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Dissecting the 2007-2009 Real Estate Market Bust: Systematic Pricing Correction or Just a Housing Fad?*

Daniele Bianchi† Massimo Guidolin‡ Francesco Ravazzolo§

Abstract

We propose a flexible Bayesian model averaging method to estimate a factor pricing model characterized by structural uncertainty and instability in factor loadings and idiosyncratic risks. We use such a framework to investigate key differences in the pricing mechanism that applies to residential vs. non-residential real estate investment trusts (REITs). An analysis of cross-sectional mispricings reveals no evidence of a pure housing/residential real estate bubble inflating between 1999 and 2007, to subsequently burst. In fact, all REITs sectors record increasing alphas during this period, and show important differences in the dynamic evolution of risk factor exposures.

Keywords: I-CAPM, Mispricing, REIT, Model Uncertainty, Stochastic Breaks, Bayesian Econometrics

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1 Introduction

Most macroeconomic and policy commentaries between 2007 and 2010 have been dominated by one obsessively worrisome news item: the U.S. real estate sector was in the middle of a convulsive bust characterized by downward spiraling prices and transaction volumes. As Glaeser (2013) has recently emphasized, such a bust was not the first and possibly not even the largest among those recorded in the history of the United States, but what he calls the "Great Convulsion" was sufficiently strong to produce one of the deepest and longest recessions of the last two centuries and a full-blown financial crisis.

A number of authors (see e.g., Case and Shiller 2003; Smith and Smith 2006; Wheaton 2008; Arce and López-Salido 2011; Hendershott, Hendershott, and Shilling 2010; Dell'Ariccia and Laeven 2011; Demyanyk and Van Hemert 2011) and commentators reached a simple conclusion: the big bust was simply the epilogue of an enormous housing bubble that would have been caused by rational (see e.g., Favilukis, Ludvigson, and Van Nieuwerburgh 2011) as well irrational (see e.g., Case and Shiller 2003; Glaeser 2013 and references therein) behaviors by households and banks. This emphasis is less than surprising because a vast literature has pointed out that, within the real estate asset class, housing would be more prone to bubbles.[1]

In this paper, we use advanced time series methods applied to a well established Merton (1973) Intertemporal CAPM (I-CAPM) setting to ask two simple questions that appear to have been neglected so far. First, we investigate whether the dominant view (often, an instinctive reflection of the ways events have unfolded and news has been broadcast during the 2007-2008 sub-prime crisis, see e.g., Cecchetti 2009; Gorton 2010; Mian and Sufi 2009) of the 2007-2010 real estate bust as predominantly consisting of a house price deflation phenomenon has any foundations from a rational pricing perspective. Equivalently, we ask whether asset market transaction data are compatible with the

[1] Using the words by Case and Shiller (2003), "Expectations of future appreciation of the home are a motive for buying that deflects consideration from how much one is paying for housing services. That is what a bubble is all about: buying for the future price increases, rather than simply for the pleasure of occupying the home." (pag. 321). Clearly, these two complementary motives to invest in real estate are largely absent in categories that differ from housing, when the pleasure of occupying (say) a factory building, a parcel of land, or an empty shop are generally absent. Mian and Sufi (2011) find that a large fraction of the home equity loans that were taken during the housing boom were used to finance consumption, which also appears to be a phenomenon specific to the housing choice.

hypothesis of any abnormal or exceptional dynamics having affected either the housing/residential or the mortgage financing sectors, differentially from other, non-residential segments of the U.S. real estate market. As a result, our first testable hypothesis is whether—assuming the literature has correctly identified the sub-prime sector as the origin of real estate busts —residential REITs were affected by the sub-prime crisis earlier and more strongly than other categories.[2] The second panel of Figure 1 supports our development of formal tests of this hypothesis: the valuations of residential and mortgage real estate led other sectors between early 2007 and Summer 2008; yet, they also recovered before most other sectors after 2009 and appear to display dynamics that is different from business-related real estate indexes.

Our second question is whether the tumble in real estate prices derived from either a correction of a previous large mispricing of real estate (or parts of it) as an asset class or whether it was an irrationally precipitated event, that is difficult to rationalize. The two perspectives show of course an interesting intersection as in this paper we also study whether any differential dynamics between the residential and the non-residential, business-specialized sectors of the U.S. real estate market may derive from a heterogeneous evolution of risk exposures, and whether these implied any correction of a mispricing that had endogenously emerged in the residential sector but that had not occurred in the non-residential segment of the market.

In methodological terms, we make two key choices. First, supported by a recent real estate finance literature (see, e.g., Cotter 2011; Gyourko 2009) that establishes robust links between publicly traded securities and underlying real assets, we use closing market price data at monthly frequency of real estate investment trusts (REITs) to measure real estate valuations ensuring sufficient liquidity and homogeneity over time (see the discussion in Himmelberg, Mayer, and Sinai 2005). Because REITs offer abundant, high-quality data for a variety of sectors, they give us the chance to perform tests that distinguish among portfolios of residential (hence, housing-related), of mortgage, and of non-residential real estate investments, as required by our first question. Such tests would be impossible should one use appraisal-based or repeat-sale data that are subject to upward biases and quality homogeneity issues, respectively, and generally available for houses only.

[2]While residential (in particular, apartment-investing) REITs represent commercial property, the key distinction in this paper is between real estate assets that are directly related to business activities (industrial buildings, offices, shopping malls, and free-standing shops) vs. residential equity REITs that invest in manufactured homes and apartments, as well as mortgage REITs that are involved with purchasing housing-related loans and mortgage-backed securities.

Second, we analyze the pricing of U.S. real estate assets in an encompassing no-arbitrage dynamic multi-factor framework by training a model to jointly price stocks, government bonds, corporate bonds, as well as REITs, using a large set of macro-financial risk factors that are capable of pricing the cross-section of U.S. securities (see Del Negro and Otrok 2007 for an example on dynamic factor models in real estate contexts). As discussed by Smith and Smith (2006), to gauge the existence of misspricings in the real estate sector, it is fundamental to incorporate also cross-sectional data on other assets. Because the implementation of such an APT-style framework requires data on liquid assets traded in a frictionless market, proxying real estate valuations with REITs seems natural. The model emphasizes the existence of no-arbitrage conditions between real estate and other financial assets, in the tradition of Case and Shiller (1989).

Our estimation approach based on Bayesian model averaging techniques allows us to handle contemporaneously a very large number of models and incorporate uncertainty about which combination of macroeconomic variables most effectively summarizes the dynamic properties of the pricing kernel. Indeed, existing asset pricing theories are not explicit about which risk factor should enter as explanatory variable in a factor pricing model. This aspect is undesirable, as it renders the empirical evidence ad-hoc and subject to data over-fitting concerns. Also, the multiplicity of potential macro-financial risk factors makes the empirical evidence difficult to interpret. For instance, one may find credit risk statistically significant based on a particular collection of explanatory variables (e.g. Petkova 2006), but often not based on a competing specification. To address these issues, we propose a Bayesian approach for exact finite sample inference on flexible multi-factor ICAPM-style models in which uncertainty on the "correct" set of macro-financial risk variables can be accommodated, risk exposures (the "betas") are time-varying and subject to discrete breaks, and also idiosyncratic non-diversifiable risk follows a stochastic process (see Engle and Smith 1999 for a discussion on stochastic break models).

We report few novel findings. First, an analysis of cross-sectional mispricing reveals that all the indicators (Jensen's alphas) implied by REITs were positive and relatively large. Ex-post, we obtain evidence that the *entire* real estate asset class has been long and persistently over-priced in the U.S. Realized excess returns have been (on average) up to 2 percent higher than what would have been justified by their exposure to systematic risk factors. Additionally, and with the partial exception of mortgage investments, all sector REITs describe a homogeneous dynamics over time. Between

1999 and 2007, all alphas climb up, in some cases going from a few basis point per month to as high as 2.5 percent. This was the great U.S. real estate bubble, with trading volumes, borrowing, and prices all exploding at the same time. Pricing errors slowly decline between 2007 and 2009, often returning to zero, when macro factors effectively explain average returns. From 2009 to 2011 there are again evidence of overpricing for all REIT sectors except for industrial- and regional malls-specialized investments. However, there are no longer evidence of mispricing across real estate assets by the end of 2013.

Second, we show that few factors carry most of the explanatory power among the large set of macro-financial variables considered. While market risk shows the highest relevance in explaining excess returns for equity REITs portfolios, the slope of the yield curve, unexpected inflation and unemployment risk immediately rank second in terms of their importance to approximate the dynamic properties of the pricing kernel. Except occasional nuances, widely used macroeconomic risk factors such as aggregate credit- and default-spread, liquidity, human capital, industrial production and consumption growth, do not sensibly contribute to the pricing of real estate assets. Interestingly, together with the market risk, the most important risk factor priced across industry portfolios is liquidity.

Third, we find differences in the structure as well the dynamics of risk factor exposures across residential vs. industrial, office, and retail REITs. This means that, indeed, residential REITs, most related to housing, were "special" during our sample, and in particular during the years in which the alleged housing bubble in early 2000s built up. For instance, residential REITs are characterized by significant exposures to unemployment risk, by massive and quickly increasing betas vs. market risk, and by quickly retreating exposures to inflation risk which turns out to be highly significant at the end of our testing sample. REITs that specialize in industrial and office investments carry instead a neutral exposure to unemployment risk, still increasing exposure to unexpected inflation and negative exposure to the slope of the yield curve.

Finally, our multi-factor pricing exercise reports no evidence of a *pure* housing/residential real estate bubble inflating between 1999 and 2007, to subsequently burst. All REIT sectors record a climb-up in alphas during this period. In fact, it is the alpha of the retail/distribution-investing REIT portfolios that shows the steepest ascent. As such, U.S. real estate would have been grossly and systematically over-priced between 1999 and 2007. Over-pricing is indicated by the fact that the posterior estimates of the real estate alphas are positive, increasing, and precisely estimated; large and positive alphas signal that after taking into account the risk exposures and premia of a large class of factors, real estate yielded “too high” a return that cannot be justified. This contradicts the occasionally reported conclusions that financial models would be able to justify the real estate valuations that were witnessed between 2004 and 2007 (see e.g., Glaeser, Gottlieb, and Gyourko 2013, Smith and Smith 2006). In this sense, the real estate fad has been pervasive. Also the claim that the great real estate bubble would have been a debt/mortgage-fueled one is consistent with the fact that the posterior median alpha of Mortgage REITs, which was the highest in the early 2000s, sensibly dropped in 2005 anticipating an extensive valuation correction in the real estate market as a whole.

The paper is structured as follows. Section 2 lays out the research design and methodology. Section 3 outlines the model estimation strategy. Section 4 presents a battery of economic tests used in the empirical analysis. Next, Section 5 describes the data and introduce results to assess the most suitable model specification. Section 6 represents the heart of the paper and contains our findings on heterogeneous mispricing across different segments of the real estate universe, with special emphasis on the dichotomy residential vs. business REITs. Section 7 concludes.

2 Research Design and Methodology

2.1 Can REITs Represent Valuations in the Real Estate Market?

One crucial assumption that backs our research is that REITs may be used to proxy the valuations in the U.S. real estate market. Even though testing this connection is beyond the scope of our paper, luckily there is a well developed real estate finance literature that has examined exactly this research question. While the early literature had reported mixed findings (see e.g., Clayton and MacKinnon 2003; Ling and Naranjo 2003; Seck 1996; but see Gyourko and Keim 1992, for early findings that the public market reliably leads the private market in commercial real estate over the cycle), most recent results are largely consistent with the claim that REITs are informative of the state of the real estate market in its various components and disaggregations. For instance, Chiang (2009) shows that past returns on public markets can forecast returns in real, physical markets. This result is coherent with the notion that public markets are more efficient in processing information than private markets. Moreover, the early literature had relied almost exclusively on appraisal-based measures of private real estate returns.

Recent research by Boudry, Coulson, Kallberg, and Liu (2012) using the novel NCREIF (National Council of Real Estate Investment Fiduciaries) MIT transaction-based indices, show that the relation between REIT and direct (privately-held properties) real estate returns appears to be strong, especially at long horizons.[3] More specifically, using a co-integration framework, they find robust evidence that REITs and the underlying real estate share a long run equilibrium; both REITs and direct real estate returns adjust towards this long run relationship. Gyourko (2009) also finds clear statistical association in the way housing, residential commercial real estate, and non-residential income-producing properties behave over time. He also notices a deterioration in underwriting standards similar to what has been reported for the housing sector. These results motivate the use of REIT valuations in our paper as representative of the general, aggregate conditions of the U.S. real estate market.

2.2 The Asset Pricing Framework

Our research design builds on a discrete-time I-CAPM framework originally developed in Merton (1973). According to the I-CAPM, if investment opportunities change over time, then assets exposures are important determinants of average returns in addition to the market beta. In its conditional version, risk exposures are not constant but time-varying as a consequence of macroeconomic and/or asset-specific news. We follow Campbell (1996) and proxy variations in the investment opportunity set by using shocks to state variables that capture business cycle effects on beliefs and/or preferences, as characterized by a pricing kernel with time-varying properties.

If we define shocks to risk factors as $u_{j,t}$ $(j = 1, ..., K)$ and $r_{i,t}$ to be the *excess* return on portfolio $i = 1, ..., N$, the I-CAPM can be implemented through a Multi-Factor Asset Pricing Model (MFAPM) defined as

$$r_{i,t} = \beta_{i0,t} + \beta_{iM,t} R_{M,t} + \sum_{j=1}^{K} \beta_{ij,t} u_{j,t} + \epsilon_{i,t}, \tag{1}$$

where $E[\epsilon_{i,t}] = E[\epsilon_{i,t} R_{M,t}] = E[\epsilon_{i,t} u_{j,t}] = 0$ for all $i = 1, ..., N$ and $j = 1, ..., K$. The $r_{i,t}$ are returns in excess of the risk-free rate proxied by the 1-month T-bill, $R_{M,t}$ the excess return on the market

[3] Additionally, since REITs tend to invest in institutional quality real estate, an ideal index would be constructed based on a similar set of properties. In this regard, the NCREIF universe of properties would make an excellent match to the set of REIT properties, since both groups tend to invest in institutional quality real estate.

portfolios and $u_{j,t}$ the innovation to the jth macroeconomic risk factor at the end of period t. Favilukis et al. (2011) discuss the importance of focusing on risk premia instead of long-term riskless rate to characterize the recent real estate bust. Importantly, even though the notation implies time-varying factor loadings, such patterns of variation is in general left unspecified by the asset pricing theory.

The factor model in (1) describes a general conditional pricing framework that is known to hold under mild conditions. It is important to specify a process for the time-series dynamics of the innovations variables $u_{j,t}$. We adopt the approach of Campbell (1996) and assume that the macroeconomic factors follow a first-order Vector Auto-Regressive (VAR) process.[4] To ensure that betas in model (1) are fully conditional, the VAR(1) is estimated recursively at each time t. Thus, for a collection of market portfolio and macroeconomic factors $x_t = (R_{M,t}, F_{1,t}, ..., F_{K,t})'$, we estimate $x_\tau = A_0 + A_1 x_{\tau-1} + u_\tau$ for $\tau = 1, ..., t$, and $t = t_0, ..., T$ with t_0 an initial set of observations. Following Petkova (2006), innovations are orthogonalized to the excess return on the market portfolio $R_{M,t}$ and scaled to have the same variance.

2.2.1 Mimicking Portfolios. One problem with an I-CAPM implementation as (1) is the difficulty with interpreting $\beta_{i0,t}$ when some (or all) risk factor is not itself a traded portfolio. In fact, unless all the factors are themselves tradable, it is impossible to interpret non-zero $\beta_{i0,t}$s as abnormal returns on portfolio i "left on the table" after all risks exposures have been considered. If some of the factors are not replicated by traded portfolios (i.e., their values cannot be written as portfolio returns), there may be an important difference between the theoretical alpha that the model uncovers, and the actual alpha that an investor may achieve by trading assets on the basis of the MFAPM. To avoid such a situation, we follow the literature (see e.g., Ferson and Korajczyk 1995; Lamont 2001; Vassalou 2003) and proceed as follows. If an economic risk factor is measured or can be easily deterministically converted in the form of an excess return, such as the U.S. market portfolio, credit- and default-spread variables, we use the corresponding excess returns as a mimicking portfolio; Shanken (1992) shows that under some conditions, such an approach delivers highly efficient risk premia estimates.

Instead, if a risk factor is not an excess return, such as unemployment and money growth, we construct the corresponding $K' \leq K$ mimicking portfolios by projecting the non-tradable factors onto

[4] The VAR(1) dynamics is rather general as any VAR(p) can be rearranged as a VAR(1) in its companion form.

a set of predictors and the space of excess returns of base assets:

$$F_{j,t} = \alpha_{0j} + \alpha'_{1j} x_t + c'_j z_{t-1} + \varepsilon_{j,t} \qquad \varepsilon_{j,t} \sim N(0, \omega),$$

where x_t is a vector of excess returns on the base assets and z_{t-1} defines a vector of predictors. The returns on the ith factor mimicking portfolio are then defined as $\hat{F}_{j,t} = \hat{\alpha}_{0j} + \hat{\alpha}'_{1j} x_t$ and collect the fitted value of a factor that may be replicated by trading base assets using weights estimated by $\hat{\alpha}_{1j}$. The base assets consist of six equity zero net investment portfolios sorted on size and book-to-market as well as the returns spread between long-term and short term government bonds and the return spread on long-term corporate bonds minus long-term government bonds. As usual, these assets are assumed to span a large fraction of the returns space. The set of instruments includes lagged values of widely known stock returns predictors such as the earnings-to-price and the dividend-payout ratios, and lagged credit yield spread (see Goyal and Welch 2008).

2.3 Model Uncertainty and Instability

A long history of evidence shows that modeling carefully the dynamics of risk exposures can provide a crucial contribution to solve typical anomalies associated with unconditional multi-factor models. For instance, Ang and Chen (2007) and Jostova and Philipov (2005) find that the CAPM is rejected when using OLS rolling window beta estimates, and the opposite result emerges allowing for stochastic variation (in the form of a standard AR(1) process) in the conditional market betas. Time-variation on risk exposures sensibly depends on macroeconomic and/or firm-specific news which do not necessarily hits investors' information set at each time t. Moreover, a recent macroeconomic literature tends to find discrete instability in the elasticities that connect real estate valuations to business cycle shocks (see e.g., Iacovello and Neri 2010). Therefore, in this paper we propose a flexible parametric model that capture both any instability in systematic risks exposures and in idiosyncratic risks, allowing for changes at any point on time but not restricting them to change at all points.

The MFAPM in (1) is conditional on having fully specified the set of macroeconomic risk factors. However, existing equilibrium pricing theories are not explicit about which variables should exactly capture changes in the investment opportunity set. In fact, the true set of macroeconomic factors is virtually unknown. We incorporate *model uncertainty* in the factor model (1), acknowledging uncertainty about which combination of macroeconomic variables most effectively summarizes the dynamic properties of the pricing kernel.[5] In particular, we introduce $K+1$ variables $\delta_{ij} \in [0,1]$, which describe the inclusion of the jth state variable for the ith portfolio, with $j = M, 1, ..., K$.

$$r_{i,t} = \beta_{i0,t} + \delta_{i,M}\beta_{iM,t}R_{M,t} + \sum_{j=1}^{K} \delta_{ij}\beta_{ij,t}u_{j,t} + \sigma_{i,t}\epsilon_{i,t}, \qquad \epsilon_{i,t} \sim IIDN(0,1) \tag{2}$$

where $E[\epsilon_{i,t}] = E[\epsilon_{i,t}R_{M,t}] = E[\epsilon_{i,t}u_{j,t}] = 0$ for all $i = 1, ..., N$ and $j = 1, ..., K$. We specify the relationship between excess returns, factors and time-varying factor loadings and idiosyncratic volatilities in a state-space form (henceforth, Bayesian Model Averaging with Stochastic Break Betas and with Stochastic Break Volatility, BMA-SBB-SBV), where the observation equation is (2), and time varying factor loadings and idiosyncratic risks are described by the state equations

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{ij,t}\eta_{ij,t} \qquad j = 0, M, 1, ..., K, \tag{3}$$

$$\ln\left(\sigma^2_{i,t}\right) = \ln\left(\sigma^2_{i,t-1}\right) + \kappa_{i\nu,t}\nu_{i,t} \qquad i = 1, ..., N, \tag{4}$$

where $\eta_{i,t} \equiv (\eta_{i0,t}, \eta_{iM,t}, \eta_{i1,t}, ..., \eta_{iK,t}, \nu_{i,t})' \sim N(0, \mathbf{Q}_i)$ with $\mathbf{Q}_i$ a diagonal matrix defined by the parameters $q^2_{i0}, q^2_{iM}, q^2_{i1}, ..., q^2_{iK}, q^2_{i\nu}$. Instability in the level of both risk exposures and of the residual variance σ^2_{it} are introduced and modeled through a mixture innovation approach as in Ravazzolo, Paap, van Dijk, and Franses (2007), Giordani and Kohn (2008) and Bianchi, Guidolin, and Ravazzolo (2015).[6] The latent binary random variables $\kappa_{ij,t}$ and $\kappa_{i\nu,t}$ capture the presence of stochastic changes in betas and/or idiosyncratic variance. For the sake of simplicity, these latent breaks are assumed to be independent across factors, portfolios, and over time.

$$\Pr\left[\kappa_{ij,t} = 1\right] = \pi_{ij} \qquad \Pr\left[\kappa_{i\nu,t} = 1\right] = \pi_{i\nu} \qquad i = 1, ..., N \quad j = 0, M, 1, ..., K. \tag{5}$$

This specification is very flexible as we allow breaks to occur independently across assets, and generalize more regular change-point processes such as Markov regime switching dynamics (see e.g. Kim and Nelson 1999). Empirically we are not preventing breaks from occurring simultaneously across portfolios and/or factor exposures. Also, (3)-(4) captures the idea that exposures to risk factors do not necessarily change at each time t, allowing for periods in which betas can be constant. More

[5] See Faust, Gilchrist, Wright, and Zakrajssek 2013 and Moral-Benito 2012 for related examples of Bayesian model averaging for forecasting and investigate determinants of economic growth, respectively.

