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WORKING PAPER SERIES

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Working Paper n. 97

First Published: March 1996
Revised: January 1997

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http://www.igier.uni-bocconi.it

Does the PPP need the UIP ?

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IGIER WP No. 97 (second revised version)

First version: March 1996
This version: January 1997

Abstract

In this paper we focus on the post Bretton Woods period and analyze whether a PPP relationship holds and what is the speed of adjustment to it. We adopt a multivariate system approach in which, initially, we test for cointegration and then we try to identify a cointegration space in which we have the PPP relationship (the "Johansen approach"). The studies that have adopted this approach have always rejected the PPP in favour of a long run relationship between the real exchange rate and the interest rate differential. On the contrary, our conclusions are in favour of the PPP for all the cases considered when we allow for a structural break in the data. We arrive to this conclusion, after having identified the cointegration space in two different ways: one in which we have the PPP as a cointegrated vector and one in which the real exchange rate plus the interest rate differential is a cointegrated vector. Adopting a dominance criterion we choose the former identification. We also address the Rogoff's (1996) puzzle on the excess volatility of real exchange rate and the slow convergence to PPP. On the basis of persistence profiles obtained from a constrained VECM, we don't find any evidence in favour of the puzzle since we estimate a relatively fast speed of adjustment.

Keywords: purchasing power parity, uncovered interest parity, cointegration, persistence profiles.
JEL classification: C32, F31, F41.

* We wish to thank Gianni Amisano and Carlo Favero for helpful comments. The usual disclaimer applies.

1. Introduction

The relative version of purchasing power parity is represented as:

$$e = k + p - p^* \qquad [1a]$$

where e is the logarithm of the exchange rate measured in units of currency A per unit of currency B, p is the logarithm of the price level in country A, p* is the logarithm of the price level in country B and k is a constant term.

This relationship has been empirically analized either imposing homogeneity restrictions on all the coefficients (unit root tests of the real exchange rate), on some of the coefficients (cointegration test between the two prices in common currencies) or without imposing any restriction on the coefficients. The conclusions obtained by the previous literature varies mainly as a function of the period analyzed and of the empirical methodology adopted (for a survey, Froot and Rogoff (1995)).

In this paper we focus on the post Bretton Woods period and analyze whether a relationship like (1a) is accepted by the data for Italy, Switzerland, United States, Germany, United Kingdom, Japan. We adopt a multivariate system approach in which, initially, we test for cointegration and then we try to identify a cointegration space in which we have the PPP relationship (the "Johansen approach"). The studies that have adopted this approach have always rejected the PPP in favour of a long run relationship between the real exchange rate and the interest rate differential (for example, Juselius (1995), Sjoo (1995)).

On the contrary, our conclusions are in favour of the PPP for all the cases considered when we allow for a structural break in the data. We arrive to this conclusion, after having identified the cointegration space in two different ways: one in which we have the PPP as a cointegrated vector and one in which the real exchange rate plus the interest rate differential is a cointegrated vector. Adopting a dominance criterion we choose the former identification.

Finally, we address the Rogoff's (1996) puzzle on the excess volatility of real exchange rate and the slow convergence to PPP. For the currencies considered, the analysis of persistence profiles performed in a constrained VECM framework does not reveal any evidence of the puzzle. In fact, we estimate a relatively fast speed of adjustment.

2. The data and their univariate properties

The variables we use to test the PPP and its relationship with interest rates are quarterly sampled for the period 74:1, 92:4. For all countries we use consumer price indexes (P_i) and three-month treasury bills interest rates (I_i)[1]; the exchange rates are spot bilateral rates (E_{ij}) with two pivotal currencies: the Italian Lira and the Swiss Franc[2]. Prices and exchange rates are in logarithms, while the interest rates have been trasformed as: $i_j = \log(1+I_j)$.

A preliminary univariate analysis of the series, performed by the Augmented Dickey-Fuller (ADF) test (table 1), emphasizes, as expected, the presence of one unit root in all variables. The null of a second unit root cannot be rejected at a significance level of 5% for all price series with the exception of the Switzerland case (ADF test on first differences) . However, the presence of a second root is rejected if we adopt the SM (Schmidt and Phillips) test or the PP (Phillips and Perron) test. This ambiguity is common in the literature on unit root tests on price series[3]. We adopt the following solution to this ambiguity: the graphical inspection of price series (Graphs 1 to 5) reveals the existence of a drift and of a break in 1982, determining a reduction in the slope of the drift, as a consequence of the beginning of a period of lower inflation in the industrialized economies, essentially due to the stabilization after the two oil price crisis.

This break might cause a misleading evidence of I(2)-ness in a I(1) series (Perron (1990)). As a consequence we adopt the Perron approach to perform a unit root test

[1] For Japan we use a 3-month interest rate which has been constructed with the rate on the gensaki market up to 1979Q2 and with the CD interest rate after that date. In this way we obtain a regulation-free interest rate variable. We have also utilized the interest rate on 60-day treasury bill, which shows the effects of high regulation during the first part of the period. The two variables don't produce substantially different results.

[2] All data have been obtained from Datastream

[3] For example, for the Italian case, Hamilton (1994) considers quarterly data for the period 73-89 as having only one unit root. On the other hand, Paruolo (1993) cannot reject the presence of two unit roots for the same variable on the period 70-91.

controlling for an exogeneously imposed structural break. The test is performed on the first difference of each price series adopting the additive outlier specification with a dummy variable of the tipe $D_t = \begin{cases} 0 & \text{if } t < 82{:}3 \\ 1 & \text{if } t \geq 82{:}3 \end{cases}$. The results (table 1, AO-ADF) reject the I(2) hypothesis for all price series. As a consequence, in the following analysis we will introduce the deterministic variable D_t in the specification of the system.

3. Does the PPP hold in isolation ?

To analyze the existence of a long-run relationship among the variables in equation (1a), we adopt the Full Information Maximum Likelihood (FIML) cointegration approach developed by Johansen (1995). This approach is more efficient than the various single equation methods unless there is a unique cointegration vector and the variables appearing on the right hand side are weakly exogenous[4]. In the light of the considerations of the previous section, we have constructed and estimated, for each pair of countries, a VAR(p) (in a reparametrized ECM form), including the two prices, the exchange rate, an unrestricted constant δ, that describes the presence of a drift in the series in levels, and a dummy variable D_t, that describes the 82.3 break in the price series:

$$\Delta Y_t = \delta + \psi K_t + \gamma D_t + \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \ldots\ldots + \Gamma_{p-1} \Delta Y_{t-p-1} + \Pi Y_{t-1} + \varepsilon_t \qquad [2]$$

where the terms ΔY_t , δ and γ are 3x1 vectors, Γ_j are 3x3 matrices, ψ is a 3x(s-1) matrix, K_t is an (s-1)x1 vector of centred seasonal dummies, where *s* (= 4) is the number of periods in a year, D_t is a scalar, $\varepsilon_t \sim$i.i.d. $N(0,\Omega)$ and, under the cointegration hypothesis, the 3x3 matrix Π can be factorized as $\Pi=\alpha\beta'$ where α and β are 3xr matrices of rank r.

[4] Results supporting this approach in terms of asymptotic and finite sample properties can be found in Gonzalo (1994). For partially different results see Haug (1996).

The matrix β contains the r cointegrating vectors, while the matrix α contains the so-called loading factors that measure the speed of adjustment of each equation of the system to the different long-run relationships.

Since the critical values for the cointegration rank tests depend on the specification of the VAR and the tabulated values in Osterwald-Lenum (1992) are produced for a model including only a constant or a drift and not an intervention dummy among the deterministic term , we obtained the correct critical values by simulation[5].

Then, we performed the standard Johansen's analysis whose results are presented in table 2[6]. The cointegrating rank trace tests (conducted along the method of Pantula (1989) and Johansen (1995), starting from the hypothesis of rank=0 and increasing the assumed rank step by step), reveal the existence of one cointegrating relationship among prices and exchange rates in the Italy/Usa, Italy/Germany, Switzerland/Germany cases; two cointegrating relationships are found in the Italy/Uk, Italy/Switzerland, Italy/Japan[7], Switzerland/Uk and Switzerland/Japan cases, while in the Germany/Usa, Switzerland/Usa cases the hypothesis of cointegrating rank equal to zero, against the alternative of 3, cannot be rejected (i.e. there is no cointegration).

However, existence of cointegration is not a sufficient condition for PPP. From equation [1a] it is evident that also proportionality and simmetry restrictions should be satisfied[8]. In our context, this implies the imposition of specific restrictions on the cointegrating space. In fact, the Johansen estimation procedure produces only an exact identification of the cointegrating space by means of a simple algebraic procedure of normalization; the resulting β-vectors spanning the cointegrating space usually don't have an economic interpretation. Johansen (1995) has provided some useful criteria to test for

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5 We have utilized DisCo by Johansen and Nielsen (1993). The simulation was performed with 10.000 iterations and the number of the discretizations of the Brownian Motions, representing the asymptotic non standard theoretical distribution of the test, has been set at 600; the obtained critical values are reported in table 2 .

6 The analysis has been performed in RATS with MALCOLM (Mosconi, 1996) and with PCFIML (Doornik and Hendry, 1994).

7 For this case the inference is not clear cut. According to the trace test there are three vectors, implying the stationarity of all the variables: this is in contrast with the univariate unit root tests. To solve this puzzle we resort to the analysis of the roots of the companion form estimated under the hypothesis of rank three: this reveals that one root is very near to one (0,987). We use this evidence to conclude in favour of the rank two hypothesis.

