 1]. Clearly, increasing the correlations between sectoral shocks unambiguously reduces the welfare gains. Further, the gains are much more sensitive to the within-country correlations (i.e., correlations of shocks between the traded and the non-traded sector within each country, denoted by  $\rho_{TN}$ ) than to cross-country correlations (i.e., correlations of shocks between traded sectors denoted by  $\rho_{TT}$  and between non-traded sectors denoted by  $\rho_{NN}$ ). In the extreme case with perfectly correlated domestic shocks, optimal monetary policy under either the Nash regime or the cooperating regime can replicate the flexible-price allocations, and thus there are no gains from coordination despite the structural asymmetry across countries. In general, if domestic shocks are imperfectly correlated across sectors, optimal monetary policy under the Nash regime cannot replicate the flexible-price allocations and there are potential gains from coordination. This, coupled with structural asymmetry and the associated terms-of-trade externality, give rise to the gains from coordination.

#### 5.4.3 Price stickiness

In our baseline analysis, we have assumed that firms in different sectors face identical durations of Calvo pricing contracts (i.e.,  $\gamma_T = \gamma_N$ ). We now relax this assumption and examine the implications on the welfare results.

Figure 7 plots the gains from coordination as the price-stickiness in one sector varies, holding the stickiness in the other sector fixed at its calibrated value. In particular, the solid line denotes the gains from coordination when  $\gamma_T$  varies in the interval [0.1, 0.9], while fixing  $\gamma_N = 0.75$ ; and the dashed line represents the other case when  $\gamma_N$  varies and  $\gamma_T = 0.75$ . Evidently, holding one sector's price rigidity fixed, the welfare gains unambiguously increase with the rigidity in the other sector. Further, the gains are more sensitive to variations in traded price rigidity than to non-trade price rigidity, since such gains arise mainly from the terms-of-trade externality, and increased nominal rigidity and the resulting price-dispersion among firms in the traded sector tend to result in disproportionately larger distortions in the terms of trade, leaving a larger room for welfare improvement by the planner through internalizing the terms-of-trade externality.

## 6 Conclusions

We have revisited the issue of gains from international monetary policy coordination in a framework that generalizes the standard model in the NOEM literature by introducing both traded and non-traded goods, and more importantly, by allowing for an asymmetry across countries in the size of the traded sector. We have shown that this more general framework enables us to discover a new source of gains from coordination that arises from a terms-oftrade externality in the presence of asymmetric structures. If acting independently, a country's central bank tends to overlook the terms-of-trade externality identified in this paper, and the terms-of-trade movements would affect unfavorably the country that is more exposed to international trade. In contrast, if countries cooperate in their monetary policy-making, this externality would be properly recognized and efforts would be made to internalize it. Thus, there is a scope for policy coordination. We have shown that, under plausible parameter values, the gains from coordination are sizeable, and increase with the degree of structural asymmetry measured by the cross-country difference between the size of the traded sector. Further, the gains from coordination are larger if the countries have a greater share of imported goods in their traded basket, if the domestic sectoral shocks are less correlated, or if the duration of pricing contracts in either sector is longer.

The terms-of-trade externality identified in this paper should not be confused with the usual sense of terms-of-trade spill-over effects described in the NOEM literature. In the special case where countries are symmetric, there are no important sources of interdependence in our framework that might give rise to significant welfare gains from coordination. A case for policy coordination can be made only when the countries involved have asymmetric production and trading structures. Such cross-country asymmetry, in our view, is an essential feature that characterizes the modern world economy. To the extent that the asymmetric production and trading structure in our model captures some of the differences between developed economies and developing ones, our work sheds some light on the welfare consequences of international monetary policy coordination between countries at different stages of development.

