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Abstract

The effects of public debt and redistribution are intimately related. We illustrate this in a model with heterogenous agents and imperfect credit markets. Our setup differs from the classic Savers-Spenders model of fiscal policy in that all agents engage in intertemporal optimization, but a fraction of them is subject to a borrowing limit. We show that, despite the credit frictions, Ricardian equivalence holds under flexible prices if the steady-state distribution of wealth is degenerate: income effects on labor supply deriving from a tax redistribution are entirely symmetric across agents. When the distribution of wealth is non-degenerate, a tax cut is, somewhat paradoxically, contractionary. Conversely, sticky prices generate empirically plausible deviations from Ricardian equivalence, even in the case of degenerate wealth distribution. A revenue-neutral redistribution from unconstrained to constrained agents is expansionary, while debt-financed tax cuts have effects that go beyond their redistributional component: the present-value multiplier of a tax cut is positive due to an interplay of intertemporal substitution by those who hold the public debt and income effects on those who do not.

"If fiscal policy is used as a deliberate instrument for the more equal distribution of incomes, its effect in increasing the propensity to consume is, of course, all the greater." (Keynes, 1936, Book III, Chapter 8, Section II).

1 Introduction

The aftermath of the Great Recession has revived a classic debate on the effects of so-called fiscal stimulus programs. This debate has often focused on the role of government debt. Less prominent in the debate is the fact that the rise in public debt in many countries has ensued from stimulus packages that have taken the form of transfers to specific income groups, rather than purchases of goods and services (see, e.g., Oh and Reis, 2011; Giambattista and Pennings, 2011; Mehrotra, 2011). This suggests that redistributional issues might be of primary importance when assessing both the size of the tax/transfers multipliers and the desirability of the upward trajectory of debt.

In this paper we study fiscal stimulus policies in the form of temporary tax cuts. We interpret redistribution as *revenue-neutral* tax cuts to a fraction of the population financed by a tax rise to another; we interpret public debt as a form of *intertemporal* redistribution.

We conduct our analysis in a framework with heterogenous agents and imperfect financial markets. Our setup resembles the classic *Savers-Spenders* (SS henceforth) model of fiscal policy (Mankiw 2000) in which "myopic" household, who merely consume their income, coexist with standard, intertemporally optimizing households. That model has been extended by, among others, Galí et al. (2004, 2007) and Bilbiie (2008) to include nominal rigidities and other frictions in order to study questions ranging from the effects of government spending to monetary policy analysis and equilibrium determinacy.[1]

Ours is a variant of these models in two respects: first, *both* agents are intertemporal maximizers—so that borrowing and lending take place in equilibrium—but a fraction of agents face a suitably defined borrowing limit; second, the *distribution* of debt/saving

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[1]See also Bilbiie and Straub (2004), Bilbiie, Meier and Mueller (2008) and Lopez-Salido and Rabanal (forthcoming) for analyses of fiscal policy issues in models with savers and spenders.

across agents is endogenous. In some respect, our model can be thought of as a simplified version of classic equilibrium models with incomplete markets, such as Bewley (1980), Aiyagari (1993), and Hugget (1998). The main difference is that we add New Keynesian features such as imperfectly competitive goods markets and price rigidity.[2]

Since our model features credit market imperfections, it is tempting to think that Ricardian equivalence readily fails, so that (lump-sum) tax cuts produce positive (and possibly large) effects on aggregate demand. We first show that this reasoning can be misleading, because the conclusion hinges on two crucial elements: (i) whether or not the steady-state distribution of wealth across agents is degenerate; (ii) whether or not labor supply is endogenous.

In fact, the baseline version of our model with perfectly flexible prices produces two paradoxical results. First, and despite the presence of borrowing frictions, a tax redistribution that favors the constrained agents (a tax cut to the borrowers financed by a rise in taxes to the savers) is completely neutral on *aggregate* consumption if the steady-state distribution of wealth is degenerate (e.g., if the borrowing limit is zero—as implicit in the traditional SS model—and profit income is either zero or redistributed uniformly across agents).[3]

Second, even if the distribution of wealth is not degenerate (so that, e.g., a fraction of agents hold private debt and another fraction a corresponding amount of savings), a tax redistribution generates a *contraction* in aggregate spending, in stark contrast with the traditional SS model.

The intuition for these results is that the steady-state distribution of wealth and the (intensity of the) income effect on labor supply are intimately linked in our setup. When

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[2]A key difference is that who borrows and lends is predetermined by preferences in our model whereas it is determined by idiosyncratic uncertainty in the Bewley-Ayagari-Hugget setup. Recent examples of models along the lines of the current paper are Monacelli and Perotti (2011) and Eggertson and Krugman (2011). Curdia and Woodford (2009) allow agents to differ in their impatience to consume, but (differently from our framework) limit the ability to borrow by assuming that agents can have access to financial markets (in the form of purchase of state contingent securities) only randomly.

[3]Throughout the paper, we abstract from the accumulation of physical capital in order to focus on one source of failure of Ricardian equivalence: sticky prices. We hint to some of the possible implications of capital accumulation in the concluding section.

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steady-state consumption levels are equalized, the income effects on the agents' individual labor supplies are symmetric. In response to a tax redistribution, borrowers choose to work less and savers to work more in an exactly offsetting way. When the distribution of wealth is such that, realistically, borrowing-constrained agents consume relatively less in steady state, their reduction in labor supply more than compensates the increase in labor supply by the savers, leading to an overall contraction in spending and output.

This result does not hinge upon the tax cut being revenue-neutral. To understand this, consider the opposite (extreme) case of a uniform tax cut today financed by issuing public debt (held by the savers), which is then fully repaid in the following period by uniform taxation. *De facto*, through public-debt market clearing, this amounts to redistributing from savers to borrowers today, and reversing that redistribution in the next period (when debt is repaid). Within each period, the same logic of redistribution described above applies, so that either the redistribution is neutral, or it generates paradoxical results: the tax cut today is contractionary and the tax increase tomorrow is expansionary, in an *exactly* symmetric way. In this vein, under flexible prices, a *long-run* version the Ricardian equivalence still holds, although *short-run* Ricardian equivalence fails.

Matters are different with nominal price rigidity, and even in the case of a degenerate steady-state distribution of wealth. Two elements are typical of the sticky-price environment. First, as firms cannot optimally adjust prices, the increase in borrowers' consumption ensuing from the tax cut generates an increase in labor demand. Second, the rise in the real wage that results from the expansion in labor demand generates, for one, a further income effect on borrowers and hence a further expansion in their consumption; and also a fall in profits, with an additional negative income effect on the savers' labor supply, that is absent under flexible prices.

In this scenario, we obtain two results. First, a revenue-neutral tax redistribution is expansionary on aggregate spending, as well as inflationary. Second, a debt-financed uniform tax cut (with debt fully repaid in the following period) generates a current expansion in aggregate spending, followed by a contraction. Crucially, however, the two effects are not symmetric: the *present-value* multiplier of debt-financed tax cut is positive. In other

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words, a tax cut financed by issuing debt generates effects that are over and above the mere sum of its distributional effects.

We show that the additional effect of public debt stems from the interplay of two forces: first, intertemporal substitution by the agents who hold that debt (the savers); second, an income effect on the agents who do not (the borrowers). Finally, we show that when the tax cut is financed by debt that is not immediately repaid, it has effects that feature endogenous persistence: the recession and deflation stemming from the increase in taxes—necessary to repay public debt—are long-lived.

Our paper adds to a very large literature studying the effects of public debt (see Elmendorf and Mankiw, 1998 for a survey). An additional novel element of our analysis is that it relates the study of the effects of public debt to redistribution through fiscal policy. The latter issue is also a classic one, whose origins can be traced at least back to Keynes, who argued in the *General Theory* that redistribution through fiscal policy is one of the main determinants—at least of equal importance to interest rates— of the marginal propensity to consume, which was in turn the key determinant of fiscal policy multipliers.[4]

2 The model

Households

There is a continuum of households $[0,1]$ indexed by j, all having the same utility function

$$U\left(C_{j,t}, N_{j,t}\right) = \ln C_{j,t} - \chi_j N_{j,t}^{1+\varphi} / \left(1 + \varphi\right)$$

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[4] "The principal objective factors which influence the propensity to consume appear to be the following: [...] 5. Changes in fiscal policy. In so far as the inducement to the individual to save depends on the future return which he expects, it clearly depends [...] on the fiscal policy of the Government. Income taxes, especially when they discriminate against "unearned" income [...] are as relevant as the rate of interest; whilst the range of possible changes in fiscal policy may be greater, in expectation at least, than for the rate of interest itself. If fiscal policy is used as a deliberate instrument for the more equal distribution of incomes, its effect in increasing the propensity to consume is, of course, all the greater." (Keynes, 1936, Book III, Chapter 8, Section II).

where $\varphi > 0$ is the inverse of the labor supply elasticity. The agents differ in their discount factors $\beta_j \in (0,1)$ and possibly in their preference for leisure χ_j. Specifically, we assume that there are two types of agents $j = s, b$, and

$$\beta_s > \beta_b$$

All households (regardless of their discount factor) consume an aggregate basket of individual goods $z \in [0,1]$, with constant elasticity of substitution ε: $C_t = \left(\int_0^1 C_t(z)^{(\varepsilon-1)/\varepsilon} \, dz \right)^{\varepsilon/(\varepsilon-1)}$, $\varepsilon > 1$. Standard demand theory implies that total demand for each good is $C_t(z) = (P_t(z)/P_t)^{-\varepsilon} C_t$, where $C_t(z)$ is total demand of good z, $P_t(z)/P_t$ its relative price and C_t aggregate consumption.[5] The aggregate price index is $P_t^{1-\varepsilon} = \int_0^1 P_t(z)^{1-\varepsilon} \, dz$.

