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Real Rigidity, Nominal Rigidity, and the Social Value of Information

George-Marios Angeletos, Luigi Iovino, Jennifer Lao

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Real Rigidity, Nominal Rigidity,
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George-Marios Angeletos
MIT

Luigi Iovino
Bocconi University and IGIER

Jennifer La'O
Columbia University

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Abstract

Does welfare improve when firms are better informed about the state of the economy and can better coordinate their decisions? We address this question in an elementary business-cycle model that highlights how the dispersion of information can be the source of both nominal and real rigidity. Within this context we develop a taxonomy for how the social value of information depends on the two rigidities, on the sources of the business cycle, and on the conduct of monetary policy.

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1 Introduction

Economic agents have access to a variety of sources of information about the state of the economy, some of which are private (such as firm- or industry-specific signals) and some of which are public (such as macroeconomic statistics and central-bank communications). By informing each agent about the activity of others, public information can ease coordination, whereas private information can hinder it. In this paper, we study the welfare consequences of this mechanism within the context of a business-cycle model in which firms make their employment, production, and pricing choices under incomplete information about one another's choices and about the state of the economy.

Background. We are not the first to study how information affects coordination and welfare. In an influential article, Morris and Shin (2002) used a "beauty contest" game—a linear-quadratic game in which actions were strategic complements—to formalize the coordinating role of public information and to study its welfare implications. In such a game, public signals have a disproportionate effect on equilibrium outcomes relative to what is warranted on the basis of their informational content regarding fundamentals alone. This is due to the fact that the players use such signals not only to predict fundamentals but also to coordinate their actions. In this regard, public signals can play a role akin to that of sunspots, possibly contributing to higher volatility and lower welfare.

Because strategic complementarity emerges naturally from the aggregate demand externalities that are embedded in macroeconomic models, Morris and Shin's analysis was used to inform the debate on the pros and cons of central-bank transparency.[1] However, subsequent work raised questions about the validity of applying Morris and Shin's lessons to a macroeconomic context.

On the one hand, Angeletos and Pavan (2007) highlighted, on the basis of a broader game-theoretic framework, that Morris and Shin's welfare conclusion hinges on the assumption that coordination is socially harmful—an assumption that need not be valid in workhorse macroeconomic models. On the other hand, a line of applied work that includes Baeriswyl and Cornand (2010), Hellwig (2005), Lorenzoni (2010), Roca (2005), and Walsh (2007) found different welfare effects than those suggested by Morris and Shin in variants of the New-Keynesian model in which nominal rigidity originates from incomplete information rather than Calvo-like sticky prices.

This applied work has pushed the analysis of the question of interest from abstract games to workhorse macroeconomic models. This is crucial step, as "anything goes" without the discipline of specific micro-foundations: different assumptions about the payoff structure of a game can justify any sort of welfare effect. This work has therefore identified important mechanisms through which information can affect productive efficiency and welfare, some of which we review in Section 6.

Yet, this work faces certain limitations. By equating the informational friction to a particular form of nominal rigidity, it abstracts from the bite that the informational friction can have on productive efficiency *regardless* of nominal rigidity. Formally, it lets the informational friction

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[1]See, e.g., the follow-up *AER* articles by Svensson (2006), Morris et al. (2006), and James and Lawler (2011).

impose a measurability constraint on prices, but abstracts from any such constraint on quantities. It is thus as if employment and production choices, in contrast to pricing choices, are made under complete information. Furthermore, this work intertwines the welfare effects of information with those of particular monetary policies, often confounding the informational incompleteness of the firms with frictions in the conduct of monetary policy—a point that we formalize in due course.

Our contribution. Seeking to overcome the aforementioned limitations, in this paper we consider a framework in which firms make not only their pricing choices but also certain employment and production choices on the basis of dispersed private information about the underlying aggregate shocks. In this sense, we allow the incompleteness of information to be the source of both *real* and *nominal* rigidity, that is, to impose a measurability constraint on both quantities and prices.

In addition, we dissect how the welfare effects of information depend on whether monetary policy coincides with or deviates from two benchmarks. The first corresponds to a policy that replicates flexible prices, in the sense of implementing the same allocation as the one that would have obtained in the absence of the nominal rigidity. The second identifies the unconstrained optimal monetary policy, meaning the solution to the Ramsey problem in which the planner can set the nominal interest rate as an arbitrary function of the underlying state of nature.

This approach leads to a certain taxonomy for how the answer to the question of interest depends on the conduct of monetary policy, on the nature of the underlying business-cycle shocks, and on the aforementioned two types of rigidity.[2]

Isolating the real rigidity. In the first part of the paper (Sections 2-4), we study the polar opposite case than the one in prior work: we assume that firms choose employment on the basis of incomplete information, thus accommodating real rigidity, but let prices adjust to the realized state of nature, thus abstracting from nominal rigidity and shutting down the pivotal role that monetary policy plays once nominal rigidity is present. This part therefore serves as a stepping stone towards the second part of the paper, which ultimately allows for both types of rigidity.

We first study how information affects two familiar welfare components: the volatility of the aggregate output gap and the inefficient cross-sectional dispersion in relative prices (and quantities). For each component separately, we show how the sign of these effects is governed by three sets of factors: (i) preference and technology parameters that pin down the coordination motives; (ii) whether the information is private or public; and (iii) the underlying sources of the business cycle.

We next show that, despite non-monotone and often conflicting effects on these two components, the sign of the overall welfare effect of *either* type of information is governed *solely* by the sources of the business cycle. When the business cycle is driven by non-distortionary forces such as technology

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[2]In this paper, we focus on the distinct normative implications of the two types of rigidity. However, the two also have distinct *positive* implications. For example, when the rigidity is nominal, the response of macroeconomic outcomes to the underlying noise shock can take *any* sign, depending on the conduct of monetary policy. Furthermore, there is a Philips curve: any deviation of the level of real output from the complete-information point is *necessarily* associated with a commensurate movement in the price level. None of this is true when the rigidity is real.

shocks, welfare unambiguously increases with either private or public information. When instead the business cycle is driven by distortionary forces such as shocks to monopoly markups, welfare unambiguously decreases with either type of information.[3]

To some extent, this result is a priori intuitive: in the case of technology shocks, one may expect information to be welfare-improving because the firms' reaction to such shocks is socially desirable, while the converse is true in the case of markup shocks. However, this basic intuition can fall apart when information is dispersed: in Morris and Shin (2002), the equilibrium is first-best efficient when information is commonly shared, resembling what happens in our setting in the absence of monopoly distortions, yet welfare can decrease with the precision of public information when, and only when, information is dispersed. As we explain in due course, the sharpness of our results therefore hinges to the following property of our micro-founded setting: the private value that the firms assign to the coordination of their choices coincides with the corresponding social value. In the absence of such a coincidence, the welfare effects of either type of information could have been reverted.

Adding nominal rigidity. In the second part of the paper (Section 5), we study a more general framework, in which we let the informational friction impede not only the firms' employment and production choices but also their price-setting behavior. As noted before, we anchor our analysis to two benchmarks that help dissect the role of monetary policy. The first identifies policies that replicate flexible prices; the second identifies the unconstrained Ramsey optimum. As in the baseline New-Keynesian model, these two benchmarks coincide in the case of technology shocks, but not in the case of markup shocks. Importantly, the scenario studied in the related prior work assumes not only the absence of real rigidity but also specific deviations from these benchmarks.

Consider the first benchmark. When monetary policy replicates flexible prices, the question of interest admits essentially the same answer as in our baseline model: welfare increases (respectively, decreases) with either type of information when the business cycle is driven by technology shocks (respectively, markup shocks). Furthermore, information matters at this benchmark *only* because of the real rigidity: in the absence of real rigidity, the aforementioned policy implements the complete-information outcome, irrespective of how limited the firms' information might be.

Away from this benchmark, an additional effect emerges: information affects not only the bite of the real rigidity but also the firms' ability to forecast, and thus preempt, any action of the monetary authority that attempts to move the economy away from its flexible-price outcomes. The welfare contribution of this additional effect then depends on whether such deviations are desirable or not—this question answered by the second of the aforementioned two policy benchmarks.

When the business cycle is driven by technology shocks, *any* deviation from flexible prices is welfare-deteriorating. Increasing the information that is available to firms may then improve

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[3]In the latter case, a countervailing effect is also at work unless one assumes, as is often done in the literature, that a non-contingent subsidy is used to eliminate the mean (or "steady-state") distortion in economic activity. We characterize this effect in Proposition 4 but, in line with the literature, abstract from it in the rest of our analysis.

welfare not only by alleviating the real rigidity but also by helping the firms forecast, and undo, the "mistakes" in monetary policy. In this sense, transparency is good.

When, instead, the business cycle is driven by markup shocks or other distortions, an appropriate deviation from flexible prices is desirable, for reasons once again familiar from the New-Keynesian framework: the optimal policy now seeks to *exploit* the nominal rigidity in order to substitute for a missing tax instrument, namely, the state-contingent subsidy that would have offset the markup shock. More information in the hands of the private sector can then be detrimental for welfare, not only for the reasons highlighted in our baseline model, but also by reducing the effectiveness of monetary policy. In this sense, opacity becomes preferable.

To recap, the taxonomy we develop in this paper provides sharp answers to our question under certain benchmarks, but it also provides a roadmap for understanding the welfare effects of information away from them. We elaborate on the details in Section 5. As an application of this roadmap, in Section 6 we revisit the prior contributions of Baeriswyl and Cornand (2010), Hellwig (2005), Lorenzoni (2010), and Walsh (2007), shedding further light on the key mechanisms in these papers and facilitating a certain synthesis.

2 The baseline model

The baseline model builds on Angeletos and La'O (2009). The economy consists of a "mainland" and a continuum of "islands". Each island is inhabited by a continuum of workers and a continuum of monopolistic firms. Firms employ local workers through a competitive labor market and produce differentiated commodities, which they ultimately sell in a centralized market in the mainland. The latter is inhabited by a continuum of consumers, each of whom is tied to one worker and one firm from *every* island in the economy. Along with the fact that there will be no heterogeneity *within* islands, this guarantees that the economy admits a representative household: we can think of the latter as a "big family" that is comprised of all agents, collects all income, and consumes all output in the economy. Nevertheless, the aforementioned geography introduces an informational friction: we assume that firms and workers observe the fundamentals on their own island, but face incomplete information about the underlying aggregate shocks and the choices that other agents (their "siblings") make on other islands. Finally, islands are indexed by $i \in I = [0,1]$; firms, workers, and commodities by $(i,j) \in I \times J = [0,1]^2$; and periods by $t \in \{0,1,2,...\}$.

Fundamentals. The utility of the representative household is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - \int_I \int_J \chi_{it} V(n_{ijt}) dj di \right],$$

where $U(C) = \frac{1}{1-\gamma} C^{1-\gamma}$, $V(n) = \frac{1}{1+\epsilon} n^{1+\epsilon}$, and $\gamma, \epsilon \geq 0$. Here, n_{ijt} is the labor input in firm j of island i (or the effort of the corresponding worker), χ_{it} is an island-specific shock to the disutility of

labor, and C_t is aggregate consumption. The latter is given by the following nested CES structure:

$$C_t = \left[\int_I (c_{it})^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}, \qquad \text{with} \qquad c_{it} = \left[\int_J (c_{ijt})^{\frac{\eta_{it}-1}{\eta_{it}}} dj \right]^{\frac{\eta_{it}}{\eta_{it}-1}} \forall i,$$

where c_{ijt} denotes the consumption of commodity j from island i, c_{it} represents a composite of all the goods of island i, and ρ and η identify the elasticities of substitution, respectively, across and within islands. In equilibrium, ρ ends up controlling the strength of aggregate demand externalities, while η controls the degree of monopoly power. We let $\eta \neq \rho$ so as to isolate the distinct roles of these two forces; we then let η_{it} be random so as to accommodate markup, or cost-push, shocks.

Recall that the representative household receives labor income and profits from all islands in the economy. Its budget constraint is thus given by the following:

$$\int_I \int_J p_{ijt} c_{ijt} dj dj + B_{t+1} \leq \int_I \int_J \pi_{ijt} di dj + \int_I (1 - \tau_{it}) w_{it} n_{it} di + (1 + R_t) B_t + T_t,$$

Here, p_{ijt} is the period-t price of the commodity produced by firm j on island i, π_{ijt} is the period-t nominal profit of that firm, w_{it} is the period-t nominal wage on island i, R_t is the period-t nominal net rate of return on the riskless bond, and B_t is the amount of bonds held in period t.

The variables τ_{it} and T_t satisfy $T_t = \int_I \tau_{it} w_{it} n_{it} di$. One can thus interpret τ_{it} as an island-specific distortionary tax and T_t as the lump-sum transfers needed to balance the budget. Alternatively, we can consider a variant of our model with monopolistic labor markets as in Blanchard and Kiyotaki (1987), in which case τ_{it} could re-emerge as an island-specific markup between the wage and the marginal revenue product of labor. In line with much of the DSGE literature, we can thus introduce exogenous variation in $1 - \tau_{it}$ and interpret this variation as shocks to the "labor wedge".

Finally, the output of firm j on island i during period t is given by

$$y_{ijt} = A_{it} n_{ijt}$$

where A_{it} is the island-specific TFP, and the firm's realized profit is given by $\pi_{ijt} = p_{ijt} y_{ijt} - w_{it} n_{ijt}$.

Information structure. Different authors have motivated informational frictions on the basis of either market segmentation (Lucas, 1972; Lorenzoni, 2009; Angeletos and La'O, 2013) or some form of inattention (Sims, 2003, Mankiw and Reis, 2002, Woodford, 2002). In either case, the key friction is an agent-specific measurability constraint, reflecting the dispersed private information upon which certain economic decisions are conditioned. In this paper we wish to understand the welfare effects of relaxing this constraint, not its possible micro-foundations. Furthermore, we seek to isolate the information about aggregate, as opposed to idiosyncratic, shocks, because it is only the former that have non trivial general-equilibrium effects.

With these points in mind, we assume that the firms and workers of any given island know the local fundamentals, but have incomplete information about the aggregate state of the economy. We then model the available information as a combination of private and public signals and proceed to

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characterize equilibrium welfare as a function of the precisions of these signals. The details, and a justification, are provided in Section 4. For now, we note that the results of Section 3 use only the weaker assumption that the stochastic structure is Gaussian.

3 Equilibrium, Welfare, and Coordination

The equilibrium is defined in a familiar manner: prices clear markets and quantities are (privately) optimal given the available information. Following the same steps as in Angeletos and La'O (2009),[4] one can show that equilibrium output is pinned down by the following fixed-point relation:

$$\chi_{it} V'\left(\frac{y_{it}}{A_{it}}\right) = \frac{1}{\mathcal{M}_{it}} \mathbb{E}_{it}\left[U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\right] A_{it}, \tag{1}$$

where $\mathcal{M}_{it} \equiv \frac{1}{1-\tau_{it}}\frac{\eta_{it}}{\eta_{it}-1}$ measures the overall wedge due to monopoly power, taxes, and/or labor-market distortions, $\mathbb{E}_{it}$ denotes the expectation conditional on the information that is available to island i, and Y_t denotes aggregate output (with $Y_t = C_t$, since there is no capital).

In the absence of informational frictions, condition (1) holds without the expectation operator; in its presence, equilibrium outcomes diverge from their complete-information counterparts insofar as aggregate output, Y_t, is not commonly known. Building on this observation, the following lemma helps reveal a formal connection between the *positive* properties of our model and those of the class of beauty-contest games studied by Morris and Shin (2002), Angeletos and Pavan (2007), and Bergemann and Morris (2013).

Lemma 1. *The equilibrium level of output is pinned down by the following fixed-point relation:*

$$\log y_{it} = \phi_0 + \phi_a a_{it} + \phi_\mu \mu_{it} + \alpha \, \mathbb{E}_{it}[\log Y_t] \tag{2}$$

where ϕ_0, $\phi_a > 0$, and $\phi_\mu < 0$ are scalars, $a_{it} \equiv \log A_{it} - \frac{1}{1+\epsilon}\log\chi_{it}$ and $\mu_{it} \equiv \log \mathcal{M}_{it}$ capture the local shocks, and

$$\alpha \equiv \frac{1-\rho\,\gamma}{1+\rho\,\epsilon} < 1. \tag{3}$$

Condition (2), which is simply a log-linear transformation of condition (1), is formally identical to the best-response condition that characterizes the aforementioned class of beauty-contest games. In the context of these games, the scalar α identifies the degree of strategic complementarity and encapsulates the *private* value of coordination: it measures how much the players in the game (the firms in our model) care to align their actions (their production levels).

In an abstract game, this scalar can be a free variable. In our setting, it is pinned down by the underlying micro-foundations and it reflects the balance of two forces. On the one hand, an increase in aggregate income raises the demand faced by each firm, which stimulates firm profits,

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[4]The characterization of the equilibrium of the baseline model, and a variant of Lemma 1 below, can also be found in Angeletos and La'O (2009). Our contribution starts with the welfare decomposition in Lemma 2.

production, and employment; this effect captures the "aggregate demand externality". On the other hand, an increase in aggregate income discourages labor supply and raises real wages, which has the opposite effect on firm profits, production, and employment. In our view, the most plausible scenario is one in which the former effect dominates, so that $\alpha > 0$. To simplify the exposition, the comparative statics of volatility and dispersion in Proposition 2 focus on this case. However, our key welfare results (Theorems 1, 2, 3, and 4) hold true regardless of the sign and value of α.

Lemma 1 permits one to characterize the positive properties of our baseline model as a direct translation of the positive properties of the aforementioned class of beauty-contest games. For example, one can readily show that a higher α maps to higher sensitivity of equilibrium production to noisy public news and therefore also to higher non-fundamental volatility; this mirrors a similar result in Morris and Shin (2002). Alternatively, following Bergemann and Morris (2013), one can show that the entire set of equilibrium allocations that obtain under arbitrary Gaussian information structures can be spanned with the two-dimensional signal structure we specify in the next section.

None of these facts, however, informs us about the *normative* properties of our model. To understand these properties, we start by developing a certain decomposition of the welfare losses that obtain in equilibrium relative to the first best. Thus let y_{it}^* and Y_t^* denote the first-best levels of, respectively, local and aggregate output, and define the corresponding output gaps by, respectively, $\log \widetilde{y}_{it} \equiv \log y_{it} - \log y_{it}^*$ and $\log \widetilde{Y}_t \equiv \log Y_t - \log Y_t^*$. Next, let

$$\Sigma \equiv \mathrm{Var}\left(\log \widetilde{Y}_t\right) \qquad \text{and} \qquad \sigma \equiv \mathrm{Var}\left(\log \widetilde{y}_{it} - \log \widetilde{Y}_t\right).$$

measure, respectively, the volatility of the aggregate output gap and the cross-sectional dispersion in local output gaps.[5] Finally, consider, as a reference point, the allocation that obtains when the mean wedge $\bar{\mu}$ is chosen so as to maximize welfare and let $\hat{Y}$ denote the mean level of output that obtains in this allocation; this identifies the optimal "steady-state" level of output, which can always be attained with the introduction of an appropriate non-contingent subsidy on employment or income. We can then reach the following characterization of equilibrium welfare.

Lemma 2. *There exists functions $v, w : \mathbb{R}_+ \to \mathbb{R}$, which are invariant to the information structure, such that equilibrium welfare is given by*

$$\mathcal{W} = v(\Delta)w\left(\Lambda\right)$$

where

$$\Delta \equiv \frac{\mathbb{E}[Y]}{\hat{Y}} \qquad and \qquad \Lambda \equiv \Sigma + \frac{1}{1-\alpha}\sigma. \tag{4}$$

Furthermore, $\mathcal{W}$ attains its maximum (the first-best level) at $\Delta = 1$ and $\Lambda = 0$, is strictly concave in Δ and strictly decreasing in Λ.

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[5]In the literature, it is customary to recast σ as a measure of dispersion in relative prices. Such a transformation is valid in our setting but is not needed for our purposes.

To interpret this lemma, note that $v(\Delta)$ captures the welfare loss caused by any distortion in the mean level of economic activity, whereas $w(\Lambda)$ captures the loss due to volatility in the aggregate output gap and/or due to cross-sectional misallocation. The first loss disappears when $\Delta = 1$ (equivalently, $\mathbb{E}[Y] = \hat{Y}$), the second when $\Lambda = 0$ (equivalently, $\Sigma = \sigma = 0$).[6]

This lemma and a set of companion results we provide in Section 5 extend the kind of welfare decompositions that are familiar in the New-Keynesian framework (Woodford, 2003, Gali, 2008) to the incomplete-information economies we are interested in. While these decompositions need not be surprising on their own right, and variants of them have appeared in all the related prior work, they serve two purposes. First, they help identify the different channels through which information can affect welfare. Second, they complete the mapping between the macroeconomic models of interest and the abstract games studied in Morris and Shin (2002), Angeletos and Pavan (2007), and Bergemann and Morris (2013), thus also clarifying whether there is any discrepancy between the private and the social value of coordination in these models.

The first point will become evident as we proceed, especially once we add nominal rigidity. To understand the second point, consider any of the games studied in the aforementioned papers and momentarily recast Σ and σ as, respectively, the volatility and the dispersion of the gaps between the equilibrium and the first-best actions in that game. Following Angeletos and Pavan (2007), the combined welfare loss due to these gaps can be shown to be proportional to the following sum:

$$\Lambda = \Sigma + \frac{1}{1 - \alpha^*}\sigma,$$

where α^* is a scalar that depends on the payoff structure of the game and that encapsulates the *social* value of coordination.[7] In general, this scalar may defer from the one that measures the degree of strategic complementarity, reflecting a divergence between private and social motives to coordinate. In the light of Lemmas 1 and 2, however, our economy maps to a game in which $\alpha = \alpha^*$, meaning that there is no such divergence.[8]

Property 1. *In our setting, the private value of coordination coincides with its social counterpart.*

This property underscores a crucial difference between our setting and that of Morris and Shin (2002): in their game, $\alpha > 0$ but $\alpha^* = 0$, meaning that coordination is socially wasteful. As we

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[6]In addition, we normalize v so that $v(1) = 1$. It follows $w(0)$ coincides with the first-best level of welfare.