To help exposition, we have restricted our attention to specifications of preferences and technologies that are simple enough to allow for analytical derivations of the welfare objectives facing policy-makers. But these simplifications, absent any asymmetry between countries, preclude a role for the terms-of-trade gap in affecting the welfare gains from coordination. We have also assumed that policymakers possess complete information, and that the size of the non-traded sector is exogenous. A more realistic model should allow for an elasticity of substitution between traded and non-traded goods and between domestic traded goods and imported goods to take non-unitary values. Naturally, tradedness should be endogenous and be a function of transport costs and some institutional arrangements such as trade regulations and policies; policy makers may not be certain about the sources of shocks or even the sources of nominal rigidities in the economy. Incorporating endogenous non-tradedness, more general assumptions about preferences and technologies and about the central bank's information sets should undoubtedly enrich the dynamics in the model and should be a promising avenue to study gains from policy coordination. In such an extended framework, it is also natural to study the desirability of forming a monetary union between countries with asymmetric production and trading structures, such as countries at different stages of development. We conjecture that future research along these lines should be both promising and fruitful. The current paper represents a small step toward this direction.

## A Appendix

#### A.1 Second-order approximation to the period utility functions

We first take a second-order approximation to the household's utility function  $U_t = \ln C_t - \Psi L_t$ around the balanced-trade steady state. Imposing the steady-state condition that  $\Psi L = 1$ , the approximated utility is given by

$$U_t - U_{ss} = \hat{c}_t - \hat{l}_t - \frac{1}{2}\hat{l}_t^2 + O\left(\|\xi\|^3\right), \qquad (A.1.1)$$

where  $O\left(\|\xi\|^3\right)$  represents terms that are of third or higher order in an appropriate bound on the amplitude of the shocks. We next show that, using equilibrium conditions, the firstorder terms  $\hat{c}_t$  and  $\hat{l}_t$  in the approximated utility can be expressed as functions of second- or higher-order terms and terms independent of policy.

We begin with expressing  $\hat{l}_t$  in second- or higher-order terms. Labor market clearing implies that  $L_t = L_{Nt} + L_{Tt}$ , a second-order approximation of which results in

$$\hat{l}_t = (1 - \alpha)\hat{l}_{Nt} + \alpha\hat{l}_{Tt} + \mu_{Lt} + O\left(\|\xi\|^3\right),$$
(A.1.2)

where  $\mu_{Lt} = \frac{1}{2} \left[ \alpha \hat{l}_{Tt}^2 + (1-\alpha) \hat{l}_{Nt}^2 - \hat{l}_t^2 \right]$  and we have imposed the steady state conditions that  $L_T/L = \alpha$  and  $L_N/L = 1 - \alpha$ .

We now relate the sectoral employment  $\hat{l}_{Nt}$  and  $\hat{l}_{Tt}$  to aggregate variables. The loglinearized demand function for labor in the non-traded sector is given by

$$\hat{l}_{Nt} = \hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt} = \hat{\bar{p}}_t + \hat{c}_t - \hat{\bar{p}}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}$$
(A.1.3)

where the second equality follows from the household's demand for non-traded goods, as in (2.8), and the term  $\hat{G}_{Nt}$  is the log-deviation of the price dispersion within the non-traded sector given by  $G_{Nt} = \int_0^1 (P_{Nt}(j)/\bar{P}_{Nt})^{-\theta_N} dj$ . The demand for labor in the traded good sector is given by  $\hat{l}_{Tt} = \hat{y}_{Tt} - \hat{a}_{Tt}$ , where  $\hat{y}_{Tt}$  denotes the log-deviation of  $\bar{Y}_{Tt} = \int_0^1 [Y_{Ht}(j) + Y_{Ht}^*(j)] dj$  from steady state, which, in light of the demand schedules for  $Y_{Ht}(j)$  and  $Y_{Ht}^*(j)$ , can be written as