[6] Unlike this literature we consider a comprehensive, multi-variate, modeling setting focusing on the real estate market dynamics.

deeply, when $\kappa_{ij,\tau} = \kappa_{i\nu,\tau} = 0$ for some $t = \tau$, then (2) reduces to (1) when the factor loadings and the quantity of idiosyncratic risk are assumed to be constant, as $\beta_{ij,\tau} = \beta_{ij,\tau-1}$ and $\ln \sigma^2_{i,\tau} = \ln \sigma^2_{i,\tau-1}$. However, when $\kappa_{ij,\tau} = 1$ and/or $\kappa_{iv,\tau} = 1$, then news hits either betas or idiosyncratic variances or both, according to the random walk dynamics $\beta_{ij,\tau} = \beta_{ij,\tau-1} + \eta_{ij,\tau}$ and $\ln(\sigma^2_{i,\tau}) = \ln(\sigma^2_{i,\tau-1}) + \nu_{i,t}$ (or $\sigma^2_{i,\tau} = \sigma^2_{i,\tau-1} \exp(\nu_{i,\tau})$). When a break affects the betas and/or variances, the size of random shift is measured by $\mathbf{Q}_i$.

The BMA-SBB-SBV model presented in (2)-(4) is the most general specification we consider in this paper. However, such a framework is highly parameterized and we cannot rule out that problems related to over-parameterization could arise. Therefore, for benchmarking purposes, we also estimated models derived by imposing a number of restrictions on the dynamics of the state equation. First we consider the case with $\kappa_{i\nu,t} = 0 \; \forall i, t$, i.e. a constant idiosyncratic volatility model. We will call this model a Bayesian Model Averaging with Stochastic Break Betas model and constant idiosyncratic risks, i.e., BMA-SBB. Second, we consider a standard random walk dynamics for both the betas and idiosyncratic risks, i.e. $\kappa_{ij,t} = 1 \; \forall i, \; j, \; t$ and $\kappa_{i\nu,t} = 1 \; \forall i, t$. Such specification is common to some of the reference literature (Koop and Potter 2007, and Jostova and Philipov 2005), and assumes a unit probability of breaks in the dynamics of $\beta_{ij,t}$ and $\sigma^2_{i,t}$ over time. This is fairly restrictive, and is not necessarily supported by the data, as we will document in our empirical analysis. We call this model Bayesian Model Averaging with Random Walk Betas and with Random Walk Volatility (BMA-RWB-RWV). Trivially, the symmetric case of $\kappa_{ij,t} = \kappa_{iv,t} = 0 \; \forall t$ implies that $\beta_{ij,t} = \beta_{ij,t-1} = \beta_{ij}$ and $\ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) = \ln(\sigma^2_i)$ and consists of the classical case with constant betas and idiosyncratic variances.

For both the general dynamics (2)-(4) and each of the restrictions above, we investigate the benefit of considering model uncertainty by alternatively imposing $\delta_{ij} = 1$ for $j = M, 1, ..., K$ and $i = 1, ...N$, i.e. each risk factor enters in the pricing equation with probability one for each asset/portfolio, on each dynamics specification. The constant volatility specification is used to highlight the effects of instabilities in residual variances. The BMA-RWB-RWV model is used as a competing specification to show the benefits of considering the parsimony of occasional, and possibly large, breaks in the dynamics of parameters as opposed to frequent breaks.

3 Estimation Strategy

Consideration of all linear data-generating processes in the presence of 13 risk factors, independently for each of the 33 portfolios, involves inference on $33 \times 2^{13} = 270,336$ models. Moreover, each model allows for stochastic changes in betas and/or idiosyncratic variance. Our Bayesian model averaging estimation scheme provides a formal way of handling inference in the presence of such large model space.[7]

Bayesian estimation methods allow to incorporate both parameter and model uncertainty in testing our I-CAPM implementation in a natural way, by characterizing the posterior distribution of virtually any function of the model parameters. For instance, we can characterize the posterior distribution of $\kappa_{ij,t}$ and $\kappa_{i\nu,t}$ for $i = 1, ..., N$, $j = 0, M, 1, ..., K$ and $t = 1, ..., T$, which can be used to incorporate uncertainty on the timing of structural breaks. Also, Bayesian inference on $D_i = (\delta_{iM}, \delta_{i1}, ..., \delta_{iK})$ can be used to estimate the posterior inclusion probability of each risk factor and to estimate the dimensionality of the vector of state variables in the pricing relation (1). For each of the *ith* asset/portfolio, the parameters of the model (1)-(5), are the structural break probabilities $\pi_i = (\pi_{i0}, \pi_{iM}, \pi_{i1}, ..., \pi_{iK}, \pi_{i\nu})'$, the vector of the size of the breaks $q_i^2 = \left(q_{i0}^2, q_{iM}^2, q_{i1}^2, ..., q_{iK}^2, q_{i\nu}^2\right)'$, and the inclusion vector $D_i = (\delta_{iM}, \delta_{i1}, ..., \delta_{iK})$. We collect the model parameters in a $(3K+3)$-dimensional vector $\theta_i = \left(\pi_i', q_i^{2'}, D_i'\right)'$.

3.1 Prior Specification

For the Bayesian algorithm to work, we need to specify the prior distributions for each of the parameters. For the structural break probabilities, we take Beta distributions

$$\pi_{ij} \sim Beta(a_{ij}, b_{ij}) \quad \pi_{i\nu} \sim Beta(a_{i\nu}, b_{i\nu}) \qquad \text{for } i = 1, ...N \quad j = 0, M, 1, ..., K, \tag{6}$$

The parameters a_{ij}, b_{ij} and $a_{i\nu}, b_{i\nu}$ represent the shape hyper-parameters and can be set according to our prior beliefs about the occurrence of structural breaks. The expected prior probability of a

[7] In a frequentist framework it would not be feasible to estimate a model with the features of the BMA-SBB-SBV specification. More prominently, above and beyond the curse of dimensionality due to model uncertainty, it would be difficult to identify the stochastic breaks $\kappa_{ij,t}$ and $\kappa_{iv,t}$ from the continuous shocks $\eta_{ij,t}$ and $\nu_{i,t}$ without specifying some highly restrictive parametric process.

break in $\beta_{ij,t}$ and $\ln(\sigma^2_{i,t})$ is given by $a_{ij}/(a_{ij}+b_{ij})$ and $a_{i\nu}/(a_{i\nu}+b_{i\nu})$, respectively. For the variable inclusion probabilities, we take a Bernoulli distribution with

$$P[\delta_{ij}=1]=\lambda_{ij}, \qquad \text{for } i=1,...N \quad j=M,1,...,K, \tag{7}$$

Therefore, λ_{ij} reflects our prior belief about the inclusion of the jth risk factor for the ith portfolio (see George and McCulloch 1993). We assume that prior beliefs about the size of the structural breaks are distributed as an Inverted-Gamma distribution;

$$q^2_{ij} \sim IG(s_{ij}S_{ij}, S_{ij}) \quad q^2_{i\nu} \sim IG(s_{i\nu}S_{i\nu}, S_{i\nu}) \quad \text{for} \quad i=1,...N \quad j=0,M,1,...,K \tag{8}$$

The expected prior break size for the betas (log-volatility) equals the square root of $s_{ij}S_{ij}/(S_{ij}-2)$ for $S_{ij}>2$ ($s_{i\nu}S_{i\nu}/(S_{i\nu}-2)$ for $S_{i\nu}>2$). The density for the joint prior is given by the product of the priors as these are independent across assets/portfolios. In order to mitigate the impact of the calibration of hyper-parameters, an initial five-year worth of observation is used to empirically calibrate the priors and the analysis is implemented over the remaining 180 observations, per each series, over the interval 1999:01-2013:12. A more extensive discussion on prior choices is provided in Appendix B (see Giannone, Lenza, and Primiceri 2015 for a discussion on prior selection in multivariate time series contexts).

3.2 Posterior Approximation

Posteriors distributions for parameters, latent betas and idiosyncratic risks, can be approximated by using the principle of data augmentation which relies on the complete likelihood function, namely, the product of the data and state variable densities, given the structural parameters. In our framework, the latent states are represented by the conditional intercept and risk exposures, i.e. $\beta_t=(\beta_{1,t},...,\beta_{N,t})$ with $\beta'_{i,t}=(\beta_{i0,t},\beta_{iM,t},\beta_{i1,t},...,\beta_{iK,t})'$, the idiosyncratic risks $\sigma^2_t=\left(\sigma^2_{1,t},...,\sigma^2_{N,t}\right)$, the breaks on the betas $\kappa_t=(\kappa_{1,t},...,\kappa_{N,t})$ with $\kappa_{i,t}=(\kappa_{i0},\kappa_{iM},\kappa_{i1},...,\kappa_{iK})$ and the breaks on (the log of) idiosyncratic risks $\kappa_{\nu,t}=(\kappa_{1\nu,t},...,\kappa_{N\nu,t})$ at each time $t=1,...,T$. The structural parameters are specified above.

Although we use a conjugate prior setting, the joint posterior distribution of the latent states and the structural parameters is not available in closed form. We use a Markov Chain Monte Carlo (MCMC) approach and develop an efficient Gibbs sampling scheme (see Geman and Geman 1984).

Defining $\boldsymbol{\theta} = \{\theta_i\}_{i=1}^{N}$, $\boldsymbol{\beta} = \{\beta_t\}_{t=1}^{T}$, $\boldsymbol{R} = \{r_{it}\}_{i=1}^{N}{}_{t=1}^{T}$, $\boldsymbol{X} = \{X_t\}_{t=1}^{T}$ with $X_t' = (R_{M,t}, u_{1t}, ..., u_{K,t})$, $\boldsymbol{\mathcal{K}}_\beta \equiv \{\kappa_t\}_{t=1}^{T}$, $\boldsymbol{\mathcal{K}}_\sigma = \{\kappa_{\nu,t}\}_{t=1}^{T}$, and $\boldsymbol{\Sigma} = \left\{\sigma_t^2\right\}_{t=1}^{T}$, the complete likelihood for the BMA-SBB-SBV model can be written as;

$$p\left(\boldsymbol{R}, \boldsymbol{\beta}, \boldsymbol{\mathcal{K}}, \boldsymbol{\Sigma} | \boldsymbol{\theta}, \boldsymbol{X}\right) = \prod_{t=1}^{T} \left\{ \prod_{i=1}^{N} p(r_{it}|X_t, \beta_{it}, \sigma_{it}^2, D) p(\sigma_{it}^2 | \sigma_{it-1}^2, \kappa_{i\nu,t}, q_{i\nu}^2) \pi_{i\nu}^{\kappa_{i\nu t}} (1-\pi_{i\nu})^{1-\kappa_{i\nu t}} \times \right.$$
$$\left. \times \left[\prod_{j=0}^{K} p(\beta_{ij,t}|\beta_{ij,t-1}, \kappa_{ij,t}, q_{ij}^2) \times \pi_{ij}^{\kappa_{ijt}} (1-\pi_{ij})^{1-\kappa_{ijt}} \right] \right\}, \tag{9}$$

where $\boldsymbol{\mathcal{K}} = (\boldsymbol{\mathcal{K}}_\beta, \boldsymbol{\mathcal{K}}_\sigma)$. Combining the prior specifications (6)-(8) with the complete likelihood, we obtain the posterior density $p(\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\mathcal{K}}, \boldsymbol{\Sigma} | \boldsymbol{R}, \boldsymbol{X}) \propto p(\boldsymbol{\theta}) p\left(\boldsymbol{R}, \boldsymbol{\beta}, \boldsymbol{\mathcal{K}}, \boldsymbol{\Sigma} | \boldsymbol{\theta}, \boldsymbol{X}\right)$. Marginal posterior distributions of quantities of interest are computed as mixtures of the model-dependent marginal distributions weighted by the posterior model probabilities. Suppose, for instance, we want to make inference on the Jensen's alpha for the ith stock, which has similar interpretation on each model. Bayesian model averaging marginal implies that the posterior distribution of $\beta_{i0,t}$ is a weighted average of its model-specific marginal posterior distributions $p\left(\beta_{i0,t}|\boldsymbol{R}, \boldsymbol{X}, D\right)$, and the weights are the model posterior probabilities $p\left(D|\boldsymbol{R}, \boldsymbol{X}\right)$;

$$p\left(\beta_{i0,t}|\boldsymbol{R}, \boldsymbol{X}\right) = \sum_{D} p\left(\beta_{i0,t}|\boldsymbol{R}, \boldsymbol{X}, D\right) p\left(D|\boldsymbol{R}, \boldsymbol{X}\right). \tag{10}$$

Sufficient statistics such as the expected value of the Jensen's alpha can be computed as

$$E\left[\beta_{i0,t}|\boldsymbol{R}, \boldsymbol{X}\right] = \int \beta_{i0,t} \cdot p\left(\beta_{i0,t}|\boldsymbol{R}, \boldsymbol{X}\right) d\beta_{i0,t}.$$

Implementation of such integration is impractical because the sum of the 2^K possible models is difficult for each portfolio if K is large. We overcome this difficulty with our MCMC estimation strategy which under mild regularity conditions provide consistent estimates of the model latent betas, idiosyncratic risks and structural parameters (see Smith and Roberts 1993). This approach enables construction of posterior probability intervals that take into account variability due to model uncertainty, and gives more reliable inference method than using a single model (Madigan and Raftery 1994). A more detailed description of the Gibbs sampler is given in Appendix A. Results on convergence properties of the estimates are provided in a separate online appendix.

4 Economic Tests

The main underlying assumption of our methodology is that while there are some degrees of freedom in modeling model time-variation, capturing instability in risk exposures and stochastic volatility is crucial. In this section we propose some economic tests to understand the economic implications of our BMA-SBB-SBV model, against the set of alternative restrictions.

4.1 Predictable Variance Decomposition

The factor structure in (1) implies that the predictable variance in stock returns is directly linked to the predictable variation captured by the factor pricing model. Following Ferson and Harvey (1991), Ferson and Korajczyk (1995) and Karolyi and Sanders (1998), we use the estimated time series of posterior factor loadings to decompose excess asset returns in each time period in a component related to risk $\beta_{i,t}' X_t$, plus a residual $\beta_{i0,t} + e_{i,t}$, with $e_{i,t} = \sigma_{i,t}\epsilon_{i,t}$. We adopt the following approach. First, we regress the excess return on each portfolio onto a set of I predictors that proxy for investors' information at time $t-1$, $\mathbf{Z}_{t-1}$,

$$r_{i,t} = \varphi_{i0} + \varphi_i' \mathbf{Z}_{t-1} + \xi_{i,t}, \tag{11}$$

to compute the sample variance of the resulting fitted values,

$$Var[P(r_{it}|\mathbf{Z}_{t-1})] \equiv Var\left[\hat{\varphi}_{i0} + \hat{\varphi}_i' \mathbf{Z}_{t-1}\right], \tag{12}$$

where the notation $P(r_{it}|\mathbf{Z}_{t-1})$ means "linear projection" of r_{it} on a set of instruments, $\mathbf{Z}_{t-1}$. Second, for each asset $i = 1, ..., N$, the risk exposures $\beta_{i,t}$ are sampled from their (marginal) posterior distribution $p(\boldsymbol{\beta}|\boldsymbol{R}, \boldsymbol{X})$. Then we compute a time series of fitted risk compensations, $\beta_{i,t}' X_t$ for each draw and regressed onto the instrumental variables,

$$\beta_{i,t}' X_t = \varphi_{i0}^{\star} + \varphi_i^{\star\prime} \mathbf{Z}_{t-1} + \xi_{i,t}^{\star} \tag{13}$$

to compute the sample variance of fitted posterior risk compensations:

$$Var\left[P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)\right] \equiv Var\left[\hat{\varphi}_{i0}^{\star} + \hat{\varphi}_i^{\star\prime}\mathbf{Z}_{t-1}\right]. \tag{14}$$

At this point, the predictable variance in the risk premia that is attributed to the model, relative to the total predictable variance in the excess returns, can be computed for each marginal draw as (see Appendix C for more details)

$$\mathcal{VR}1 \equiv \frac{Var\left[P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)\right]}{Var[P(r_{it}|\mathbf{Z}_{t-1})]} \geq 0 \tag{15}$$

The variance ratio is equal to one if the I-CAPM implementation is perfectly specified, which means that all the predictable variation in excess returns is captured by variation in risk exposures. When these tests are implemented using the output from our MCMC estimation scheme we retain complete consistency. In fact, drawing from the marginal posterior densities of the factor loadings $\beta_{ij,t}$, for $i = 1, ..., N$, $j = M, 1, ..., K$, and $t = 1, ..., T$, and holding the instruments constant over time, it becomes feasible to generate a posterior distribution for the statistics $\mathcal{VR}1$. This allows to make robust inference on the properties of $\mathcal{VR}1$ in our sample.

Based on Hoeting, Madigan, Raftery, and Volinsky (1999) and Geweke and Amisano (2014), one can show that the predictable variance in the risk premia that is attributed to the model can be decomposed in three different components: (i) intrinsic variance due to sample variation, (ii) extrinsic variation due to parameter uncertainty and (iii) extra source of variation due to model uncertainty. Appendix C shows that, by using the law of total variance, the cross-sectional variation of the risk premia predicted by the model can be decomposed as follows;

$$\begin{aligned} Var\left[P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)\right] \equiv \sum_{k=1}^{2^K} p\left(D_{i,k}\right) \Big\{ & E\left[Var\left(P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)|D_{i,k},\boldsymbol{\theta}\right)\right] \\ +Var\left[E\left(P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)|D_{i,k},\boldsymbol{\theta}\right)\right] & + E\left[P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}\right]^2 - E\left[P\left(\beta_{i,t}'X_t|\mathbf{Z}_{t-1}\right)\right]^2 \Big\} \end{aligned} \tag{16}$$

where $p\left(D_{i,k}\right)$ is the marginal posterior distribution of the k_{th} model $D_{i,k}$ for the ith portfolio/asset. The first component represents the variance in the projection that would exists if one would know the structural parameters, averaged across the marginal posterior distribution $p\left(\boldsymbol{\theta}|\boldsymbol{R},\boldsymbol{X}\right)$. The second component represents the contribution of parameter uncertainty, and finally;

$$\sum_{k=1}^{2^K} p\left(D_{i,k}\right)\left\{E\left[P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}\right]^2 - E\left[P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)\right]^2\right\} \tag{17}$$

represents the contribution of model uncertainty to the total explained predictable variation, with $E\left[P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)\right]$ the expected value of the projection (13), i.e. when both parameter and model uncertainty have been integrated out.