[7]Formally, α^* is defined as the degree of strategy complementarity in a fictitious game whose equilibrium strategy coincides with the strategy that maximizes welfare in the economy under consideration; it therefore reflects how much agents *should* care to coordinate, as opposed to how much they actually do care in equilibrium.

[8]The mapping between our model and Angeletos and Pavan (2007) is complicated by the fact that the Δ term can vary with the available information in our setting, a kind of effect that is not accommodated by the linear-quadratic framework of Angeletos and Pavan (2007). This complication turns out to be inconsequential in the case of technology shocks, but not in the case of markup shocks. See Proposition 4 and the discussion surrounding this proposition. Also, for the case of technology shocks, the coincidence of α and α^* was first pointed out in Angeletos and La'O (2009) by comparing directly the equilibrium to the constrained efficient allocation. That paper, however, did not arrive at the precise mapping between the welfare effects of information in our setting and those in Angeletos and Pavan (2007), nor did it consider the extension with nominal rigidity we consider in Section 5.

emphasize in due course, this property is also key to understanding why the *combined* effect of information on Λ turns out to be unambiguous, even though its component effects on volatility and dispersion are ambiguous in general and are often in conflict with one another.

4 The effects of information on volatility, dispersion, and welfare

In this section, we characterize the comparative statics of the volatility measure Σ, the dispersion measure σ, and overall welfare $\mathcal{W}$ with respect to the information structure. We do so by distinguishing two polar cases. In the first, the underlying fundamental uncertainty is over technology or preferences. In the second, it is over monopoly power or labor wedges. The first case captures the scenario in which the business cycle would have been efficient had information been complete; in this case, Σ and σ are non-zero only due to the incompleteness of information. The second case captures the scenario in which the business cycle originates from distortions in product and labor markets; in this case, Σ and σ reflect the combination of the informational friction with such distortions.

Efficient fluctuations. In this part, we fix $\mathcal{M}_{it} = \bar{\mathcal{M}}$ for all (i,t) and concentrate on the case of technology shocks; the case of preference shocks is identical in terms of welfare properties.

To facilitate sharp comparative statics, we specify the stochastic structure of the economy as follows. First, we let local productivity be $a_{it} \equiv \log A_{it} = \bar{a}_t + \xi_{it}$, where $\bar{a}_t$ is the aggregate productivity shock and ξ_{it} is an idiosyncratic productivity shock. The aggregate shock $\bar{a}_t$ is i.i.d. over time, drawn from $\mathcal{N}(0, \sigma_a^2)$, while the idiosyncratic shock ξ_{it} is i.i.d. across both t and i, independent of $\bar{a}_t$, and drawn from $\mathcal{N}(0, \sigma_\xi^2)$. Next, we summarize all of the private (local) information of island i regarding the underlying aggregate shock $\bar{a}_t$ in an island-specific signal x_{it} given by

$$x_{it} = \bar{a}_t + u_{it}, \tag{5}$$

where the noise term u_{it} is i.i.d. across i and t, orthogonal to $\bar{a}_t$, and drawn from $\mathcal{N}(0, \sigma_x^2)$.[9] Similarly, we summarize all of the public (aggregate) information in a public signal z_t given by

$$z_t = \bar{a}_t + \varepsilon_t, \tag{6}$$

where the noise term ε_t is i.i.d. across t, orthogonal to all other shocks, and drawn from $\mathcal{N}(0, \sigma_z^2)$. Finally, to ease notation, we let $\kappa_a \equiv \sigma_a^{-2}$, $\kappa_\xi \equiv \sigma_\xi^{-2}$, $\kappa_x \equiv \sigma_x^{-2}$, and $\kappa_z \equiv \sigma_z^{-2}$.

The subsequent analysis focuses on the comparative statics of the equilibrium volatility, dispersion, and welfare with respect to the scalars κ_x and κ_z, which measure the precisions of, respectively, the available private and public information. When interpreting our results, however, it is worth

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[9]Note that local productivity is itself a private signal of aggregate productivity. The sufficient statistic x_{it} is meant to include this information. More precisely, $x_{it} \equiv (1-\omega)a_{it} + \omega x'_{it}$, where: $x'_{it} = \bar{a} + u'_{it}$ is a signal that captures any private information *other* than the one contained in local productivity; u'_{it} is the noise in that signal, which is i.i.d. across i and t, orthogonal to $\bar{a}_t$ and ξ_{it}, and drawn from $\mathcal{N}(0, \sigma_x'^2)$; $\sigma_x^{-2} \equiv \sigma_\xi^{-2} + \sigma_x'^{-2}$; and $\omega \equiv \sigma_x'^{-2}/\sigma_x^{-2}$.

keeping in mind the following point. As noted before, the results of Bergemann and Morris (2013) guarantee that the equilibrium allocation obtained by *any* Gaussian information structure can always be replicated with an information structure like the one specified above. This means that the adopted specification is without serious loss of generality and that the scalars κ_x and κ_z represent more generally a convenient parameterization of the information structure.

Prior work has often emphasized the different effects that each type of information can have on volatility and dispersion. We thus start by revisiting these effects in the context of our model.

Proposition 1. *(i) An increase in κ_z necessarily reduces dispersion σ, whereas it reduces volatility Σ iff κ_z is high enough. (ii) Symmetrically, an increase in κ_x necessarily reduces Σ, whereas it reduces σ iff κ_x is high enough.*

To understand part (i), note that an increase in the precision of public information induces firms and workers to reduce their reliance on their private signals, which in turn reduces the contribution of idiosyncratic noise to cross-sectional dispersion. At the same time, because these agents increase their reliance on public signals, the contribution of public information to aggregate output gaps is ambiguous: the reduction in the level of the noise itself tends to reduce Σ, while the increased reaction of the agents tends to raise Σ. Which effect dominates depends on how large the noise is, which explains part (i). The intuition for part (ii) is symmetric.

Although each type of information can have a negative effect on either volatility or dispersion, the *combined* welfare effect is unambiguously positive: as shown in the appendix, Λ necessarily decreases with either κ_x or κ_z. Along with the fact that Δ turns out to be invariant to the information structure,[10] this gives us the following result.

Theorem 1. *Suppose the business cycle is driven by technology shocks. Welfare necessarily increases with the precision of either public or private information, no matter the value of α. Moreover, when $\alpha > 0$, the marginal welfare benefit of public information increases with α, that is, $\frac{\partial^2 \mathcal{W}}{\partial \alpha \partial \kappa_z} > 0$.*

As anticipated, this result owes its sharpness to the coincidence of the private and social values of coordination and can thus can be seen as a variant of Proposition 6 in Angeletos and Pavan (2007). If the scalar that governs the relative contribution of volatility and dispersion in Λ were lower from the one that governs the strategic complementarity ($\alpha^* < \alpha$), then *public* information would have a non-monotone welfare effect, in line with the result of Morris and Shin (2002); and if the converse were true ($\alpha^* > \alpha$), then it would be *private* information that would have a non-monotone welfare effect. It is thus Property 1 that explains why both types of information have a similar and unambiguously welfare effect in our setting.[11]

Inefficient fluctuations. We now shift focus to the case of inefficient fluctuations, which we capture with shocks to monopoly markups (or, equivalently, to labor wedges). We thus fix

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[10]The intuition for this particular property is discussed in the context of Proposition 4 below.

[11]Theorem 1 can also be inferred from the result in Angeletos and La'O (2009) that, in the absence of markup shocks, the equilibrium is constrained efficient. This, however, does not apply to Theorems 2, 3, or 4.

$A_{it} = \chi_{it} = 1$ for all (i, t) and let the log of the local wedge be given by

$$\mu_{it} \equiv \log \mathcal{M}_{it} = \bar{\mu}_t + \xi_{it},$$

where $\bar{\mu}_t$ is an aggregate component and ξ_{it} is an idiosyncratic component. The former is i.i.d. across t, drawn from $\mathcal{N}(\bar{\mu}, \sigma_\mu^2)$; the latter is i.i.d. across both t and i, independent of $\bar{\mu}_t$, and drawn from $\mathcal{N}(0, \sigma_\xi^2)$. Finally, we let $\kappa_\mu \equiv \sigma_\mu^{-2}$ and model the information structure in the same way as in the previous section: the available signals are given by (5) and (6), replacing $\bar{a}_t$ with $\bar{\mu}_t$.

In the case of the technology shocks, equilibrium allocations could fluctuate away from the first best *only* because of the incompleteness of information. Here, by contrast, the *entire* variation in equilibrium allocations represents a deviation from the first best, no matter whether this variation originates in the noise or in the fundamentals themselves. The comparative statics of the resulting volatility and dispersion measures are described below.

Proposition 2. *Suppose $\alpha > 0$. (i) Volatility Σ increases with either κ_x or κ_z. (ii) Dispersion σ decreases with κ_z, and is generally non-monotone in κ_x.*

In spite of the possible conflict between the component effects, Property 1 guarantees that the combined effect is once again unambiguous—but now of the opposite sign than in the case of technology shocks.

Proposition 3. *The combined welfare loss due to volatility and dispersion, as captured by Λ, increases with either κ_x or κ_z, no matter α.*

As before, it is useful to relate the above finding to Angeletos and Pavan (2007). Corollary 9 of that paper uses an abstract example in which $\alpha^* = \alpha = 0$ to illustrate the basic insight that information can be detrimental for welfare when it regards shocks that only move the complete-information equilibrium away from the first best. However, by leaving open the possibility that $\alpha^* \neq \alpha$ in workhorse macroeconomic models, and in fact conjecturing that $\alpha < \alpha^*$, that paper also left open the door for ambiguous welfare effects. Similarly to Theorem 1, the above result therefore owes its sharpness to Property 1, the coincidence of the private and social values of coordination.

Welfare depends not only on Λ, which we characterized above, but also on Δ. In the case of technology shocks, Δ was pinned down by the mean wedge $\bar{\mathcal{M}}$, and was invariant to the information structure. Here, instead, Δ varies with the level of noise.

Proposition 4. *Δ increases with either κ_x or κ_z, regardless of α.*

This finding can be explained as follows. The uncertainty that firms face in predicting aggregate demand impacts the mean level of economic activity, due to curvature at both the firm level (curvature of the profit function) and the aggregate level (imperfect substitutability across products). This effect is present irrespective of the nature of the underlying aggregate shocks. Its welfare consequences, however, hinge on the nature of the shocks. In the case of technology

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shocks, the equilibrium use of information is socially optimal and the aforementioned effect does not represent a distortion, which explains why Δ does not vary with the information structure. In the case of markup shocks, instead, the planner would prefer the agents not to respond to the underlying uncertainty and the aforementioned effect is thus associated with an increase in Δ.

Recall that welfare is a strictly convex in Δ, with a maximum attained at $\Delta = 1$. It follows that the aforementioned effect represents a welfare loss when $\Delta > 1$ and a welfare gain when $\Delta < 1$. In the former case, this effect therefore complements the one of Λ. In the latter case, instead, the two effects conflict with each other. Which one dominates then depends on the distance of Δ from the bliss point $\Delta = 1$. Finally, this point is itself attained if the planner has at his disposal a fiscal instrument that permits him to control the mean level of output, such as a non-contingent subsidy on employment, output, or sales. We thus reach the following result.

Theorem 2. *Suppose the business cycle is driven by markup shocks. There exists a threshold $\hat{\Delta} \in (0, 1)$ such that welfare decreases with the precision of either public or private information if and only if $\Delta > \hat{\Delta}$. Furthermore, the latter condition holds, with $\Delta = 1$, if a non-contingent subsidy is available and set optimally.*

It is interesting to note that the threshold $\hat{\Delta}$ is pinned down solely by preference and technology parameters and it is the same whether we consider the effect of private information or that of public information. This kind of symmetry between the two types of information is yet another symptom of Property 1: no matter which case we have considered, this property has guaranteed that the distinction between private and public information is inconsequential for the question of interest.

We conclude this section by noting that is it is customary in the literature to shut down any "steady-state" distortion (that is, to set $\Delta = 1$) by assuming from scratch the presence of the aforementioned non-contingent subsidy. Although the effect on Δ documented above may be of interest in its own right, in the sequel we also opt to abstract from it and, instead, extend the analysis in the direction of adding nominal rigidity and studying the role of monetary policy.

5   Nominal rigidity and monetary policy

In the preceding analysis we isolated the role of the informational friction as a source of real rigidity. We now extend the analysis to the more realistic scenario in which the informational friction is also a source of nominal rigidity: firms set their nominal prices on the basis of the kind of noisy private and public signals that were featured in our preceding analysis.

Setup. As usual, the introduction of nominal rigidity requires that we allow a margin of adjustment in quantities: at least one input must be free to adjust to realized demand, or else markets would fail to clear at the posted prices. Accordingly, we allow for two types of labor: one that is chosen on the basis of incomplete information, thus preserving the type of real rigidity that

was at the core of our baseline model; and another that adjusts freely to the underlying state of nature, thus preserving market clearing in the presence of the nominal rigidity.

More specifically, we assume that the output of the typical firm in island i is now given by

$$y_{it} = A_{it} n_{it}^{\theta} \ell_{it}^{\eta}$$

where n_{it} is the labor input that is chosen on the basis of incomplete information (as in our baseline analysis), ℓ_{it} is the alternative input that adjusts to the realized state (so that markets can clear), and θ and η are positive scalars, with $\theta + \eta \leq 1$. One may think of n_{it} as bodies of employed workers whom the firm hires on the basis of incomplete information and of ℓ_{it} as labor utilization, overtime work, or other margins that adjust to realized demand. The precise interpretation of these inputs, however, is not essential. Rather, the key is that this specification helps accommodate the combination of the two types of rigidity we are interested in. Another useful feature of this specification is that it permits us to nest the scenario studied in prior work as the limit case in which $\theta = 0$ (meaning that all output is free to adjust to the state of nature).

We next let the per-period utility of the representative household be given by the following sum:

$$\frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{1}{1+\epsilon_n} \int_I n_{it}^{1+\epsilon_n} di - \frac{1}{1+\epsilon_\ell} \int_I \ell_{it}^{1+\epsilon_\ell} di,$$

where ϵ_n and ϵ_ℓ are positive scalars that parameterize the Frisch elasticities of the two types of labor. To simplify the algebra, and without serious loss, we let $\epsilon_n = \epsilon_\ell = \epsilon$.

Consider now the specification of monetary policy. In general, this opens the door to delicate modeling issues. What is the information upon which the monetary authority acts? Does this contain only signals of the exogenous shocks or also signals of endogenous economic outcomes? What are the objectives, targets, or policy rules that guide the policy maker? How one chooses to answer these questions is bound to affect the welfare properties of the model. In what follows, we develop a taxonomy that seeks to dissect the role of different monetary policies, without however getting into the granular details of how policy is conducted.

We assume that the policy instrument is the nominal interest rate and, to start with, allow the latter to follow a possibly arbitrary stochastic process. We only require that this process is log-normal in order to maintain the Gaussian structure of the equilibrium. Following the tradition of the Ramsey literature, we then follow an approach that permit us to span directly the set of all the allocations that can obtain in equilibrium under such an arbitrary monetary policy. The benefit of this approach is its flexibility; the cost is that it suppresses the question of what exactly it takes for the policy maker to be able to implement a particular allocation.

To economize on space, the characterization of the set of allocations that can be implemented with arbitrary monetary policies is delegated to the appendix. (See Section B.1 and Lemmas 6-8 in Appendix B.) To facilitate the subsequent analysis, we nevertheless need a "topography" of this set, that is, a way to index the different points in it. We provide such a topography in the next lemma.

Lemma 3. *(i) In any equilibrium, nominal GDP satisfies*

$$\log M_t = \lambda_s \bar{s}_t + \lambda_z z_t + m_t, \tag{7}$$

where λ_s and λ_z are scalars, $\bar{s}_t$ stands for either the technology or markup shock, and m_t is a random variable that is drawn from $\mathcal{N}(0, \sigma_m^2)$, for some $\sigma_m \geq 0$, and is orthogonal to both $\bar{s}_t$ and z_t.

(ii) Suppose that the interest rate satisfies

$$\log(1 + R_t) = \rho_s \bar{s}_t + \rho_z z_t + r_t, \tag{8}$$

where ρ_s and ρ_z are scalars and r_t is a random variable that is drawn from $\mathcal{N}(0, \sigma_r^2)$ for some $\sigma_r \geq 0$, and is orthogonal to both $\bar{s}_t$ and z_t. For any triplet $(\lambda_s, \lambda_z, \sigma_m)$, there exists a monetary policy as in (8) such that (7) holds in the equilibrium induced by this policy.

(iii) A policy as in (8) can replicate the equilibrium allocation induced by any other policy.

Part (i) follows from regressing the equilibrium value of nominal GDP on the fundamental and the public signal, and letting (λ_s, λ_z) be the projection coefficients and m_t the residual. This part is therefore trivial, but it is useful for our purposes because in conjunction with the rest of the lemma it permit us to index different equilibria with different values for the triplet $(\lambda_s, \lambda_z, \sigma_m)$. Parts (ii) and (iii) then provide us with a class of monetary policies that can implement *any* value for this triplet and that span the entire set of the allocations that obtain under arbitrary monetary policies.

Although it is possible to interpret condition (8) as a policy rule, it is also possible to arrive to it from a different specification of how policy is conducted. For instance, suppose that the monetary authority follows the following Taylor rule:

$$\log(1 + R_t) = r_z z_t + r_y(\log Y_t + \epsilon_t^y) + r_p(\log P_t + \epsilon_t^p) + \tilde{r}_t,$$

where (r_z, r_y, r_p) are policy coefficients, ϵ_t^y and ϵ_t^p are measurement errors in the monetary authority's observation of real output and the price level, and $\tilde{r}_t$ is a monetary shock. Once one solves for equilibrium output and prices, the above reduces to condition (8), with the scalars $(\rho_{\bar{s}}, \rho_z)$ being functions of the policy coefficients (r_z, r_y, r_p) and the random variable r_t being a mixture of the monetary shock $\tilde{r}_t$ and the measurement errors $(\epsilon_t^y, \epsilon_t^p)$. In a nutshell, condition (8) can always be recast as a representation of the equilibrium implemented by any given policy rule.

Furthermore, although condition (8) requires that the interest rate react to the current technology or markup shock, such a contemporaneous reaction is not strictly needed for the policy maker to implement a particular response of macroeconomic activity to the shock. Rather, it suffices that monetary policy reacts at some point in the future: Lemma 7 in Appendix B establishes that the entire set of implementable allocations remains the same whether monetary policy responds within the same period or with an arbitrary lag. The reason is simple: the power of the monetary

authority to control real allocations rests on the dependence of aggregate demand on the nominal interest rate, but it makes no difference whether the desired movements in aggregate demand are implemented by moving the current interest rate or by committing to move future rates.

These points underscore that conditions (7) and (8) are equivalent representations of all the equilibrium allocations that can obtain under arbitrary monetary policies. We have found (7) to be most convenient for our purposes, for reasons that will become evident in the statement of the formal results in this section, as well as in the discussion of the related literature in the next section.[12]

To close the model, we must specify the information upon which firms can condition their production and pricing decisions. As in the baseline model, we assume that this is summarized by a pair of signals about the underlying fundamental: the public signal z_t and the private signal x_t. We proceed to investigate the comparative statics of welfare with respect to the corresponding precisions, κ_x and κ_z. Note that this rules out the possibility that the firms also have information about the shock m_t, which can be interpreted as a monetary shock. This alternative kind of information is the subject matter of Hellwig (2005) and is briefly discussed at a later point.

A familiar benchmark. Consider, as a reference point, the hypothetical scenario in which the nominal rigidity is removed, by which we mean the case in which p_{it} is free to adjust to the realized state. This scenario is henceforth referred to as "flexible prices" and the equilibrium allocation that obtains under it as the "flexible-price allocation." The next lemma identifies a set of monetary policies that implement this allocation when the nominal rigidity is present.

Lemma 4. *There exists a λ_s^* and a ρ^* such that a monetary policy replicates flexible prices if (7) holds with $\lambda_s = \lambda_s^*$ and $\sigma_m = 0$ or, equivalently, if (8) holds with $\rho_s = \rho^*$ and $\sigma_r = 0$.*

This lemma is a special case of a more general result in Angeletos and La'O (2014): just as in the baseline New-Keynesian framework there are monetary policies that can undo the nominal rigidity induced by Calvo-like sticky prices, in the class of incomplete-information models studied here (and in related papers) there are monetary policies that can undo the nominal rigidity induced by informational frictions. These policies presume that the policy maker can observe the aggregate state perfectly, although perhaps with a time lag, and that she has perfect control over aggregate demand. They are therefore not meant to be realistic. Nevertheless, they represent a useful benchmark, separating the informational friction of the market from any friction on the policy maker's side, and facilitating sharp welfare conclusions.[13]

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[12]Some papers, such as Woodford (2002) and Hellwig (2005), treat M_t as an exogenous random variable. Others, such as Baeriswyl and Conrand (2010), assume that the policy instrument is M_t rather than the interest rate. Our approach can accommodate both these possibilities, but is not limited to them.

[13]Our analysis also abstracts from any interference the nominal rigidity may have with the response to sectoral or idiosyncratic shocks, or from other types of relative-price distortions that monetary policy may be unable to correct even under the assumption that the policy maker observes perfectly the state of the economy.