$$\begin{split} \bar{Y}_{Tt} &= \int_0^1 [Y_{Ht}(j) + Y_{Ht}^*(j)] dj \\ &= G_{Ht} [C_{Ht} + C_{Ht}^*] \\ &= G_{Ht} \left[ \omega \alpha \frac{P_t C_t}{\bar{P}_{Ht}} + (1-\omega) \alpha^* \frac{\mathcal{E}_t P_t^* C_t^*}{\bar{P}_{Ht}} \right] \\ &= G_{Ht} \frac{P_t C_t}{\bar{P}_{Ht}} \left[ \omega \alpha + (1-\omega) \alpha^* Q_t \frac{C_t^*}{C_t} \right] \\ &= G_{Ht} \frac{P_t C_t}{\bar{P}_{Ht}} \left[ \omega \alpha + (1-\omega) \alpha^* \phi_0 \right], \end{split}$$

where the term  $G_{Ht} = \int_0^1 (P_{Ht}(j)/\bar{P}_{Ht})^{-\theta_T} dj$  denotes the price dispersion in the traded sector. The second equality here follows from the demand functions for the traded varieties as described in (2.16) and its foreign counterpart, the third from the demand functions for traded goods (2.10), the fourth from the definition of the real exchange rate, and the last from the risksharing condition (2.26). The log-linearized version of this equation is given by

$$\hat{\bar{y}}_{Tt} = \hat{G}_{Ht} + \hat{\bar{p}}_t + \hat{c}_t - \hat{\bar{p}}_{Ht},$$
 (A.1.4)

Thus, the labor demand in the traded sector is given by

$$\hat{l}_{Tt} = \hat{\bar{p}}_t + \hat{c}_t - \hat{\bar{p}}_{Ht} + \hat{G}_{Ht} - \hat{a}_{Tt}.$$
(A.1.5)

Given the sectoral demand for labor (A.1.3) and (A.1.5), we can rewrite the linear terms in (A.1.2) as

$$\begin{aligned} (1-\alpha)\hat{l}_{Nt} + \alpha\hat{l}_{Tt} &= (1-\alpha)[\hat{p}_t + \hat{c}_t - \hat{p}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}] + \alpha[\hat{p}_t + \hat{c}_t - \hat{p}_{Ht} + \hat{G}_{Ht} - \hat{a}_{Tt}] \\ &= \hat{p}_t + \hat{c}_t - (1-\alpha)\hat{p}_{Nt} - \alpha\hat{p}_{Ht} + (1-\alpha)(\hat{G}_{Nt} - \hat{a}_{Nt}) + \alpha(\hat{G}_{Ht} - \hat{a}_{Tt}) \\ &= \hat{c}_t + \alpha(\hat{p}_{Tt} - \hat{p}_{Ht}) + (1-\alpha)(\hat{G}_{Nt} - \hat{a}_{Nt}) + \alpha(\hat{G}_{Ht} - \hat{a}_{Tt}) \\ &= \hat{c}_t + \alpha(1-\omega)\hat{s}_t + (1-\alpha)(\hat{G}_{Nt} - \hat{a}_{Nt}) + \alpha(\hat{G}_{Ht} - \hat{a}_{Tt}), \end{aligned}$$

where the third equality follows from the log-linearized version of the price level relation in (2.9), the last from the log-linearized traded price index relation (which implies that  $\hat{\bar{p}}_{Tt} - \hat{\bar{p}}_{Ht} = (1 - \omega)\hat{s}_t$ ). Thus, aggregate employment is given by

$$\hat{l}_t = \hat{c}_t + \alpha (1 - \omega) \hat{s}_t + (1 - \alpha) (\hat{G}_{Nt} - \hat{a}_{Nt}) + \alpha (\hat{G}_{Ht} - \hat{a}_{Tt}) + \mu_{Lt} + O\left( \|\xi\|^3 \right).$$
(A.1.6)

Applying the same logic to the foreign country, we can obtain the foreign counterpart to (A.1.6) given by

$$\hat{l}_{t}^{*} = \hat{c}_{t}^{*} - \alpha^{*}(1-\omega)\hat{s}_{t} + (1-\alpha^{*})(\hat{G}_{Nt}^{*} - \hat{a}_{Nt}^{*}) + \alpha^{*}(\hat{G}_{Ft}^{*} - \hat{a}_{Tt}^{*}) + \mu_{Lt}^{*} + O\left(\|\xi\|^{3}\right).$$
(A.1.7)