A $1 - \lambda$ share is represented by households who are patient: we label them *savers*, discounting the future at β_s. Consistent with the equilibrium outcome (discussed below) that patient agents are savers (and hence will hold the bonds issued by impatient agents), we impose that patient agents also hold all the shares in firms.

Each saver chooses consumption, hours worked and asset holdings (bonds and shares), solving the standard intertemporal problem:

$$\max \mathbb{E}_t \left\{ \sum_{i=0}^\infty \beta_s^i U(C_{s,t+i}, N_{s,t+i}) \right\}$$

subject to the sequence of constraints:

$$C_{s,t} + B_{s,t+1} + \frac{1 + I_{t-1}}{1 + \Pi_t} D_{s,t} + \Omega_{s,t+1} V_t \leq \frac{1 + I_{t-1}}{1 + \Pi_t} B_{s,t} + D_{s,t+1} + \Omega_{s,t}(V_t + \mathcal{P}_t) + W_t N_{s,t} - \tau_{s,t},$$

$$(1)$$

where $\mathbb{E}_t\{.\}$ is the expectations operator, $C_{s,t}, N_{s,t}$ are consumption and hours worked by the patient agent, W_t is the real wage, $D_{s,t}$ is the *real* value at beginning of period t of total borrowing outstanding in period t ($1 + \Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate), a portfolio of one-period bonds issued in $t-1$ on which the household pays nominal interest I_{t-1}. V_t is the real market value at time t of shares in intermediate good firms, $\mathcal{P}_t$ are real

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[5]This equation holds in aggregate because the same static problem is solved by both types of households.

dividend payoffs of these shares, $\Omega_{s,t}$ are share holdings, and $B_{s,t}$ the savers' holdings of nominal public bonds which deliver the same nominal interest as private bonds.

The intratemporal optimality conditions and Euler equations for bond and share holdings respectively are:[6]

$$C_{s,t}^{-1} = \beta_s \mathbb{E}_t \left(\frac{1 + I_t}{1 + \Pi_{t+1}} C_{s,t+1}^{-1} \right) \text{ and } V_t = \beta_s \mathbb{E}_t \left(\frac{C_{s,t}}{C_{s,t+1}} \frac{V_{t+1} + \mathcal{P}_{t+1}}{1 + \Pi_{t+1}} \right) \tag{2}$$

$$\chi N_{s,t}^{\varphi} = \frac{1}{C_{s,t}} W_t. \tag{3}$$

The rest of the households on the $[0, \lambda]$ interval are impatient (*and will borrow in equilibrium,* hence we index them by b for borrowers) face the intertemporal constraint:

$$C_{b,t} + \frac{1 + I_{t-1}}{1 + \Pi_t} D_{b,t} \leq D_{b,t+1} + W_t N_{b,t} - \tau_{b,t}, \tag{4}$$

as well as the additional borrowing constraint (on borrowing in real terms) at all times t :

$$D_{b,t+1} \leq \bar{D},$$

$$\chi N_{b,t}^{\varphi} = \frac{1}{C_{b,t}} W_t, \tag{5}$$

$$C_{b,t}^{-1} = \beta_b \mathbb{E}_t \left(\frac{1 + I_t}{1 + \Pi_{t+1}} C_{b,t+1}^{-1} \right) + \psi_t, \tag{6}$$

where ψ_t takes a positive value whenever the constraint is binding. Indeed, because of our assumption on the relative size of the discount factors, the borrowing constraint will bind in steady state (we discuss this in more detail below). For the remainder of the paper, we assume that aggregate shocks are small enough such that the constraint keeps binding out of (but still in the neighborhood of) the steady state.

Firms Each individual good is produced by a monopolistic competitive firm, indexed by z, using a technology given by: $Y_t(z) = N_t(z)$. Cost minimization taking the wage as given, implies that real marginal cost is W_t. The profit function in real terms is given

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[6]These conditions must hold along with the usual transversality conditions.

by: $\mathcal{P}_t(z) = [P_t(z)/P_t] Y_t(z) - W_t N_t(z)$, which aggregated over firms gives total profits $\mathcal{P}_t = [1 - W_t \Delta_t] Y_t$. The term Δ_t is relative price dispersion defined following Woodford (2003) as $\Delta_t \equiv \int_0^1 (P_t(z)/P_t)^{-\varepsilon} dz$.

Monetary authority   A monetary authority sets the nominal interest rate in response fluctuations in expected inflation (we assume for simplicity that target inflation is zero):

$$1 + I_t = \Phi(1 + \mathbb{E}_t \Pi_{t+1})$$

where $\Phi(1) = \beta_s^{-1} > 1$.

Government   The government issues B_{t+1} one-period bonds, which are held only by the savers. We assume, without loss of generality, that government spending is zero. Hence the government budget constraint reads:

$$B_{t+1} = \left(\frac{1 + I_{t-1}}{1 + \Pi_t} \right) B_t - \tau_t \tag{7}$$

where τ_t are *total* tax revenues, i.e., $\tau_t = \lambda \tau_{b,t} + (1 - \lambda) \tau_{s,t}$.

Notice that the assumption that government spending is fixed implies that exogenous variations in taxes will readily constitute a test of whether Ricardian Equivalence holds in our model.

Equilibrium   In an equilibrium of this economy, all agents take as given prices (with the exception of monopolists who reset their good's price in a given period), as well as the evolution of exogenous processes. A rational expectations equilibrium is then as usually a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for all agents and all markets clear at any given time t. Specifically, labor market clearing requires that labor demand equal total labor supply, $N_t = \lambda N_{b,t} + (1 - \lambda) N_{s,t}$. Private debt is in zero net supply $\int_0^1 D_{j,t+1} = 0$, and hence,

since agents of a certain type make symmetric decisions:

$$\lambda D_{b,t+1} + (1 - \lambda) D_{s,t+1} = 0$$

Equity market clearing implies that share holdings of each saver are:

$$\Omega_{s,t+1} = \Omega_{s,t} = \Omega = \frac{1}{1 - \lambda}.$$

Finally, by Walras' Law the goods market also clears. The resource constraint specifies that all produced output will be consumed:

$$C_t = Y_t = \frac{N_t}{\Delta_t} \tag{8}$$

where $C_t \equiv \lambda C_{b,t} + (1 - \lambda) C_{s,t}$ is aggregate consumption and Δ_t is relative-price dispersion.

All bonds issued by the government will be held by savers. Market clearing for public debt implies:

$$(1 - \lambda) B_{s,t+1} = B_{t+1} \tag{9}$$

To close the model we need to specify a law of adjustment for taxes in response to public debt, as well as the distribution of that adjustment between savers and borrowers. But before, it is worthwhile to emphasize that the mechanism of transmission of fiscal policy changes in this economy; in particular, we now show that fiscal policy (and, more specifically: public debt) can potentially play a role in this economy only through the lump-sum taxes on borrowers. In other words, we underline the key redistribution dimension of fiscal policy that is inherent in our model.

2.1 Redistribution and Public Debt

Substituting equations (7), (9)and the definition of total taxes in the savers' budget constraint , we obtain:

$$C_{s,t} + \frac{1 + I_{t-1}}{1 + \Pi_t} D_{s,t} + \Omega_{s,t+1} V_t \leq \frac{\lambda}{1 - \lambda} \tau_{b,t} + D_{s,t+1} + \Omega_{s,t} (V_t + \mathcal{P}_t) + W_t N_{s,t}. \tag{10}$$

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Equation (10) already shows that the only way in which fiscal policy matters in this model is through the impact of taxes on (transfers to) borrowers; this happens because savers internalize the government budget constraint through their public debt holdings, and so recognize that a transfer to borrowers today effectively implies a tax on themselves, today or in the future. More specifically, the higher the fraction of borrowers, the more sensitive the consumption of savers to a change in the tax on borrowers (*ceteris paribus*).

Notice also that no other equilibrium condition will be directly affected by lump-sum taxes on borrowers—but the tax process itself needs to be endogenous and respond to public debt in order to ensure sustainability. To close the model we need to specify how fast this adjustment takes place, and how the burden of readjustment is shared between savers and borrowers.

