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Debt and the effects of fiscal policy *

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Abstract

Empirical investigations of the effects of fiscal policy shocks share a common weakness: taxes, government spending and interest rates are assumed to respond to various macroeconomic variables but not to the level of the public debt; moreover the impact of fiscal shocks on the dynamics of the debt-to-GDP ratio are not tracked. We analyze the effects of fiscal shocks allowing for a direct response of taxes, government spending and the cost of debt service to the level of the public debt. We show that omitting such a feedback can result in incorrect estimates of the dynamic effects of fiscal shocks. In particular the absence of an effect of fiscal shocks on long-term interest rates–a frequent finding in research based on VAR's that omit a debt feedback–can be explained by their mis-specification, especially over samples in which the debt dynamics appears to be unstable. Using data for the U.S. economy and the identification assumption proposed by Blanchard and Perotti (2002) we reconsider the effects of fiscal policy shocks correcting for these shortcomings.

Keywords: fiscal policy, public debt, government budget constraint, VAR models
JEL Classification: H60, E62

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1 Introduction

The effects on the economy of exogenous shifts in fiscal policy remain an area of wide disagreement. While in the case of monetary policy disagreements are limited to the persistence of the effects of a change in interest rates, in the case of fiscal policy different theoretical models and different empirical methodologies produce results that differ in the sign of the effects on the economy, not only in their magnitude or persistence.

The reason why, in empirical work, different estimation approaches often produce widely different results is the difficulty of identifying shifts in taxes or government spending that are truly exogenous and indeed the research in this area has mostly concentrated on comparing different identification assumptions. [1]

This paper points to a different problem. Existing empirical tests of the effects of fiscal policy shocks share a common weakness: the models that are typically used to estimate the effects of fiscal shocks omit the response of taxes, spending and of the cost of debt service to the level of the public debt. Taxes and government spending are assumed to respond to various macroeconomic variables (output, inflation, the rate of interest) but not to the level of the public debt. This has two consequences. First, the error terms in the equations that are estimated for various macroeconomic variables (such as output and private consumption) include, along with truly exogenous shocks, the responses of G and T to the level of the public debt: the coefficients that are estimated are thus typically biased. Second, when the analysis of shocks to G and T is carried out by studying the characteristics of impulse responses, these might imply "incredible" paths for the ratio of debt-to-GDP.

Note that allowing taxes and spending to respond to the variables that enter the government budget constraint (output, inflation, interest rates), as is typically assumed in models used to study the effects of fiscal policy shocks, is not enough to avoid such a bias: one needs to explicitly include the level of the debt. Consider for example a fiscal shock that moves output, inflation and the interest rate: the change in the debt level that such a shock will produce depends–via the equation that determines the dynamics of the debt–on the initial level of the debt-to-GDP ratio. The change in the primary surplus required to stabilize the debt will thus depend on the size of the shock

[1] See Blanchard and Perotti (2002) and Perotti (2006) for a review of the implications of different theoretical models and for a discussion of alternative identification assumptions.

but also on the level of the debt ratio before the shock.

Omission of a feedback from the level of debt to interest rates could be the reason–especially in samples in which the debt dynamics appears to be unstable–why fiscal shocks do not seem to affect long-term rates.

The point we make could also shed light on a common empirical finding: the effects of fiscal policy shocks seem to change over time. For instance, Perotti (2004) finds that the effect on U.S. output of a shock to government spending is positive and statistically significant in the 1960's and 1970's, but becomes insignificant in the 1980's and 1990's. We find a sharp difference in the way U.S. fiscal authorities responded to the accumulation of debt in the two samples: since the early 1980's, following a shock to spending or taxes, both fiscal policy instruments are adjusted over time in order to stabilize the debt-to-GDP ratio. This does not appear to have happened in the first period (the 1960's and 1970's) when there is no evidence of a stabilizing response of fiscal policy.

Our findings are also related to the evidence of a nonlinearity in the response of macroeconomic variables (private saving and consumption in particular) to fiscal shocks [2]: it should not be surprising that consumers respond differently to an innovation in government spending depending on whether or not they expect the government to meet its intertemporal budget constraint by adjusting tax rates and spending in the future.

Since the main purpose of this paper is not the identification of fiscal policy shocks, we take off the shelf the identification assumption proposed by Blanchard and Perotti (2002). The point we make, however, is independent of the particular identification assumption.

The plan of the rest of this paper is as follows. In Section 2 we explain why estimating the effects of fiscal policy shocks omitting the response of taxes, spending and interest rates to the level of the public debt is problematic. Sections 3 and 4 evaluate the empirical relevance of our point by estimating on U.S. data, the effects of fiscal policy shocks using a vector autoregression in which taxes and spending respond to the level of the debt and which also includes (in a way that we shall explain) the intertemporal government budget constraint. We then compare the impulses responses we obtain with those computed using a standard model. The final section hints at further possible applications of this idea.

[2] See e.g. Giavazzi, Jappelli and Pagano (2000).

2 Why standard fiscal policy VAR's are misspecified

The study of the dynamic response of macroeconomic variables to shifts in fiscal policy is typically carried out estimating a "standard" vector autoregression of the form

$$\mathbf{Y}_t = \sum_{i=1}^{k} \mathbf{C}_i \mathbf{Y}_{t-i} + \mathbf{u}_t \tag{1}$$

where $\mathbf{Y}_t$ typically includes government spending, taxes, output, inflation and interest rates. To analyze the effects of fiscal policy shocks on the variables included in $\mathbf{Y}$, the first step consists in identifying such shocks starting from the observed reduced form innovations $\mathbf{u}$. Different identification methods have been proposed by various authors: since the result presented in this paper is independent of the particular identification assumption we refer to Perotti (2006) for a review and a discussion of such assumptions. [3]

The level of the debt-to-GDP-ratio, d_t, is not included in (1). This variable, however, is an important factor in determining the the stance of fiscal policy for two reasons at least:

- a feedback from the debt ratio to the primary deficit is necessary in order to guarantee stability, i.e. to avoid that d_t embarks on an exploding path. Such a feedback is also a feature of the data: for instance Bohn

[3] The vector $\mathbf{u}_t$ contains three terms: (i) the discretionary response of fiscal policy to macroeconomic variables, such as output; (ii) the response implied by the presence of automatic stabilizers and (iii) truly exogenous shifts in fiscal taxes and spending, the ones that we would wish to identify. Blanchard and Perotti (2002) exploit the fact that it typically takes longer than a quarter for discretionary fiscal policy to respond to shocks in macroeconomic variables. At quarterly frequencies the discretionary response of fiscal policy to macroeconomic data can thus be assumed to be zero. To identify the component of $\mathbf{u}_t$ which depends on automatic stabilizers they use institutional information on the elasticities of fiscal variables to macroeconomic variables implied by the stabilizers. Edelberg, Eichenbaum and Fisher (1999) and Burnside, Eichenbaum and Fisher (2004) use a different approach: they apply to fiscal policy the methodology proposed by Romer and Romer (1989) to identify monetary policy shocks and construct a dummy variable which characterizes episodes of significant and exogenous increases in government spending (typically wars). Mountford and Uhlig (2002) identify government spending and revenue shocks by imposing restrictions on the sign of impulse responses. Fatas and Mihov (2001) rely on Choleski ordering to identify fiscal shocks.

(1998) shows that a century of U.S. data reveals a positive correlation between the government surplus-to-GDP ratio and the government debt-to-GDP ratio;

- the average cost of debt financing depends on future expected monetary policy and on the risk premium: both may be affected by debt dynamics–for instance if a growing stock of debt raises fears of future monetization or, in the extreme case, of debt default.

If the level of d is significant in explaining at least some of the variables included in $\mathbf{Y}_t$ it is important that, in simulating the effect of fiscal policy shocks, such feedbacks are included. But once the level of the debt ratio is explicitly included in (1), one must allow for the fact that taxes, government spending, output, inflation and the rate of interest–in other words the variables entering $\mathbf{Y}_t$–are linked by an identity, the equation that determines how the debt ratio moves over time.

