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Demographics and The Behaviour of Interest Rates

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Abstract

In this paper we relate the very persistent component of interest rates to a specific demographic variable, MY_t , the proportion of middle-aged to young population. We first reconsider the results in Fama (2006) to document how MY_t captures the long run component identified by Fama in his analysis of the one-year spot rate. Using MY_t to model this low frequency component of interest rates is particularly useful for forecasting the term structure as the demographic variable is exogenous and highly predictable, even at very long horizons. We then study the forecasting performance of a no-arbitrage affine term structure model that allows for the presence of a persistent component driven by demographics. This performance is superior to that of a traditional affine term structure model with macroeconomic factors (e.g. Ang, Dong and Piazzesi, 2005).

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1 Introduction

The behavior of interest rates (Fama, 2006) reveals that spot rates are the sum of two processes, (i) a very persistent long term expected value and (ii) a mean-reverting component. This paper has two main objectives. First, we relate the very persistent component of interest rates to a specific demographic variable, i.e. the proportion of middle-aged to young population, second we show how an affine term structure model that incorporates a demographics driven very persistent component can be built and used for long-run forecasting of the entire term structure. Figure 1 illustrates the existence of a persistent component in interest rates and the possibility of relating it to demographic trends. The figure reports the Yield to maturity of 1-Year US Treasury bond, the persistent component as identified by Fama (a 5-year backward looking moving average) and MY, defined as the ratio of middle-aged (40-49) to young (20-29) population in the US.

Insert Figure 1 about here

The Figure shows that MY not only strongly comoves with the estimate of the persistent component proposed by Fama, but it is also capable of matching exactly the observed peak in yields at the beginning of the eighties, while, by its nature, the backward looking moving average lags behind.

As a matter of fact the very persistent component identified by Fama is common to the entire term structure of interest rates: Figure 2 shows this point by reporting for US post-war data, the relationship between nominal interest rates and MY_t .

Insert Figure 2 about here

The presence of a very persistent long-term expected value common to the entire term structure has important implications for affine term structure models in which factors driving the term structure are modelled via stationary VAR representations. In fact, the stationary VAR representations cannot capture the persistent component in the term structure and such omission might explain the unsatisfactory forecasting performance of affine term structure models noted in the literature (see, Duffee (2002), Moench (2008)). Using MY_t to model the persistent

component of interest rates is particularly useful for forecasting the term structure as the demographic variable is exogenous and highly predictable even for very long-horizons (the Bureau of Census currently publishes on its website projections for the age structure of the population with a forecasting horizon up to fifty years ahead).

Demographic trends are a slow-moving information variable, whose forecasting power for interest rates is low at high frequency but becomes high at low frequencies, when the effect of the fluctuations of stationary component of yield curve fluctuations subsides. Intuitive reasoning and formal modeling hints at demography as an important variable to determine the long-run behavior of the financial markets. In fact, abundant evidence is available on the impact of the demographic structure of the population on long-run stock-market returns (Ang and Maddaloni (2005), Bakshi and Chen (1994), Goyal (2004), Della Vigna and Pollet (2007) and Favero, Gozluklu and Tamoni (2010)). To our knowledge the empirical study on the empirical relation between the bond market and demographics is much more limited (the only study we are aware of is McMillan, H. and J. B. Baesel, 1988), despite the strong interest for co-movements between the stock and the bond markets (Lander et al.(1997), Campbell and Vuoltenaho (2004), Bekaert and Engstrom (2010)). This paper fills this gap by presenting evidence on the relation between demographics and the persistent component of the yield curve and on the importance of using this information to de-trend the term structure before producing VAR-based forecasts.

The rest of this paper is organized as follows.

In the next section we place our contribution in the literature and discuss the rationale for the relationship between the demographic variable MY_t and the persistent component in interest rates. The dataset and the results in Fama (2006) are then reconsidered to document how MY_t captures the long-run component identified by Fama in his analysis of the one-year spot rate. The evidence naturally leads to pursue the consequences of the permanent-transitory decomposition for forecasting the term structure. In particular, we propose a no-arbitrage affine term structure model that allows for the presence of a low-frequency component driven by demographics. The forecasting performance of our model is then examined and compared with that of a traditional affine term structure model with macroeconomic factors (e.g. Ang, Dong and Piazzesi, 2005). We then devote a section to exploit the peculiar feature of an affine

term structure with exogenous demographic factors to produce long-term forecast for the yield curve up to 2040. The last section concludes.

2 Related Literature

This paper brings together three different strands of the literature: i) the one analyzing the implication of the presence of a persistent component for spot rates predictability, ii) the affine no-arbitrage term structure models with observable macro factors and latent variables and iii) the one linking demographic fluctuations with asset prices.

The literature on spot rates predictability has emerged from a view in which forecastability is determined by the slowly mean-reverting nature of the relevant process. Recently, it moved to a consensus that modelling a persistent component is a necessary requirement for a good predictive performance.

Traditionally, yield curve modelling in finance is governed by parsimony principle; all the relevant information to price bonds at any given point in time is summarized by a small number of stationary factors (Litterman and Scheinkman, 1991). Both macro and finance term structure models agree on the role of at least two factors, i.e. the level and the slope, that capture a slow mean-reverting component and a more rapid (business-cycle-length) mean-reverting component. Besides providing a structural interpretation, this literature also documents the role of forward interest rates in forecasting future spot rates for longer horizons (e.g. Fama and Bliss, 1979). Particularly linear combinations of forward rates are successful in predicting term premia (Cochrane and Piazzesi, 2005). Early literature attributes this predictability to the mean reversion of the spot rate toward a constant expected value. This view has been recently challenged; the predictability of the spot rate captured by forward rates is either attributed to a slowly moving, yet still stationary, mean (Balduzzi, Das, and Foresi, 1998) or a time-varying very persistent long-term expected value (Fama, 2006). Following Hamilton (1988), other papers estimate regime-switching models for the spot rate (for example, Gray (1996), Ang and Bekaert (2002)) to capture the time variation of the long run mean.

The importance of such a persistent component for modelling and forecasting the term

structure is our main focus. The existence of such a component raises two immediate issues. First, the validity of all forecasting models based on a stationary representation of factors is not warranted. Second, some investigation is needed on the determinants of persistence.

The affine no-arbitrage term structure models with observable factors are commonly based on a stationary representation of factors, where the following specification is generally adopted:

$$y_{t,t+n} = \frac{-1}{n} (A_n + B'_n X_t) + \varepsilon_{t,t+n} \quad \varepsilon_{t,t+n} \sim i.i.d.N(0, \sigma_n^2) \quad (1)$$

$$X_t = \mu + \Phi X_{t-1} + v_t \quad v_t \sim i.i.d.N(0, \Omega) \quad (2)$$

where $y_{t,t+n}$ denotes the yield at time t of a zero-coupon government bond maturing at time $t+n$, bonds are priced via a vector X_t of stationary state variables, that can be observable or unobserved. The no-arbitrage assumption imposes the following structure on the coefficients of the measurement equation (for $n \geq 1$):

$$\begin{aligned} A_{n+1} &= A_n + B'_n (\mu - \Omega \lambda_0) + \frac{1}{2} B'_n \Omega B_n + A_1 \\ B'_{n+1} &= B'_n (\Phi - \Omega \lambda_1) + B'_1 \end{aligned}$$

The potential problem with this general structure is that while yields contain a persistent component, the state evolves as a stationary VAR which, independently from imposing the no-arbitrage restrictions, is designed to model a mean-reverting process and cannot capture the time series behavior of persistent variables.

In fact, several papers indicate that macroeconomic variables have strong effects on future movements of the yield curve (among others, Ang and Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2005) and Rudebusch and Wu (2008)). Ang and Piazzesi (2003) show that a mixed factor model (with three latent financial factors plus output and inflation) performs better than a yields-only model in terms of one step ahead forecast at quarterly frequency. Others (e.g. Bekaert, Cho, and Moreno (2003), Gallmeyer, Hollifield, and Zin (2005), Rudebusch and Wu (2008), and Hordahl, Tristani, and Vestin (2006)) estimate structural models with interest rates and macro variables. Ang, Dong and Piazzesi (2005), and Dewachter and Lyrio (2006),

investigate how no-arbitrage restrictions can help estimate different policy rules. In all these macro-finance models the process determining the macro variables is taken to be stationary. This assumption is not consistent with the presence of a low-frequency component in determining the behavior of interest rates and might therefore explain the, somewhat disappointingly, mixed results from the forecasting performance of affine term structure models (Duffee (2002), Favero, Niu and Sala (2010)). Forecasting interest rates in the presence of a highly persistent component in rates requires the existence of a factor capable of modelling the persistence. Inflation is an obvious candidate as far as the persistence is concerned, but it is an endogenous factor, since it is influenced by monetary policy, and its turning points are very difficult to predict.

Any factor built on the basis of demographic information is instead exogenous and predictable, given the nature of demographic variable. The interesting question here is why should demographics influence the behavior of interest rates and why the relevant effect of demographics can be captured by the relative size of the middle aged to young population.

Fluctuations in nominal interest rates must be due to either movements in real interest rates, expected inflation, or the inflation risk premium. There is available literature relating demographic factors to the first two components.