8 The failure of these restrictions might be theoretically rationalized with measurement errors in prices. Empirically, however, the estimated coefficients found in the literature vary a lot, and are very difficult to

the over-identifying restrictions suggested by economic theory; his approach is based, on one side on the satisfaction of necessary and sufficient rank conditions (formal or generical identification) and, on the other side, on likelihood ratio test checking the plausibility of the restrictions (empirical identification).

The results of the identification analysis (table 3) show that the PPP is rejected in all cases but two (Italy/Switzerland and Italy/Japan). Therefore, we can conclude that in the post Bretton Woods period, one cannot find evidence supporting the standard PPP hypothesis for most of the cases considered in the "small" VAR model.

In the light of this frequent failure for PPP to hold as an isolated relationship, we follow part of the recent empirical literature (Johansen and Juselius 1992, Johansen 1992, Jore, Skjerpen and Swensen 1992, Juselius 1995, Sjoo 1995, Caporale, Kalyvitis and Pittis 1995), that has adopted a different approach by introducing into the VAR also interest rate variables. The idea is to control for the interaction between the asset market equilibrium and the good market equilibrium. A theoretical basis for this approach comes from the sticky prices model of exchange rate by Dornbush (1976).

Allowing for short run deviations from the PPP and assuming the uncovered interest parity (UIP) condition it is possible to obtain, within the Dornbush's model, the following equation[9]:

$$(e - p + p^*) = \gamma (i - i^*) \qquad [1b]$$

Note that within this framework both [1a] and [1b] are equilibrium conditions: the former is valid in the long run and the latter also in the short run. Both are candidate for cointegration[10].

reconcile with the available quantitative evidence on the biases in measurement (Froot and Rogoff, 1995).

9 A similar and more famous relationship can be obtained, for example, from the Frankel's (1979) version of the sticky price model. Allowing for expected inflation differential in the equation determining the expected change in the exchange rate, it is possible to obtain a relationship between the real exchange rate and real interest rate differential.

10 Often the statistical concept of cointegration has been linked to long run equilibrium.It is probably a question of terminology, but also short run relationships can satisfy the cointegration condition (see Hatanaka, 1996).

4. The augmented system: allowing for interest rate differential

For each pair of countries we specify a VAR with five variables (p_i, p_j, e_{ij}, i_i, i_j), an unrestricted constant, the intervention dummy in 1982.3 and centered seasonal dummies.

The cointegration rank trace tests (table 4) show the existence (at a 5% significance level) of two cointegrating vectors for the bilateral cases of Ger-Usa, Swi-Usa, three cointegrating vectors for the cases of Ita-Usa[11], Ita-Ger, Ita-Swi, Ita-Jap[12], Swi-Ger, finally, four cointegrating vectors for the cases of Ita-Uk, Swi-Jap and Swi-Uk.

To analyze the validity of the PPP, we have imposed overidentifying restrictions on the cointegrating space (sp(β)). We have followed two alternative approaches: in the first, the restrictions are suggested by [1b] (approach 1b) and in the second by [1a] (approach 1a). In both cases, we have tested for formal and empirical identification.

Focusing on the Ita-Usa case with approach 1b, the linear restrictions utilized to identify the three cointegrating vectors are defined in their explicit form by the following three matrices[13] :

$$Q_1^1 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \qquad Q_2^1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad Q_3^1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad [3]$$

The first matrix defines β_1 (i.e. the first cointegrating vector) as the Dornbusch equilibrium relationship [1b]; the second one describes β_2 as the weak version of PPP

[11] In this case the test seems to suggest that at 5% there exist four long run stationary relationships between Italy and USA, while at a significance level of 2.5% the evidence is in favour of a three dimensional cointegration space; our final choice of rank 3 is motivated by two different considerations. Firstly, from the empirical point of view, the exam of the roots of the companion matrix of the system, after the imposition of rank four (tab 5a), suggests the existence of more than one unit root; the same analysis performed under rank three, seems to exclude the existence of more than two unit roots (tab. 5b). Secondly, Reimers (1992) found that the Johansen cointegration test, in small systems and with small samples, over-rejects the null hypothesis. As a consequence he suggests a small sample correction for the test: the value of the adjusted statistic is 8,57 and doesn't allow the rejection of rank 3 at 5%.

[12] In this case the inference is based on the trace test adjusted for small sample bias (see note 11); the value of the test is 10,96 and the rank three hypothesis cannot be rejected at 2,5%.

[13] The row ordering of the variables is: own price, foreign price, exchange rate, own interest rate and foreign interest rate.

(cointegration among prices and exchange rate without any restrictions on the coefficients) and the third matrix represents a cointegrating relationship between the Italian interest rate, the Usa interest rate and the exchange rate. The set of hypothesis in [3] formally (over)identifies the sp(β) (it satisfies the necessary and sufficient conditions for formal identification (Johansen, (1995)) and is empirically accepted (empirical identification (Johansen, 1995)), since the corresponding value of the Likelihood Ratio test statistic is $\chi^2(3) = 2.610$, with a P-value of 0.455 (table 6)[14].

The alternative set of overidentifying restrictions implies only the PPP (approach 1a): the interest rates are not allowed to have any role in the long run. These restrictions are represented by the following matrices:

$$Q_1^1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad Q_2^1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Q_3^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad [4]$$

The first restricted vector is a PPP, the second describes a relation between p_{usa}, i_{ita} and i_{usa}, while p_{ita}, e_{itusa} and i_{ita} are the cointegrated variables in the third vector. The new set of linear restrictions (over)identifies formally and empirically the cointegration space (LR test = 3.6) (table 6).

In summary, we have identified the cointegration space in two different ways. On one side, [1b], combining PPP and UIP, is one of the identified cointegrating vectors; on the other side, [1a] (i.e. PPP alone) is one of the cointegrating vectors.

To discriminate between the two competing identified structures, we adopt the Likelihood Dominance Criterion by Pollak and Wales (1991). The idea is that, given two nonnested hypothesis H_1 and H_2 regarding the specification of the sp(β), we can select the dominant one by simply comparing their associated adjusted likelihood values.

[14] To reduce the amount of tables, we don't report the coefficients of the over identified cointegrating vectors (however, we specify in the main text the variables involved); we also don't report the corresponding loading factors (we report more detailed information on the speed of adjustment in the persistence profiles). All these results are available upon request.

By referring to a range of composite hypothesis H_c with different parametric sizes, the Dominance Criterion acts as follow :

- H_1 is preferred to H_2 if L_2-L_1 <[$\chi^2(n_2+1)$- $\chi^2(n_1+1)$]/2
- H_2 is preferred to H_1 if L_2-L_1 >[$\chi^2(n_2 - n_1 + 1)$- $\chi^2(1)$]/2
- The criterion is indecisive if [$\chi^2(n_2 - n_1 + 1)$- $\chi^2(1)$]/2 > L_2-L_1 > [$\chi^2(n_2+1)$- $\chi^2(n_1+1)$]/2

where n_1 and n_2 are respectively the degrees of freedom related to the H_1 and H_2 and L_1 and L_2 are their respective likelihood values .

From now on, we refer to the specification derived from [1a] as H_1 and to the specification derived from [1b] as H_2 . Using the likelihood values reported in table 6, we have: L_2-L_1 = 0.49 , [$\chi^2(n_2+1)$- $\chi^2(n_1+1)$]/2 = [$\chi^2(4)$- $\chi^2(3)$]/2 = 0.84 (at the 5% level) and [$\chi^2(n_2 - n_1 + 1)$- $\chi^2(1)$]/2 = [$\chi^2(2)$- $\chi^2(1)$]/2 = 1.075.

Therefore, the Pollak and Wales criterion establishes the dominance of the “overidentifying restrictions set“ involving the PPP relationship.

It should be stressed that this result in favour of PPP has been obtained in a context in which the interest rates are allowed to influence the short run dynamics[15].

For the Ita-Ger case, the starting identification structure follows approach 1b and is composed by eq. [1b], a vector containing p_{ita} and e_{itager} and another containing e_{itager} and i_{ita}. This structure is not rejected at 5%.

A different identification (approach 1a) imposes the PPP on the first vector, a relationship between p_{ita}, e_{itager}, i_{ger} on the second and a relationship between p_{ger}, i_{ita} and i_{ger} on the third. Also this one satisfies the necessary and sufficient conditions for the formal identification and cannot be rejected by the empirical LR test (table 6).

Again, we have two competing identified structures and to select one we use the Dominance Criterion. In this case (see table 7), we have L_2-L_1 =-0.44, [$\chi^2(n_2+1)$- $\chi^2(n_1+1)$]/2 = [$\chi^2(4)$- $\chi^2(3)$]/2 = 0.84 (at the 5% level) and [$\chi^2(n_2 - n_1 + 1)$- $\chi^2(1)$]/2 = [$\chi^2(2)$- $\chi^2(1)$]/2 = 1.075. and therefore the overidentification hypothesis based on the PPP is again preferred to the one based on [1b].

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[15] The FIML Johansen estimates are obtained by a multi-step concentration of the likelihood function of the system with respect to different blocks of parameters and the long run coefficients estimates are function of the short run ones.