These two observations naturally lead to replacing (1) with

$$\begin{aligned} \mathbf{Y}_t &= \sum_{i=1}^{k} \mathbf{C}_i \mathbf{Y}_{t-i} + \sum_{i=1}^{k} \gamma_i d_{t-i} + \mathbf{u}_t \\ d_t &= \frac{1+i_t}{(1+\Delta p_t)(1+\Delta y_t)} d_{t-1} + \frac{\exp(g_t) - \exp(t_t)}{\exp(y_t)} \end{aligned} \tag{2}$$

$$\mathbf{Y}_t = \begin{bmatrix} g_t \\ t_t \\ y_t \\ \Delta p_t \\ i_t \end{bmatrix}$$

where i is the nominal rate of interest (the cost of debt financing), Δy is real GDP growth, Δp is inflation, t and g are, respectively, (the logs of) government revenues and government expenditure net of interest. (We use logs because it is the log of output, taxes and spending that enters $\mathbf{Y}_t$).

The presence of d_{t-i} amplifies the dynamic effect of shocks because they cumulate in (2), while they do not in (1): the difference between impulse responses computed using (2) and (1) might thus diverge as the horizon increases. The analysis of the effects of fiscal shocks using (1) can thus be problematic if we wish to track the effect of a shock on variables that typically respond to the level of the debt, for instance the yield on government bonds. The impact of a given fiscal shock on i will be very different depending on whether the shock produces a path of debt that is stable or tends to become explosive.

Before discussing how fiscal policy shocks can be studied in the context of (2) we pause and ask a question. $\mathbf{Y}_t$ already contains all the variables that enter the government intertemporal budget constraint (2). Why can the impulse responses be biased if the model does not explicitly include d and the identity describing debt accumulation?

The reason is that it is unlikely that the short lags of g, t, Δp, Δy and i that enter (linearly) (1) can replace the level of the debt ratio accurately enough. To convince yourself notice that d_t is the expression of a long and non-linear lag dynamics

$$\begin{aligned} d_t &= \sum_{i=0}^{K}\left(\frac{\exp\left(g_{t-i}\right)-\exp\left(t_{t-i}\right)}{\exp\left(y_{t-i}\right)}\right)^{i}\prod_{i=o}^{K}\left(\frac{1+i_{t-i}}{\left(1+\Delta p_{t-i}\right)\left(1+\Delta y_{t-i}\right)}\right)+ \\ &+\prod_{i=o}^{K}\left(\frac{1+i_{t-i}}{\left(1+\Delta p_{t-i}\right)\left(1+\Delta y_{t-i}\right)}\right)d_{t-i-1} \end{aligned}$$

But the best way to convince the reader of our point is to show how different the impulses responses can be under (1) and under (2)–and correspondingly how different is the implied path for d. We shall do this using U.S. data–the same (with the qualifications to be discussed in the next section) analyzed in Blanchard and Perotti (2002) and extended in Perotti (2006). This will make our results directly comparable with those derived in standard VAR's.

3 Data

We use quarterly data for the U.S. economy over the years from 1960:1 to 2006:2. Our approach requires that the debt-dynamics equation in (2) tracks the path of d_t accurately: we thus need to define the variables in this equation with some care.

The source for the different components of the budget deficit and for all macroecononomic variables are the NIPA accounts (available on the Bureau of Economic Analysis website). y_t is (the log of) real GDP per capita, Δp_t is the log difference of the GDP deflator. Data for the stock of U.S. public debt and for population are from the FRED database (available on the Federal Reserve of St.Louis website). Our measure for g_t is (the log of) real per capita primary government expenditure: nominal expenditure is obtained subtracting from total Federal Government Current Expenditure (line 39, NIPA Table 3.2) net interest payments at annual rates (obtained as the difference between line 28 and line 13 on the same table). Real per capita expenditure is then obtained by dividing the nominal variable by population times the GDP chain price deflator. Our measure for t_t is (the log of) real per capita government receipts at annual rates (the nominal variable is reported on line 36 of the same NIPA table).

The cost servicing the debt, i_t, is obtained by dividing net interest payments by the federal government debt held by the public (FYGFDPUN in the Fred database) at time $t-1$.The federal government debt held by the public is smaller than the gross federal debt, which is the broadest definition of public debt. However, not all gross debt represents past borrowing in the credit markets since a portion of the gross federal debt is held by trust funds–primarily the Social Security Trust Fund, but also other funds: the trust fund for unemployment insurance, the highway trust fund, the pension fund of federal employees, etc.. The assets held by these funds consist of non-marketable federal debt: as explained for instance in Cashell (2006) "this debt exists only as a book-keeping entry, and does not reflect past borrowing in credit markets." We thus exclude it from our definition of federal public debt.

Figure 1 reports this measure of the debt held by the public as a fraction of GDP, along with the series constructed using the debt dynamics equation in (2) and, as initial value, the level of debt in 1970:1. The figure shows that the simulated series is virtually super-imposed to the actual one: the small differences are due to approximation errors in computing inflation and growth rates as logarithmic differences, and to the fact that the simulated series are obtained by using seasonally adjusted measures of expenditures and revenues. Based on this evidence we have used the debt dynamics equation to extend d_t (which on the FRED website is only available starting in 1970:1) back to the beginning of our sample.

4 Estimating the effects of fiscal policy shocks using a standard VAR

We set the stage by estimating a standard VAR such as (1). To make our results comparable with previous studies we keep as close as possible to two of them, Blanchard and Perotti (2002) and Perotti (2006) [BP&P] especially in the identification assumption. Our choice of variables differs slightly however, because, as discussed above, we need to use variables that allow the debt dynamics equation to track the path of d closely. Our measure of i is thus the average cost of financing the debt described in the Data section: [BP&P] use the yield to maturity on long-term government bonds. Our definitions of g and t are also different: we follow the NIPA definitions by considering net transfers as part of government expenditure, rather than subtracting them from taxes. Importantly, since our definition of debt refers, as we discussed, to Federal government debt, we only consider Federal government spending, omitting expenditures by the States.and othe local governments. One of the purposes of this Section is to check that these differences in data definitions do not alter significantly the estimates of the effects of fiscal policy shocks in the standard VAR.

[BP&P] identify the shocks to g and t by imposing on the $\mathbf{A}$ and $\mathbf{B}$ matrices in $\mathbf{Au} = \mathbf{Be}$ (where $\mathbf{u}$ are the VAR innovations and $\mathbf{e}$ are the structural shocks, orthogonal to each other) the conditions

$$\begin{bmatrix} 1 & 0 & a_{gy} & a_{g\Delta p} & a_{gi} \\ 0 & 1 & a_{ty} & a_{t\Delta p} & a_{ti} \\ a_{31} & a_{32} & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{bmatrix} \begin{bmatrix} u_t^g \\ u_t^t \\ u_t^y \\ u_t^{\Delta p_t} \\ u_t^i \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix} \begin{bmatrix} e_t^g \\ e_t^t \\ e_t^1 \\ e_t^2 \\ e_t^3 \end{bmatrix}$$

where e_t^i $(i = 1, 2, 3)$ are non-fiscal shocks that have no direct interpretation. Since a_{gy}, $a_{g\Delta p}$, a_{gi}, a_{ty}, $a_{t\Delta p}$ and a_{ti} are identified using external information [4], there are only 15 parameters to be estimated. As there are also 15 different elements in the variance-covariance matrix of the VAR innovations, the model is just identified. Note that the e_t^i $(i = 1, 2, 3)$ are constructed imposing a recursive scheme on the bottom three rows of **A** and **B;** however, the identification of the two fiscal shocks–the only ones that we shall use to compute impulse responses–is independent of this assumption. Finally, the identification assumption imposes $b_{12} = 0$. [BP&P] provide robustness checks for this assumption by setting $b_{21} = 0$ and estimating b_{12}. We have also experimented with this alternative option: in practice, as the top left corner of the **B** matrix is not statistically different from a diagonal matrix, the assumption $b_{12} = 0$ is irrelevant to determine the shape of impulse response functions.