In the remainder of the paper, we will focus on two extreme scenarios that allow us to obtain analytical solutions:

1. *Pure redistribution.* In this scenario $\tau_{b,t}$ is exogenous, implying that public debt is irrelevant (savers, who hold all public debt, internalize the government budget constraint). Taxes on savers will need to adjust to ensure public debt sustainability, but this is irrelevant for the allocation. This experiment is equivalent to having a pure 'Robin Hood' policy that taxes savers and redistributes the proceeds to borrowers within the period.

2. *Uniform taxation with one-period debt repayment* ($\tau_{b,t} = \tau_{s,t} = \tau_t$). For instance, a uniform cut in taxes today, issuing debt, and fully repaying debt tomorrow by taxing both agents. This is equivalent to a transfer from savers to borrowers within the period, and a transfer (including interest payments) in the opposite direction next period. In this scenario the government "deleverages" within one period; the shock itself may be persistent, but this persistence is purely exogenous (i.e., not a function of outstanding debt).

Finally, we shall study a more general financing scheme that taken to either limit–no stabilization or perfect debt stabilization, respectively–would deliver the two extreme scenarios described above. More specifically, we will assume that taxes ($\tau_{b,t} = \tau_{s,t}$) will

increase to repay outstanding debt, but only gradually so. Public debt will then influence the endogenous persistence of the tax process, but it still matters for the aggregate allocation *only* through its impact on taxes on borrowers.

2.2 Steady state

A steady state is found by evaluating the optimality conditions in the absence of shocks and assuming that all variables are constant (and inflation is zero). Defining the steady-state net mark-up as $\mu \equiv (\varepsilon - 1)^{-1}$, the pricing condition of firms implies (regardless of whether prices are sticky or not) that the steady-state real wage is $W = 1/(1 + \mu) = 1 - \varepsilon^{-1}$, which is also the share of labor income in total income WN/Y. The share of profits in total output is: $\mathcal{P}_Y = \mu/(1 + \mu) = \varepsilon^{-1}$. We also assume that steady-state public debt is zero and (unless specified otherwise) that there is no steady-state redistribution: transfers are also zero.

Since the constraint binds in steady state ($\psi = C_b^{-1}[1 - (\beta_b/\beta_s)] > 0$ whenever $\beta_s > \beta_b$), patient agents are net borrowers and steady-state private debt is $D_b = \overline{D}$; by debt market clearing, then the patient agents are net lenders and $D_s = -\lambda \bar{D}/(1 - \lambda)$. The budget constraint in steady state for each group of agents is, respectively (R is the net real interest rate obtained from the Euler equation of savers $R = I = \beta_s^{-1} - 1$):

$$C_s = \frac{1}{1+\mu} N_s + \frac{1}{1-\lambda} \frac{\mu}{1+\mu} Y + \frac{\lambda}{1-\lambda} R\bar{D};$$

$$C_b = \frac{1}{1+\mu} N_b - R\bar{D}$$

These two equations, together with the intratemporal optimality conditions $\chi_s N_s^\varphi C_s = \chi_b N_b^\varphi C_b = (1 + \mu)^{-1}$ and $Y = C = N = \lambda N_b + (1 - \lambda) N_s = \lambda C_b + (1 - \lambda) C_s$ fully determine the steady-state of our model as a function of its deep parameters.

In the interest of analytical simplicity, we make the further assumption that agents work the same number of hours in steady state, regardless of whether they are borrowers/impatient or savers/patient: $N_b = N_s = N$. Besides making the model solution more transparent (while being innocuous for our conclusions), this assumption is consistent

with the view that there are no wealth effects on long-run hours worked. Specifically, the relative weight of leisure in the utility function needs to be different across agents, $\chi_s \neq \chi_b$ by precisely the amount needed to make (only) steady-state hours identical across groups, $N_b = N_s = N$ namely:

$$\chi_s = \frac{1}{N^{1+\varphi}\left(\frac{1}{1+\mu} + \frac{1}{1-\lambda}\frac{\mu}{1+\mu} + \frac{\lambda}{1-\lambda}R\bar{D}_Y\right)} < \chi_b = \frac{1}{N^{1+\varphi}\left(\frac{1}{1+\mu} - R\bar{D}_Y\right)}$$

The second equation determines N as a function of χ_b, and the first determines the χ_s that delivers the equalization of hours. Note that $\chi_s < \chi_b$ (to work the same steady-state hours, savers need to dislike labor less). The per-group shares of consumption in total consumption are

$$\frac{C_b}{C} = \frac{1}{1+\mu} - R\bar{D}_Y; \quad \frac{C_s}{C} = \frac{1}{1+\mu} + \frac{1}{1-\lambda}\frac{\mu}{1+\mu} + \frac{\lambda}{1-\lambda}R\bar{D}_Y$$

In the remainder, we use the notation

$$\frac{C_b}{C} \equiv \gamma = \frac{1}{1+\mu} - R\bar{D}_Y \leq 1$$
$$\frac{C_s}{C} = \frac{1-\lambda\gamma}{1-\lambda} \geq 1$$

Note for future use that $\gamma < 1$ if and only if either goods are imperfect substitutes (steady-state markup is positive) or the debt limit is zero.

We solve our model locally by loglinearizing it around this steady state; a small letter denotes log-deviations of a variable from its steady-state value, with two exceptions: taxes/transfers and public debt are in deviations from steady state, as a share of steady-state output Y $(t_{j,t} \equiv (\tau_{j,t} - \tau_j)/Y; \; b_t \equiv (B_t - B)/Y)$ and interest and inflation rates are in absolute deviations from their steady-state values. All loglinearized equilibrium conditions are outlined in Table 1. Notice that we specify a log-linear tax rule thereby taxes on each agent are allowed to react to stabilize government debt:

$$t_{j,t} = \phi_B^j b_t - \epsilon_{j,t} \quad (j = b, s) \tag{11}$$

where $\epsilon_{j,t}$ is a random (and possibly persistent) innovation, and $\phi_B^j \geq 0$ is the debt feedback coefficient.

Table 1. Summary of the Log-Linear Model

Euler equation, S	$\mathbb{E}_t c_{s,t+1} - c_{s,t} = i_t - \mathbb{E}_t \pi_{t+1}$
Labor supply, S	$\varphi n_{s,t} = w_t - c_{s,t}$
Labor supply, B	$\varphi n_{b,t} = w_t - c_{b,t}$
Budget constraint, B	$\gamma c_{b,t} + \bar{D}_Y (i_{t-1} - \pi_t) = \frac{WN_b}{C}(w_t + n_{b,t}) - t_{b,t}$
Production function	$y_t = n_t$
Phillips curve	$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} w_t$
Government debt	$\beta_s b_{t+1} = B_Y (i_{t-1} - \pi_t) + b_t - t_t$
Lump-sum taxes	$t_t = \lambda t_{b,t} + (1-\lambda) t_{s,t}$
Tax rule	$t_{j,t} = \phi_B^j b_t - \epsilon_{j,t}$, where $j = b, s$
Labor market clearing	$n_t = \lambda n_{b,t} + (1-\lambda) n_{s,t}$
Aggregate cons.	$c_t = \lambda\gamma c_{b,t} + (1-\lambda\gamma) c_{s,t}$
Resource constraint	$y_t = c_t$
Monetary policy	$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1}$

Note: Savers' budget constraint replaced with aggregate resource constraint (Walras' Law)

3 Flexible Prices and Ricardian Equivalence

We begin by assuming that prices are fully flexible. We show that, in an environment in which the steady-state levels of consumption of borrowers and savers are different, Ricardian equivalence fails: changes in lump-sum taxes affect the real allocation. However, the predictions concerning the effect of tax cuts (and interest rate cuts) are counterintuitive and contrary to empirical findings—which motivates our further analysis of other deviations from Ricardian equivalence.

Log-linearizing (4) and (5) around the steady state, and combining, we obtain:

$$n_{b,t} = \frac{\gamma(1+\mu)-1}{\varphi\gamma(1+\mu)+1} w_t + \frac{1+\mu}{\varphi\gamma(1+\mu)+1}\left(\bar{D}_Y r_{t-1} + t_{b,t}\right), \tag{12}$$

$$c_{b,t} = \frac{1+\varphi}{\varphi\gamma(1+\mu)+1} w_t - \frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu)+1}\left(\bar{D}_Y r_{t-1} + t_{b,t}\right), \tag{13}$$

where $r_{t-1} \equiv i_{t-1} - \pi_t$.

Starting from the steady state, and in response to an increase in taxation, borrowers' hours worked decrease (in equilibrium) with the real wage because of a positive income effect (which disappears when the debt limit is zero and $\gamma(1+\mu) = 1$), and increase with taxes and interest payments.