4.2 Cross-Sectional Pricing Error

Under the factor pricing model (1), the expected excess return on the ith asset over the interval $[t, t+1]$ can be related to its exposures to each of the shocks to systematic risk factors and the associated risk premia (see Ferson and Harvey 1991)

$$E_t[r_{i,t+1}] = \gamma_{0,t} + \gamma_{M,t}\beta_{iM,t} + \sum_{j=1}^{K}\gamma_{j,t}\beta_{ij,t}, \tag{18}$$

where both the risk premia and the betas are conditional on the information available at time t. Such no-arbitrage condition is derived with respect to the investors' information set at time t and is known to hold approximately under weaker conditions. Rather than testing directly this pricing relationship, we follow Geweke and Zhou (1996), measuring the reliability of the pricing approximation by using an average of the squared pricing errors at each time t and across test portfolios;

$$Q_t^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\beta_{i0,t} - \gamma_{0,t} - \gamma_{M,t}\beta_{iM,t} - \sum_{j=1}^{K}\gamma_{j,t}\beta_{ij,t}\right)^2, \qquad t = 1, ..., T$$

Conditional on alphas and betas at time t, and given the residual covariance structure is diagonal, the minimized squared average pricing error can be computed as

$$Q_t^2 = \beta_{0,t}'\left(I_N - \tilde{\beta}_t\left(\tilde{\beta}_t'\tilde{\beta}_t\right)^{-1}\tilde{\beta}_t'\right)\beta_{0,t}/N, \qquad t = 1, ..., T \tag{19}$$

with I_N an N-dimensional identity matrix, $\beta_{0,t} = (\beta_{10,t}, ..., \beta_{N0,t})'$ the N-dimensional vector of intercepts and $\tilde{\beta}_t$ the $N \times K$ matrix of risk exposures plus a constant term. The sampling distribution of these squared average pricing errors is difficult to determine. However, our Bayesian estimation strategy allows to generate the entire distribution of (19) since our Gibbs sampling scheme derives posteriors for all the objects that enter $\beta_{0,t}$ and $\tilde{\beta}_t$, respectively.

5 Data and Model Assessment

5.1 Data and Descriptive Statistics

Our paper is based on a large panel of monthly time series sampled over the period 1994:01-2013:12. Although the choice of portfolios or individual securities in tests of multi-factor models is a researched topic in the empirical finance literature, in our case it is the economic questions that best advise us to use portfolios of securities. The 1994:01 starting date derives from the availability of monthly return series for all the sector REIT total return indexes used in this paper. An initial five-year worth of observations is used to set priors and the analysis is implemented over the remaining 180 observations, per each series, over the interval 1999:01-2013:12.

The series belong to two main categories. The first group, "Portfolio Returns", includes stocks, bonds and real estate, organized in portfolios. The stocks are publicly traded firms listed on the NYSE, AMEX and Nasdaq and sorted according to their four-digit SIC code. Industry-based classification makes stock portfolios sufficiently unrelated to each other to avoid particular bias in testing the significance of common risk factors (see Lewellen, Nagel, and Shanken 2010). Also, clustering stocks according to the industry where they operates is consistent with the investment specialization used to construct REIT portfolios.

Data on government bond returns are from Ibbotson, while the 1-month T-bill and 10-year government bond yields are from FREDII at the Federal Reserve Bank of St. Louis. Data on high-yield investment grade bond returns (Baa average corporate bond yields, 10-to-20 year maturity) are from Moody's and converted into returns using Shiller (1979) approximation formula. The data on sector tax-qualified REIT total returns are obtained from the North American Real Estate Investment Trust (NAREIT) Association and consist of data on 11 portfolios, i.e., Industrial, Office, Shopping Centers, Regional Malls, Free Standing shops, Apartments, Manufactured Homes, Healthcare, Lodging/Resorts, Self-Storage and Mortgage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. These eleven portfolios are formed when REITs are classified on the basis of their main focus of activity. Mortgage REITs specialize in mortgage-backed security (MBS) investments. These are breakdowns common in the literature (see e.g., Payne 2007). As usual, excess return series are computed as the difference between total returns and 1-month T-bill rates.

Second, we use a range of macroeconomic variables as standard proxies for systematic, economy-wide risk factors potentially priced in asset returns. We employ thirteen economic factors which have been previously studied separately in the literature: the excess return on a value-weighted market portfolio that includes all stocks traded on the NYSE, AMEX, and Nasdaq (MKT); the aggregate dividend yield on the CRSP value-weighted stock market portfolio (DY); the unexpected inflation rate (UI), computed as the residual of a simple ARIMA(1,1,1) model applied to (seasonally adjusted) log CPI index; the unemployment rate (UNEMP); the 1-month real T-bill rate of return (RF), computed as the difference between the 1-month T-bill nominal return and realized CPI inflation rate; the term premium (TERM), measured as the difference between 10-year and 1-month Treasury yields; money growth (MG), computed as changes in the money base; the credit spread (PREM), measured as the difference between Baa and Aaa Moody's yields; the default risk premium (DEF), approximated as the difference between Baa Moody's and 10-year Treasuries yields; the growth of (year-on-year, seasonally adjusted) industrial production (IP); the growth of (year-on-year, seasonally adjusted) real per-capita personal consumption expenditures on non-durables and services (CONS); the traded Liquidity factor from Pastor and Stambaugh (2003) (LIQ); and return on Human capital (HC), measured as the growth rate of per-capita labor income.[8]

Table 1 presents summary statistics for the time series under investigation over our overall 1994-2013 sample.[9] The summary statistics show no unexpected stylized facts. Starting with the REIT sectors, the equity-like groups imply largely similar sample means and yield comparable monthly Sharpe ratios that fall between 0.064 and 0.20. As one would expect, mortgage REITs are characterized by lower expected returns; however, because their volatility is similar to that of equity REITs, their realized sample Sharpe ratio is relatively low, only 0.067 per month. Mortgage REITs are characterized by a large negative skewness in line with high-yield bonds returns.

[Insert Table 1 about here]

[8]The per-capita labor income is constructed as the difference between total personal income and dividend payments, divided by total population (from the Bureau of Economic Analysis). The growth rate then is computed by taking a 2-month moving average of per-capita labor income minus one (see Jagannathan and Wang 1996).

[9]Summary statistics on predictors are not reported given they either represents lagged value of current risk factors, or are widely known in the empirical asset pricing literature.

The REIT panel of Table 1 reveals few differences between Industrial and Office REITs. On the contrary, the realized risk-return performance of Retail REITs appears to be driven by Free Standing REITs with a monthly Sharpe ratio of 0.187, to be contrasted to the comparably poor performance of Shopping Center-specialized REITs. Finally, and in spite of the recent housing bust, the Residential sector reveals a good risk-reward trade-off, mostly driven by the Apartment-specialized sector, as it is characterized by strong average realized returns (1.05% per month), albeit its high volatility (5.82% per month); Manufactured Home REIT returns give instead more stable, but lower expected returns. Most equity Sharpe ratios are in the 0.10-0.20 range. Bond Sharpe ratios are relatively low.

Figure 1 provides a visual summary of the movements of the REIT total return indexes under investigation. As a benchmark, we also plot the total return index for the value-weighted market portfolio (black solid line). To favor comparability across different sectors, all indexes are standardized to equal 100 in correspondence with the end of January 2007. This date is chosen because most of the literature (see e.g., Aït-Sahalia, Andritzky, Jobst, and Nowak 2009) has dated the onset of the sub-prime crisis to early to mid-2007. To limit the number of series plotted, Industrial and Office REITs are aggregated in a "Industrial/Office" (I&O) sector, Shopping Centers, Regional Malls, Free Standing shops REITs into a "Retail" sector, and Apartments, Manufactured Homes into a "Residential" one.

[Insert Figure 1 about here]

Figure 1 provides motivation for our analysis because it shows that the residential sector exactly peaks in correspondence to the end of 2006 and leads the aggregate stock market through all of 2007 and 2008. In fact, the mortgage REIT sector had already boomed between 2003 and 2005 and—consistently with most anecdotal accounts of the onset of the sub-prime crisis (e.g., Mian and Sufi 2009)—subsequently tumbled starting in late Spring 2007.

Panel B of Figure 1 shows that, from Fall 2008—approximately after the demise of Lehmann Brothers—the I&O and retail sectors started to lead (and fall at higher rate than) residential and mortgage REITs. This is consistent with the policy debate and the financial press accounts of the time (see e.g., Greenlee 2009). Interestingly, starting in Spring 2009, all four sectors recovered somewhat, with their total return indexes approximately returning to the levels of late 2003, but the residential REIT index displays a "V-shaped" bounce-back that has no equivalent in the case of the other sectors. In fact, a simple calculation for the period January 2007 - December 2013 reveals that residential REIT is the only portfolio in Figure 1 for which average returns are positive, albeit small. Our goal is to explain these differential dynamics.

5.2 Model Assessment

The I-CAPM implementation (2)-(4) represents the most general specification we considered in this paper. However, one may argue the results can be driven by the specific dynamics of risk exposures and idosyncratic risks, rather than by the presence of heteroskedasticity versus constant idiosyncratic risks, or again by the fact that indeed the "right" set of risk factors is known ex-ante. Although our general specification does not rule out a priori any of these possibility, it is worth to carefully investigate if a potential richly parameterized specification as (2)-(4) may in fact hurts from a pure asset pricing perspective. Therefore, the first question we ask, as is common in all empirical papers, concerns the model specification that should be used in the investigation of pricing mechanism of our test portfolios. In the following we carefully address model assessment both from a statistical and an economic perspective.

We first compute the marginal likelihood for each model specification to perform a comparison that takes into account their overall, in-sample, statistical performance. The marginal likelihood of a model is known to penalize model complexity taking into account both uncertainty about the size and presence of structural breaks, and uncertainty concerning the parameters and the set of significant risk factors. The marginal likelihood is computed as

$$p\left(\boldsymbol{R}|\boldsymbol{X}\right) = \int \dots \int \sum_{\boldsymbol{\mathcal{K}}} \sum_{\boldsymbol{D}} p\left(\boldsymbol{R}|\boldsymbol{\beta}, \boldsymbol{\mathcal{K}}, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{X}\right) p(\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\mathcal{K}}, \boldsymbol{\Sigma}|\boldsymbol{R}, \boldsymbol{X}) d\boldsymbol{\beta} d\boldsymbol{\Sigma} d\boldsymbol{\pi} d\boldsymbol{q}^2, \tag{20}$$

where the posterior density $p(\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\mathcal{K}}, \boldsymbol{\Sigma}|\boldsymbol{R}, \boldsymbol{X})$ is computed as in Section 3.2. Following Chib (1995), we compute the marginal likelihood by replacing the unobservable breaks and parameters in the likelihood of the data generating process (DGP) for each draw. The DGP changes for each of the specifications we used (see section 2.3 for a discussion on alternative model restrictions). Table 2 reports the (log of) marginal likelihoods for each specification of the dynamics in risk exposures and residual variances, with and without acknowledging model uncertainty.

[Insert Table 2 about here]

Top panel shows that the BMA-SBB-SBV model implies the highest log marginal likelihood across our REITs test portfolios. Surprisingly, the BMA-RWB-RWV model ranks second outperforming the BMA-SBB specification. This result indicates that fully acknowledging instability in the idiosyncratic risks plays a relevant role that cannot be simply surrogated by latent change-point models for the betas, similarly to the findings in Bianchi et al. (2015) and Nardari and Scruggs (2007) with reference to alternative applications. Interestingly, by disregarding the fact that the "correct" set of risk factors is unknown, sensibly deteriorates the in-sample performance of the models. The log marginal likelihood of the same specifications with a fixed set of risk factors are sensibly lower (around 30%) than by considering model uncertainty. The superior performance of our latent discrete breaks specification is confirmed across both bond and industry-classified equity portfolios (bottom panel).

Table 2 establishes the superiority of the flexible BMA-SBB-SBV model over all other competitors using an in-sample statistical metric such as the marginal likelihood. Yet, one would also have the comfort of some economic distance measure. We extend our model assessment exercise including a different criterion, which is luckily rather straightforward: given our objectives, a model is as good as its realized pricing performance. In particular, since Section 6 will examine our research question using a common notion of mispricing index (i.e. Jensen's alpha), it is crucial that a model be able to price our test portfolios as well as possible before any mispricings, i.e. $\beta_{i0,t}s$, are estimated.

Table 3 reports a set of sufficient statistics of posterior average cross-sectional pricing errors $Q_{t,N}$ computed as the square root of (19), for different sample periods and across different specifications of time-variation in risk exposures and idiosyncratic risks.

[Insert Table 3 about here]

Our BMA-SBB-SBV model yields the lowest average pricing error across the full sample period, with an expected posterior average error of 0.35% per month. As one would expect, because pricing financial portfolios in times of crisis is harder than in "normal" times, the mean pricing error increases towards the end of our sample (that refers to the 2007-2013 sub-sample), reaching 0.40% on a monthly basis. However, the BMA-SBB-SBV model keeps substantially out-performing all other models, often cutting average errors by almost a third (this is for instance the case vs. the BMA-SBB model, with an average of 1.24% per month across the whole sample). Again, neglecting uncertainty on the set of risk factors significantly reduces the pricing performance of our I-CAPM implementation. For instance, over the full sample, while BMA-SBB-SBV yields a mean pricing error of 0.35% per month, the same dynamics with fixed macroeconomic factors, i.e. SBB-SBV yields a much higher 0.73%. Table 3 reports encouraging evidence that our BMA-SBB-SBV specification may be consistent with the data.

We now assess the performance of each model specification on the basis of predictable variance decomposition tests. In particular, we first compute the amount of predictable variation captured by each specification starting from our general BMA-SBB-SBV model. Table 4 shows posterior medians of the explained predictable variation (15) derived under all the model dynamics entertained in this paper.

[Insert Table 4 about here]

Variance ratios results are encouraging. Approximately 65% of the predictable variation in excess returns is captured by the BMA-SBB-SBV model, on average across REITs. As far as real estate assets are concerned, these relatively high $\mathcal{VR}1$ ratios characterize especially apartment and manufactured homes-specialized REITs, with a $\mathcal{VR}1$ ratio of 82% and 86%, respectively. Also, Office and HealthCare specialized REITs are characterized by a highly significant explained predictable variation of 79% and 80%, respectively. Such percentages decrease for both government and corporate bond portfolios, being around 30%, on average. Consistent with Bianchi et al. (2015), equity portfolios show a satisfactory fit offered by a model with parsimonious time-variation in both risk exposures and idiosyncratic risks.

Table 5 disentangles the sources of explained predictable variation in excess returns that our BMA-SBB-SBV captures. In particular, we decompose the variance ratio $\mathcal{VR}1$ in three main component according to (16). The first component represents the variance in the projection that would exists if one would know the structural parameters, averaged across the marginal posterior distribution $p\left(\boldsymbol{\theta}|\boldsymbol{R},\boldsymbol{X}\right)$. We label this as risk factors-specific explained variation (i.e. intrinsic variation). The second component component represents the contribution of parameter uncertainty (i.e. extrinsic variation). Finally the third component represents the contribution of model uncertainty to the total explained predictable variance. For obvious reasons, we report the results for those model specifications that acknowledge model uncertainty.

[Insert Table 5 about here]

Table 5 shows that, as we would expect, most of the explained predictable variation comes from the implicit in-sample variation of the risk factors. Looking at the last three columns, i.e. BMA-SBB-SBV, intrinsic variation is anywhere above 70% with the only exception of Food industry stocks and the 10-year government bond, where the relative importance of intrinsic variation drops to 65% and 66%, respectively. As far as REIT portfolios are concerned, model uncertainty plays a fairly relevant role, with a relative contribution higher than 10% except for Industry- and Regional Malls- and Lodging/Resorts-specialized REITs. Also for industry-classified and bond portfolios, acknowledging model uncertainty play a fairly relevant role in capturing the in-sample cross-sectional variation of returns. As shown in equation (17), the contribution of model uncertainty increases the greater the dispersion of projections across the models. Such dispersion crucially depends upon fluctuations of risk factors around their historical means. In fact, in absence of variability in risk factors, conditional projections turn out to be identical across models, and the model uncertainty component becomes negligible.

Table 5 shows that parameter uncertainty plays a minor role, albeit still significant. In fact, the relative contribution of parameter uncertainty is almost anywhere below 5% for the BMA-SBB-SBV model, although non-negligible for the alternative BMA-RWB-RWV specification and the homoskedastic BMA-SBB model. Equation (17) shows that the contribution of parameter uncertainty increases the greater the variability of the betas across the models. Such variability intuitively tend to be higher under a random walk dynamics of the parameters as this implies highly unstable risk exposures and idiosyncratic risks. Given the evidence in Section 5.2, in the rest of the paper we exclusively devote our attention to the economic insights derived from the BMA-SBB-SBV model.

6 Time-Varying Mispricing and Risk Exposures

6.1 Heterogeneous Mispricing

Figure 2 reports the marginal posterior median estimates of $\beta_{i0,t}$. In an I-CAPM implementation as (1), $\beta_{i0,t} \neq 0$ shows evidence of a non-zero risk premium for a portfolio i with zero exposures to the K risk factors. This implies the existence of arbitrage opportunities and clashes with first principles (e.g., non-satiation). Equivalently, the Jensen's alphas, $\beta_{i0,t}$s, are measures of abnormal (excess) returns. The figure reports both the posterior medians (solid blue line) and the 95% credibility intervals (dot-dashed black lines) for a set of REIT investment categories. For the ease of exposition, and given our research questions, we focus our attention on REITs portfolios. Apartments and Manufactured Homes represent the "Residential" real estate sector.[10]

[Insert Figure 2 about here]

Figure 2 offers a rather stark view of a number of asset pricing trends that have involved real estate over the past decade and a half: all the Jensen's alpha related to REITs are positive and relatively large. Ex-post, we have evidence that—even in the light of a no-arbitrage multi-factor model driven by macroeconomic risks—real estate as an asset class has been long and persistently over-priced in the U.S., in the sense that realized excess returns have been (on average) higher than what their exposure to systematic risk would have justified between 1999 and 2013. Additionally, and with the partial exception of mortgage investments, all REIT portfolios describe a rather homogeneous dynamics over time: the alphas start out relatively low (in fact, near zero in the case of apartments and manufactured homes), and climb up, in some cases (Industrials, Shopping Centers, Regional Malls) going from a few basis points per month in late 2001 to as high as 2 percent per month (which is a massive annualized abnormal performance of around 24%). This was the great U.S. real estate bubble, with trading, borrowing volumes and prices all exploding at the same time.

However, the alphas for most sectors then slowly declined between 2007 and 2009, settling to average levels around zero percent, when macro factors can perfectly explain average excess returns. Finally, mortgage REITs present a rather peculiar behavior over time: although the mispricing of mortgages seems to have been rather large and accurately estimated with reference to the 2001-2005 period (when the corresponding posterior median $\beta_{mortgages,0,t}$ touched 3% per month), since 2005 the mortgage alphas have been declining to reach on average just a few dozens basis points below zero between 2005 and 2008.

Figure 2 shows no evidence of a *pure* housing/residential real estate bubble—as measured by the mispricing of apartment and manufactured home-investing REITs—inflating between 2001 and 2007, to subsequently burst. All REIT sectors record a climb-up in alphas during this period. In fact, it is the alphas of the retail/distribution-investing REIT sectors that show the steepest ascent, with an increase in posterior medians between 2001 and 2007 in the range of 1.2-2.5 percent for

[10] Additional results on standard stocks and bond portfolios are available upon request.

Shopping Centers, Regional Malls and Lodging/Resort. Yet, Figure 2 shows that none of the real estate sectors are sensibly characterized by mispricing at the end of our sample, possibly due by the massive interventions in the mortgage-backed securities markets by the Federal Reserve between 2009 and 2011 that slowed down properties overvaluations.

In conclusion, Figure 2 tells us a story that only partially matches the tale of the financial crisis often reported by the popular press and portions of the literature. On the one hand, it is a fact that U.S. real estate would have been grossly and systematically over-priced between 2001 and 2007. Over-pricing is indicated by the fact that the posterior estimates of the real estate alphas are positive, climbing, and precisely estimated; large and positive alphas signal that after taking into account the risk exposures and premia captured by the rich set of macroeconomic factors entertained in our paper, real estate yielded "too high" a return, that cannot be rationally justified. In this sense, the real estate fad has been pervasive. Also, the claim that the real estate bubble would have been a debt/mortgage-fueled one (see e.g., Brueckner, Calem, and Nakamura 2012; Coleman, LaCour-Little, and Vandell 2008) is consistent with the fact that drop in mortgage REITs valuations in 2005 sensibly led other real estate sectors. On the other hand, there is no evidence of a larger bubble in the residential vs. the I&O and retail sectors because the alphas of manufactured homes are actually estimated to be not statistically different from zero for a large fraction of the pre-2007 sample. The actual real estate over-pricing occurred instead in the industrial and retail commercial sectors: in particular, the posterior median alphas of commercial real estates (I&O and retails) remain persistently at levels around 1.5 percent per month throughout our sample. In this sense, the 2007-2008 real estate bust did not simply consist of a temporary residential real estate (housing) and mortgage-driven fad, but occurred as a result of a large-scale, widespread correction of substantial mispricings of the entire real estate asset class.

[Insert Figure 3 about here]

Figure 3 shows the marginal posterior estimates of the Jensen's alpha for a set of Industry portfolios. There is no strong evidence of a significant departure from conditional mean-variance efficiency. Except Mines from 2003 and 2007 and the Oil industry across the same period, there is no systematic evidence of over-pricing across Industry portfolios. Interestingly, the increasing alpha for the Oil industry, that subsequently declines across the great financial crisis is consistent with the alleged boom-bust cycle that characterize the oil commodity market across the years 2000.