## A.2 The Approximated Welfare Objective Functions

#### A.2.1 Independent central banks

We now derive a welfare criterion for the monetary authority under the Nash regime (i.e., with independent central banks). We present derivations of the welfare objective for the home country and note that the case for the foreign country is similar. Under the Nash regime, the home central bank takes foreign equilibrium variables and monetary policy as given, and thus foreign terms are all treated as terms independent of policy (i.e., the *t.i.p.* term). Evidently, from (A.1.7), we see that the home country's terms of trade  $\hat{s}_t$  can be expressed as a function of foreign variables alone and is thus treated as a term independent of home monetary policy under the Nash regime.

We can now replace the linear term  $\hat{c}_t - \hat{l}_t$  in the approximated period utility function (A.1.1) by second- or higher-order terms, using the equilibrium relation (A.1.6), while recognizing that  $\hat{s}_t$  is part of the "t.i.p." term. The resulting approximated period utility function is given by

$$U_t - U_{ss} = -(1 - \alpha)\hat{G}_{Nt} - \alpha\hat{G}_{Ht} - \frac{1}{2} \left[ \alpha \hat{l}_{Tt}^2 + (1 - \alpha)\hat{l}_{Nt}^2 \right] + t.i.p. + O\left( \|\xi\|^3 \right).$$
(A.2.1)

Rewriting the sectoral employments in terms of gaps, we obtain

$$\hat{l}_{Nt} = \hat{p}_{t} + \hat{c}_{t} - \hat{p}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}, 
= \hat{c}_{t} - \alpha \hat{q}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}, 
= \tilde{c}_{t} - \alpha \tilde{q}_{Nt} + \hat{G}_{Nt},$$
(A.2.2)

and

$$\hat{l}_{Tt} = \hat{p}_t + \hat{c}_t - \hat{p}_{Ht} + \hat{G}_{Ht} - \hat{a}_{Tt}, 
= \hat{c}_t + (\hat{p}_t - \hat{p}_{Tt}) + (\hat{p}_{Tt} - \hat{p}_{Ht}) + \hat{G}_{Ht} - \hat{a}_{Tt}, 
= \hat{c}_t + (1 - \alpha)\hat{q}_{Nt} + (1 - \omega)\hat{s}_t + \hat{G}_{Ht} - \hat{a}_{Tt}, 
= \tilde{c}_t + (1 - \alpha)\tilde{q}_{Nt} + (1 - \omega)\tilde{s}_t + \hat{G}_{Ht}.$$
(A.2.3)

Finally, following Woodford (2003), we can show that the price dispersion terms  $\hat{G}_{Nt}$  and  $\hat{G}_{Ht}$  can be related to the variabilities in the sectoral inflation rates. In particular, we have

$$\sum_{t=0}^{\infty} \beta^t \hat{G}_{jt} = \frac{1}{2} \frac{\theta_j \gamma_j}{(1 - \beta \gamma_j)(1 - \gamma_j)} \sum_{t=0}^{\infty} \beta^t \pi_{jt}^2 + \underline{tip}^{\pi} + O\left(\|\xi\|^3\right), \quad j = N, H,$$
(A.2.4)

Thus, the expected life-time utility of the representative household in the home country is given by

$$W_t^{Nash} = E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U_{ss}) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t^{Nash} + t.i.p. + O\left(\|\xi\|^3\right),$$
(A.2.5)

where

$$L_t^{Nash} = (1 - \alpha) \left[ (\tilde{c}_t - \alpha \tilde{q}_{Nt} + \hat{G}_{Nt})^2 + \frac{\theta_N}{\kappa_N} \pi_{Nt}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \omega) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \omega) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \omega) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \omega) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{Tt} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 + (1 - \omega) \tilde{s}_t + \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t + \hat{G}_{Ht})^2 \right] + \alpha \left[ (\tilde{c}_{TT} + (1 - \omega) \tilde{s}_t$$