When these policies are in place, information matters for welfare only through the real rigidity. This suggests a possible connection to our baseline analysis, which we formalize next. Let $q_{it} \equiv A_{it} n_{it}^{\theta}$ denote the component of output that is determined on the basis of incomplete information. Next, define the corresponding aggregate as

$$Q_t \equiv \left[\int_I (q_{it})^{\frac{\hat{\rho}-1}{\hat{\rho}}} di \right]^{\frac{\hat{\rho}}{\hat{\rho}-1}},$$

and finally let

$$\hat{\alpha} \equiv \frac{1 - \hat{\rho}\,\hat{\gamma}}{1 + \hat{\rho}\,\hat{\epsilon}}, \tag{9}$$

where $\hat{\epsilon} \equiv \frac{1+\epsilon-\theta}{\theta}$, $\hat{\gamma} \equiv 1 - \frac{(1-\gamma)(1+\epsilon)}{1+\epsilon-\eta(1-\gamma)}$ and $\hat{\rho} \equiv \frac{\rho(1+\epsilon-\eta)+\eta}{1+\epsilon+\eta(1-\rho)}$ are transformations of the underlying preference and technology parameters. (One can verify that $\hat{\alpha} < 1$.) We can then obtain the following characterization of the flexible-price allocation.

Proposition 5. *There exist scalars $\hat{\phi}_a > 0, \hat{\phi}_\mu < 0$, and $\hat{\phi}_{\bar{\mu}}$, and a decreasing function w, such that the following are true at the flexible-price allocation for any information structure:*

(i) The equilibrium value of q_{it} is determined by the solution to the following fixed-point relation:

$$\log q_{it} \;=\; \hat{\phi}_a \, a_{it} \;+\; \hat{\phi}_\mu \, \mu_{it} \;+\; \hat{\phi}_{\bar{\mu}} \, \mathbb{E}_{it}[\bar{\mu}_t] \;+\; \hat{\alpha} \, \mathbb{E}_{it}[\log Q_t], \tag{10}$$

(ii) Welfare is given by $\mathcal{W} = w(\Lambda)$, where

$$\Lambda = \Sigma + \frac{1}{1-\hat{\alpha}} \sigma + \omega, \tag{11}$$

where Σ and σ are defined in the same way as in the baseline model, modulo replacing output y with the component q defined above and the scalar α with the scalar $\hat{\alpha}$, and where ω is a scalar that does not depend on either κ_x or κ_z and that vanishes in the absence of markup shocks.

This proposition extends Lemmas 1 and 2 from our baseline model to the flexible-price allocation of the extended model. If we compare condition (10) to the corresponding condition in the baseline model (2), we see three differences. First, q_{it} and Q_t have taken the place of, respectively, y_{it} and Y_t. This is because it is only q_{it} (or n_{it}), not y_{it} (or ℓ_{it}), that is restricted to depend on incomplete information. Second, the hatted scalars $\hat{\alpha}$, $\hat{\rho}$, etc. have taken the place of the corresponding un-hatted scalars in the baseline model. This reflects the more general specification of preferences and technologies allowed in the extended model. Finally, a new term has emerged: in addition to the firm's own markup, the expected aggregate markup enters the firm's best-response condition. This is because the realized aggregate markup affects the realized aggregate output for any given Q_t, implying in turn that a firm's optimal choice of q_{it} depends directly on its expectation of $\bar{\mu}_t$.

A new term shows up also in the definition of Λ. Even if we hold constant the firms' ex-ante input choices (and therefore the allocation of q_{it}), the realized markup distorts the firms' ex-post choices (namely, ℓ_{it}). This explains why Λ contains not only the terms Σ and σ but also the term ω in condition (11), which is proportional to the volatility of the markup shock.

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Notwithstanding these differences in the micro-foundations and the interpretations of conditions (10) and (11), the scalar $\hat{\alpha}$ that captures the strategic complementarity in the one condition continues to determine the relative welfare costs of volatility and dispersion in the other. As in the baseline model, this facilitates a direct mapping to the taxonomy of games developed in Angeletos and Pavan (2007) and proves the following point, which has already been anticipated.

Property 2. *The private and the social value of coordination coincide as long as monetary policy replicates flexible prices.*

This observation leads once again to a sharp answer to the question of interest. Insofar as monetary policy replicates flexible prices, ℓ_{it} choice adjusts to the realized supply and demand conditions *as if* information were complete. The ω term in (11) is therefore invariant to κ_x and κ_z, and can be ignored for our purposes. Consider now the case of technology shocks. In this case, q_{it} solves *exactly* the same fixed-point relation as y_{it} did in the baseline model, guaranteeing that the mapping from the information structure to the equilibrium values of Σ, σ, and Λ are also the same, modulo the replacement of the scalars α and ϕ_a with their hatted counterparts. When instead the business cycle is driven by markup shocks, the presence of the aggregate markup in condition (10) breaks the equivalence between the two models. Nevertheless, because this term is just a different facet of the distortionary effects of markup shocks, it does not interfere with the essence of the lessons of the baseline model. We thus arrive to the following extension of Theorems 1 and 2.

Theorem 3. *Suppose that monetary policy replicates flexible prices. Welfare increases with both private and public information in the case of technology shocks, and decreases with both in the case of markup shocks.*

Away from the benchmark. We now turn to policies, and equilibria, that deviate from replicating flexible prices. Following Lemma 3, any such deviation can contain at most three components: one that is correlated with the underlying fundamental; one that is correlated with the public signal; and a residual, which can be interpreted as a monetary shock. The second component has no welfare consequences, reflecting the fact that the response of monetary policy to public information is predictable when firms set prices and can thus have no real effect. We thus reach the following decomposition of welfare for *any* monetary policy.

Proposition 6. *There exists a decreasing function w such that, for any monetary policy and any information structure, the equilibrium level of welfare is given by*

$$\mathcal{W} = w(\Lambda + \mathcal{K} + \mathcal{T}),$$

where the following are true:
    (i) Λ is the welfare loss at the flexible-price allocation.

(ii) $\mathcal{K}$ is the welfare effect of $\lambda_s \neq \lambda_s^*$, which can be expressed as follows:

$$\mathcal{K} = \mathcal{K}(\lambda_s) \equiv \begin{cases} \Theta(\lambda_s - \lambda_s^*)^2 \sigma_a^2 & \text{in the case of technology shocks} \\ -2\Theta_1(\lambda_s - \lambda_s^*)\sigma_\mu^2 + \Theta_2(\lambda_s - \lambda_s^*)^2\sigma_\mu^2 & \text{in the case of markup shocks} \end{cases} \tag{12}$$

where Θ, Θ_1, and Θ_2 are scalars, which depend on (κ_x, κ_z) and vanish as $\kappa_x \to \infty$ or $\kappa_z \to \infty$.

(iii) $\mathcal{T}$ is the welfare loss caused by $\sigma_m \neq 0$.

This result complements our earlier welfare decompositions and formalizes the sense in which the two forms of rigidity map into two channels through which information affects welfare: the role of the real rigidity is captured by Λ; the additional effect of the nominal rigidity is captured by the sum $\mathcal{K} + \mathcal{T}$. This sum is non-zero only insofar as monetary policy deviates from the benchmark of replicating flexible prices. By contrast, Λ is necessarily positive.[14]

As already mentioned, $\mathcal{T}$ captures the welfare consequences of m_t, or equivalently of r_t. The latter represents a deviation that is orthogonal to both the underlying fundamental and the public signal–a deviation that can be interpreted as a monetary shock or a policy "mistake". Whatever the interpretation, $\mathcal{T}$ is necessarily non-negative and independent of the available information about $\bar{s}_t$. By contrast, $\mathcal{K}$ depends on that information precisely because it captures the deviations that are correlated with $\bar{s}_t$. Furthermore, both the sign and the comparative statics of this term with respect to the available information depend on the nature of the underlying business-cycle forces.

From part (ii), we see that the minimum of $\mathcal{K}$ is zero and it is attained at $\lambda_s = \lambda_s^*$ when the business cycle is driven by technology shocks, whereas it is positive and it is attained at $\lambda_s \neq \lambda_s^*$ when the business cycle is driven by markup shocks. This verifies that a familiar policy lesson extends from the New-Keynesian setting to the present framework.

Property 3. *A monetary policy that replicates flexible prices is optimal in the case of technology shocks, but not in the case of markup shocks.*

Notwithstanding the similarity to the New-Keynesian framework, the following difference is worth mentioning in the case of technology shocks: unless the real rigidity is shut down ($\theta = 0$), replicating flexible prices does *not* implement the first-best allocation, nor is it synonymous to targeting price stability.[15] Turning to the case of markup shocks, certain deviations from the flexible-price allocation are welfare-improving because they substitute for a missing tax instrument, namely the state-contingent subsidy that would have offset the markup shock. The only key difference from the New-Keynesian framework then is that, since the nominal rigidity originates in an informational friction rather than Calvo-like sticky prices, the ability of the monetary authority to counter the markup shock hinges on its ability to respond to information that is not available to the firms when the latter set their prices. This suggests that more precise information in the firms' hands may contribute towards lower welfare not only by reducing the "base" level of welfare that

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[14]With the exemption, of course, of the extreme case in which $\theta = 0$ (no real rigidity), in which case $\Lambda = 0$.

[15]For more details on these points, see Angeletos and La'O (2014).

obtains in the flexible-price allocation but also by limiting the ability of the monetary authority to counteract the markup shock.[16] As it turns out, this intuition is only partially correct.

Lemma 5. *Suppose the business cycle is driven by markup shocks. The optimal policy corresponds to $\lambda_s = \lambda_s^{**}$ and $\sigma_m = 0$, where $\lambda_s^{**} \equiv \arg\min_{\lambda_s} \mathcal{K}(\lambda_s)$. Let $K(\kappa_x, \kappa_z)$ be this minimum.*

(i) Suppose $\theta = 0$. Then, $K(\kappa_x, \kappa_z)$ is increasing in both κ_x and κ_z

(ii) Suppose $\theta > 0$. There are values of the preferences and technology parameters for which $K(\kappa_x, \kappa_z)$ is non-monotone in either κ_x or κ_z.

(iii) Suppose $\theta > 0$ and let $\bar{K}(\kappa, \varrho)$ be the function defined by $K(\kappa_x, \kappa_z)$ along the locus of (κ_x, κ_z) such that $\kappa_x + \kappa_z = \kappa$ and $\frac{\kappa_x}{\kappa_\mu + \kappa_z} = \varrho$. Then, $\bar{K}(\kappa, \varrho)$ increases in κ, but is non-monotone in ϱ.

Part (i) verifies the aforementioned intuition in the special case in which the real rigidity is absent; as already mentioned, this is the case considered in prior work. Part (ii) establishes that the intuition can be overturned once the real rigidity is present. Part (iii) concludes by providing a qualified variant of the intuition that holds true irrespective of the real rigidity: any additional information at the hands of the firms necessarily reduces the welfare contribution of the optimal monetary policy if that comes without a change to the degree to which information is common (meaning an increase in the overall precision, κ, without a change in the relative precision, ϱ).

We do not fully comprehend the non-monotonicities documented in part (ii). At this point, what we know for sure, thanks to parts (i) and (iii), is only that these non-monotonicities derive exclusively from the interaction of the real rigidity with the degree to which information is correlated across the firms.[17] We thus reach the following summary.

Theorem 4. *Suppose that monetary policy is optimal.*

(i) When the business cycle is driven by technology shocks, more information improves welfare by, and only by, improving the efficiency of the underlying flexible-price allocation.

(ii) When instead the business cycle is driven by markup shocks, more information contributes to lower welfare both by exacerbating the inefficiency of the underlying flexible-price fluctuations and by reducing the ability of the monetary authority to combat these fluctuations. Nevertheless, when and only when the real rigidity is present, an ambiguous effect can obtain with changes in the composition of information.

This result refers to the solution of a Ramsey problem where the planner is free to select an arbitrary Gaussian process for the interest rate or, equivalently, to induce any triplet $(\lambda_s, \lambda_z, \sigma_m)$ he

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[16]The latter possibility is also highlighted in Baeriswyl and Cornand (2010), albeit in a model that adds a certain friction in the conduct of monetary policy. As explained in Section 6, this friction is the key to understanding why that paper does not reach the kind of unambiguous result we obtain in part (i) of Lemma 5 below.

[17]In part (iii), the ratio ϱ is defined as the ratio of the precision of the private information to that of the public information. However, following Angeletos and Pavan (2007) and Bergemann and Morris (2013), this ratio can be interpreted more generally as a measure of the extent to which information is correlated across the agents.

wishes in condition (7). We now discuss what happens when the monetary policy falls short of this "unconstrained" optimum. This could be because the monetary authority has imperfect control of aggregate demand, because it observes the underlying shocks and/or the endogenous economic outcomes with noise, because its objectives diverge from the welfare criterion in the model, or because of any other reason that is left outside our model.

Not surprisingly, not much can be said if one puts no structure whatsoever on the deviation from optimality. To understand the logic, let us concentrate on the case of technology shocks. When the nominal rigidity is shut down, the informational friction represents a real distortion that moves the equilibrium away from the first best. Increasing the precision of the available information necessarily reduces the welfare cost of this distortion. When this is the only distortion, more information is unambiguously welfare-improving. But if an additional distortion is present due to the combination of nominal rigidity and suboptimal monetary policy, a second-best result applies: reducing information may increase welfare by having one distortion offset the other.

The opposite scenario, however, is also possible and seems relevant for the following reason. To the extent that monetary policy is guided by standard New-Keynesian lessons, the policy maker may fail to incorporate how the informational friction affects the nature of the optimal allocation and the corresponding policy targets. In so doing, the policy maker may inadvertently introduce distortions in addition to those induced by the informational friction. But when the latter vanishes, the policy maker's "mistake" also vanishes. Under this scenario, more precise information may help increase welfare not only by attenuating the real rigidity but also by alleviating the policy suboptimality. We illustrate this logic in Section B.3 of Appendix B, with a numerical example that examines the welfare implications of policies that target either price-level or output-gap stabilization.

Monetary shocks. Consider now $\mathcal{T}$, the welfare term corresponding to deviations from the flexible-price benchmark that are orthogonal to the fundamental and the public signal. As already noted, these deviations can be interpreted as monetary shocks. So far, we have assumed that the firms have no information about them. But now suppose the contrary. How does this distinct type of information matter for welfare?

This question is the subject matter of Hellwig (2005). The answer is as follows. Consider first the case of public information. In equilibrium, any *commonly* predictable variation in m_t can have no real effect. Furthermore, any residual variation in m_t necessarily contributes to welfare losses, because m_t is orthogonal to the underlying preferences and technologies. It follows that more precise public information about m_t necessarily improves welfare (it reduces $\mathcal{T}$).

Consider next the case of private information. As with public information, private information dampens the aggregate real effect of any given monetary shock. But unlike public information, private information does so in an imperfect manner, because the lack of common knowledge hinders the coordination of the firms' pricing decisions. At the same time, an increase in the precision of private information can exacerbate the cross-sectional misallocation of resources. It follows that

more precise private information about monetary shocks can have a non-monotone welfare effect. We refer the reader to Hellwig (2005) for a more detailed analysis of this particular effect.

Remarks. We conclude this section with four remarks regarding the possible endogeneity of the information structure and and applicability of our results.

1. Although our analysis has treated the information structure of the firms as an exogenous object, this does not necessarily limit the usefulness of our results. Suppose, for example, that some of the available public information obtains from the release of macroeconomic indicators or from policy actions. Alternatively, suppose that the available private information is the product of costly information acquisition, i.e. the informational friction is a symptom of inattention.[18] Under these scenarios, the precisions κ_x and κ_z become endogenous to the behavior of the firms, as well as to that of the monetary authority. How this endogeneity shapes the mapping from "deeper" parameters to the precisions κ_x and κ_z, or how it impacts the nature of the optimal monetary policy, is beyond the scope of this paper. We refer the reader to Amador and Weill (2010), Baeriswyl and Conrand (2010), and Paciello and Wiederholt (2013) for certain explorations in this direction. But no matter how κ_x and κ_z are determined in the first place, the anatomy of the welfare effects of information that our paper has offered remains valid in the following regard: the mapping from the precisions (κ_x, κ_z) to equilibrium welfare is invariant to the micro-foundations of the former.

2. The reinterpretation of the informational friction in terms of limited attention also explains why the policy maker may be unable to eliminate the friction even if she happens to know the state and can reveal it to the firms before the latter make their production and pricing decision. But even if the informational friction cannot itself be eliminated, whether it ultimately has a bite on real allocations and welfare still hinges on whether the rigidity is real or nominal: when the rigidity is only nominal, a monetary policy that replicates flexible prices, and *only* this policy, guarantees that the friction ceases to have a bite on welfare. This underscores, once again, the distinct normative implications of the two rigidities and the pivotal role of monetary policy vis-à-vis the nominal one.

3. The two policy benchmarks we characterized in Theorems 3 and 4 require that the policy maker observe the state of the economy. But as already mentioned, they do not presume that policy maker has an informational advantage over the firms at any time. It suffices that the policy maker commits to act in the future, after the state of the economy has become public information, provided of course that the market expects this to happen. This underscores the role of the policy maker in "managing expectations".

4. Our analysis has orthogonalized the information structure in three dimensions: one corresponding to preference and technologies; one corresponding to monopoly markups and other real distortions; and one corresponding to pure monetary shocks. When translating our results to cer-

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[18]One can then contemplate various micro-foundations of how agents allocate attention. See, e.g., Pavan (2014) for a flexible approach. But as long as one maintains a Gaussian specification (which is the golden standard in the related literature), the results of Bergemman and Morris (2013) imply that the resulting equilibrium allocations can always be replicated with an information structure like the one we assume.

tain applied contexts, however, it may be natural to consider signals that confound two or more of these distinct types of information. For instance, to the extent that business cycles are driven by a mixture of technology, markup, and monetary shocks, macroeconomic statistics will serve as a mixed signal of all these shocks, and their combined welfare effect will itself be a mixture of the effects we have documented. A similar point applies to central-bank communications insofar as they may contain only an overall assessment of the state of the economy. But if central-bank communications contain detailed explanations of the data and of the reasons underlying certain policy actions in a manner that informs the public about the underlying nature of the shocks, then such communications are more likely to have an unambiguous welfare effect along the lines of our theorems.

6   Related literature

In this section we revisit the prior works of Hellwig (2005), Walsh (2007), Lorenzoni (2010), and Baeriswyl and Cornand (2010), under the lens of our analysis.

As already noted, the aforementioned works rule out real rigidity, a scenario nested in our framework by letting $\theta = 0$. In this limit case, a monetary policy that replicates flexible prices implements the complete-information allocation. This is because the absence of real rigidity guarantees that the informational friction ceases to have a bite on real allocations and welfare once the "right" monetary policy is in place. Of course, such a policy may be unrealistic or undesirable. But the point we wish to make, for pedagogical reasons, is that the welfare effects reported in these prior works hinge entirely on deviations from the benchmark of replicating flexible prices: had monetary policy replicated flexible prices, the complete-information outcomes would have obtained.

Consider Hellwig (2005). Because that paper models M_t as an exogenous process and rules out shocks in preferences and technologies, all the volatility in M_t represents a monetary shock of the type described in Lemma 3. As noted in the previous section, the exercise conducted in Hellwig (2005) therefore boils down to studying the comparative statics of $\mathcal{T}$ with respect to the information that firms have about this shock, and the key unexpected finding is that private information has a non-monotone effect on relative-price dispersion and thereby on welfare as well. But unlike the intuition proposed in Section 6.3 of Angeletos and Pavan (2007), these non-monotonicities do not reflect any kind of *innate* discrepancy between the private and the social value of coordination.

Consider next Walsh (2007). That paper allows monetary policy to react systematically to shocks in preferences and markups (which Walsh interprets as, respectively, "demand shocks" and "cost-push shocks"). A deviation from the unconstrained optimum we studied in Section 5, however, obtains because of two types of policy frictions: the restriction that monetary policy can respond only to contemporaneous and noisy signals of the state of the economy, and the assumption that policy objectives differ from the model's ex-ante utility. Unable to obtain analytic results, Walsh employs numerical simulations and arrives at a somewhat inconclusive answer to the question of

interest, namely the welfare effects of central-bank transparency. If instead one abstracts from the aforementioned policy frictions, a particularly sharp answer becomes available on the basis of our results: maximal transparency is desirable in the face of benign forces such as technology shocks, maximal opacity is desirable in the face of distortionary forces such as markup shocks.

Consider next Lorenzoni (2010). In that paper, there are no markup shocks, the policy maker observes perfectly the state of the economy (with a lag), and monetary policy is set so as to maximize welfare. On the basis of our results, one may have expected monetary policy to replicate flexible prices and, in conjunction with the right non-contingent subsidy, to implement the first best. However, this is not the case because of the presence of an additional friction, a certain segmentation in consumer markets: each firm is matched with a random subset of consumers in the economy, and each consumer gets to see only a random subset of the prices in the economy. In the presence of this friction, a policy that implements the first-best response to the underlying aggregate shocks is still feasible, yet a distortion remains because prices cannot adjust to the idiosyncratic shocks induced by the random matching between the firms and the consumers. It is this distortion that drives the distinct welfare results reported in that paper.

Finally, consider Baeriswyl and Cornand (2010). Under the lens of our analysis, this paper makes two key assumptions. First, it requires that nominal GDP, $M_t \equiv P_t Y_t$, satisfy

$$\log M_t = \lambda_a(\bar{a}_t + \epsilon_{a,t}) + \lambda_\mu(\bar{\mu}_t + \epsilon_{\mu,t}), \tag{13}$$

where $\bar{a}_t$ and $\bar{\mu}_t$ are the underlying technology and markup shocks, $\epsilon_{a,t}$ and $\epsilon_{\mu,t}$ are exogenous noises, and λ_a and λ_μ are scalars under the control of the monetary authority. Second, it allows each firm to observe a noisy private signal of $\log M_t$. Baeriswyl and Cornand (2010) interpret M_t as the policy instrument, condition (13) as a policy rule, the noises $(\epsilon_{a,t}, \epsilon_{\mu,t})$ as measurement errors in the policy maker's *contemporaneous* observation of the underlying shocks, and the firms' signal of M_t as a signal of the policy action. The key contribution of that paper is then to study how the signaling role of monetary policy interacts with its stabilization role.