For comparison with Perotti (2006) we compute impulse responses with respect to shocks to g and t over the full sample (1960:1-2006:2) and two subsamples: 1960:1-1979:4 (S1), and 1980:1-2006:2 (S2). The choice is motivated by the evidence of a break in macroeconomics and financial time-series at the beginning of the 1980's. The dynamics of the debt-to-GDP ratio is in fact quite different in the two sub-samples. One reason are the different paths of nominal GDP growth and the cost of debt financing (see Figure 2). In the sixties and seventies nominal GDP growth was higher than the average cost of financing the debt: this meant that the debt ratio could fall–as in fact it did–even in the presence of a primary deficit. This changed starting in the early 1980's when the growth rate of the economy fell below the average cost of debt financing and the debt ratio started rising, at least until the mid 1990's. The observation that over the two sub-samples the debt ratio has opposite trends suggests that the differences in impulse responses observed in the two periods might be related to the different paths of the debt.

The impulse responses are shown in Figures 3.1 to 3.3, respectively for

[4]The elasticities of ouptut, inflation and interest rates used by Perotti (2006) to identify the structural shocks are

Elasticities of revenues and expenditures						
	a_{gy}	$a_{g\Delta p}$	a_{gi}	a_{ty}	$a_{t\Delta p}$	a_{ti}
Entire sample	0	-0.5	0	1.85	1.25	0
1960:1-1979:4	0	-0.5	0	1.75	1.09	0
1980:1-2006:2	0	-0.5	0	1.97	1.40	0

the first and second sub-sample and for the full sample. In each Figure the left-hand panels refer to a shock to g equivalent to one percent of output; the righ-hand side panels refer to an equivalent shock to t. Along the rows the graphs show, from top to bottom, the impulse response of g, t, y, Δp, and i. The results are consistent with those reported in [BP&P]: in particular:

- an increase in public expenditure has an expansionary effect on output, while an increase in revenues is contractionary. The impact of fiscal policy weakens in the second sub-sample, in particular the effects of revenue shocks become insignificant;
- the persistence of fiscal variables is lower in the second subsample. In particular we observe a reduction in the volatility of spending shocks;
- the effect of fiscal shocks on i is insignificant in the first sub-sample; it is small, significant but counterintuitive in the second subsample when an expansionary fiscal policy lowers the cost of servicing the debt;
- fiscal shocks have consistently no significant effect on inflation.

The debt dynamics implied by a standard VAR

The standard VAR excludes d. To assess the importance of this omission we start with a simple exercise: we append to (1) the identity which describes debt accumulation. Then, after having estimated parameters $\mathbf{C}_i$, we simulate the system out-sample for twenty years to see what path for d_t it implies.

When the VAR is estimated over the first sub-sample (1960:1-1979:4) the simulated out-of-sample path for d_t is explosive (Figure 4). When the VAR is estimated over the second sub-sample (1980:1-2006:2) the simulated debt ratio tends, eventually, to fall below zero. Finally, when we use the entire sample, the simulated debt ratio appears to be stable.

This evidence naturally raises a number of questions

- it is obviously difficult to interpret impulse response functions when they are computed along unstable paths for the debt ratio, as they would eventually diverge. This is not a big problem when identification is obtained imposing restrictions on the simultaneous effects of fiscal policy shocks and when impulse responses are computed over relatively short horizons. But an unstable dynamics becomes problematic when identification is obtained imposing long run restrictions on the shape of impulse responses and the effects of fiscal shocks are computed over relatively long horizons;

- does instability depend on the underlying behaviour of the government or is it simply the result of a mis-specified VAR? Debt stabilization requires that the primary budget surplus reacts to the accumulation of debt, but such a reaction is not allowed in a standard VAR–and we discussed how (1) cannot fully capture the response of spending and taxes to the debt ratio. Hence the simulated path may very well be the result of mis-specification of the empirical model rather than a description of the actual behaviour of the government;

- how much of the heterogeneity observed in the impulse response functions over the three samples (S1, S2 and the entire sample) can in fact be explained by the different dynamics of the debt ratio implied by the VAR estimated over each period ?

- in a standard VAR the response of i_t to a fiscal shock is puzzling. Consider for example the response to an expansionary fiscal shock over the first sub-sample. The path of the debt ratio eventually becomes explosive: how can this be reconciled with the evidence that the estimated response of i_t is small and negative?

One final observation is suggested by the frequent use of these impulse responses to decide between competing DSGE models, or to provide evidence on the stylized facts to include in theoretical models used for policy analysis. It is obviously impossible to compare the empirical evidence from a VAR model that delivers an explosive path for the debt, with the path of variables produced by forward looking models, since such models do not have a solution when the debt dynamics is unstable.

We now turn to model (2).

5 Estimating the effects of fiscal policy shocks in a model with debt dynamics

Estimating (2) involves three steps.

- identify the structural shocks $\mathbf{e}_t$. Since we treat the debt-deficit relationship as an identity, the inclusion of the debt dynamics does not alter the dimension of $\mathbf{e}_t$: in other words, the number of shocks considered is the same as in a standard VAR. Therefore the solution to the identification problem discussed in the previous Section can be readily applied to our extended VAR. Since there are no parameters to be estimated in the identity, (2) can be estimated excluding the debt-deficit dynamics. Once the reduced form is estimated, (2) can thus be re-written as

$$\begin{aligned} \mathbf{Y}_t &= \sum_i \mathbf{C}_i \mathbf{Y}_{t-i} + \sum_i \boldsymbol{\gamma}_i d_{t-i} + \mathbf{A}^{-1}\mathbf{B}\mathbf{e}_t \\ d_t &= \frac{1+i_t}{(1+\Delta p_t)(1+y_t-y_{t-1})} d_{t-1} + \frac{\exp(g_t) - \exp(t_t)}{\exp(y_t)} \end{aligned} \tag{3}$$

- compute the responses of the variables in $\mathbf{Y}_t$ to innovations in $\mathbf{e}_t$. The presence of the intertemporal budget constraint makes this step different from traditional impulse response analysis. Impulse responses comparable to those obtained from the traditional moving average representation of a VAR can be obtained going through the following steps:

  - generate a baseline simulation for all variables by solving (3) dynamically forward (this requires setting to zero all shocks for a number of periods equal to the horizon up to which impulse responses are needed),
  - generate an alternative simulation for all variables by setting to one–just for the first period of the simulation–the structural shock of interest, and then solve dynamically forward the model up to the same horizon used in the baseline simulation,
  - compute impulse responses to the structural shocks as the difference between the simulated values in the two steps above. Note that these steps, if applied to a standard VAR, would produce

standard impulse responses. In our case they produce impulse responses that allow for both the feedback from d_{t-i} to $\mathbf{Y}_t$ and for the debt dynamics,

- compute confidence intervals, for example by bootstrapping.[5]

Table 1 shows the coefficients of the first and the second lags of d_t in all the equations in (2) for the full sample and for the two subsamples. In the equations for g_t and t_t the coefficients on d_{t-1} and d_{t-2} are significant in the full sample and in the second subsample. The restriction that the two coefficients are of equal magnitude and of opposite sign cannot be rejected, suggesting that fiscal policy reacts to the lagged change in the debt ratio, $(d_{t-1} - d_{t-2})$. This is interesting since $(d_{t-1} - d_{t-2})$ measures the gap between the actual primary surplus (as a fraction of GDP) and the surplus that would stabilize d. In the samples in which the two coefficients are significant (the full sample and the second sub-sample) their magnitude indicates that the gap between the surplus that would stabilize the debt ratio and the actual surplus acts as an error correction mechanism in the fiscal reaction function: current expenditures are decreased and current taxation increased when last period's primary surplus was below the level that would have kept the debt stable.