The Ita-Swi and Ita-Jap cases are those for which, in the previous paragraph, we have already found evidence in favour of PPP. Nevertheless, for comparison with the rest of the analysis, we have conducted the analysis also on the basis of the augmented VAR.

In both cases, the cointegrating space has been succesfully identified both with approach 1b and with 1a. Specifically, for Ita-Swi, according to 1b, the set of restrictions implies a vector corresponding to [1b], another implying a relationship between p_{ita} and e_{itaswi} and, finally, a third with p_{ita} and i_{ita}. This formally identified structure cannot be empirically rejected by the LR test (table 6). The same conclusion holds for approach 1a, with the PPP imposed on the first vector, a relationship among p_{ita}, p_{swi} and i_{ita} in the second, the third vector has the coefficients of e_{itaswi} and i_{ita} restricted to zero..

In the Ita-Jap case, the set of restriction matrices according to approach 1b implies [1b], a relationship between p_{ita} and e_{itajap} and one between p_{jap} and i_{jap} . With approach [1a] the first identified relationship is [1a], the second is between $p_{jap,}$, e_{itajap} and i_{ita} and the third among e_{itajap} and the two interest rates. In both cases approach 1a dominates over 1b (table 7).

Finally, for the Ita-Uk case we don't find any identifying structure containing neither [1a] nor [1b].

Moving now to the bilateral cases for Switzerland, in the Swi-Usa and Swi-Uk cases, we don't find any support for PPP, but only for eq. [1b]. In the former case the identifying structure contains also a relationship among e_{swiusa} and the two interest rates. In the latter case, the restrictions implied by [1b] together with any other set of three linearly independent vectors exactly identify the cointegrating space since we have found four stationary long run relationships; as a consequence no test for empirical identification is required.

In the other two cases, both approaches cannot be rejected and the one including PPP is dominant. In the Swi-Ger case the second and the third cointegrating vectors are identical under both approaches: one restricts to zero p_{ger} and i_{swi} coefficients and the second has zero restrictions on p_{swi} and e_{swiger}. In the Swi-Jap case, for approach [1b] the results are identical to those for the Swi-Uk case. Under [1a], beyond the ppp, we find a relationship between p_{swi} and e_{swijap}, one between p_{jap} and i_{swi} and one between e_{swijap} and i_{jap} .

For the Ger-Usa case, no set of restrictions implying [1b] was able to formally and empirically identify the cointegration space. On the contrary, another overidentifying structure defining the PPP, and relationships among p_{ger}, e_{gerusa}, i_{ger}, and i_{usa} cannot be rejected by the Likelihood Ratio test for empirical identification (table 6).

In summary, adopting the augmented framework, in seven out of the ten bilateral cases considered, we conclude in favour of PPP as a stationary long run relationship. In two of the remaining cases we find the long run relationship among prices, exchange rate and interest rates implied jointly by the PPP and the UIP.

5. The speed of adjustment to PPP: is there any puzzle ?

In a recent article, Rogoff (1996) underlines the existence of a puzzle: on one hand, the very high short run volatility of real exchange rates and, on the other, the very low estimated speed of adjustment to the PPP. The former stylized fact is usually explained on the basis of monetary and financial shocks. Under this condition and in presence of sticky prices we don't expect to find a very fast adjustment to equilibrium; however, the very slow speed of adjustment identified by the empirical literature (half life of three to five years, Froot and Rogoff 1995) cannot be explained simply by nominal rigidities, but one should also introduce into the analysis real shocks to productivity and/or preferences[16].

In this paper we argue that one of the plausible explanations of the puzzle might be in the methodology adopted to measure the speed of adjustment. Most of the literature extracts information on it in the context of a single equation approach to cointegration. This implies the omission of potentially important short run interactions among the

[16] Clarida and Gali (1994; CG henceforth), introduce new elements into the picture: they find that the main source of the real exchange rate fluctuations (US$/DM and US$/Yen cases) is given by real demand shocks. However, their impulse response analysis shows that the effects of structural nominal shocks to the real exchange rate are persistent (they die out only after five years in the US$/DM case and after four years in the Japan/USA case).

variables during the adjustment process. In other terms, this approach is correct only if the right hand side variables are strongly exogenous[17].

Moreover, we prefer to compute persistence profiles rather than synthetic measures (for example, the median lag) to have a full description of system dynamics[18].

Graphs 6 to 12 present the scaled persistence profiles[19] proposed by Pesaran and Shin (P-S henceforth) (1996) and by Lee and Pesaran (1993), for the cases in which PPP has been found in the previous section.

They describe the responses of an equilibrium relationship to system-wide shocks and, differently from the impulse response functions[20], they are unique and do not depend on the specifically defined shocks orthogonalization procedure.

The scaled persistence profiles in presence of multiple cointegrating vectors are obtained as follows:

- given the ML estimates of Γ_i, Π and Ω in [2], the ML estimates of Φ_j in [A1] are obtained by the usual isomorphic reparametrization of a VECM(p-1) into a VAR(p).
- with the estimated matrices Φ_j one can obtain the ML estimators of the following H_s:

$$H_s = \Phi_1 H_{s-1} + \Phi_2 H_{s-2} + \ldots\ldots + \Phi_p H_{s-p} \qquad s = 1,2,\ldots. \qquad [5]$$
$$\text{where} \quad H_0 = I_n \ \text{ and } \ H_s = 0 \quad \text{for} \quad s < 0$$

  The nxn matrices H_s are defined as : $H_s = \sum_{j=0}^{i} A_j$ and the A_j's are the nxn matrices containing the coefficients of the fundamental Wold representation of the VAR in first-differences.

- from the estimated H_s one can derive the estimated unscaled persistence profile of the cointegrating vector $Z_i = \beta' Y_t$ as follows:

$$P_i(s) \equiv \beta_i' H_s \Omega H_s' \beta_i \qquad \text{for } s = 0,1,2,\ldots. \qquad [6]$$

  where *s* represents the horizon at which we evaluate the persistence of a shock occurred in *t-s*.

[17] Not only weak exogeneity is required (i.e. no consideration of potential feedbacks from the ecm in the other equations, when all the variables are jointly modelled), but, in this case, also the condition of Granger non-causality should be satisfied.

[18] Note that only if the adjustment profile is a straight line, the median lag is a sufficient statistic.

[19] They have been computed over a forty quarter horizon starting from our restricted VECM (i.e. constrained to satisfy the PPP).

[20] In a previous version of this paper, we have computed impulse response functions for the Ita-Usa, Ita-Ger and Ger-Usa cases after having structuralized the var. The information obtained in this context might be useful to analize the contribution of each variable to the adjustment process.

- The corresponding scaled measure is the following:

$$p_i(s) \equiv \beta_i' H_s \Omega H_s' \beta_i \,/\, \beta_i' \Omega \beta_i \qquad \text{for } s = 0,1,2,... \qquad [7]$$

The persistence profile describes the dynamic behaviour of the shocked cointegrating relationship both in the long and in the short run. When s=0, [7] is equal to 1; it should go to zero in the long run (since cointegration is imposed).

The evidence from the graphs clearly points to a speed of adjustment which is higher than the one usually obtained in the empirical literature: half life is never larger than 7 quarters. In five out of seven cases, the median lag is less than one year. Corresponding to this homogeneity, a relevant heterogeneity arises if we look at the behaviour at the longer horizon; for example, in the Ita-Ger case, approximatively 90% of the shock is absorbed after 18 quarters, while in the Ita-Jap case this adjustment takes place within only 5 quarters. This highlights the importance of having information on the whole shape of the speed of adjustment path.

Another feature of the graphs is the lack of monotonicity of the persistence profiles in some cases. In three cases (Ita-Usa, Swi-Jap and Ger-Usa), the real exchange rate depreciates for some quarters immediately after the shock and then it monotonically appreciates up to the final adjustment. This inverted U-shape is obtained also by P-S (1996) and, with a different approach, by C-G (1994)[21]. A possible explanation refers to the overshooting of the nominal exchange rate in the context of a sticky-price environment. Another rationale (P-S, 1996) lies in the J-effect characterizing the adjustment path of the current account in presence of monetary shocks. Moreover, in the Ita-Ger case the persistence profile is characterized by frequent and persistent inversion of the slope: the real exchange rate at first depreciates, then appreciates and then depreciates again before starting the final and definitive appreciating step.

[21] In the context of real exchange rate impulse response analysis.

6. Conclusions

In this paper we have tested the hypothesis that the PPP holds as a long run stationary relationship. The analysis has been performed for the post Bretton Woods period on a set of bilateral cases having Italy and Switzerland as pivotal countries.

We have adopted a multivariate full information maximum likelihood approach to investigate the existence of a cointegrating relationship. In the analysis we have found some indication of I(2)-ness of the price series. Rather than adopting an I(2) system and, consequently, a multicointegration approach, we have interpreted the mixed evidence in favour of a second unit root as a consequence of the presence of one exogeneous structural break in the series corresponding to the generalized reduction in inflation rates at the beginning of the 80s. The VAR model includes therefore a step dummy controlling for this break.

Differently from the recent literature adopting an approach similar to ours, we have been able to identify a valid cointegrating relationship that corresponds to PPP in most cases. This has been done within a system in which, from the point of view of the PPP, interest rate variables play a role only in the short run. Moreover, we also find an alternative identification structure in which, similarly to part of the recent literature, there is a cointegrating vector involving the real exchange rate and the interest rates differential. Applying a likelihood dominance criterion we found evidence in favour of the former identification. Finally, differently from most of the previous results, we found that shocks to PPP are relatively quickly absorbed: the median lag never exceeds seven quarters.