Our results qualify that paper's analysis in the following regard. Interpreting (13) as the policy rule overlooks the ability of the monetary authority to control current outcomes by committing to move interest rates in the future. Such commitment would not only improve the stabilization role of monetary policy by utilizing additional information that may arrive in the future but would also mute the signaling effect of current policy actions. The absence of commitment is therefore a key unstated assumption (although probably a realistic one) behind the core result of that paper regarding the tradeoff between the stabilization and the signaling roles of monetary policy.

Putting aside this point and the interpretation of condition (13), this condition represents a restriction on the set of implementable allocations. This restriction drives the optimal policy in that paper away from the unconstrained optimum we characterized in Section 5. This fact in turn is the key to understanding why welfare in that paper depends on the firms' information about the technology shock despite the absence of real rigidity, as well as why welfare is a non-monotone

function of the firms' information about the markup shock, in contrast to the monotone effect we obtained in part (i) of Lemma 5. Finally, because condition (13) is nested in Lemma 3 of our paper by letting $m_t = \lambda_a \epsilon_{a,t} + \lambda_\mu \epsilon_{\mu,t}$, the welfare effects of the signal that a firm receives about M_t can be understood under the lens of our analysis as the mixture of three kinds of information: information about the technology shock; information about the markup shock; and information about the policy "mistake" caused by measurement error.[19]

Let us close this section by noting the obvious: none of the preceding discussion is meant to downplay the contribution the aforementioned papers. The mechanisms that have been identified by these papers seem both intriguing and relevant. We nevertheless hope that our discussion has shed additional light on the inner workings of these mechanisms and on the assumptions that underly them, thus also illustrating more generally how our paper can facilitate a useful anatomy of the welfare effects of information in baseline macroeconomic models.

7   Conclusion

By assuming away incomplete information and strategic uncertainty, standard macroeconomic models presume that firms can perfectly coordinate their production and pricing decisions. By contrast, in this paper we allow an informational friction to inhibit this coordination and we study how this shapes the social value of information within an elementary business-cycle model. The key lessons can be summarized as follows:

- The welfare effects of information can be decomposed into two channels: the real rigidity that emerges as firms make production choices on the basis of incomplete information and the nominal rigidity that emerges as firms also set prices on the basis of such information.

- The first channel is present irrespective of the conduct of monetary policy. It also has sharp comparative statics: more information is welfare-improving through this channel if the business cycle is driven by "benign" forces such as technology shocks and welfare-deteriorating if it is driven by distortionary forces such as markup shocks.

- By contrast, the second channel hinges on the conduct of monetary policy. As in the New-Keynesian framework, there is a policy that neutralizes the nominal rigidity. At the flexible-price benchmark, the welfare effects of information are shaped solely by the real-rigidity channel. Away from it, they hinge on whether the provision of more information dampens or amplifies the deviation of monetary policy and on whether that deviation was desirable to begin with.

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[19]The welfare effects of the first two types of information are those mentioned above. The third one has a non-monotone effect for the reason first explained in Hellwig (2005).

- When the business cycle is driven by technology shocks, a monetary policy that replicates flexible prices is optimal. When, instead, the business cycle is driven by markup shocks, a deviation from this benchmark is desirable. More information then tends to decrease welfare not only because it exacerbates the inefficiency of the underlying flexible-price fluctuations but also because it curtails the monetary authority's ability to combat these fluctuations.

We view the sharpness of these lessons and their close connection to familiar normative properties of RBC and New-Keynesian models as the main strengths of our contribution. This sharpness, however, comes at a cost. By narrowing the analysis within the context of an elementary model, we preclude any quantitative assessment. By treating the information structure as exogenous, we bypass the question of either how information gets collected or what policy instruments can affect it. Finally, while we allow the informational friction to inhibit the coordination of production and pricing decisions of firms, we assume away any such friction in, say, the consumption and saving choices of households or the trades of financial investors. The bite of incomplete information on the social efficiency of the latter kind of economic decisions is an important research question; it is, however, beyond the scope of this paper.

Appendix A: Proofs for the Baseline Model

This appendix contains the proof of all the results that appear in Sections 3-4.

Derivation of equation (1). Let p_{it} be the price index for the consumption basket of the goods produced in island i. The optimal consumption decision satisfy the following conditions:

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\rho} C_t,$$

for the aforementioned basket; and

$$c_{ijt} = \left(\frac{p_{ijt}}{p_{it}}\right)^{-\eta_{it}} c_{it},$$

for the particular good produced by firm j in island i. In equilibrium, consumption coincides with production. It follows that the inverse demand function faced by firm j in island i is given by

$$p_{ijt} = D_{it} \, y_{ijt}^{-\frac{1}{\eta_{it}}}, \tag{14}$$

where

$$D_{it} \equiv p_{it} y_{it}^{\frac{1}{\eta_{it}}} = P_t Y_t^{\frac{1}{\rho}} y_{it}^{\frac{1}{\eta_{it}} - \frac{1}{\rho}}$$

is exogenous to the firm but endogenous to the island.

Consider now the optimal behavior of the aforementioned firm. Given that marginal value of (nominal) income for the representative household is $U'(Y_t)/P_t$, the firm's objective is simply the local expectation of its profit times $U'(Y_t)/P_t$. Using (14), this can be expressed as follows:

$$\mathbb{E}_{it}\left[\frac{U'(Y_t)}{P_t}\left(D_{it} y_{ijt}^{1-\frac{1}{\eta_{it}}} - w_{it} n_{ijt}\right)\right]$$

Using $y_{ijt} = A_i n_{ijt}$ and taking the FOC with respect to n_{ijt} gives

$$\mathbb{E}_{it}\left[\left(1 - \frac{1}{\eta_{it}}\right) A_{it} U'(Y_t) \frac{D_{it} y_{ijt}^{-\frac{1}{\eta_{it}}}}{P_t} - U'(Y_t) \frac{w_{it}}{P_t}\right] = 0.$$

By the fact that all firms within a given island are symmetric, we have that, in equilibrium, $n_{ijt} = n_{it}$, $y_{ijt} = y_{it}$, and $p_{ijt} = p_{it}$. It follows that $D_{it} y_{ijt}^{-\frac{1}{\eta_{it}}} = P_t Y_t^{\frac{1}{\rho}} y_{it}^{-\frac{1}{\rho}}$ and the above condition reduces to

$$\mathbb{E}_{it}\left[U'(Y_t) \frac{w_{it}}{P_t}\right] = \mathbb{E}_{it}\left[\left(1 - \frac{1}{\eta_{it}}\right) U'(Y_t) Y_t^{\frac{1}{\rho}} y_{it}^{-\frac{1}{\rho}} A_{it}\right]$$

Finally, consider the optimal labor supply in island i. The relevant FOC for the household gives

$$\chi_{it} V'(n_{it}) = (1 - \tau_{it}) \mathbb{E}_{it}\left[U'(Y_t) \frac{w_{it}}{P_t}\right]$$

Combining the above two conditions and letting $\mathcal{M}_{it} \equiv \frac{1}{1-\tau_{it}} \frac{\eta_{it}}{\eta_{it}-1}$ gives condition (1). ∎

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Proof of Lemma 1. Taking logs of both sides of (1) and rearranging gives

$$\left(\frac{1}{\rho} + \epsilon\right) \log y_{it} = -\mu_{it} + \log \mathbb{E}_{it}\left[Y_t^{\frac{1}{\rho}-\gamma}\right] + (1+\epsilon) a_{it}.$$

Assuming that Y_t is log-normal (we verify this below) the latter can be rewritten as

$$\left(\frac{1}{\rho} + \epsilon\right) \log y_{it} = -\mu_{it} + \left(\frac{1}{\rho} - \gamma\right) \mathbb{E}_{it}\left[\log Y_t\right] + \frac{1}{2}\left(\frac{1}{\rho} - \gamma\right)^2 Var\left(\log Y_t\right) + (1+\epsilon) a_{it}$$

or equivalently as

$$\log y_{it} = \phi_0 + \phi_a a_{it} + \phi_\mu \mu_{it} + \alpha \mathbb{E}_{it}\left[\log Y_t\right],$$

where

$$\phi_0 \equiv \frac{1}{2}\frac{\rho}{1+\rho\epsilon}\left(\frac{1}{\rho} - \gamma\right)^2 Var\left(\log Y_t\right), \quad \phi_a \equiv \frac{\rho(1+\epsilon)}{1+\rho\epsilon}, \quad \phi_\mu \equiv -\frac{\rho}{1+\rho\epsilon}, \quad \alpha \equiv \frac{1-\rho\gamma}{1+\rho\epsilon}.$$

Note that $\phi_a > 0$ and $\phi_\mu < 0$, reflecting the fact that local output increases with local productivity and decreases with the local level of monopoly power. Finally, note that α could be either positive or negative, but it is necessarily less than 1. ■

Proof of Lemma 2. Welfare is given by

$$\mathcal{W} = \sum \beta^t W_t$$

where

$$W_t \equiv \mathbb{E}\left[\frac{Y_t^{1-\gamma}}{1-\gamma} - \frac{1}{1+\epsilon}\int \chi_{it}\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right]$$

measures the unconditional expectation of the welfare flow in period t. Because the aggregate shocks are i.i.d. across time and all second moments are time-invariant,[20] the unconditional expectations of all the objects that enter in W_t are time-invariant, and hence W_t is itself a time-invariant function of the underlying preference, technology, and information parameters. To simplify the notation, we thus drop the time index t in the rest of this proof and proceed to develop a certain decomposition of the welfare flow W for an arbitrary period.

Before doing this, we highlight a property of log-normal distributions that is utilized repeatedly in this appendix. When a variable X is log-normal with $\ln X \sim \mathcal{N}\left(\bar{x}, \sigma^2\right)$, then, for any $\delta \in \mathbb{R}$, we have that

$$\mathbb{E}[X^\delta] = \exp\left(\delta\bar{x} + \frac{1}{2}\delta^2\sigma^2\right) = \left(\exp\left(\bar{x} + \frac{1}{2}\sigma^2\right)\right)^\delta \exp\left(\frac{1}{2}(\delta-1)\delta\sigma^2\right)$$

and therefore

$$\mathbb{E}[X^\delta] = (\mathbb{E}[X])^\delta \exp\left(\frac{1}{2}(\delta-1)\delta\sigma^2\right). \tag{15}$$

---

[20] These assumptions are for expositional simplicity; otherwise, the welfare results we document would have to be restated simply by distinguishing the information structure period by period.

We use this property again and again in the derivations that follow, for various X and δ.

Consider the first component of W, which corresponds to the utility of consumption and which is given by $\frac{1}{1-\gamma}\mathbb{E}\left(Y^{1-\gamma}\right)$. Noting that equilibrium Y is log-normal and using the aforementioned property, we have that

$$\mathbb{E}\left(Y^{1-\gamma}\right) = \left[\mathbb{E}\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} \tag{16}$$

Consider now the second component of W, which corresponds to the disutility of labor. Defining $b_i \equiv A_i^{1+\epsilon}/\chi_i$, letting B denote the cross-sectional mean of b_i, noting that $Y = \left(\int y_i^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$, and using once again the aforementioned property of log-normal distribution, we can express the *realized* disutility of labor, in any given state, as follows:

$$\int \chi_i \left(\frac{y_i}{A_i}\right)^{1+\epsilon} di = \mathbb{E}\left[\int \frac{y_i^{1+\epsilon}}{b_i}\right] = \frac{Y^{1+\epsilon}}{B}\exp(H)$$

where

$$H \equiv \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)Var\left(\log y_i|\Theta\right) + \tfrac{1}{2}Var\left(\log b_i|\Theta\right) - (1+\epsilon)Cov\left(\log y_i, \log b_i|\Theta\right)$$

and where $\Theta \equiv (Y, B)$ encapsulates the aggregate state of the economy. It follows that the *expected* disutility of labor is given by

$$\mathbb{E}\left[\int \chi_i \left(\frac{y_i}{A_i}\right)^{1+\epsilon} di\right] = \mathbb{E}\left[\frac{Y^{1+\epsilon}}{B}\right]\exp(H) = \frac{\mathbb{E}[Y]^{1+\epsilon}}{\mathbb{E}[B]}\exp(G) \tag{17}$$

where we have used once again the property from (15) to obtain

$$G \equiv H + \tfrac{1}{2}\epsilon(1+\epsilon)Var\left(\log Y\right) + Var\left(\log B\right) - (1+\epsilon)Cov\left(\log Y, \log B\right).$$

Because of our Gaussian specification, the variance and covariance terms that enter in H and G above are constants (non-random and time-invariant), and hence H and G are themselves constants.

Combining (16) and (17), we infer that the per-period welfare flow is given by

$$W = \tfrac{1}{1-\gamma}\left[\mathbb{E}\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} - \tfrac{1}{1+\epsilon}\frac{\left[\mathbb{E}\left(Y\right)\right]^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G).$$

Next, let us define $\hat{Y}$ as the value of $\mathbb{E}\left(Y\right)$ that maximizes the aforementioned expression for W, taking as given B, G, and $Var\left(\log Y\right)$. Clearly, this is given by taking the FOC of the above with respect to $\mathbb{E}(Y)$ and equating this with 0, or equivalently by the solution to the following condition:

$$\hat{Y}^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} = \frac{\hat{Y}^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G) \tag{18}$$

We can then restate W as follows:

$$W = \left\{\tfrac{1}{1-\gamma}\left[\frac{\mathbb{E}\left(Y\right)}{\hat{Y}}\right]^{1-\gamma} - \tfrac{1}{1+\epsilon}\left[\frac{\mathbb{E}\left(Y\right)}{\hat{Y}}\right]^{1+\epsilon}\right\}\frac{\hat{Y}^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G)$$

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If $\mathbb{E}(Y)$ happens to equal $\hat{Y}$, then $W = \hat{W}$, where

$$\hat{W} \equiv \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)} \frac{\hat{Y}^{1+\epsilon}}{\mathbb{E}(B)} \exp(G). \tag{19}$$

Letting

$$\Delta \equiv \frac{\mathbb{E}(Y)}{\hat{Y}} \qquad \text{and} \qquad v(\Delta) \equiv \frac{U(\Delta) - V(\Delta)}{U(1) - V(1)} = \frac{\frac{1}{1-\gamma}\Delta^{1-\gamma} - \frac{1}{1+\epsilon}\Delta^{1+\epsilon}}{\frac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}},$$

we conclude that

$$W = v(\Delta)\hat{W}. \tag{20}$$

The term $v(\Delta)$ therefore identifies the wedge between actual welfare, W, and the reference level $\hat{W}$ that a planner could have afforded if he had a non-contingent subsidy that permitted him to scale up and down the mean level of output and could use to maximize welfare. To see this more clearly, note that $v(\Delta)$ is strictly concave in Δ and reaches its maximum at $\Delta = 1$ when $\gamma < 1$, whereas it is strictly convex and reaches its minimum at $\Delta = 1$ when $\gamma > 1$. Along with the fact that $\hat{W} > 0$ when $\gamma < 1$ but $\hat{W} < 0$ when $\gamma > 1$ (this fact will be come clear in the sequel), this means that $\hat{W}v(\Delta)$ is always strictly concave in Δ, with the maximum attained at $\Delta = 1$.

So far, we have decomposed the per-period welfare flow as $W = \hat{W}v(\Delta)$. In the sequel, we proceed to decompose the reference level $\hat{W}$ itself into the product of two terms: the first-best level W^*; and a function of Λ, which encapsulates the welfare losses of volatility and dispersion.

From (18), we have that

$$\hat{Y} = [\mathbb{E}(B)]^{\frac{1}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma)Var(\log Y)\right]\right\},$$

which together with (19) gives

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{G - \tfrac{1+\epsilon}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma)Var(\log Y)\right]\right\}$$

Equivalently,

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\hat{\Omega})\right\} \tag{21}$$

where

$$\begin{aligned}
\hat{\Omega} &\equiv \tfrac{2}{1+\epsilon}G + \gamma Var(\log Y) \\
&= (\epsilon+\gamma)Var(\log Y) + \tfrac{2}{1+\epsilon}Var(\log B) - 2Cov(\log Y, \log B) \\
&\quad + \left(\epsilon + \tfrac{1}{\rho}\right)Var(\log y_i|\Theta) + \tfrac{1}{1+\epsilon}Var(\log b_i|\Theta) - 2Cov(\log y_i, \log b_i|\Theta)
\end{aligned}$$

Now, note that the first-best levels of output are given by the fixed point to the following:

$$\log y_i^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}\log b_i + \alpha\log Y^*.$$

It follows that, up to some constants that we omit for notational simplicity,

$$\log Y^* = \frac{1}{\epsilon+\gamma}\log B \qquad \text{and} \qquad \log y_i^* - \log Y^* = (1-\alpha)\frac{1}{\epsilon+\gamma}(\log b_i - \log B)$$

Using this result towards replacing the terms in $\hat{\Omega}$ that involve b_i and B, we get

$$
\begin{aligned}
\hat{\Omega} &= (\epsilon+\gamma)Var\left(\log Y\right) + 2\frac{(\epsilon+\gamma)^2}{(1+\epsilon)}Var\left(\log Y^*\right) - 2(\epsilon+\gamma)Cov\left(\log Y, \log Y^*\right) \\
&\quad + \left(\epsilon + \frac{1}{\rho}\right)Var\left(\log y_i|\Theta\right) + \frac{(\epsilon+\gamma)^2}{(1+\epsilon)(1-\alpha)^2}Var\left(\log y_i^*|\Theta\right) - 2\frac{\epsilon+\gamma}{1-\alpha}Cov\left(\log y_i, \log y_i^*|\Theta\right)
\end{aligned}
$$

Furthermore, the first-best level of welfare is given by

$$W^* = \frac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}\left(B\right)]^{\frac{1-\gamma}{\epsilon+\gamma}}\exp\left\{-\frac{1}{2}\frac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\Omega^*\right\} \tag{22}$$

where Ω^* obtains from $\hat{\Omega}$ once we replace y_i and Y with, respectively, y_i^* and Y^* (which have themselves been obtained above as functions of the exogenous objects b_i and B). We conclude that

$$\hat{W} = W^*\exp\left\{-\frac{1}{2}\frac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\left(\hat{\Omega}-\Omega^*\right)\right\} \tag{23}$$

Finally, using the definitions of $\hat{\Omega}$ and Ω^* together with the fact that $1-\alpha = \frac{\epsilon+\gamma}{\epsilon+1/\rho}$, we have

$$
\begin{aligned}
\frac{\hat{\Omega}-\Omega^*}{\epsilon+\gamma} &= \{Var\left(\log Y\right) + Var\left(\log Y^*\right) - 2Cov\left(\log Y, \log Y^*\right)\} \\
&\quad + \frac{1}{1-\alpha}\{Var\left(\log y_i|\Theta\right) + Var\left(\log y_i^*|\Theta\right) - 2Cov\left(\log y_i, \log y_i^*|\Theta\right)\} \\
&= Var\left(\log Y - \log Y^*\right) + \frac{1}{1-\alpha}Var\left(\log y_i - \log y_i^*|\Theta\right)
\end{aligned}
$$

Note that conditioning on $\Theta \equiv (\log Y, \log B)$ is equivalent to conditioning on $(\log Y, \log Y^*)$. Furthermore, because $\log Y$ and $\log Y^*$ are the cross-sectional means (expectations) of, respectively, $\log y_i$ and $\log y_i^*$, we have that

$$
\begin{aligned}
Var\left(\log y_i - \log y_i^*\,\middle|\,\log Y, \log Y^*\right) &= Var\left((\log y_i - \log Y) - (\log y_i^* - \log Y^*)\,\middle|\,\log Y, \log Y^*\right), \\
&= Var\left(\log \widetilde{y}_i - \log \widetilde{Y}\right)
\end{aligned}
$$

Combining the above results with the definitions of Σ, σ and Λ, gives

$$\frac{\hat{\Omega}-\Omega^*}{\epsilon+\gamma} = \Sigma + \frac{1}{1-\alpha}\sigma = \Lambda,$$

and therefore (23) can be restated as

$$\hat{W} = W^*\exp\left\{-\frac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}, \tag{24}$$

which gives the sought-after decomposition of $\hat{W}$.

Note from (22) that the sign of W^* is the same as the sign of $(1-\gamma)$. It follows that the sign of $\hat{W}$ is also the same as that of $(1-\gamma)$, which in turn verifies the claim made earlier on that the product $\hat{W}v(\Delta)$ is strictly convex in Δ with a maximum value of 1 attained at $\Delta = 1$.