Lags of d_t are also important in determining the average cost of debt financing. The cost of debt financing depends–as was the case for g_t and t_t–on the gap between the actual surplus and the debt stabilizing surplus. This result is particular strong in the second sub-sample. Finally, the direct effect of lags in d_t on inflation and output growth is never significant in any of the samples.

[5]Bootstrapping requires saving the residuals from the estimated VAR and then iterating the following steps:

a) re-sample from the saved residuals and generate a set of observation for $\mathbf{Y}_t$ and d_t,

b) estimate the VAR and identify strucutral shocks,

c) compute impulse responses going thorough the steps described in the text,

d) go back to step 1.

By going thorugh a sufficient number of iterations one can produce a bootstrapped distribution for impulse responses and thus compute confidence intervals.

Table 1 *Feedbacks from* d_{t-i}

		g_t	t_t	y_t	Δp_t	i_t
	1960:1-1979:4	-5.83 (5.14)	-3.55 (2.17)	-1.59 (2.17)	-0.88 (0.71)	0.079 (0.25)
d_{t-1}	*1980:1-2006:2*	-3.94 (2.58)	1.63 (4.27)	0.83 (1.06)	0.13 (0.34)	0.62 (0.32)
	Full sample	-4.68 (2.00)	4.18 (2.94)	-0.16 (0.85)	0.15 (0.29)	-0.057 (0.22)
	1960:1-1979:4	5.90 (5.11)	4.18 (5.89)	1.75 (2.16)	0.87 (0.72)	-0.049 (0.25)
d_{t-2}	*1980:1-2006:2*	3.82 (2.60)	-1.59 (4.30)	-0.85 (1.06)	-0.14 (0.34)	-0.63 (0.33)
	Full sample	4.62 (2.00)	-4.14 (2.95)	0.16 (0.85)	-0.16 (0.29)	0.064 (0.22)

The results in Table 1 naturally raise a question. We argued that the standard VAR is mis-specified because it omits the possibility that fiscal policy reacts to the level of the debt ratio. In other words the mis-specification would arise from the omission of a low-frequency variable. But according to Table 1 what matters is the change in the debt, thus again a high-frequency variable. Does this invalidate our claim ? It doesn't because the first difference of the debt is itself a (non-linear) function of the debt level. Differencing the equation that describes the debt dynamics we obtain

$$\Delta d_t = \frac{(i_t - \Delta p_t - \Delta y_t - \Delta y_t \Delta p_t)}{(1+\Delta p_t)(1+\Delta y_t)} d_{t-1} + \frac{\exp(g_t) - \exp(t_t)}{\exp(y_t)} \quad (4)$$

or, in other words, the change in the debt ratio is equal to the difference between the actual surplus-to-GDP-ratio and the ratio that would keep the debt stable–and such ratios are a function of the level of the debt. Hence, the change in debt ratio depends on the level of the debt via a time-varying relationship–because the first term on the right hand side of (4) varies over time and, as shown in Figure 2, this time variation is empirically relevant in the U.S. case over the sample we consider.[6] Therefore, including the

[6] Note that stability of the debt ratio requires stationarity of the gap between the debt stabilizing surplus and actual surplus, and therefore a cointegrating relation between the debt stabilizing surplus, government spending and government revenues (both as a ratio to GDP). Such cointegrating relation is different from those exerimented in the standard fiscal plicy VAR. In particular, the cointegrating relation implied by (4) is different from

change of the debt-to-GDP ratio in the fiscal VAR is virtually equivalent to augmenting such a VAR with a time-varying function of the level of the debt-to-GDP ratio, which is indeed a slow moving variable [7].

We now turn to the results.

Debt dynamics in a model with feedbacks

How does the debt ratio respond to fiscal shocks when the variables in $\mathbf{Y}_t$ react to past debt growth? Figure 5 reports out-sample simulations of d_t obtained from (2) and compares them with those obtained from a traditional VAR–already shown in Figure 4. In the sub-second sample, following any fiscal policy shock if $\mathbf{Y}_t$ is allowed to respond to past debt growth the path of d_t is stabilized. This is not the case in the first sub-sample–not surprisingly, since the feedbacks from the debt ratio to g_t and t_t only start being significant since the early 1980's.

Thus omitting a feedback from the debt level to fiscal policy can result in impulse responses to fiscal shocks that are computed along implausible paths for the debt ratio. Whether including such a feedback is sufficient to produce stable debt paths obviously depends on the size of the feddbacks. If they are too small unstable paths will not be eliminated.

The effects of fiscal shocks in a model with feddbacks

Figure 6 compares the impulse responses obtained from (2) with those obtained from the standard VAR, using, in both cases, the same identifying

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the cointegrating relations between g_t and t_t, with a cointegrating vector $(1, -1)$ proposed in their robustness check by BP&P. (This could explain why a cointegrated model or a simple model in differences do not make any substantial difference for the evidence reported by BP&P). Of course, if the debt stabilizing surplus were stationary, the data would support, up to a logarithmic transformation, the cointegrating vector in BP&P, but the long-run solution of their cointegrating system would still be different from the one implied by a system in which there is tight relation between the actual surplus and the debt stabilizing surplus. The cointegrating relation implied by (4) is also different from the error correction model proposed by Bohn (1988): Bohn includes the level of the debt ratio in the fiscal reaction function but does so without allowing for the time variation of the coefficient multiplying the debt level.

[7] As a robustness check we have re-run our VAR augmenting it with the debt stabilizing surplus-to-GDP ratio lagged once and twice. The coefficients of the two lags have identical signs and their sum is not statistically different from the coefficient on the first difference of the debt ratio in our model. Note that if the two coeffcients had opposite signs and similar magnitudes the variable entering the VAR would indeed be the change in d, but this is not the case.

assumptions. Impulse responses are shown in Figure 6 exactly as they were in Figure 3: Figures 6.1 to 6.3 refer, respectively, to the first and second sub-sample and for the full sample;.left-hand panels refer to a shock to g equivalent to one percent of output; righ-hand panels refer to an equivalent shock to t; the rows show, from top to bottom, the impulse response of g, t, y, Δp, and i.

- In the first sub-sample (1960:1-1979:4, Figure 6.1)
  - a shock to government spending
    * the cumulative response of output to a spending shock increases from 0.44 in the standard model to 0.54 in the model with feedbacks. The responses computed assuming a feedback lie often outside the commonly chosen confidence interval (ninetyfive per cent confidence bounds around the impulse response computed without feedbacks are shown by the two darker continuous lines). Fiscal shocks computed assuming a feedback are more persistent than in the standard VAR– a persstence which translates into higher persistence of the effects of such shocks,.
    * the effect of a spending shock on the cost of debt service is also larger in the presence of feedbacks. Importantly, the difference between the impulse responses computed with and without feedbacks tends to increase over time. This is because the model with feedbacks tracks the cumulative effect of the shock on the stock of debt and its impact on the variables included in the dynamic model,
  - government revenues shocks display a pattern that mirrors that of expenditure shocks. Interestingly, the impact of a revenue shock on output is smaller when one allows for feedbacks, since we observe a compensating movement in government expenditure that dampens the restrictive effect of higher taxes on output,
- In the second sub-sample (1980:1-2006:2, Figure 6.2)
  - the output response to a spending shock is smaller in the presence of feedbacks. This is because, in this sample, the persistence of

spending shocks is lower (in the presence of feedbacks) and the compensating shift in taxes larger. As a consequence, the evidence that the response of output to an expenditure shock is weaker in this sample–a finding in Perotti (2006)–is strengthened when one allows for feedbacks,

  - the response of the cost of debt service to fiscal shocks is much weaker in this sample This evidence is consistent with the observation that since the early 1980's the debt ratio has been on a stable path, while its path was unstable in the previous period,
  - as in the case of spending shocks, the effect of revenue shocks on output is again dampened by compensating movements in expenditure.