6.2 Risk Factors Loadings and Significance

At the outset of the paper we argue that macroeconomic and firm specific news do not necessarily hits investors information set at each time t. Thus, systematic risks exposures and in idiosyncratic risks although may change at any point on time, they do not have to be restricted to change at all points. One way to assess the plausibility of our assumption is to observe the "degree" of instability, namely the probability of having a break, for each risk exposure across portfolios. Figure 4 reports the posterior average probabilities over our sample of observing a break in the factor loadings for each REIT, stock and bond portfolio computed from our general BMA-SBB-SBV model specification.

[Insert Figure 4 about here]

Bond portfolios show the highest instability in portfolio betas, with an average probability of having a change in risk exposures around 30% across risk factors. Such instability is rather homogeneous on macro-financial risk factors. While betas on stocks show an intermediate level of instability, betas for REIT portfolios appear to be quite stable over time. Indeed, the average probability of having a break in risk exposures fluctuates between 10% to 15% across factors and portfolios. As a whole, figure 4 shows that infrequent and, possibly, large breaks in betas (as well as Jensens alphas) are often isolated by our estimation procedure, making less parsimonious, although benchmarking, dynamics such as random walks arguably noisy and inefficient.

Our flexible BMA-SBB-SBV not only allows for stochastic and unequally spaced news affecting risk exposures, but also can be used to rank each risk factor in order of their importance to characterize the dynamics of the pricing kernel (1). Indeed, from the output of our MCMC estimation scheme, the probability that a risk factor k enters the factor structure for the ith portfolio is equal to

$$\hat{\lambda}_{ij} = \frac{1}{G}\sum_{g=1}\delta_{ij}^{(g)}, \qquad i = 1, ..., N \qquad j = M, 1, ..., K$$

with G the number of MCMC draws. In other words, the posterior probabilities $\hat{\lambda}_{ij}$ measure the importance of the jth risk factor in the pricing of the ith portfolio. Figure 5 shows the posterior probability of inclusion in the set of risk factors for each state variable across the REIT portfolios (left panel).

[Insert Figure 5 about here]

Few factors are crucial for the pricing of real estate assets, while aggregate dividend yield, real per-capita consumption growth, aggregate credit and default risk and human capital do not appear to sensibly affect the dynamics of the pricing kernel for REIT portfolios. Money growth turns out to be relevant for Regional Malls only. Market risk appear to be the most relevant pricing factor for REIT portfolios, and unexpected inflation and the term spread rank consistently second across real estate investment categories. Also, unemployment represents a fairly relevant priced risk factor for real estate assets, although has less impact on office- and retails-specified REITs. Aggregate liquidity plays a marginal role being significant for Manufactured Homes only. However, we should bear in mind that we are investigating REITs, i.e., publicly traded vehicles that may be seen as derivatives linked to actual properties. In fact, although infrastructure and real estate may represent rather illiquid investments, REITs are not.

Right panel of Figure 5 shows the importance of risk factors across Industry portfolios. Market risk turns out to be the most important risk factor priced in the cross-section. Occasionally, also the real risk-free rate represents an important source of macroeconomic risk. This is true for Financials and Durables. Likewise both the Term-spread and Money growth turn out to be relevant factors in the pricing mechanism of the Oil industry. Interestingly, unlike REIT, liquidity risk plays a major role in explaining the cross-sectional variation of industry-classified portfolios.

Based on the results of Figures 5, we report in 6-9 plots of the posterior medians for the betas on market risk, shocks to unemployment rate, term spread and unexpected inflation. We show factor loadings for the REIT sectors at the heart of our empirical investigation, sampled from their marginal posterior distributions characterized as in (10).[11] In each plot, besides the posterior medians estimated over time (solid blue line), we also show the associated 95% credibility region (dot-dashed black lines)

[11] Our plots do not focus on results for the industry portfolios, even though these have been used in estimating the MFAPM and especially its implied risk premia. Complete results are available upon request.

characterized by the 2.5th and 97.5th percentiles of the posterior density of $\beta_{ij,t}$.[12]

[Insert Figure 6 about here]

Figure 6 shows that over our 1999-2013 sample, REIT portfolios have a market beta that follows a similar path over time, with a marked increase between 2003 and 2007, and the corresponding 95% confidence regions all come to exclude zero by 2005. Market risk prove to be positive and statistically significant for each REIT investment category by the end of the sample, indicating that while before the collapse of the sub-prime market, none of the REITs offered partial hedging against aggregate market risk. This is consistent with Chen, Roll, and Ross (1986) and simple valuation principles. Intuitively, higher market returns in the equity market have a positive effect, in aggregate, on the income of tenants and potential house buyers, increasing the probability of the rents being paid and houses being sold; both of these positively affect index REIT levels which can be thought of a function of discounted profits stemming from rents and sales properties. The positive and significant market betas therefore match standard valuation predictions.

[Insert Figure 7 about here]

Figure 7 shows that, except occasional nuances all REIT portfolios have negative exposures to the slope of the yield curve; these betas are large with a posterior distribution tilted away from zero, especially in the case of non-residential REITs (e.g. Industrial, Office, FreeStand). This negative relationship is similar to a "flight-to-quality" effect typical in treasury and bond portfolios, in the sense that REITs command high prices and low risk premia exactly when the risk-less yield curve is flat or inverted, as typical of the early stages of recessions. This is not entirely surprising. Equity REITs often hold long-term fixed leases, and they have to pay out most of their earnings to investors. Thus, REITs may be exposed to variations in the yield curve based on their inherent investment characteristics. In fact, the fixed nature of the underlying cash flows and the limited growth opportunity of their assets, makes equity REITs resemble investments in bond portfolios (see e.g. Graff 2001). This finding is

[12] Pinning down the " statistical significance" of coefficients on the basis of 95% credibility intervals represents a rather stringent criterion because the Bayesian posterior density will reflect not only the uncertainty on the individual coefficient but also the overall uncertainty on the entire model (e.g., the uncertainty on structural instability of all the coefficients as well as uncertainty on the probability of inclusion of a single risk factor).

consistent with the traditional view often discussed in the literature that posits real estate would represent a "composite" asset class that inherits mixed features (here, factor exposures) from both stocks and bonds, (see e.g., Simpson and Ramchander 2007) and references therein. Also, a significant correlation with the term structure is consistent with a well developed literature that has related real estate valuations to interest rate levels and monetary policy (see e.g., Iacovello 2005; Iacovello and Neri 2010).

[Insert Figure 8 about here]

Figure 8 shows that all sector REIT portfolios display a strong and statistically significantly positive exposure to unexpected inflation. This is consistent with the view that increasing inflation increase the nominal future cash flows from underlying properties and therefore have a positive correlation with returns on real estate assets (see e.g., Hoesli, Lizieri, and MacGregor 2008). As a whole, infrastructure and real estate tend to have non-trivial positive betas on unexpected inflation, which makes them desirable investments when inflation surprises on the upside.

Interestingly, exposure to unemployment risk is highly heterogeneous across different REIT investment categories. Figure 9 shows that while residential REITs carry a highly significant and negative exposure on shocks to unemployment, core-commercial real estate sectors such as industrial, office, and retails such as regional malls and shopping centers have essentially neutral exposure. Again, these findings can be interpreted having in mind a simple valuation model. The impact of unemployment on REITs is two-fold; first, unemployment rate tend to be counter-cyclical, therefore higher periods of unemployment correspond to lower discount rates increasing the discounted profits stemming from rents and sales of property. Second, increasing unemployment has a negative effect on the income of tenants and potential house buyers, increasing the probability of foreclosures and rents being not paid. Figure 9 tells us that these two forces tend to compensate within I&O, Retails and Lodging/Resort, while the negative income effects of increasing unemployment prevails on residential- and mortgage-specialized REITs.

[Insert Figure 9 about here]

Figures 6-9 show that for most factors and portfolios the BMA-SBB-SBV model reveals interesting variation in betas. However, we emphasize that such time variation is not forced upon the data, in the sense that a casual look at the plots reveals that combinations of test assets and factors can be found for which the $\beta_{ij,t}$s implies little or no instability. For instance, in the bottom-right panel of

Figure 7, concerning the exposure of Lodging/Resort REIT to the yield spread, the plot reveals a posterior median of $\beta_{Lodg,Term,t}$ that is almost flat at approximately -0.5 throughout the entire sample period. Interestingly, for most factors the REIT sectors tend to share a common dynamics in their exposures, even when such betas are characterized by different means. For instance, in Figure 6, the $\beta_{ij,t}$s with respect to market risk all generally increase (the only exception is mortgage REITs), but a comparison between sectors show that magnitudes and rate of growths are sensibly different.

Generally speaking, we make clear that different REITs are characterized by a substantially heterogeneous dynamics in estimated beta posteriors. Residential REITs are different from retail and industrial REITs; mortgage REITs have a risk factor structure that is very specific and that diverges from equity REITs. For instance, residential REITs, unlike industrial and retails-specialized sectors, are characterized by a significant negative exposure on unemployment risk. All sectors are characterized by increasing exposure to market risk, although housing-driven and mortgage REITs have a lower exposure. Manufactured Homes are almost neutral across the sample with respect to changes in the slope of the yield curve, while both I&O and retails sectors are highly negatively exposed.

A growing literature (see e.g., Campbell, Lettau, Malkiel, and Xu 2001 and Engle, Ghysels, and Sohn 2013) has pointed out that idiosyncratic risks, σ^2_{it}, have sustained important shifts over the last decades. Figure 10 plots marginal posterior medians (solid blue line) for σ^2_{it} estimated from our BMA-SBB-SBV model, along with 95% credibility intervals (dot-dashed black lines).

[Insert Figure 10 about here]

The financial crisis of 2008-2009 induces idiosyncratic risk increase when the model had temporarily reduced its ability to price REIT, stock and bond portfolios. The fact that idiosyncratic is countercyclical was largely expected in the light of the literature (see Campbell et al. 2001). Increasing idiosyncratic risk are more pronounced for industrial- and retails-specialized (e.g. Shopping, Regional Malls) REITs. However, no clear trend is detected, which is consistent with recent findings reported by Bekaert, Hodrick, and Zhang (2012).

7 Conclusions

In this paper we have asked a simple question: can a rational multi-factor asset pricing model in which macroeconomic factors measure risk shed any light on the actual or alleged differences in the pricing mechanism underlying residential vs. non-residential real estate? Equivalently, has it been fair to place most of the burden of the recent real estate bust on over-pricing and misconduct that would have taken place mainly in the private, residential housing sector? To provide an answer to this question, we have made two critical choices. First, we have estimated using flexible Bayesian model averaging methods a rich parametric multi-factor stochastic volatility model with discrete stochastic breaks in factor loadings and idiosyncratic risks. Such a choice is intended to deal with the widespread evidence that asset pricing relationships are both unstable, in the sense that the exposures of different portfolios to risk variables change over time, and unknown, in the sense that assuming the "right" set of macro-financial risk factors is observable arguably leads to misleading inference. Second, we have addressed this question resorting to abundant and detailed data on publicly traded REIT total return indices for disaggregated sector portfolios to distinguish between residential, business-related (i.e., industrial, office, and retail) investments, and mortgage specializations.

We uncover three key results. First, an analysis of cross-sectional mispricing reveals that all the Jensen's alpha implied by REITs were positive and relatively large over parts of our sample. Additionally, and with the partial exception of mortgage investments, all sector REITs described a homogeneous dynamics over time: all alphas climb up, in some cases going from a few basis points to as high as 2.5 percent per month across 1999-2007. This was the great U.S. real estate bubble. Such mispricing corrected between 2007-2009, often returning to zero percent, when macro risk factors can perfectly explain average excess returns. Moreover, the claim that the real estate "bubble" would have been a debt/mortgage-fueled one is consistent with our result that between 1999 and 2005 mortgage REITs implied the largest, positive median alphas, whom drops sensibly led other sectors. Yet, there is no evidence of a pure housing/residential real estate bubble inflating between 1999 and 2007, to subsequently burst. All REIT sectors record a climb-up in alphas during this period. In fact, the real estate over-pricing occurred across the board and also involved the non-residential real estate sectors.

Second, we show that few factors carry most of the dynamic properties of the pricing equations. As far as equity REITs portfolios are concerned, market risk and unexpected inflation show the highest relevance in explaining excess returns. Also, the slope of the yield curve, unexpected inflation and unemployment risk immediately rank second in terms of their importance to approximate the pricing kernel for real estate assets. Liquidity, instead, plays a crucial role to explain the cross-sectional variation of industry-classified portfolios.

Third, there are differences in the structure and dynamic evolution of risk factor exposures across residential and non-residential REITs. Residential REITs—according to most of the literature, the area from which the sub-prime crisis would have originated—are characterized by a negative exposure to unemployment risk, by quickly increasing, albeit modest, exposures to market risk, negative correlation with term premium and by massive and quickly increasing beta on unexpected inflation. REITs that specialize in industrial and office investments carry instead neutral exposure to unemployment risk, and larger exposure to aggregate financial wealth as measured by the excess return on the market portfolio. Retail- (shopping, regional malls and free-standing) specialized REITs also display a neutral exposure to unemployment risk. As a whole, a comparison among residential vs. I&O and retail REITs, sheds light on one potential cause of their diverging behavior in the aftermath of the 2007-2009 crisis.

This finding of deeply rooted and persistent overpricing of specific types of commercial real estate properties, has important policy implications. Indeed, in the measure in which – as sometimes discussed in policy circles (see e.g. Bernanke 2012, Greenlee 2009, and Gyourko 2009) – core commercial real estate property investments sit in large amounts on the balance sheets of nationally- and regionally-relevant U.S. banks, their exposure to macroeconomic and inflation risks may end up hindering the correct transmission of monetary policy impulses.

References

Aït-Sahalia, J., J. Andritzky, A. Jobst, and a. T. N. Nowak, S. 2009. How to Stop a Herd of Running Bears? Market Response to Policy Initiatives during the Global Financial Crisis. *IMF working paper* 204.

Ang, A., and J. Chen. 2007. CAPM Over the Long-Run: 1926-2001. *Journal of Empirical Finance* pp. 1–40.

Arce, O., and D. López-Salido. 2011. Housing Bubbles. *American Economic Journal: Macroeconomics* pp. 212–241.

Bekaert, G., R. Hodrick, and X. Zhang. 2012. Aggregate Idiosyncratic Volatility. *Journal of Financial and Quantitative Analysis* 47:1155–1185.

Bernanke, B. 2012. Banks and Bank Lending: The State of Play. *Remarks to the 48th Annual Conference on Bank Structure and Competition Sponsored by the Federal Reserve Bank of Chicago* .

Bianchi, D., M. Guidolin, and F. Ravazzolo. 2015. Macroeconomic Factors Strike Back: A Bayesian Change-Point Model of Time-Varying Risk Exposures and Premia in the U.S. Cross-Section. *Journal of Business and Economic Statistics* .

Boudry, W., N. Coulson, J. Kallberg, and C. Liu. 2012. On the Hybrid Nature of REITs. *Journal of Real Estate Finance and Economics* pp. 230–249.

Brueckner, J., P. Calem, and L. Nakamura. 2012. Subprime Mortgages and the Housing Bubble. *Journal of Urban Economics* pp. 230–243.

Campbell, J. 1996. Understanding Risk and Return. *Journal of Political Economy* pp. 298–345.

Campbell, J., M. Lettau, B. Malkiel, and Y. Xu. 2001. Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *Journal of Finance* 56:1–43.

Carter, C., and R. Kohn. 1994. On Gibbs sampling for state-space models. *Biometrika* pp. 541–553.

Case, K., and R. Shiller. 1989. The Efficiency of the Market for Single-Family Homes. *American Economic Review* pp. 125–137.

Case, K., and R. Shiller. 2003. Is There a Bubble in the Housing Market? *Brookings Papers on Economic Activity* pp. 299–342.

Cecchetti, S. 2009. Crisis and Responses: The Federal Reserve in the Early Stages of the Financial Crisis. *Journal of Economic Perspectives* 23:51–75.

Chen, N., R. Roll, and S. Ross. 1986. Economic Forces and the Stock Market. *Journal of Business* 59:383–403.

Chiang, K. 2009. Discovering REIT Price Discovery: A New Data Setting. *Journal of Real Estate Finance and Economics* 39:74–91.

Chib, S. 1995. Marginal Likelihood from the Gibbs Output. *Journal of the American Statistical Association* 432:1313–1321.

Clayton, J., and G. MacKinnon. 2003. The Relative Importance of Stock, Bond and Real Estate Factors in Explaining REIT Returns. *Journal of Real Estate Finance and Economics* 27:39–60.

Coleman, M., M. LaCour-Little, and K. Vandell. 2008. Subprime Lending and the Housing Bubble: Tail Wags Dog? *Journal of Housing Economics* pp. 272–290.

Cotter, R. R., J. and. 2011. A Comparative Anatomy of REITs and Residential Real Estate Indexes: Returns, Risks and Distributional Characteristics. *Working Paper, UCLA* .

Del Negro, M., and C. Otrok. 2007. 99 Luftballons: Monetary Policy and the House Price Boom Across U.S. States. *Journal of Monetary Economics* 54:1962–1985.

Dell'Ariccia, I. D., G., and L. Laeven. 2011. Boom and Lending Standards: Evidence from the Subprime Mortgage Market. *Journal of Money, Credit and Banking* 44:367–384.

Demyanyk, Y., and O. Van Hemert. 2011. Understanding the Subprime Mortgage Crisis. *Review of Financial Studies* 24:1848–1880.

Engle, R., E. Ghysels, and B. Sohn. 2013. Stock Market Volatility and Macroeconomic Fundamentals. *Review of Economics and Statistics* 95:776–797.

Engle, R., and A. Smith. 1999. Stochastic Permanent Breaks. *Review of Economics and Statistics* 81:553–574.

Faust, J., S. Gilchrist, J. Wright, and E. Zakrajssek. 2013. Credit Spreads as Predictors of Real-Time Economic Activity: A Bayesian Model-Averaging Approach. *Review of Economics and Statistics* 95:1501–1519.

Favilukis, J., S. Ludvigson, and S. Van Nieuwerburgh. 2011. The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium. *NYU Working Paper* .

Ferson, W., and C. Harvey. 1991. The Variation of Economic Risk Premiums. *Journal of Political Economy* 99:385–415.

Ferson, W., and R. Korajczyk. 1995. Do Arbitrage Pricing Models Explain the Predictability of Stock Returns? *Journal of Business* pp. 309–349.

Frühwirth-Schnatter, S. 1994. Data Augmentation and Dynamic Linear Models. *Journal of Time Series Analysis* 15:183–202.

Geman, S., and D. Geman. 1984. Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images. *IEEE Transactions* 6:721–741.

George, E., and R. McCulloch. 1993. Variable Selection via Gibbs Sampling. *Journal of the American Statistical Association* pp. 881–889.

Gerlach, R., C. Carter, and R. Kohn. 2000. Efficient Bayesian Inference for Dynamic Mixture Models. *Journal of the American Statistical Association* pp. 819–828.

Geweke, J., and G. Amisano. 2014. Analysis of Variance for Bayesian Inference. *Econometric Reviews* 33:270–288.

Geweke, J., and G. Zhou. 1996. Measuring the Pricing Error of the Arbitrage Pricing Theory. *Review of Financial Studies* 9:557–587.

Giannone, D., M. Lenza, and G. Primiceri. 2015. Prior Selection for Vector Autoregressions. *Review of Economics and Statistics* 97:436–451.

Giordani, P., and R. Kohn. 2008. Efficient Bayesian Inference for Multiple Change-Point and Mixture Innovation Models. *Journal of Business and Economic Statistics* 26:66–77.

Glaeser, E. 2013. A Nation of Gamblers: Real Estate Speculation and American History. *American Economic Review P&P* pp. 1–42.

Glaeser, E., J. Gottlieb, and J. Gyourko. 2013. *Can Cheap Credit Explain the Housing Boom?* In "Housing and the Financial Crisis", edited by Edward Glaeser and Todd Sinai, University of Chicago Press.

Gorton, G. 2010. The Panic of 2007. *European Financial Management* 15:10–46.

Goyal, A., and I. Welch. 2008. A Comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies* 4:1455–1508.

Graff, R. 2001. Economic Analysis Suggests That REIT Investment Characteristics are not as Advertised. *Journal of Real Estate Portfolio Management* pp. 99–124.

Greenlee, J. 2009. Testimony on Residential and Commercial Real Estate before the Subcommittee on Domestic Policy. *Committee on Oversight and Government Reform, U.S. House of Representatives, Atlanta, GA* .