Let's denote with a star a variable under flexible prices. Notice that, under flexible prices and constant returns to scale in production, the marginal cost is constant, so $w_t^* = 0$. Hence (12) reduces to:

$$
\begin{aligned}
n_{b,t}^* &= \frac{(1+\mu)}{\varphi\gamma(1+\mu)+1}\left(\bar{D}_Y r_{t-1} + t_{b,t}\right), \\
c_{b,t}^* &= -\frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu)+1}\left(\bar{D}_Y r_{t-1} + t_{b,t}\right).
\end{aligned}
\tag{14}
$$

Replacing (14) into the aggregate consumption definition, solving for savers' consumption at flexible prices, and using (5), we obtain the following expression for *aggregate* consumption (output) under flexible prices:[7]

$$
c_t^* = \zeta\left(\bar{D}_Y r_{t-1} + t_{b,t}\right)
\tag{15}
$$

where

$$
\zeta \equiv \frac{\lambda(1-\gamma)}{1-\lambda+\varphi(1-\lambda\gamma)}\frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu)+1} \geq 0.
$$

Equation 15) contains a reduced form expression for aggregate consumption as a function of the exogenous tax process for borrowers, $t_{b,t}$, and the predetermined real interest rate, r_{t-1}. Direct inspection of (15) in the case $\gamma = 1$ (equal steady state consumption shares) suggests the following proposition:

Proposition 1 *When steady-state consumption of savers and borrowers are equal, Ricardian equivalence holds—regardless of how high the fraction of borrowers λ and how tight the debt constraint $\bar{D}_Y$ are.*

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[7]Using also aggregate hours and the equilibrium expression for the hours of borrower, as well as goods market clearing $c_t = n_t$.

The intuition for this result is simple: when steady-state consumption levels are equalized, income effects on agents' individual labor supplies (effects which are governed precisely by the steady-state consumption levels) are fully symmetric; to take one example that we elaborate on below: in response to an increase in their taxes $t_{b,t}$, borrowers want to work exactly as many hours more as savers are willing to work less when their taxes fall in order to balance the budget ($t_{s,t} = -\lambda (1-\lambda)^{-1} t_{b,t}$).

This symmetry breaks up when steady-state consumption levels are different. In the more general case $\gamma < 1$, three features of the solution are worth emphasizing and are developed below. First, Ricardian equivalence fails (and so does, incidentally, monetary neutrality): any given change in lump-sum taxes on borrowers produce an effect on aggregate consumption. Second, with $\zeta > 0$, the effect on aggregate spending is paradoxical: a rise (fall) in taxes generates a rise (fall) in consumption. Third, even when the debt limit is zero ($\bar{D}_Y = 0$), there is still steady-state consumption inequality and Ricardian Equivalence still fails. In order to better understand the effects of redistribution and public debt under flexible prices, consider in turn the two extreme fiscal policy experiments described above, assuming for simplicity that $\bar{D}_Y = 0$.

3.1 Pure redistribution ("Robin Hood")

In order to isolate the role of pure redistribution, we first study the effect of the first policy experiment outlined in Section 2.1 above: a transfer to borrowers financed by taxes on savers. By means of example, we assume that the transfer takes place *within the period*–so that the budget is balanced every period; but recall that the effects are exactly identical if the transfer is debt-financed, insofar as the tax process *for borrowers* remains strictly exogenous (i.e., it does not adjust endogenously in response to public debt accumulation, which obtains in the case $\phi_B = 0$)[8]:

$$t_{b,t} = -\epsilon_t, \ \ t_{s,t} = \frac{\lambda}{1-\lambda}\epsilon_t. \tag{16}$$

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[8]The tax process for savers would, of course, adjust in order to ensure that debt converges—but the path of taxation of savers is irrelevant for the allocation.

The multiplier of the tax cut on consumption is simply (replacing (16) into (15) with $\bar{D}_Y = 0$):

$$c_t^* = -\zeta \epsilon_t < 0.$$

Consumption, output, and hours worked fall (when $\gamma < 1$) because the negative income effect on savers is larger in absolute value than the positive income effect on borrowers.

3.2 Public debt

Next we assume that the fiscal policy experiment takes the form of a *uniform tax cut* of size ϵ_t^B to both agents, financed via public debt. To start with, we take the extreme scenario of full debt stabilization: debt is repaid entirely in the next period by taxing both agents. As we discussed in Section 2.1, this is equivalent to having a tax cut on borrowers, which is reversed next period: the borrowers need to repay it inclusive of interest payments on the debt used to finance it. Effectively, the government lends to borrowers. A tax rule that ensures debt stabilization within the period requires $\phi_B = 1$ in (11), i.e.:

$$t_t = b_t - \epsilon_t^B, \tag{17}$$

where ϵ_t^B is the size of the debt-financed uniform tax cut.

Combining this with the government budget constraint log-linearized around a steady state with zero public debt ($B_Y = 0$):

$$\beta_s b_{t+1} = b_t - t_t, \tag{18}$$

we obtain:

$$b_{t+1} = \beta_s^{-1} \epsilon_t^B \rightarrow t_{t+1} = \beta_s^{-1} \epsilon_t^B - \mathbb{E}_t \epsilon_{t+1}^B \tag{19}$$

If the shock has zero persistence, consumption is given by (assuming for simplicity that the initial level of public debt is zero)

$$c_t^* = -\zeta \epsilon_t^B; \ c_{t+1}^* = \zeta \beta_s^{-1} \epsilon_t^B; \ c_{t+i}^* = 0 \ \forall i \geq 2.$$

Consumption falls on impact for the same reason why it falls on impact under pure redistribution (recall that a deficit-financed tax cut amounts to redistribution today).

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And by the same token—since repaying the debt tomorrow by uniform taxation amounts to redistributing from borrowers to savers—consumption tomorrow increases with the increase in taxes. The positive income effect on savers next period dominates, in absolute value, the negative income effect on borrowers.

Notice, however, that the *present-value multiplier* (denoted $\mathcal{M}^*_{debt}$) of this debt-financed tax cut is exactly zero:

$$\mathcal{M}^*_{debt} \equiv \frac{\partial \left(c_t^* + \beta_s c_{t+1}^* \right)}{\partial \epsilon_t^B} = 0.$$

The two effects exactly offset each other; in this sense, although Ricardian equivalence fails, a *long-run* version of it still holds under flexible prices.

3.2.1 Debt-financed uniform tax cut with endogenously persistent debt

In the more general case where debt is not repaid immediately, the aggregate tax rule $t_t = \phi_B b_t - \epsilon_t^B$ replaced in the government budget constraint (18) delivers the debt accumulation equation:

$$b_{t+1} = (1 - \phi_B) \beta_s^{-1} b_t + \beta_s^{-1} \epsilon_t^B. \tag{20}$$

In order to ensure *debt sustainability*, the response of taxes to debt needs to obey:

$$\phi_B > 1 - \beta_s. \tag{21}$$

Notice, however, that condition (21) need not hold for taxation of *both* agents, but for *the aggregate response*. Indeed, it would be sufficient if for instance taxes of savers fulfilled (21), i.e. $\phi_B^s > 1 - \beta_s$ and taxes of borrowers did not respond to debt at all, $\phi_B^b = 0$. When the condition is fulfilled, (20) can be solved independently of the rest of the model to determine the path of public debt; this is due to our assumption of zero public debt in steady state, which makes interest payments irrelevant to first order.

Under this experiment, equation (15), combined with the aggregate tax rule $t_t = \phi_B b_t - \epsilon_t^B$, implies that aggregate consumption obeys (assuming $\bar{D}_Y = 0$ for simplicity):

$$c_t^* = \zeta \phi_B b_t - \zeta \epsilon_t^B.$$

16

Since $\zeta > 0$, the prediction of this model under flexible prices is once again that tax cuts cause a *contraction* in aggregate consumption on impact. Moreover, as taxes increase in the future in order to repay public debt even when the shock is purely transitory:

$$c_{t+i}^* = \zeta t_{t+i} = \zeta \phi_B b_{t+i} \ \forall i \geq 1,$$

the model also predicts that *future* consumption will *increase* along with future taxes.

These implications of the model are inconsistent with a large empirical literature documenting that positive tax shocks (as well as interest rate increases) are contractionary, rather than expansionary. The reason why tax increases are expansionary in the model is as follows: the income effect on savers' labor supply deriving from any given tax change is larger than that on borrowers' labor supply. Therefore, in response to a change of equal size (but of opposite sign) in their taxes, savers wish to increase their labor input more than borrowers want to decrease it.

Any given increase in the real interest rate works in the same way, as it redistributes wealth from borrowers to lenders; but the latter's reaction (through labor supply and consumption) is larger in relative terms if they consume on average a higher share of income. These paradoxical effects of lump-sum tax changes and interest rates on aggregate consumption under flexible prices motivate our further analysis, which consists of studying a model in which price adjustment is imperfect.

4 Sticky Prices: an Analytically Solvable Special Case

To introduce realistic departures from both monetary and debt neutrality we follow Calvo (1983) and Yun (1996) and assume that intermediate good firms adjust their prices infrequently, θ being both the history-independent probability of keeping the price constant and—by the law of large numbers—the fraction of firms that keep their prices unchanged.