- Finally, the evidence from the entire sample (Figure 6.3) confirms that the reason why, in the presence of feedbacks, the response of output to a fiscal shock is smaller, is the dampening movement of revenues after a spending shock and of spending after a revenue shock. The response of the cost of debt financing to fiscal shocks is highly dependent on the path of the debt ratio. Unsurprisingly, an unstable path generates much stronger responses of interest rates to fiscal shocks.

6 Conclusions

We have analyzed the effects of fiscal shocks allowing for a direct response of taxes, government spending and the cost of debt service to the level of the public debt. We have shown that omitting such a feedback can result in incorrect estimates of the dynamic effects of fiscal shocks. We suggested in particular that the absence of an effect of fiscal shocks on long-term interest rates–a frequent finding in research based on standard VAR's, that is VAR's that omit a debt feedback–can be explained by their mis-specification, especially over samples in which the debt dynamics appears to be unstable.

The effects of fiscal shocks that we estimate, using U.S. data, often lie outside the commonly chosen confidence interval for a model that does not allow for feedbacks. We also find that the effect of fiscal shocks on interest rates depends on the path of the debt-to-GDP ratio: they are weaker when the debt ratio is on stable path, that is when taxes and spending react to the debt so as to keep it on a stable path. Allowing for feedbacks from the debt level we also find that the effect of revenue shocks on output is dampened by compensating movements in government expenditure.

The methodology described in this paper to analyze the impact of fiscal shocks by taking into account the stock-flow relationship between debt and fiscal variables could be extended to other dynamic models which include similar identities. For instance, the recent discussions on the importance of including capital as a slow-moving variable to capture the relation between productivity shocks and hours worked (see e.g. Christiano et al, 2005 and Chari et al. 2005) could benefit from an estimation technique that tracks the dynamics of the capital stock generated by the relevant shocks.

A further natural extension of our approach is the analysis of the effects of fiscal shocks on debt sustainability, an issue which cannot be addressed in the context of a standard VAR. Stochastic simulations of (2) could also be used to evaluate the sustainability of current systematic fiscal policy and to compute the risk of an unstable debt dynamics implied by the current policy regime.

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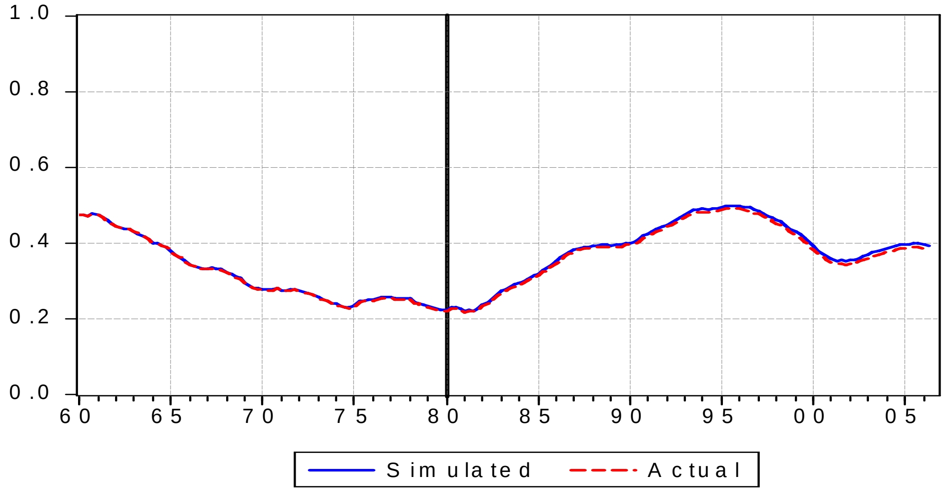

Figure 1: Actual and Simulated (ex-post) debt/GDP dynamics

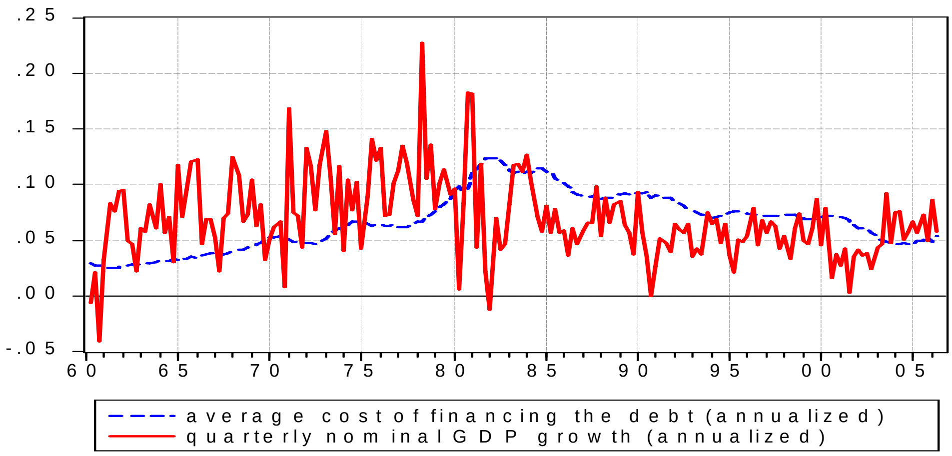

Figure 2: Average cost of debt financing and quarterly (annualized) nominal GDP growth