Gyourko, J. 2009. Understanding Commercial Real Estate: Just How Different from Housing Is It? *NBER working paper No. 14708.* .

Gyourko, J., and D. Keim. 1992. What Does the Stock Market Tell Us about Real Estate Returns? *AREUEA Journal* 20:457–485.

Hendershott, P., R. Hendershott, and J. Shilling. 2010. The Mortgage Finance Bubble: Causes and Corrections. *Journal of Housing Research* pp. 1–16.

Himmelberg, C., C. Mayer, and T. Sinai. 2005. High House Prices: Bubbles, Fundamentals and Misperceptions. *Journal of Economic Perspectives* pp. 67–92.

Hoesli, M., C. Lizieri, and B. MacGregor. 2008. The Inflation Hedging Characteristics of U.S. and U.K. Investments: A Multi-Factor Error Correction Approach. *Journal of Real Estate Finance and Economics* pp. 183–206.

Hoeting, J., D. Madigan, A. Raftery, and C. Volinsky. 1999. Bayesian Model Averaging: A Tutorial. *Statistical Science* 14:328–417.

Iacovello, M. 2005. House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review* 95:739–764.

Iacovello, M., and S. Neri. 2010. Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics* 2:125–164.

Jagannathan, R., and Z. Wang. 1996. The Conditional CAPM and the Cross-Section of Expected Returns. *The Journal of Finance* 51:3–53.

Jostova, G., and A. Philipov. 2005. Bayesian Analysis of Stochastic Betas. *Journal of Financial and Quantitative Analysis* 40:747–778.

Karolyi, G., and A. Sanders. 1998. The Variation of Economic Risk Premiums in Real Estate Returns. *Journal of Real Estate Finance and Economics* 17:245–262.

Kim, C., and C. Nelson. 1999. Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle. *Review of Economics and Statistics* 81:608–616.

Koop, G., and S. Potter. 2007. Estimation and Forecasting in Models with Multiple Breaks. *Review of Economic Studies* 74:763–789.

Kuo, L., and B. Mallick. 1998. Variable Selection for Regression Models. *The Indian Journal of Statistics* pp. 65–81.

Lamont, O. 2001. Economic Tracking Portfolios. *Journal of Econometrics* 105:161–184.

Lewellen, J., S. Nagel, and J. Shanken. 2010. A Skeptical Appraisal of Asset Pricing Tests. *Journal of Financial Economics* 96:175–194.

Ling, D., and A. Naranjo. 2003. The Dynamics of REIT Capital Flows and Returns. *Real Estate Economics* pp. 405–434.

Madigan, D., and A. Raftery. 1994. Model Selection and Accounting for Model Uncertainty in Graphical Models using Occam's Window. *Journal of the American Statistical Association* 89:1535–1546.

McCulloch, R., and P. Rossi. 1991. Posterior, Predictive and Utility Based Approaches to Testing Arbitrage Pricing Theory. *Journal of Financial Economics* 28:7–38.

Merton, R. 1973. An Intertemporal Capital Asset Pricing Model. *Econometrica* 41:867–887.

Mian, A., and A. Sufi. 2009. The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis. *Quarterly Journal of Economics* pp. 1449–1496.

Mian, A., and A. Sufi. 2011. House Prices, Home-Equity Based Borrowing, and the U.S. Household Leverage Crisis. *American Economic Review* pp. 2132–2156.

Moral-Benito, E. 2012. Determinants of Economic Growth: A Bayesian Panel Data Approach. *Review of Economics and Statistics* 94:566–579.

Nardari, F., and J. Scruggs. 2007. Bayesian Analysis of Linear Factor Models with Latent Factors, Multivariate Stochastic Volatility, and APT Pricing Restrictions. *Journal of Financial and Quantitative Analysis* 42:857–891.

Omori, Y., S. Chib, N. Shepard, and J. Nakajima. 2007. Stochastic Volatility with Leverage: Fast and Efficient Likelihood Inference. *Journal of Econometrics* 140:425–449.

Pastor, L., and F. Stambaugh, R. 2003. Liquidity Risk and Expected Stock Returns. *Journal of Political Economy* pp. 642–685.

Payne, a. W. G., J. 2007. Have Equity REITs Experienced Periodically Collapsing Bubbles? *Journal of Real Estate Finance and Economics* pp. 207–224.

Petkova, R. 2006. Do the Fama-French Factors Proxy for Innovations in Predictive Variables? *Journal of Finance* pp. 581–612.

Ravazzolo, F., R. Paap, D. van Dijk, and P. Franses. 2007. *Bayesian Model Averaging in the Presence of Structural Breaks*. Frontiers of economics and globalization ed. Elsevier.

Seck, D. 1996. The Substitutability of Real Estate Assets. *Real Estate Economics* pp. 75–96.

Shanken, J. 1992. On the Estimation of Beta Pricing Models. *Review of Financial Studies* pp. 1–34.

Shiller, R. 1979. The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure. *Journal of Political Economy* 87:1190–1219.

Simpson, M., and a. W. J. Ramchander, S. 2007. The Asymmetric Response of Equity REIT Returns to Inflation. *Journal of Real Estate Finance and Economics* 34:513–529.

Smith, A., and G. Roberts. 1993. Bayesian Computation via Gibbs Sampler and Related Markov Chain Monte Carlo Methods. *Journal of the Royal Statistical Society: Series B* pp. 3–24.

Smith, M., and G. Smith. 2006. Bubble, Bubble, Where is the Housing Bubble? *Brookings Papers on Economic Activity* pp. 1–67.

Vassalou, M. 2003. News Related to future GDP growth as a risk factor in equity returns. *Journal of Financial Economics* pp. 47–73.

West, M., and J. Harrison. 1997. *Bayesian forecasting and dynamics models*. Springer.

Wheaton, a. N. G., W.C. 2008. The 1998-2005 Housing 'Bubble' and the Current 'Correction': What's Different This Time? *Journal of Real Estate Research* pp. 1–26.

Appendix

A The Gibbs Sampling Algorithm

Our Gibbs sampler is a combination of the Forward Filtering Backward Sampling of Carter and Kohn (1994), Omori, Chib, Shepard, and Nakajima (2007) and the efficient sampling algorithm for the random breaks proposed in Gerlach, Carter, and Kohn (2000). We use a burn-in period of 2,000 and draw 10,000 observations storing every second observation to simulate the posterior of parameters and latent variables. Here, we derive the full conditional posterior distributions of the latent variables and the model parameters discussed in Section 2 of the main text. Before to describe in details the efficient Gibbs sampler, we need to define the densities that make up the joint density of the data and the latent variables (9). For the *ith* portfolio, the probability density functions for each portfolios, betas and log variances can be written from (2)-(4) as follows:

$$p(r_{it}|X_t,\beta_{it},\sigma_{it}^2,D) = \frac{1}{\sqrt{2\pi\sigma_{i,t}^2}}\exp\left(-\frac{\left(r_{it}-\beta_{i0,t}-\delta_{i,M}\beta_{iM,t}R_{M,t}-\sum_{j=1}^{K}\delta_{ij}\beta_{ij,t}u_{j,t}\right)^2}{2\sigma_{i,t}^2}\right)$$

$$p(\beta_{i,t}|\beta_{i,t-1},\kappa_{i,t},q_i^2) = \prod_{j=0}^{K}\left(\frac{1}{\sqrt{2\pi q_{ij}^2}}\exp\left(-\frac{\left(\beta_{ij,t}-\beta_{ij,t-1}\right)^2}{2q_{ij}^2}\right)\right)^{\kappa_{ij,t}}\left(\boldsymbol{\delta}\left(\beta_{ij,t}-\beta_{ij,t-1}\right)\right)^{1-\kappa_{ij,t}}$$

$$p(\ln\sigma_{it}^2|\ln\sigma_{it-1}^2,\kappa_{i\nu,t},q_{i\nu}^2) = \left(\frac{1}{\sqrt{2\pi q_{i\nu}^2}}\exp\left(-\frac{\left(\ln\sigma_{i,t}^2-\ln\sigma_{i,t-1}^2\right)^2}{2q_{i\nu}^2}\right)\right)^{\kappa_{i\nu,t}}\left(\boldsymbol{\delta}\left(\ln\sigma_{i,t}^2-\ln\sigma_{i,t-1}^2\right)\right)^{1-\kappa_{i\nu,t}}$$

where $\boldsymbol{\delta}(\cdot)$ is a Dirac delta function. The densities for $\beta_{i,t}$ and $\ln\sigma_{it}^2$ each consist of two parts. First one where breaks occurs and these are drawn from their corresponding distributions. The second component is the case of no break which results in a degenerate distribution of either the $\beta_{ij,t}$ or $\ln\sigma_{it}^2$, which can be represented as a Dirac function. For the ease of exposition we summarize the Gibbs sampler for the i_{th} asset.

Step 1. Sampling the Variable Selection Parameters D

We follow George and McCulloch (1993), Kuo and Mallick (1998) and citeRFSGroen:etal:2013 to address model uncertainty while estimating the model dynamics. The variable D is drawn at each iteration from its full conditional distribution. We compute the value of the posterior density for $\delta_{ij}=0$ and $\delta_{ij}=1$ given the value of the other parameters which results in p_{j0} and p_{j1}, respectively. The full conditional posterior simplifies to:

$$\begin{aligned}&p\left(\delta_{ij}=1|D_{i[-j]},\boldsymbol{\beta},\boldsymbol{\mathcal{K}},\boldsymbol{\Sigma},\boldsymbol{\theta},\boldsymbol{X},\boldsymbol{R}\right)=\\&=\frac{\lambda_{ij}\prod_{t=1}^{T}p(r_{it}|X_t,\beta_{it},\sigma_{it}^2,D_{i[-j]})|_{\delta_{ij}=1}}{(1-\lambda_{ij})\prod_{t=1}^{T}p(r_{it}|X_t,\beta_{it},\sigma_{it}^2,D_{i[-j]})|_{\delta_{ij}=0}+\lambda_{ij}\prod_{t=1}^{T}p(r_{it}|X_t,\beta_{it},\sigma_{it}^2,D_{i[-j]})|_{\delta_{ij}=1}},\end{aligned}\tag{A.1}$$

for $i=1,...,N$ and $j=M,1,...,K$, where $D_{i[-j]}=(\delta_{i1},...,\delta_{ij-1},\delta_{ij+1},...,\delta_{iK})'$ and where the density of r_{it} is given above. We randomly choose the order in which we sample the K δ_{ij} parameters. As starting value of the Gibbs sampler we consider a model which includes all K risk factors.

Step 2. Sampling $\mathcal{K}_\beta$.

The structural breaks in the conditional dynamics of the factor loadings $\boldsymbol{\beta}$, measured by the latent binary state $\mathcal{K}_\beta$, are drawn using the algorithm of Gerlach et al. (2000). This algorithm increases the efficiency of the sampling procedure since allows to generate $\kappa_{ij,t}$, without conditioning on the relative regression parameters $\beta_{ij,t}$. The conditional posterior density for $\kappa_{ij,t}$, $t = 1, ..., T, j = 0, M, 1, ..., K$, is defined as

$$p\left(\kappa_{0t,...,}\kappa_{Kt}|\mathcal{K}_{\beta[-t]}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right) \propto p\left(\boldsymbol{R}|\mathcal{K}_{\beta t}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right) p\left(\kappa_{0t,...,}\kappa_{Kt}|\mathcal{K}_{\beta[-t]}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right)$$
$$\propto p\left(r_{t+1,...,}r_T|r_{1,...,}r_t, \mathcal{K}_{\beta t}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right)$$
$$p\left(r_t|r_{1,...,}r_{t-1}, \kappa_{0t,...,}\kappa_{Kt}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right) p\left(\kappa_{0t,...,}\kappa_{Kt}|\mathcal{K}_{\beta[-t]}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right) \tag{A.2}$$

where $\mathcal{K}_{\beta[-t]} = \left\{\{\kappa_{js}\}_{j=0}^{K}\right\}_{s=1,s\neq t}^{T}$. We assume that each of the κ_{js} breaks are independent from each other such that the joint density is defined is defined as $\prod_{j=0}^{K} \pi_{ij}^{\kappa_{jt}} \left(1 - \pi_{ij}\right)^{1-\kappa_{jt}}$.

The remaining densities $p\left(r_{t+1,...,}r_T|r_{1,...,}r_t, \mathcal{K}_{\beta t}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \theta, F\right)$ and $p\left(r_t|r_{1,...,}r_{t-1}, \kappa_{0t,...,}\kappa_{Kt}, \mathcal{K}_\sigma, \boldsymbol{\Sigma}, \boldsymbol{\theta}, \boldsymbol{R}, \boldsymbol{X}\right)$ are evaluated as in Gerlach et al. (2000). Notice that, since κ_{jt} is a binary state the integrating constant is easily evaluated.

Step 2. Sampling the Factor Loadings B

The full conditional posterior density for the time-varying factor loadings is computed using a forward filtering backward sampling as in Carter and Kohn (1994). For each of the $i = 1, ..., N$ assets, the prior distribution of the $\beta_{i0}, ..., \beta_{iK}$ loadings is a multivariate normal with the location parameters corresponding to the OLS parameter estimates and a covariance structure which is diagonal and defined by the variances of the OLS estimates. The initial prior are sequentially updated via the Kalman Filtering recursion, then the parameters are drawn from the posterior distribution which is generated by a backward recursion (see Frühwirth-Schnatter 1994, Carter and Kohn 1994, and West and Harrison 1997).

Step 3 and 4. Sampling the Breaks and the Values of the Idiosyncratic Volatility.

In order to draw the structural breaks $\mathcal{K}_\sigma$ and the idiosyncratic volatilities S we follow a similar approach as above. The stochastic breaks $\mathcal{K}_\sigma$ are drawn by using the Gerlach et al. (2000) algorithm. The conditional variances $\ln \sigma_{it}^2$, does not show a linear structure even though still preserving the standard properties of state space models. The model is rewritten as

$$\ln\left(r_{it} - \beta_{i0,t} - \delta_{i,M}\beta_{iM,t}R_{M,t} - \sum_{j=1}^{K}\delta_{ij}\beta_{ij,t}u_{j,t}\right)^2 = \ln \sigma_{it}^2 + u_t$$
$$\ln \sigma_{it}^2 = \ln \sigma_{it-1}^2 + \kappa_{\nu it}\nu_{it} \tag{A.3}$$

where $u_t = \ln \varepsilon_t^2$ has a $\ln \chi^2(1)$. Here we follow Omori et al. (2007) and approximate the $\ln \chi^2(1)$ distribution with a finite mixture of ten normal distributions, such that the density of u_t is given by

$$p(u_t) = \sum_{l=1}^{10} \varphi_l \frac{1}{\sqrt{\varpi_l^2 2\pi}} \exp\left(-\frac{(u_t - \mu_l)^2}{2\varpi_l}\right) \tag{A.4}$$

with $\sum_{l=1}^{10} \varphi_l = 1$. The appropriate values for μ_l, φ_l and ϖ_l^2 can be found in Omori et al. (2007). Mechanically in each step of the Gibbs Samplers we simulate at each time t a component of the mixture. Now, given the mixture component we can apply the standard Kalman filter method, such that $\boldsymbol{\mathcal{K}}_\sigma$ and S can be sampled in a similar way as $\boldsymbol{\mathcal{K}}_\beta$ and $\boldsymbol{\beta}$ in the first and second step. The initial prior of the log idiosyncratic volatility $\ln \sigma_0^2$ is normal with mean -2 and conditional variance equal to 10.

Step 5b. Sampling the Stochastic Breaks Probabilities.

The full conditional posterior densities for the breaks probabilities $\pi = (\pi_{i1,...,}\pi_{iK})$ is given by

$$p\left(\pi|q^2, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mathcal{K}}_\beta, \boldsymbol{R}, \boldsymbol{X}\right) \propto \prod_{j=0}^{K} \pi_{ij}^{a_{ij}-1} (1-\pi_{ij})^{b_{ij}-1} \prod_{t=1}^{T} \pi_{ij}^{\kappa_{ijt}} (1-\pi_{ij})^{1-\kappa_{ijt}} \tag{A.5}$$

and hence the individual π_{ij} parameter can be sampled from a Beta distribution with shape parameters $a_{ij} + \sum_{t=1}^{T} \kappa_{ijt}$ and $b_{ij} + \sum_{t=1}^{T}(1-\kappa_{ijt})$ for $j = 0, ..., K$. Likewise the full conditional posterior distribution for the breaks probabilities in the idiosyncratic volatilities π_ν is given by

$$p\left(\pi_\nu|, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mathcal{K}}_\sigma, \boldsymbol{R}, \boldsymbol{X}\right) \propto \pi_{i\nu}^{a_{i\nu}-1} (1-\pi_{i\nu})^{b_{i\nu}-1} \prod_{t=1}^{T} \pi_{i\nu}^{\kappa_{i\nu t}} (1-\pi_{i\nu})^{1-\kappa_{i\nu t}}$$

such that the individual $\pi_{i\nu}$ can be sampled from a Beta distribution with shape parameters $a_{i\nu} + \sum_{t=1}^{T} \kappa_{i\nu t}$ and $b_{i\nu} + \sum_{t=1}^{T}(1-\kappa_{i\nu t})$ for $i = 1, ..., N$.

Step 5c. Sampling the Conditional Variance of the States.

The prior distributions for the conditional volatilities of the factor loadings β_{ijt} for $j = 0, ..., K$ are inverse-gamma

$$p\left(q_{ij}^2|\boldsymbol{\pi}, \boldsymbol{B}, \boldsymbol{\Sigma}, \boldsymbol{\mathcal{K}}_\beta, \boldsymbol{\mathcal{K}}_\sigma, \boldsymbol{R}, \boldsymbol{X}\right) \propto q_{ij}^{-\nu_{ij}} \exp\left(-\frac{\delta_{ij}}{2q_{ij}^2}\right) \prod_{t=1}^{T} \left(\frac{1}{q_{ij}} \exp\left(\frac{-\left(\beta_{ijt}-\beta_{ijt-1}\right)^2}{2q_{ij}^2}\right)\right)^{\kappa_{ijt}} \tag{A.6}$$

hence q_{ij}^2 is sampled from an inverse-gamma distribution with scale parameter $\nu_{ij} + \sum_{t=1}^{T} \kappa_{ijt} \left(\beta_{ijt}-\beta_{ijt-1}\right)^2$ and degrees of freedom equal to $\nu_{ij} + \sum_{t=1}^{T} \kappa_{ijt}$. Likewise the full conditional of the variance for the idiosyncratic log volatility $q_{i\nu}^2$ is defined as

$$p\left(q_{i\nu}^2|\boldsymbol{\pi}, \boldsymbol{B}, \boldsymbol{\Sigma}, \boldsymbol{\mathcal{K}}_\beta, \boldsymbol{\mathcal{K}}_\sigma, \boldsymbol{R}, \boldsymbol{X}\right) \propto q_{i\nu}^{-\nu_{i\nu}} \exp\left(-\frac{\delta_{i\nu}}{2q_{i\nu}^2}\right) \prod_{t=1}^{T} \left(\frac{1}{q_{i\nu}} \exp\left(\frac{-\left(\ln\sigma_{it}^2 - \ln\sigma_{it-1}^2\right)^2}{2q_{i\nu}^2}\right)\right)^{\kappa_{i\nu t}} \tag{A.7}$$

such that $q_{i\nu}^2$ is sampled from an inverted Gamma distribution with scale parameter $\nu_{i\nu} + \sum_{t=1}^{T} \kappa_{i\nu t} \left(\ln\sigma_{it}^2 - \ln\sigma_{it-1}^2\right)^2$ and degrees of freedom equal to $\nu_{i\nu} + \sum_{t=1}^{T} \kappa_{i\nu t}$.

B Choice of Priors Hyper-Parameters

Realistic values for the different prior distributions obviously depend on the problem at hand.[13] In general, we use weak priors, excluding the size of the breaks $\mathbf{Q}_i$ and the probabilities $\Pr(\kappa_{ij,t} = 1)$ and $\Pr(\kappa_{i\nu,t} = 1)$ for which our priors are quite informative. This is also important because these priors restrict the maximum number of breaks of maximum magnitude and therefore help to identify the factor exposures, which is otherwise rather problematic because linear multi-factor models are subject to well-known indeterminacy problems upon rotations of factors and risk premia (see e.g., McCulloch and Rossi 1991). The prior shape parameters for the probability of breaks in the dynamics of the price sensitivitites is set to be $a_{ij} = 20$ and $b_{ij} = 80$. As such,

$$E\left[\pi_{ij}\right] = \frac{20}{20+80} = 0.2 \qquad \text{and} \qquad Std\left[\pi_{ij}\right] = \left(\frac{20 \times 80}{(20+80)^2(20+80+1)}\right)^{1/2} = 0.04$$

which means an expected 20% prior probability of a random shock in the dynamics of factor loadings. With respect to the idiosyncratic volatility, the shape hyperparameters are set to be $a_{i\nu} = 10$ and $b_{i\nu} = 70$, such that

$$E\left[\pi_{i\nu}\right] = \frac{10}{10+70} = 0.12 \qquad \text{and} \qquad Std\left[\pi_{i\nu}\right] = \left(\frac{10 \times 70}{90^2 \times 91}\right)^{1/2} = 0.03$$

which set the expected prior probability of having a break in the dynamics of idiosyncratic risks to be equal to around 12%. These small prior probabilities makes the modelling dynamics more parsimonious, mitigating the magnitude of prior information, letting the data speak about the likelihood of random breaks. The prior beliefs on the size of the breaks are inverse-gamma distributed. The scale and degrees of freedom parameters are calibrated to match the OLS asymptotic standard errors of the betas obtained from the training sample.