Finally, combining (24) with (20), we conclude that

$$\mathcal{W} = v(\Delta)w(\Lambda)$$

where $w(x) \equiv \mathcal{W}^* \exp\left\{-\frac{1}{2}(1+\epsilon)(1-\gamma)x\right\}$ for every x and where $\mathcal{W}^* \equiv \frac{1}{1-\beta}W^*$ is the first-best level of (life-time) welfare. The proof is then completed by noting once again that W^* has the same sign as $1-\gamma$ and therefore that w is a strictly decreasing function of Λ, regardless of the sign of γ. The fact that $\mathcal{W}$ is strictly concave in Δ, with a maximum attained at $\Delta = 1$, follows directly from our earlier observation that $W = v(\Delta)\hat{W}$ has these exact properties. $\blacksquare$

Equilibrium with productivity shocks. Suppose the equilibrium production strategy takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_a a_{it} + \varphi_x x_{it} + \varphi_z z_t, \tag{25}$$

for some coefficients $(\varphi_a, \varphi_x, \varphi_z)$. Aggregate output is then given by

$$\log Y_t = \varphi_0 + X + (\varphi_a + \varphi_x)\bar{a}_t + \varphi_z z_t$$

where

$$X \equiv \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)Var(\log y_{it}|\Theta) = \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\left[\frac{\varphi_a^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_a\varphi_x}{\kappa_x}\right]$$

adjusts for the curvature in the CES aggregator. It follows that Y_t is log-normal, with

$$\mathbb{E}_{it}[\log Y_t] = \varphi_0 + X + (\varphi_a + \varphi_x)\mathbb{E}_{it}[\bar{a}_t] + \varphi_z z_t \tag{26}$$

$$Var_{it}[\log Y_t] = (\varphi_a + \varphi_x)^2 Var_{it}[\bar{a}_t] \tag{27}$$

where, by standard Gaussian updating,

$$\mathbb{E}_{it}[\bar{a}_t] = \frac{\kappa_x}{\kappa_a + \kappa_x + \kappa_z}x_{it} + \frac{\kappa_z}{\kappa_a + \kappa_x + \kappa_z}z_t \tag{28}$$

$$Var_{it}[\bar{a}_t] = \frac{1}{\kappa_a + \kappa_x + \kappa_z} \tag{29}$$

Because of the log-normality of Y_t, the fixed-point condition (1) reduces to following:

$$\log y_{it} = (1-\alpha)(\Psi a_{it} - \Psi' \log \bar{\mathcal{M}}) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma \tag{30}$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma} > 0$, $\Psi' \equiv \frac{1}{\epsilon+\gamma} > 0$, $\log \bar{\mathcal{M}} \equiv -\log\left[\left(\frac{\bar{\eta}-1}{\bar{\eta}}\right)(1-\bar{\tau})\right] \approx \bar{\mu} + \bar{\tau} > 0$ is the overall distortion caused by the monopoly markup and the labor wedge (which are both constant because we are herein focusing on the case with only productivity shocks), and

$$\Gamma = \frac{1}{2}\alpha\left(\frac{1}{\rho} - \gamma\right)Var_{it}[\log Y_t] = \frac{1}{2}\alpha^2\left(\frac{1}{\rho} + \epsilon\right)Var_{it}[\log Y_t] > 0$$

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Next, combining (30) with (26) and (28), we obtain

$$\begin{aligned}
\log y_{it} = {} & \Gamma - (1-\alpha)\Psi'\lambda_s + (1-\alpha)\,\Psi a_{it} + \alpha\left(\varphi_0 + X + \varphi_z z_t\right) \\
& + \alpha\left(\varphi_a + \varphi_x\right)\left(\frac{\kappa_x}{\kappa_a + \kappa_x + \kappa_z}x_{it} + \frac{\kappa_z}{\kappa_a + \kappa_x + \kappa_z}z_t\right)
\end{aligned}$$

For this to coincide with our initial guess in (25) for every event, it is necessary and sufficient that the coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ solve the following system:

$$\begin{aligned}
\varphi_0 &= \Gamma - (1-\alpha)\Psi'\log\bar{\mathcal{M}} + \alpha(\varphi_0 + X) \\
\varphi_a &= (1-\alpha)\,\Psi \\
\varphi_x &= \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_x}{\kappa_a + \kappa_x + \kappa_z} \\
\varphi_z &= \alpha\varphi_z + \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_z}{\kappa_a + \kappa_x + \kappa_z}
\end{aligned}$$

The unique solution to this system is given by the following:

$$\varphi_a = (1-\alpha)\,\Psi > 0, \quad \varphi_x = \frac{(1-\alpha)\,\kappa_x}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi,$$

$$\varphi_z = \frac{\kappa_z}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi, \quad \text{and} \quad \varphi_0 = -\Psi'\lambda_s + \tfrac{1}{1-\alpha}\left(\alpha X + \Gamma\right)$$

Note then that the coefficients φ_x and φ_z, which capture the individual response to expectations of the aggregate state, are positive if and only if $\alpha > 0$. $\blacksquare$

Proof of Proposition 1. Using the characterization of the equilibrium allocation in the preceding proof along with that of the first best in the proof of Lemma 2, we can calculate the equilibrium value of the aggregate and local output gaps as follows:

$$\begin{aligned}
\log Y_t - \log Y_t^* &= \left(\varphi_a + \varphi_x + \varphi_z\right)\bar{a}_t + \varphi_z\varepsilon_t - \Psi\bar{a}_t \\
\log y_{it} - \log y_{it}^* &= \varphi_x u_{it}
\end{aligned}$$

It follows that the volatility of the aggregate output gap is

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{\left(\varphi_a + \varphi_x + \varphi_z - \Psi\right)^2}{\kappa_a} = \frac{\alpha^2\left(\kappa_a + \kappa_z\right)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^2}\Psi^2$$

and the cross-sectional dispersion of the local output gaps is

$$\sigma = \frac{\varphi_x^2}{\kappa_x} = \frac{\alpha^2\left(1-\alpha\right)^2\kappa_x}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^2}\Psi^2.$$

Taking the derivative of Σ with respect to the precision of public information gives

$$\frac{\partial\Sigma}{\partial\kappa_z} = \frac{(1-\alpha)\kappa_x - \left(\kappa_a + \kappa_z\right)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}\alpha^2\Psi^2$$

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which is negative if and only if $\kappa_z > (1-\alpha)\kappa_x - \kappa_a$, while taking the derivative of σ gives

$$\frac{\partial \sigma}{\partial \kappa_z} = -2 \frac{\alpha^2 (1-\alpha)^2 \kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \Psi^2$$

which is necessarily negative.

Similarly, taking the derivatives of Σ and σ with respect to the precision of private information, we obtain

$$\frac{\partial \Sigma}{\partial \kappa_x} = -\frac{2(1-\alpha)(\kappa_a + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \alpha^2 (\Psi')^2$$

which is necessarily negative and

$$\frac{\partial \sigma}{\partial \kappa_x} = \frac{\kappa_z + \kappa_a - (1-\alpha)\kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} (1-\alpha)^2 \alpha^2 (\Psi')^2$$

which is negative if and only if $(1-\alpha)\kappa_x > \kappa_z + \kappa_a$. ■

Proof of Theorem 1. From the proof of Proposition 1, we can rewrite Λ as

$$\Lambda = \Sigma + \frac{1}{1-\alpha}\sigma = \frac{\alpha^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)} \Psi^2$$

from which it is immediate that Λ is decreasing in the precision of either public or private information, regardless of the sign of α. Furthermore,

$$\frac{\partial^2 \Lambda}{\partial \kappa_z \partial \alpha} = -\frac{2\alpha(\kappa_x + \kappa_z + \kappa_a)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \Psi^2$$

which is itself negative if and only if $\alpha > 0$. Finally, note that the distortion in the mean level of output is given by

$$\Delta = \bar{\mathcal{M}}^{\frac{1}{\epsilon+\gamma}} \equiv \left[\left(\frac{\bar{\eta}-1}{\eta}\right)(1-\bar{\tau})\right]^{\frac{1}{\epsilon+\gamma}} < 1$$

where $\frac{\bar{\eta}-1}{\eta}$ is the monopoly wedge (the reciprocal of the markup) and $1-\bar{\tau}$ is the labor wedge. Since Δ is invariant to the information structure, the welfare effects of either type of information are captured by the comparative statics of Λ alone, which have been established above. ■

Equilibrium with markup shocks. This follows very similar steps as the characterization of equilibrium in the case with productivity shocks. Suppose equilibrium output takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t,$$

for some coefficients $(\varphi_\mu, \varphi_x, \varphi_z)$. This guarantees that aggregate output is log-normal, which in turn implies that the fixed-point condition (1) now reduces to

$$\log y_{it} = (1-\alpha)(\Psi \bar{a} - \Psi' \mu_{it}) + \alpha \mathbb{E}_{it}[\log Y_t] + \Gamma$$

where Ψ, Ψ', and Γ are defined as in the case with productivity shocks. Following similar steps as in that case, we can then show that the unique equilibrium coefficients are given by the following:

$$\varphi_\mu = -(1-\alpha)\Psi' < 0, \quad \varphi_x = -\frac{(1-\alpha)\kappa_x}{\kappa_\mu+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi',$$

$$\varphi_z = -\frac{\kappa_z}{\kappa_\mu+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi', \quad \text{and} \quad \varphi_0 = \Psi\bar{a} + \frac{1}{1-\alpha}(\alpha X + \Gamma)$$

Note that the sign of the coefficients φ_x and φ_z is once again pinned down by the sign of α. ■

Proof of Proposition 2. With only markup shocks, the first-best levels of output are constant. The volatility of aggregate output gaps and the dispersion of local output gaps are thus given by the following:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_\mu + \varphi_x + \varphi_z)^2}{\kappa_\mu} = \frac{\alpha^2\kappa_\mu\kappa_z + ((1-\alpha)\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z)^2}{\kappa_\mu(\kappa_\mu+(1-\alpha)\kappa_x+\kappa_z)^2}(\Psi')^2 \tag{31}$$

$$\sigma = \frac{\varphi_\mu^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_\mu\varphi_x}{\kappa_x} = \frac{(1-\alpha)^2}{\kappa_\xi}(\Psi')^2 + \frac{\alpha(1-\alpha)^2(2\kappa_\mu + (2-\alpha)\kappa_x + 2\kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(\Psi')^2 \tag{32}$$

Next, taking the derivatives with respect to the precision of public information, we obtain

$$\frac{\partial\Sigma}{\partial\kappa_z} = \frac{(2+\alpha)(1-\alpha)\kappa_x + (\kappa_z+\kappa_\mu)(2-\alpha)}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^3}\alpha(\Psi')^2$$

which is positive if $\alpha > 0$ and

$$\frac{\partial\sigma}{\partial\kappa_z} = -\frac{2(1-\alpha)^2(\kappa_\mu+\kappa_x+\kappa_z)}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^3}\alpha(\Psi')^2$$

which is negative if (and only if) $\alpha > 0$.

Similarly, taking the derivatives of Σ and σ with respect to the precision of private information, we obtain

$$\frac{\partial\Sigma}{\partial\kappa_x} = \frac{2(1-\alpha)^2(\kappa_x+\kappa_z+\kappa_\mu)}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^3}\alpha(\Psi')^2$$

which is positive if (and only if) $\alpha > 0$ and

$$\frac{\partial\sigma}{\partial\kappa_x} = -\frac{(1-\alpha)^2[(2-\alpha)(1-\alpha)\kappa_x + (2-3\alpha)(\kappa_\mu+\kappa_z)]}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^3}\alpha(\Psi')^2,$$

which is in general ambiguous. ■

Proof of Proposition 3. Using (31) and (32), we can obtain the equilibrium value of Λ as

$$\Lambda = \frac{1-\alpha}{\kappa_\xi}(\Psi')^2 + \frac{(1-\alpha)\kappa_x + \kappa_z + (1-\alpha^2)\kappa_\mu}{\kappa_\mu((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)}(\Psi')^2$$

(Note that the first term captures the distortion caused by cross-sectional dispersion in actual markups, whereas the second terms captures the distortion caused by the firms' response to their information about the aggregate markup.) It follows that, regardless of the sign of α,

$$\frac{\partial \Lambda}{\partial \kappa_z} = \frac{\alpha^2 (\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2} > 0 \tag{33}$$

$$\frac{\partial \Lambda}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2 (\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2} > 0, \tag{34}$$

which proves that Λ increases with the precision of either public or private information, regardless of the sign of α. ∎

Proof of Proposition 4. Recall that Δ is given by the ratio of the equilibrium value of expected output, $\mathbb{E}[Y]$, to the corresponding optimal value, $\hat{Y}$. The former can be computed from the preceding equilibrium characterization and the latter from condition (18). After some tedious algebra (which is available upon request), we can thus show that

$$\Delta \equiv \frac{\mathbb{E}[Y]}{\hat{Y}} = \exp\left[-\Psi'\left(\bar{\mu} + \tfrac{1}{2}D\right)\right],$$

where

$$
\begin{aligned}
D &\equiv Var(\mu_i) + 2(1+\epsilon)Cov(y_i, \mu_i) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} + 2(1+\epsilon)\left(\frac{\varphi_\mu}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{\varphi_\mu + \varphi_x + \varphi_z}{\kappa_\mu}\right) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} - 2(1+\epsilon)\left(\frac{1-\alpha}{\kappa_\xi} + \frac{1}{\kappa_\mu} - \frac{\alpha^2}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\right)\Psi'
\end{aligned}
$$

It follows that

$$\frac{\partial \Delta}{\partial \kappa_z} = -\tfrac{1}{2}\Delta\Psi'\frac{\partial D}{\partial \kappa_z} = \frac{\alpha^2 (\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(1+\epsilon)\Delta > 0 \tag{35}$$

$$\frac{\partial \Delta}{\partial \kappa_x} = -\tfrac{1}{2}\Delta\Psi'\frac{\partial D}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2 (\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(1+\epsilon)\Delta > 0. \tag{36}$$

That is, Δ is increasing in the precision of either public or private information, irrespective of whether α is positive or negative. ∎

Proof of Theorem 2. To obtain the overall welfare effect, recall that welfare is given by

$$\mathcal{W} = v(\Delta)w(\Lambda) = \mathcal{W}^* v(\Delta)\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}$$

Consider first the case of public information. From the above, we have that

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\left(v'(\Delta)\frac{\partial \Delta}{\partial \kappa_z} - \tfrac{1}{2}v(\Delta)(1+\epsilon)(1-\gamma)\frac{\partial \Lambda}{\partial \kappa_z}\right)$$

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From (33) and (35), we have that
$$\frac{\partial \Delta}{\partial \kappa_z} = \frac{\partial \Lambda}{\partial \kappa_z}(1+\epsilon)\Delta$$

It follows that
$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \frac{\partial \Lambda}{\partial \kappa_z} H$$

where
$$H \equiv v'(\Delta)(1+\epsilon)\Delta - \tfrac{1}{2}(1-\gamma)(1+\epsilon)v(\Delta) = \frac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[(1+\epsilon)\Delta^{1-\gamma} - (1+2\epsilon+\gamma)\Delta^{1+\epsilon}\right],$$

and, therefore,
$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \frac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \frac{\partial \Lambda}{\partial \kappa_z}\Delta^{1-\gamma}\left[(1+\epsilon) - (1+2\epsilon+\gamma)\Delta^{\epsilon+\gamma}\right]$$

Note then that the sign of $\mathcal{W}^*$ is the same as that of $(1-\gamma)$, which together with the facts that $\frac{\partial \Lambda}{\partial \kappa_z} > 0$ and $\Delta > 0$ implies that the sign of $\frac{\partial \mathcal{W}}{\partial \kappa_z}$ is the same as the sign of $(1+\epsilon) - (1+2\epsilon+\gamma)\Delta^{\epsilon+\gamma}$. We conclude that
$$\frac{\partial \mathcal{W}}{\partial \kappa_z} < 0 \quad \text{iff} \quad \Delta > \hat{\Delta},$$

where
$$\hat{\Delta} \equiv \left(\frac{1+\epsilon}{1+2\epsilon+\gamma}\right)^{\frac{1}{\epsilon+\gamma}} \in (0,1).$$

Consider next the case of private information. From (34) and (36), we have that
$$\frac{\partial \Delta}{\partial \kappa_x} = \frac{\partial \Lambda}{\partial \kappa_x}(1+\epsilon)\Delta,$$

as in the case of public information. It follows that
$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \kappa_x} &= \mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\left(v'(\Delta)\frac{\partial \Delta}{\partial \kappa_x} - \tfrac{1}{2}v(\Delta)(1+\epsilon)(1-\gamma)\frac{\partial \Lambda}{\partial \kappa_x}\right)\\ &= \mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \frac{\partial \Lambda}{\partial \kappa_x} H\end{aligned}$$

where H is defined as before. By direct implication,
$$\frac{\partial \mathcal{W}}{\partial \kappa_x} < 0 \quad \text{iff} \quad \Delta > \hat{\Delta},$$

where $\hat{\Delta}$ is the same threshold as the one in the case of public information.

Clearly, when a non-contingent subsidy is set optimally, $\mathbb{E}[Y] = \hat{Y}$ or $\Delta = 1 > \hat{\Delta}$. ∎

Appendix B: Auxiliary Results and Proofs for Extended Model

This appendix contains the proofs for Section 5, along with a number of auxiliary results. In Section B.1, we provide a characterization of the set of implementable allocations, that is, the set of all allocations that can be part of an equilibrium for *some* monetary policy; we also show that this set remains the same whether monetary policy responds to the state within the same period or with a lag. In Section B.2, we develop a preliminary welfare decomposition, which forms the basis of the particular decompositions that appear in the main text. In Section B.4, we use a numerical example to illustrate an argument made in the main text. In Section B.4, we conclude with the proofs for all the results that appear either in the main text or in parts B.1 and B.2 here.

B.1 Equilibrium and Implementable Allocations.

The equilibrium is defined in a similar manner as in the baseline model, modulo the fact that prices are now set on the basis of incomplete information. Consider the FOCs of firm i, who chooses q_{it} and p_{it} so as to maximize the expected valuation of its profit. Combining these conditions with the household's FOC for labor supply and for the demand of the different commodities, we obtain the following conditions:

$$0 = n_{it}^{1+\epsilon} - \mathbb{E}_{it}\left[\mathcal{M}_{it}U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\theta c_{it}\right] \tag{37}$$

$$0 = \mathbb{E}_{it}\left[\left(l_{it}^{1+\epsilon} - \mathcal{M}_{it}U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\eta y_{it}\right)\right] \tag{38}$$

where

$$Y_t = \left[\int (q_{it}l_{it}^{\eta})^{\frac{\rho-1}{\rho}}\,di\right]^{\frac{\rho}{\rho-1}}; \tag{39}$$

These conditions are the analogue of condition (1) from the baseline model and identify two of the four key implementability conditions of the general model. The third condition follows form the household's optimal demand for the different commodities and ties relative prices to relative quantities:

$$\frac{p_{it}}{P_t} = \left(\frac{q_{it}l_{it}^{\eta}}{Y_t}\right)^{-\frac{1}{\rho}}; \tag{40}$$

The last condition follows from the Euler condition of the household and ties the nominal interest rate to output growth and inflation:

$$\log(1+R_t) = const + \gamma\left(\mathbb{E}_t[\log Y_{t+1}] - \log Y_t\right) + \left(\mathbb{E}_t[\log P_{t+1}] - \log P_t\right) \tag{41}$$

To recap, a combination of quantities and prices are part of an equilibrium if and only if (i) the quantities and prices satisfy conditions (37) through (40) and (ii) monetary policy satisfies (41).

We now proceed to restate these conditions in a manner that facilitates our subsequent analysis. For expositional purposes, this is done in three steps. First, in Lemma 6, we restrict attention to

the subset of equilibria in which the interest rate is measurable in the current fundamental and the current public signal. Next, in Lemma 7, we show that exactly the same real outcomes as those in Lemma 6 obtain if we instead consider the subset of equilibria in which the interest rate is measurable in the value of the shock at some past period (i.e., if policy reacts with the lag). It is then immediate that the set of implementable allocations remain the same if we also consider the more general case in which the interest rate is arbitrary function of the entire history of the shock. Clearly, the same applies for the public signal. We thus conclude the characterization in Lemma 8 by considering the residual case in which the interest rate depends also on a shock that is orthogonal to the entire history of the fundamental and the public signal. Throughout, we let s_{it} and $\bar{s}_t$ denote the idiosyncratic and aggregate shocks to fundamentals (technology or markups).

Lemma 6. *Suppose that the nominal interest rate satisfies*

$$\log\left(1 + R_t\right) = \rho_s \bar{s}_t + \rho_z z_t,$$

for some coefficients $r_{\bar{s}}$ and r_z, and consider the following pair of strategies:[21]

$$\log q_{it} = \varphi_0 + \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \qquad and \qquad \log l_{it} = l_0 + l_{\bar{s}} \bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t. \qquad (42)$$

(i) When prices are set on the basis of incomplete information, a pair of strategies as in (42) can be implemented as part of an equilibrium if and only if the following conditions are satisfied:[22]

$$\varphi_s = \hat{\phi}_s \qquad (43)$$

$$\varphi_x = \Gamma_x + \Gamma'_x l_{\bar{s}} \qquad (44)$$

$$\varphi_z = \Gamma_z + \Gamma'_z l_{\bar{s}} \qquad (45)$$

$$l_s = \frac{1}{\theta}(\varphi_s - \mathbf{1}_{s=a}) \qquad (46)$$

$$l_x = \frac{1}{\theta}\varphi_x - \frac{\kappa_x}{\kappa_s + \kappa_x + \kappa_z} l_{\bar{s}} \qquad (47)$$

$$l_z = \frac{1}{\theta}\varphi_z - \frac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z} l_{\bar{s}}, \qquad (48)$$

where $\hat{\phi}_s$ is given in (51), $\mathbf{1}_{s=a}$ is an indicator that takes the value 1 in the case of technology shocks ($s = a$) and 0 in the case of markup shocks ($s = \mu$), Γ_x, Γ'_x, Γ_z, Γ'_z are scalars given in the proof, and $l_{\bar{s}}$ is an arbitrary coefficient.

(ii) When instead prices are flexible (i.e., free to adjust to the true sate), there exists a unique pair of strategies as in (42) that can obtain in equilibrium, and this pair is pinned down by the combination of conditions (43)-(48) with the following condition:

$$l_{\bar{s}} = l^*_{\bar{s}} \equiv \frac{(1 - \rho\gamma)\left(1 + \frac{\eta}{\theta}\right)\left(\hat{\phi}_s + \Gamma_x\right) - \eta(1 - \rho\gamma)\mathbf{1}_{s=a}}{\rho(1 + \epsilon - \eta) + \eta - (1 - \rho\gamma)\left(\left(1 + \frac{\eta}{\theta}\right)\Gamma'_x + \eta\frac{\kappa_s + \kappa_z}{\kappa_s + \kappa_x + \kappa_z}\right)}. \qquad (49)$$

---

[21] By "strategies" we refer to functions that map the information set of a firm to its employment and production.

[22] There is also a pair of restrictions on φ_0 and l_0, which we omit because these are of no interest: φ_0 and l_0 are irrelevant for the stochastic properties of the equilibrium.