Geanakoplos, Magill and Quinzii (2004, henceforth, GMQ) consider an overlapping generation model in which the demographic structure mimics the pattern of live births in the U.S., that have featured alternating twenty-year periods of boom and busts. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994), which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired, plays an important role in determining equilibrium asset prices. Consumption smoothing by the agents, given the assumed demographic structure requires that when the MY_t ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged (young). The model predicts that the price of all financial assets should be positively related to MY_t and it therefore also predicts the negative correlations between yields and MY_t exhibited in Figure 1. In a related paper (Favero, Tamoni, Gozluklu, 2010) we have illustrated how

the persistent component in the dividend/price ratio is captured by MY_t . In the GMQ model bond and stock are perfect substitutes, therefore the evaluation of the performance of MY_t in determining the persistent component of interest rates is a natural step within this framework¹. The GMQ model provides an explicit foundation for the relationship between MY_t and real interest rates. In a separate strand of literature, Lindh and Malberg (2000) investigate the hypothesis that inflation pressures covary with the age distribution unless accommodated by monetary policy. The results of the estimation of a relation between inflation and age structure on annual OECD data covering 1960–1994 for 20 countries suggest the existence of an age pattern of inflation effects. It is consistent with the hypothesis that increases in the population of net savers dampen inflation, whereas especially the younger retirees fan inflation as they start consuming out of accumulated pension claims. This confirms the positive correlation between the young-to-middle aged ratio and inflation illustrated in Figure 1, which is implied by the life-cycle saving behavior (Bakshi and Chen, 1994). The importance of demographic projections is also documented in Gozluklu (2010), where the role of time variation in the age structure in shaping the long-run co-movement between inflation and financial markets is analyzed.

We merge these three strands of the literature by using MY_t to model the persistent component in spot rates and by then proposing a no-arbitrage affine term structure model capable of accommodating a long run component in the term structure via the low frequency time-series properties of the age structure of population.

3 Demographics and the permanent component of spot rates

Fama (2006) explains the evidence that forward rates forecast future spot rates in terms of a mean reversion of spot rates towards a non-stationary long-term mean which is subject to sequence of permanent shocks. The forward-spot spread is then interpreted as an empirical proxy for the deviation of spot rates from their time-varying long-run trend. As a matter of fact, when future spot rates are predicted via a model that includes the forward-spot spread

¹Note that the existence of a relation between the dividend price ratio and MY implies a similar relation between MY and interests rates also when the perfect substitutability assumption is relaxed by maintaining the hypothesis that the relative risk premium on stock and bonds is stationary. In fact a stationary relative risk term premium cannot determine the persistent components of interest rates and the dividend/price ratio.

and deviations of the spot rate from a five-year backward looking moving average (together with a step dummy taking a value of one for the first part of the sample up to August 1981 and zero otherwise) the second predictor swamps the significance of the first one. This evidence is consistent with the fact the dominant feature in the spot rates of an upward movement from the fifties to mid 1981 and a downward movement from 1981 onwards is not matched by any similar movement in the forward-spot spread which looks like a mean reverting process over the sample 1952-2004. Fama proposes a decomposition of the spot interest rates, $y_{t,t+n}$ ² in two processes: a long term expected value $K_{t,t+n}$, that is subject to permanent shocks, and a mean reverting component $X_{t,t+n}$.

$$y_{t,t+n} = K_{t,t+n} + X_{t,t+n}$$

On the basis of this decomposition Fama addresses the question of predicting changes in the one-year spot rate at different horizons (from 1-year to 4-year ahead) using monthly data. The predictive model is designed to evaluate the forecasting ability of the spread between forward and spot rates and the deviations of the spot rates from its long term expected value.

The following model is estimated on a sample of monthly data 1958:6-1984:12:

$$\begin{aligned} y_{t+12x,t+12x+12} - y_{t,t+12} &= a^x + b^x D_t + c^x [f_{t,t+12x,t+12x+12} - y_{t,t+12}] + d^x [y_{t,t+12} - K_{t,t+12}] + \varepsilon_{t+12x} \\ K_{t,t+12} &= \frac{1}{60} \sum_{i=1}^{60} y_{t-i-1,t+12-i-1} \end{aligned} \quad (3)$$

where $f_{t,t+12x,t+12x+12}$ is the one-year forward rate at time t of an investment with settlement after x years and maturity in $x + 1$ years, $y_{t,t+12}$ is the one-year spot interest rate and $K_{t,t+12}$ is measured as the moving average of the most recent past five years of the spot rates.

²We adopt Cochrane and Piazzesi (2005) notation for log bond prices:

$$p_{t,t+n} = \log \text{ price of } n\text{-year discount bond at time } t$$

where the parenthesis in the superscript refers to time to maturity. Then the continuously compounded spot rate is

$$y_{t,t+n} \equiv -\frac{1}{n} p_{t,t+n}$$

Note that the model also includes a dummy variable D_t that is equal to one for the first part of the sample up to August 1981 and zero otherwise. This variable captures the turning point in the behavior of interest rates from a positive upward trend to a negative upward trend occurred in mid 1981 and clearly detectable from Figure 1.³

A model with the restriction $d^x = 0$ is first estimated to obtain a positive and significant estimate of c^x with a significance increasing with the horizon x . However, when the restriction $d^x = 0$ is relaxed, then the null hypothesis that c^x is not statistically different from zero cannot be rejected. d^x is estimated significantly negative increasing with the horizon. All results are crucially affected by the inclusion of the dummy, in that when the b^x is restricted to zero the predictive power of the regression is dramatically reduced. In the light of these results Fama concludes that there is evidence of mean reversion of the spot rates toward a time varying expected value and that the predictability of the spot rate captured by the forward rate is exclusively related to this phenomenon.

In Table 1 below we replicate the empirical results in Fama (2006) both using the same sample with monthly observations and by considering quarterly data over a sample extended to the last quarter of 2008⁴.

Insert Table 1 about here

Our results fully replicate those reported by Fama (2006) on monthly data and are robust when the sample is extended to 2008 and quarterly data are considered.

As we have already discussed in the introduction and shown in Figure 1, the demographic variable MY_t has a similar pattern to that of the Fama's proxy for the permanent component of the spot rate, with the advantage of matching exactly the peak in interest rates in 1981.

We then use the Fama (2006) setup to assess the importance of the demographic variable MY_t in capturing the long term expected value $K_{t,t+n}$. We consider the following alternative

³A possible economic rationale for this turning point is introduced with reference to the experience of Federal Reserve introducing a fiduciary currency in 1971 that created permanently high inflation expectations up to mid-1981, until Federal Reserve gained enough experience with the new system and learned how to manage inflation using a fiduciary currency.

⁴1-year Treasury bond yields are computed from the (monthly and quarterly) price series from the Fama CRSP zero coupon files. Middle-young ratio data is available at annual frequencies from Bureau of Census (BoC) and it has been interpolated to obtain monthly and quarterly series.

specification to (3):

$$\begin{aligned}
y_{t+12x,t+12x+12} - y_{t,t+12} &= a^x + b^x D_t + c^x [f_{t,t+12x,t+12x+12} - y_{t,t+12}] + d^x [y_{t,t+12} - K_{t,t+12}] + \varepsilon_{t+12x} \\
K_{t,t+12} &= e^x MY_t
\end{aligned} \tag{4}$$

The results from the estimation, reported in Panel B of Table 1, show that the coefficient on MY_t is strongly significant in all regressions at the different frequencies and horizons. The performance of the forecasting model improves as the forecasting horizon increases (with an R^2 going from 0.24 at the one-year horizon to 0.64 at the four-year horizon). Importantly, when $K_{t,t+12}$ is modelled via demographics the coefficients on the dummy variable capturing the turning points in the underlying trend is not significant anymore, witnessing the capability of demographics of capturing the change in the underlying trend affecting spot rates.

To further test the forecasting performance of the model, we conduct a (pseudo) out-of-sample forecasting comparison of the models (3) and (4). We generate out-of-sample forecasts for the period 1994Q1-2008Q4, based on a model estimated with a rolling window of 140 observations.

Insert Table 2 about here

The results are reported in Table 2. The first column reports the out-of-sample R^2 statistics (Campbell and Thomson, 2008) which is computed as

$$R_{OS,x}^2 = 1 - \frac{\sum_{t=t_0}^T [(y_{t+12x,t+12x+12} - y_{t,t+12}) - (y_{t+12x,t+12x+12} - y_{t,t+12})^{pr}]^2}{\sum_{t=t_0}^T [(y_{t+12x,t+12x+12} - y_{t,t+12}) - (y_{t+12x,t+12x+12} - y_{t,t+12})^{av}]^2}$$

where $(y_{t+12x,t+12x+12} - y_{t,t+12})^{pr}$ are the model-based projections at horizon x and $(y_{t+12x,t+12x+12} - y_{t,t+12})^{av}$ is the historical average change computed with data available at the time of forecasting. In our exercise, $t_0 = 1994$ and $T = 2008$. If R_{OS}^2 is positive, it means that the predictive regression has a lower mean square error than the prevailing historical mean. Then we report mean absolute error (MAE) and root mean square error (RMSE) calculated based on the residuals in the forecasting period, namely 1994-2008. In the last column, we report the Diebold-Mariano (DM)

t -test for checking equal-forecast accuracy from two nested models for forecasting h -step ahead spot rate changes.

$$DM = \sqrt{\frac{(T+1 - 2 * h + h * (h - 1))}{T}} * \left[\frac{\bar{d}}{\widehat{se}(\bar{d})} \right]$$

where we define e_{1t}^2 as the squared forecasting error of prevailing mean, and e_{2t}^2 as the squared forecasting error of the predictive variables, $d_t = e_{1t}^2 - e_{2t}^2$, i.e. the difference between the two forecast errors, $\bar{d} = \frac{1}{T} \sum_{t=t_0}^T d_t$ and $\widehat{se}(\bar{d}) = \frac{1}{T} \sum_{\tau=-(h-1)}^{h-1} \sum_{t=|\tau|+1}^T (d_t - \bar{d}) * (d_{t-|\tau|} - \bar{d})$. A positive DM t -test statistic indicates that the predictive regression model performs better than the historical mean.