with  $\kappa_j = (1 - \beta \gamma_j)(1 - \gamma_j)/\gamma_j$  for j = N, T. Expanding the squared terms, recognizing that  $\hat{G}_{jt}$  is of second order importance and that  $\tilde{s}_t$  is independent of domestic monetary policy, we obtain the welfare objective (4.1) in the text, with the period loss function given by

$$L_t^{Nash} = \tilde{c}_t^2 + \alpha (1 - \alpha) \tilde{q}_{Nt}^2 + \alpha \frac{\theta_T}{\kappa_T} \pi_{Ht}^2 + (1 - \alpha) \frac{\theta_N}{\kappa_N} \pi_{Nt}^2, \qquad (A.2.6)$$

which is equation (4.2) in the text.

#### A.2.2 Cooperating central banks

The central authority's objective will be different from the objective of the independent central banks, since all the foreign variables that we have included in the t.i.p. in deriving the objective function (A.2.5) for the independent central banks will now matter for the social planner.

We assume the planner assigns equal weights to the national welfare of each member country, so that a second-order approximation of the planner's period utility function yields

$$U_t^P = \frac{1}{2} \left\{ \hat{c}_t + \hat{c}_t^* - \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) - \left( \hat{l}_t^* + \frac{1}{2} \hat{l}_t^{*2} \right) \right\} + O\left( \|\xi\|^3 \right).$$
(A.2.7)

By using (A.1.6) and (A.1.7), we can express the linear terms  $\hat{c}_t - \hat{l}_t + \hat{c}_t^* - \hat{l}_t^*$  as a function of the terms of trade along with second- or higher-order terms. Specifically, we have

$$U_{t}^{P} = -\frac{1}{2} \left\{ \frac{1}{2} \left[ (1-\alpha)\hat{l}_{Nt}^{2} + \alpha\hat{l}_{Tt}^{2} \right] + (1-\alpha)\hat{G}_{Nt} + \alpha\hat{G}_{Tt} + \frac{1}{2} \left[ (1-\alpha^{*})\hat{l}_{Nt}^{*2} + \alpha^{*}\hat{l}_{Tt}^{*2} \right] + (1-\alpha^{*})\hat{G}_{Nt}^{*} + \alpha^{*}\hat{G}_{Tt}^{*} + (\alpha-\alpha^{*})(1-\omega)\hat{s}_{t} \right\} + t.i.p. + O\left( ||\xi||^{3} \right).$$
(A.2.8)

To express the welfare objective for the social planner in terms of gaps and sectoral inflation rates, we use the relations between sectoral employments and the gap variables as described in (A.2.2) and (A.2.3), recognizing that the  $G_{jt}$  terms are of second order, and the infinite discounted sums of which are related to the variabilities of the sectoral inflation rates as described in (A.2.4) and its foreign counterpart. This way, we obtain the planner's welfare objective function (4.3) in the text, with the period loss function given by

$$L_{t}^{Planner} = \alpha \left[ \tilde{m}c_{Ht}^{2} + \theta_{T}\kappa_{T}^{-1}\pi_{Ht}^{2} \right] + (1-\alpha) \left[ \tilde{m}c_{Nt}^{2} + \theta_{N}\kappa_{N}^{-1}\pi_{Nt}^{2} \right] + \alpha^{*} \left[ \tilde{m}c_{Ft}^{*2} + \theta_{T}^{*}\kappa_{T}^{*-1}\pi_{Ft}^{*2} \right] + (1-\alpha^{*}) \left[ \tilde{m}c_{Nt}^{*2} + \theta_{N}^{*}\kappa_{N}^{*-1}\pi_{Nt}^{*2} \right] + 2(\alpha - \alpha^{*})(1-\omega)\tilde{s}_{t},$$
(A.2.9)