This lemma identifies the precise way in which monetary policy can control real allocations. When prices are set on the basis of incomplete information, by appropriately designing the response of the interest rate to the realized shock, the policy maker can choose at will the coefficient $l_{\bar{s}}$, that is, the response of the second-stage labor input (the margin of adjustment in quantities) to the realized technology or markup shock. Conditional on choosing this coefficient, however, the monetary authority has no further control over the real allocation. In this sense, the coefficient coefficient $l_{\bar{s}}$ is the only "free variable" at the disposal of the policy maker. Finally, when prices are flexible (free to adjust to the realized shock), this variable ceases to be free, and the policy maker has, of course, no control over real allocations (although he can still control the price level).

We now proceed to show that the set of implementable allocations remains the same whether monetary policy responds to the realized state within the same period or with an arbitrary lag.

Lemma 7. *Suppose that the nominal interest rate satisfies*

$$\log(1 + R_t) = \rho_{-k}\bar{s}_{t-k} + \rho_z z_t \tag{50}$$

for some $k \geq 1$ and some scalars ρ_{-k} and ρ_z. Parts (i) and (ii) of Lemma 6 continue to hold. That is, the set of implementable allocations remains the same.

By a similar argument as the one found in the proof of this lemma, the set of implementable allocations remains the same if we consider the more general class of policies in which the the interest rate is be an arbitrary function of the entire history of the fundamental and the public signal. We thus conclude this section by extending Lemma 6 to the only case that has not been allowed so far, namely allowing for the interest rate to contain a pure monetary shock, by which we mean a shock orthogonal to both the fundamental and the public signal (and the histories thereof). This makes no essential difference to the logic underlying the implementability constraints we derived in Lemma 6. It only introduces a mechanical response of output to the monetary shock.[23]

Lemma 8. *Suppose monetary policy satisfies*

$$\log(1 + R_t) = \rho_s \bar{s}_t + \rho_z z_t + r_t,$$

where r_t is a Normally distributed random variable that is orthogonal to both $\bar{s}_t$ and z_t and that is unpredictable by the firms. Then, the second-period labor choice satisfies:

$$\log l_{it} = l_0 + l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t - \frac{1}{\eta\gamma} r_t.$$

However, the strategy for q_{it} remains the same and the implementability conditions (43)-(48) are also not affected.

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[23]This response would itself be more complicated if firms had information about the monetary shock at the moment they make their pricing and production decisions, a possibility which we only briefly discuss in the end of Section 5.

B.2 Welfare

In this subsection, we obtain a preliminary welfare decomposition, which extends Lemma 2 from the baseline model to the more general model under consideration.

To this goal, we first introduce certain notation:

$$
\begin{aligned}
\hat{\epsilon} &\equiv \frac{1+\epsilon}{\theta} - 1, \quad \hat{\gamma} \equiv 1 - \frac{(1-\gamma)(1+\epsilon)}{1+\epsilon-\eta(1-\gamma)}, \quad \hat{\rho} \equiv \frac{\rho(1+\epsilon-\eta)+\eta}{1+\epsilon+\eta(1-\rho)} = \frac{1}{1-\rho\nu+\nu}, \\
\hat{\alpha} &\equiv \frac{1-\hat{\rho}\hat{\gamma}}{1+\hat{\rho}\hat{\epsilon}}, \quad \chi \equiv \frac{1+\epsilon}{1+\epsilon-\eta+\gamma\eta} > 0, \quad \nu \equiv \frac{1+\epsilon}{\rho(1+\epsilon-\eta)+\eta}, \\
\hat{\phi}_a &\equiv \frac{\hat{\rho}(1+\hat{\epsilon})}{1+\hat{\rho}\hat{\epsilon}}, \quad \hat{\phi}_\mu \equiv -\frac{\hat{\rho}+(\hat{\rho}-1)\frac{\eta}{1+\epsilon}}{1+\hat{\rho}\hat{\epsilon}}, \quad \bar{\alpha} \equiv \frac{(1-\rho\gamma)\eta}{\rho(1+\epsilon-\eta)+\eta}.
\end{aligned}
\tag{51}
$$

As in the main text, we also let $q_{it} \equiv A_{it} n_{it}^\theta$ denote the component of output that is fixed on the basis of the firm's incomplete information of the state of the economy, and define the corresponding aggregate as

$$
Q_t \equiv \left[\int_I (q_{it})^{\frac{\hat{\rho}-1}{\hat{\rho}}} di \right]^{\frac{\hat{\rho}}{\hat{\rho}-1}}.
$$

Next, we denote with $\log \bar{y}_{it}$ and $\log \bar{Y}_t$ the socially optimal levels of, respectively, local and aggregate output, *conditional* on an arbitrary allocation of the q's; and with $\log q_{it}^*$ and $\log Q_t^*$ the first-best levels of, respectively, $\log q_{it}$ and $\log Q_t$. Finally, we let Σ_Q and σ_q denote, respectively, the volatility of $\log Q_t - \log Q_t^*$ and the cross-sectional dispersion of $\log q_{it} - \log q_{it}^*$; and similarly we let Σ_Y and σ_y denote, respectively, the volatility of $\log Y_t - \log \bar{Y}_t$ and the cross-sectional dispersion of $\log y_{it} - \log \bar{y}_{it}$.

The first of the following two lemmas characterizes the aforementioned reference points, the first best and the allocation that is optimal conditional on q's. The second lemma then develops the desired welfare decomposition in terms of gaps relative to these reference points.

Lemma 9. *For any given distribution of q in the cross-section, the optimal output levels solve the following fixed-point relation:*

$$
\log \bar{y}_{it} = \rho\nu \log q_{it} + \bar{\alpha} \log \bar{Y}_t.
\tag{52}
$$

The first-best allocation satisfies the following fixed-point relation:

$$
\log q_{it}^* = \hat{\phi}_a a_{it} + \hat{\alpha} \log Q_t^*.
\tag{53}
$$

Lemma 10. *There exists a decreasing function w, which is invariant to the information structure, such that welfare satisfies*

$$
\mathcal{W} = w(\Lambda'),
\tag{54}
$$

with

$$
\Lambda' = \left(\Sigma_Q + \frac{1}{1-\hat{\alpha}} \sigma_q \right) + \xi \left(\Sigma_Y + \frac{1}{1-\bar{\alpha}} \sigma_y \right),
\tag{55}
$$

and where $\hat{\gamma}$ and $\hat{\epsilon}$ are given in (51) and ξ is a positive scalar pinned down by $(\gamma, \epsilon, \theta, \eta)$.

Like Lemma 2 in the baseline model, Lemma 10 is not particularly surprising. It simply decomposes the welfare losses that obtain relative to the first-best in two components. The first component, namely the sum $\Sigma_Q + \frac{1}{1-\widehat{\alpha}}\sigma_q$, capture the distortions (if any) that obtain in the first-stage production decisions, that is, those that must be set on the basis of incomplete information. The second component, namely the sum $\Sigma_Y + \frac{1}{1-\widehat{\alpha}}\sigma_y$, captures the distortions (if any) that obtain in the second-stage production decisions, that is, those that are free to adjust to the realized state. Each of these components contains a volatility and a dispersion subcomponent, reflecting the fact that some distortions are aggregate whereas others are idiosyncratic.

What is interesting, however, is how these components are affected by the information frictions and by the associated types of rigidity. To develop intuition, let us abstract from markup shocks. When information is complete, all distortions vanish, $\Sigma_Q = \Sigma_q = \Sigma_Y = \Sigma_y = 0$, and hence $\Lambda' = 0$. When, instead, information is incomplete, the nature of the distortions depends on whether the incompleteness of information is only the source of real rigidity or also the source of nominal rigidity. In the former case, Σ_Q and Σ_q are positive, reflecting the measurability constraint on quantities, but $\Sigma_Y = \Sigma_y = 0$, reflecting that the margin of adjustment to the realized state. In the latter case, by contrast, whether Σ_Y and Σ_y coincide or diverge from zero depends on whether monetary policy coincides or diverges from the benchmark of replicating flexible prices. This discussion therefore underscores how the two types of rigidity map into different kinds of potential distortions, an issue that is further explored in the main text.

B.3 An Example With Different Policy Targets

In the main text, we noted that, if monetary policy tries to stabilize either the price level or the output gap, more precise information may help increase welfare not only by attenuating the real rigidity but also by alleviating the policy suboptimality. We now illustrate the logic behind this argument in Figure 1, with the help of a numerical example. This example assumes the following parameterization: $\theta = \eta = 0.5$, $\epsilon = 1$, $\gamma = 0.25$, $\rho = 0.5$, $\sigma_a = \sigma_x = \sigma_z = .02$, and $\sigma_m = \sigma_\epsilon = 0$.
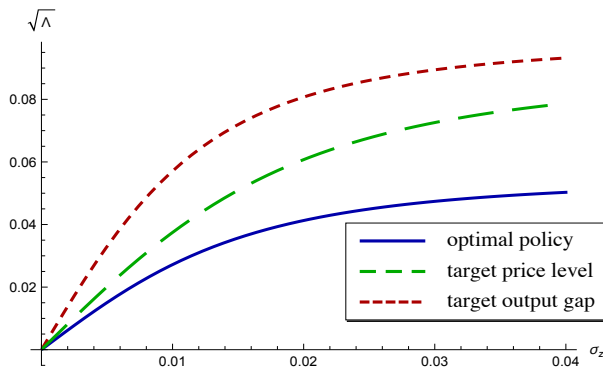

Figure 1: The welfare losses due to incomplete information, under different levels of noise in the public signal and under different policy rules.

The figure reports the welfare effects of public information under three alternative specifications of the monetary policy: the optimal policy (solid blue line), a policy that stabilizes the aggregate output gap (dotted red line), and a policy that stabilizes the price level (dashed green line). The last two policies are suboptimal in our model due to the presence of the informational friction, but are useful reference points because they are optimal in the prototypical New-Keynesian model.

For any level of noise $\sigma_z > 0$, the welfare losses associated with targeting either the price level or the output gap are higher than those associated with the optimal policy. Nevertheless, the welfare gap between these policies and the optimal one decreases with the precision of the available information, and vanishes in the limit as $\sigma_z \to 0$. It follows that, under either of these two policies, more information improves welfare via two effects: by bringing the equilibrium allocation closer to the optimal one; and by raising the welfare attained at the optimal allocation.

B.4   Proofs

In this subsection, we provide, first, the proofs of the auxiliary results we stated in parts B.1 and B.2 of this appendix and, next, the proofs of the results that appear in Section 5.

Proof of Lemma 6. *Part (i).* The proof combines the first-order conditions of the firm's problem with the resource constraint and the monetary policy to derive conditions that the coefficients in (42) have to satisfy in order for the latter to be part of an equilibrium.

We now seek to translate these properties in terms of the relevant coefficients that parameterize the allocations, prices, and policy under a log-normal specification. First notice that, since all the shocks are independent over time, $\mathbb{E}_t[\log Y_{t+1}]$ and $\mathbb{E}_t[\log P_{t+1}]$ in (41) will be constant. Thus, let (omitting unimportant constants)

$$
\begin{aligned}
\log q_{it} &= \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \\
\log l_{it} &= l_{\bar{s}} \bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t \\
\log Y_t &= c_{\bar{s}} \bar{s}_t + c_z z_t \\
\log p_{it} &= \psi_s s_{it} + \psi_x x_{it} + \psi_z z_t
\end{aligned}
$$

for some coefficients $(\varphi_s, \varphi_x, ..., \psi_z)$, and with the understanding that s stands either for a or μ, depending on whether we are considering the case of technology or markup shocks. Note that the resource constraint (39) is satisfied if and only if

$$
\begin{aligned}
c_{\bar{s}} &= (\varphi_s + \varphi_x) + \eta(l_s + l_x + l_{\bar{s}}) & (56) \\
c_z &= \varphi_z + \eta l_z & (57)
\end{aligned}
$$

while (41) is satisfied if and only if

$$
\begin{aligned}
\rho_s &= -\gamma c_{\bar{s}} - (\psi_s + \psi_x) & (58) \\
\rho_z &= -\gamma c_z - \psi_z. & (59)
\end{aligned}
$$

Consider the consumer's demand function (40), which can be expressed as follows:

$$-\rho\left(\log p_{it} - \log P_t\right) = \left(\log y_{it} - \log Y_t\right)$$

Using market clearing, the production function, and the proposed strategies, we can express the output of good i as follows:

$$
\begin{aligned}
\log y_{it} &= \log q_{it} + \eta \log l_{it} \\
&= (\varphi_s + \eta l_s)s_{it} + (\varphi_x + \eta l_x)x_{it} + (\varphi_z + \eta l_z)z_t + \eta l_{\bar{s}}\bar{s}_t,
\end{aligned}
$$

By implication, aggregate output satisfies

$$\log Y_{it} = (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t + (\varphi_z + \eta l_z)z_t.$$

Substituting the above two results in the consumer's demand function, and doing a similar substitution for p_{it} and P_t, we infer that the following must hold for all realizations of the shocks and signals:

$$-\rho(\psi_s s_{it} + \psi_x x_{it} - (\psi_s + \psi_x)\bar{s}_t) = (\varphi_s + \eta l_s)s_{it} + (\varphi_x + \eta l_x)x_{it} - (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t.$$

This is true if and only if

$$\psi_s = -\frac{1}{\rho}(\varphi_s + \eta l_s) \qquad \text{and} \qquad \psi_x = -\frac{1}{\rho}(\varphi_x + \eta l_x). \tag{60}$$

Finally, taking the logs of conditions (37) and (38) and using the properties of log-normal distributions, we can rewrite these conditions as follows:

$$\mathbb{E}_{it}[\log l_{it}] = \frac{1}{\theta}\left(\log q_{it} - a_{it}\right) \tag{61}$$

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \frac{\hat{\alpha}}{\chi}\mathbb{E}_{it}\left[\log Y_t\right]. \tag{62}$$

Clearly, condition (61) holds for all i if and only if

$$
\begin{aligned}
l_s &= \frac{1}{\theta}\left(\varphi_s - \mathbf{1}_{s=a}\right) \tag{63} \\
l_x &= \frac{1}{\theta}\varphi_x - l_{\bar{s}}\frac{\kappa_x}{\kappa_s + \kappa_x + \kappa_z} \tag{64} \\
l_z &= \frac{1}{\theta}\varphi_z - l_{\bar{s}}\frac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z}, \tag{65}
\end{aligned}
$$

while condition (62) holds for all i if and only if

$$
\begin{aligned}
\varphi_s &= \hat{\phi}_s \tag{66} \\
\varphi_x &= \frac{\hat{\alpha}}{\chi}c_{\bar{s}}\frac{\kappa_x}{\kappa_s + \kappa_x + \kappa_z} \tag{67} \\
\varphi_z &= \frac{\hat{\alpha}}{\chi}\left(c_{\bar{s}}\frac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z} + c_z\right). \tag{68}
\end{aligned}
$$

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We can now use (56)-(57) to replace $c_{\bar{s}}$ and c_z in (67) and (68) to get

$$\varphi_x = \frac{\hat{\alpha}}{\chi}\left(\varphi_s + \varphi_x + \eta\left(l_s + l_x + l_{\bar{s}}\right)\right)\frac{\kappa_x}{\kappa_s + \kappa_x + \kappa_z}$$

and

$$\varphi_z = \frac{\hat{\alpha}}{\chi}\left(\varphi_s + \varphi_x + \eta\left(l_s + l_x + l_{\bar{s}}\right)\right)\frac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z} + \frac{\hat{\alpha}}{\chi}\left(\varphi_z + \eta l_z\right),$$

with the understanding that s stands for either a or μ, depending on the case under consideration. Using (63), (64), (65), and (66) and rearranging gives

$$\left(\frac{\kappa_s + \kappa_x + \kappa_z}{\kappa_x}\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}\right)\varphi_x = \hat{\phi}_s + \frac{\eta}{\theta}\left(\hat{\phi}_s - \mathbf{1}_{s=a}\right) + \eta\frac{\kappa_s + \kappa_z}{\kappa_s + \kappa_x + \kappa_z}l_{\bar{s}}$$

and

$$\left(\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}\right)\varphi_z = \frac{\chi}{\hat{\alpha}}\frac{\kappa_z}{\kappa_x}\varphi_x - \eta\frac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z}l_{\bar{s}}.$$

or

$$\varphi_x = \Gamma_x + \Gamma_x' l_{\bar{s}},$$

and

$$\varphi_z = \Gamma_z + \Gamma_z' l_{\bar{s}},$$

where

$$\Gamma_x \equiv \frac{\hat{\phi}_s + \frac{\eta}{\theta}\left(\hat{\phi}_s - \mathbf{1}_{s=a}\right)}{\frac{\kappa_s + \kappa_x + \kappa_z}{\kappa_x}\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}, \quad \Gamma_x' \equiv \frac{\eta\kappa_x}{\kappa_s + \kappa_x + \kappa_z}\frac{\kappa_s + \kappa_z}{\left(\kappa_s + \kappa_x + \kappa_z\right)\frac{\chi}{\hat{\alpha}} - \kappa_x\left(1 + \frac{\eta}{\theta}\right)},$$

$$\Gamma_z \equiv \frac{\frac{\chi}{\hat{\alpha}}\frac{\kappa_z}{\kappa_x}}{\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}\Gamma_x, \quad \Gamma_z' \equiv \frac{\frac{\chi}{\hat{\alpha}}\frac{\kappa_z}{\kappa_x}\Gamma_x' - \eta\frac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z}}{\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}.$$

This completes the proof of the necessity of conditions (43)-(48) for an allocation to be part of an equilibrium.

We now prove sufficiency. Pick arbitrary $l_{\bar{s}}$ and let $(\varphi_s, \varphi_x, \varphi_z, l_s, l_x, l_z)$ be the unique vector that satisfies conditions (43) through (48) for the given $l_{\bar{s}}$. Next, let $(c_{\bar{s}}, c_z, \rho_{\bar{s}}, \rho_z, \psi_s, \psi_x)$ be determined as in (56)-(60) and let $\psi_z = -\gamma c_z$. By construction, the allocations, prices and policies defined in this way constitute an equilibrium, which completes the sufficiency argument.

Part (ii). This proof is the same as that of part (i), except for one key difference: now the marginal costs and returns of second-state employment must be equated state-by-state, not just in expectation. It is this additional restriction that pins down $l_{\bar{s}}$ at the value stated in the lemma. A detailed derivation is available upon request. ∎

Proof of Lemma 7. Before proceeding, it is useful to recall two key facts about Lemma 6. First, that lemma established that, when the interest rate is determined according to (41), an allocation as in (42) can be implemented as part of an equilibrium if and only if conditions (43)-(48) are

satisfied. Second, the nominal prices that supported such an allocation (i.e., that were consistent with the firm's optimal price-setting behavior) were left outside the statement of that lemma, but were constructed as part of its proof. With this context in mind, the strategy underlying the present proof is to show that the class of policies in (50) spans exactly the same set of allocations as the class of policies in (41), but now the nominal prices that support any such allocation are different.

Thus let us start by collecting, once again, the key equilibrium conditions:

$$0 = n_{it}^{1+\epsilon} - \mathbb{E}_{it}\left[\mathcal{M}_{it}U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\theta c_{it}\right] \tag{69}$$

$$0 = \mathbb{E}_{it}\left[\left(l_{it}^{1+\epsilon} - \mathcal{M}_{it}U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\eta y_{it}\right)\right] \tag{70}$$

$$Y_t = \left[\int (q_{it}l_{it}^{\eta})^{\frac{\rho-1}{\rho}}\,di\right]^{\frac{\rho}{\rho-1}}; \tag{71}$$

$$\frac{p_{it}}{P_t} = \left(\frac{q_{it}l_{it}^{\eta}}{Y_t}\right)^{-\frac{1}{\rho}}; \tag{72}$$

$$\log(1+R_t) = \gamma\left(\mathbb{E}_t[\log Y_{t+1}] - \log Y_t\right) + \left(\mathbb{E}_t[\log P_{t+1}] - \log P_t\right) \tag{73}$$

Clearly, these conditions are necessary and sufficient for equilibrium *regardless* of how R_t is determined.

Now, let us restrict R_t to be determined according to (50) and let us consider the following strategies (omitting unimportant constants):

$$\begin{aligned}
\log q_{it} &= \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \\
\log l_{it} &= l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t \\
\log p_{it} &= \psi_s s_{it} + \psi_x x_{it} + \psi_z z_t + \psi_{-k}\bar{s}_{t-k}
\end{aligned}$$

for some coefficients $\left(\varphi_s, \varphi_x, ..., \psi_{-k}\right)$. The proof proceeds by obtaining the restrictions that equilibrium imposes on these coefficients and by show that the restrictions imposed on the φ's and the l's (the quantity-related coefficients) are the same as those found in Lemma 6, whereas those that are imposed on the ψ's (the price-related coefficients) are different. The proof then concludes by obtaining the scalars ρ_{-k} and ρ_z the implement these coefficients.

First, consider conditions (69) and (70), which encapsulated the optimal behavior of the firms and the workers. These conditions can be restated as:

$$\begin{aligned}
\mathbb{E}_{it}[\log l_{it}] &= \frac{1}{\theta}\left(\log q_{it} - a_{it}\right) \\
\log q_{it} &= \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \frac{\hat{\alpha}}{\chi}\mathbb{E}_{it}[\log Y_t].
\end{aligned}$$

Because the proposed strategies for q_{it} and l_{it} are the same as those in Lemma 6, and because neither the prices nor the interest rate enter in the above conditions, these conditions reduce to exactly the same restrictions on the ϕ and the l coefficients as those in Lemma 6.

Next, consider condition (71). Under the proposed strategies, this reduces to

$$\log Y_t = c_{\bar{s}}\bar{s}_t + c_z z_t$$

with

$$c_{\bar{s}} = (\varphi_s + \varphi_x) + \eta(l_s + l_x + l_{\bar{s}}) \qquad \text{and} \qquad c_z = \varphi_z + \eta l_z \tag{74}$$

Similarly to the case of the ϕ and the l coefficients, the restrictions that pertain to the coefficients $c_{\bar{s}}$ and c_s are the therefore the same as the corresponding ones in the proof of Lemma 6.