Savers (who in equilibrium will hold all the shares in firms) maximize the value of the firm, i.e. the discounted sum of future nominal profits, choosing the price $P_t(z)$ and using the relevant stochastic discount factor (pricing kernel) for nominal payoffs:

$\max \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} (\theta^s \Lambda_{t,t+i} \left[P_t(z) Y_{t,t+i}(z) - W_{t+i} Y_{t,t+i}(z) \right] \right\}$, subject to the demand equation. The optimality conditions of this problem lead to the by now standard New Keynesian Phillips curve, that is presented in log-linearized form below.

In order to make the point that price stickiness by itself leads, in this model, to a failure of Ricardian equivalence, we take as a benchmark scenario the case in which steady-state consumption shares are equalized—which, as from Proposition 1, is precisely the case in which Ricardian Equivalence holds under flexible prices. We highlight, however, that our results do not depend in any crucial way on this assumption, which we in fact relax subsequently; but this assumption does allows us to obtain instructive analytical solutions in cases in which we otherwise cannot.

To achieve steady-state consumption equalization across agents, we assume that both the debt limit and steady-state profits are zero; the latter is achieved by assuming, for instance, that there is a sales subsidy $\sigma = \mu$, so that profits' share in total output is: $\mathcal{P}_Y = 0$.[9] Note also that under this assumption, the implied weights on leisure in the utility function are equal across agents.[10]

The debt limit being zero implies that interest payments on outstanding (private) debt are irrelevant to first order, which eliminates an endogenous state variable (lagged real interest rate). The equilibrium hours and consumption of borrowers are governed by:

$$n_{b,t} = \frac{1}{1+\varphi} t_{b,t} \text{ and } c_{b,t} = w_t - \frac{\varphi}{1+\varphi} t_{b,t}.$$

The steady-state symmetry of consumption levels implies that aggregation is simple, and allows us to isolate the role of sticky prices in generating a failure of Ricardian Equivalence. In particular, the aggregate constant-consumption labor supply curve has

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[9]This can be shown as follows: the profit function under the subsidy becomes $\mathcal{P}_t(z) = (1+\sigma)\left[P_t(z)/P_t \right] Y_t(z) - W_t N_t(z) - L_t(z)$, where L are lump-sum taxes/transfers on the firm to finance the subsidy. Under flexible prices, the relative price which solves the profit-maximization problem is $P_t(z)/P_t = \left[(1+\mu)/(1+\sigma) \right] W_t$, so the subsidy equating price and marginal cost is indeed $\sigma = \mu$. Note that under this policy and flexible prices, profits are zero, since $L_t(z) = \sigma \left[P_t(z)/P_t \right] Y_t(z) = \mu W_t N_t(z)$.

[10]The alternative to achieve this outcome would be to assume that there are steady-state transfers that redistribute asset income evenly; the assumption we use has the relative merit of being consistent with evidence pointing to the long-run share of pure economic profits being virtually zero (see Rotemberg and Woodford, 1999). We also avoid taking a stand on the amount of *steady-state* redistribution through lump-sum transfers, which is very hard to measure.

the same parameters as the individual ones: $\varphi n_t = w_t - c_t$, implying $w_t = (1 + \varphi) c_t$. Replacing these equations in the definition of aggregate consumption and solving for consumption of savers, we obtain:

$$c_{s,t} = \delta c_t + \eta t_{b,t}$$
$$\text{where } \delta \equiv 1 - \frac{\lambda \varphi}{1 - \lambda} \text{ and } \eta \equiv \frac{\lambda}{1 - \lambda} \frac{\varphi}{1 + \varphi}$$

Replacing this in the Euler equation of savers we obtain the IS curve:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1} \left(i_t - \mathbb{E}_t \pi_{t+1} \right) - \delta^{-1} \eta \left(t_{b,t} - \mathbb{E}_t t_{b,t+1} \right) \tag{22}$$

Bilbiie (2008) shows that—in a model that is equivalent to ours with $\bar{D}_Y = 0$—for values of $\lambda > 1/(1 + \varphi)$, δ becomes negative: the aggregate elasticity of intertemporal substitution changes sign, and interest rate cuts become contractionary. In that "inverted aggregate demand logic" region, the monetary policy rule needs to follow an inverted Taylor principle (i.e., be passive) in order to ensure determinacy and rule out sunspot fluctuations. In the remainder of this paper, we focus on parameter values that imply that $\delta > 0$ and standard aggregate demand logic holds.[11]

The Phillips curve and Taylor rule are entirely standard:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa c_t, \text{ where } \kappa \equiv (1 + \varphi) \frac{(1 - \beta_s \theta)(1 - \theta)}{\theta} \tag{23}$$
$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1}. \tag{24}$$

The insight that taxes on borrowers are the only channel through which fiscal policy matters in this economy—insight which we previously obtained from simply combining budget constraints—is seen most clearly here.[12] We next show how this feature of our

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[11] In our framework with non-zero debt limit, this result will depend upon the value of the debt limit $\bar{D}_Y$ (intuitively, even when $\delta < 0$ an increase in the real rate needs not necessarily be expansionary, because of the contractionary effect on aggregate demand of interest payments on outstanding debt). But the same core intuition holds, in that for values of the share of borrowers above that threshold, the effects of the type of fiscal shocks analyzed here are overturned.

[12] This result has been noticed already in models of fiscal policy with limited asset markets participation, or rule-of-thumb consumers such as: Gali, Lopez-Salido and Valles (2007), Bilbiie and Straub (2004), Bibiie, Meier and Mueller (2008) or Lopez-Salido and Rabanal (forthcoming).

model generates, in the parameter region where $\delta > 0$, positive and potentially large multipliers of lump-sum transfers to borrowing constrained agents–either in the form of pure redistribution or public debt.

4.1 Pure redistribution ("Robin Hood")

Consider first the effect of pure redistribution—a transfer ϵ_t to borrowers financed by taxes on savers *within the period*—as in Section 2.1 above, the tax processes are given by (16). It is instructive to simplify even further and first consider the case where the shock lasts only one period, $\mathbb{E}_t \epsilon_{t+1} = 0$. Since the model is entirely forward-looking, expected values of consumption and inflation are also zero: $\mathbb{E}_t c_{t+1} = \mathbb{E}_t \pi_{t+1} = 0$, and the solution is simply:

$$
\begin{aligned}
c_t &= -\delta^{-1}\eta \; t_{b,t} = \mathcal{M}_{red}\epsilon_t, \\
\pi_t &= -\kappa\delta^{-1}\eta \; t_{b,t} = \kappa\mathcal{M}_{red}\epsilon_t, \\
\text{where } \mathcal{M}_{red} &\equiv \frac{\lambda}{1 - \lambda\,(1+\varphi)}\frac{\varphi}{1+\varphi}
\end{aligned}
$$

is the consumption multiplier of redistribution. These expressions suggest the following proposition.

Proposition 2 *A wihtin-the-period revenue-neutral transitory redistribution from savers to borrowers generates an expansion in aggregate consumption and inflation, as long as the elasticity of aggregate demand to interest rate is negative ($\delta > 0$), i.e.:*

$$
\lambda < \frac{1}{1+\varphi}.
$$

To understand the intuition, recall first what happens under flexible prices, if income effects on both agents are equal (they have the same long-run consumption values): labor supply of the borrowers shifts to the left, but labor supply of the savers shifts to the right by the same amount. Labor demand does not change either—so redistribution has no effect.

With sticky prices, and even in the knife-edge case in which steady-state consumption shares are equal, two key ingredients break this neutrality. First, recall that output is demand-determined; the increase in borrowers' consumption generates a demand effect: labor demand increases as some firms are stuck with the old, suboptimally low price. The second key ingredient is the asymmetry between income effects. Faced with an increase in the real wage (marginal cost), the savers recognize that they need to undertake an extra negative income effect (that is absent with flexible prices) since their profit income fall. In equilibrium, they will therefore work more than the borrowers are willing to work less, therefore supporting the aggregate expansion in consumption.

This income effect is increasing in the fraction of borrowers and decreasing with labor supply elasticity, but only up to the threshold given in the Proposition. Beyond that threshold (if there are "too many" borrowers or if labor supply is "too inelastic") the effects of all shocks—including the ones studied here—are overturned: the slope of the aggregate IS curve changes sign and interest rate increases become expansionary; we abstract from the implications of that insight in the remainder of this paper.[13]

In the more general case when redistribution is persistent (exogenously), with $\mathbb{E}_t\epsilon_{t+1} = \rho\epsilon_t$, the responses of inflation and consumption to taxes on borrowers are emphasized in the following proposition (a proof of which can be found in the Appendix):

Proposition 3 *In response to an exogenously persistent redistribution from savers to borrowers ϵ_t, inflation and consumption (output) follow:*

$$
\pi_t = \frac{\beta^{-1}\kappa\delta^{-1}\eta\left(1-\rho\right)}{\det}\epsilon_t
$$
$$
c_t = \frac{\delta^{-1}\eta\left(1-\rho\right)\left(\beta^{-1}-\rho\right)}{\det}\epsilon_t,
$$

where $\det = (1-\rho)\left(\beta^{-1}-\rho\right) + \beta^{-1}\kappa\delta^{-1}\left(\phi_\pi - 1\right)\rho$.