We note that the forecasting performance of the two alternative models is similar (although no dummy is included in the model with demographics) and that the relative performance of the model based on demographics improves with the forecasting horizon. We interpret these results as supportive of the view that the persistent component in the 1-year interest rates can be related to demographic factors that are therefore very useful predictors, especially, at long-horizons.

4 An affine No-Arbitrage Term Structure Model with Demographics

The evidence reported in the previous section supports the hypothesis of the existence of a permanent component in 1-year spot interest rates and its relation with the demographic variable MY_t . As discussed in the introduction, in affine term structure models stationarity of the state space is assumed to justify its VAR representation and to derive long-term VAR-based projections. Therefore, the presence of a permanent component in spot-rates poses an important econometric challenge to these models. Consider, for example, a typical affine model with macroeconomic factors in a data-rich environment (Bernanke and Boivin (2003), Ang, Dong and Piazzesi (2005), Ludvigson and Ng (2009)). The term structure is described as follows:

$$\begin{aligned}
y_{t,t+n} &= \frac{-1}{n} (A_n + B'_n X_t) + \varepsilon_{t,t+n} & \varepsilon_{t,t+n} &\sim i.i.d.N(0, \sigma_n^2) \\
X_t &= \mu + \Phi X_{t-1} + v_t & v_t &\sim i.i.d.N(0, \Omega) \\
y_{t,t+1} &= \delta_0 + \delta'_1 X_t
\end{aligned} \tag{5}$$

where $y_{t,t+n}$ denotes the yield at time t of a zero-coupon government bond maturing at time $t+n$, the vector of the states $X_t = [f_t^o, f_t^u]$, where f_t^o are two observable factors extracted from large-data sets to capture all the relevant "real" and inflation information which the Fed uses to set the monetary policy rate in a data-rich environment, while f_t^u contains unobservable factor(s) needed to capture the persistent component in interest rates. Note that the one-period yield is typically the monetary policy rate which is modelled as function of the relevant information in the central bank reaction function as in a Taylor-type rule (Ang, Dong and Piazzesi, 2005). We consider the two factors estimated by Ludvigson and Ng (2009) to capture "real" and inflation information. This model is completed by assuming a linear (affine) relation between the price of risk, Λ_t , and the states X_t by specifying the pricing kernel, m_{t+1} , consistently and by imposing no-arbitrage restrictions (see, for example, Duffie and Kan (1996), Ang and Piazzesi (2003)):

$$\begin{aligned}
\Lambda_t &= \lambda_0 + \lambda_1 X_t \\
m_{t+1} &= \exp(-y_{t,t+1} - \frac{1}{2} \Lambda'_t \Omega \Lambda_t - \Lambda_t \varepsilon_{t+1}) \\
A_{n+1} &= A_n + B'_n (\mu - \Omega \lambda_0) + \frac{1}{2} B'_n \Omega B_n + A_1 \\
B'_{n+1} &= B'_n (\Phi - \Omega \lambda_1) + B'_1
\end{aligned}$$

This structure requires a stationary representation of the states that makes the model "incongruent" in the case of the presence of a persistent component in spot rates. The "incongruency" comes from the fact that highly persistent time series are linearly related to a stationary process. Table 3, which illustrates the time series properties of spot rates with different maturity and the two Ludvigson and Ng factors, highlights the empirical relevance of this problem.

Insert Table 3 about here

Insert Figure 3 about here

The evidence on the time series properties of MY_t also confirms the high persistence and it is consistent with the evidence provided in the previous section of the potential of this demographic factor for modelling the permanent component in spot rates. In light of this evidence, we propose the following affine term structure model with demographics:

$$\begin{aligned}
y_{t,t+n} &= -\frac{1}{n} (A_n + B'_n X_t + \Gamma_n MY_t^n) + \varepsilon_{t,t+n} & \varepsilon_{t,t+n} &\sim N(0, \sigma_n^2) \\
X_t &= \mu + \Phi X_{t-1} + \nu_t & \nu_t &\sim i.i.d.N(0, \Omega) \\
y_{t,t+1} &= \delta_0 + \delta'_1 X_t + \delta_2 MY_t
\end{aligned}$$

where $\Gamma_n = [\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n]$, and $MY_t^n = [MY_t, MY_{t+1} \cdots, MY_{t+n-1}]'$ and $X_t = [f_t^o, f_t^u]$.

By using the traditional relation between the price of risk and the states and by imposing the no-arbitrage restrictions we obtain (see appendix)

$$\begin{aligned}
A_{n+1} &= A_1 + A_n + B'_n (\mu - \Omega \lambda_0) + \frac{1}{2} B'_n \Omega B_n \\
B_{n+1} &= B'_n \Phi - B'_n \Omega \lambda_1 + B'_1 \\
\Gamma_{n+1} &= [-\delta_2, \Gamma_n]
\end{aligned}$$

As we include the demographic variable in the process for monetary policy rates, yields with maturity $t+n$ are function of the demographic variable between time t and time $t+n$ as they reflect the expected path of future monetary policy. MY_t is included into the model as an exogenous variable, its high degree of predictability (the Bureau of Census posts on its website projections for the composition of population with a fifty year-ahead horizon) allows to model empirically the relation between $y_{t,t+n}$ and MY_t^n without specifying a model to predict future values of MY_t .

We propose a model where the demographic variable MY_t enters linearly in the monetary policy rule, to capture the permanent component in spot rates. The price of risk is independent from the demographic variable and it is affine in the vector of state factors. The empirical

evidence discussed in the previous section points to demographics as an observable factor capable of capturing the permanent component of 1-year spot rates. This evidence can be extended to the 3-month rates (the rate with the shortest maturity in the sample of quarterly data we analyze in this section). We report the results in Table 4 the results. The estimated coefficient on MY_t is always significant with stable loadings.

Insert Table 4 about here

The no-arbitrage restrictions, appropriately modified from the baseline case with no demographics, are imposed on the system (see Appendix for a full derivation). The entire term structure is affected by the permanent component of spot rates as long-term spot rates are affected by expected monetary policy rates. Note that in our specification we do not make the price of risk function of the demographic variable consistently with the idea that risk premia are stationary and cannot explain the presence of a permanent component in the term structure. However, as the entire term structure depends on future expected monetary policy, then the future values of the demographic variable affect yields at all maturities.

5 Model Specification and Estimation

We consider as a benchmark model the discrete-time no-arbitrage term structure model with both observable and unobservable variables, first suggested by Ang and Piazzesi (2003). Following the specification analysis of Pericoli and Taboga (2008), we focus on a parsimonious model including three latent factors and only contemporaneous values of the macro variables. We use the Chen and Scott's (1993) methodology; given the set of parameters and observed yields latent variables are extracted by assuming that number of bonds which are priced exactly is equal to the number of unobserved variables. Hence we assume that 3-month, 2-year and 5-year bond prices are measured without error and estimate the model with maximum likelihood. We assume the state dynamics to follow a VAR(1). We do not impose any restriction on the VAR coefficient matrix, while we make sure that the following conditions are met in our estimation (Favero, Niu and Sala, 2010):

i) the covariance matrix Ω is block diagonal with the block corresponding to the unobservable yield factor being identity, and the block corresponding to the observable factors being unrestricted, i.e.

$$\Omega = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Omega^o \end{bmatrix}$$

ii) the loadings on the factors in the short rate equation are positive, $0 \leq -A_1$

iii) $X_0^u = 0$.

Despite its heavier parametrization, this specification allows interaction between yield factors and macro factors without sacrificing forecasting power (Favero, Niu and Sala, 2010).