Next, consider condition (72), which can be expressed as follows:

$$-\rho\left(\log p_{it} - \log P_t\right) = \left(\log y_{it} - \log Y_t\right)$$

Under the proposed strategies, this reduces to the following:

$$-\rho(\psi_s s_{it} + \psi_x x_{it} - (\psi_s + \psi_x)\bar{s}_t) = (\varphi_s + \eta l_s)s_{it} + (\varphi_x + \eta l_x)x_{it} - (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t.$$

This in turn is true if and only if

$$\psi_s = -\frac{1}{\rho}(\varphi_s + \eta l_s) \qquad \text{and} \qquad \psi_x = -\frac{1}{\rho}(\varphi_x + \eta l_x). \tag{75}$$

The restrictions on the coefficients ψ_s and ψ_x are therefore also the same as the corresponding ones in the proof of Lemma 6. (Also note that, up to this point, the coefficients ψ_{-k} and ψ_z are "free".)

Finally, consider the Euler condition (73). Let $X_t \equiv \gamma \log Y_t + \log P_t$ and rewrite (73) as follows:

$$X_t = -\log(1 + R_t) + \mathbb{E}_t[X_{t+1}].$$

Iterating this condition forward k times yields

$$X_t = -\sum_{j=0}^{k} \mathbb{E}_t\left[\log(1 + R_{t+j})\right] + \mathbb{E}_t\left[X_{t+k+1}\right] \tag{76}$$

Under the proposed strategies, the output and price level in period $t + k + 1$ satisfy

$$
\begin{aligned}
\mathbb{E}_t\left[\log Y_{t+k+1}\right] &= \mathbb{E}_t\left[c_{\bar{s}}\bar{s}_{t+k+1} + c_z z_{t+k+1}\right] = constant \\
\mathbb{E}_t\left[\log P_{t+k+1}\right] &= \mathbb{E}_t\left[(\psi_s + \psi_x)\bar{s}_{t+k+1} + \psi_z z_{t+k+1} + \psi_{-k}\bar{s}_{t+1}\right] = constant,
\end{aligned}
$$

From (50), the interest rate satisfies

$$\mathbb{E}_t\left[\log(1 + R_{t+j})\right] = \mathbb{E}_t\left[\rho_{-k}\bar{s}_{t-k+j} + \rho_z z_{t+j}\right] = \begin{cases} \rho_{-k}\bar{s}_{t-k} + \rho_z z_t, & \text{for } j = 0 \\ constant, & \text{for } j \in \{1, ..., k-1\} \\ constant + \rho_{-k}\bar{s}_t, & \text{for } j = k \end{cases}$$

It follows that (76) reduces to the following (omitting constants):

$$X_t = -\rho_{-k}\bar{s}_{t-k} - \rho_z z_t - \rho_{-k}\bar{s}_t$$

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At the same time, the proposed strategies imply

$$
\begin{aligned}
X_t \equiv \gamma \log Y_t + \log P_t &= \gamma \left(c_{\bar{s}} \bar{s}_t + c_z z_t \right) + \left(\left(\psi_s + \psi_x \right) \bar{s}_t + \psi_z z_t + \psi_{-k} \bar{s}_{t-k} \right) \\
&= \left(\gamma c_{\bar{s}} + \psi_s + \psi_x \right) \bar{s}_t + \left(\gamma c_z + \psi_z \right) z_t + \psi_{-k} \bar{s}_{t-k}
\end{aligned}
$$

The two are consistent if and only if the following hold true:

$$
\begin{aligned}
\psi_{-k} &= \gamma c_{\bar{s}} + \psi_s + \psi_x && (77) \\
\rho_{-k} &= -\psi_{-k} && (78) \\
\rho_z &= -\gamma c_z - \psi_z && (79)
\end{aligned}
$$

Now, pick any of the allocations that are implementable under (41). This is equivalent to choosing an arbitrary value for $l_{\bar{s}}$ and letting the ϕ and l coefficients be determined by the restrictions stated in that lemma, namely (43)-(48). Any such choice also pins down the values of $c_{\bar{s}}$, c_z, ψ_s, and ψ_x according to the restrictions (74)-(75). But then this pins down ψ_{-k} from (77), which in turn pins down ρ_{-k} from (78). We have thus arrived to the value of ρ_{-k} which is necessary and sufficient for the policy in (50) to implement the allocation under consideration. What we are left with then is the familiar indeterminacy of the response of the price level to the public signal: whatever the allocation that has been implemented, we can still pick an arbitrary ψ_z and let ρ_z be determined from (79). ∎

Proof of Lemma 8. Since prices are set without any information about the monetary shock (a maintained assumption throughout), employment and aggregate output will respond to $\tilde{r}_t$ one-to-one. Thus, if we conjecture

$$
\begin{aligned}
\log l_{it} &= l_{\bar{s}} \bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t + l_r r_t \\
\log Y_t &= c_{\bar{s}} \bar{s}_t + c_z z_t + c_r r_t
\end{aligned}
$$

and follow steps analogous to those in the proof of part (i) of Lemma 6, we obtain $c_r = \eta l_m = -1/\gamma$. Modulo these changes, it is immediate to see that conditions (43)-(48) remain true. ∎

Proof of Lemma 9. Similarly to the baseline model, once we use the equilibrium conditions for the wages and the prices, the first-best level of output *conditional* on $\log q_{it}$ satisfies the following FOC of the firm's profit with respect to the second input:

$$
l_{it}^\epsilon = U'(Y_t) \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}} \left(\eta \frac{y_{it}}{l_{it}} \right). \tag{80}
$$

Furthermore, the first-best level of output satisfies (80) and, in addition, the following FOC of the firm's profit with respect to the first input:

$$
n_{it}^\epsilon = U'(Y_t) \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}} \left(\theta \frac{y_{it}}{n_{it}} \right) \tag{81}
$$

Note that, by the definition of first-best level of output, the markup and the expectation operator are absent from both conditions.

Rearranging (80) to solve for $\log l_{it}$ (we omit unimportant constants)

$$(1 + \epsilon) \log l_{it} = \left(\frac{1}{\rho} - \gamma \right) \log Y_t + \left(1 - \frac{1}{\rho} \right) \log y_{it} \tag{82}$$

and using $\log y_{it} = \log q_{it} + \eta \log l_{it}$ to replace l_{it}, we can restate the above as

$$\log y_{it} = \rho \nu \log q_{it} + \bar{\alpha} \log Y_t,$$

which proves (52). Using (52) to replace y_{it} into (81), taking logs, and noting that $\log q_{it} = a_{it} + \theta \log n_{it}$, we arrive at

$$\log q_{it}^* = \hat{\phi}_a a_{it} + \frac{\hat{\alpha}}{\chi} \log Y_t, \tag{83}$$

where $\hat{\phi}_a$, $\hat{\alpha}$, χ, and ν are defined in (51). Finally, integrating (82) across different islands

$$\int \log l_{it} di = \frac{1 - \gamma}{1 + \epsilon} \log Y_t$$

and combining the latter with (39) gives

$$\log Y_t = \chi \log Q_t$$

and therefore (83) gives (53). ∎

Proof of Lemma 10. As with the proof of Lemma 2, we drop the time index t and focus on the characterization of the per-period welfare flow. The latter is now defined as

$$W = \mathbb{E} \left[\frac{Y^{1-\gamma}}{1 - \gamma} - \frac{1}{1 + \epsilon} \int \left(\frac{q_i}{A_i} \right)^{\frac{1+\epsilon}{\theta}} di - \frac{1}{1 + \epsilon} \int \left(\frac{y_i}{q_i} \right)^{\frac{1+\epsilon}{\eta}} di \right]$$

In the sequel, we then proceed to establish that the above can be expressed as follows:

$$W = W^* \exp \left(-\frac{1}{2} (1 - \hat{\gamma}) (1 + \hat{\epsilon}) \Lambda' \right).$$

where W^* is the first-best level of W.

We first focus on the first and third component of W:

$$\mathbb{E} \left[\frac{Y^{1-\gamma}}{1 - \gamma} - \frac{1}{1 + \epsilon} \int \left(\frac{y_i}{q_i} \right)^{\frac{1+\epsilon}{\eta}} di \right].$$

This expression resembles the expression for welfare in the baseline model with q_i replacing local productivity a_i. We can, thus, follows the same steps as in the proof of Lemma 2 and rewrite the latter expression as (omitting unimportant constants)

$$\mathbb{E} [Q]^{\frac{1-\gamma}{1+\epsilon} - 1 + \gamma \frac{1+\epsilon}{1-\eta}} \exp \left(-\frac{1}{2} \frac{\frac{1+\epsilon}{\eta} (1 - \gamma)}{\frac{1+\epsilon}{\eta} - 1 + \gamma} \bar{\Omega} \right) \exp \left(-\frac{1}{2} \frac{1 + \epsilon}{\eta} (1 - \gamma) \left(\Sigma_Y + \frac{1}{1 - \bar{\alpha}} \sigma_y \right) \right),$$

with
$$\bar{\Omega} \equiv \left(\frac{1+\epsilon}{\eta}\gamma + \gamma - 1\right)\frac{\frac{1+\epsilon}{\eta} + \gamma - 1}{\left(\frac{1+\epsilon}{\eta}\right)^2}Var\left(\log\bar{Y}\right) = \frac{\frac{1+\epsilon}{\eta}\gamma + \gamma - 1}{\frac{1+\epsilon}{\eta} + \gamma - 1}Var\left(\log Q\right),$$

where the second line uses the fact that (52) implies $\log\bar{Y} = \frac{\rho\nu}{1-\bar{\alpha}}\log Q$.

We can use these results to rewrite W as follows:

$$W = \mathbb{E}\left[Q\right]^{1-\hat{\gamma}}\exp\left(-\frac{1}{2}\frac{\frac{1+\epsilon}{\eta}(1-\gamma)}{\frac{1+\epsilon}{\eta} - 1 + \gamma}\bar{\Omega} - \frac{1}{2}\frac{1+\epsilon}{\eta}(1-\gamma)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right) - \frac{1}{1+\epsilon}\mathbb{E}\left[\int\left(\frac{q_i}{A_i}\right)^{\frac{1+\epsilon}{\theta}}di\right]$$

Using the property (15),

$$E\left[Q\right]^{1-\hat{\gamma}} = E\left[Q^{1-\hat{\gamma}}\right]e^{\frac{1}{2}\hat{\gamma}(1-\hat{\gamma})Var(\log Q)}$$

and, thus,

$$W = \frac{1}{1-\hat{\gamma}}\mathbb{E}\left[Q^{1-\hat{\gamma}}\right]\Xi - \frac{1}{1+\hat{\epsilon}}\int\left(\frac{q_i}{A_i}\right)^{1+\hat{\epsilon}}di$$

where

$$\Xi \equiv (1-\hat{\gamma})\theta\exp\left(\frac{1}{2}\hat{\gamma}(1-\hat{\gamma})Var(\log Q) - \frac{1}{2}\frac{\frac{1+\epsilon}{\eta}(1-\gamma)}{\frac{1+\epsilon}{\eta} - 1 + \gamma}\bar{\Omega} - \frac{1}{2}\frac{1+\epsilon}{\eta}(1-\gamma)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right),$$

or, using the definition of $\bar{\Omega}$,

$$\Xi = (1-\hat{\gamma})\theta\exp\left(-\frac{1}{2}\frac{1+\epsilon}{\eta}(1-\gamma)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right).$$

Note that then that the functional that maps the strategy q into the welfare level $\mathcal{W}$ is the same as the one in the proof of Lemma 2, provided that we make two changes: we replace ρ, γ, and ϵ with, respectively, $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\epsilon}$; and we accommodate the constant Ξ. Thus, following the same steps as in that proof, we can obtain the following characterization of the per-period welfare flow:

$$W = W^*\exp\left\{-\frac{1}{2}(1+\hat{\epsilon})(1-\hat{\gamma})\left(\Sigma_Q - \frac{1}{1-\hat{\alpha}}\sigma_q\right)\right\}\Xi^{\frac{1+\hat{\epsilon}}{\hat{\epsilon}+\hat{\gamma}}}.$$

Finally, using the definition of Ξ and rearranging gives

$$\begin{aligned}W &= W^*\exp\left\{-\frac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})\left(\Sigma_Q - \frac{1}{1-\hat{\alpha}}\sigma_q\right)\right\}\exp\left(-\frac{1}{2}\frac{1+\hat{\epsilon}}{\hat{\epsilon}+\hat{\gamma}}(1-\gamma)\frac{1+\epsilon}{\eta}\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right)\\
&= W^*\exp\left\{-\frac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})\left[\left(\Sigma_Q - \frac{1}{1-\hat{\alpha}}\sigma_q\right) + \xi\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right]\right\},\end{aligned}$$

where

$$\xi \equiv \frac{\frac{1+\epsilon}{\eta} + \gamma - 1}{\hat{\epsilon}+\hat{\gamma}}.$$

The result then follows from translating the above in terms of life-time welfare and defining the function w as $w(x) \equiv \mathcal{W}^*\exp\left\{-\frac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})x\right\}$. ∎

Proof of Lemma 3. *Part (i).* This part follows, trivially, from projecting (regressing) the equilibrium nominal GDP on the fundamental and the public signal, and letting (λ_s, λ_z) be the projection coefficients and m_t the residual.

Part (ii). From Lemma 8, the sum $c_{\bar{s}} + (\psi_s + \psi_x)$, which gives the response of $\log M_t$ to the fundamental, is a linear function of ρ_s. Furthermore, c_z is invariant to ρ_z, whereas ψ_z is inversely to it. It follows that, for any pair (λ_s, λ_z), we can find a value of the pair (ρ_s, ρ_z) so that

$$c_{\bar{s}} + (\psi_s + \psi_x) = \lambda_s \qquad \text{and} \qquad c_z + \psi_z = \lambda_z$$

The result then follows from considering the policy that is identified by the combination of this particular value for the pair (ρ_s, ρ_z) with $r_t = -\frac{1}{\gamma} m_t$.

Part (iii). This follows directly from the analysis in Section B.1. ∎

Proof of Lemma 4. From part (ii) of Lemma 6 and the proof of Lemma 3 we conclude that a monetary policy as in (7) replicates flexible-price allocations if and only if $\lambda_s = \lambda_s^*$, where $\lambda_s^* \equiv \bar{\Pi} + \Pi l_{\bar{s}}^*$ and $l_{\bar{s}}^*$ is given by (49). We can then use Lemma 3 to translate these results in terms of the interest rate rule. ∎

Proof of Proposition 5. *Part (i).* Similarly to the proof of Lemma 6, once we use the equilibrium conditions for the wages and the prices, the FOCs of the firm's profit with respect to the two inputs reduce to the following:

$$n_{it}^{\epsilon} = \mathbb{E}_{it}\left[\mathcal{M}_{it} U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\theta \frac{y_{it}}{n_{it}}\right)\right] \tag{84}$$

$$l_{it}^{\epsilon} = \mathcal{M}_{it} U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\eta \frac{y_{it}}{l_{it}}\right) \tag{85}$$

Note that the first condition is the same as (37). The second condition, by contrast, does not feature an expectation operator, because the absence of nominal rigidity means that the stage-2 input adjusts so as to equate marginal cost and marginal revenue state-by-state. Rearranging (85) to solve for $\log l_{it}$ (we omit unimportant constants)

$$(1 + \epsilon)\log l_{it} = -\mu_{it} + \left(\frac{1}{\rho} - \gamma\right)\log Y_t + \left(1 - \frac{1}{\rho}\right)\log y_{it} \tag{86}$$

and using $\log y_{it} = \log q_{it} + \eta \log l_{it}$ to replace l_{it}, we can restate the above as

$$\log y_{it} = b_q \log q_{it} - b_\mu \mu_{it} + \bar{\alpha} \log Y_t$$

where

$$b_q \equiv \rho\nu \quad \text{and} \quad b_\mu \equiv \frac{\eta}{1 + \epsilon}\rho\nu.$$

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Using the above result to replace y_{it} into (84), taking logs, and noting that $\log q_{it} = a_{it} + \theta \log n_{it}$, we arrive at

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \frac{\hat{\alpha}}{\chi} \mathbb{E}_{it}[\log Y_t],$$

where $\hat{\phi}_a$, $\hat{\phi}_\mu$, $\hat{\alpha}$, χ, and ν are defined in (51). Finally, integrating (86) across different islands

$$\int \log l_{it} di = \frac{1 - \gamma}{1 + \epsilon} \log Y_t - \frac{1}{1 + \epsilon} \bar{\mu}_t$$

and combining the latter with (39) gives

$$\log Y_t = \chi \left(\log Q_t - \frac{\eta}{1 + \epsilon} \bar{\mu}_t \right)$$

and, thus,

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \hat{\phi}_{\bar{\mu}} \mathbb{E}_{it}[\bar{\mu}_t] + \hat{\alpha} \mathbb{E}_{it}[\log Q_t],$$

where $\hat{\phi}_{\bar{\mu}} \equiv -\hat{\alpha} \frac{\eta}{1+\epsilon}$.

Part (ii). From Lemma 10 we have that, irrespective of whether the nominal rigidity is present or not, welfare is a decreasing function of

$$\Lambda' = \left(\Sigma_Q + \frac{1}{1 - \hat{\alpha}} \sigma_q \right) + \xi \left(\Sigma_Y + \frac{1}{1 - \bar{\alpha}} \sigma_y \right).$$

When the policy of Lemma 4 is in place, the equilibrium quantities satisfy the FOC (85) which, as showed in part (i), can be rearranged as

$$\log y_{it} = b_q \log q_{it} - b_\mu \mu_{it} + \bar{\alpha} \log Y_t.$$

On the other hand, Lemma 9 shows that the first-best level of output conditional on the first-period equilibrium production decision satisfies

$$\log \bar{y}_{it} = b_q \log q_{it} + \bar{\alpha} \log \bar{Y}_t.$$

Therefore, the aggregate and local gaps are given by, respectively,

$$\log Y_t - \log \bar{Y}_t = -b_{\bar{\mu}} \bar{\mu}_t$$

where $b_{\bar{\mu}} \equiv \frac{b_\mu}{1 - \bar{\alpha}}$ and

$$\log y_{it} - \log \bar{y}_{it} = -b_\mu \mu_{it} - \bar{\alpha} b_{\bar{\mu}} \bar{\mu}_t.$$

Importantly, note that b_μ and $b_{\bar{\mu}}$ are independent of the information structure. It follows that the second term in Λ' reduces to

$$\Sigma_Y + \frac{1}{1 - \bar{\alpha}} \sigma_y = b_{\bar{\mu}}^2 \sigma_{\bar{\mu}}^2 + \frac{1}{1 - \bar{\alpha}} b_\mu^2 \sigma_\xi^2$$

which is independent of the information structure. Therefore, (55) reduces to

$$\Lambda' = \Sigma_Q + \frac{1}{1 - \hat{\alpha}} \sigma_q + \omega$$

where $\omega \equiv \xi \frac{b_\mu^2}{1 - \bar{\alpha}} \left(\frac{1}{1 - \bar{\alpha}} \sigma_{\bar{\mu}}^2 + \sigma_\xi^2 \right)$, which corresponds to (11) in the main text. ∎

Proof of Theorem 3. From Proposition 5, when the policy that replicates flexible prices is in place, welfare is a decreasing function of

$$\Lambda' = \Sigma + \frac{1}{1 - \hat{\alpha}}\sigma + \omega.$$

Since ω is independent of the information structure, the effects of the precision of private and public information are determined only by the changes in the volatility and dispersion of the first-period gaps.

Consider first the case with technology shocks. In that case $\hat{\phi}_\mu = \hat{\phi}_{\bar{\mu}} = 0$ and $\omega = 0$ and the welfare effects coincide with those found in Theorem 1.

In the case of markup shocks, $\hat{\phi}_\mu$ is given in (51), $\hat{\phi}_{\bar{\mu}} \equiv -\hat{\alpha}\frac{\eta}{1+\epsilon}$, ω is given in Proposition 5, and

$$\Lambda' = \frac{\left(\hat{\alpha}\eta - (1 + \epsilon)\hat{\phi}_\mu\right)^2 ((1 - \hat{\alpha})\kappa_x + \kappa_z) - (1 - \hat{\alpha})(1 + \epsilon)\left(2\alpha\eta - (1 + \epsilon)(1 + \hat{\alpha})\hat{\phi}_\mu\right)\kappa_\mu\hat{\phi}_\mu}{(1 - \hat{\alpha})^2(1 + \epsilon)^2(\kappa_\mu + (1 - \hat{\alpha})\kappa_x + \kappa_z)\kappa_\mu} + \frac{\hat{\phi}_\mu^2}{(1 - \hat{\alpha})\kappa_\xi} + \omega.$$

Differentiating Λ' with respect to the precision of, respectively, public and private information gives

$$\frac{\partial \Lambda'}{\partial \kappa_z} = \frac{\hat{\alpha}^2\left(\eta - (1 + \epsilon)\hat{\phi}_\mu\right)^2}{(1 - \hat{\alpha})^2(1 + \epsilon)^2(\kappa_\mu + (1 - \hat{\alpha})\kappa_x + \kappa_z)^2}$$

and

$$\frac{\partial \Lambda'}{\partial \kappa_x} = \frac{\hat{\alpha}^2\left(\eta - (1 + \epsilon)\hat{\phi}_\mu\right)^2}{(1 - \hat{\alpha})(1 + \epsilon)^2(\kappa_\mu + (1 - \hat{\alpha})\kappa_x + \kappa_z)^2},$$

which are both always positive.∎

Proof of Proposition 6. While derivations are lengthy, the idea of the proof is simple. From Lemma 10 we know that welfare losses Λ' depend on the volatility and dispersion of the first-period and second-period gaps between equilibrium and first-best production. We thus take each gap and decompose it into two new gaps: the first captures the deviation of equilibrium production from production with flexible prices; and the second captures the deviation of flexible-price production from first-best production. Finally, we rewrite the volatility and dispersion of the original gaps in terms of the volatility and dispersion of the new gaps.