TABLES

TABLE 1

Unit root tests

	ADF test	PP test	SP test	AO-ADF
p_{ita}	-2.61 (2)			
p_{usa}	-1.59 (3)			
p_{ger}	-0.44 (4)			
p_{swi}	0.78 (3)			
p_{uk}	-2.50 (5)			
p_{jap}	-2.03 (4)			
e_{itusa}	-2.14 (0)			
e_{gerusa}	-1.17 (0)			
e_{itager}	-2.36 (0)			
e_{itajap}	-2.81 (0)			
e_{itauk}	-2.08 (0)			
e_{itaswi}	-2.51 (0)			
e_{swiusa}	-1.71 (1)			
e_{swiger}	-2.85 (1)			
e_{swiuk}	-2.01 (5)			
e_{swijap}	-1.72 (1)			
i_{ita}	-1.93 (0)			
i_{usa}	-0.47 (2)			
i_{ger}	-1.59 (1)			
i_{swi}	-1.76 (0)			
i_{uk}	-2.41 (0)			
i_{jap}	0.13 (1)			
Δp_{ita}	-1.81 (1)	-3.40 (5)	-3.2 (5)	-4.52 (2)
Δp_{usa}	-1.47 (2)	-3.27 (5)	-3.1 (5)	-5.00 (2)
Δp_{ger}	-1.80 (3)	-5.46 (5)	-5.54 (5)	-6.72 (2)
Δp_{swi}	-3.05 (2)	-5.27 (5)	-5.22 (5)	--
Δp_{uk}	-2.82 (3)	-5.22 (5)	-5.21 (5)	-5.07 (2)
Δp_{jap}	-1.91 (3)	-6.45 (5)	-6.79 (5)	-4.48 (1)

Notes:
-the specification adopted in the tests is the one containing a costant .
-in brackets there is the number of lags included to remove autocorrelation in residuals

TABLE 2
Cointegration rank tests: exchange rate, prices

	Rank	Constant	Trend	Statistic	Simulated Value
	0	Unrestricted	Excluded	45.51	26.88
Italy/USA (2)	1	Unrestricted	Excluded	9.18	9.54
	2	Unrestricted	Excluded	0.70	#### *
	0	Unrestricted	Excluded	34.26	26.88
Italy/Germany (2)	1	Unrestricted	Excluded	9.46	9.54
	2	Unrestricted	Excluded	0.35	#### *
	0	Unrestricted	Excluded	38.77	26.88
Italy/Switzerland (3)	1	Unrestricted	Excluded	13.76	9.54
	2	Unrestricted	Excluded	0.00	#### *
	0	Unrestricted	Excluded	61.89	26.88
Italy/Japan (3)	1	Unrestricted	Excluded	15.85	9.54
	2	Unrestricted	Excluded	3.01	#### *
	0	Unrestricted	Excluded	49.00	26.88
Italy/UK (3)	1	Unrestricted	Excluded	10.16	9.54
	2	Unrestricted	Excluded	0.46	#### *
	0	Unrestricted	Excluded	25.04	26.88
Switzerland/USA (3)	1	Unrestricted	Excluded	5.02	9.54
	2	Unrestricted	Excluded	0.04	#### *
	0	Unrestricted	Excluded	36.74	29.68
Switzerland/Germany (2) **	1	Unrestricted	Excluded	12.84	15.41
	2	Unrestricted	Excluded	0.02	3.76
	0	Unrestricted	Excluded	47.53	26.88
Switzerland/Japan (2)	1	Unrestricted	Excluded	21.07	9.54
	2	Unrestricted	Excluded	0.73	#### *
	0	Unrestricted	Excluded	45.75	26.88
Switzerland/UK (3)	1	Unrestricted	Excluded	20.53	9.54
	2	Unrestricted	Excluded	0.17	#### *
	0	Unrestricted	Excluded	14.93	26.88
Germany/USA (2)	1	Unrestricted	Excluded	3.63	9.54
	2	Unrestricted	Excluded	0.12	#### *

Notes:
-in brackets there is the number of lags included into the model
-the simulated values have been obtained by simulation with the Package DisCo.
* we treat the drift and the intervention dummy as unrestricted deterministic term and in this case, the drift term is not proportional to the value of convergence of the dummy function in the simulation interval [0,1] so that we cannot simulated the critical values for the hypothesis of rank =2 and a number of common trends less than two
** In this case we use the Osterwald-Lenum tabulated values, because we don't include the step dummy in the Var.

TABLE 3

Testing PPP

	Null hypothesis*	CHI SQUARE TEST	P-Value	Outcome
Italy/USA	[PPP]	23.32 (2)	0.0000086	rejected
Italy/Germany	[PPP]	6.81 (2)	0.033	rejected
Italy/Switzerland	[PPP];[p_{ita}+e_{itusa}]	1.56 (1)	0.211	non rejeected
Italy/Japan	[PPP];[p_{ita}+e_{itajap}]	2.94 (1)	0.086	non rejected
Italy/UK	[PPP];[p_{ita}+e_{itauk}]**	8.23 (1)	0.004	rejected
Switzerland/Germany	[PPP]	8.82 (2)	0.012	rejected
Switzerland/Japan	[PPP];[p_{swi}+e_{swijap}]**	5.80 (1)	0.016	rejected
Switzerland/UK	[PPP]+[p_{swi}+e_{swiuk}]**	5.13 (1)	0.02	rejected

Notes:

- we can't reject the null hypothesis if the P-Value exceeds 0.05
- in brackets the number of degrees of freedom.

* in square brackets we present the (over)identified cointegrating vectors

** we test, with the same results, all the alternative plausible specifications of the second cointegrating vectors

TABLE 4
Cointegration rank tests: exchange rate, prices, interest rates

	Rank	Constant	Trend	Statistic	Tabulated Value 5%	Tabulated Value 2.5%
	0	unrestricted	excluded	114.09	70.44	73.96
	1	unrestricted	excluded	66.30	46.90	50.02
Italy/Usa (3)	2	unrestricted	excluded	29.46	26.88	29.68
	3	unrestricted	excluded	10.79	9.54	11.15
	4	unrestricted	excluded	0.41	####	####
	0	unrestricted	excluded	129.69	70.44	73.96
	1	unrestricted	excluded	71.42	46.90	50.02
Italy/Germany (4)	2	unrestricted	excluded	32.94	26.88	29.68
	3	unrestricted	excluded	7.08	9.54	11.15
	4	unrestricted	excluded	1.26	####	####
	0	unrestricted	excluded	129.24	70.44	73.96
	1	unrestricted	excluded	72.26	46.90	50.02
Ita/Swi (4)	2	unrestricted	excluded	32.25	26.88	29.68
	3	unrestricted	excluded	7.03	9.54	11.15
	4	unrestricted	excluded	0.00	####	####
	0	unrestricted	excluded	140.91	70.44	73.96
	1	unrestricted	excluded	70.42	46.90	50.02
Ita/Jap (3)	2	unrestricted	excluded	33.54	26.88	29.68
	3	unrestricted	excluded	12.91	9.54	11.15
	4	unrestricted	excluded	2.94	####	####
	0	unrestricted	excluded	113.48	70.44	73.96
	1	unrestricted	excluded	58.45	46.90	50.02
Ita/UK (2)	2	unrestricted	excluded	30.54	26.88	29.68
	3	unrestricted	excluded	14.19	9.54	11.15
	4	unrestricted	excluded	0.65	####	####
	0	unrestricted	excluded	93.67	70.44	73.96
	1	unrestricted	excluded	48.91	46.90	50.02
Swi/Usa (4)	2	unrestricted	excluded	23.16	26.88	29.68
	3	unrestricted	excluded	8.08	9.54	11.15
	4	unrestricted	excluded	0.02	####	####
	0	unrestricted	excluded	111.11	68.52	71.80
	1	unrestricted	excluded	66.83	47.21	50.35
Swi/Ger (3) *	2	unrestricted	excluded	32.67	29.68	32.56
	3	unrestricted	excluded	8.15	15.41	17.52
	4	unrestricted	excluded	1.53	3.76	4.95
	0	unrestricted	excluded	132.29	70.44	73.96
	1	unrestricted	excluded	81.69	46.90	50.02
Swi/Jap (2)	2	unrestricted	excluded	51.03	26.88	29.68
	3	unrestricted	excluded	23.91	9.54	11.15
	4	unrestricted	excluded	0.21	####	####
	0	unrestricted	excluded	141.56	70.44	73.96
	1	unrestricted	excluded	90.92	46.90	50.02
Swi/UK (4)	2	unrestricted	excluded	46.88	26.88	29.68
	3	unrestricted	excluded	21.33	9.54	11.15
	4	unrestricted	excluded	0.19	####	####
	0	unrestricted	excluded	121.12	70.44	73.96
	1	unrestricted	excluded	47.05	46.90	50.02
Ger/USA (4)	2	unrestricted	excluded	21.10	26.88	29.68
	3	unrestricted	excluded	1.40	9.54	11.15
	4	unrestricted	excluded	0.01	####	####

Notes:
- for the non available tabulated value relative to the hypothesis of Rank=4 see Tab.2
- in brackets there is the number of lags caracterizing the VAR system
* in this case we use the Osterwald-Lenum tabulated values, because we don’t include the step dummy in the Var.