Qualitatively, the responses are the same as above: inflation and consumption increase when there is an exogenous tax cut to borrowers. Quantitatively, it can be easily

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[13]We refer the interested reader to Bilbiie (2008) for a theoretical investigation of the implications of that scenario, and to Bilbiie and Straub (2011) for an analysis of its empirical merits in explaining the Great Inflation.

shown that the multiplier on consumption is decreasing with the persistence parameter ρ (implying that it is always lower than the multiplier derived under zero persistence). This happens because a persistent shock generates two effects. First, it raises expected future inflation and hence, via the monetary policy rule, an increase in the real interest rate which works to reduce savers' consumption through intertemporal substitution. Second, it enhances the negative wealth effect on labor supply by the savers, inducing them, at the margin, to reduce consumption further.

4.2   Public debt

We next turn to a uniform tax cut financed via debt that is repaid entirely (and uniformly) next period—the same experiment we study under flexible prices in Section 3.2 above, consisting of the tax rule with $\phi_B = 1$ (17) that implies the tax process (19).

Substituting the tax process in the IS curve (22), recalling that we look at uniform taxation, so $t_{b,t} = t_t$, and restricting our attention without loss of generality to purely transitory shocks ($\mathbb{E}_t \epsilon^B_{t+1} = 0$), we obtain:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1} \left(i_t - \mathbb{E}_t \pi_{t+1} \right) - \delta^{-1} \eta b_t + \delta^{-1} \eta \left(1 + \beta_s^{-1} \right) \epsilon^B_t.$$

The solutions for consumption and inflation are shown in the following Proposition (the proof of which is relegated to the Appendix).

Proposition 4 *In response to a tax cut financed by public debt which is fully repaid next period by uniform taxation, consumption and inflation are given by:*

$$
\begin{aligned}
c_t &= \delta^{-1} \eta \left[1 + \delta^{-1} \kappa \left(\phi_\pi - 1 \right) \beta_s^{-1} \right] \epsilon^B_t - \delta^{-1} \eta b_t \\
\pi_t &= \kappa^2 \delta^{-2} \eta \left(\phi_\pi - 1 \right) \beta_s^{-1} \epsilon^B_t - \kappa \delta^{-1} \eta b_t \\
c_{t+1} &= -\delta^{-1} \eta \beta_s^{-1} \epsilon^B_t \\
\pi_{t+1} &= -\kappa \delta^{-1} \eta \beta_s^{-1} \epsilon^B_t \\
and \ \pi_{t+i} &= c_{t+i} = 0 \ \forall i \geq 2.
\end{aligned}
$$

These responses obtained in the foregoing proposition can be explained intuitively as follows. Consider, to start with, the equilibrium values obtained in period $t + 1$: in present-value terms, they are equal (but of opposite sign) to the responses of consumption and inflation to a pure redistribution described in the previous section, precisely because our experiment is akin to a redistribution from borrowers to savers in period $t + 1$. In period t, however, we have two effects. First, the usual effect of redistribution on consumption, summarized by the term $\delta^{-1}\eta\epsilon_t^B - \delta^{-1}\eta b_t$; second, an additional effect equal to $\delta^{-2}\eta\kappa\left(\phi_\pi - 1\right)\beta_s^{-1}\epsilon_t^B$, that is driven by intertemporal substitution by savers and that can be explained as follows.

The key insight is that the present discounted value of savers' consumption is *larger* under public debt (for instance, $\phi_B = 0$) than under redistribution ($\phi_B \to 1 - \beta_s$). In period $t + 1$ firms are faced with lower demand and cut prices, creating deflation. Savers react to this, depending upon the monetary policy rule, either by: (i) increasing their consumption at $t + 1$ relative to $t + 2$ (when the economy returns to steady state), if the real interest rate falls which in turn happens when monetary policy responds to realized inflation; or, if monetary policy responds to *expected* inflation (which is zero) by (ii) not changing their consumption at all.

In the first scenario (contemporaneous rule), the expected increase in savers' consumption tomorrow implies that an increase in inflation today—coming from the demand effect of redistribution to borrowers in the first period—will trigger a relatively *smaller* fall in consumption of savers at time t relative to the case of pure redistribution—once again, because of intertemporal substitution. In the second scenario (forward-looking rule), the mechanism is more direct: the deflation at time $t + 1$ implies a cut in the ex-ante real interest rate today, and (since tomorrow's consumption is unchanged) an increase in savers' consumption today—once more, by intertemporal substitution.

Finally, in equilibrium, and regardless of the type of monetary policy rule, firms correctly anticipate lower demand in the future and increase prices today by less than they would if redistribution were not 'reversed' in the future; so inflation increases by less, reinforcing the effect described previously (for a contemporaneous rule).

Notice that, consistent with this intuition, this effect disappears when either prices are fixed ($\theta = 1$), or there are no savers, and hence no intertemporal substitution ($\lambda = 1$ implies $\delta^{-1} \to 0$), or no endogenous movements in real interest rates ($\phi_\pi = 1$); finally, the effect also disappears when there are no borrowers ($\lambda = 1 \to \eta = 0$), consistently with Ricardian equivalence.

A complementary way of thinking about this additional, intertemporal-substitution effect, is to calculate the *present-value* aggregate consumption multiplier of a debt-financed tax cut, $\mathcal{M}_{debt}$:

$$\mathcal{M}_{debt} \equiv \frac{\partial (c_t + \beta_s c_{t+1})}{\partial \epsilon_t^B} = \delta^{-1}\eta \left[1 + \delta^{-1}\kappa (\phi_\pi - 1) \beta_s^{-1}\right] - \delta^{-1}\eta = \delta^{-2}\eta\kappa (\phi_\pi - 1) \beta_s^{-1} > 0, \tag{25}$$

where there is discounting at the steady-state real interest rate (which is determined by the savers' discount factor), and we assumed for simplicity that the initial level of public debt is equal to its steady-state value.

A simple inspection of (25) reveals that it is identical to the effect described previously, i.e. to the component of the period-t consumption multiplier of a uniform tax cut that is over and above the multiplier due to pure redistribution. We can assess the *magnitude* of $\mathcal{M}_{debt}$ by looking at a parameterization that is standard in the literature, namely: unitary inverse Frisch elasticity $\varphi = 1$, average price duration of one year ($\theta = 0.75$), steady-state markup of 0.2 ($\epsilon = 6$), and discount factor of savers $\beta_s = 0.99$. Figure 2 plots the value of this multiplier for the whole range of the share of borrowers λ for which the elasticity of aggregate demand to interest rates is positive ($\delta > 0$), namely $\lambda < 0.5$. We consider two values of the inflation elasticity of interest rates: $\phi_\pi = 1.5$ (red dashed line) and $\phi_\pi = 3$ (blue solid line) respectively; consistent with our analytical results and intuition, the multiplier is uniformly larger for the higher value of ϕ_π. At low values of λ, until about 0.4, the multiplier is very small, below 1 percent. But when approaching the threshold beyond which the economy moves to the "inverted" region, the multiplier becomes very large.[14]

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[14]For instance, under $\phi_\pi = 3$, it is about 4 percent when $\lambda = 0.45$ and about 12 percent when $\lambda = 0.47$. The reason for this abrupt increase is that the elasticity of aggregate demand to real interest rates δ^{-1}
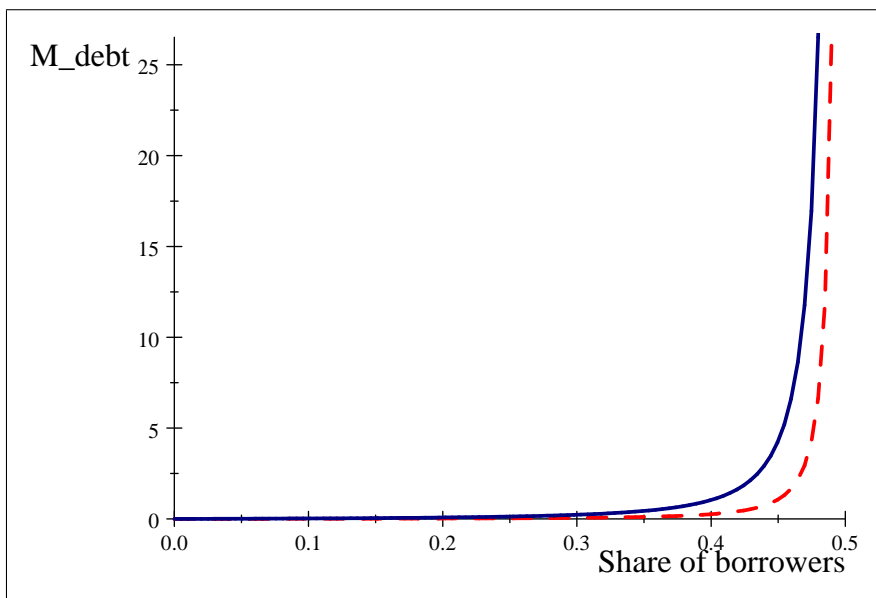
Figure 2: Present-value multiplier as a function of λ for the baseline parameterization, for $\phi_\pi = 1.5$ (red dashed) and $\phi_\pi = 3$ (blue solid) respectively.