6 Out-of Sample Forecasts

An interesting feature of MY_t is that long-run forecasts for this variable are readily available. In fact, the Bureau of Census (BoC) provide projections up to 2050 for MY_t . Therefore we focus on the properties of out-of-sample forecasts of our model at different horizons. In our multi-period ahead forecast, we choose iterated forecast procedure, where multiple step ahead states are obtained by iterating the one-step model forward

$$\begin{aligned} \hat{y}_{t+h,t+h+n|t} &= a_n + b_n \hat{X}_{t+h|t} + \Gamma_n MY_{t+h}^n \\ \hat{X}_{t+h|t} &= \sum_{i=0}^h \hat{\Phi}^i \mu + \hat{\Phi}^h \hat{X}_t \end{aligned}$$

where $a_n = -\frac{1}{n}A_n$, $b_n = -\frac{1}{n}B_n$ are obtained by no-arbitrage restrictions. Forecast are produced on the basis of rolling estimation with a rolling window of one hundred observation, the first sample used for estimation is 1964Q1-1988Q4. We consider 6 forecasting horizons (denoted by h): one quarter, one year, two years, three years, four years and five years. For the one quarter ahead forecasting horizon, we conduct our exercise for all dates in the period 1989Q1 - 2007Q4, a total of 76 periods; for the 1-year ahead forecast, we end up with a total of 73 forecasts, and so on, up to the 5-year ahead forecast, for which we end up with 57 forecasts.

Forecasting performance is measured by the ratio of the root mean squared forecast error (RMSFE) of each the affine model with demographics to the RMSFE of a random walk forecast and to the RMSFE of the benchmark yield-macro model without the demographic variable. Forecasting results from different models are reported in Table 6 and Table 7. Bold characters are used when the ratio of the model’s RMSFE to that of the random walk is in the range [0.9, 1) while bold and underline characters are used for ratios smaller than 0.9. Two specifications for risk prices are estimated

- i) Constant prices of risk: $\lambda_0 \neq 0, \lambda_1 = 0$. (Table 6)
- ii) Time-varying prices of risk, $\lambda_1 \neq 0, \lambda_1$ diagonal. (Table 7)

Insert Table 6 about here (constant risk)

Insert Table 7 about here (time-varying risk)

Table 6 reports the RMSFE of the model with demographics relative to the random walk and benchmark yield-macro model in the case of constant risk pricing ($\lambda_1 = 0$). The evidence shows clearly that the inclusion of the demographic variable in the model improves the forecasting performance for the entire term structure as the forecasting horizon gets larger. The results become much stronger when time-varying risk prices are allowed. We also report in parentheses the p-values of the forecasting accuracy test due to Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level.

Insert Figure 4 here

The intuition behind the superior forecasting performance of the model augmented with demographics is illustrated in Figure 4, where we report the actual time-series for different yields along with the dynamically simulated values for the same yields based on the benchmark affine model with macro factors and the model augmented with demographics. Figure 4 reports the results of dynamic simulation from the beginning of the sample given estimation over the full-sample. The dynamic simulation of the benchmark model converges rather rapidly to the unconditional mean and it is not capable of capturing the persistent component in the term structure. The

picture is very different for the simulated values from the model with demographics where the middle-to-young ratio allows to keep the simulated value remarkably close to the long-run trend of the term structure.

7 Long-Term Projections

One of the appealing features of an affine term structure model with demographics factors is that the availability of long-term projections for the age-structure of the population which can be exploited to produce long-term projections for the yield curve. In our specification yields at time $t + j$ with maturities $t + j + n$ are functions of all realization of MY between $t + j$ and $t + j + n$. The exogeneity of the demographic variable and the availability of long term projections is combined in the affine model with a parsimonious parameterization generated by the no-arbitrage restrictions that allow to weight properly all future values of MY with the estimation of few coefficients. As a result future paths up to 2045 can be generated for the entire term structure, given the availability of demographic projections up to 2050 (the Bureau of Census websites provides projections for demographics variable up to 2050 and the current 5-year yield depends on the values of MY over the next five years). Note that an affine model with demographics allow for a very different out-of-sample dynamics from those that can be generated by a Fama (2006) type model based on the decomposition of interest rates in long-term expected mean and mean-reverting component and on the measurement of the long-term expected mean as the moving average of the most recent past five years of the spot rates. In fact, long-term projections from the Fama model would not differ from those of an autoregressive process, they would just converge more slowly to the unconditional mean. To illustrate this feature of the model we report in Figure 5 all yields reported in Figure 1 by also drawing their predicted out-of-sample path up to 2045 .

Insert Figure 5 here

The predicted path of the age structure of the population drives the forecast of the term structure up in the range between six per cent and eight per cent over the next twenty years. An affine

model with stationary macro factors estimated over the last twenty years could never produce such forecasts, as they are clearly far away from the unconditional sample mean.

8 Conclusions

There is a general agreement on the existence of a common persistent component in the entire term structure of interest rates. In this paper we have shown that it is possible to model such a persistent component using a demographic variable, to ratio of middle aged to young population, MY_t . We have first provided evidence that MY_t is a better measure of the persistent component of the 1-year rate than the moving-average of the most recent past five years of the spot rates used by Fama (2006). We have then explored the implication of this evidence for the affine model of the term structure of interest rates. Affine models are built by assuming that the relevant state variable to price bonds evolve as a stationary VAR. The problem arises from the fact that a stationary VAR, independently from imposing the no-arbitrage restrictions, is designed to model a mean-reverting process and cannot capture the time series behavior of persistent variables. We propose to include MY_t in affine model by considering it as a determinant of the trend in the short-term rate. As the entire term structure is function of the short-term rate this allows to have a common persistent component in all yields. Importantly this happens without changing the traditional model for the price of risk, where no persistent component is introduced. Our results show that the forecasting performance of an affine model with demographics uniformly outperforms that of traditional affine models with stationary factors. We believe that our evidence is important both for long-term projections of the yield curve and for modelling its fluctuations around the persistent component.

To illustrate this point we have produced model-based forecast for the term structure up to 2045. The demographics bases forecast of the yield curve over the next twenty year are significantly higher than the sample mean observed over the course of the last twenty years.

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APPENDIX

We consider the following model specification for pricing bonds with macro and demographic factors:

$$\begin{aligned}
 y_{t,t+n} &= -\frac{1}{n} (A_n + B_n' X_t + \Gamma_n \text{MY}_t^n) + \varepsilon_{t,t+1} & \varepsilon_{t,t+n} &\sim N(0, \sigma_n^2) \\
 X_t &= \mu + \Phi X_{t-1} + \nu_t & \nu_t &\sim i.i.d.N(0, \Omega) \\
 y_{t,t+1} &= \delta_0 + \boldsymbol{\delta}'_1 X_t + \delta_2 \text{MY}_t \\
 \Lambda_t &= \lambda_0 + \lambda_1 X_t \\
 m_{t+1} &= \exp(-y_{t,t+1} - \frac{1}{2} \Lambda_t' \Omega \Lambda_t - \Lambda_t' \nu_{t+1}) \\
 P_t^{(n)} &\equiv \left[\frac{1}{1 + Y_{t,t+n}} \right]^n, \quad y_{t,t+n} \equiv \ln(1 + Y_{t,t+n}) \\
 \Gamma_n \text{MY}_t^n &\equiv [\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n] \begin{bmatrix} \text{MY}_t \\ \text{MY}_{t+1} \\ \vdots \\ \text{MY}_{t+n-1} \end{bmatrix}
 \end{aligned}$$

Bond prices can be recursively computed as:

$$\begin{aligned}
P_t^{(n)} &= E_t[m_{t+1}P_{t+1}^{(n-1)}] \\
&= E_t[m_{t+1}m_{t+2}P_{t+2}^{(n-2)}] \\
&= E_t[m_{t+1}m_{t+2}\cdots m_{t+n}P_{t+n}^{(0)}] \\
&= E_t[m_{t+1}m_{t+2}\cdots m_{t+n}1] \\
&= E_t[\exp(\sum_{i=0}^{n-1}(-y_{t+i,t+i+1} - \frac{1}{2}\Lambda'_{t+i}\Omega\Lambda_{t+i} - \Lambda'_{t+i}\nu_{t+i+1}))] \\
&= E_t[\exp(A_n + B'_n X_t + \Gamma'_n M Y_t^n)] \\
&= E_t[\exp(-ny_{t,t+n})] \\
&= E_t^Q[\exp(-\sum_{i=0}^{n-1}y_{t+i,t+i+1})]
\end{aligned}$$

where E_t^Q denotes the expectation under the risk-neutral probability measure, under which the dynamics of the state vector X_t are characterized by the risk neutral vector of constants μ^Q and by the autoregressive matrix Φ^Q

$$\begin{aligned}
\mu^Q &= \mu - \Omega\lambda_0 \\
\Phi^Q &= \Phi - \Omega\lambda_1
\end{aligned}$$

To derive the coefficients of the model, let us start with $n = 1$:

$$\begin{aligned}
P_t^{(1)} &= \exp(-y_{t,t+1}) \\
&= \exp(-\delta_0 - \delta'_1 X_t - \delta_2 M Y_t)
\end{aligned}$$

$$A_1 = -\delta_0, B_1 = -\delta_1 \text{ and } \Gamma_1 = \gamma_0^1 = -\delta_2,$$