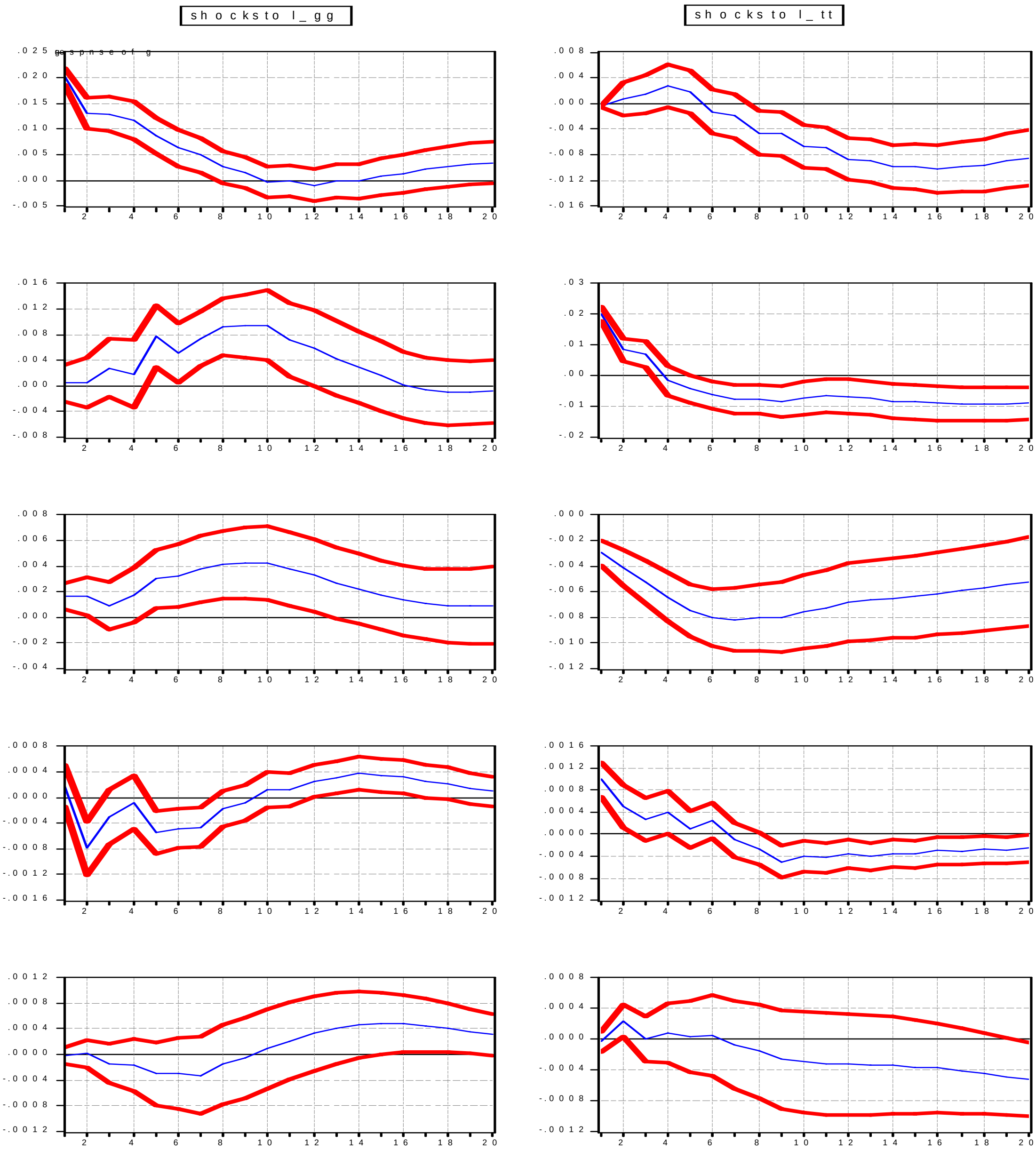

Figure 3.1: Fiscal policy in a standard VAR. Sample: 1960:1-1979:4. The first column shows responses to shocks to g_t; the second column to shocks to t_t.The responses reported along the rows refer, respectively, to the effects on g_t, t_t, y_t, Δp_t, i_t.