Finally, it is worth noticing that consumption only increases *above the steady-state level* when:

$$\epsilon_t^B > \frac{1}{1 + \delta^{-1}\kappa\left(\phi_\pi - 1\right)\beta_s^{-1}}b_t$$

In other words, if the initial public debt level is high compared to its long-run level, a tax cut (more debt) will not be successful in curing a recession.

4.2.1 Debt-financed uniform tax cut with endogenously persistent debt

When debt stabilization does not take place within the period (so that $\phi_B < 1$) there is more endogenous persistence because of (endogenous) debt accumulation as implied by equation (20), derived for the case with flexible prices above. The Euler equation under endogenously persistent debt becomes:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) - \delta^{-1}\eta\phi_B\left(b_t - b_{t+1}\right) + \delta^{-1}\eta\epsilon_t^B \tag{26}$$

approaches infinity when λ approaches that threshold value; see Bilbiie (2008) for an elaboration of that point.

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The first observation is that when ϕ_B tends to its lower bound given by $1 - \beta_s$, cf. (21), the effects of a debt-financed uniform tax cut are almost identical to the effects of *pure redistribution*. Indeed, this result holds *precisely* when ϕ_B^b is zero and ϕ_B^s (which does not enter the aggregate Euler equation *per se*) satisfies (21). The intuition for this result is simple: when debt repayments are pushed into the far future, savers fully internalize the government budget constraint; taxation in the future is, for them, equivalent to taxation today. But for the borrowers, a tax cut today is disposable income. Therefore, a uniform tax cut is equivalent to a pure redistribution within the period when the uniform tax cut is financed with very persistent debt.

When ϕ_B takes on intermediate values, so that debt stabilization is neither immediate nor postponed into the far future, the interplay of income effects, intertemporal substitution by savers and the demand effect due to sticky prices generates different responses that feature endogenous persistence.[15]

Figure 3 illustrates these findings by plotting the responses of consumption, inflation and public debt to a purely transitory uniform tax cut—for the baseline parameterization described above—under three scenarios. The (blue) solid line corresponds to the "full debt stabilization" scenario, in which debt is repaid entirely in the following period. As our analytical results have already shown, aggregate consumption increases on impact but falls next period; there is a small positive inflation on impact, and deflation in the second period. The other extreme scenario (as close as possible to the one of "no debt stabilization") is pictured by the red circle line: consistent with our analytical results and intuition above, the effects are almost identical to those obtained under pure redistribution: despite the (endogenously) highly persistent response of debt, consumption and inflation increase only in the first period. For intermediary values of ϕ_B, the fall in consumption and inflation in the second period is smaller that under full debt stabilization, but the recession and deflation last longer.

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[15] The analytical expressions are difficult to interpret and are presented for completeness in the Appendix; we rely on a numerical simulation of the model in order to illustrate the main properties of the solution.
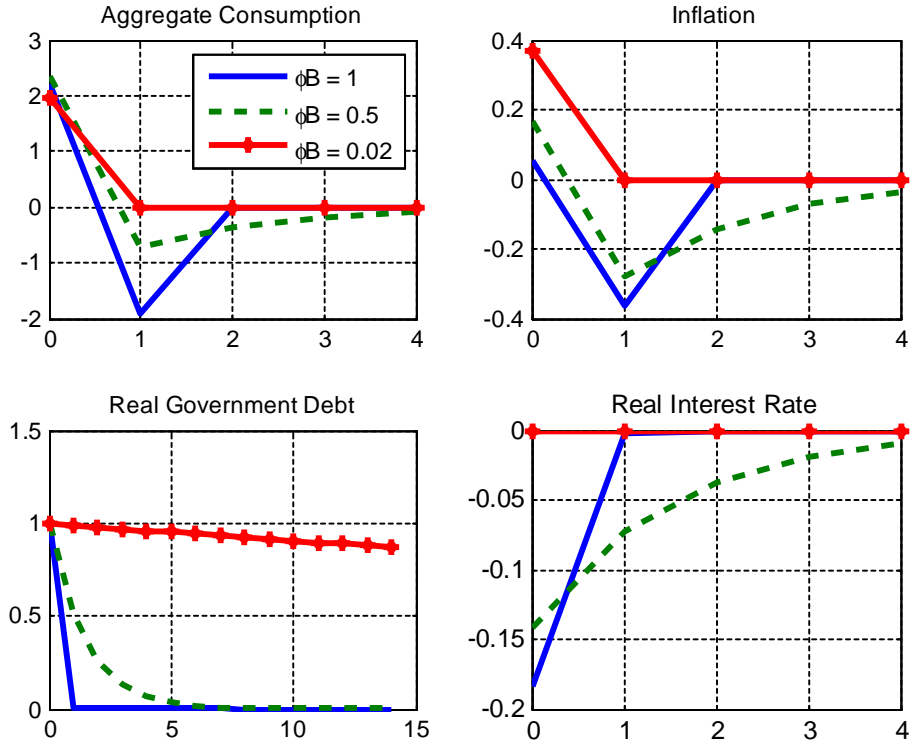
Figure 3: Impulse responses to a debt-financed uniform tax cut, for different debt stabilization coefficients ϕ_B.

4.2.2 Robustness: non-zero debt limit and non-zero steady-state public debt

Finally, while the results above were derived under the special assumptions that the private debt limit is zero, and steady-state public debt is also zero, we emphasize that they are robust to relaxing those assumptions. The reason is that the main difference, when relaxing either of those assumptions, is related to interest payments—on either private or public debt, respectively—which turn out to be quantitatively negligible. As an illustration, Figure 4 shows that when the debt limit is increased rather dramatically (from 0 to 0.8 of steady-state income) the responses of inflation and output (consumption) are virtually indistinguishable across the different values of the debt limit (Results for all other robustness experiments are available upon request).
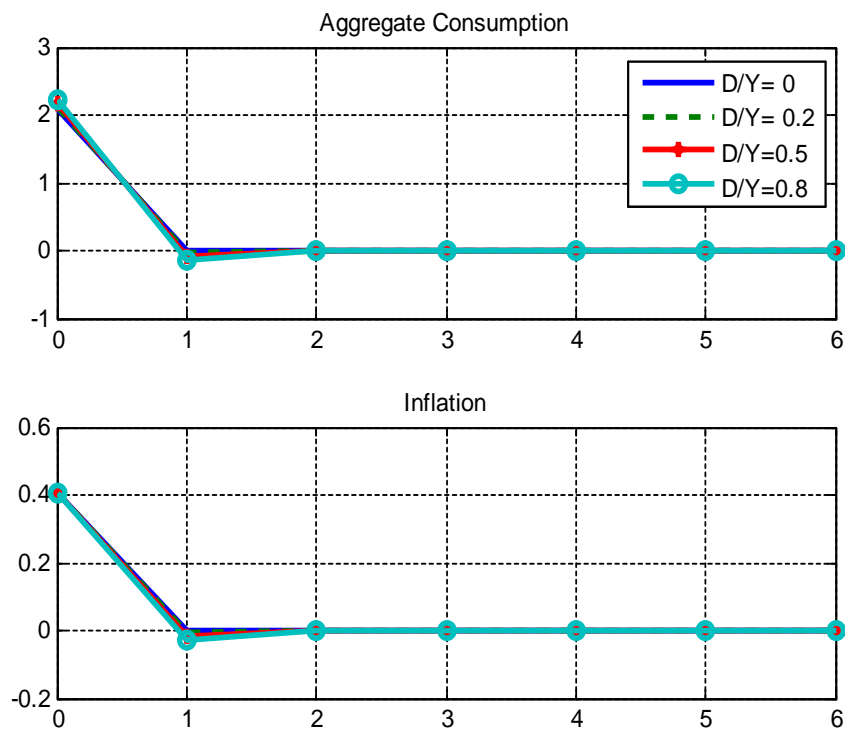
Figure 4: Response to pure redistribution varying the exogenous debt limit, for transitory shock.