Then for the general case $n + 1$, we have:

$$\begin{aligned}
P_t^{(n+1)} &= E_t[m_{t+1}P_{t+1}^{(n)}] \\
&= E_t[\exp(-y_{t,t+1} - \frac{1}{2}\Lambda'_t\Omega\Lambda_t - \Lambda'_t\nu_{t+1}) \exp(A_n + B'_nX_{t+1} + \Gamma_nMY_{t+1}^n)] \\
&= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda'_t\Omega\Lambda_t + A_n) E_t[\exp(-\Lambda'_t\nu_{t+1} + B'_nX_{t+1} + \Gamma_nMY_{t+1}^n)] \\
&= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda'_t\Omega\Lambda_t + A_n + \Gamma_nMY_{t+1}^n) \\
&\quad E_t[\exp(-\Lambda'_t\nu_{t+1} + B'_n(\mu + \Phi X_t + \nu_{t+1}))] \\
&= \exp[-\delta_0 - \delta'_1X_t - \delta_2MY_t - \frac{1}{2}\Lambda'_t\Omega\Lambda_t + A_n + \Gamma_nMY_{t+1}^n + B'_n(\mu + \Phi X_t)] \\
&\quad E_t[\exp(-\Lambda'_t\nu_{t+1} + B'_n\nu_{t+1})] \\
&= \exp[-\delta_0 - \delta'_1X_t - \frac{1}{2}\Lambda'_t\Omega\Lambda_t + A_n - \delta_2MY_t + B'_n(\mu + \Phi X_t) \\
&\quad + \Gamma_nMY_{t+1}^n] \exp\{E_t[(-\Lambda'_t + B'_n)\nu_{t+1}] + \frac{1}{2}var[(-\Lambda'_t + B'_n)\nu_{t+1}]\} \\
&= \exp[-\delta_0 - \delta'_1X_t - \frac{1}{2}\Lambda'_t\Omega\Lambda_t + A_n + B'_n(\mu + \Phi X_t) \\
&\quad + [-\delta_2, \gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n] MY_t^{n+1}] \exp\{\frac{1}{2}var[(-\Lambda'_t + B'_n)\nu_{t+1}]\}
\end{aligned}$$

To simplify the notation we define

$$[-\delta_2, \Gamma_n] \equiv [-\delta_2, \gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n]$$

$$\begin{aligned}
&= \exp[-\delta_0 - \delta'_1 X_t - \frac{1}{2} \Lambda'_t \Omega \Lambda_t + A_n + B'_n (\mu + \Phi X_t) \\
&\quad + [-\delta_2, \Gamma_n] \text{MY}_t^{n+1}] \exp\{\frac{1}{2} E_t[(-\Lambda'_t + B'_n) \nu_{t+1} \nu'_{t+1} (-\Lambda_t + B_n)]\} \\
&= \exp[-\delta_0 - \delta'_1 X_t - \frac{1}{2} \Lambda'_t \Omega \Lambda_t + A_n + B'_n (\mu + \Phi X_t) \\
&\quad + [-\delta_2, \Gamma_n] \text{MY}_t^{n+1}] \exp\{\frac{1}{2} [\Lambda'_t \Omega \Lambda_t - 2B'_n \Omega \Lambda_t + B'_n \Omega B_n]\} \\
&= \exp[-\delta_0 + A_n + B'_n \mu + (B'_n \Phi - \delta'_1) X_t - B'_n \Omega \Lambda_t + \frac{1}{2} B'_n \Omega B_n \\
&\quad + [-\delta_2, \Gamma_n] \text{MY}_t^{n+1}] \\
&= \exp[-\delta_0 + A_n + B'_n \mu + (B'_n \Phi - \delta'_1) X_t - B'_n \Omega (\lambda_0 + \lambda_1 X_t) + \frac{1}{2} B'_n \Omega B_n \\
&\quad + [-\delta_2, \Gamma_n] \text{MY}_t^{n+1}] \\
&= \exp[A_1 + A_n + B'_n (\mu - \Omega \lambda_0) + \frac{1}{2} B'_n \Omega B_n + (B'_n \Phi - B'_n \Omega \lambda_1 + B'_1) X_t \\
&\quad + [-\delta_2, \Gamma_n] \text{MY}_t^{n+1}]
\end{aligned}$$

Then we can find the coefficients following the difference equations

$$\begin{aligned}
A_{n+1} &= A_1 + A_n + B'_n (\mu - \Omega \lambda_0) + \frac{1}{2} B'_n \Omega B_n \\
B'_{n+1} &= B'_n \Phi - B'_n \Omega \lambda_1 + B'_1 \\
\Gamma_{n+1} &= [-\delta_2, \Gamma_n]
\end{aligned}$$

TABLE 1. Regressions to explain one-year to four-year changes in the spot rate
 Predictive Model: $y_{t+px,t+px+p} - y_{t,p} = \alpha^x + b^x D_t + c^x [f_{t+px,t+px+p} - y_{t,p}] + d^x [y_{t,p} - K_{t,p}] + \varepsilon_{t+px}$

Panel A. Fama's Model												
$K_{t,p} = \frac{1}{5p} \sum_{i=1}^{5p} y_{t-i-1,t+p-i-1}$, p=12(4), if monthly (quarterly) data												
	α^x	b^x	c^x	d^x	R^2	α^x	b^x	c^x	d^x	R^2		
	(t-stat)	(t-stat)	(t-stat)	(t-stat)		(t-stat)	(t-stat)	(t-stat)	(t-stat)			
Exclude Dummy	June,1958-December 2004					1958Q2-2008Q4						
x = 2	0.178 (0.68)	-0.460 (-1.18)	-0.283 (-1.52)	0.05	0.156 (0.57)	-0.380 (-0.98)	-0.286 (-1.82)	0.05				
x = 3	0.380 (0.63)	-0.551 (-1.27)	-0.583 (-3.27)	0.09	0.437 (0.78)	-0.561 (-1.32)	-0.644 (-4.09)	0.12				
x = 4	-0.172 (-0.25)	0.133 (0.29)	-0.385 (-1.80)	0.10	-0.180 (-0.26)	0.218 (0.54)	-0.426 (-2.41)	0.14				
x = 5	-0.506 (-0.84)	0.584 (1.11)	-0.205 (-0.64)	0.12	-0.401 (-0.66)	0.484 (0.95)	-0.282 (-0.90)	0.15				
Include Dummy	June,1958-December 2004					1958Q2-2008Q4						
x = 2	-0.743 (-1.90)	1.576 (3.07)	-0.216 (-0.65)	-0.417 (-2.86)	0.21	-0.574 (-1.54)	1.405 (2.91)	-0.216 (-0.65)	-0.398 (-2.96)	0.18		
x = 3	-1.207 (-1.82)	2.808 (3.43)	-0.391 (-0.97)	-0.859 (3.45)	0.36	-0.886 (-1.44)	2.479 (3.61)	-0.393 (-1.04)	-0.854 (-6.68)	0.34		
x = 4	-2.519 (-3.33)	3.969 (4.29)	0.413 (1.11)	-0.716 (-2.73)	0.51	-2.289 (-3.79)	3.814 (4.94)	0.439 (1.49)	-0.787 (-5.57)	0.50		
x = 5	-3.076 (-4.51)	4.730 (5.40)	0.632 (1.86)	-0.729 (-2.63)	0.62	-3.047 (-5.85)	4.746 (5.47)	0.714 (2.75)	-0.752 (-4.35)	0.62		
Panel B. Demographic Model												
$K_{t,p} = e^x MY_t$												
	α^x	b^x	c^x	d^x	e^x	R^2	α^x	b^x	c^x	d^x	e^x	R^2
	(t-stat)	(t-stat)	(t-stat)	(t-stat)	(t-stat)		(t-stat)	(t-stat)	(t-stat)	(t-stat)	(t-stat)	
	June,1958-December 2004					1958Q2-2008Q4						
x = 2	7.388 (4.08)	-0.335 (-1.39)	-0.475 (-4.88)	0.109 (3.23)	0.25	7.576 (4.37)	-0.255 (-1.00)	-0.493 (-5.51)	0.109 (3.41)	0.24		
x = 3	14.261 (4.63)	-0.440 (-1.26)	-0.938 (-6.43)	0.106 (3.83)	0.47	14.617 (4.73)	-0.375 (-1.11)	-0.974 (-6.42)	0.104 (3.97)	0.47		
x = 4	16.947 (4.51)	-0.142 (-0.53)	-1.117 (-5.99)	0.108 (3.93)	0.56	17.004 (4.61)	-0.003 (-0.01)	-1.138 (-5.83)	0.106 (4.12)	0.58		
x = 5	19.375 (4.79)	-0.134 (-0.70)	-1.295 (-6.39)	0.106 (4.17)	0.64	19.539 (4.94)	-0.127 (-0.69)	-1.320 (-6.50)	0.103 (4.32)	0.64		

Table 1. Fama Regressions. $r(t)$ is the one-year spot rate observed at time t . $f(x;t)$ is the forward rate observed at t for the year from $t+x-1$ to $t+x$. D is a dummy variable that is 1 for June 1958 to August 1981. $K(t)$ is the average value of the spot rate for the 60 months ending in month $t-1$. In the left panel we replicate Fama's results using the same monthly sample, June 1958- December 2004. The right panel updates the sample up to December 2008 using quarterly data. The standard errors of the regression coefficients are adjusted for autocorrelation due to overlap of monthly observations with the method of Hansen and Hodrick (1980). The t-statistics are reported in parentheses. The regression R^2 are adjusted for degrees of freedom.