TABLE 5

Eigenvalues of companion matrix

a Rank 4	b Rank 3
1.00	1.00
0.99	1.00
-0.99	0.90
0.92	-0.90
0.69	0.73
-0.69	0.68
0.68	-0.68
-0.68	0.65
0.56	-0.65
-0.56	0.58
0.53	-0.58
-0.53	0.42
0.49	-0.42
-0.49	0.34
0.43	0.27

TABLE 6

Empirical identification of sp(β) : Likelihood Ratio tests

Bilateral	Approach	Likelihood of the restricted model	Degrees of freedom	Test statistic	P-Value	Outcome
Italy/Usa	1a	1384.89	2	χ^2=3.60	0.16	non rejected
	1b	1385.38	3	χ^2=2.610	0.455	non rejected
Italy/Germany	1a	1485.04	2	χ^2=5.49	0.064	non rejected
	1b	1484.60	3	χ^2=6.37	0.095	non rejected
Ita/Swi	1a	1470.50	2	χ^2=0.37	0.832	non rejected
	1b	1469.15	3	χ^2=3.06	0.383	non rejected
Ita/Jap	1a	1252.76	2	χ^2=4.34	0.114	non rejected
	1b	1251.98	3	χ^2=5.91	0.116	non rejected
Swi/Usa	1a	--	--	--	--	rejected
	1b	1325.44	3	χ^2=5.80	0.121	non rejected
Swi/Ger	1a	1403.14	2	χ^2=5.60	0.061	non rejected
	1b	1402.94	3	χ^2=5.99	0.111	non rejected
Swi/Jap	1a	1398.38	1	χ^2=0.11	0.740	non rejected
	1b**					non rejected
Swi/UK	--	--	--	--	--	rejected
	1b**					non rejected
Ger/USA	1a	1459.13	3	χ^2=7.45	0.059	non rejected
	1b	--	--	--	--	rejected

Notes:
- ** no effective restrictions are imposed, in this case, on the basis of the cointegrating space

TABLE 7

Comparison of alternative over-identifying structures on the basis of Likelihood Dominance Criterion

Case.	Hp	Approach	L_2-L_1	$[\chi^2(n_2+1)- \chi^2(n_1+1)]/2$	$[\chi^2(n_2 - n_1 + 1)-\chi^2(1)]/2$	Dominant approach
Italy/Usa	H_1	1a	0.49	0.84	1.075	H_1
	H_2	1b				
Italy/Germany	H_1	1a	-0.44	0.84	1.075	H_1
	H_2	1b				
Ita/Swi	H_1	1a	-1.35	0.84	1.075	H_1
	H_2	1b				
Ita/Jap	H_1	1a	-0.78	0.84	1.075	H_1
	H_2	1b				
Swi/Ger	H_1	1a	-0.20	0.84	1.075	H_1
	H_2	1b				