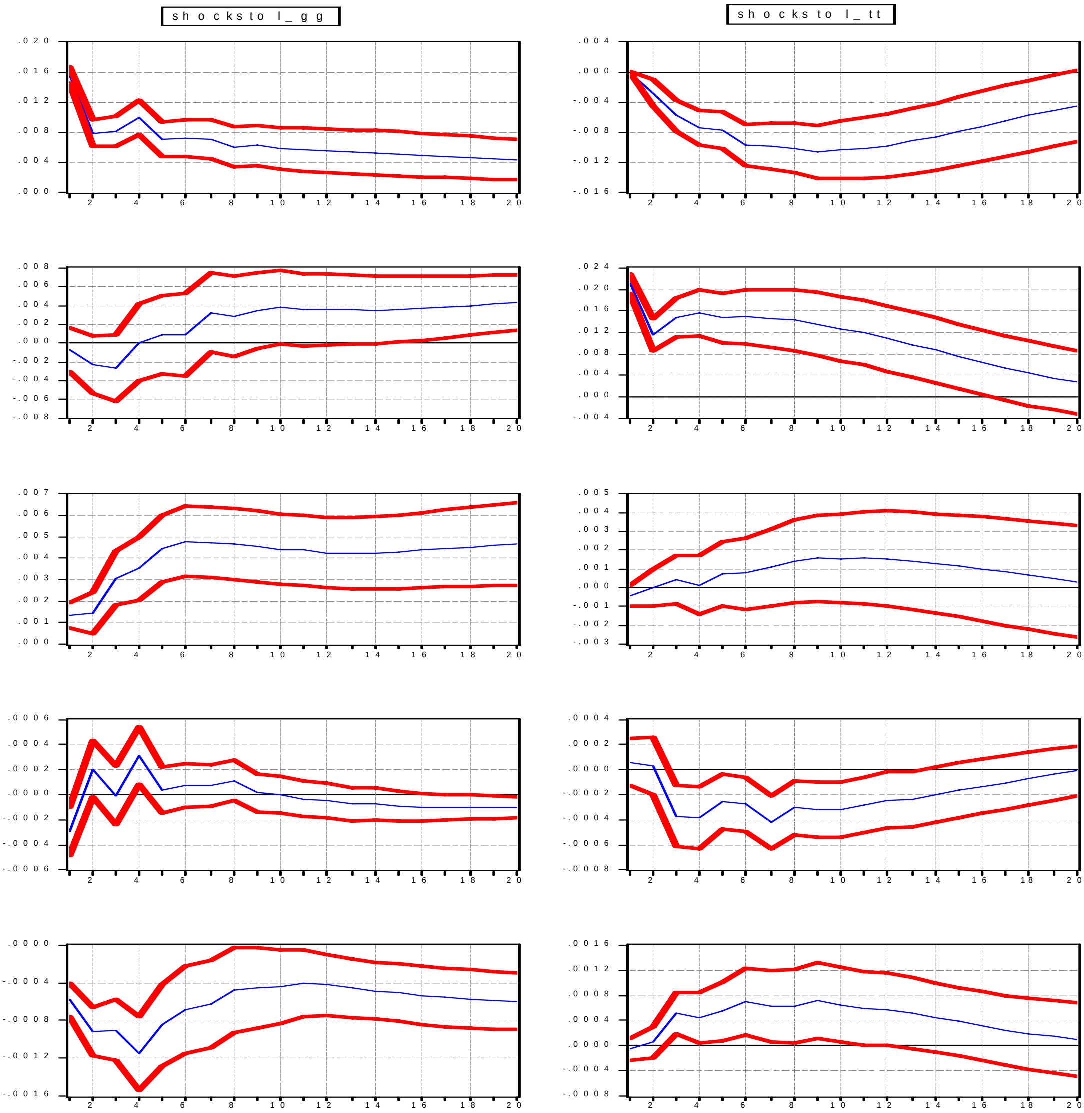

Figure 3.2: Fiscal policy in a standard VAR. Sample 1980:1 2006:2

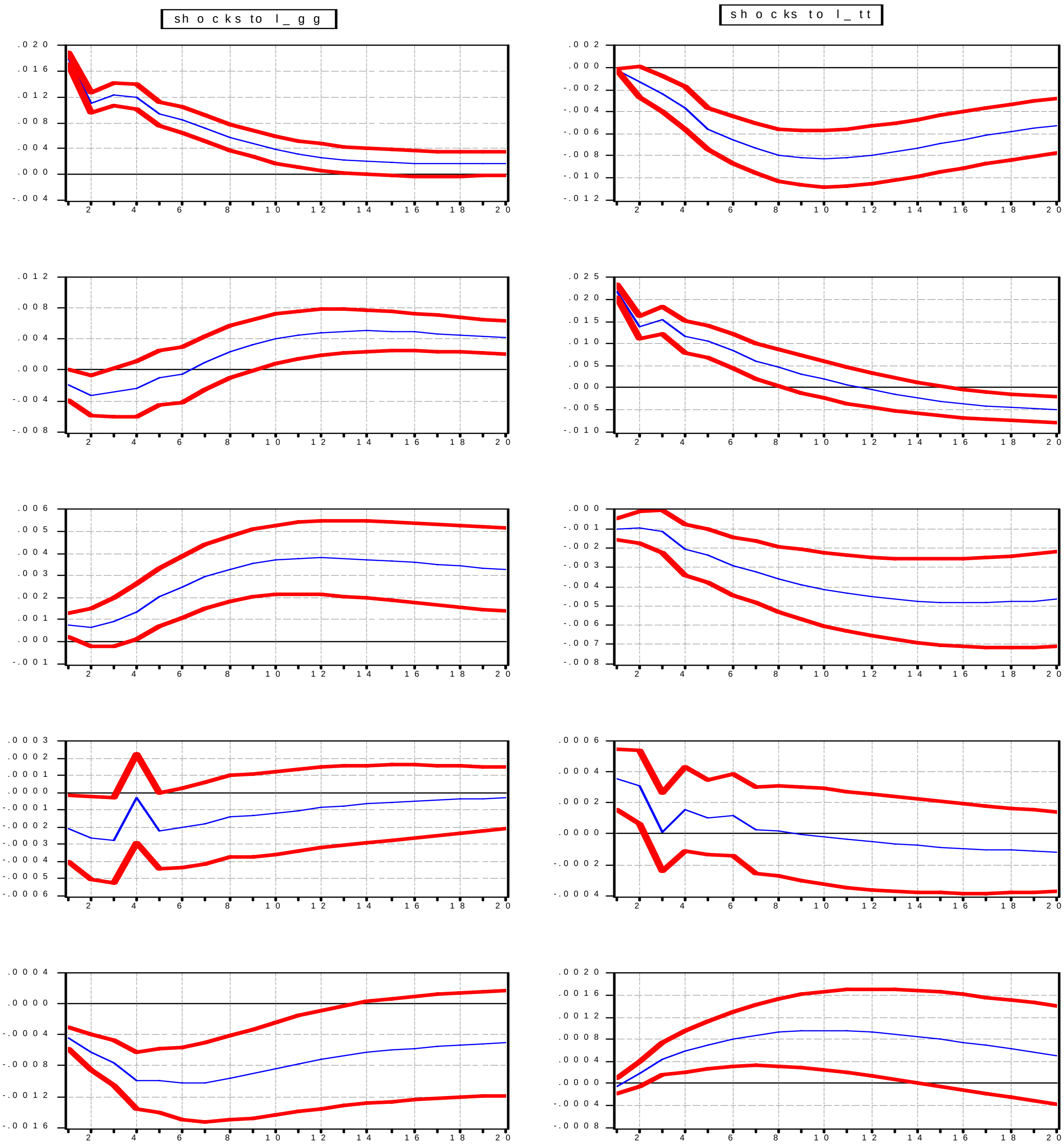

Figure 3.3: Fiscal policy in standard VAR. Sample 1960:1 2006:2.

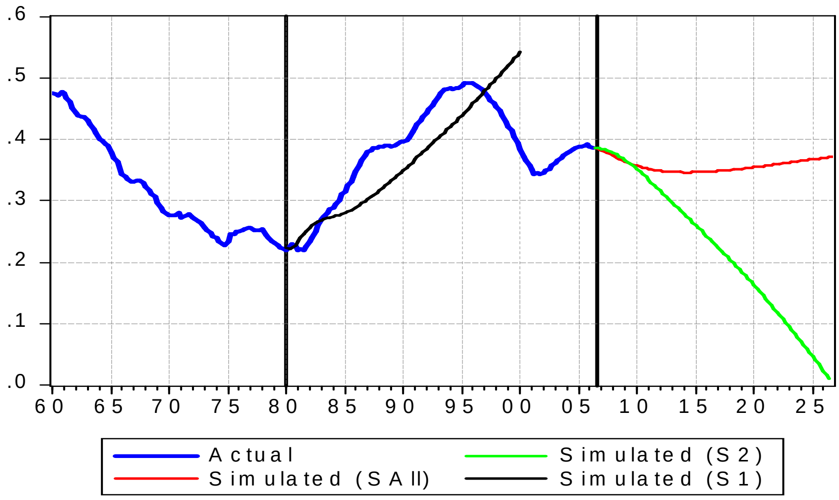

Figure 4: Actual and Simulated debt-GDP dynamics

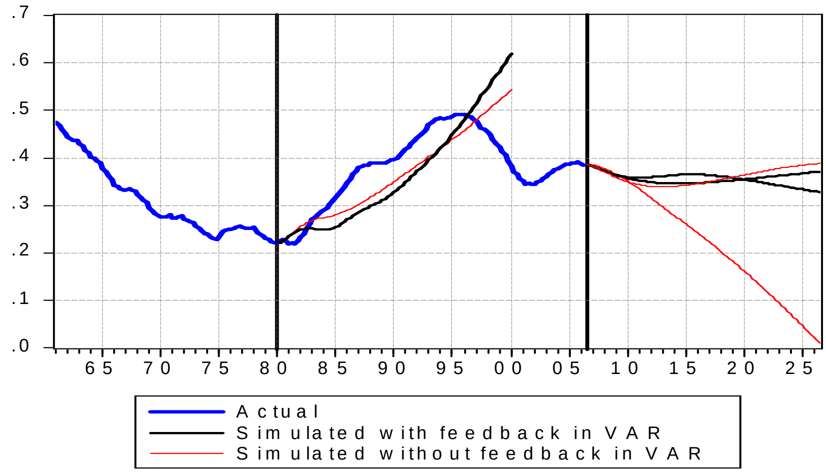

Figure 5: Actual and simulated debt-GDP dynamics (with and without debt feedbacks in the VAR)

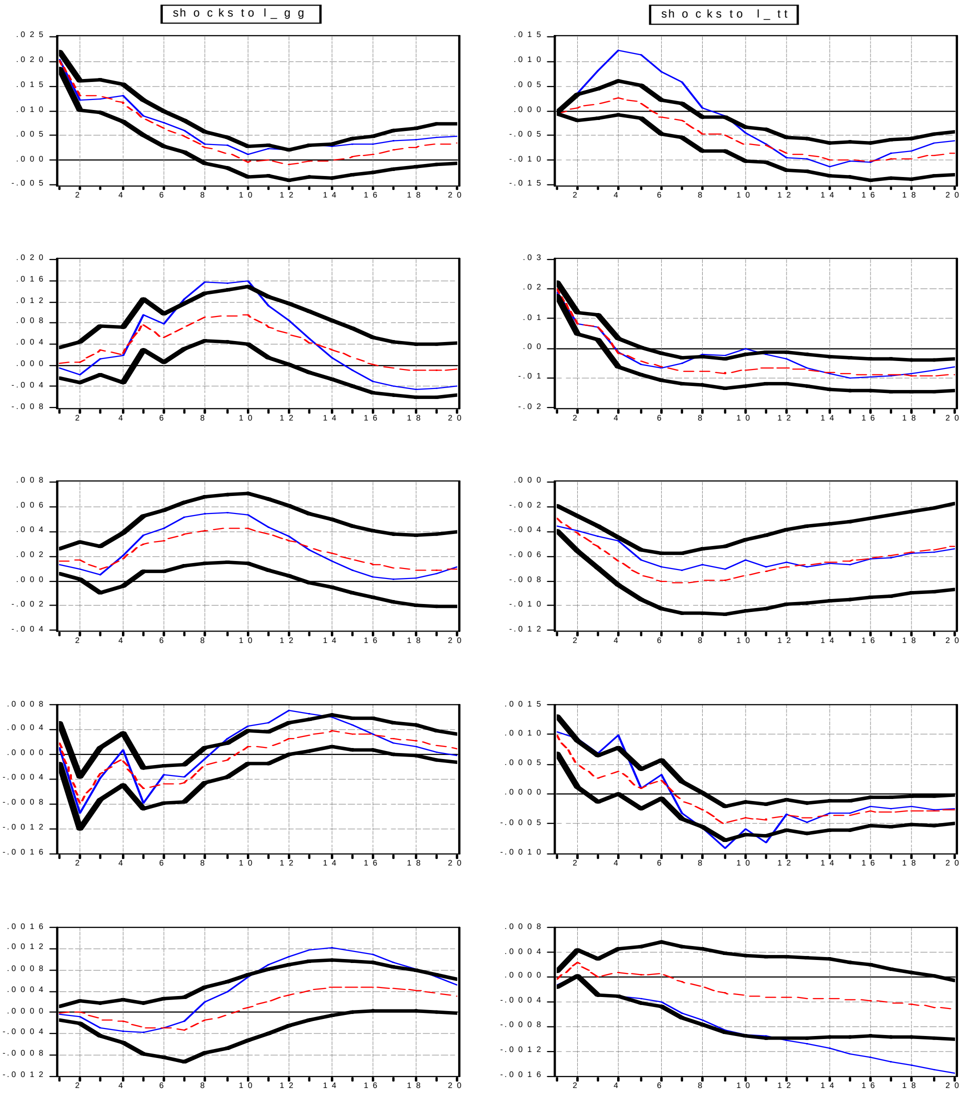

Figure 6.1: Fiscal policy in a standard VAR(dotted line) and in a model with feedbacks (solid line). Sample 1960:1 1979:4. The first column shows responses to shocks to g_t; the second column to shocks to t_t.The responses reported along the rows refer, respectively, to the effects on g_t, t_t, y_t, Δp_t, i_t.