TABLE 2. Predictive Performance (Pseudo) Out-of-sample 1994Q1-2008Q4

TABLE 2. Predictive Performance (Pseudo) Out-of-sample 1994Q1-2008Q4								
FAMA					MY			
Panel A	Ros^2	MAE	$FRMSE$	DM	Ros^2	MAE	$FRMSE$	DM
$x = 2$	-0.4383	1.6004	1.8853	-0.0242	-0.5623	1.6352	1.9649	-0.0490
$x = 3$	-0.2019	2.1945	2.5085	-0.0017	-0.1751	2.2370	2.4804	-0.0002
$x = 4$	-0.0531	2.4416	2.7362	0.0010	0.1115	2.1563	2.5133	0.0024
$x = 5$	0.2303	1.9553	2.4541	0.0029	0.3102	1.8226	2.3234	0.0035
MAE_{MY}/MAE_{FAMA}				$RMSE_{MY}/RMSE_{FAMA}$				
Panel B	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 2$	$x = 3$	$x = 4$	$x = 5$
	1.0217 (0.003)	1.0194 (0.074)	0.8832 (0.133)	0.9321 (0.002)	1.0422 (0.073)	0.9888 (0.042)	0.9186 (0.029)	0.9467 (0.000)

Table 2 presents statistics on x-year ahead out-of-sample forecast errors for changes in spot rates. The first column lists the out-of-sample R_{OS}^2 (Campbell and Thomson, 2008) which compares the forecast error of the historical mean with the forecast from predictive regressions. FRMSE is the forecast root mean square error, MAE is the mean absolute error. DM is the Diebold and Mariano (1995) t-statistic for difference in MSE of the unconditional forecast and the conditional forecast. Panel B reports the ratio of mean absolute errors and forecast root mean square errors between two models. A ratio less than 1 indicates that the model using demographic information performs better (highlighted in the table) in forecasting relative to Fama's model. We report in parentheses the p-values of the forecasting test due to Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level. The sample starts in 1958Q2 and we construct the first forecast in 1994Q1. Forecast performance is evaluated over the period 1994Q1-2008Q4.

TABLE 3. Summary Statistics of the Data in Affine Term Structure Models

	Central Moments				Autocorrelations		
	mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
3-month	5.850	2.778	1.061	4.775	0.917	0.866	0.843
1-year	6.282	2.748	0.817	4.107	0.925	0.881	0.841
2-year	6.483	2.698	0.853	3.980	0.935	0.891	0.852
3-year	6.643	2.615	0.876	3.915	0.941	0.899	0.860
4-year	6.774	2.571	0.928	3.950	0.945	0.904	0.865
5-year	6.851	2.521	0.910	3.680	0.949	0.910	0.875
inflation factor	0.116	0.922	2.149	16.463	0.337	0.100	0.170
real factor	-0.026	0.965	1.225	6.631	0.681	0.450	0.248
middle-young	0.836	0.211	0.102	1.500	0.995	0.989	0.981

Table 3. Summary Statistics of data. We report the mean, standard deviation, skewness and kurtosis together with the autocorrelation coefficients up to 3 lags. The 1, 4, 8, 12, 16, 20 quarter yields are annualized (in percentage) zero coupon bond yields from the Fama-Bliss CRSP bond files. Inflation and real activity refer to the price and output factors extracted from large dataset provided by Ludvigson and Ng (2009). Sample 1964Q1-2007Q4. Quarterly data.

TABLE 4. The Short-rate and Demographic Variable				
	$y_{t+4x,t+4x+4}^{3m}$	$-y_{t,t+4}^{3m}$	$=a^x+b^x[y_{t,t+4}^{3m}-K_{t,t+4}]+\varepsilon_{t+4x}$	$K_{t,t+4}=c^xMY_t$
	a^x	b^x	c^x	R^2
	(t-stat)	(t-stat)	(t-stat)	
Panel A. Sample 1958Q2-2008Q4				
$x = 2$	6.451 (3.72)	-0.451 (-4.50)	0.105 (2.93)	0.23
$x = 3$	12.573 (5.05)	-0.888 (-6.78)	0.102 (3.86)	0.45
$x = 4$	16.501 (5.06)	-1.167 (-7.02)	0.102 (4.11)	0.59
$x = 5$	17.895 (4.64)	-1.280 (-6.73)	0.100 (3.92)	0.64
Panel B. Sample 1971Q1-2008Q4				
$x = 2$	6.154 (3.48)	-0.433 (-3.95)	0.102 (4.08)	0.21
$x = 3$	12.555 (5.08)	-0.897 (-6.62)	0.098 (4.08)	0.45
$x = 4$	16.840 (5.14)	-1.204 (-6.89)	0.098 (4.08)	0.61
$x = 5$	18.379 (4.61)	-1.326 (-6.32)	0.098 (4.08)	0.66
Panel C. Sample 1982Q1-2008Q4				
$x = 2$	5.941 (2.45)	-0.538 (-4.61)	0.070 (1.85)	0.35
$x = 3$	10.700 (3.59)	-0.923 (-5.64)	0.074 (2.67)	0.56
$x = 4$	12.906 (3.91)	-1.126 (-5.22)	0.074 (3.02)	0.68
$x = 5$	13.147 (4.60)	-1.205 (-8.10)	0.069 (3.34)	0.75

Table 4. We regress three-month T-bill, y_t^{3m} , on a constant and deviations of the short rate from its long-run mean captured by middle-aged to young ratio. In each panel, we provide evidence for the longest sample (post-war period) and other sub-samples representing different monetary policy regimes. We report the Newey-West corrected t-statistics in parentheses. Quarterly data.

TABLE 5a. Model Estimates (constant risk price). Sample 1969Q1-2007Q4.

Macro Model with MY _t						Macro Model				
	π_t	g_t	$f_{1,t}^u$	$f_{2,t}^u$	$f_{3,t}^u$	π_t	g_t	$f_{1,t}^u$	$f_{2,t}^u$	$f_{3,t}^u$
π_{t-1}	0.286 (0.068)	-0.037 (0.046)	-0.079 (0.056)	-0.019 (0.164)	0.068 (0.034)	0.270 (0.070)	-0.020 (0.098)	-0.073 (0.042)	0.042 (0.068)	0.055 (0.042)
g_{t-1}	0.190 (0.085)	0.531 (0.120)	-0.117 (0.042)	-0.020 (0.149)	0.020 (0.048)	0.162 (0.112)	0.588 (0.095)	-0.117 (0.036)	0.059 (0.049)	-0.019 (0.054)
$f_{1,t-1}^u$	0.269 (0.140)	0.419 (0.215)	0.832 (0.614)	0.140 (0.286)	-0.194 (0.088)	0.292 (0.149)	0.397 (0.096)	0.977 (0.055)	-0.064 (0.063)	-0.128 (0.082)
$f_{2,t-1}^u$	-0.000 (0.167)	0.187 (0.356)	0.300 (0.251)	0.789 (0.357)	0.110 (0.397)	-0.005 (0.265)	0.160 (0.121)	0.116 (0.119)	0.862 (0.156)	0.164 (0.131)
$f_{3,t-1}^u$	0.186 (0.167)	0.141 (0.070)	-0.143 (0.302)	0.042 (0.285)	0.857 (0.311)	0.223 (0.158)	0.141 (0.169)	-0.222 (0.095)	0.111 (0.142)	0.636 (0.150)
$y_{t,t+1}$	$y_{t,t+1} = \delta_0 + \boldsymbol{\delta}'_1 X_t + \delta_2 MY_t$					$y_{t,t+1} = \delta_0 + \boldsymbol{\delta}'_1 X_t$				
δ_1	-0.156 (0.087)	-0.519 (0.226)	2.621 (1.271)	1.039 (2.890)	0.000 (0.000)	-0.141 (0.103)	-0.543 (0.221)	2.815 (0.397)	0.000 (0.000)	0.000 (0.000)
δ_2			-0.021 (0.004)							

Table 5a. Parameters estimates. This table reports the parameter estimates of maximum likelihood estimation of the no-arbitrage models, including macro factors (Macro Model) and middle-aged-young ratio (Macro Model with MY_t). We report the standard errors in parentheses. The model is estimated assuming constant risk price $\lambda_1 = 0$, i.e. Sample 1969Q1-2007Q4.

TABLE 5b. Model Estimates (time-varying risk price). Sample 1969Q1-2007Q4.