FIGURE 1 : Italian consumer price index

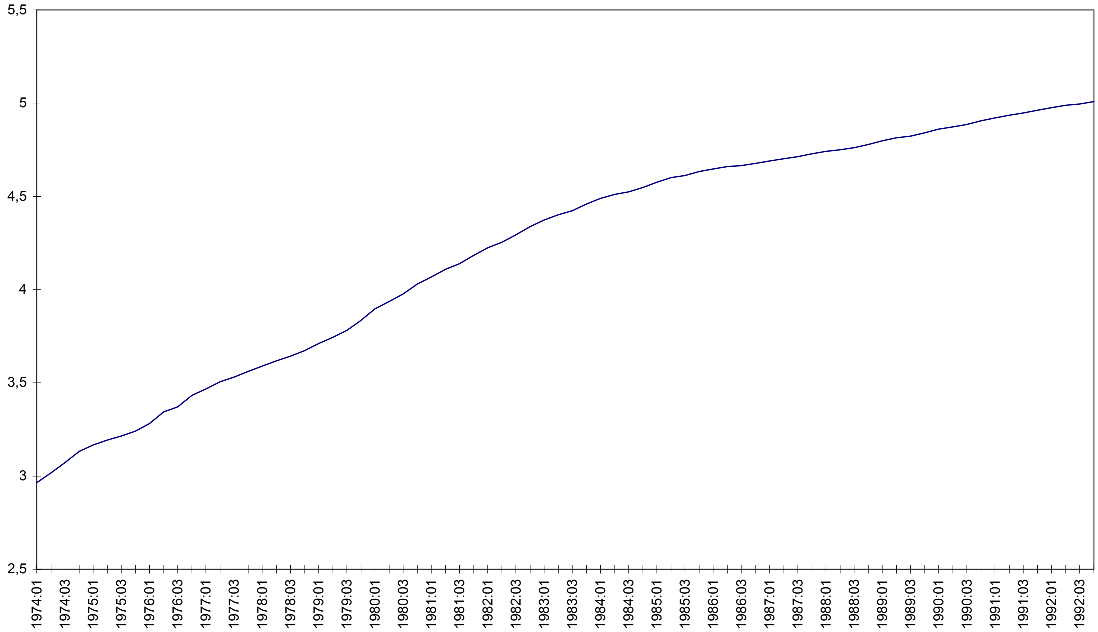


FIGURE 2 : United States consumer price index

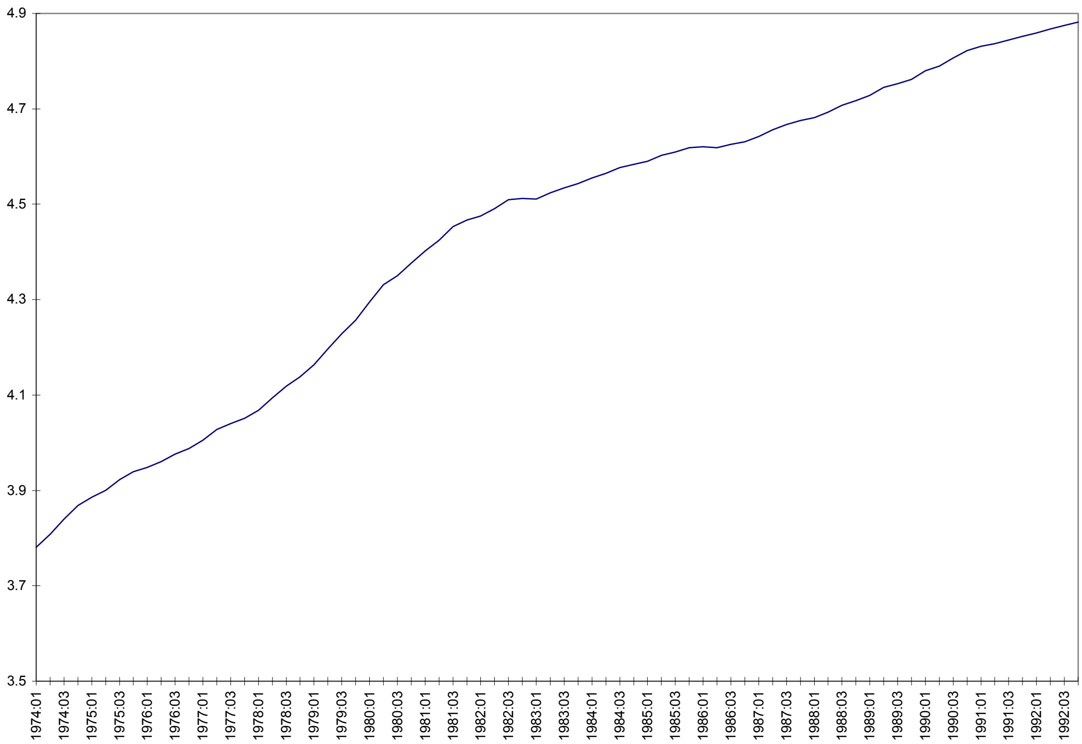


FIGURE 3 : German consumer price index

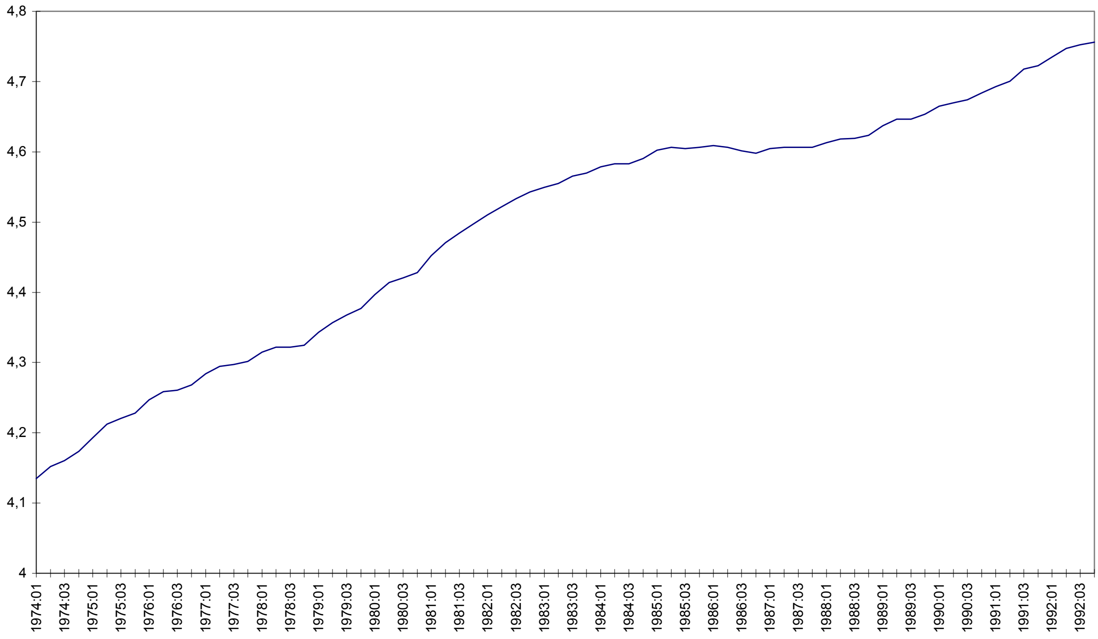


FIGURE 4 : UK consumer price index

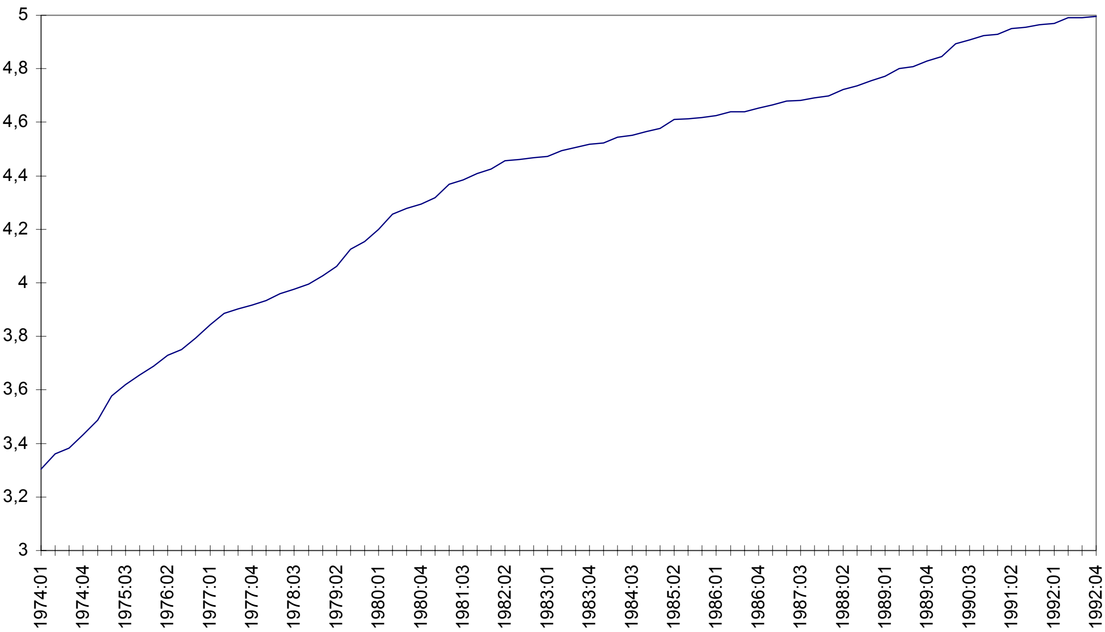


FIGURE 5 : Japan consumer price index

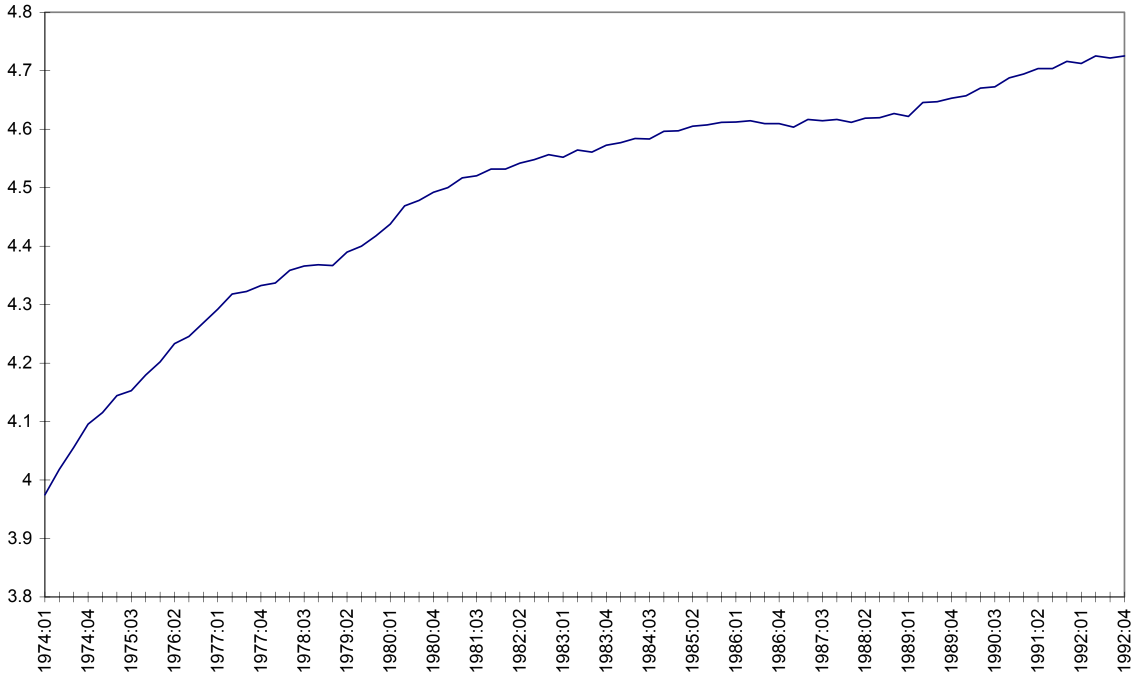


FIGURE 6 - Persistence Profile of PPP: Italy/USA

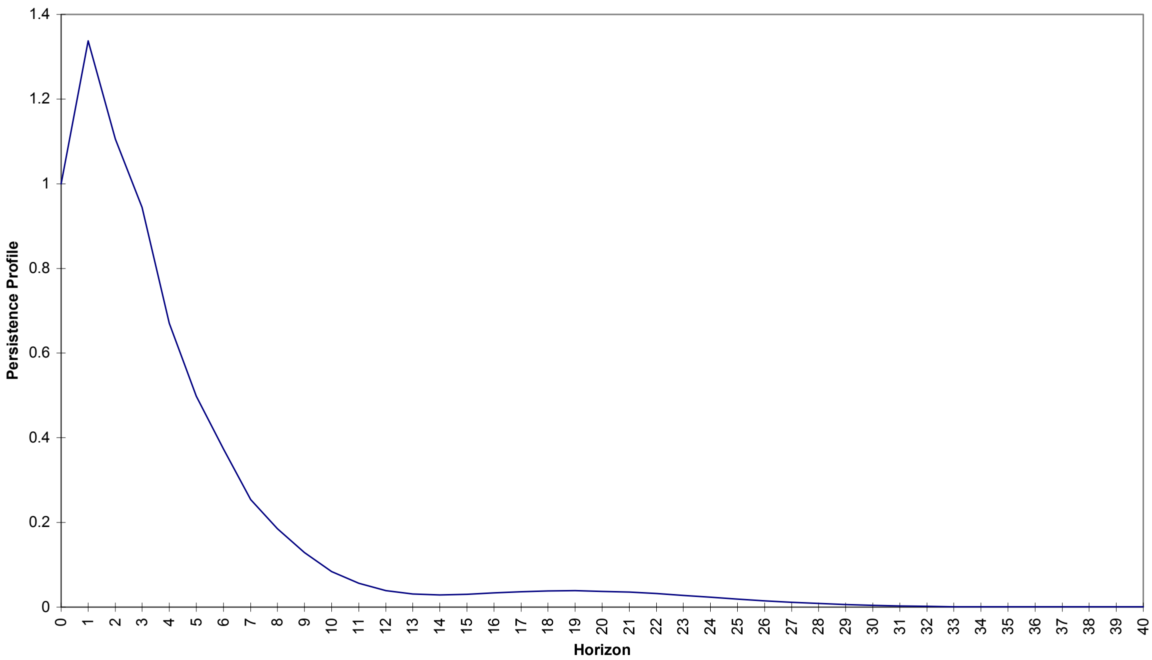


FIGURE 7 - Persistence Profile of PPP: Italy/Switzerland

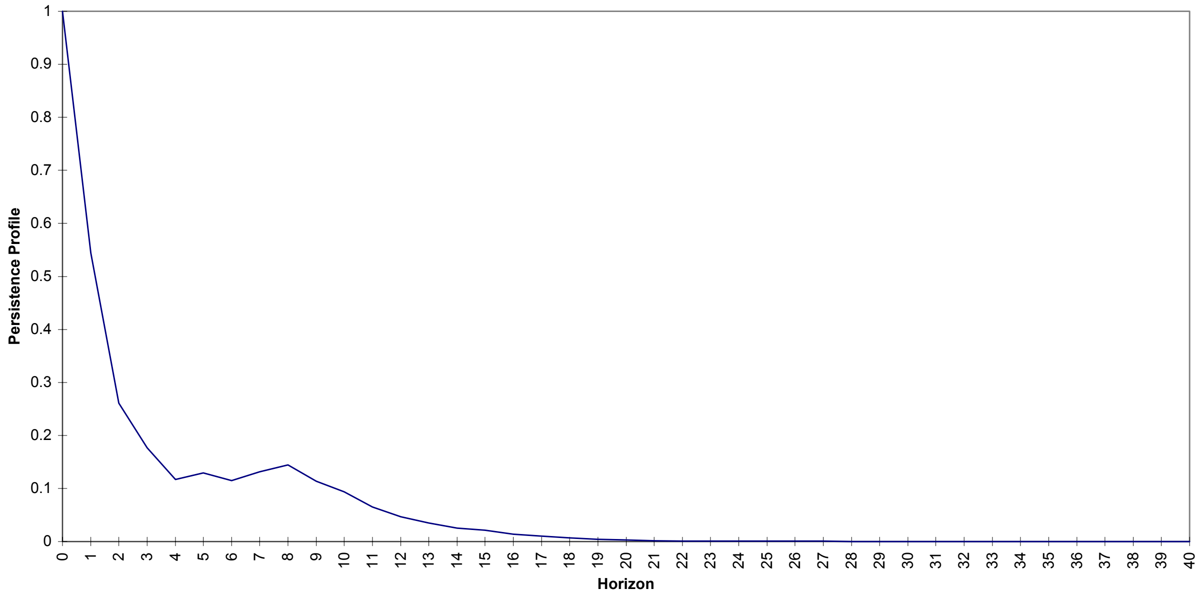


FIGURE 8 - Persistence Profile of PPP: Italy/Japan

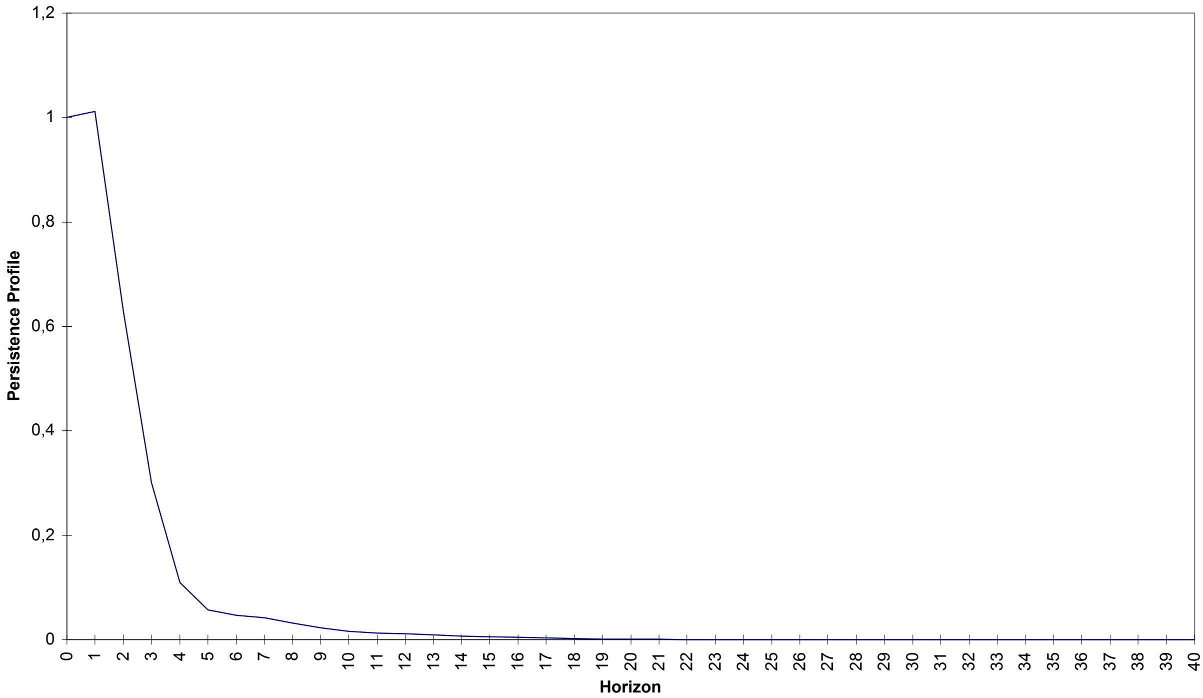


FIGURE 9 - Persistence Profile of PPP: Italy/Germany

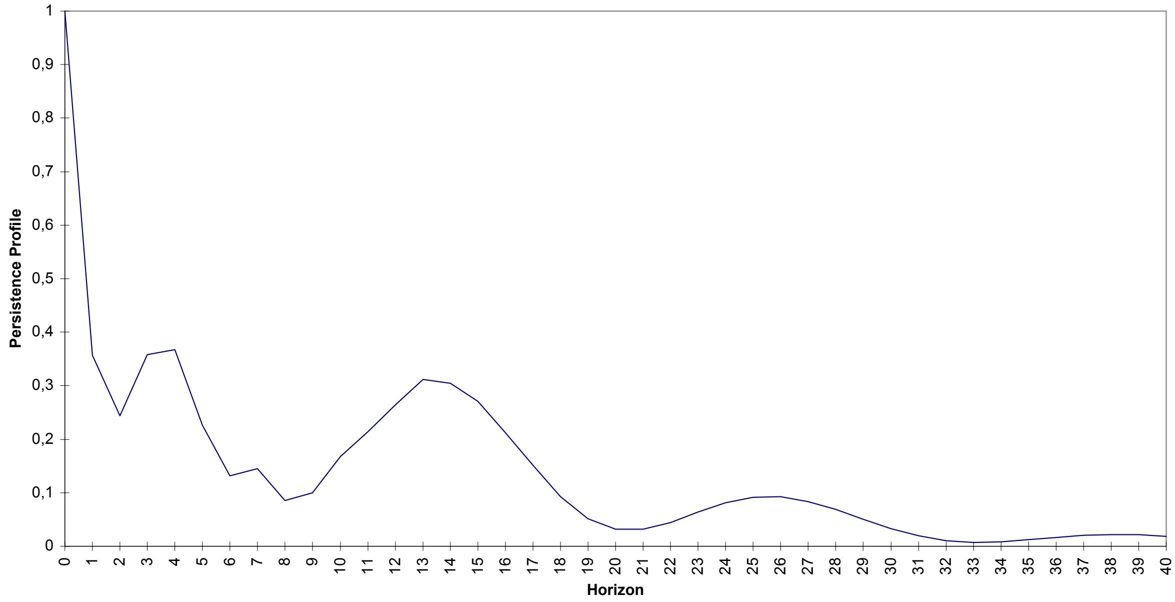


FIGURE 10 - Persistence Profile of PPP: Switzerland/Germany

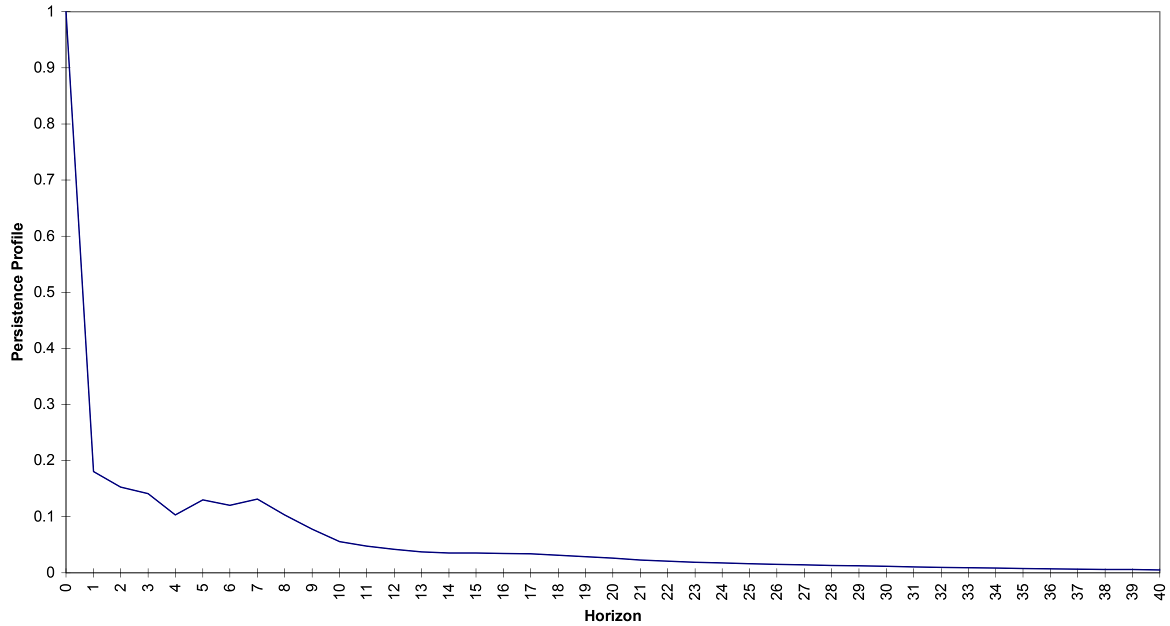


FIGURE 11 - Persistence Profile of PPP: Switzerland/Japan

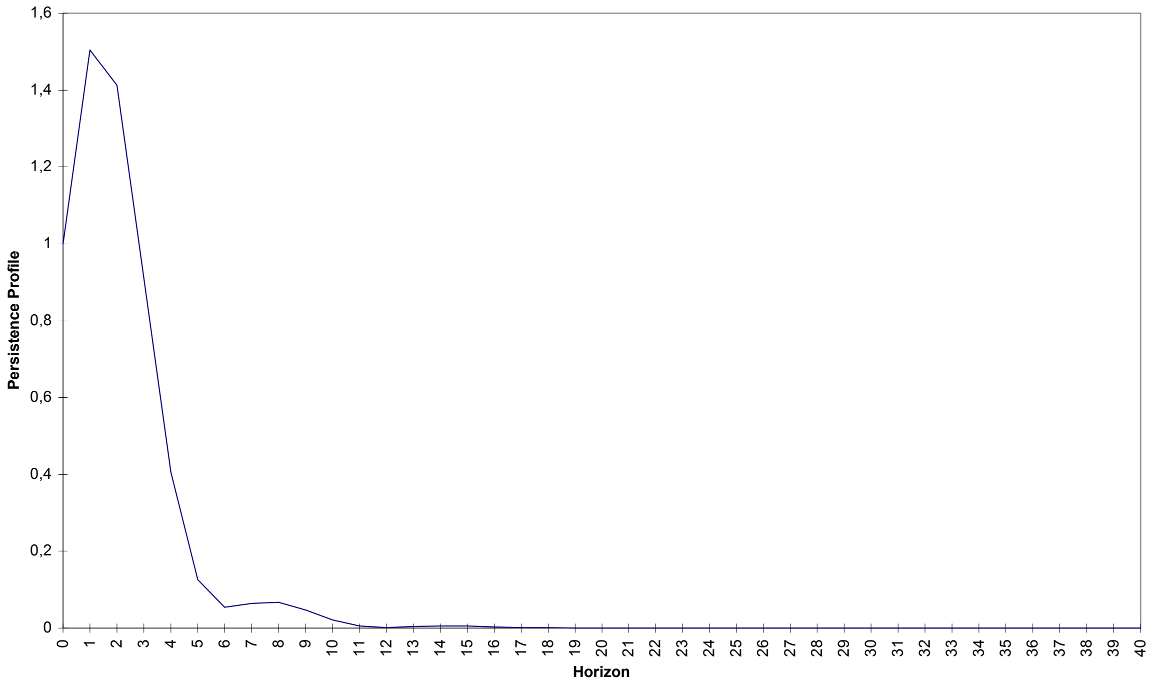
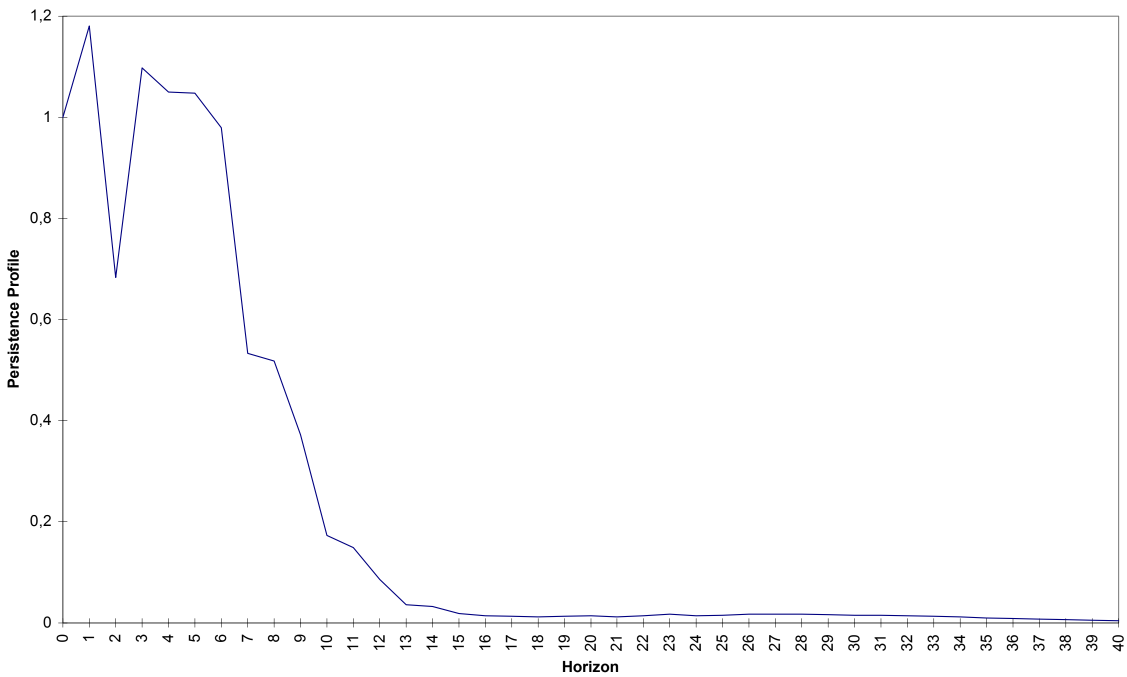


FIGURE 12 - Persistence Profile of PPP: Germany/USA



References

Banerjee A., Dolado J., Galbraith J.W and Hendry D.F. (1993), "*Co-Integration, error correction, and the econometric analysis of non-stationary data*", Advanced Texts in Econometrics, Oxford University Press, Oxford .

Caporale G.M., Kalyvitis S. and Pittis N. (1995) "Testing for PPP and UIP in FIML framework: some evidence for Germany and Japan", Centre for Economic Forecasting, London Business School, Discussion Paper n.30.

Clarida R. and Galì J. (1994) "Sources of real exchange rate fluctuations: how important are nominal shocks ?" *Carnegie Rochester Conf. Ser. Public Pol.* 41, 1-56.

Doornik J. and Hendry D.F. (1994) "*PC-GIVE 8.0. An interactive econometric modelling system*",Institute of Economic and Statistics, University of Oxford.