which is equation (4.4) in the text, where the marginal cost terms are given by

$$\tilde{mc}_{Ht} = \tilde{c}_t + (1-\alpha)\tilde{q}_{Nt} + (1-\omega)\tilde{s}_t, \quad \tilde{mc}_{Nt} = \tilde{c}_t - \alpha\tilde{q}_{Nt}, \quad (A.2.10)$$

$$\tilde{m}c_{Ft}^* = \tilde{c}_t^* + (1 - \alpha^*)\tilde{q}_{Nt}^* - (1 - \omega)\tilde{s}_t, \quad \tilde{m}c_{Nt}^* = \tilde{c}_t^* - \alpha^*\tilde{q}_{Nt}^*, \quad (A.2.11)$$

as in the text.

#### A.2.3 Expressing the terms-of-trade gap in second order terms

With asymmetric structures, the planner's the objective function contains a linear term involving the terms-of-trade gap. To cast the optimal policy problem into an LQ problem, we need to express the linear terms-of-trade gap in terms of period-0 variables and second- or higher-order terms. For this purpose, we follow a procedure similar to that described in Benigno and Woodford (2004) by taking second-order approximations to the pricing equations in the traded sectors. The only difference is that, in the Benigno-Woodford model, a linear term in the welfare measure arises from steady-state distortions; whereas in our model, there are no such distortions and the linear term in our welfare measure appears because of the cross-country asymmetry. By taking second-order approximations to the pricing equation (2.21) for firms in the home traded sector and to the price index relation for home traded intermediate goods, along with their foreign counterparts, we obtain a pair of analogous expressions to that in the closed-economy model in Benigno and Woodford (2004, equation (2.6)):

$$V_{H0} = \kappa_T E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \hat{c}_t + (1-\alpha)\hat{q}_{Nt} + (1-\omega)\hat{s}_t - \hat{a}_{Tt} \right\} \\ + \kappa_T E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} [\hat{c}_t + (1-\alpha)\hat{q}_{Nt} + (1-\omega)\hat{s}_t - \hat{a}_{Tt}]^2 + \frac{\kappa_T^{-1}\theta_T}{2} \pi_{Ht}^2 \right\} + O\left( \|\xi\|^3 \right),$$
(A.2.12)

$$V_{F0}^{*} = \kappa_{T}^{*} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \hat{c}_{t}^{*} + (1 - \alpha^{*}) \hat{q}_{Nt}^{*} - (1 - \omega) \hat{s}_{t} - \hat{a}_{Tt}^{*} \right\} \\ + \kappa_{T}^{*} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{1}{2} [\hat{c}_{t}^{*} + (1 - \alpha^{*}) \hat{q}_{Nt}^{*} - (1 - \omega) \hat{s}_{t} - \hat{a}_{Tt}^{*}]^{2} + \frac{\kappa_{T}^{*-1} \theta_{T}^{*}}{2} \pi_{Ft}^{*2} \right\} + O\left( \|\xi\|^{3} \right),$$
(A.2.13)

where the V terms are defined by

$$V_{H0} = \pi_{H0} - \frac{1 - \theta_T}{2(1 - \gamma_T)} \pi_{H0}^2 + \frac{1 - \beta \gamma_T}{2} \pi_{H0} Z_0 + \frac{\theta_T}{2} \pi_{H0}^2, \qquad (A.2.14)$$

$$V_{F0}^{*} = \pi_{F0}^{*} - \frac{1 - \theta_{T}^{*}}{2(1 - \gamma_{T}^{*})} \pi_{F0}^{*2} + \frac{1 - \beta \gamma_{T}^{*}}{2} \pi_{F0}^{*} Z_{0}^{*} + \frac{\theta_{T}^{*}}{2} \pi_{F0}^{*2}, \qquad (A.2.15)$$

with the Z's given by

$$Z_t = \tilde{m}c_{Ht} - \frac{\beta\gamma_T}{1 - \beta\gamma_T} (1 - 2\theta_T) \mathbf{E}_t \pi_{H,t+1} + \beta\gamma_T \mathbf{E}_t Z_{t+1}, \qquad (A.2.16)$$

$$Z_t^* = \tilde{m}c_{Ft}^* - \frac{\beta\gamma_T^*}{1 - \beta\gamma_T^*} (1 - 2\theta_T^*) \mathbf{E}_t \pi_{F,t+1}^* + \beta\gamma_T^* \mathbf{E}_t Z_{t+1}^*.$$
(A.2.17)