Macro Model with MY_t						Macro Model				
	π_t	g_t	$f_{1,t}^u$	$f_{2,t}^u$	$f_{3,t}^u$	π_t	g_t	$f_{1,t}^u$	$f_{2,t}^u$	$f_{3,t}^u$
π_{t-1}	0.222 (0.090)	-0.118 (0.080)	-0.043 (0.050)	0.007 (0.092)	-0.038 (0.157)	0.213 (0.937)	-0.095 (1.796)	-0.044 (0.072)	0.011 (1.032)	-0.049 (0.533)
g_{t-1}	0.207 (0.086)	0.521 (0.111)	-0.102 (0.115)	0.021 (0.063)	-0.054 (0.364)	0.179 (1.850)	0.583 (0.327)	-0.083 (0.572)	0.039 (1.348)	-0.105 (0.188)
$f_{1,t-1}^u$	0.223 (0.115)	0.500 (0.129)	0.907 (0.239)	-0.083 (0.335)	0.063 (0.864)	-0.010 (2.285)	-0.047 (1.340)	0.962 (0.175)	0.002 (1.264)	0.027 (0.579)
$f_{2,t-1}^u$	0.186 (0.363)	0.114 (0.374)	-0.103 (0.435)	0.766 (1.229)	-0.153 (0.699)	-0.212 (0.358)	-0.018 (4.459)	0.010 (1.490)	0.554 (0.232)	0.258 (1.372)
$f_{3,t-1}^u$	0.144 (0.602)	0.116 (0.446)	-0.091 (0.402)	-0.234 (0.476)	0.679 (1.243)	0.316 (0.247)	0.482 (2.200)	0.137 (0.392)	0.081 (2.002)	0.805 (0.418)
$y_{t,t+1}$	$y_{t,t+1} = \delta_0 + \delta_1' X_t + \delta_2 MY_t$					$y_{t,t+1} = \delta_0 + \delta_1' X_t$				
δ_1	-0.240 (0.109)	-0.542 (0.602)	2.565 (1.057)	0.000 (0.000)	0.901 (2.562)	-0.255 (2.473)	-0.569 (1.323)	0.981 (1.948)	0.000 (0.000)	2.520 (1.129)
δ_2			-0.022 (0.017)							

Table 5b. Parameters estimates. This table reports the parameter estimates of maximum likelihood estimation of the no-arbitrage models, including macro factors (Macro Model) and middle-aged-young ratio (Macro Model with MY_t). We report the standard errors in parentheses. The model is estimated assuming constant risk price $\lambda_1 \neq 0$, i.e. Sample 1969Q1-2007Q4.

TABLE 6. Yield Forecasts (constant risk price)						
Panel A. FRMSE (Random-walk)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.1408 (0.001)	0.9514 (0.000)	0.9403 (0.000)	<u>0.8344</u> (0.000)	<u>0.8020</u> (0.000)	<u>0.8579</u> (0.000)
$y_{t,t+4}$	1.1602 (0.000)	1.0342 (0.008)	0.9391 (0.001)	<u>0.8321</u> (0.006)	<u>0.8125</u> (0.001)	<u>0.8852</u> (0.005)
$y_{t,t+8}$	1.1126 (0.000)	1.0479 (0.062)	0.9474 (0.003)	<u>0.8576</u> (0.010)	<u>0.8623</u> (0.003)	0.9573 (0.015)
$y_{t,t+12}$	1.0799 (0.000)	1.0397 (0.055)	0.9531 (0.464)	<u>0.8812</u> (0.039)	0.9002 (0.000)	1.0122 (0.033)
$y_{t,t+16}$	1.0924 (0.000)	1.0392 (0.015)	0.9641 (0.198)	0.9108 (0.350)	0.9395 (0.022)	1.0559 (0.000)
$y_{t,t+20}$	1.0986 (0.000)	1.0405 (0.004)	0.9822 (0.052)	0.9437 (0.074)	0.9808 (0.268)	1.0997 (0.024)
Panel B. FRMSE (Yield-Macro Model)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.2734 (0.005)	1.0230 (0.120)	1.0167 (0.261)	1.0241 (0.335)	1.0342 (0.380)	1.0467 (0.383)
$y_{t,t+4}$	1.1618 (0.001)	1.0148 (0.025)	1.0071 (0.049)	1.0107 (0.038)	1.0194 (0.040)	1.0265 (0.021)
$y_{t,t+8}$	1.1024 (0.000)	1.0083 (0.000)	1.0016 (0.000)	1.0029 (0.000)	1.0132 (0.000)	1.0189 (0.000)
$y_{t,t+12}$	1.0896 (0.000)	1.0034 (0.008)	0.9936 (0.018)	0.9939 (0.035)	1.0031 (0.006)	1.0067 (0.001)
$y_{t,t+16}$	1.0908 (0.002)	0.9980 (0.011)	0.9848 (0.000)	0.9848 (0.004)	0.9907 (0.008)	0.9899 (0.000)
$y_{t,t+20}$	1.0863 (0.000)	0.9884 (0.000)	0.9721 (0.001)	0.9737 (0.000)	0.9750 (0.000)	0.9703 (0.000)

Table 6. Out-of Sample Yield Forecasts ($\lambda_1 = 0$). h indicates 1, 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model to the RMSFE of a random walk forecast and the benchmark yield-macro model. We show the comparison of forecasting results from different models in Table 3. The table shows better forecasts with respect to the random walk and yield-macro model with bold characters for the range of $[0.9, 1)$ and with added underline for ratios smaller than 0.9. We report in parentheses the p-values of the forecasting test due to Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level. Sample 1964Q1-2007Q4.

TABLE 7. Yield Forecasts (time-varying risk price)

Panel A. FRMSE (Random-walk)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.3641 (0.214)	0.9951 (0.000)	0.8620 (0.000)	0.7711 (0.000)	0.7476 (0.000)	0.8027 (0.000)
$y_{t,t+4}$	1.3681 (0.000)	1.0758 (0.032)	0.8936 (0.000)	0.8020 (0.006)	0.7863 (0.001)	0.8669 (0.001)
$y_{t,t+8}$	1.2335 (0.000)	1.0737 (0.154)	0.9251 (0.004)	0.8550 (0.011)	0.8543 (0.010)	0.9646 (0.000)
$y_{t,t+12}$	1.1409 (0.000)	1.0499 (0.049)	0.9452 (0.441)	0.8912 (0.057)	0.9005 (0.010)	1.0264 (0.039)
$y_{t,t+16}$	1.1290 (0.000)	1.0385 (0.015)	0.9710 (0.184)	0.9376 (0.430)	0.9487 (0.000)	1.0805 (0.023)
$y_{t,t+20}$	1.1251 (0.000)	1.0251 (0.005)	0.9935 (0.063)	0.9725 (0.106)	0.9836 (0.229)	1.1151 (0.02)
Panel B. FRMSE (Yield-Macro Model)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.0909 (0.003)	0.8574 (0.046)	0.8261 (0.079)	0.8280 (0.076)	0.8017 (0.102)	0.7443 (0.056)
$y_{t,t+4}$	1.0193 (0.001)	0.8674 (0.027)	0.8428 (0.047)	0.8411 (0.064)	0.8090 (0.054)	0.7568 (0.024)
$y_{t,t+8}$	1.0022 (0.000)	0.8677 (0.005)	0.8442 (0.007)	0.8442 (0.009)	0.8094 (0.008)	0.7614 (0.011)
$y_{t,t+12}$	1.0075 (0.000)	0.8683 (0.000)	0.8413 (0.000)	0.8401 (0.000)	0.8028 (0.000)	0.7571 (0.000)
$y_{t,t+16}$	1.0099 (0.000)	0.8645 (0.000)	0.8342 (0.001)	0.8304 (0.001)	0.7904 (0.000)	0.7465 (0.000)
$y_{t,t+20}$	1.0069 (0.000)	0.8594 (0.000)	0.8225 (0.006)	0.8180 (0.010)	0.7740 (0.000)	0.7324 (0.000)

Table 6. Out-of Sample Yield Forecasts ($\lambda_1 \neq 0$). h indicates 1, 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model to the RMSFE of a random walk forecast and the benchmark yield-macro model. We show the comparison of forecasting results from different models in Table 3. The table shows better forecasts with respect to the random walk and yield-macro model with bold characters for the range of [0.9, 1,) and with added underline for ratios smaller than 0.9. We report in parentheses the p-values of the forecasting test due to Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level. Sample 1964Q1-2007Q4.

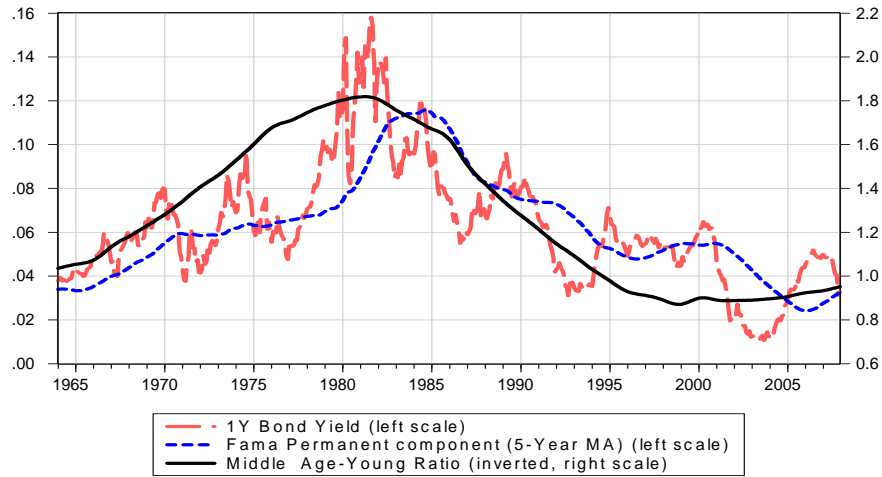


Figure 1. Permanent Component of the US Spot Rate. The figure plots Nominal U.S Treasury 1-Year Bond Yields, Fama's proxy for the permanent component of the US spot rate, $\bar{K}_{t,t+12}$ which is measured as the moving average of the most recent past five years of the spot rates and inverted Middle-aged to Young population ratio. Sample 1964-2007.