Dornbush R. (1976) "Expectations and exchange rate dynamics", *Journal of Political Economy*, 84, 1161-1176.

Frankel J. (1979) "On the Mark: a theory of floating exchange rates based on real interest differentials", *American Economic Review*, 69, 601-622.

Froot K.A. and Rogoff K. (1995) "Perspectives on PPP and long-run real exchange rates", in Grossman G. and Rogoff K. (eds.), *Handbook of International Economics* , vol.III, Elsevier Science.

Gonzalo J. (1994) "Five alternative methods of estimating long-run equilibrium relationships", *Journal of Econometrics*, 60, 203-223.

Hatanaka M. (1996) "*Time-series-based econometrics*", Oxford University Press.

Hamilton J.D. (1994) "*Time series analysis*", Princeton University Press.

Haug A.A. (1996) "Tests for cointegration: a Monte Carlo comparison", *Journal of Econometrics*, 71, 89-115.

Johansen S. (1992) "An I(2) cointegration analysis of the purchasing power parity between Australia and the United States" in Hargreaves C. (ed.), *Macroeconomic modelling of the long run*, Edward Elgar.

Johansen S. (1995) "*Likelihood-based inference on cointegration : theory and applications*", Oxford University Press.

Johansen S. and Juselius K. (1992) "Testing structural hypothesis in a multivariate cointegration analysis of the PPP and UIP for UK", *Journal of Econometrics*, 53, 211-24.

Johansen S. and Nielsen B. (1993) " Manual for the simulation program DisCo" version 1.0, mimeo.

Jore A.S., Skierpen T. and Swensen A.R. (1992) "Testing for Purchasing Power Parity and interest rate parities on Norvegian data", Discussion paper no.101, Central Bureau of Statistics, Norway.

Juselius K. (1995) "Do purchasing power parity and uncovered interest rate parity hold in the long run? An example of likelihood inference in a multivariate time series model", *Journal of Econometrics*, 69, 211-240.