In order to mitigate the impact of the calibration of hyper-parameters, an initial five-year worth of observations is used to empirically calibrate the priors and the analysis is implemented over the remaining 180 observations, per each series, over the interval 1999:01-2013:12. A more extensive discussion on the sensitivity of posterior estimates to prior hyper-parameters calibration is provided in a separate online appendix.

C Predictable Variance Decomposition

We use the posterior densities of the time series of factor loadings and risk premia to perform a number of tests that allow us to assess whether a posited asset pricing framework may explain an adequate percentage of excess asset returns. Equation (2) decomposes excess asset returns in a component related to risk, represented by $\beta_{i,t}' X_t$ plus a residual $\beta_{i0,t} + e_{i,t}$, with $e_{i,t} = \sigma_{i,t}\epsilon_{i,t}$. In principle, a multi-factor model is as good as the implied percentage of total variation in excess returns explained by its first component. We therefore follow earlier literature, such as Ferson and Harvey (1991), Ferson and Korajczyk (1995) and Karolyi and Sanders (1998), and adopt the following approach. First, the excess return on each asset is regressed onto a set of M instrumental variables that proxy for available information

[13] We extensively discuss a prior sensitivity analysis and MCMC convergence tests in a seprate online appendix.

at time $t-1$, $\mathbf{Z}_{t-1}$

$$r_{i,t} = \varphi_{i0} + \varphi_i' \mathbf{Z}_{t-1} + \xi_{i,t}, \tag{A.8}$$

to compute the sample variance of the resulting fitted values,

$$Var[P(r_{it}|\mathbf{Z}_{t-1})] \equiv Var\left[\hat{\varphi}_{i0} + \hat{\varphi}_i' \mathbf{Z}_{t-1}\right], \tag{A.9}$$

where the notation $P(r_{it}|\mathbf{Z}_{t-1})$ means "linear projection" of r_{it} on a set of instruments, $\mathbf{Z}_{t-1}$. Second, for each asset $i = 1, ..., N$, the risk exposures $\beta_{i,t}$ are sampled from their (marginal) posterior distribution $p\left(\boldsymbol{\beta}|\boldsymbol{R}, \boldsymbol{X}\right)$. Then we compute a time series of fitted risk compensations, $\beta_{i,t}^{g,\prime} X_t$ for each draw $g = 1, ..., G$, and regressed onto the instrumental variables,

$$\beta_{i,t}^{g,\prime} X_t = \varphi_{i0}^{g,\star} + \varphi_i^{g,\star\prime} \mathbf{Z}_{t-1} + \xi_{i,t}^{g,\star} \qquad g = 1, ..., G \tag{A.10}$$

to compute the sample variance of fitted posterior risk compensations:

$$Var\left[P\left(\beta_{i,t}^{g,\prime} X_t|\mathbf{Z}_{t-1}\right)\Big|\boldsymbol{R}, \boldsymbol{X}\right] \equiv Var\left[\hat{\varphi}_{i0}^{g,\star} + \hat{\varphi}_i^{g,\star\prime} \mathbf{Z}_{t-1}\right]. \tag{A.11}$$

At this point, the predictable variance in the risk premia that is attributed to the model, relative to the total predictable variance in the excess returns, can be computed for each marginal draw as

$$\mathcal{VR}1^g \equiv \frac{Var\left[P\left(\beta_{i,t}^{g,\prime} X_t|\mathbf{Z}_{t-1}\right)\Big|\boldsymbol{R}, \boldsymbol{X}\right]}{Var[P(r_{it}|\mathbf{Z}_{t-1})]} \geq 0 \qquad g = 1, ..., G \tag{A.12}$$

$\mathcal{VR}1$ should be equal to 1 if the multi-factor model is perfectly specified, which means that all the predictable variation in excess returns ought to be captured by variation in risk compensations (see Ferson and Harvey 1991).

Based on the weighted posterior distributions for each model the predictable variance in the risk premia that is attributed to the model can be decomposed with respect to the model space and using the law of iterated expectations (see for instance Hoeting et al. 1999 and Geweke and Amisano 2014);

$$\begin{aligned} Var\left[P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)\right] \equiv \sum_{k=1}^{2^K} p\left(D_{i,k}\right) \Bigg\{ & Var\left(P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}\right) + \\ +E\left[P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}\right]^2 - E\left[P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)\right]^2 & \Bigg\} \end{aligned} \tag{A.13}$$

where $p\left(D_{i,k}\right)$ is the marginal posterior distribution of the k_{th} model $D_{i,k}$ for the ith portfolio/asset. Decomposing the within-model explained predictable variation yields

$$Var\left(P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}\right) = E\left[Var\left(P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}, \boldsymbol{\theta}\right)\right] + Var\left[E\left(P\left(\beta_{i,t}' X_t|\mathbf{Z}_{t-1}\right)|D_{i,k}, \boldsymbol{\theta}\right)\right] \tag{A.14}$$

where $\boldsymbol{\theta}$ the vector of parameters. Plugging (A.14) in (A.13) yields the variance decomposition formulation in the main text. The first component represents the variance in the projection that would exists if one would know the structural parameters, averaged across the marginal posterior distribution $p\left(\boldsymbol{\theta}|\boldsymbol{R}, \boldsymbol{X}\right)$. We label this as risk factors-specific explained variation, i.e. intrinsic variation. The second component component represents the contribution of parameter uncertainty, i.e. explicit variation. Finally the third component represents the contribution of model uncertainty to the total explained predictable variation.

Table 1. Descriptive Statistics

Test assets and risk factors. This table reports descriptive statistics for the portfolios and risk factors used in the empirical analysis. Data on sector tax-qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. Data on government bond returns are from Ibbotson while yields are from the FREDII at the Federal Reserve Bank of St. Louis and from CRSP. Data on high-yield investment grade bond returns are approximated from Moody's (10-to-20 year maturity) Baa average corporate bond yields. Finally, data on industry classified portfolios are taken from the Kenneth French's website. Returns in excess of the risk free rate are computed monthly and cover the sample period 1994:01 - 2013:12.

Portfolio/Risk Factor Name	Mean	Median	St.Dev.	Skewness	Kurtosis	Sharpe Ratio
	Real Estate Returns - Sub-Sectors					
NAREIT - Industrial	1.132	1.440	9.099	0.277	25.290	0.099
NAREIT - Office	1.083	1.585	6.359	-0.475	9.031	0.133
NAREIT - Shopping Centers	0.983	1.420	6.440	-0.597	14.574	0.116
NAREIT - Regional Malls	1.312	1.560	7.699	0.473	20.668	0.140
NAREIT - Free Standing	1.197	1.535	5.124	-0.263	4.807	0.187
NAREIT - Apartments	1.046	1.285	5.832	-0.714	7.010	0.139
NAREIT - Manufactured Homes	0.877	1.040	5.283	-0.474	5.696	0.121
NAREIT - Healthcare	1.167	1.260	6.055	-0.334	6.700	0.154
NAREIT - Lodging/Resorts	0.825	0.945	9.138	0.950	15.801	0.064
NAREIT - Self-Storage	1.404	1.755	5.752	-0.479	5.181	0.203
NAREIT - Mortgage TR	0.652	1.600	6.236	-1.140	5.505	0.067
	17 Industry Portfolios - Value-Weighted					
Food	0.938	1.065	3.868	-0.467	4.781	0.181
Mining and Minerals	1.025	1.270	8.394	-0.427	4.362	0.092
Oil and Petroleum Products	1.138	1.095	5.571	-0.067	3.783	0.165
Textiles, Apparel & Footware	1.000	1.525	6.194	-0.202	4.940	0.126
Consumer Durables	0.683	1.095	5.954	-0.246	6.720	0.080
Chemicals	1.014	1.415	6.031	-0.150	5.110	0.130
Drugs, Soap, Prfums, Tobacco	1.084	1.421	4.172	-0.399	3.053	0.200
Construction and Materials	0.984	1.290	6.026	-0.205	3.732	0.120
Steel Works	0.778	0.680	8.911	-0.301	4.695	0.062
Fabricated Products	0.958	1.590	5.681	-0.506	5.027	0.132
Machinery and Business Equip.	1.159	1.825	7.783	-0.525	4.273	0.123
Automobiles	0.783	1.200	6.963	-0.014	5.492	0.084
Transportation	0.993	1.730	5.123	-0.600	4.344	0.156
Utilities	0.804	1.290	4.184	-0.601	3.759	0.141
Retail Stores	0.931	1.190	4.698	-0.278	3.693	0.154
Banks, Insurance, and Other Financials	0.917	1.300	5.783	-0.698	5.272	0.125
Other	0.783	1.750	5.255	-0.543	3.711	0.106
	Bond Returns					
10-Years T-Notes	0.540	0.891	3.341	0.271	5.011	0.091
5-Years T-Notes	0.530	0.892	3.671	-0.131	3.222	0.082
3-Years T-Notes	0.520	0.754	3.693	-0.111	3.143	0.084
1-Years T-Notes	0.460	0.201	3.343	0.550	4.905	0.070
Baa Corporate Bonds (10-20)	0.700	0.812	2.513	-1.570	16.153	0.180
	Economic Risk Variables					
Excess Value-Weighted Mkt	0.618	1.335	4.521	-0.737	4.074	0.137
Dividend Yield	1.872	1.831	0.458	0.807	4.455	
Unexpected Inflation	-0.002	0.006	0.191	-1.353	13.091	
Unemployment Rate	6.008	5.600	1.710	0.955	2.724	
Real T-Bill Interest Rate	0.077	0.071	0.252	0.286	3.776	
Term Spread	1.433	1.520	1.091	0.061	1.723	
Money Growth	0.964	0.480	3.006	5.493	47.091	
Credit Risk Premium	0.977	0.870	0.447	2.933	13.730	
Default Risk Premium	2.426	2.370	0.847	1.435	6.334	
Output Growth	2.271	3.191	4.439	-1.945	7.379	
Real Consumption Growth	2.864	3.011	1.759	-0.942	4.442	
Liquidity	0.601	0.272	3.959	0.596	5.879	0.152
Human Capital	0.297	0.294	0.343	-0.374	7.208	

Table 2. Marginal Likelihoods

Log of Marginal Likelihoods. This table reports the (log of) marginal likelihoods across assets. Top panel reports the log marginal likelihood concerning the REITs classified in underlying investable properties as explained in section 5 in the main text (from column 2 to 13), and the log marginal likelihood for government and corporate bonds (from column 14 to the end). Bottom panel reports the results for the industry classified portfolios. The log marginal likelihood are obtained from the MCMC estimation output by integrating out uncertainty on parameters, the "right" set of risk factors as well as uncertainty on time-varying risk exposures and idiosyncratic risks. The table reports the results for our Bayesian Model Averaging with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). We considered several restrictions to our general framework; Bayesian Model Averaging with Stochastic Breaks with constant idiosyncratic risks (BMA-SBB) and Bayesian Model Averaging with Random Walk Betas and Random Walk Volatility (BMA-RWB-RWV). Finally, we also consider full information about the "right" set of risk factors (SBB-SBV, SBB, and RWB-RWV). Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors for all the models.

	REIT											Bond Portfolios				
	Ind	Office	Shop	Malls	FreeSt	Apts	Homes	Health	Lodg	Storage	Mortg	10-yrs	5-yrs	3-yrs	1-yrs	High-Yield
BMA-SBB	-641.9	-626.0	-636.4	-628.4	-630.4	-636.4	-607.3	-655.1	-670.4	-640.0	-671.0	-591.73	-539.89	-553.33	-605.41	-361.48
BMA-RWB-RWV	-444.9	-440.9	-442.6	-449.6	-441.0	-442.3	-438.1	-443.1	-452.2	-445.3	-459.6	-434.14	-435.05	-435.76	-432.18	-429.53
BMA-SBB-SBV	-251.0	-238.5	-246.9	-272.2	-225.5	-246.1	-229.6	-256.1	-273.1	-268.3	-297.0	-201.30	-204.35	-205.86	-181.46	-167.01
SBB	-821.9	-827.8	-823.1	-820.0	-824.8	-814.9	-819.1	-810.2	-816.8	-807.6	-830.3	-820.99	-820.34	-832.04	-837.84	-839.23
RWB-RWV	-589.0	-600.8	-598.0	-589.9	-602.2	-596.4	-600.9	-595.1	-587.5	-595.4	-580.2	-603.43	-604.13	-601.86	-601.03	-603.63
SBB-SBV	-462.4	-470.3	-468.3	-463.1	-469.6	-468.3	-470.3	-465.2	-463.3	-466.9	-459.9	-470.90	-470.33	-469.16	-468.44	-471.99

	Industry Portfolios																
	Food	Mines	Oil	Clths	Durbl	Chems	Cnsum	Cnstr	Steel	FabPr	Machn	Cars	Trans	Utils	Rtail	Finan	Other
BMA-SBB	-536.2	-669.1	-618.4	-640.4	-453.6	-578.5	-541.0	-590.6	-640.9	-561.5	-624.3	-640.7	-432.4	-631.3	-510.3	-505.0	-403.7
BMA-RWB-RWV	-432.3	-458.4	-446.1	-440.7	-433.5	-435.6	-432.8	-437.4	-441.8	-435.6	-436.4	-439.1	-433.2	-437.5	-435.8	-432.6	-428.5
BMA-SBB-SBV	-178.3	-301.2	-243.8	-219.4	-204.8	-202.7	-206.7	-216.1	-233.8	-212.6	-215.0	-218.1	-195.3	-227.9	-191.5	-194.8	-175.2
SBB	-823.2	-813.4	-807.6	-807.7	-817.1	-806.2	-820.3	-805.6	-824.8	-819.6	-827.4	-820.5	-816.7	-819.3	-816.5	-821.6	-823.0
RWB-RWV	-604.6	-588.4	-601.6	-596.7	-601.2	-603.5	-603.8	-600.5	-595.7	-602.7	-603.9	-600.1	-603.5	-601.3	-604.2	-602.2	-607.7
SBB-SBV	-470.9	-461.8	-470.1	-468.6	-470.3	-470.4	-470.2	-470.2	-468.2	-471.1	-470.9	-469.5	-471.6	-470.6	-472.9	-471.4	-473.7

Table 3. Pricing Errors

Squared average pricing errors. This table reports the median squared average cross-sectional pricing error computed as in (19). Conditional intercepts and risk exposures are sampled from their marginal posterior distribution obtained integrating out both parameter and model uncertainty. Median values of the pricing error measure are taken at each time t from the output of the MCMC estimation scheme detailed in appendix A. The table reports the results for our Bayesian Model Averaging with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). We considered several restrictions to our general framework; Bayesian Model Averaging with Stochastic Breaks with constant idiosyncratic risks (BMA-SBB) and Bayesian Model Averaging with Random Walk Betas and Random Walk Volatility (BMA-RWB-RWV). Finally, we also consider full information about the "right" set of risk factors (SBB-SBV, SBB, and RWB-RWV). Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors for all the models.

Models	Sample 1999:01-2013:12				Sample 1999:01-2007:01				Sample 2007:01-2013:12			
	Mean	Std	5%	95%	Mean	Std	5%	95%	Mean	Std	5%	95%
BMA-SBB	1.101	0.303	0.844	1.651	0.894	0.036	0.831	0.945	1.351	0.289	0.965	1.932
BMA-SBB-SBV	0.351	0.062	0.291	0.492	0.314	0.019	0.286	0.356	0.401	0.063	0.305	0.517
BMA-RWB-RWV	0.841	0.098	0.701	0.992	0.816	0.071	0.697	0.942	0.875	0.111	0.735	1.123
SBB	1.241	0.309	0.876	1.799	1.201	0.271	0.895	1.691	1.281	0.334	0.871	1.871
SBB-SBV	0.734	0.153	0.601	1.109	0.638	0.025	0.591	0.688	0.856	0.156	0.701	1.245
RWB-RWV	0.978	0.273	0.687	1.471	0.955	0.251	0.651	1.281	0.997	0.298	0.686	1.561

Table 4. Explained Predictable Variation

Variance Ratios. This table reports the fraction of predictable variation explained by the model across real estate, stock and bond portfolios computed as explained in (15). The table reports the median values obtained from the marginal distribution of the risk exposures across assets. Draws from the marginal distribution are simulated from the MCMC estimation output by integrating out both parameter and model uncertainty (see apendix). We report the results for our Bayesian Model Averaging with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). We considered several restrictions to our general framework; Bayesian Model Averaging with Stochastic Breaks with constant idiosyncratic risks (BMA-SBB) and Bayesian Model Averaging with Random Walk Betas and Random Walk Volatility (BMA-RWB-RWV). Finally, we also consider full information about the "right" set of risk factors (SBB-SBV, SBB, and RWB-RWV). Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors for all the models.

	REIT												Bond Portfolios				
	Ind	Office	Shop	Malls	FreeSt	Apts	Homes	Health	Lodg	Storage	Mortg		10-yrs	5-yrs	3-yrs	1-yrs	High-Yield
BMA-SBB	0.241	0.602	0.245	0.252	0.291	0.521	0.491	0.501	0.590	0.360	0.241		0.176	0.411	0.137	0.117	0.118
BMA-SBB-SBV	0.522	0.791	0.524	0.520	0.514	0.825	0.860	0.804	0.610	0.565	0.400		0.300	0.350	0.450	0.150	0.250
BMA-RWB-RWV	0.361	0.703	0.442	0.415	0.418	0.748	0.652	0.717	0.605	0.474	0.295		0.491	0.260	0.350	0.205	0.422
SBB	0.201	0.104	0.141	0.121	0.125	0.267	0.333	0.421	0.360	0.281	0.160		0.101	0.129	0.150	0.089	0.304
SBB-SBV	0.272	0.377	0.310	0.280	0.377	0.535	0.562	0.691	0.505	0.365	0.314		0.287	0.269	0.157	0.051	0.198
RWB-RWV	0.263	0.495	0.221	0.153	0.157	0.591	0.510	0.520	0.407	0.271	0.124		0.150	0.250	0.350	0.150	0.350

	Industry Portfolios																
	Food	Mines	Oil	Clths	Durbl	Chems	Cnsum	Cnstr	Steel	FabPr	Machn	Cars	Trans	Utils	Rtail	Finan	Other
BMA-SBB	0.593	0.864	0.816	0.639	0.741	0.828	0.332	0.841	0.764	0.863	0.863	0.729	0.753	0.417	0.524	0.706	0.921
BMA-SBB-SBV	0.620	0.876	0.805	0.641	0.796	0.964	0.674	0.793	0.759	0.903	0.848	0.684	0.940	0.730	0.600	0.723	0.980
BMA-RWB-RWV	0.510	0.890	0.724	0.638	0.782	0.846	0.578	0.837	0.710	0.871	0.857	0.711	0.830	0.480	0.480	0.700	0.920
SBB	0.486	0.257	0.303	0.169	0.141	0.067	0.537	0.088	0.032	0.380	0.067	0.173	0.378	0.436	0.164	0.100	0.381
SBB-SBV	0.708	0.579	0.437	0.422	0.558	0.573	0.671	0.683	0.407	0.686	0.556	0.457	0.559	0.659	0.321	0.391	0.413
RWB-RWV	0.516	0.459	0.314	0.411	0.431	0.488	0.387	0.638	0.321	0.460	0.433	0.412	0.410	0.131	0.287	0.443	0.686

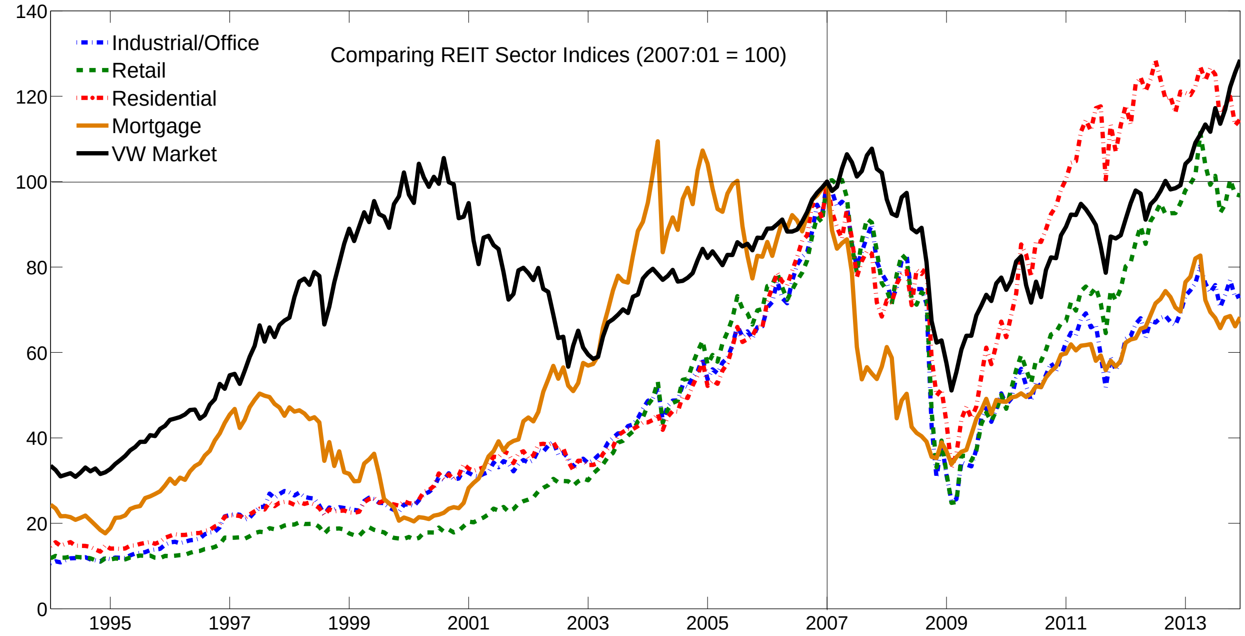
Table 5. Predictable Variance Decomposition

Variance decomposition. This table shows the marginal contribution of each source of variation to the amount of explained predictable variation captured by our Bayesian model averaging model specifications. *ModelU* stands for model uncertainty and represents the contribution of model uncertainty to the total explained predictable variation. *Extrinsic* measures the contribution gained capturing parameter uncertainty, while *Intrinsic* measures the fraction of predictable variation can be attributed to the risk factors as it exists if one would know the structural parameters. We report the results for our Bayesian Model Averaging with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). We considered several restrictions to our general framework; Bayesian Model Averaging with Stochastic Breaks with constant idiosyncratic risks (BMA-SBB) and Bayesian Model Averaging with Random Walk Betas and Random Walk Volatility (BMA-RWB-RWV). Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors for all the models.