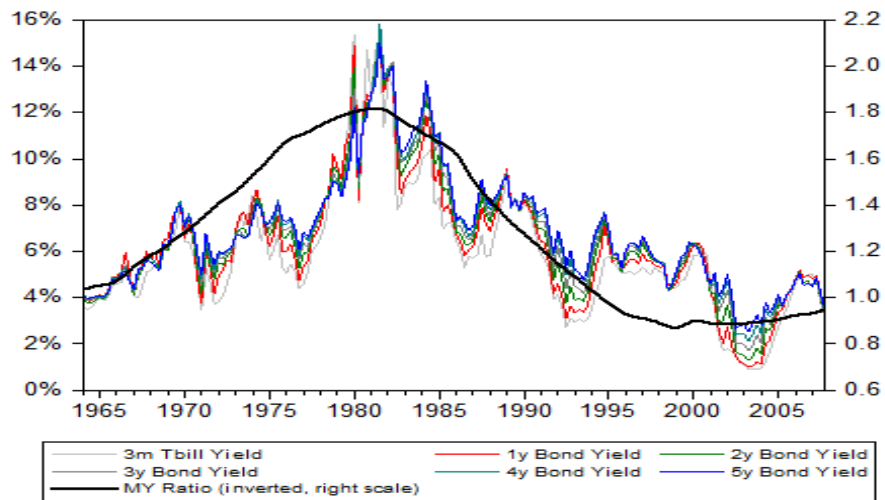


Figure 2. Nominal U.S Treasury Bond Yields and MY Ratio. The figure plots the quarterly (annualized) zero coupon bond yields of maturity 3-month, 1-year, 2-year, 3-year, 4-year and 5-year and inverted Middle-aged to Young population ratio. Sample 1964-2007.

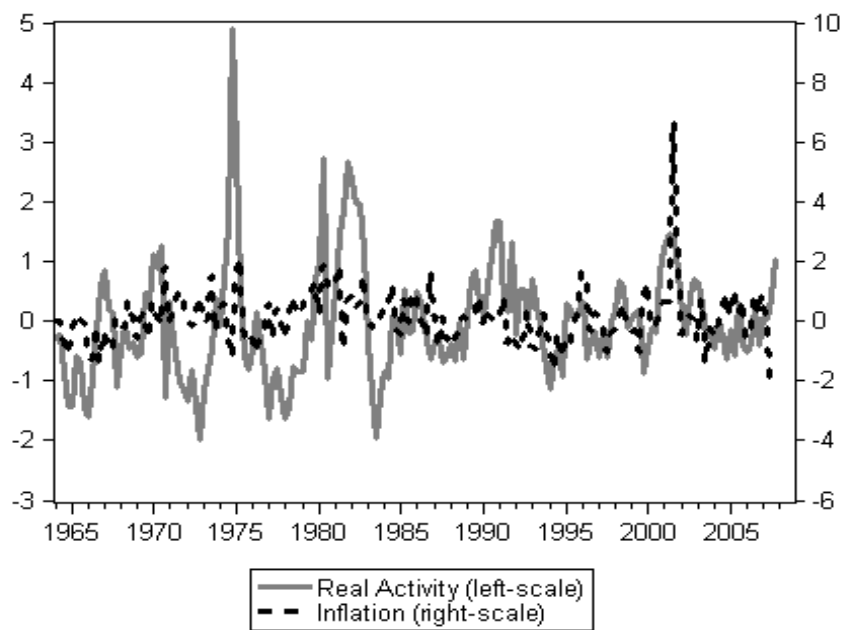


Figure 3. Nominal and Real factors. These are the factors extracted from large dataset provided by Ludvigson and Ng (2009). Sample 1964Q1-2007Q4.

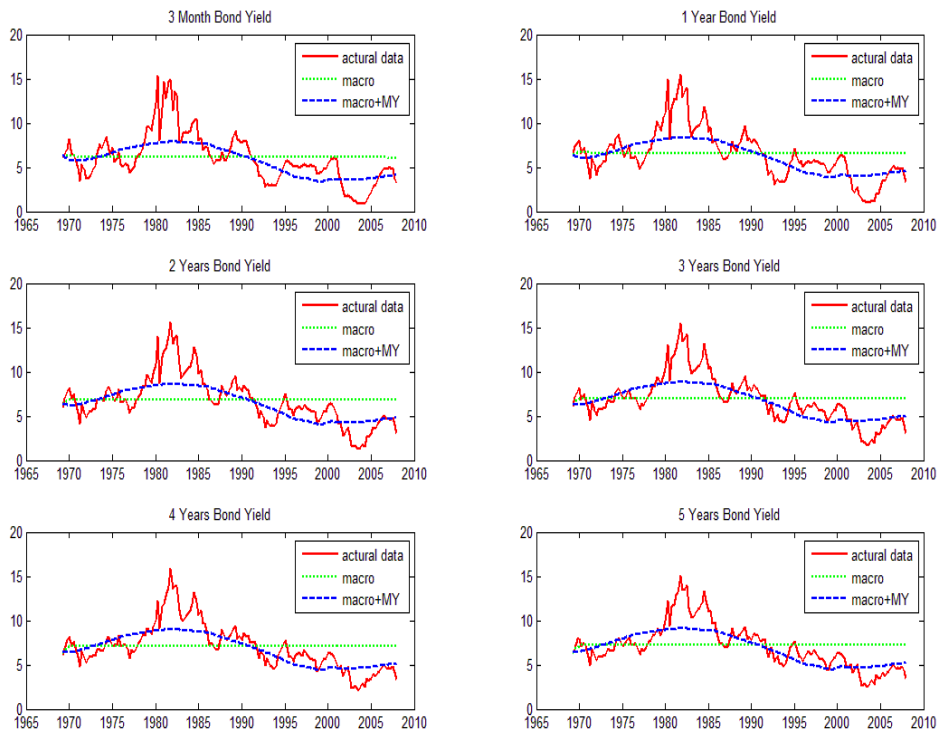


Figure 4. Dynamic Simulations. This figure plots the historical time series for bond yields (maturity: 3m, 1y, 2y, 3y, 4y, 5y) alongwith those dynamically simulated from the benchmark affine model with macro factors (dotted green line) and that augmented with demographics (dashed blue line). The no-arbitrage term structure models are estimated over the whole sample 1969Q1-2007q4. Using the estimated model parameters, models are solved dynamically forward from 1968Q4.

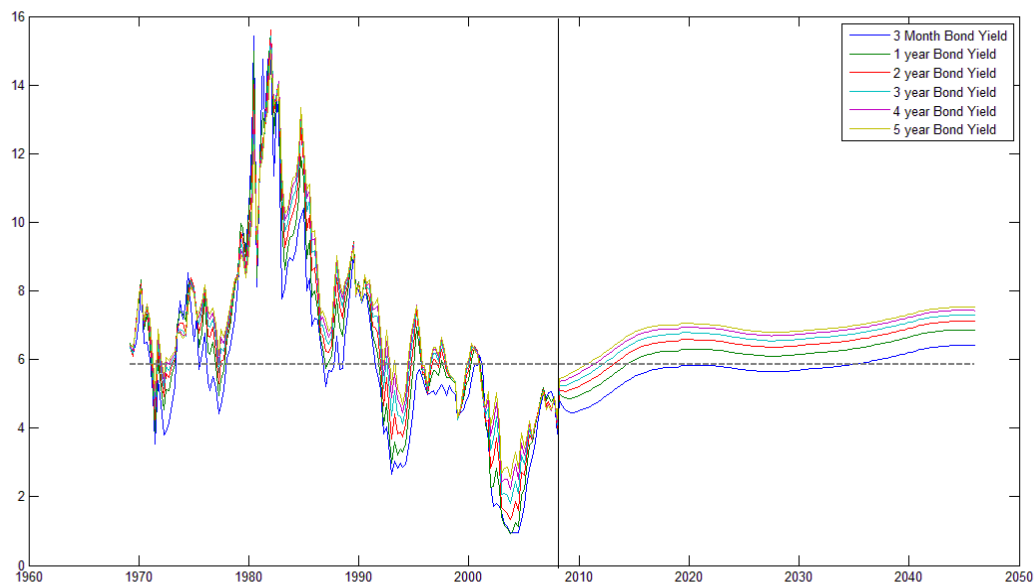


Figure 5. In Sample Estimation and out of Sample Prediction. Here we use no arbitrage term structure model with demographics. This figure plots the in sample estimated value 1969Q1-2007Q4 and out-of-sample prediction path 2008Q1-2045Q4 with all maturities: 3m, 1y, 2y, 3y, 4y, 5y. The no-arbitrage term structure models are estimated over the whole sample 1969Q1-2007q4. Using the estimated model parameters, models are solved dynamically forward from 1968Q4. Meanwhile the grey dash line is in sample average of short rate (3 month), and the grey solid line shows end of in sample estimation.