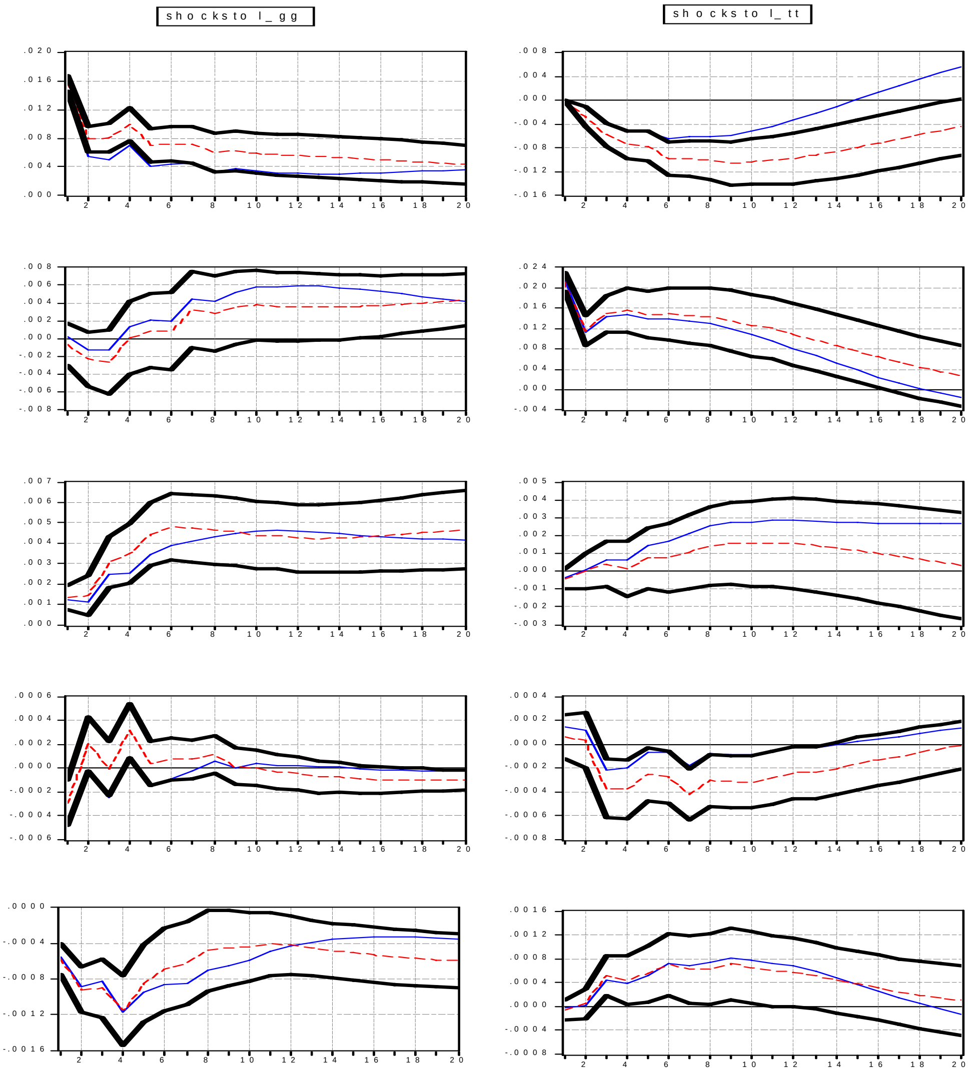

Figure 6.2: Fiscal policy in a standard VAR(dotted line) and in a model with feedbacks (solid line). Sample 1960:1 1979:4.

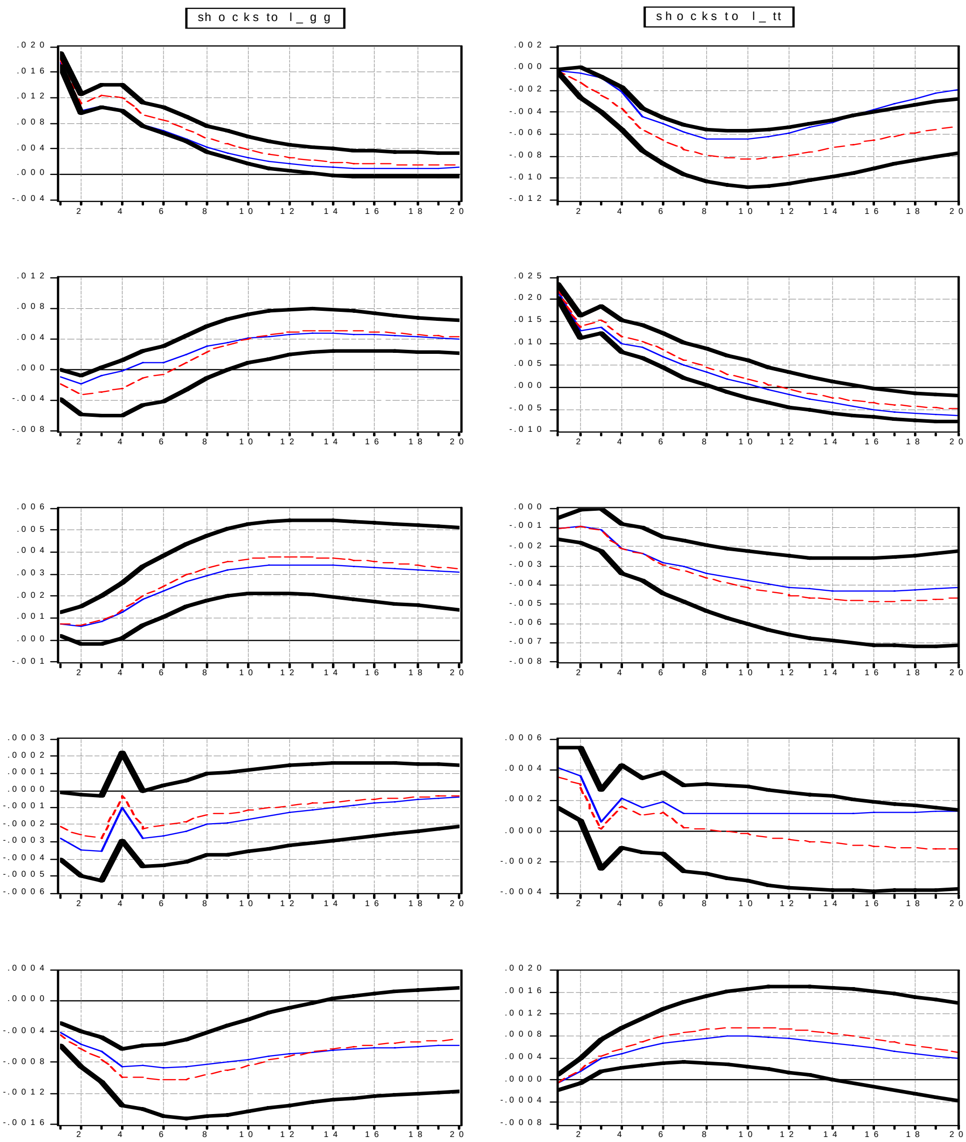

Figure 6.3: Fiscal policy in a standard VAR(dotted line) and in a model with feedbacks (solid line). Sample 1960:1 1979:4.