		BMA-SBB			BMA-RWB-RWV			BMA-SBB-SBV		
		ModelU	Extrinsic	Intrinsic	ModelU	Extrinsic	Intrinsic	ModelU	Extrinsic	Intrinsic
	Ind	0.149	0.087	0.764	0.117	0.034	0.849	0.086	0.023	0.892
	Office	0.099	0.016	0.885	0.123	0.037	0.839	0.131	0.007	0.862
	Shop	0.095	0.060	0.845	0.142	0.023	0.835	0.135	0.016	0.849
	Malls	0.112	0.033	0.855	0.099	0.032	0.869	0.079	0.022	0.899
	FreeSt	0.069	0.033	0.899	0.286	0.045	0.669	0.173	0.020	0.807
REIT	Apts	0.087	0.013	0.899	0.173	0.021	0.806	0.169	0.014	0.817
	Homes	0.070	0.059	0.871	0.207	0.188	0.605	0.146	0.027	0.828
	Health	0.075	0.081	0.844	0.216	0.089	0.695	0.245	0.019	0.735
	Lodg	0.152	0.008	0.841	0.098	0.005	0.897	0.090	0.006	0.904
	Storage	0.061	0.054	0.886	0.236	0.081	0.683	0.188	0.025	0.787
	Mortg	0.066	0.027	0.908	0.328	0.054	0.618	0.228	0.041	0.731
	Food	0.039	0.385	0.576	0.165	0.148	0.687	0.328	0.014	0.658
	Mines	0.093	0.023	0.883	0.174	0.098	0.727	0.161	0.007	0.832
	Oil	0.065	0.008	0.927	0.275	0.096	0.629	0.217	0.010	0.773
	Clths	0.114	0.011	0.874	0.137	0.046	0.817	0.147	0.023	0.831
	Durbl	0.113	0.003	0.885	0.143	0.102	0.755	0.178	0.010	0.812
	Chems	0.105	0.003	0.892	0.192	0.102	0.706	0.148	0.008	0.844
	Cnsum	0.095	0.665	0.240	0.371	0.247	0.382	0.217	0.041	0.741
Industry	Cnstr	0.113	0.010	0.877	0.190	0.188	0.622	0.159	0.018	0.823
	Steel	0.188	0.006	0.807	0.081	0.025	0.894	0.088	0.001	0.911
	FabPr	0.107	0.004	0.890	0.149	0.079	0.772	0.122	0.013	0.865
	Machn	0.183	0.009	0.807	0.056	0.013	0.931	0.070	0.002	0.929
	Cars	0.124	0.020	0.857	0.133	0.056	0.810	0.119	0.012	0.869
	Trans	0.090	0.005	0.905	0.206	0.038	0.757	0.187	0.007	0.807
	Utils	0.049	0.092	0.859	0.138	0.131	0.731	0.149	0.013	0.838
	Rtail	0.088	0.009	0.903	0.003	0.102	0.896	0.169	0.014	0.817
	Finan	0.107	0.004	0.889	0.039	0.039	0.922	0.140	0.007	0.853
	Other	0.125	0.002	0.873	0.181	0.030	0.788	0.097	0.002	0.901
	10-yrs	0.021	0.132	0.847	0.448	0.429	0.123	0.177	0.160	0.663
	5-yrs	0.045	0.206	0.750	0.123	0.091	0.485	0.051	0.055	0.893
Bond	3-yrs	0.055	0.309	0.636	0.074	0.056	0.870	0.102	0.112	0.786
	1-yrs	0.036	0.159	0.804	0.100	0.118	0.782	0.161	0.016	0.824
	High-Yield	0.042	0.378	0.581	0.308	0.297	0.395	0.192	0.090	0.718

Figure 1. Comparing the Dynamics of Sector REIT Indexes Over Time

REIT Indexes. This figure plots the REIT total return indexes for different underlying investment categories. Industrial and Office REITs are aggregated in a "Industrial/Office" (I&O) sector, Shopping Centers, Regional Malls, Free Standing shops REITs into a "Retail" sector, and Apartments, Manufactured Homes into a "Residential" one. As a benchmark, we also plot the total return index for the value-weighted market portfolio (black solid line). To favor comparability across different sectors, all indexes are standardized to equal 100 in correspondence with the end of January 2007. Data are monthly and cover the period 1994:01-2013:12.



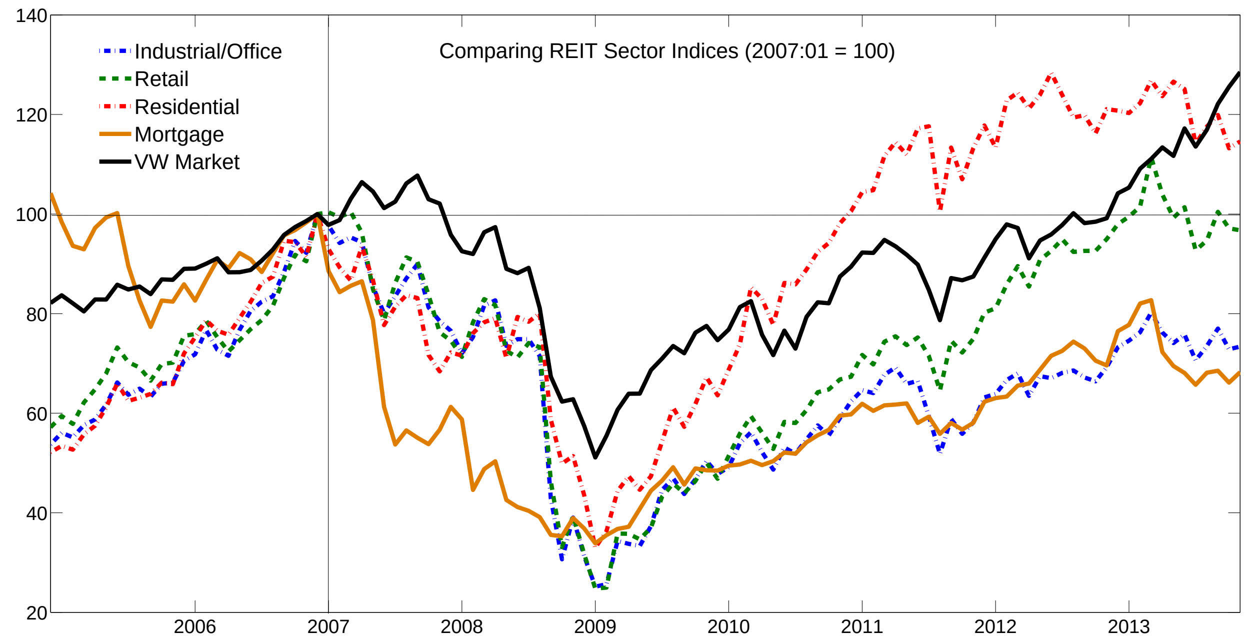


Figure 2. Mispricing for REITs

Time series pricing errors. This figure plots the conditional intercepts, i.e. Jensen's alphas, across different REIT investment categories. The data on sector tax qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. For the ease of exposition the figure does not report Healthcare and Self-Storage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. Intercepts are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

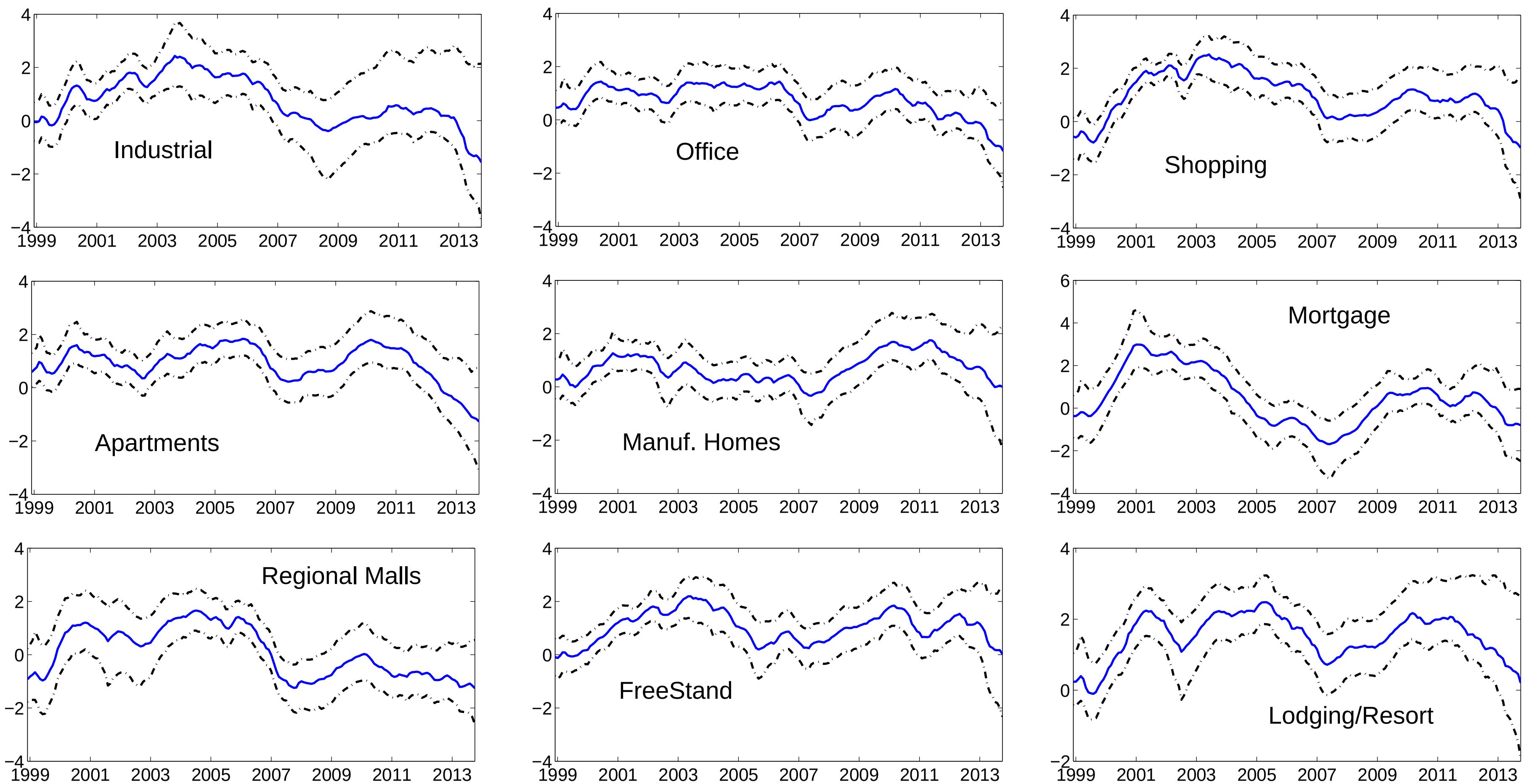


Figure 3. Mispricing for Industry Portfolios

Time series pricing errors for Industry Portfolios. This figure plots the conditional intercepts, i.e. Jensen's alphas, across different Industry portfolios. For the ease of exposition the figure reports a subset of the 17 portfolios considered. Intercepts are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

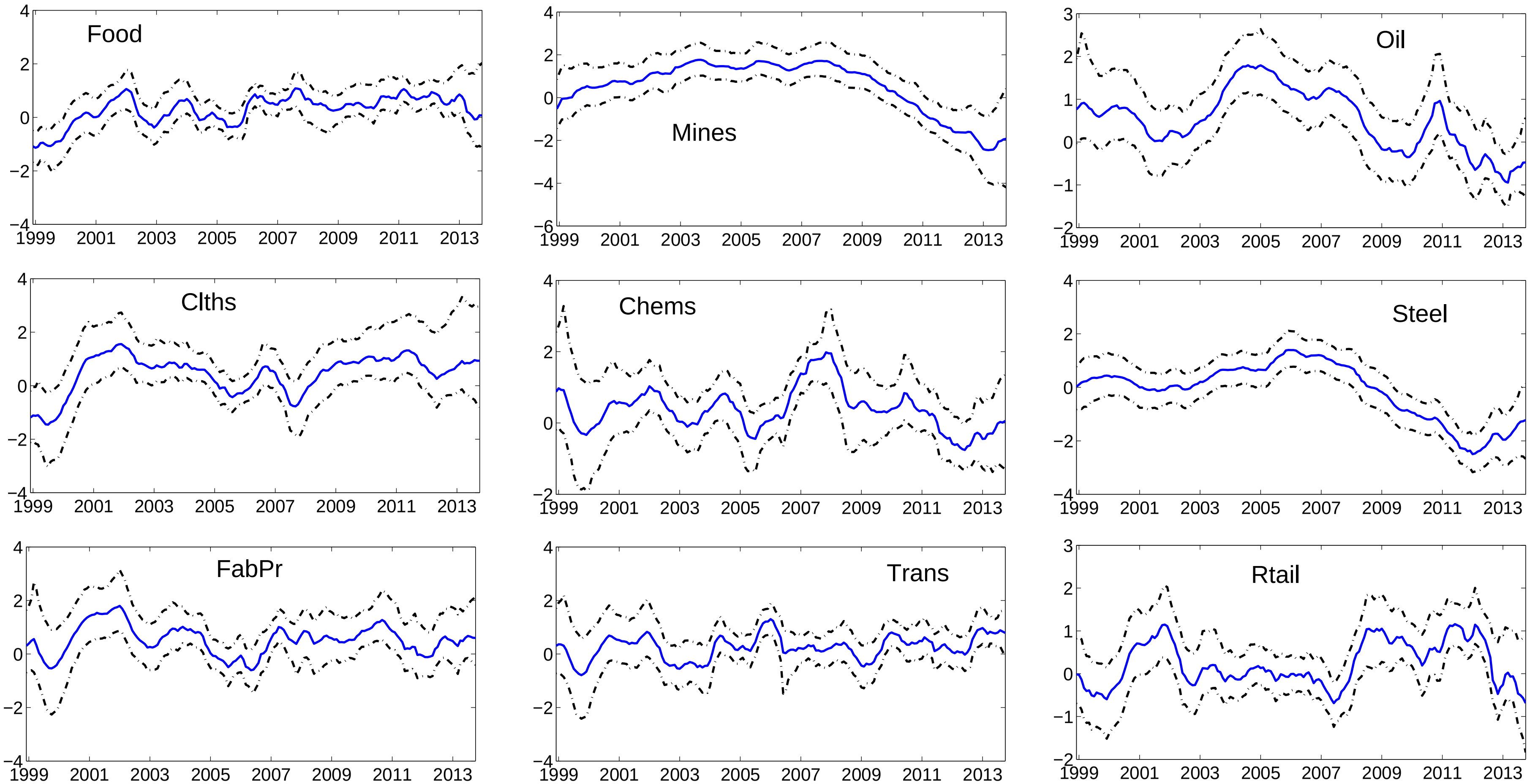


Figure 4. Posterior Probability of Breaks in Factor Loadings

Break Probabilities. This figure reports the median estimates of the marginal posterior probability across different REIT investment categories. Break probabilities are sampled from their marginal distributions fully acknowledging uncertainty on the "right" set of risk factors. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

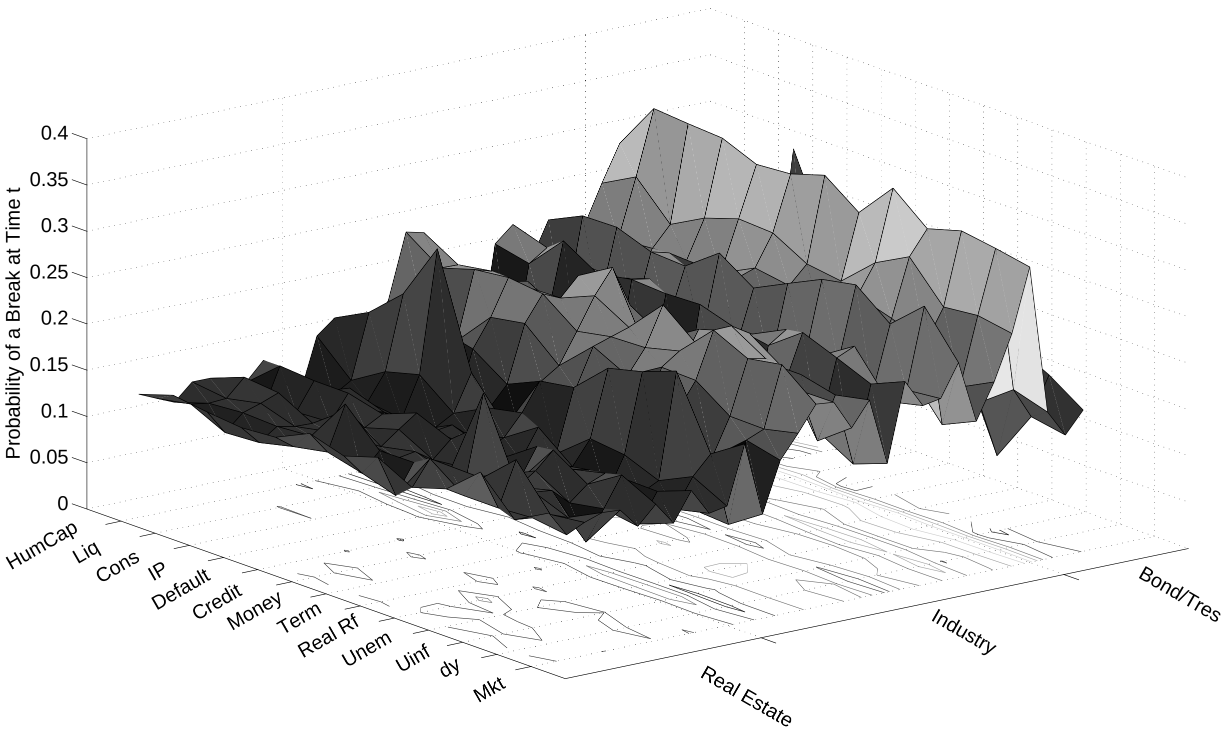


Figure 5. Relevance of Risk Factors in the Pricing Equations

Risk Factors Inclusion Probabilities. This figure reports the median estimates of the marginal posterior probability that a certain risk factor effectively enters in the I-CAPM implementation (2), computed across different REIT investment categories. Inclusion probabilities are sampled from their marginal distributions fully acknowledging uncertainty on both latent betas, idiosyncratic risks and their structural parameters. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). Panel A shows the results for the REITs and panel B shows the results for Industry returns. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