5 Conclusions

In this paper, we have studied two interrelated issues: the macroeconomic effects of public debt and redistribution. In a model with financial imperfections —some agents are patient and some impatient, the latter being subject to a borrowing constraint—we show that, somewhat surprisingly at first glance, Ricardian equivalence holds when prices are free to adjust and the steady-state (or initial) wealth distribution is degenerate; the explanation is that income effects in that setup are symmetric, so someone's decision to consume less and work more is exactly compensated by somebody else's decision to consume more and work less. When the steady-state distribution of wealth is not degenerate, Ricardian equivalence does fail period-by-period—and so does monetary neutrality; but the predic-

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tions pertaining to the effects both taxes and interest rates are counterfactual: both fiscal and monetary expansions are in fact contractionary. Moreover, we show that a long-run version of Ricardian equivalence still holds. We explain this result by emphasizing the redistributional dimension of public debt: a uniform tax cut financed by issuing debt amounts to a redistribution from savers (the holders of that debt) to borrowers today, coupled with a reversal of that redistribution in the future, when debt is repaid by uniform taxation. With flexible prices, the effects of those redistributions are entirely symmetric, so the overall change in macroeconomic variables is nil.

Imperfect price adjustment breaks this symmetry, even when the steady-state distribution of wealth is degenerate. It does so by introducing a usual demand effect (hence linking inflation to demand) and by linking that demand effect to the income effect on borrowers (for whom variations in income translate into variations in demand within the period); and finally, by effectively activating the intertemporal substitution channel for savers (hence linking inflation to their intertemporal allocation of demand through real interest rates). The result is that a mere redistribution of income from savers to borrowers will now be expansionary and inflationary. Moreover, a uniform tax cut financed by issuing debt has an aggregate effect that goes beyond the mere sum of its redistributive components. In other words, the present-value multiplier of a tax cut is positive, and long-run Ricardian equivalence fails.

In order to focus on one source of failure of Ricardian equivalence (sticky prices) we abstracted from another modeling feature that would no doubt generate realistic departures from Ricardian equivalence even under flexible prices, namely endogenous investment (for instance, in physical capital). The implications of that assumption have been explored in models with two types of agents elsewhere (see for instance Mankiw, 2000). The interaction of endogenous investment and endogenous borrowing limits is certainly worth exploring, but is beyond the scope of this paper.

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A Appendix

Proof of Proposition 3. Rewrite the system as

$$
\begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t c_{t+1} \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} + \Upsilon_R \epsilon_t, \tag{27}
$$

$$
\text{where } \mathbf{\Gamma} = \begin{bmatrix} \beta_s^{-1} & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & 1 - \beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1) \end{bmatrix}, \quad \Upsilon_R = \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1-\rho) \end{bmatrix}
$$

The impulse response functions are calculated as

$$
\begin{aligned}
\Omega &= [\rho I - \mathbf{\Gamma}]^{-1} \Upsilon_R \\
&= \frac{1}{\det} \begin{bmatrix} \rho - 1 + \beta_s^{-1}\kappa\delta^{-1}(\phi_\pi - 1) & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & \rho - \beta_s^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1-\rho) \end{bmatrix} \\
&= \frac{1}{\det} \begin{bmatrix} \beta_s^{-1}\kappa\delta^{-1}\eta(1-\rho) \\ \delta^{-1}\eta(1-\rho)(\beta_s^{-1} - \rho) \end{bmatrix}.
\end{aligned}
$$

Proof of Proposition 4. Note that although the exogenous shock has zero persistence, there is endogenous persistence due to the presence of a state variable, public debt; but that endogenous persistence takes a very special form under our assumption that debt is repaid next period: the effects of the shock will live for two periods only. Therefore, in order to solve the model we must solve for the endogenous variables in periods t and $t+1$. We do this by solving the model backwards as follows: next period's consumption is determined by the Euler equation at $t+1$:

$$
c_{t+1} = \mathbb{E}_{t+1} c_{t+2} - \delta^{-1}(i_{t+1} - \mathbb{E}_{t+1}\pi_{t+2}) - \delta^{-1}\eta b_{t+1} + \delta^{-1}\eta(1 + \beta_s^{-1})\epsilon_{t+1}^B - \delta^{-1}\eta \mathbb{E}_t \epsilon_{t+2}^B,
$$

which under zero persistence and the assumption that debt is repaid at $t+1$ (and so $i_{t+1} = \mathbb{E}_{t+1}\pi_{t+2} = \mathbb{E}_{t+1}c_{t+2} = 0$) delivers:

$$
c_{t+1} = -\delta^{-1}\eta\beta_s^{-1}\epsilon_t^B = \mathbb{E}_t c_{t+1},
$$

where the second equality holds because the shock ϵ_t^B is in the information set at time t. From the Phillips curve at $t+1$, imposing $\mathbb{E}_{t+1}\pi_{t+2} = 0$, we have:

$$
\pi_{t+1} = -\kappa\delta^{-1}\eta\beta_s^{-1}\epsilon_t^B = \mathbb{E}_t\pi_{t+1}.
$$

The impact multiplier, substituting these expressions in the Euler equation at time t is:

$$c_t = \delta^{-1}\eta \left[1 + \delta^{-1}\kappa \left(\phi_\pi - 1\right)\beta_s^{-1}\right]\epsilon_t^B - \delta^{-1}\eta b_t$$

and inflation is, from the Phillips curve:

$$\pi_t = \kappa^2\delta^{-2}\eta\left(\phi_\pi - 1\right)\beta_s^{-1}\epsilon_t^B - \kappa\delta^{-1}\eta b_t$$

Analytical solution with endogenously persistent debt $\phi_B < 1$.

Replacing the debt accumulation equation (20) into the Euler equation, we obtain:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}\left(i_t - \mathbb{E}_t\pi_{t+1}\right) + \delta^{-1}\eta\phi_B\left[\beta_s^{-1}\left(1 - \phi_B\right) - 1\right]b_t + \delta^{-1}\eta\left(1 + \phi_B\beta_s^{-1}\right)\epsilon_t^B \quad (28)$$

This is a reduced-form IS curve for a given level of public debt; together with (23) and (24) it can be solved to determine consumption and output as a function of outstanding debt and the fiscal shock. The system to be solved is:

$$\begin{bmatrix} \mathbb{E}_t\pi_{t+1} \\ \mathbb{E}_t c_{t+1} \end{bmatrix} = \boldsymbol{\Gamma}\begin{bmatrix} \pi_t \\ c_t \end{bmatrix} + \boldsymbol{\Psi}b_t + \boldsymbol{\Upsilon}_B\epsilon_t^B, \quad (29)$$

$$\text{where } \boldsymbol{\Gamma} = \begin{bmatrix} \beta_s^{-1} & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}\left(\phi_\pi - 1\right) & 1 - \beta_s^{-1}\delta^{-1}\kappa\left(\phi_\pi - 1\right) \end{bmatrix},$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0 \\ \delta^{-1}\eta\phi_B\left[1 - \beta_s^{-1}\left(1 - \phi_B\right)\right] \end{bmatrix}; \quad \boldsymbol{\Upsilon}_B = \begin{bmatrix} 0 \\ -\delta^{-1}\eta\left(1 + \phi_B^2\beta_s^{-1}\right) \end{bmatrix} \quad (30)$$

Using the method of undetermined coefficients, we can guess and verify that the solution takes the form:

$$\begin{bmatrix} \pi_t \\ c_t \end{bmatrix} = \mathbf{A}_B b_t + \mathbf{A}_\epsilon\epsilon_t^B,$$

which substituted in the original system (using also the public debt dynamics equation) delivers:

$$\mathbf{A}_B\left(1 - \phi_B\right)\beta_s^{-1}b_t + \mathbf{A}_B\beta_s^{-1}\epsilon_t^B = \boldsymbol{\Gamma}\mathbf{A}_B b_t + \boldsymbol{\Gamma}\mathbf{A}_\epsilon\epsilon_t^B + \boldsymbol{\Psi}b_t + \boldsymbol{\Upsilon}_B\epsilon_t^B.$$

Identifying coefficients:

$$\mathbf{A}_B = \left[\left(1 - \phi_B\right)\beta_s^{-1}\mathbf{I} - \boldsymbol{\Gamma}\right]^{-1}\boldsymbol{\Psi},$$

$$\mathbf{A}_\epsilon = \boldsymbol{\Gamma}^{-1}\left(\mathbf{A}_B\beta_s^{-1} - \boldsymbol{\Upsilon}_B\right).$$

$$\mathbf{A}_B = \frac{1}{\det} \begin{bmatrix} (1-\phi_B)\,\beta_s^{-1} - 1 + \beta_s^{-1}\kappa\delta^{-1}\,(\phi_\pi - 1) & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}\,(\phi_\pi - 1) & -\phi_B\beta_s^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ \delta^{-1}\eta\phi_B\left[1 - \beta_s^{-1}\,(1-\phi_B)\right] \end{bmatrix}$$

$$\mathbf{A}_B = -\frac{\delta^{-1}\eta\phi_B\left[1 - \beta_s^{-1}\,(1-\phi_B)\right]}{\det} \begin{bmatrix} \beta_s^{-1}\kappa \\ \beta_s^{-1}\phi_B \end{bmatrix}$$

$$\det = \left(1 - (1-\phi_B)\,\beta_s^{-1}\right)\beta_s^{-1}\phi_B + \beta_s^{-2}\kappa\delta^{-1}\,(\phi_\pi - 1)\,(1-\phi_B) > 0$$

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