We introduce some notation which will simplify the expressions in the proof. First, we let $\mathring{q}_{it}$ denote first-period output when monetary policy replicates flexible prices. Next, we decompose the first-period output gap as follows

$$\log \tilde{q}_{it} \equiv \log q_{it} - \log q_{it}^* = \log \hat{q}_{it} + \log \check{q}_{it},$$

where $\log \hat{q}_{it} \equiv \log q_{it} - \log \mathring{q}_{it}$ denotes the deviation of first-period equilibrium production from the flexible-price benchmark, and where $\log \check{q}_{it} \equiv \log \mathring{q}_{it} - \log q_{it}^*$ denotes the usual output gap with

flexible prices. With the same notation we obtain a similar decomposition for the second-period output gap:

$$\log \tilde{y}_{it} \equiv \log y_{it} - \log \bar{y}_{it} = \log \hat{y}_{it} + \log \check{y}_{it}.$$

Finally, in the proof we use analogous decompositions for the first and second-period aggregate gaps, $\log \tilde{Q}_t$ and $\log \tilde{Y}_t$.

The volatility of the first-period aggregate gap can thus be expressed as follows

$$\Sigma_Q \equiv Var\left(\log \tilde{Q}_t\right) = Var\left(\log \hat{Q}_t\right) + 2Cov\left(\log \hat{Q}_t, \log \check{Q}_t\right) + \check{\Sigma}_Q,$$

where $\check{\Sigma}_Q$ is the aggregate volatility of the first-period aggregate gap with flexible prices. Similarly, the dispersion of the first-period local gaps can be rewritten as

$$
\begin{aligned}
\sigma_q &\equiv Var\left(\log \tilde{q}_{it} - \log \tilde{Q}_t\right) \\
&= Var\left(\log \hat{q}_{it} - \log \hat{Q}_t\right) + 2Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right) + \check{\sigma}_q,
\end{aligned}
$$

where $\check{\sigma}_q$ is the dispersion of the first-period local gaps with flexible prices.

From part (i) of Lemma 6, first-period production satisfies (42) with φ_x and φ_z given by (44) and (45), respectively. Denote by $\mathring{\varphi}_x$ and $\mathring{\varphi}_z$ the coefficients φ_x and φ_z when the policy replicating flexible prices is in place. Combining parts (i) and (iii) of Lemma 6, $\mathring{\varphi}_x$ and $\mathring{\varphi}_z$ satisfy

$$
\begin{aligned}
\mathring{\varphi}_x &= \Gamma_x + \Gamma_x' l_{\bar{a}}^* = \frac{\hat{\phi}_a}{\kappa_a + (1-\hat{\alpha})\,\kappa_x + \kappa_z}\hat{\alpha}\kappa_x, \\
\mathring{\varphi}_z &= \Gamma_z + \Gamma_z' l_{\bar{a}}^* = \frac{\hat{\phi}_a}{\kappa_a + (1-\hat{\alpha})\,\kappa_x + \kappa_z}\frac{\hat{\alpha}\kappa_z}{1-\hat{\alpha}}.
\end{aligned}
$$

Finally, first-best production is given by (53). We can thus compute all the gaps defined above as follows

$$
\begin{aligned}
\log \hat{Q}_t &= \hat{Q}_{\bar{a}}^a \bar{a}_t + \hat{Q}_z^a \varepsilon_t \\
\log \check{Q}_t &= \check{Q}_{\bar{a}}^a \bar{a}_t + \check{Q}_z^a \varepsilon_t,
\end{aligned}
$$

where $\hat{Q}_{\bar{a}}^a \equiv \Gamma_x' + \Gamma_z'$, $\hat{Q}_z^a \equiv \Gamma_z'$, $\check{Q}_{\bar{a}}^a \equiv -\frac{\hat{\alpha}}{1-\hat{\alpha}}\hat{\phi}_a + \Gamma_x + \Gamma_x' l_{\bar{a}}^* + \Gamma_z + \Gamma_z' l_{\bar{a}}^*$, and $\check{Q}_z^a \equiv \Gamma_z + \Gamma_z' l_{\bar{a}}^*$. Therefore,

$$
\begin{aligned}
Var\left(\log \hat{Q}_t\right) &= \left(\hat{Q}_{\bar{a}}^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_a} + \left(\hat{Q}_z^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_z} \\
Cov\left(\log \hat{Q}_t, \log \check{Q}_t\right) &= \hat{Q}_{\bar{a}}^a \check{Q}_{\bar{a}}^a (l_{\bar{a}} - l_{\bar{a}}^*) \frac{1}{\kappa_a} + \hat{Q}_z^a \check{Q}_z^a (l_{\bar{a}} - l_{\bar{a}}^*) \frac{1}{\kappa_z}.
\end{aligned}
$$

Similarly, the terms capturing the dispersion of the first-period local gaps can be expressed as

$$
\begin{aligned}
\log \hat{q}_{it} - \log \hat{Q}_t &= \hat{q}_x^a (l_{\bar{a}} - l_{\bar{a}}^*) u_{it} \\
\log \check{q}_{it} - \log \check{Q}_t &= \check{q}_x^a u_{it}.
\end{aligned}
$$

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where $\hat{q}_x^a \equiv \Gamma_x'$ and $\check{q}_x^a \equiv \Gamma_x + \Gamma_x' l_{\bar{a}}^*$. Therefore,

$$Var\left(\log \hat{q}_{it} - \log \hat{Q}_t\right) = (\hat{q}_x^a)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_x}$$

$$Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right) = \hat{q}_x^a \check{q}_x^a (l_{\bar{a}} - l_{\bar{a}}^*) \frac{1}{\kappa_x}.$$

Furthermore, it turns out that, in the case of technology shocks, the covariance terms do not contribute to the welfare losses, that is,

$$Cov\left(\log \hat{Q}_t, \log \check{Q}_t\right) + \frac{1}{1 - \hat{\alpha}} Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right)$$

$$= \left(\hat{Q}_{\bar{a}}^a \check{Q}_{\bar{a}}^a \frac{1}{\kappa_a} + \hat{Q}_z^a \check{Q}_z^a \frac{1}{\kappa_z} + \frac{1}{1 - \hat{\alpha}} \hat{q}_x^a \check{q}_x^a \frac{1}{\kappa_x}\right) (l_{\bar{a}} - l_{\bar{a}}^*) = 0.$$

Collecting all the remaining terms together, the second-order welfare losses associated with the first-period gaps can be rewritten as

$$\Sigma_Q + \frac{1}{1 - \hat{\alpha}} \sigma_q = \vartheta_a (l_{\bar{a}} - l_{\bar{a}}^*)^2 + \check{\Sigma}_Q + \frac{1}{1 - \hat{\alpha}} \check{\sigma}_q, \tag{87}$$

where

$$\vartheta_a \equiv \left(\hat{Q}_{\bar{a}}^a\right)^2 \frac{1}{\kappa_a} + \left(\hat{Q}_z^a\right)^2 \frac{1}{\kappa_z} + \frac{1}{1 - \hat{\alpha}} (\hat{q}_x^a)^2 \frac{1}{\kappa_x}.$$

Let's now turn to the second-period output gaps. With technology shocks flexible-price allocations satisfy (85) with a constant markup. If we rearrange (85) we can then obtain an expression similar to (52), except for a constant capturing the markup. Thus, with technology shocks, flexible-price allocations and first-best allocations conditional on equilibrium first-period production differ only by a constant and, thus, there is no use in decomposing the second-period gaps as we did for the first-period gaps. Using the conditions in Lemma 6, the second-period aggregate and local gaps are, respectively,

$$\log \tilde{Y}_t = \tilde{Y}_{\bar{a}}^a (l_{\bar{a}} - l_{\bar{a}}^*) \bar{a}_t + \tilde{Y}_z^a (l_{\bar{a}} - l_{\bar{a}}^*) \varepsilon_t$$

$$\log \tilde{y}_{it} = \tilde{Y}_x^a (l_{\bar{a}} - l_{\bar{a}}^*) u_{it}$$

where $\tilde{Y}_{\bar{a}}^a \equiv \left(1 + \frac{\eta}{\theta}\right) \Gamma_x' + \left(1 + \frac{\eta}{\theta}\right) \Gamma_z' + \frac{\eta \kappa_a}{\kappa_a + \kappa_x + \kappa_z}$, $\tilde{Y}_z^a \equiv \left(1 + \frac{\eta}{\theta}\right) \Gamma_z' - \frac{\eta \kappa_z}{\kappa_a + \kappa_x + \kappa_z}$, and $\tilde{Y}_x^a \equiv \left(1 + \frac{\eta}{\theta}\right) \Gamma_x' - \frac{\eta \kappa_x}{\kappa_a + \kappa_x + \kappa_z}$. Thus, volatility and dispersion of second-period gaps are, respectively,

$$\Sigma_Y = \left(\tilde{Y}_{\bar{a}}^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_a} + \left(\tilde{Y}_z^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_z} + \sigma_m^2$$

$$\sigma_y = \left(\tilde{Y}_x^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_x}.$$

Collecting all the terms, the second-order welfare losses associated with the second-period gaps can be rewritten as

$$\Sigma_Y + \frac{1}{1 - \hat{\alpha}} \sigma_y = \vartheta_a' (l_{\bar{a}} - l_{\bar{a}}^*)^2 + \sigma_m^2,$$

where

$$\vartheta'_a \equiv \left(\tilde{Y}^a_{\bar{a}}\right)^2 \frac{1}{\kappa_a} + \left(\tilde{Y}^a_z\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\bar{\alpha}} \left(\tilde{Y}^a_x\right)^2 \frac{1}{\kappa_x}.$$

Finally, in the proof of Lemma 8 we show that we can rewrite $l_{\bar{a}} - l^*_{\bar{a}}$ as $(\lambda_s - \lambda^*_s)/\Pi$, thus, if we let $\Lambda = \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\check{\sigma}_q$, $\mathcal{T} = \sigma^2_m$, and

$$\Theta \equiv \frac{\vartheta_a + \xi\vartheta'_a}{\Pi^2\sigma^2_a},$$

the statement of the proposition follows directly from the welfare decomposition in Lemma 10.

The proof for the case with markup shocks closely resembles the proof with technology shocks, so here we simply report the terms required to derive $\mathcal{K}$ and $\mathcal{T}$. Since markups are absent from first-best allocations, the latter are constant when the business cycle is driven by markup shocks. Using parts (i) and (iii) of Lemma 6, the first-period aggregate gap $\log \check{Q}_t$ is given by the sum of the following gaps

$$\log \check{Q}_t = \hat{Q}^\mu_{\bar{\mu}} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \bar{\mu}_t + \hat{Q}^\mu_z \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \varepsilon_t$$
$$\log \hat{Q}_t = \check{Q}^\mu_{\bar{\mu}} \bar{\mu}_t + \check{Q}^\mu_z \varepsilon_t.$$

where $\hat{Q}^\mu_{\bar{\mu}} \equiv \Gamma'_x + \Gamma'_z$, $\hat{Q}^\mu_z \equiv \Gamma'_z$, $\check{Q}^\mu_{\bar{\mu}} \equiv \hat{\phi}_\mu + \Gamma_x + \Gamma'_x l^*_{\bar{\mu}} + \Gamma_z + \Gamma'_z l^*_{\bar{\mu}}$, and $\check{Q}^\mu_z \equiv \Gamma_z + \Gamma'_z l^*_{\bar{\mu}}$. Therefore,

$$Var\left(\log \check{Q}_t\right) = \left(\hat{Q}^\mu_{\bar{\mu}}\right)^2 \frac{1}{\kappa_\mu} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + \left(\hat{Q}^\mu_z\right)^2 \frac{1}{\kappa_z} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2$$
$$Cov\left(\log \check{Q}_t, \log \hat{Q}_t\right) = \hat{Q}^\mu_{\bar{\mu}}\check{Q}^\mu_{\bar{\mu}} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \frac{1}{\kappa_\mu} + \hat{Q}^\mu_z\check{Q}^\mu_z \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \frac{1}{\kappa_z}.$$

Similarly, the terms capturing the dispersion of the first-period local gaps can be expressed as

$$\log \hat{q}_{it} - \log \hat{Q}_t = \hat{q}^\mu_x \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) u_{it}$$
$$\log \check{q}_{it} - \log \check{Q}_t = \check{q}^\mu_\xi \xi_{it} + \check{q}^\mu_x u_{it},$$

where $\hat{q}^\mu_x \equiv \Gamma'_x$, $\check{q}^\mu_\xi \equiv \hat{\phi}_\mu$, and $\check{q}^\mu_x \equiv \Gamma_x + \Gamma'_x l^*_{\bar{\mu}}$. Therefore,

$$Var\left(\log \hat{q}_{it} - \log \hat{Q}_t\right) = (\hat{q}^\mu_x)^2 \frac{1}{\kappa_x} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2$$
$$Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right) = \hat{q}^\mu_x \left(\check{q}^\mu_\xi + \check{q}^\mu_x\right) \frac{1}{\kappa_x} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right).$$

Collecting all the terms, the second-order welfare losses associated with the first-period gaps can be rewritten as

$$\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q = \vartheta_\mu \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + 2\tilde{\vartheta}_\mu \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) + \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\check{\sigma}_q,$$

where

$$\vartheta_\mu \equiv \left(\hat{Q}^\mu_{\bar{\mu}}\right)^2 \frac{1}{\kappa_\mu} + \left(\hat{Q}^\mu_z\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}} (\hat{q}^\mu_x)^2 \frac{1}{\kappa_x}$$
$$\tilde{\vartheta}_\mu \equiv \hat{Q}^\mu_{\bar{\mu}}\check{Q}^\mu_{\bar{\mu}} \frac{1}{\kappa_\mu} + \hat{Q}^\mu_z\check{Q}^\mu_z \frac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}}\hat{q}^\mu_x \left(\check{q}^\mu_\xi + \check{q}^\mu_x\right) \frac{1}{\kappa_x}.$$

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Let's now turn to the second-period output gaps. With markup shocks, flexible-price output and first-best output conditional on first-period equilibrium production are no longer related only by a constant. In particular, the latter satisfies (52) and the corresponding aggregate level is obtained by aggregating (52) across islands:

$$\log \bar{Y}_t = \frac{\rho \nu}{1 - \bar{\alpha}} \log Q_t.$$

In contrast, equilibrium and flexible-price allocations can be obtained using the conditions in parts (i), (iii), and (iv) of Lemma 6.

Combining all the terms, the second-period aggregate gaps are given by the following:

$$\begin{aligned}
\log \hat{Y}_t &= \hat{Y}_{\bar{\mu}}^{\mu} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^* \right) \bar{\mu}_t + \hat{Y}_z^{\mu} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^* \right) \varepsilon_t + m_t \\
\log \check{Y}_t &= \hat{Y}_x^{\mu} \bar{\mu}_t.
\end{aligned}$$

where $\hat{Y}_{\bar{\mu}}^{\mu} \equiv \left(1 + \frac{\eta}{\theta}\right)\left(\Gamma_x' + \Gamma_z'\right) + \frac{\eta \kappa_\mu}{\kappa_\mu + \kappa_x + \kappa_z}$, $\hat{Y}_z^{\mu} \equiv \left(1 + \frac{\eta}{\theta}\right)\Gamma_z' - \frac{\eta \kappa_z}{\kappa_\mu + \kappa_x + \kappa_z}$, and $\hat{Y}_x^{\mu} \equiv -\frac{\eta}{1+\varepsilon}\frac{\rho \nu}{1-\bar{\alpha}}$. Therefore,

$$\begin{aligned}
Var\left(\log \hat{Y}_t\right) &= \left(\hat{Y}_{\bar{\mu}}^{\mu}\right)^2 \frac{1}{\kappa_\mu} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2 + \left(\hat{Y}_z^{\mu}\right)^2 \frac{1}{\kappa_z} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2 + \sigma_m^2 \\
Cov\left(\log \hat{Y}_t, \log \check{Y}_t\right) &= \hat{Y}_x^{\mu} \hat{Y}_{\bar{\mu}}^{\mu} \frac{1}{\kappa_\mu} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right).
\end{aligned}$$

Similarly, the terms capturing the dispersion of the second-period local gaps can be expressed as

$$\begin{aligned}
\log \hat{y}_{it} - \log \hat{Y}_t &= \hat{y}_x^{\mu} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right) u_{it} \\
\log \check{y}_{it} - \log \check{Y}_t &= \check{y}_\xi^{\mu} \xi_{it},
\end{aligned}$$

where $\hat{y}_x^{\mu} \equiv \left(1 + \frac{\eta}{\theta}\right)\Gamma_x' - \frac{\eta \kappa_x}{\kappa_\mu + \kappa_x + \kappa_z}$ and $\check{y}_x^{\mu} \equiv -\frac{\eta}{1+\varepsilon}\rho\nu$. Therefore,

$$\begin{aligned}
Var\left(\log \hat{y}_{it} - \log \hat{Y}_t\right) &= \left(\hat{y}_x^{\mu}\right)^2 \frac{1}{\kappa_x} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2 \\
Cov\left(\log \hat{y}_{it} - \log \hat{Y}_t, \log \check{y}_{it} - \log \check{Y}_t\right) &= \check{y}_\xi^{\mu} \hat{y}_x^{\mu} \frac{1}{\kappa_x} \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right).
\end{aligned}$$

Collecting all the terms together, the second-order welfare losses associated with the second-period gaps can be rewritten as

$$\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y = \vartheta_\mu' \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2 + 2\tilde{\vartheta}_\mu' \left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right) + \check{\Sigma}_Y + \frac{1}{1-\bar{\alpha}}\check{\sigma}_y + \sigma_m^2,$$

where

$$\begin{aligned}
\vartheta_\mu' &\equiv \left(\hat{Y}_{\bar{\mu}}^{\mu}\right)^2 \frac{1}{\kappa_\mu} + \left(\hat{Y}_z^{\mu}\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\bar{\alpha}}\left(\hat{y}_x^{\mu}\right)^2 \frac{1}{\kappa_x} \\
\tilde{\vartheta}_\mu' &\equiv \hat{Y}_x^{\mu} \hat{Y}_{\bar{\mu}}^{\mu} \frac{1}{\kappa_\mu} + \frac{1}{1-\bar{\alpha}}\check{y}_\xi^{\mu} \hat{y}_x^{\mu} \frac{1}{\kappa_x}.
\end{aligned}$$

Finally, in the proof of Lemma 8 we show that we can rewrite $l_{\bar{\mu}} - l_{\bar{\mu}}^*$ as $(\lambda_s - \lambda_s^*)/\Pi$, thus, if we let

$$
\Lambda = \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\check{\sigma}_q + \xi\left(\check{\Sigma}_Y + \frac{1}{1-\bar{\alpha}}\check{\sigma}_y\right), \quad \mathcal{T} = \sigma_m^2,
$$

$$
\Theta_1 = \frac{\tilde{\vartheta}_\mu + \xi\tilde{\vartheta}_\mu'}{\Pi^2\sigma_\mu^2}, \quad \Theta_2 = -\frac{\vartheta_\mu + \xi\vartheta_\mu'}{\Pi^2\sigma_\mu^2},
$$

the statement of the proposition follows directly from the welfare decomposition in Lemma 10. ∎

Proof of Lemma 5. From Proposition 6, the optimal policy can be found by minimizing the term $\Lambda + \mathcal{K} + \mathcal{T}$. The only terms that depend on the monetary policy are $\mathcal{K}$ and $\mathcal{T}$. The minimum value for $\mathcal{T}$ is clearly achieved when $\sigma_m^2 = 0$. From (12), $\lambda_s^{**} = \arg\min_{\lambda_s} \mathcal{K}(\lambda_s) = \Theta_1/\Theta_2$ and the minimum value is $K(\kappa_x, \kappa_z) = -\Theta_1^2/\Theta_2$. Finally, note that, since $\mathcal{K}(0) = 0$, it has to be the case that $K(\kappa_x, \kappa_z) \leq 0$.

Part (i). When real rigidities are absent ($\theta = 0$),

$$
K(\kappa_x, \kappa_z) = -\frac{(1 - \gamma\rho)^2}{(\gamma + \epsilon)^2(1 + \epsilon\rho)((1 + \epsilon\rho)(\kappa_\mu + \kappa_z) + \rho(\gamma + \epsilon)\kappa_x)},
$$

which is clearly increasing in both κ_x and κ_z.

Part (ii). When $(\gamma, \epsilon, \eta, \theta, \rho, \kappa_s, \kappa_\xi, \kappa_x, \kappa_z) = (1, 1, .5, .5, 2, 0, .1, .1, .9)$, we have that $K(\kappa_x, \kappa_z)$ is increasing in both κ_x and κ_z. On the contrary, when $(\gamma, \epsilon, \eta, \theta, \rho, \kappa_s, \kappa_\xi, \kappa_x, \kappa_z) = (2, 1, .5, .5, 2, 0, .5, .5, .5)$, $K(\kappa_x, \kappa_z)$ is decreasing in κ_x and when $(\gamma, \epsilon, \eta, \theta, \rho, \kappa_s, \kappa_\xi, \kappa_x, \kappa_z) = (.1, .45, .03, .36, .09, 0, .1, .1, .9)$, $K(\kappa_x, \kappa_z)$ is decreasing in κ_z.

Part (iii). First note that both Θ_1 and Θ_2 are linear in $\kappa = \kappa_x + \kappa_z$. Thus, $\bar{K}(\kappa, \varrho)$ is linear in κ and the statement follows from the fact that $-\Theta_1^2/\Theta_2 \leq 0$. ∎

Proof of Theorem 4. *Part (i).* This follows from Proposition 5 along with the fact that, in the case of technology shocks, the optimal policy replicates flexible prices.

Part (ii). This follows from Proposition 5, Lemma 5, and the discussion in the main text. ∎

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