Lee K.C. and Pesaran M.H. (1993) "Persistence profiles and business cycle fluctuations in a disaggregated model of UK output growth", *Ricerche Economiche*, 47, 293-322.

Lütkepohl H. and Reimers H.E. (1992) "Impulse response analysis of cointegrated systems with an application to German money demand", *Journal of Economic Dynamics and Control*, 16, 53-78.

Mosconi R. (1996) "*MALCOLM. The theory and practice of cointegration analysis in RATS*", Cafoscarina, Venezia, forthcoming.

Osterwald-Lenum M. (1992) "Recalculated and extended tables of the asymptotic distribution of some important maximum likelihood cointegration test statistics", *Oxford Bulletin of Economics and Statistics*, 54, 461-71.

Pantula S.G. (1989) "Testing for unit roots in time series data", *Econometric Theory*, 5, 256-271.

Paruolo P. (1993) "Analisi di multicointegrazione in sistemi VAR: alcune prospettive" , paper presented at the meeting, Dynamic models of short and long-run period, Florence, 5-6 November 1992.

Perron P. (1990) "Testing for a unit root in a time series with a changing mean", *Journal of Business and Economic Statistics*, Vol.8, No.2.

Pesaran M.H. and Shin Y. (1996) "Cointegration and the speed of convergence to equilibrium", *Journal of Econometrics*, 71, 117-143.

Pollak R.A. and Wales T.J. (1991) "The likelihood dominance criterion: a new approach to model selection", *Journal of Econometrics*, 47, 227-242.

Reimers H.E. (1992) "Comparisons of tests for multivariate co-integration", *Statistical Papers*, 33, 335-59.

Rogoff K. (1996) "The purchasing power parity puzzle", *Journal of Economic Literature*, XXXIV, 669-700.

Sjoo B. (1995) "Foreign trasmission effects in Sweden: do PPP and UIP hold in the long run?", *Advances in International Banking and Finance*, 1 129-149.