Multiply through (A.2.12) by  $\kappa_T^{-1}$  and (A.2.13) by  $\kappa_T^{*-1}$ , and subtract the latter from the former, we obtain

$$\begin{split} V_{0} &\equiv \kappa_{T}^{-1}V_{H0} - \kappa_{T}^{*-1}V_{F0}^{*} \\ &= E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\left[\hat{c}_{t} + (1-\alpha)\hat{q}_{Nt} + (1-\omega)\hat{s}_{t} - \hat{a}_{Tt}\right] - \left[\hat{c}_{t}^{*} + (1-\alpha^{*})\hat{q}_{Nt}^{*} - (1-\omega)\hat{s}_{t} - \hat{a}_{Tt}^{*}\right]\right\} \\ &+ \frac{1}{2}E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\left[\hat{c}_{t} + (1-\alpha)\hat{q}_{Nt} + (1-\omega)\hat{s}_{t} - \hat{a}_{Tt}\right]^{2} - \left[\hat{c}_{t}^{*} + (1-\alpha^{*})\hat{q}_{Nt}^{*} - (1-\omega)\hat{s}_{t} - \hat{a}_{Tt}^{*}\right]^{2}\right\} \\ &+ E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\frac{\kappa_{T}^{-1}\theta_{T}}{2}\pi_{Ht}^{2} - \frac{\kappa_{T}^{*-1}\theta_{T}^{*}}{2}\pi_{Ft}^{*2}\right\} + O\left(\|\xi\|^{3}\right) \\ &= E_{0}\sum_{t=0}^{\infty}\beta^{t}\tilde{s}_{t} + \frac{1}{2}E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\tilde{m}c_{Ht}^{2} - \tilde{m}c_{Ft}^{*2}\right\} + E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\frac{\kappa_{T}^{-1}\theta_{T}}{2}\pi_{Ht}^{2} - \frac{\kappa_{T}^{*-1}\theta_{T}^{*}}{2}\pi_{Ft}^{*2}\right\} + O\left(\|\xi\|^{3}\right) \end{split}$$

,

where, in obtaining the last equality, we have used the risk-sharing condition (3.15), the definition of the terms-of-trade gap  $\tilde{s}_t = \hat{s}_t - \hat{s}_t^n$ , and the definitions of the marginal-cost gaps in (A.2.10) and (A.2.11). Clearly, the infinite sum of the linear terms-of-trade gap can now be expressed in terms of  $V_0$  and second- or higher-order terms, as in equation (4.6) in the text.

Finally, from the definitions of the V terms in (A.2.14) and (A.2.15), we obtain, to a firstorder approximation, that  $\hat{V}_{H0} = \pi_{H0}$  and  $\hat{V}_{F0}^* = \pi_{F0}^*$ , so that the log-linearized  $V_0$  term is given by  $\hat{V}_0 = \kappa_T^{-1} \pi_{H0} - \kappa_T^{*-1} \pi_{F0}^*$ , as in the text (see the end of Section 4.2).

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Data source: the World Bank.

Figure 1: --- Value-added shares and tradedness of services



Figure 2:—Welfare losses of alternative monetary policy regimes: symmetric structures.



Figure 3:—Welfare gains from coordination: sizes of the traded sectors



Figure 4:—Welfare gains from coordination: asymmetric structures



Figure 5:—Welfare losses under alternative regimes and gains from coordination: home bias.



Figure 6:—Welfare gains from coordination: correlations of sectoral shocks.



Figure 7:—Welfare gains from coordination: sectoral price stickiness.