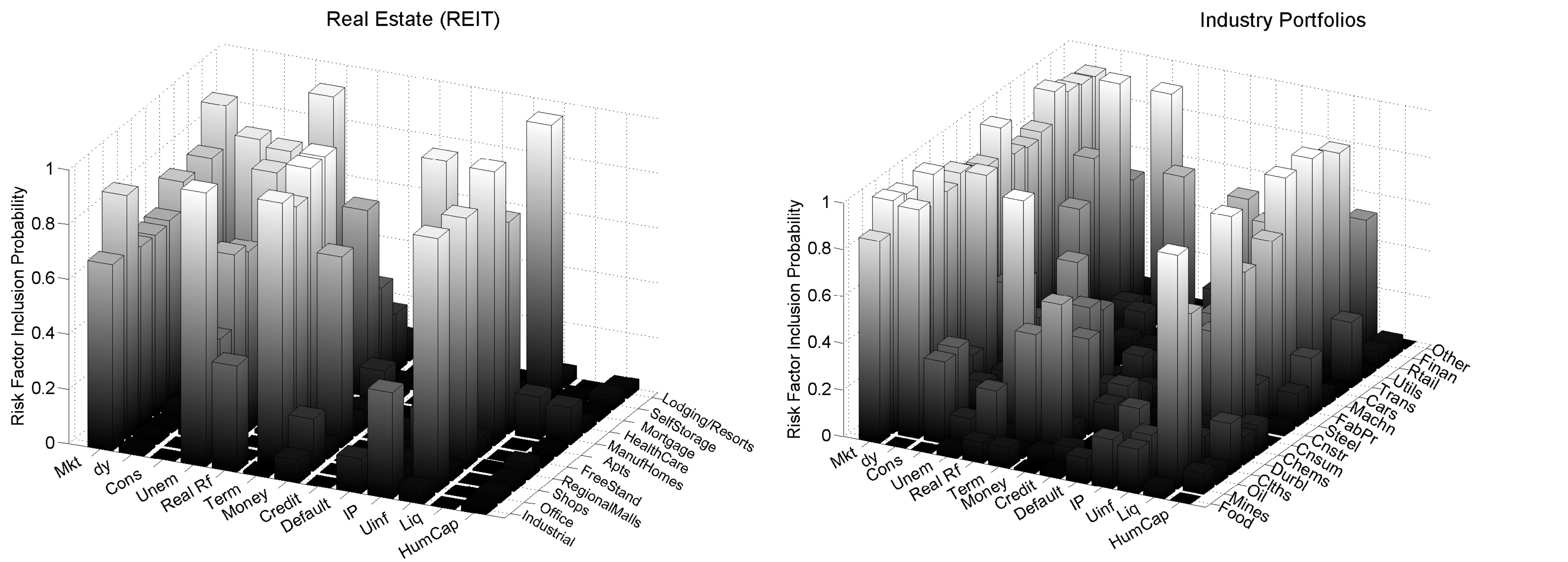


Figure 6. Exposures to Market Risk for REITs

Exposures to market risk. This figure plots the conditional exposures to market risk, i.e. CAPM, across different REIT investment categories. Market risk is measured as the returns on the aggregate market portfolio in excess of the risk free rate. The data on sector tax qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. For the ease of exposition the figure reports Healthcare and Self-Storage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. Betas are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

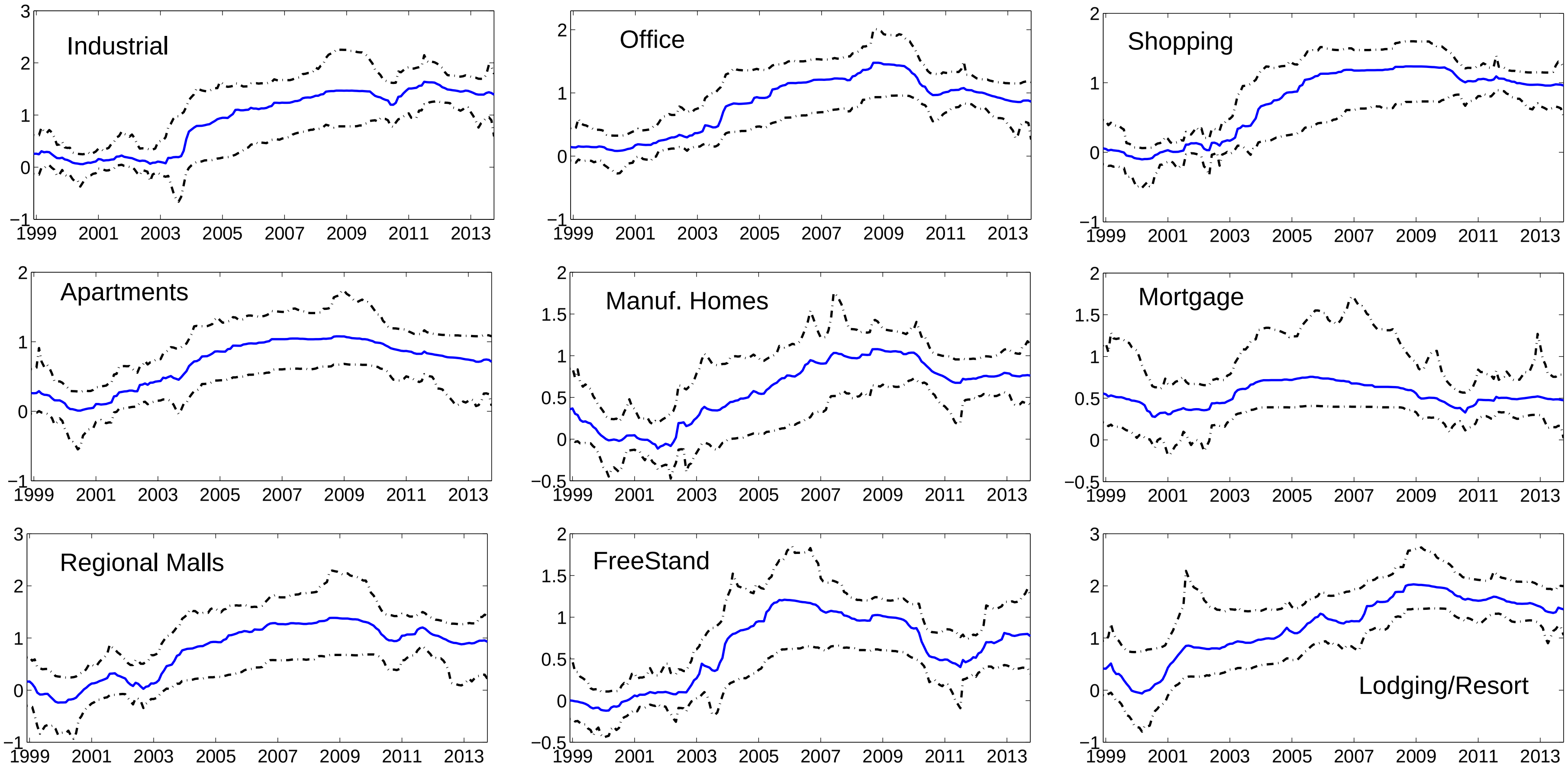


Figure 7. Exposures to Term Spread for REITs

Exposures to term spread. This figure plots the conditional exposures to shocks on term spread across different REIT investment categories. Such spread is measured as the yields difference between 10-year and a 1-year government bonds. Shocks are computed as the residuals from a recursive VAR(1). The data on sector tax qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. For the ease of exposition the figure reports Healthcare and Self-Storage REITs. Apartments and Manufactured Homes represent the “Residential” real estate sector. Betas are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

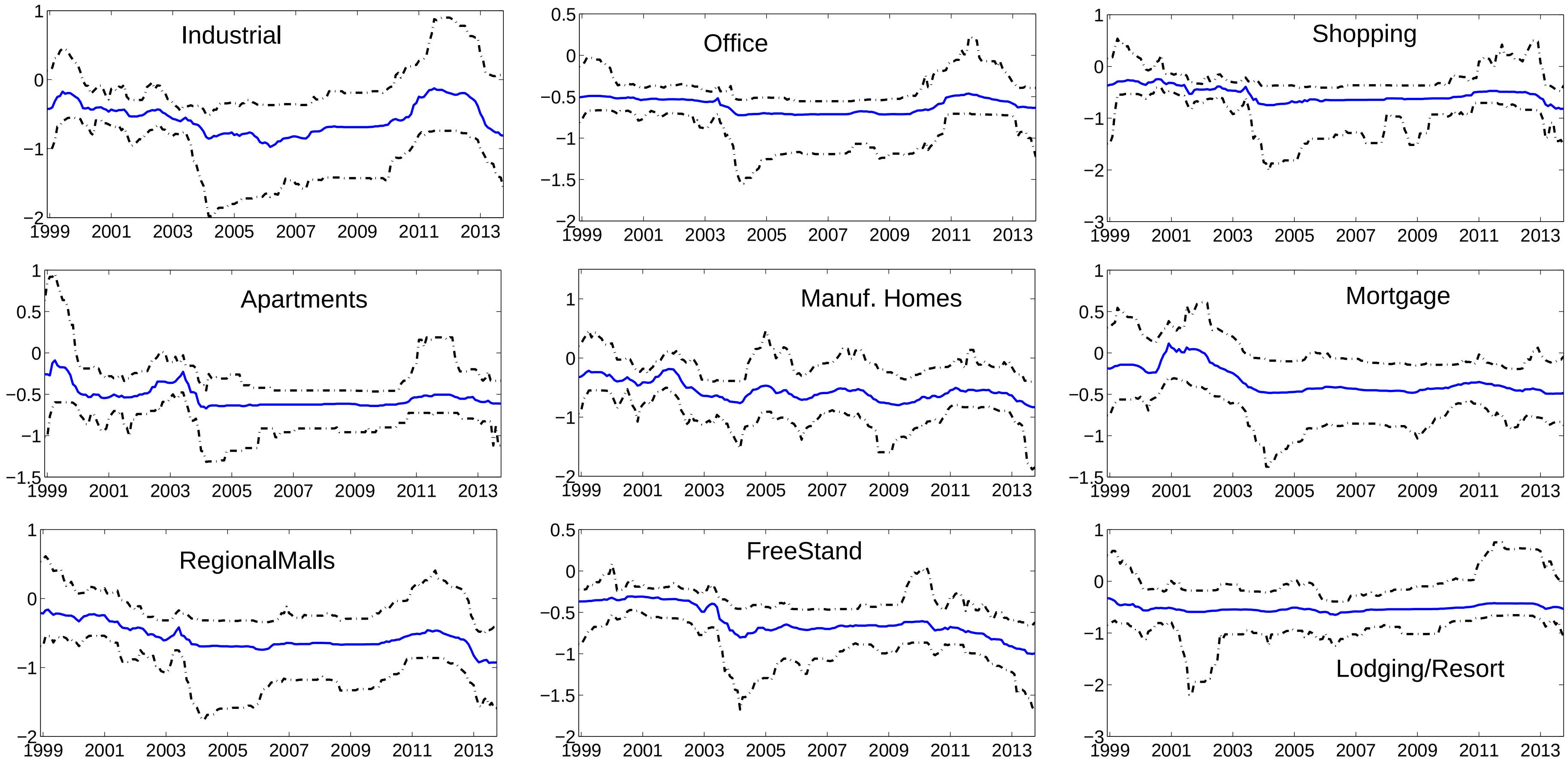


Figure 8. Exposures to Inflation Risk for REITs

Exposures to unexpected inflation. This figure plots the conditional exposures to shocks on unexpected inflation across different REIT investment categories. Unexpected inflation is measured as the residuals from an ARIMA(1,1,1) model fitted to the (seasonally adjusted) CPI index. Shocks to unexpected inflation are computed as the residuals from a recursive VAR(1). The data on sector tax qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. For the ease of exposition the figure reports Healthcare and Self-Storage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. Betas are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

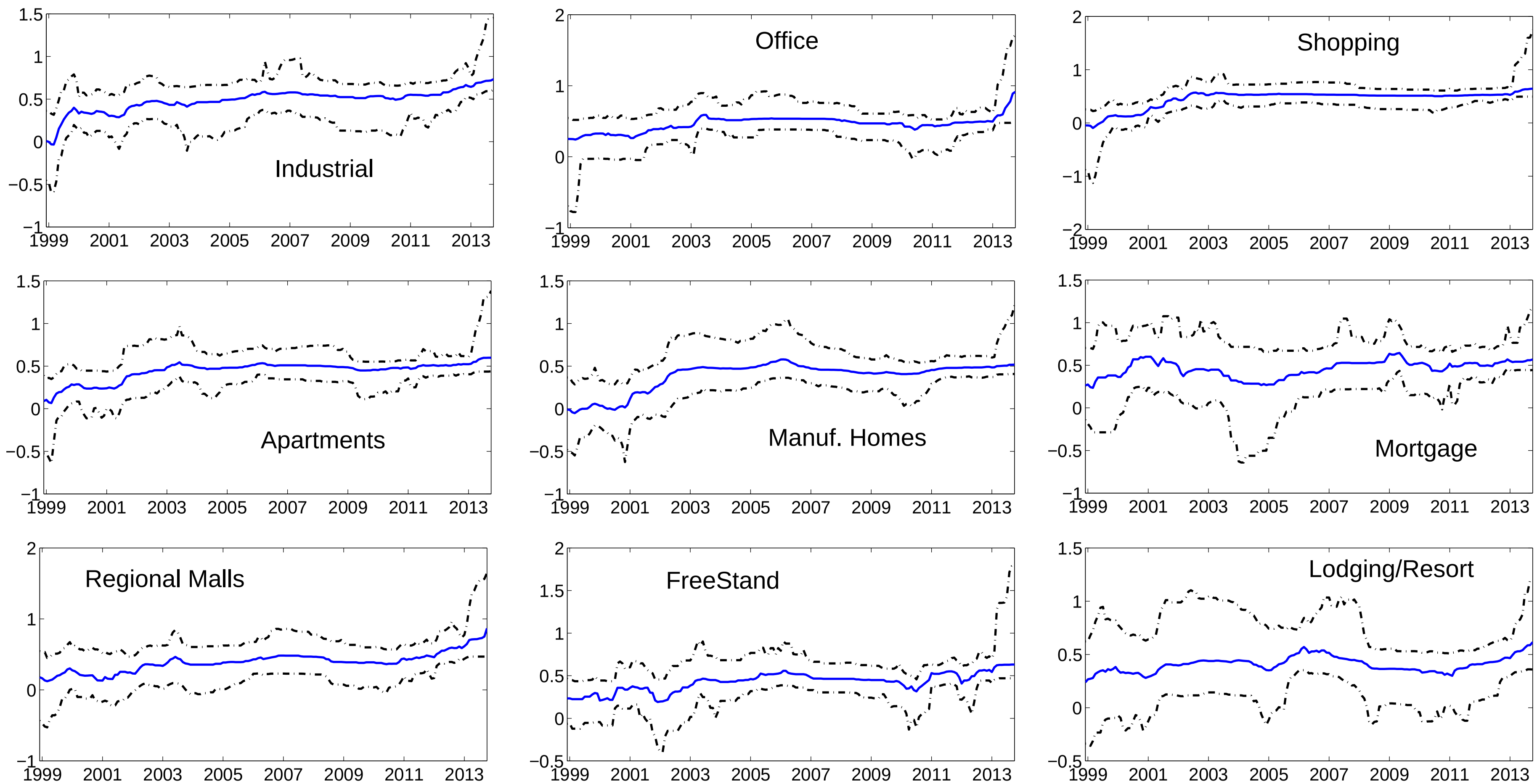


Figure 9. Exposures to Unemployment Risk for REITs

Exposures to unemployment. This figure plots the conditional exposures to shocks on unemployment across different REIT investment categories. Shocks to unemployment are computed as the residuals from a recursive VAR(1). The data on sector tax qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. For the ease of exposition the figure reports Healthcare and Self-Storage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. Betas are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.

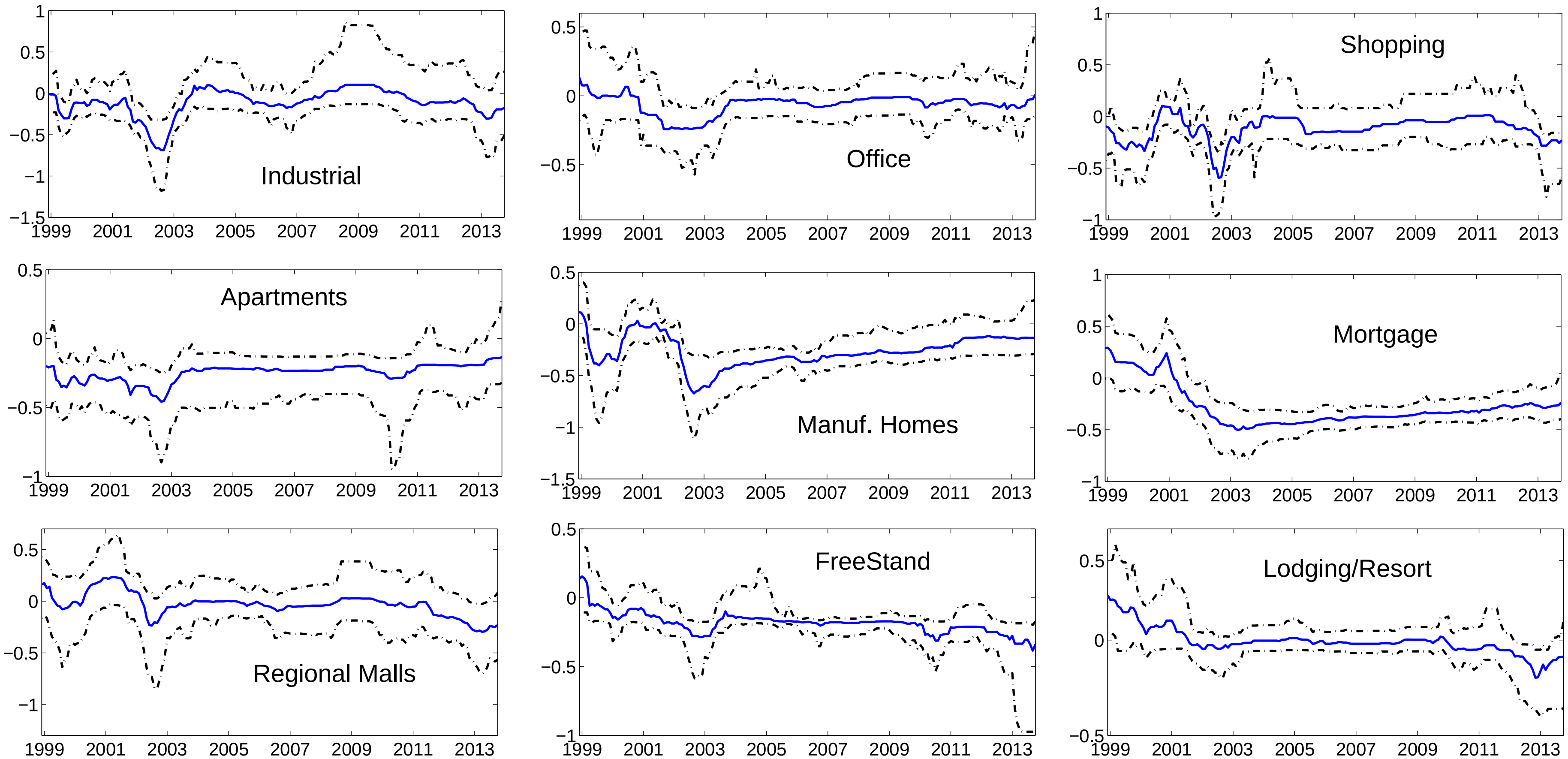


Figure 10. Idiosyncratic Risk

Conditional Idiosyncratic Volatility. This figure plots the conditional idiosyncratic risk across different REIT investment categories. Data on sector tax qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association. For the ease of exposition the figure reports Healthcare and Self-Storage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. Idiosyncratic risks are sampled from their marginal distributions once parameter and model uncertainty have been integrated out. Marginal distributions are approximated through the MCMC estimation scheme detailed in the appendix. The figure shows the result for our general Bayesian Model Averaging specification with Stochastic Breaks in Betas and Stochastic Breaks in Volatility (BMA-SBB-SBV). The solid blue line represents the median value of the alpha at time t and the dot dashed black lines are the 95% confidence intervals. Data are monthly and cover the sample period 1994:01 - 2013:12. The first five years of monthly data are used to calibrate the priors.